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"Interactive Boundary Layer" and "Triple Deck":

models for high Reynolds number flow with flow separation

http://www.lmm.jussieu.fr/~lagree/COURS/CISM/blasius_CISM.pdf

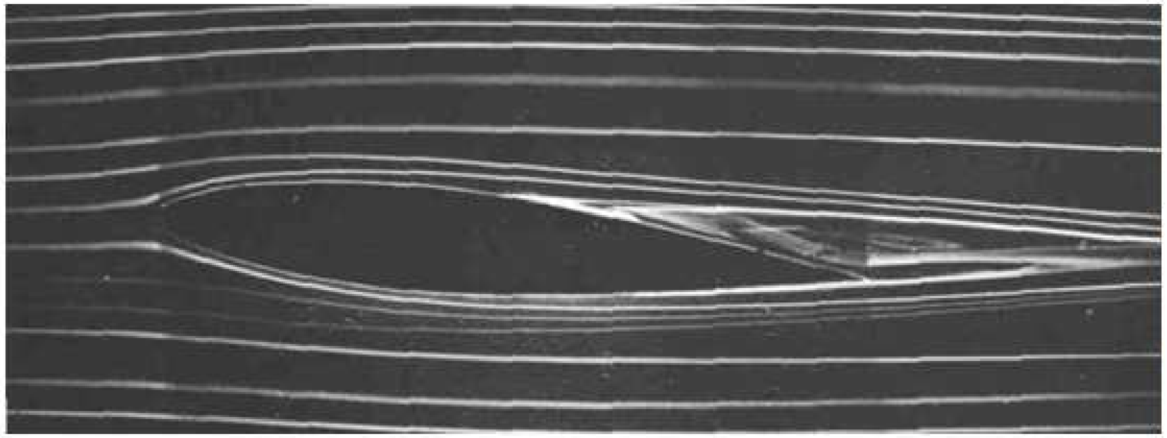
http://www.lmm.jussieu.fr/~lagree/COURS/CISM/TriplePont_CISM.pdf

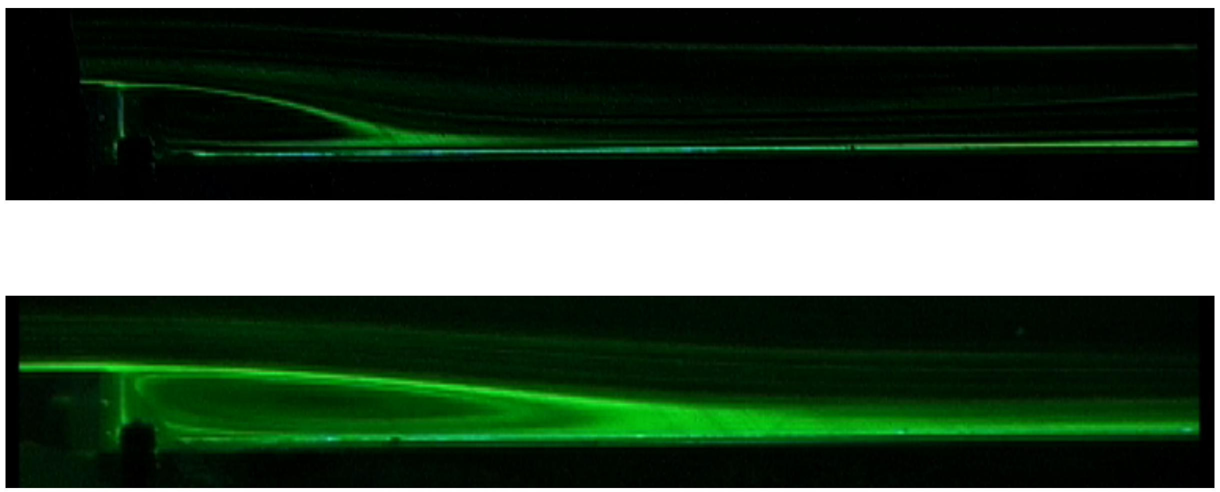
https://vimeo.com/98712197

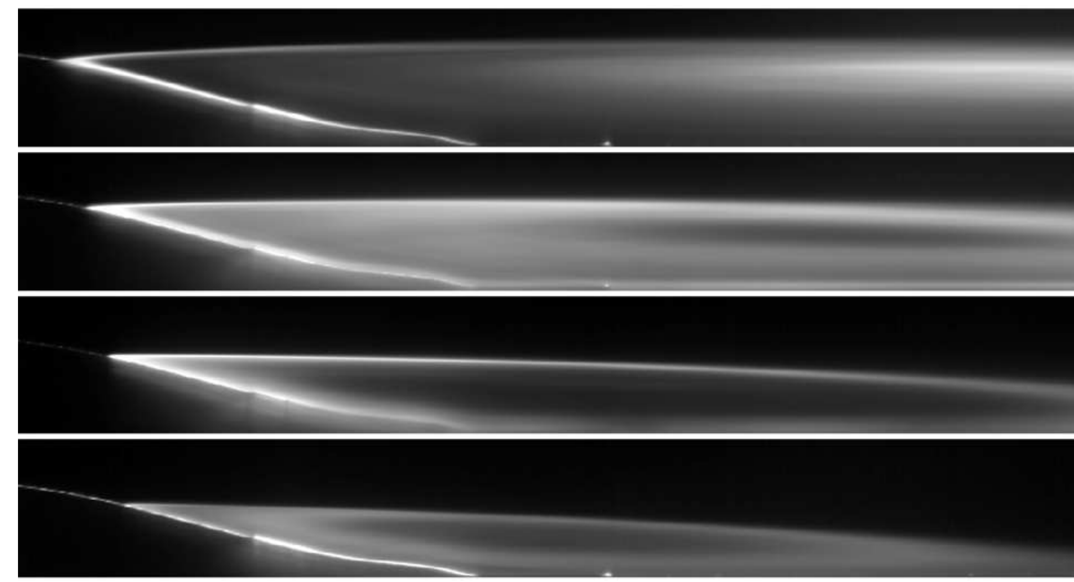
outline

- the classical Boundary Layer
- second order Boundary Layer
- Interactive Boundary Layer
- some examples of numerical resolution with some comparaisons with Navier Stokes
- the Triple Deck, example of numerical solution
- the Double Deck, example of numerical solution FD FE
- summary

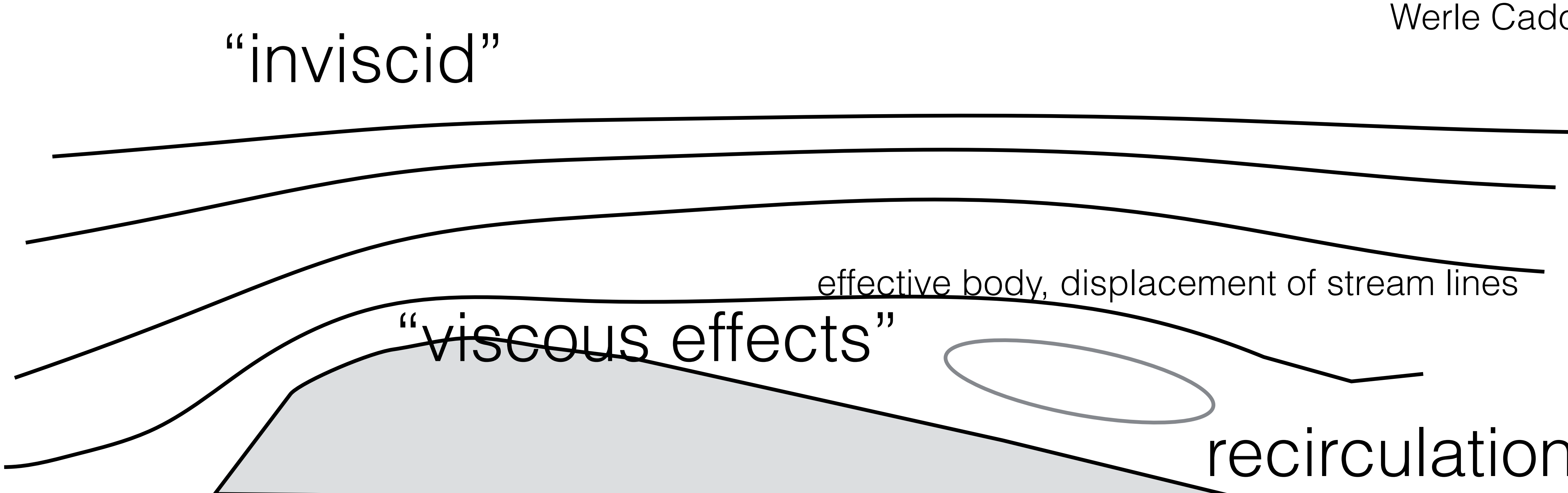
examples of flow separation







Werle Cadot Boujo



What we will see:

with various scales

dominant equations are "Prandtl" equations

$$\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0 \qquad u\frac{\partial}{\partial x}u + v\frac{\partial}{\partial y}u = -\frac{\partial}{\partial x}p + \frac{\partial^2}{\partial y^2}u \qquad 0 = -\frac{\partial}{\partial y}p$$

with no slip conditions
first profile given
with various boundary conditions at the top

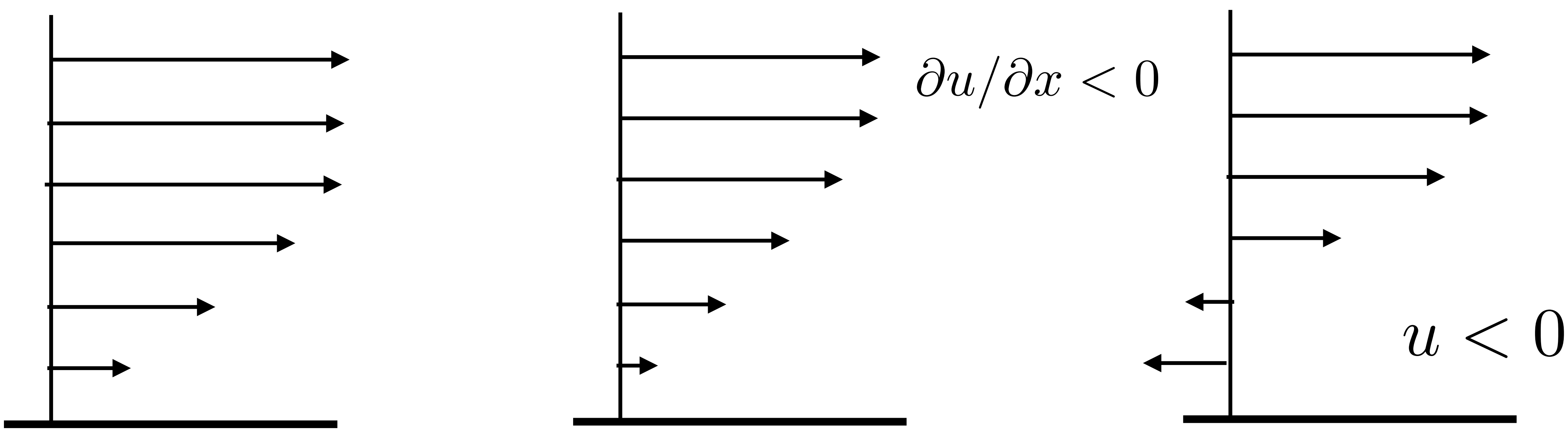
parabolic
sometimes coupled with an external ideal fluid
which makes a global retroaction

those equations are a good model for flow separation

Heuristic condition for Boundary Layer separation

$$u\frac{\partial u}{\partial x} \simeq -\frac{\partial p}{\partial x}$$

counter pressure gradient
decreases the velocity
(mostly inviscid)



$$u\frac{\partial u}{\partial x} \simeq \frac{1}{Re}\frac{\partial^2 u}{\partial y^2}$$

The thin viscous layer near the wall
is more sensitive to pressure changes

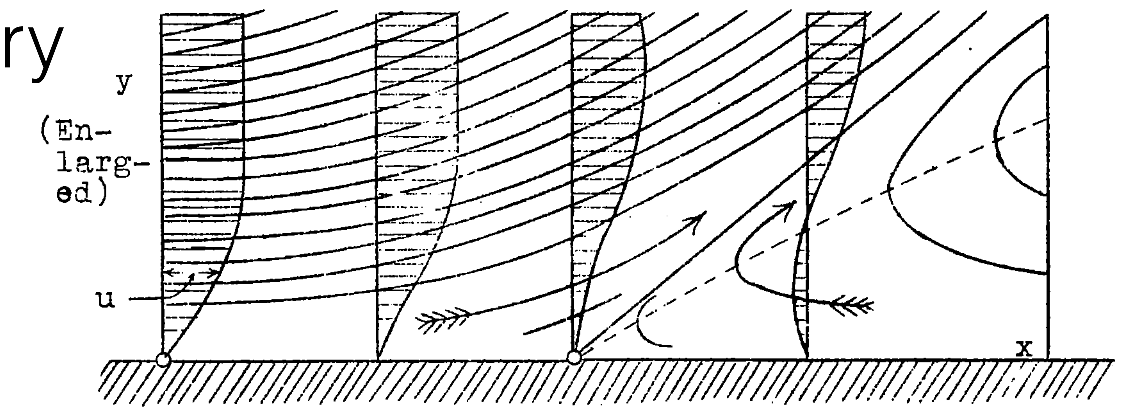
History

Prandtl 1904
Blasius 1908
Von Kármán

Decomposition of the flow
in an inviscid domain
and a viscous domain near the wall

Goldstein 48

singularity of boundary
layer separation



Landau 50'
Lighthill 53

first attempts

Neiland 69
Stewartson 69
Sychev 72

triple deck

Le Balleur 78
Carter 79,
Cebeci 70s
Veldman 81

Interactive Boundary Layer

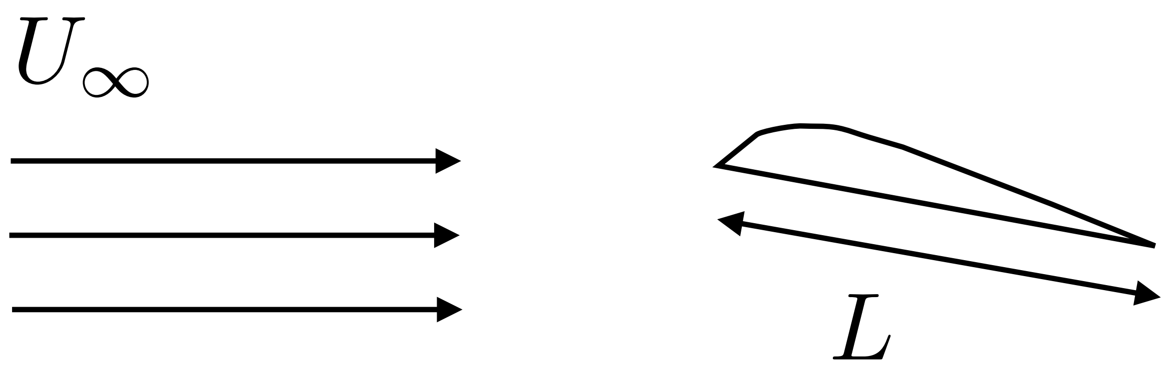
Smith 77-

double deck, triple deck stability

“Classical” text book

A good way to “understand” some flows
and to “feel” the relative influence of the terms in NS
it is a step after “Bernoulli”

Navier Stokes



$$Re = \rho \frac{U_\infty L}{\mu}$$

Real Full 3D unsteady flows
Direct Numerical Simulations : DNS

Reynolds Number controls transition from laminar to turbulent

turbulence modeling

Very complicated and serious problems

Small Reynolds number: viscosity dominates

Micro fluidics, some biological flows

flow is laminar

$$Re = \rho \frac{U_\infty L}{\mu}$$

Large Reynolds number: inertia dominates

Aerodynamics, most of classical industrial flows

flow is turbulent

Question :

what is the laminar flow in the limit of large Reynolds number?

steady -> Basic flow for instability theory

we do not care about turbulence (laminar)

2D / Axi laminar, steady

Question :
what is the flow in the limit of
large Reynolds number?

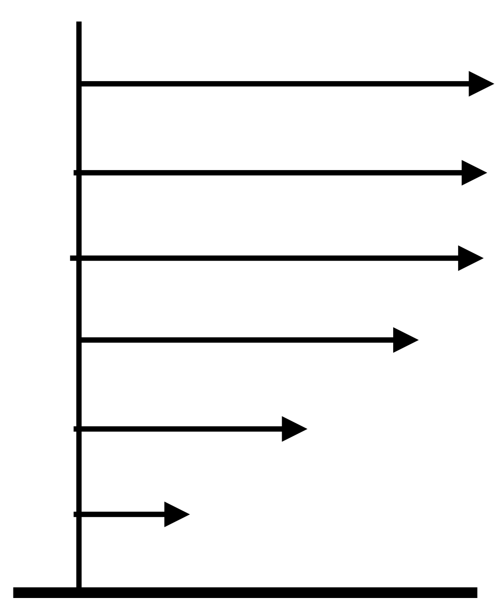
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$Re = \rho \frac{U_\infty L}{\mu}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

zero velocities at the wall



Question :
what is the flow in the limit of large Reynolds number?

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y}$$

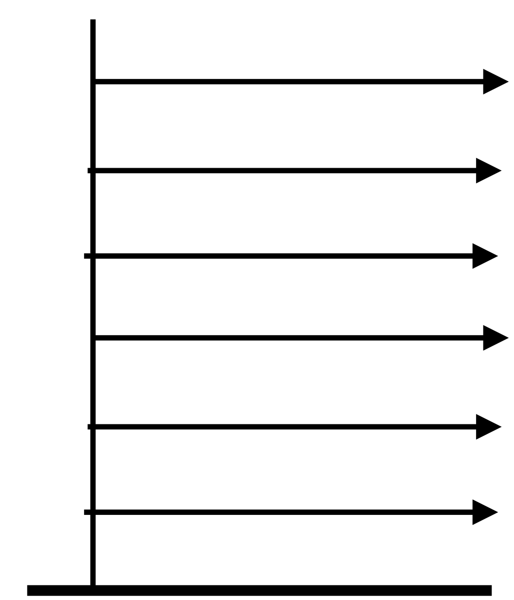
$$Re = \rho\frac{U_\infty L}{\mu}$$

$$\frac{1}{Re} \to 0$$

an order of derivation disappears

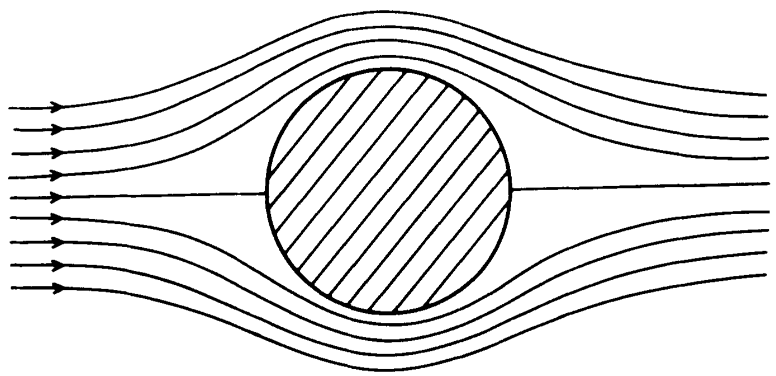
only zero transverse velocity at the wall

singular perturbation problem



Question :
what is the flow in the limit of
large Reynolds number?

(a)



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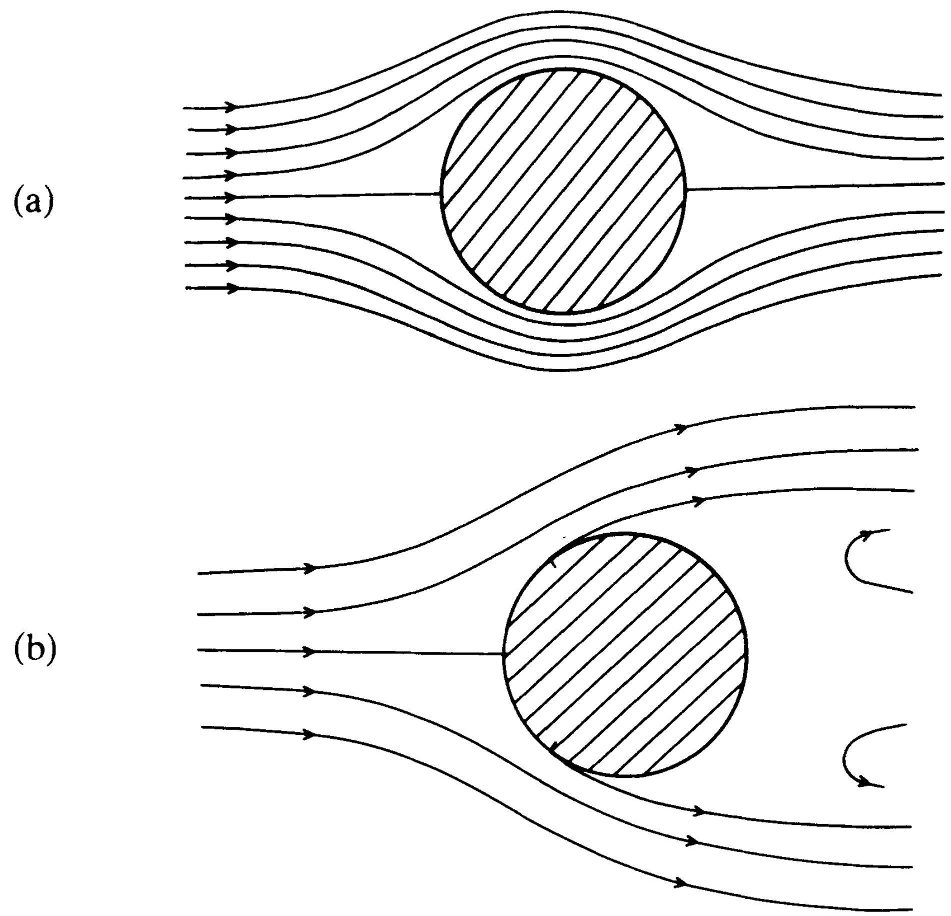

$$\frac{1}{Re} \to 0$$

FIG. 1. *Two of the candidates for the steady solution of the Navier–Stokes equations for flow past a circular cylinder at* $R \gg 1$. (a) *attached potential flow.* (b) *Kirchhoff free-streamline flow.*

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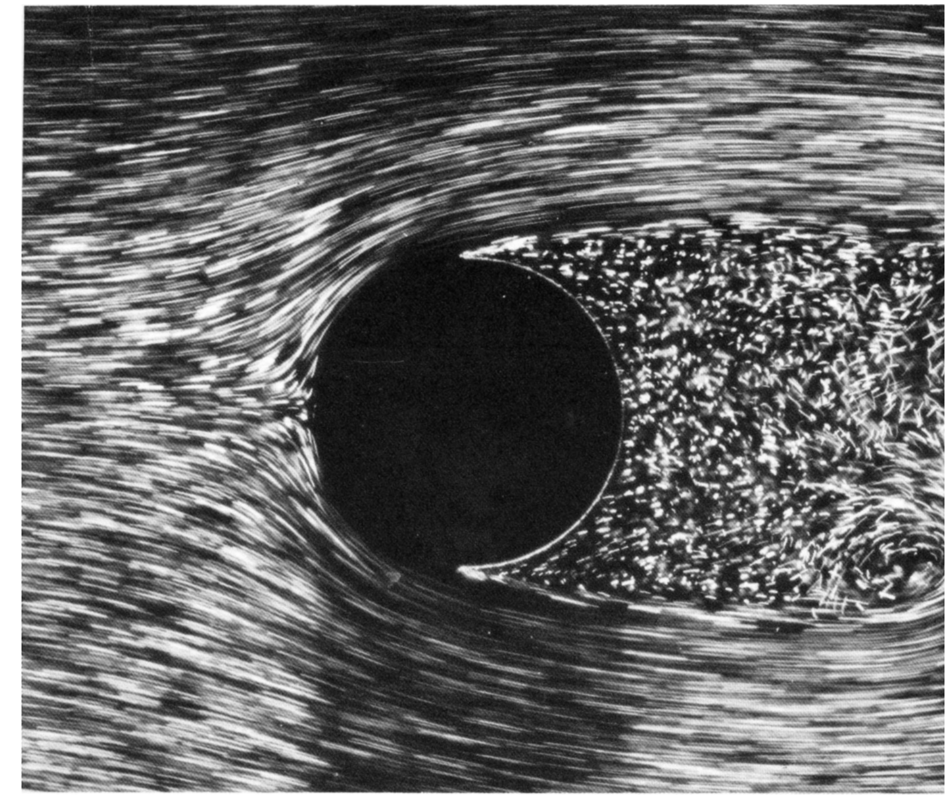
D'ALEMBERT'S PARADOX*

KEITH STEWARTSON†

- Kirchhoff - Helmholtz

Question :
what is the flow in the limit of large Reynolds number?



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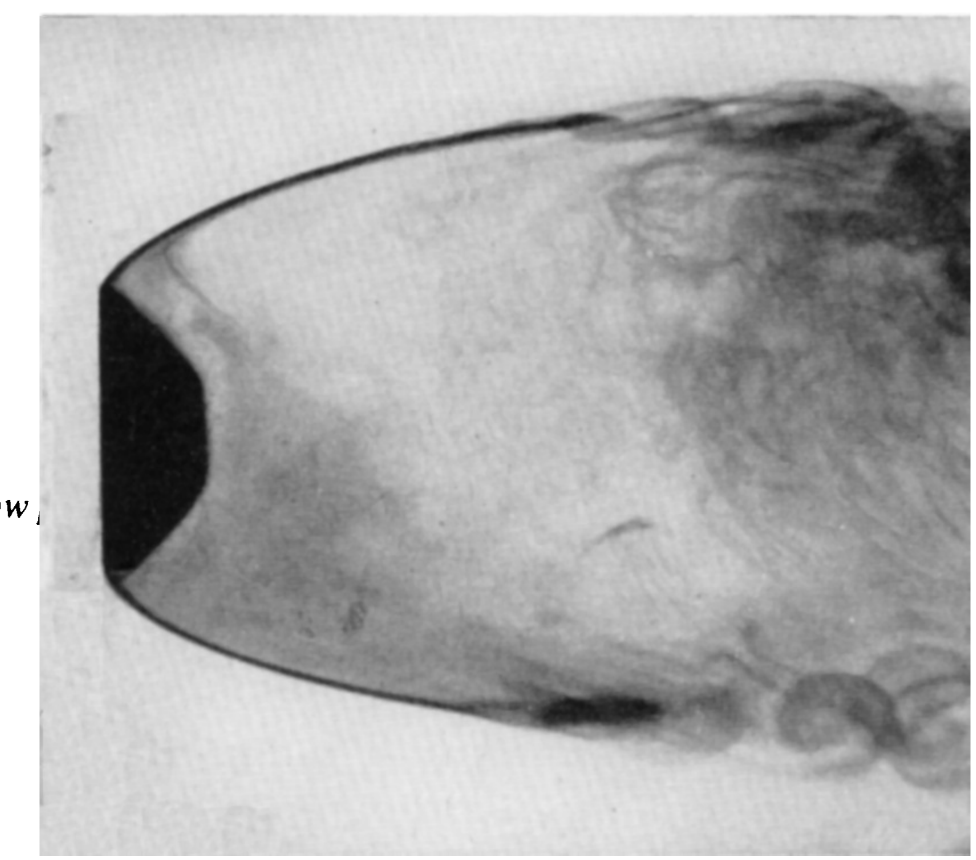

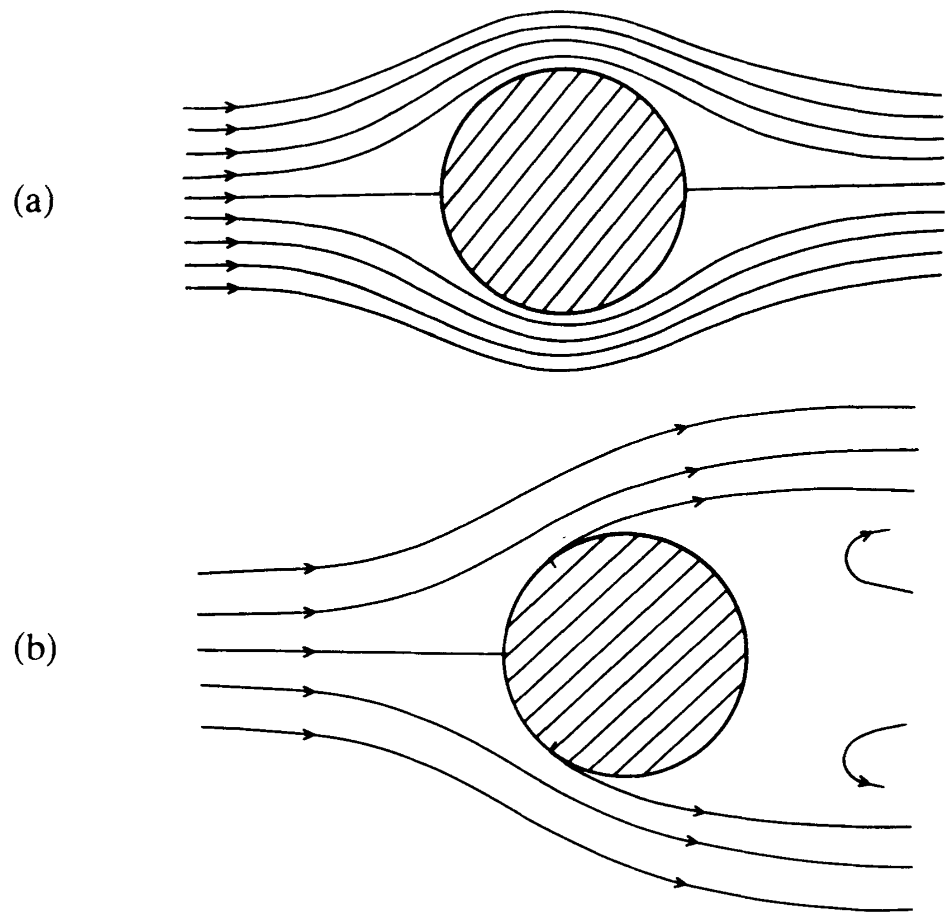

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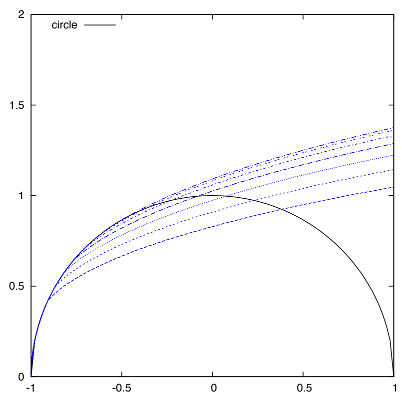
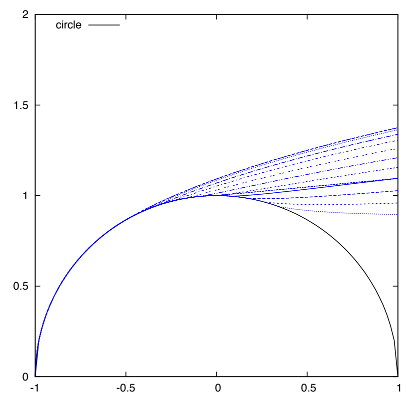




решение, обладающее всеми необходимыми свойствами. Рассмотрим это более подробно.

Решение задачи обтекания гладкого препятствия, например кругового цилиндра по схеме Кирхгофа, вообще говоря, неоднозначно. Оно может быть построено при различных положениях точки отрыва нулевой линии тока от поверхности тела (рис. 1.4). Если начало криволинейной ортогональной системы координат Oxy поместить в точку отхода нулевой линии тока от поверхности тела, то кривизна свободной линии тока будет определяться выражением

$$\varkappa = -kx^{-1/2} + \varkappa_0 + O(x^{1/2}), \quad x \to +0 \tag{3.1}$$

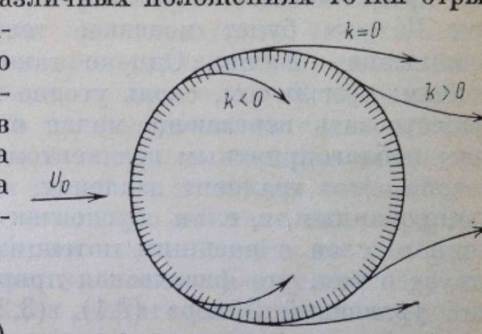


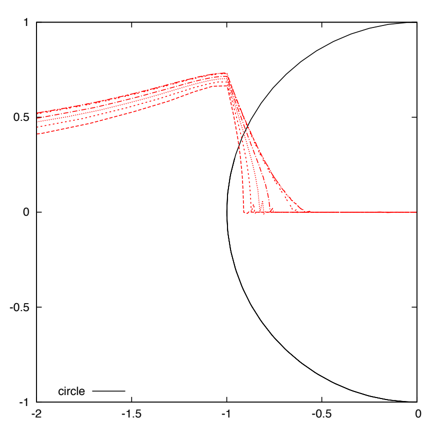
Рис. 1.4. Форма свободных линий тока в течении Кирхгофа при различных положениях точки отрыва

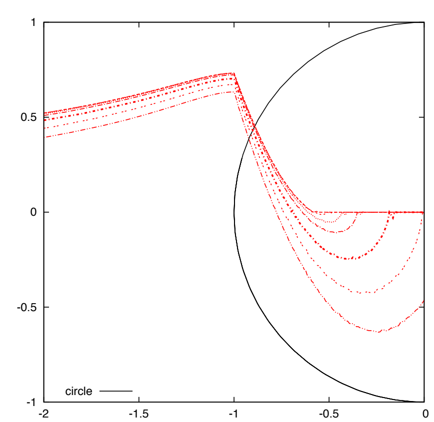
($\varkappa_0$ — безразмерная кривизна поверхности тела в точке отрыва, полагаемая конечной), а величина градиента давления на поверхности тела в окрестности этой точки будет равна

$$\frac{dp}{dx} = k(-x)^{-1/2} + \frac{16}{3}k^2 + O[(-x)^{1/2}], \quad x \to -0;$$
$$\frac{dp}{dx} = 0, \quad x > 0. \tag{3.2}$$

curvature $(\frac{d\theta}{ds})$ of the free stream line is $\frac{k}{2\sqrt{x - x_s}}$, where x_s is the point of separation.

$$\frac{1}{Re} \to 0$$



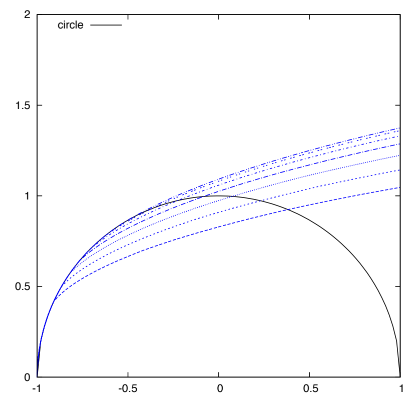


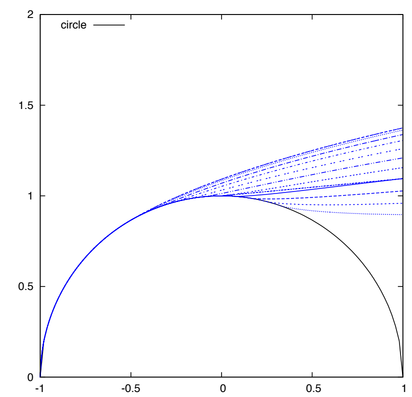
$p = p_0 - k\sqrt{x_s - x} + ...$ before separation, and after $p = p$

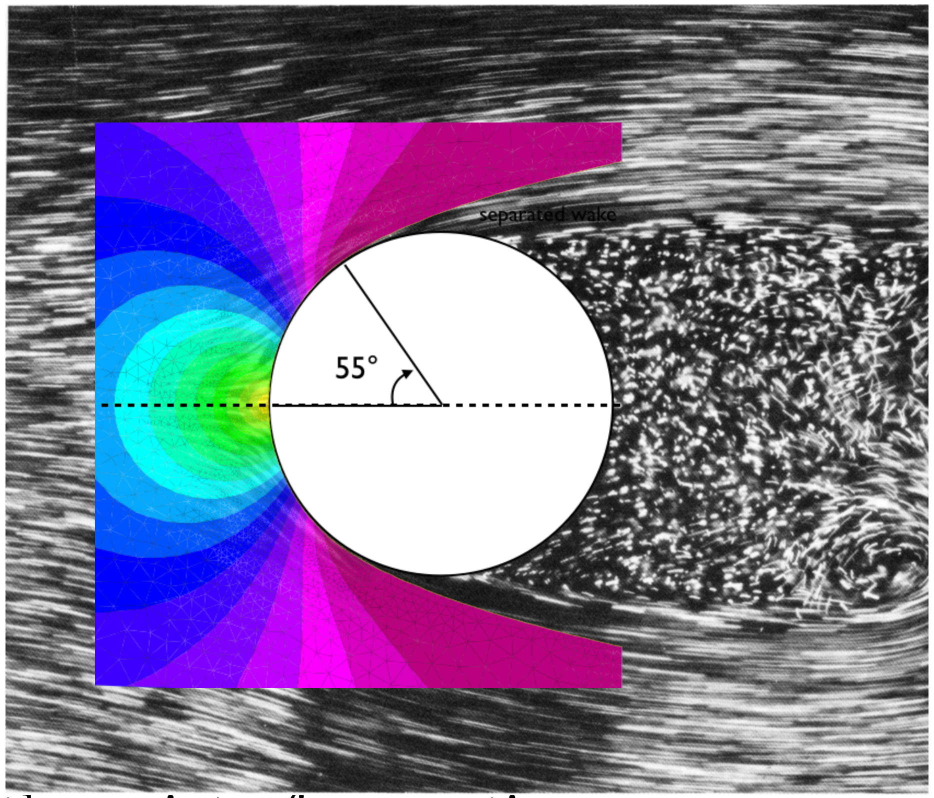
The sole solution is *k=0,* this is the Brillouin-Villat condition: the curvature of the free streamlines is tangent to the body at the "separation point" But the flow is smooth, there is no counter pressure. So there is no separation. This is the "Brillouin-Villat" paradox

- **Kirchhoff - Helmholtz**

Question :
what is the flow in the limit of large Reynolds number?







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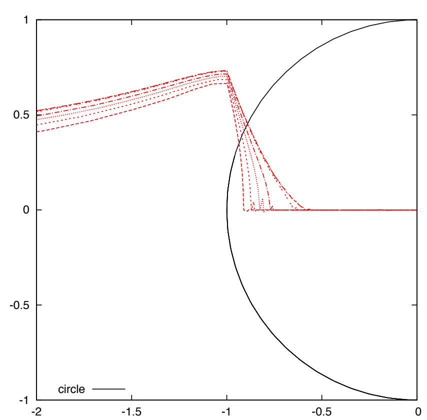

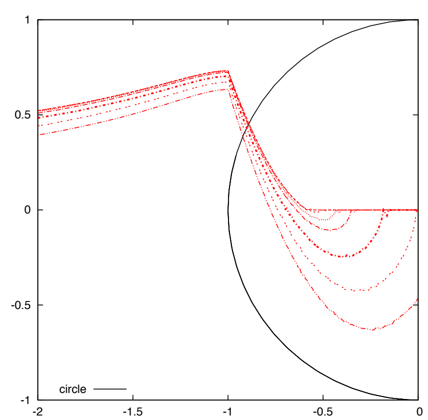

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Question :
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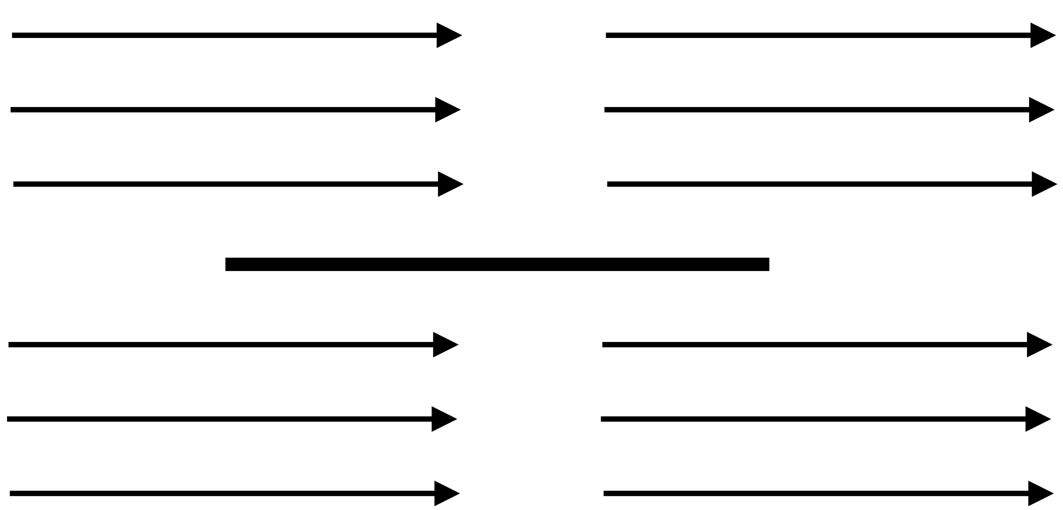
$$\frac{1}{Re} \to 0$$



Flat plate

Prandtl

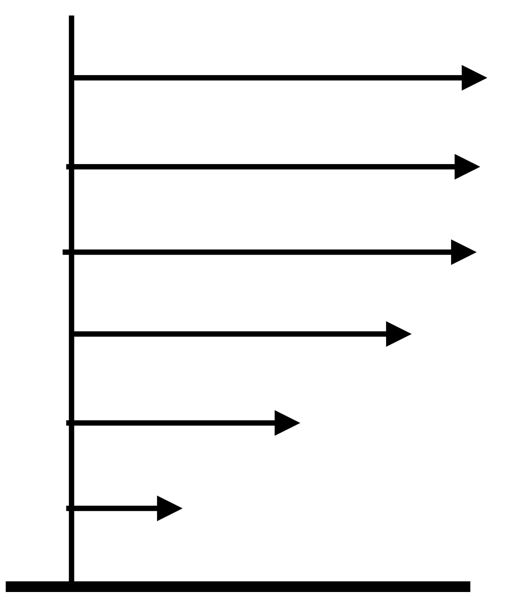
Come back to Navier Stokes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

zero velocity at the wall



Ideal Fluid: Euler equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x}$$

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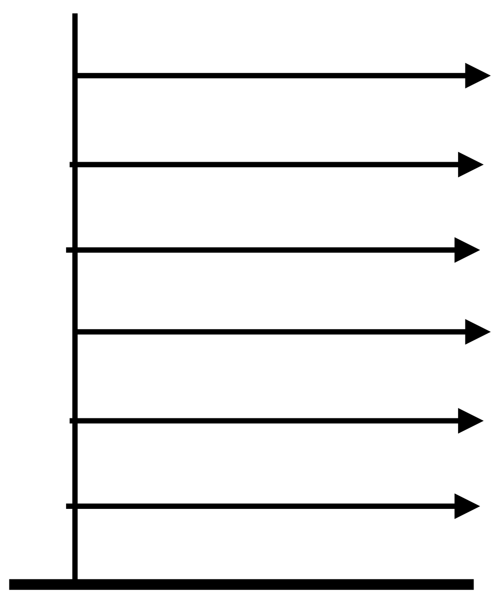

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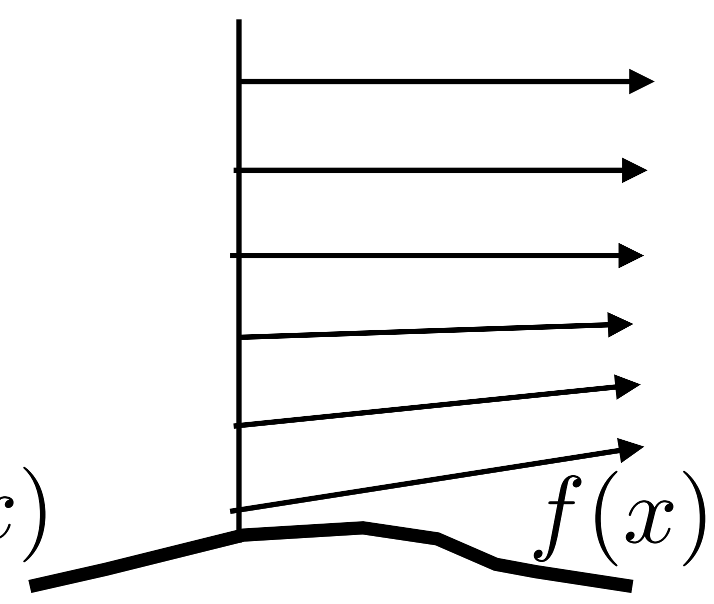
$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y}$$

$$\frac{1}{Re} \to 0$$

Linearized solution for the slip velocity

$$u(x,0) = 1 + \frac{1}{\pi} fp \int \frac{f'(x)}{x-\xi} d\xi$$

$u_e(x)$ slip velocity on a wall of shape $f(x)$



singular perturbation problem

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

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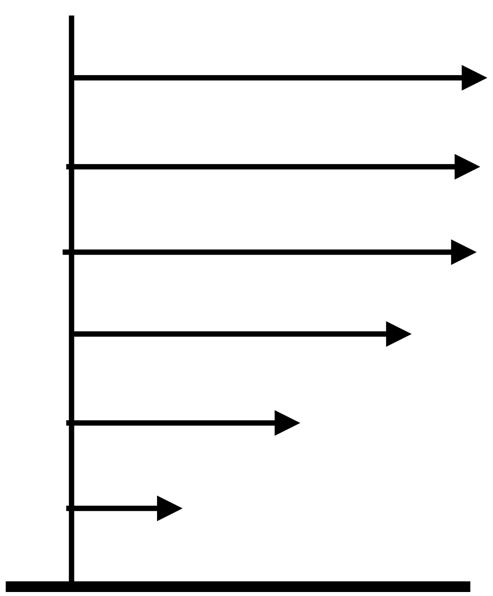

Classical Boundary Layer

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

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$$u = \tilde{u}$$

$$x = \tilde{x}$$

$$y = \varepsilon\tilde{y}$$

Classical Boundary Layer

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

dominant balance

$u = \tilde{u}$ $\qquad$ $v = \varepsilon \tilde{v}$

$x = \tilde{x}$

$y = \varepsilon \tilde{y}$

Classical Boundary Layer

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$$\varepsilon(\tilde{u}\frac{\partial \tilde{v}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{v}}{\partial \tilde{y}}) = -\frac{1}{\varepsilon}\frac{\partial \tilde{p}}{\partial \tilde{y}} + \frac{1}{Re}\left(\varepsilon\frac{\partial^2 \tilde{v}}{\partial \tilde{x}^2} + \frac{1}{\varepsilon^2}\frac{\partial^2 \tilde{v}}{\partial \tilde{y}^2}\right)$$

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dominant balance

$$u = \tilde{u} \qquad v = \varepsilon\tilde{v}$$

$$x = \tilde{x} \qquad p = \tilde{p} \qquad \varepsilon = \frac{1}{\sqrt{Re}}$$

$$y = \varepsilon\tilde{y}$$

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$$y = \varepsilon\tilde{y}$$

Classical Boundary Layer

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0$$

$$\tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}} = -\frac{\partial \tilde{p}}{\partial \tilde{x}} + \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2}$$

$$\varepsilon^2(\tilde{u}\frac{\partial \tilde{v}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{v}}{\partial \tilde{y}}) = -\frac{\partial \tilde{p}}{\partial \tilde{y}} + \left(\varepsilon^4\frac{\partial^2 \tilde{v}}{\partial \tilde{x}^2} + \varepsilon\frac{\partial^2 \tilde{v}}{\partial \tilde{y}^2}\right)$$

dominant balance

$$u = \tilde{u} \qquad v = \varepsilon\tilde{v}$$

$$x = \tilde{x} \qquad p = \tilde{p} \qquad \varepsilon = \frac{1}{\sqrt{Re}}$$

$$y = \varepsilon\tilde{y}$$

Classical Boundary Layer

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0$$

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$$0 = -\frac{\partial \tilde{p}}{\partial \tilde{y}}$$

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$$y = \varepsilon \tilde{y}$$

Classical Boundary Layer

"Matched Asymptotic Expansion"

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0$$

$$\tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}} = -\frac{\partial \tilde{p}}{\partial \tilde{x}} + \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2}$$

$$0 = -\frac{\partial \tilde{p}}{\partial \tilde{y}}$$

Matching

$$\tilde{u}(\tilde{x}, \infty) = u(x, 0)$$

$$\tilde{p}(\tilde{x}) = p(x, 0)$$

dominant balance

$$u = \tilde{u} \qquad v = \varepsilon \tilde{v}$$

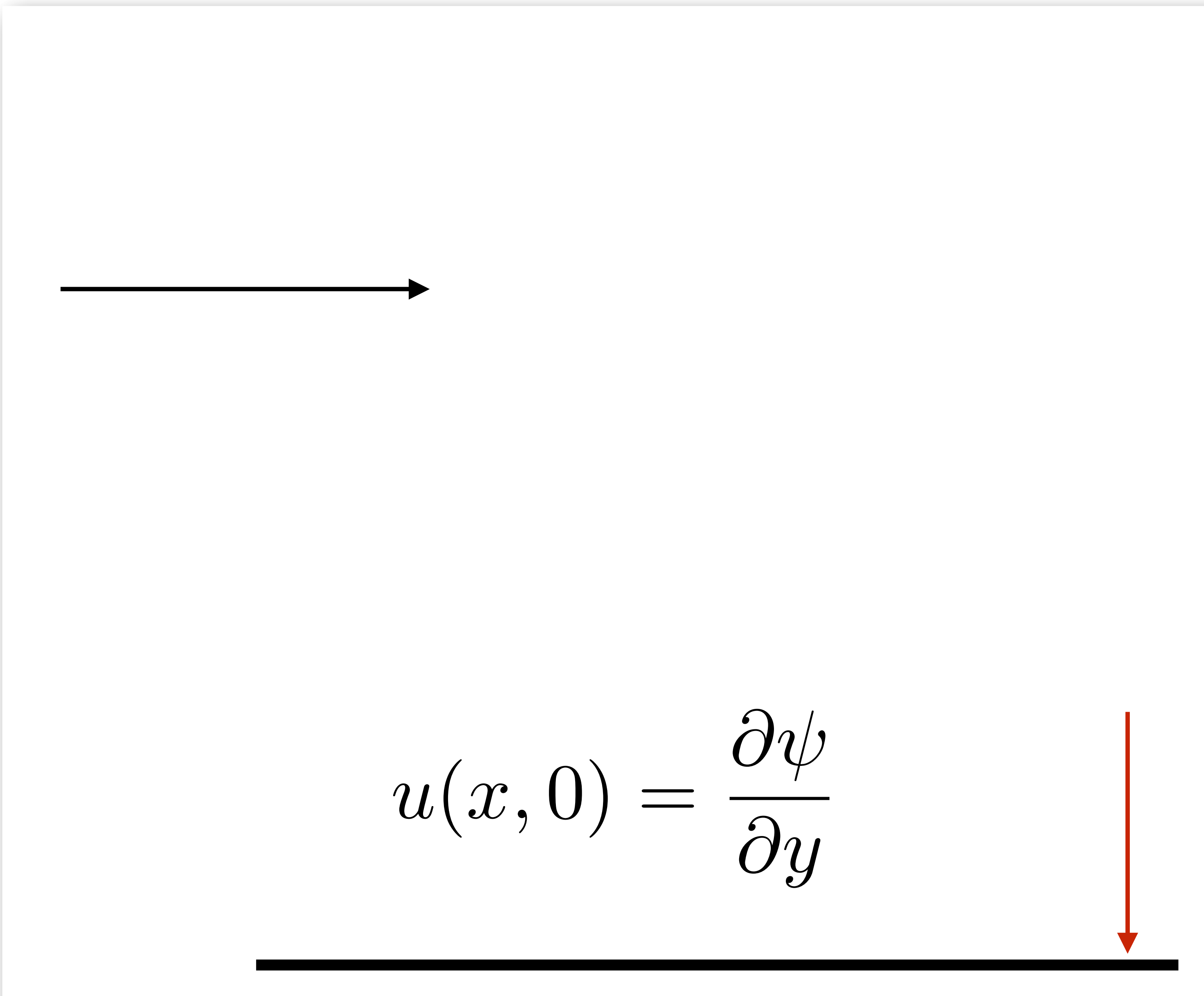
$$x = \tilde{x} \qquad p = \tilde{p}$$

$$y = \varepsilon \tilde{y}$$

$$\varepsilon = \frac{1}{\sqrt{Re}}$$

Classical Boundary Layer

"Matched Asymptotic Expansion"



ideal fluid:
Euler Equations

$$\nabla^2 \psi = 0$$

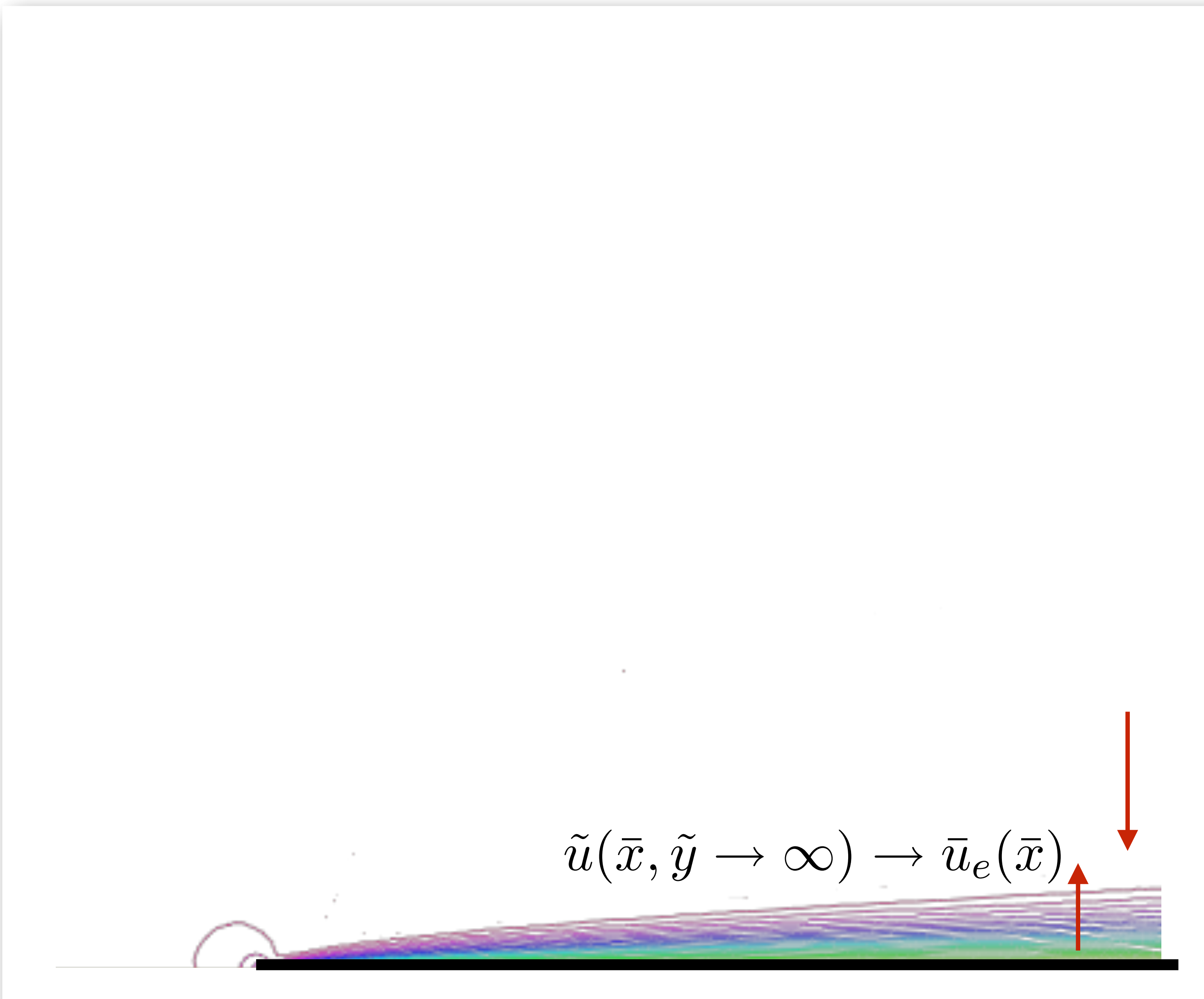
elliptic

$$\frac{\partial \psi}{\partial y} = 1$$ far from the body

$$\psi = 0$$ on the body

Classical Boundary Layer

"Matched Asymptotic Expansion"



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As long as the boundary layer is “attached” (no strong deceleration for the ideal fluid velocity, or weak counter pressure), every thing is OK

$$\varepsilon = \frac{1}{\sqrt{Re}}$$

As long as the boundary layer is “attached” (no strong deceleration for the ideal fluid velocity, or weak counter pressure), every thing is OK

problem: separation

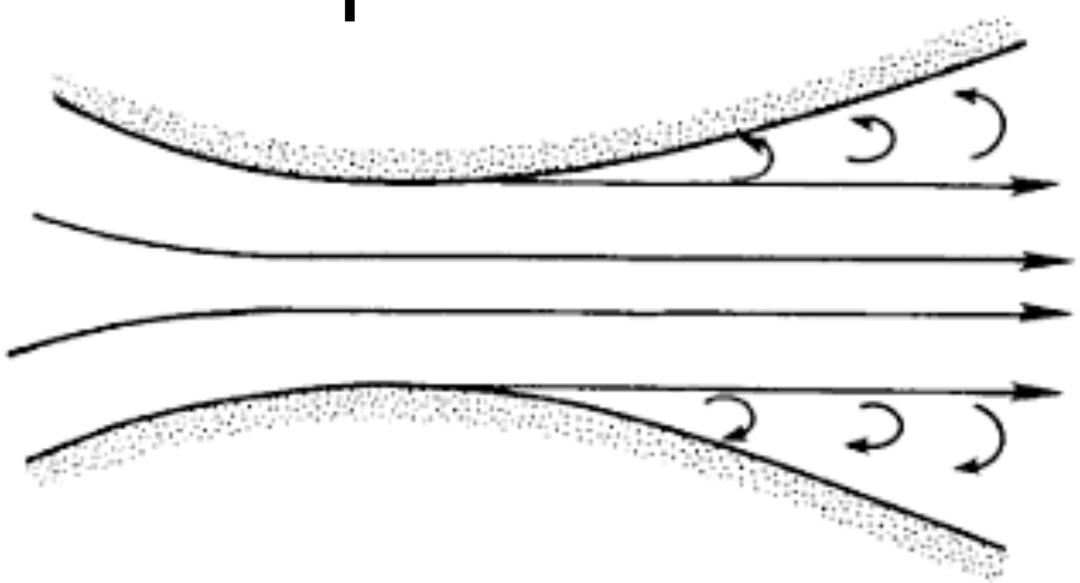
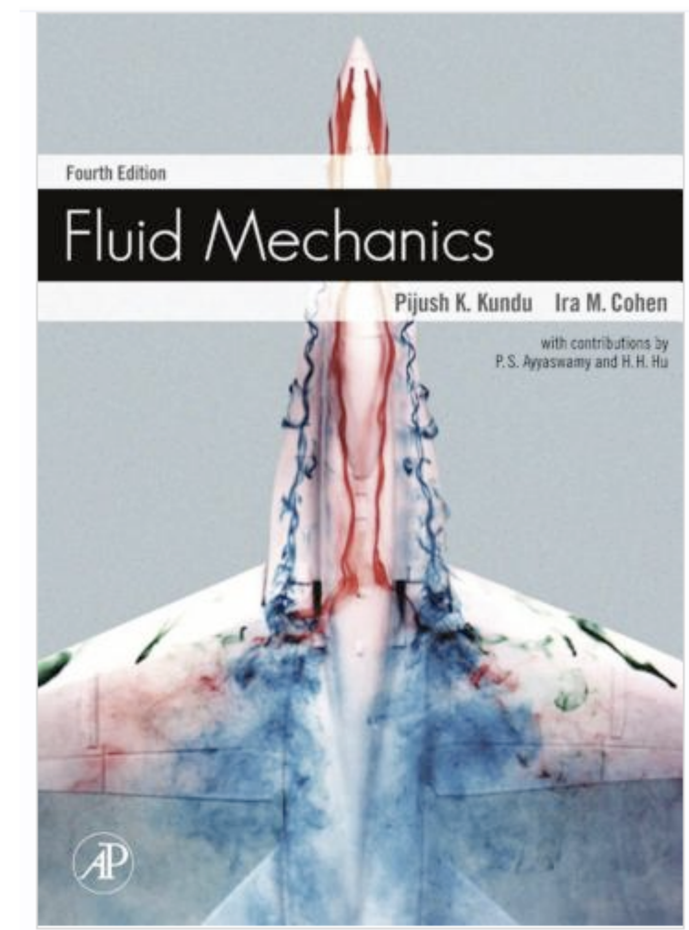


Figure 10.16 Separation of flow in a highly divergent channel.

gradient is favorable and the flow adheres to the wall. Downstream of the throat a large enough adverse pressure gradient can cause separation.

The boundary layer equations are valid only as far downstream as the point of separation. Beyond it the boundary layer becomes so thick that the basic underlying assumptions become invalid. Moreover, the parabolic character of the boundary layer equations requires that a numerical integration is possible only in the direction of advection (along which information is propagated), which is *upstream* within the reversed flow region. A forward (downstream) integration of the boundary layer equations therefore breaks down after the separation point. Last, we can no longer apply potential theory to find the pressure distribution in the separated region, as the effective boundary of the irrotational flow is no longer the solid surface but some unknown shape encompassing part of the body plus the separated region.



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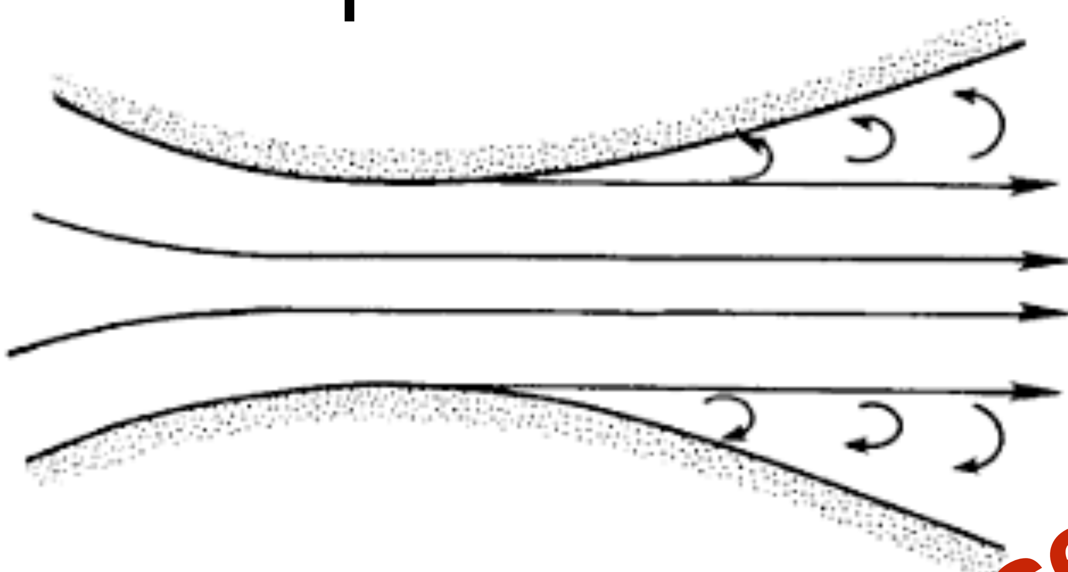

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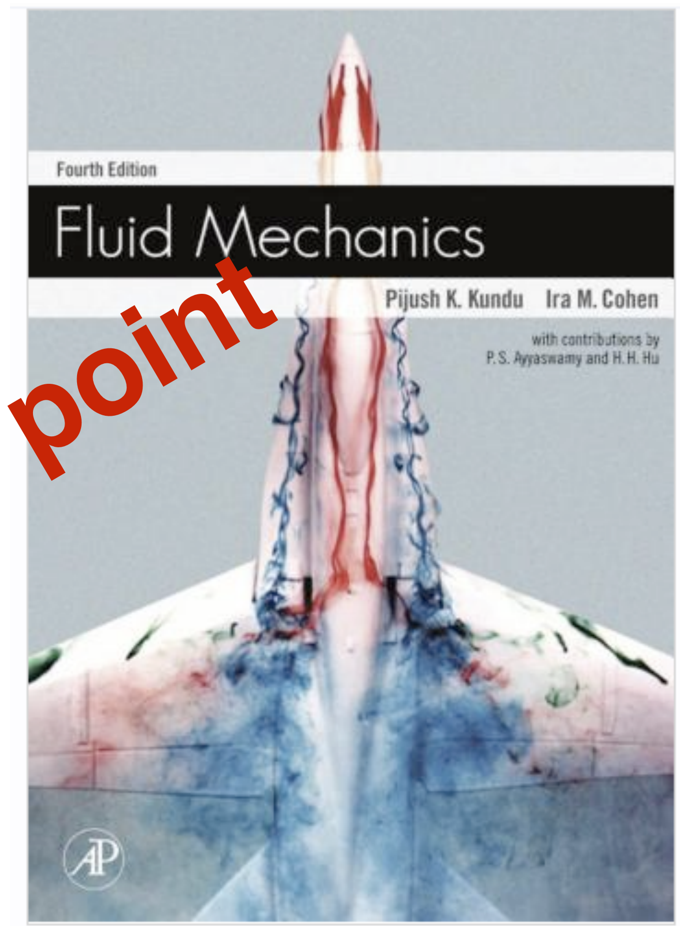
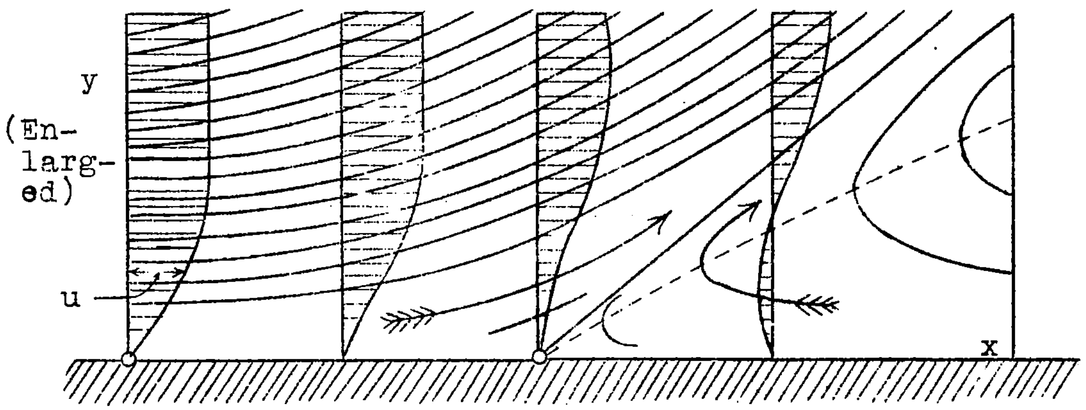
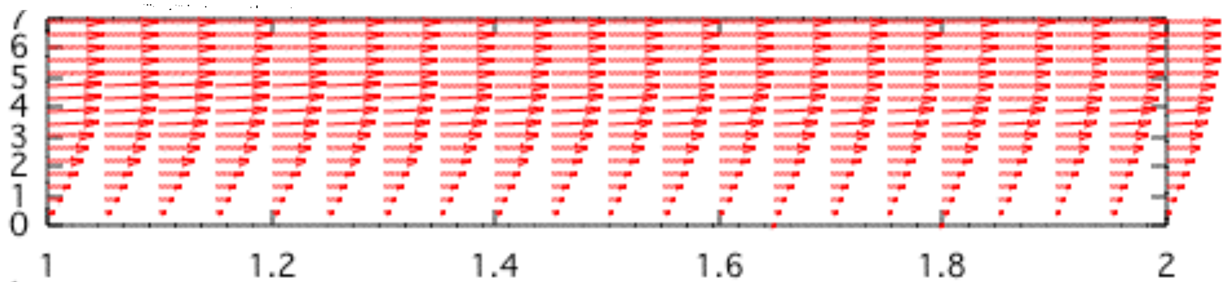
Impossible to cross separation point

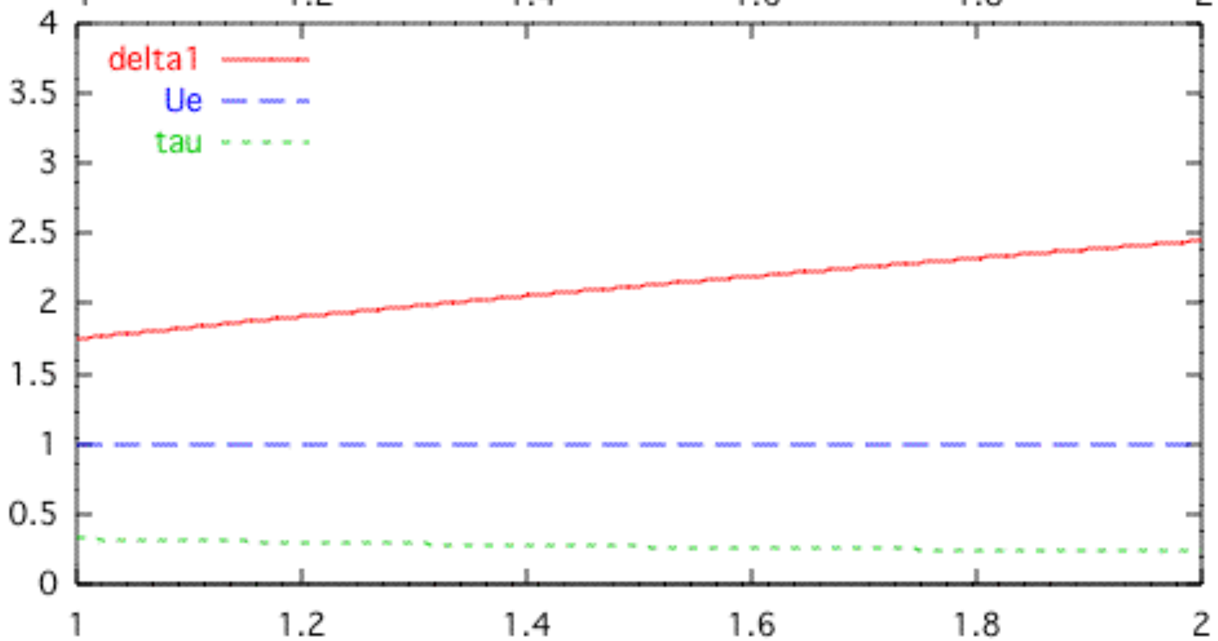


$$\varepsilon = \frac{1}{\sqrt{Re}}$$

some problems: separation







direct resolution

prescribed $u_e(x)$

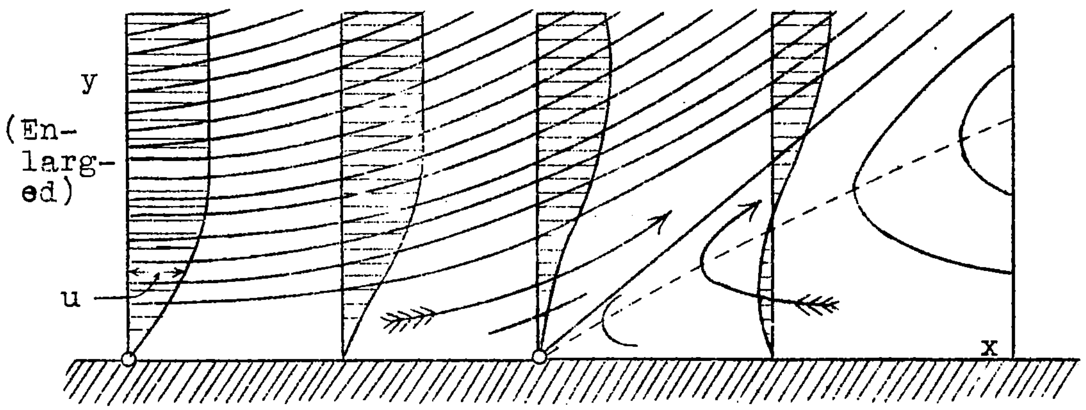
$$\frac{\partial u}{\partial y} \sim \sqrt{x_s - x}$$

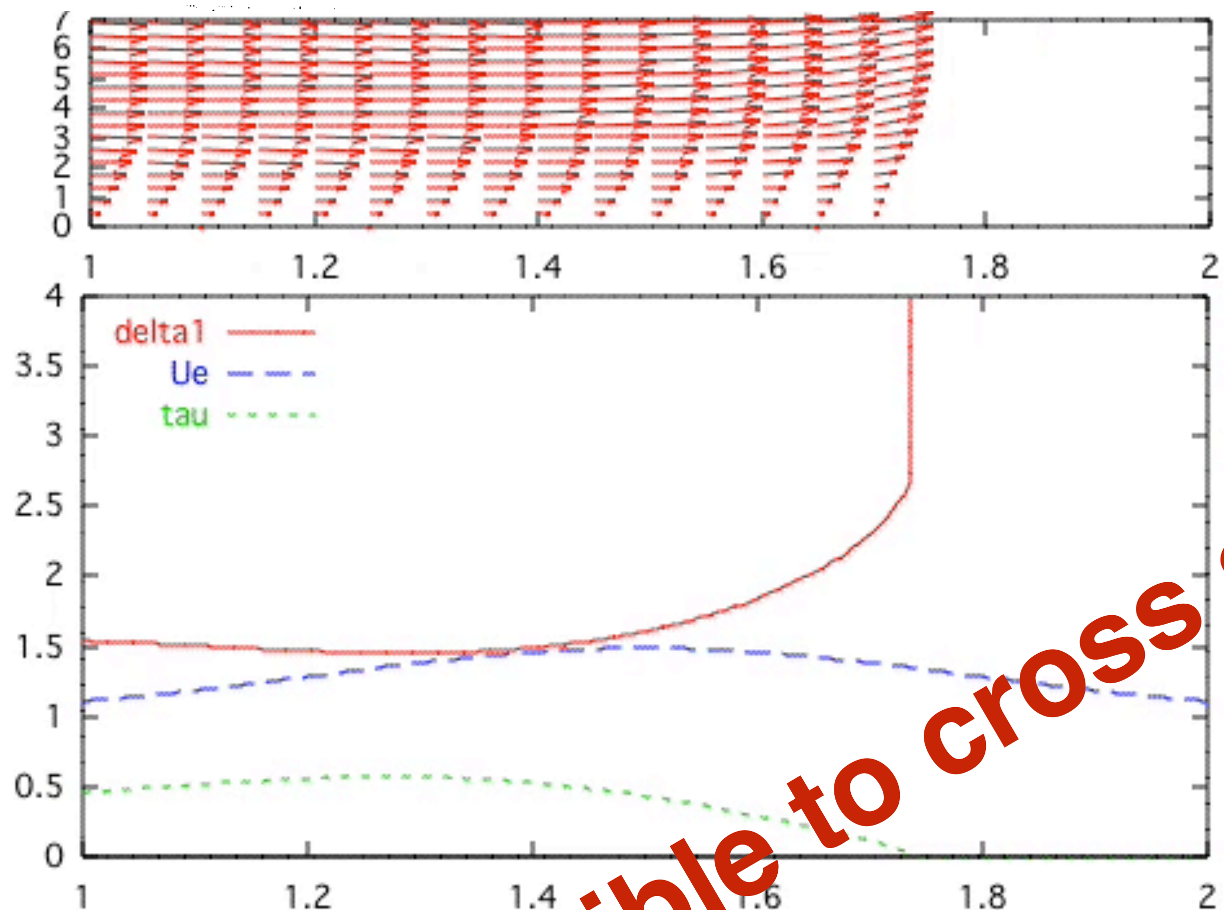
Landau 50'
Goldstein 48

$$\varepsilon = \frac{1}{\sqrt{Re}}$$

Singularity at the point of separation:
we can not cross $\frac{\partial \tilde{u}}{\partial \tilde{y}} = 0$

some problems: separation





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Landau 50'
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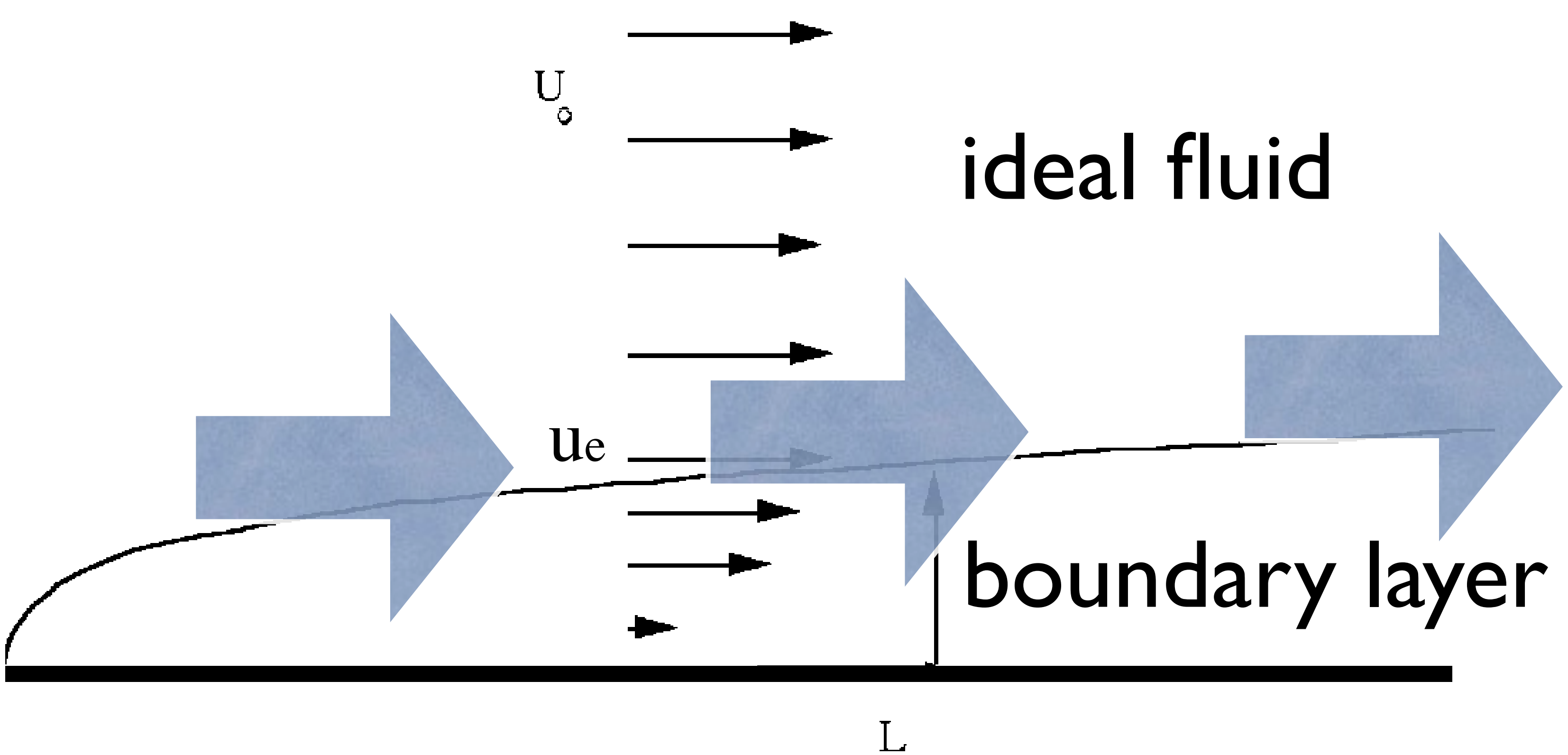
Singularity at the point of separation:
we can not cross $\quad \dfrac{\partial \tilde{u}}{\partial \tilde{y}} = 0$

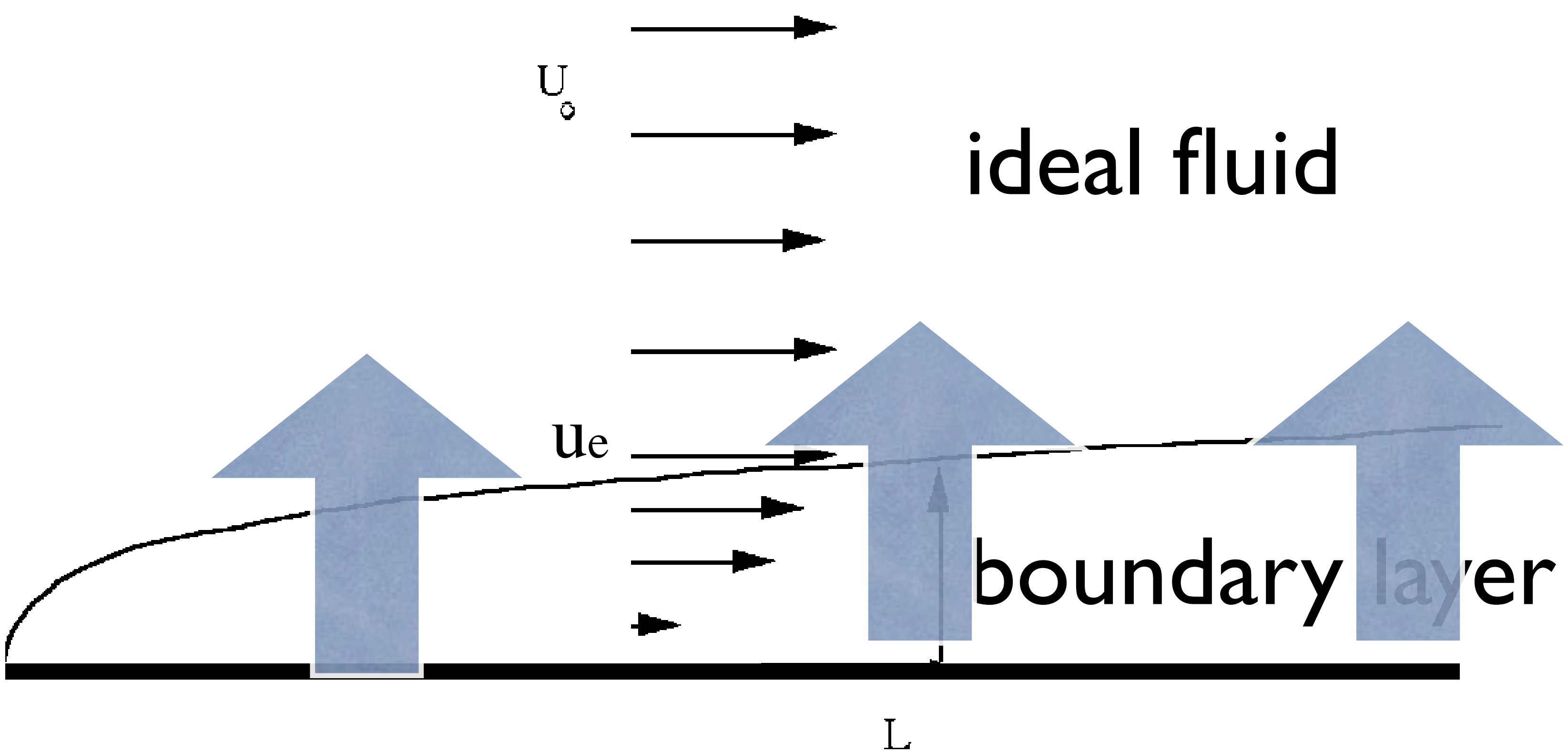
2nd order BLT



Van Dyke 62

2nd order BLT

Perturbation of the Ideal fluid at the next order



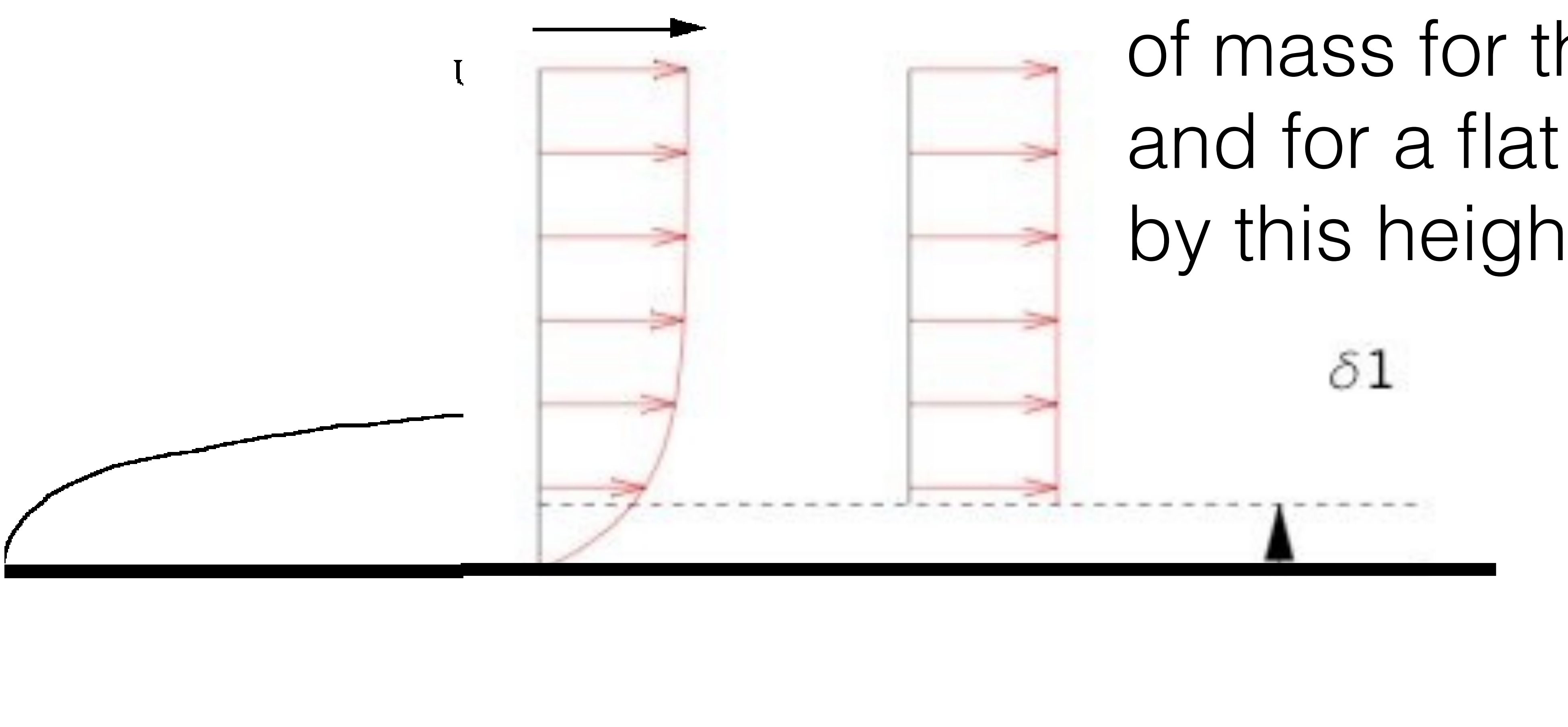
Van Dyke 62

2nd order BLT

Perturbation of the Ideal fluid at the next order

$$\tilde{\delta}_1 = \int_0^\infty (1 - \tilde{u}/\bar{u}_e)\, d\tilde{y}$$

the displacement thickness corresponds to the same flux of mass for the actual velocity and for a flat one displaced by this height



Van Dyke 62

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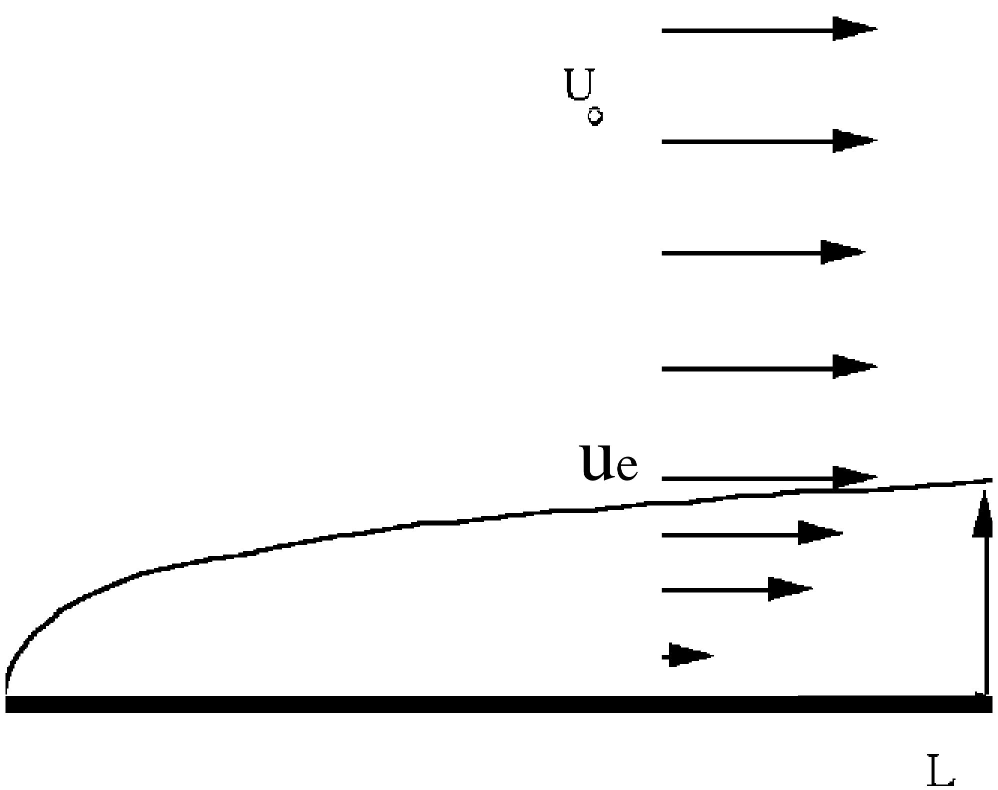

Van Dyke 62

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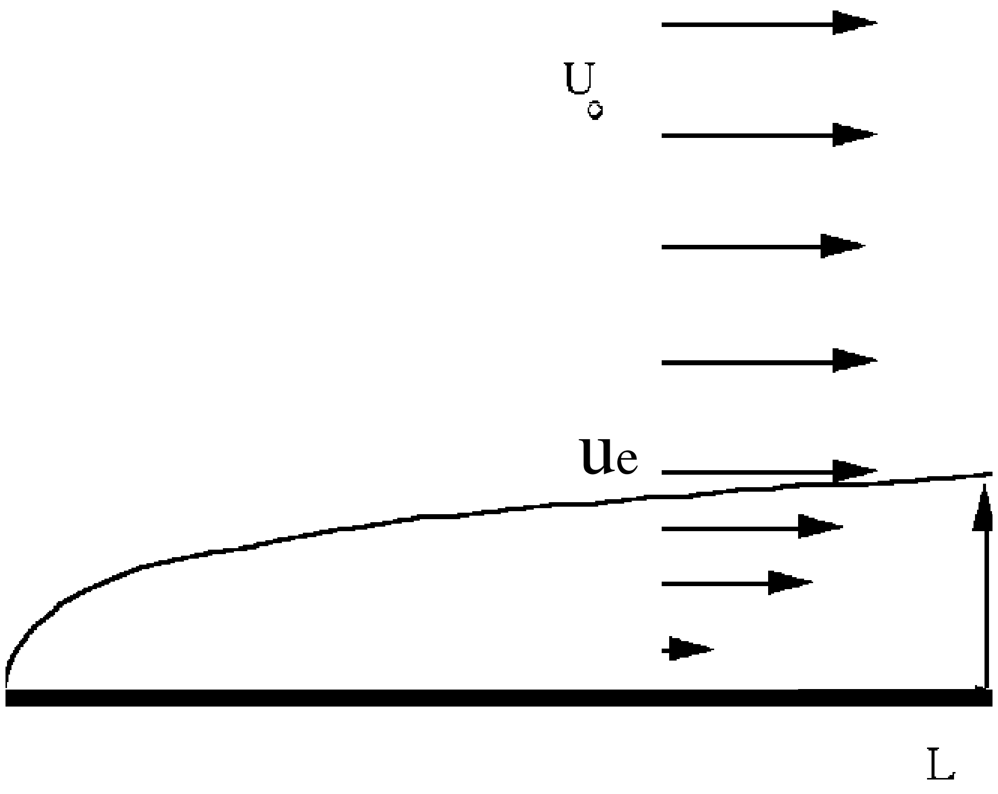


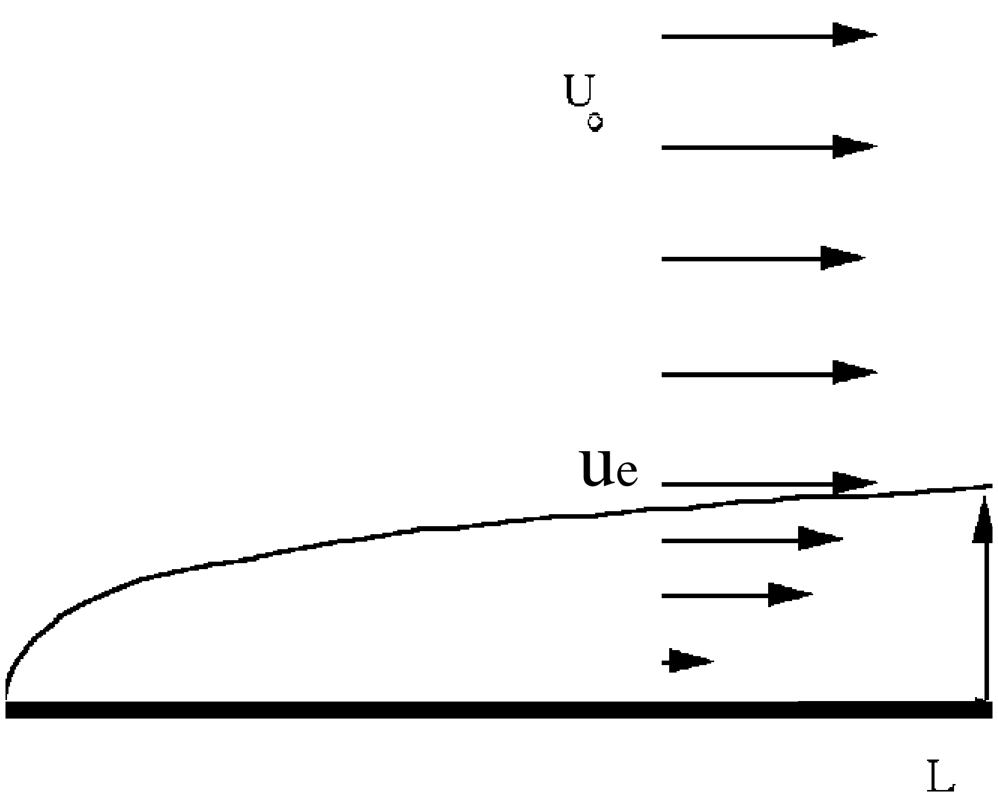
$$\frac{\partial \tilde{v}}{\partial \tilde{y}} = (-\frac{\partial \tilde{u}}{\partial x} + \frac{\partial \bar{u}_e}{\partial \bar{x}}) - \frac{\partial \bar{u}_e}{\partial \bar{x}},$$

effect of the displacement thickness

2nd order BLT

Perturbation of the Ideal fluid at the next order

$$\tilde{\delta}_1 = \int_0^\infty (1 - \tilde{u}/\bar{u}_e) d\tilde{y}$$



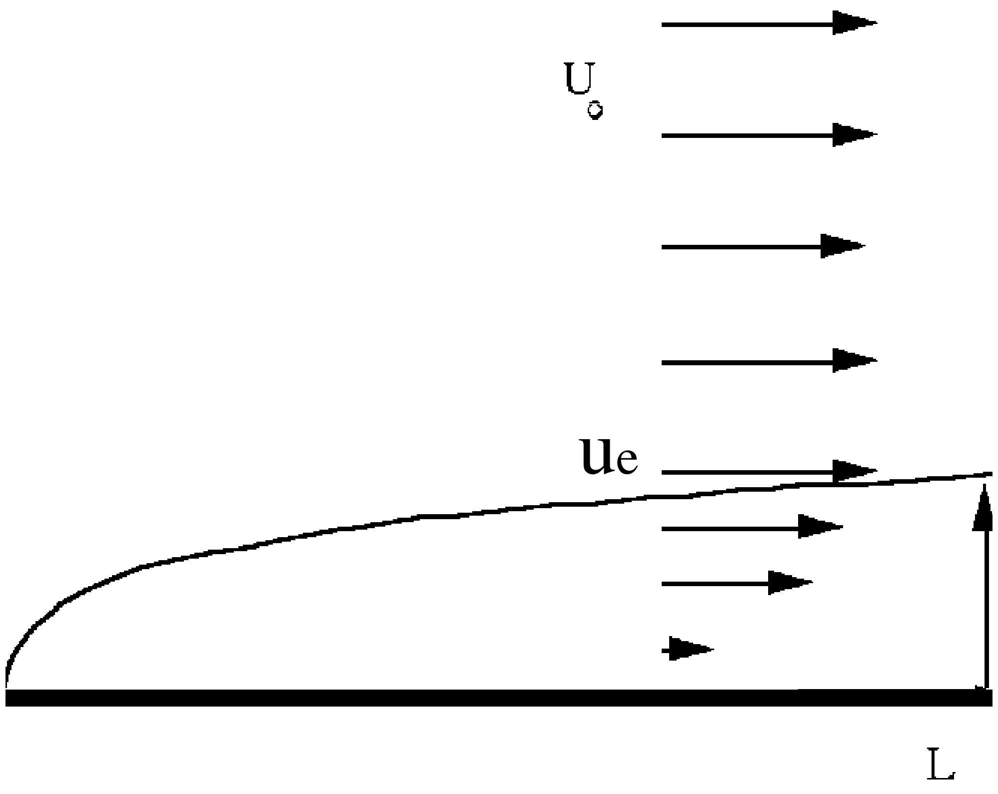
$$\tilde{v}(\tilde{y}) - \tilde{v}(0) = -\frac{\partial}{\partial \bar{x}} \int_0^{\tilde{y}} (\tilde{u} - \bar{u}_e) d\tilde{y} - \tilde{y} \frac{\partial \bar{u}_e}{\partial \bar{x}}$$

effect of the displacement thickness

2nd order BLT

Perturbation of the Ideal fluid at the next order

$$\tilde{\delta}_1 = \int_0^\infty (1 - \tilde{u}/\bar{u}_e) d\tilde{y}$$



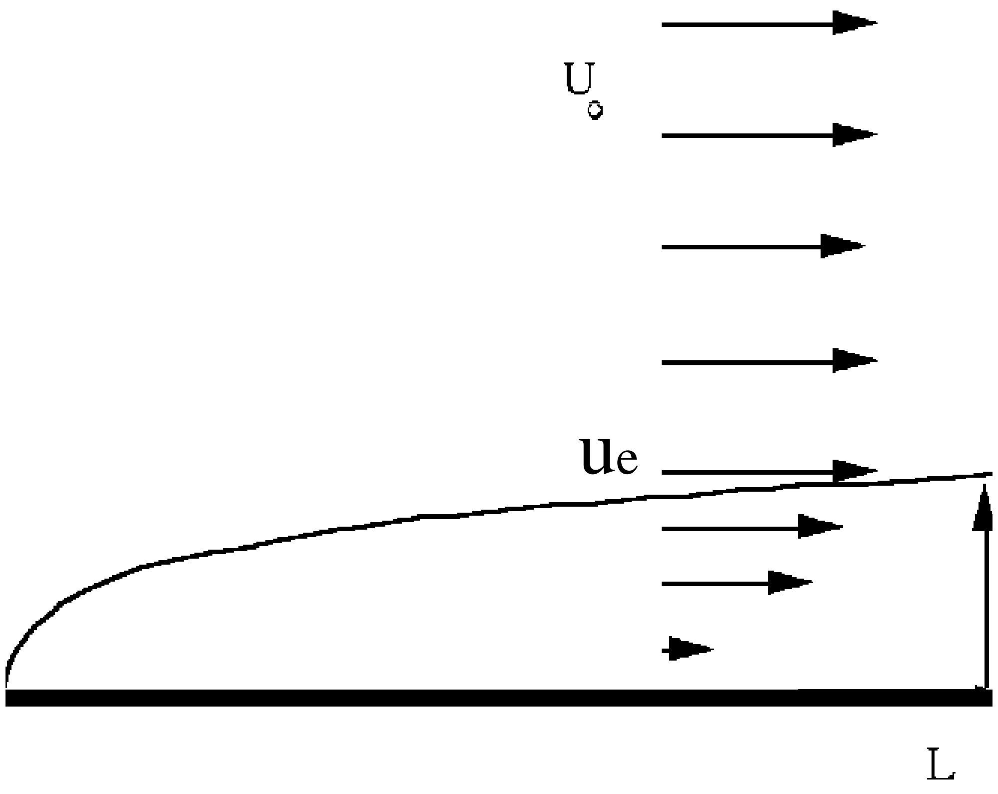
$$\tilde{v}(\tilde{y}) \simeq \frac{\partial}{\partial \bar{x}}(\bar{u}_e \tilde{\delta}_1) - \tilde{y}\frac{\partial \bar{u}_e}{\partial \bar{x}}$$

effect of the displacement thickness

2nd order BLT

Perturbation of the Ideal fluid at the next order

$$\tilde{\delta}_1 = \int_0^\infty (1 - \tilde{u}/\bar{u}_e) d\tilde{y}$$



$$\bar{v} = \bar{v}(\bar{x}, 0) + \bar{y}\frac{\partial \bar{v}}{\partial \bar{y}} + ... = \bar{v}(\bar{x}, 0) - \bar{y}\frac{\partial \bar{u}_e}{\partial \bar{x}} + ...$$

$$\tilde{v}(\tilde{y}) \simeq \frac{\partial}{\partial \bar{x}}(\bar{u}_e \tilde{\delta}_1) - \tilde{y}\frac{\partial \bar{u}_e}{\partial \bar{x}}$$

effect of the displacement thickness

2nd order BLT

Perturbation of the Ideal fluid at the next order

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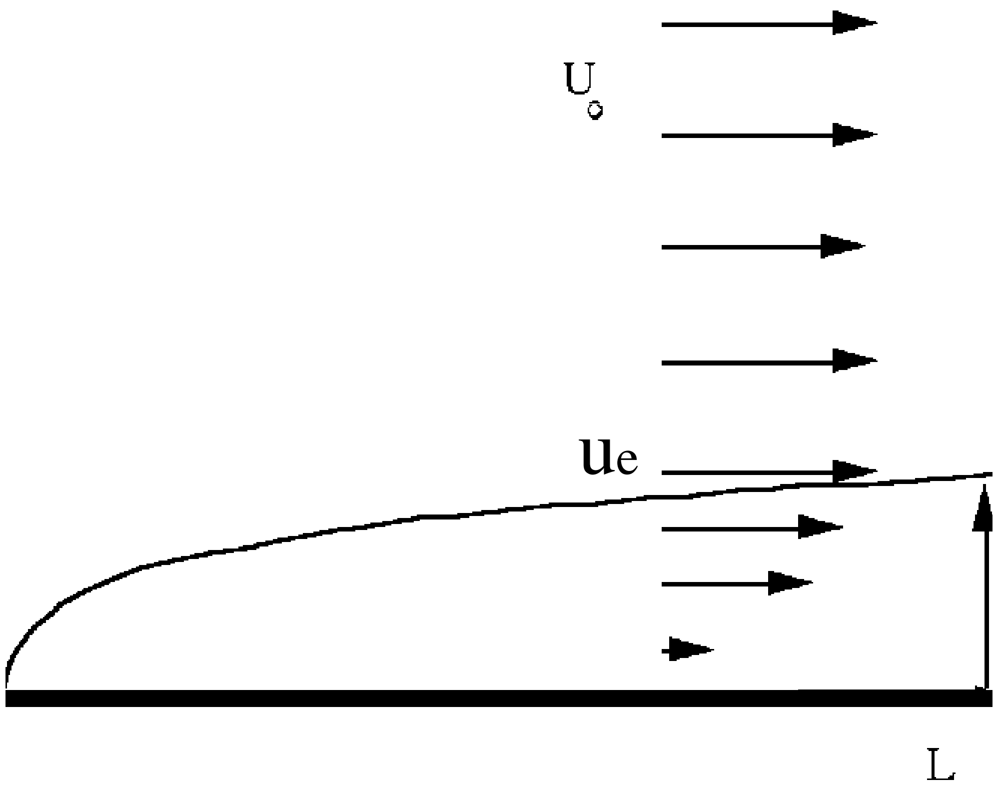

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$$\tilde{v}(\tilde{y}) \simeq \frac{\partial}{\partial \bar{x}}(\bar{u}_e \tilde{\delta}_1) - \tilde{y}\frac{\partial \bar{u}_e}{\partial \bar{x}}$$

$$\bar{v}(\bar{x},0) = Re^{-1/2}\frac{\partial}{\partial \bar{x}}(\bar{u}_e \tilde{\delta}_1)$$

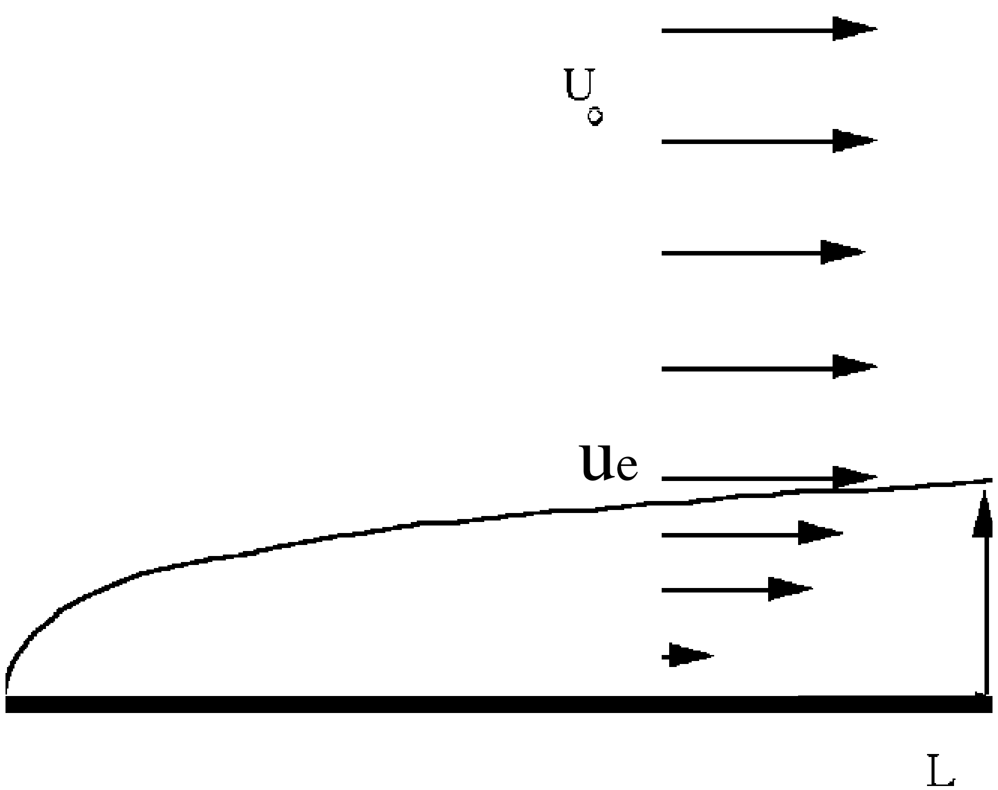
effect of the displacement thickness

2nd order BLT

Perturbation of the Ideal fluid at the next order

ideal fluid at next order

$$\bar{u} = \bar{u}_1 + Re^{-1/2}\bar{u}_2, \quad \bar{v} = \bar{v}_1 + Re^{-1/2}\bar{v}_2 \; \bar{p} = \bar{p}_1 + Re^{-1/2}\bar{p}_2....$$



$$\bar{v}(\bar{x},0) = Re^{-1/2}\frac{\partial}{\partial \bar{x}}(\bar{u}_e\tilde{\delta}_1)$$

effect of the displacement thickness

weak effect of the displacement thickness

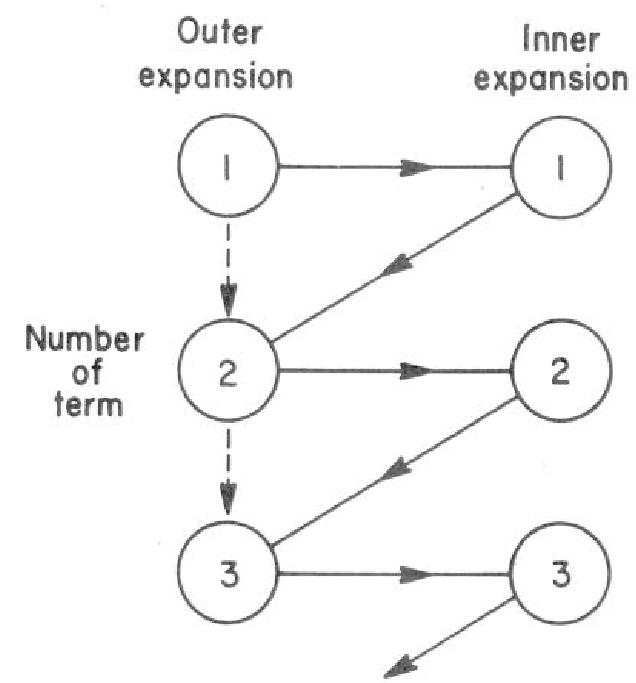


Fig. 5.6. Matching order for inner and outer expansions.

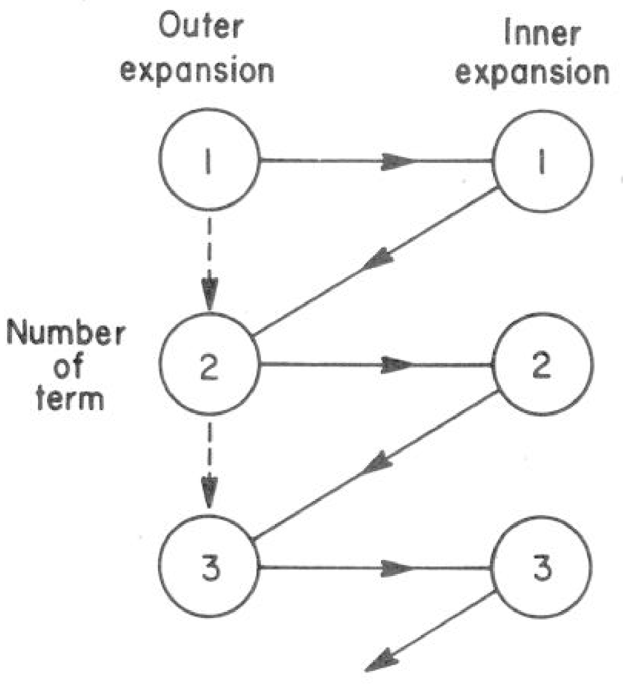
Van Dyke

$$\varepsilon = \frac{1}{\sqrt{Re}}$$

$$\varepsilon^2 = \frac{1}{Re}$$

$$\varepsilon^3 = \frac{1}{Re^{3/2}}$$

weak effect of the displacement thickness



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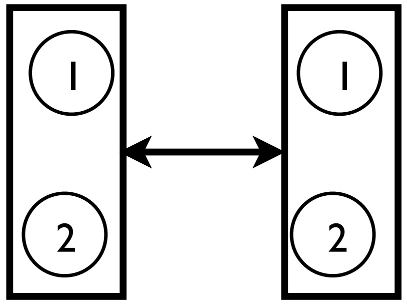
$$\varepsilon^3 = \frac{1}{Re^{3/2}}$$

strong effect of the displacement thickness

$$\bar{v}(\bar{x}, 0) = Re^{-1/2} \frac{\partial}{\partial \bar{x}} (\bar{u}_e \tilde{\delta}_1)$$



ideal fluid is modified at first order

INTERACTIVE BOUNDARY LAYER

VISCOUS INVISCID INTERACTIONS

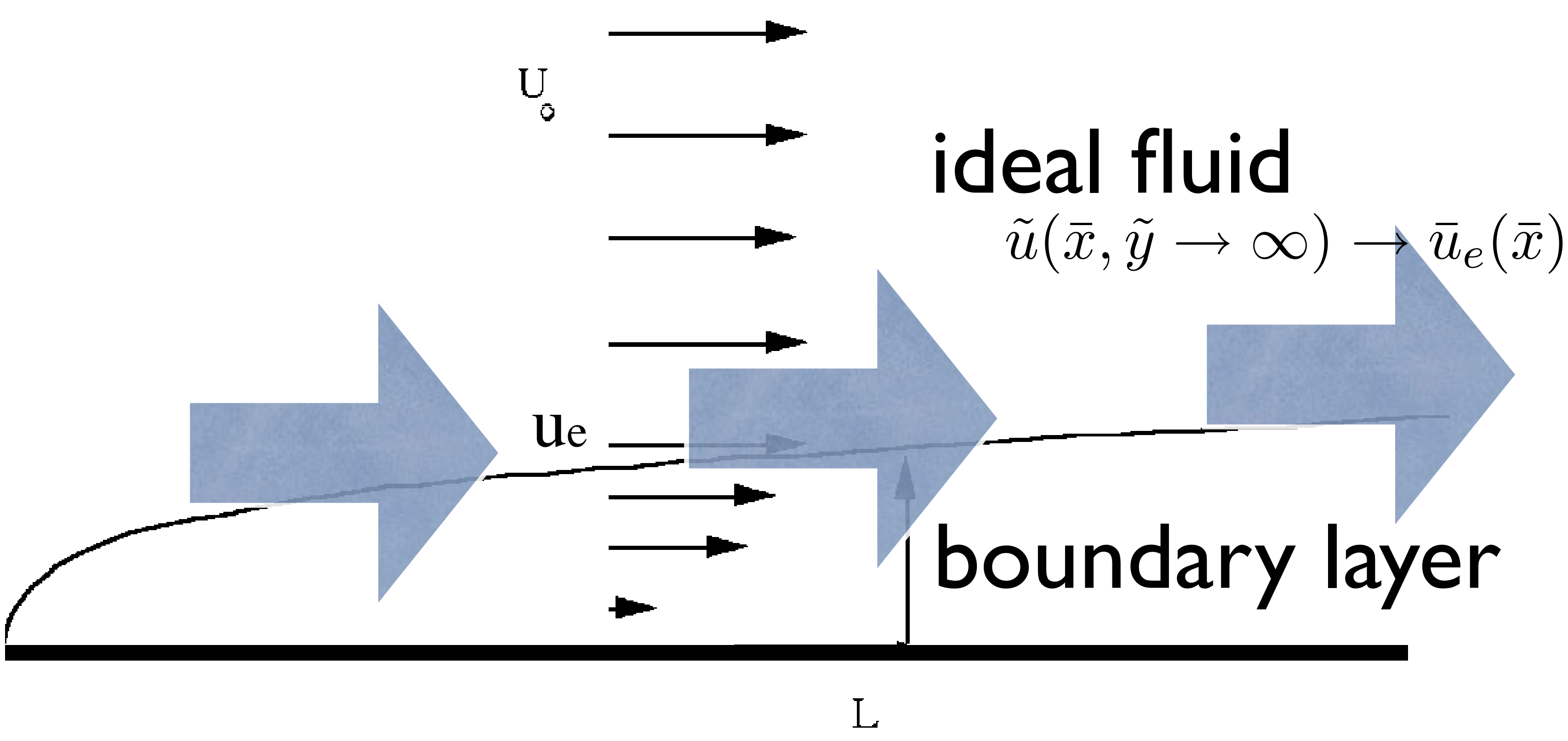
Cebecci Smith

Mauss Cousteix:
Asymptotic Analysis and Boundary Layers
Scientific Computation Springer 2007,

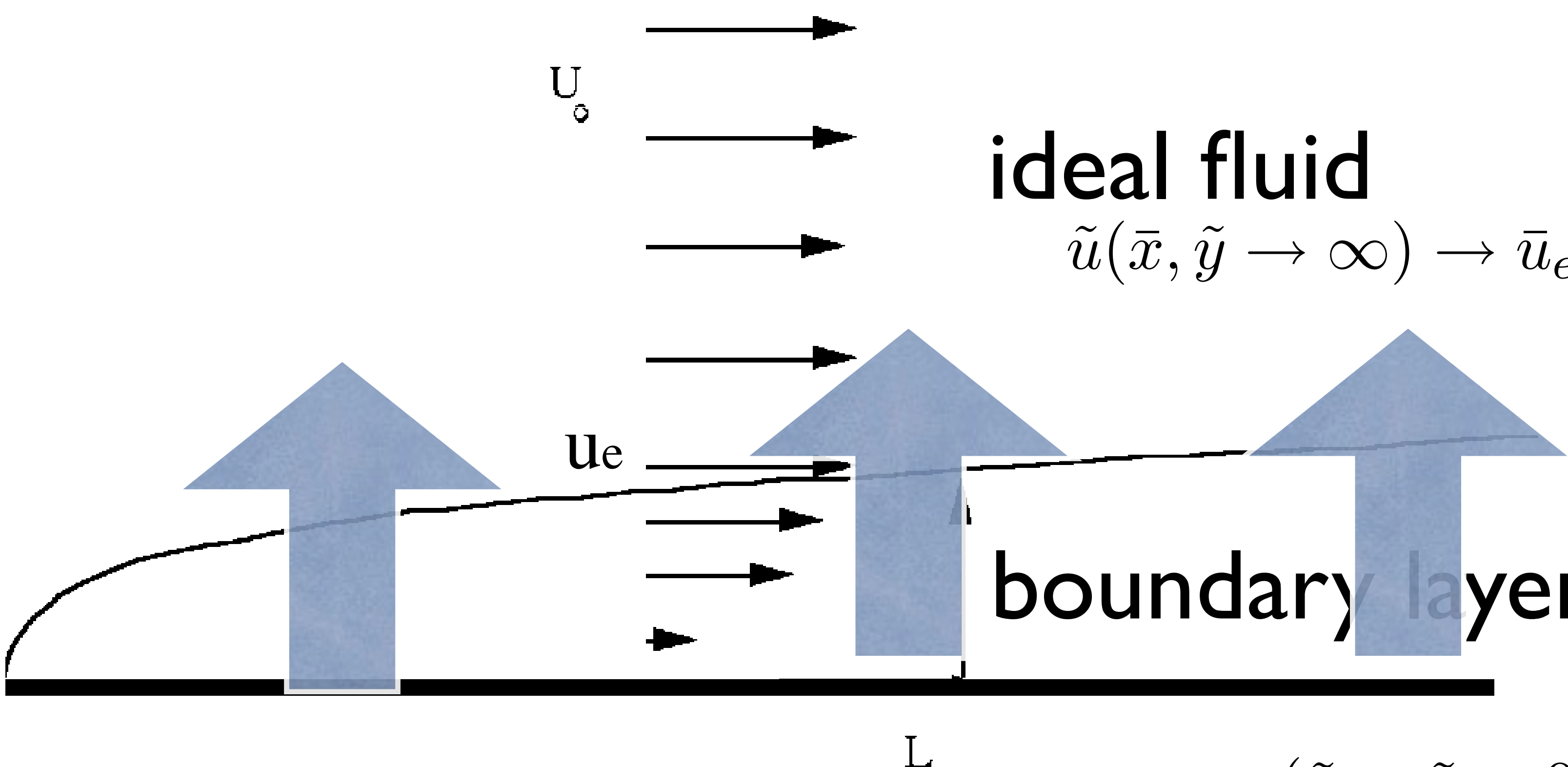
"Successive Complementary Expansion Method"

is preferable to "Matched Asymptotic Expansion"

construct an uniform expansion in which epsilon is not so small



U_o
ideal fluid
$\tilde{u}(\bar{x}, \tilde{y} \to \infty) \to \bar{u}_e(\bar{x})$
u_e
boundary layer
L



$$\tilde{u}(\bar{x}, \tilde{y} \to \infty) \to \bar{u}_e(\bar{x}) \qquad \bar{v}_e = Re^{-1/2} \frac{d(\tilde{\delta}_1 \bar{u}_e)}{d\bar{x}}$$

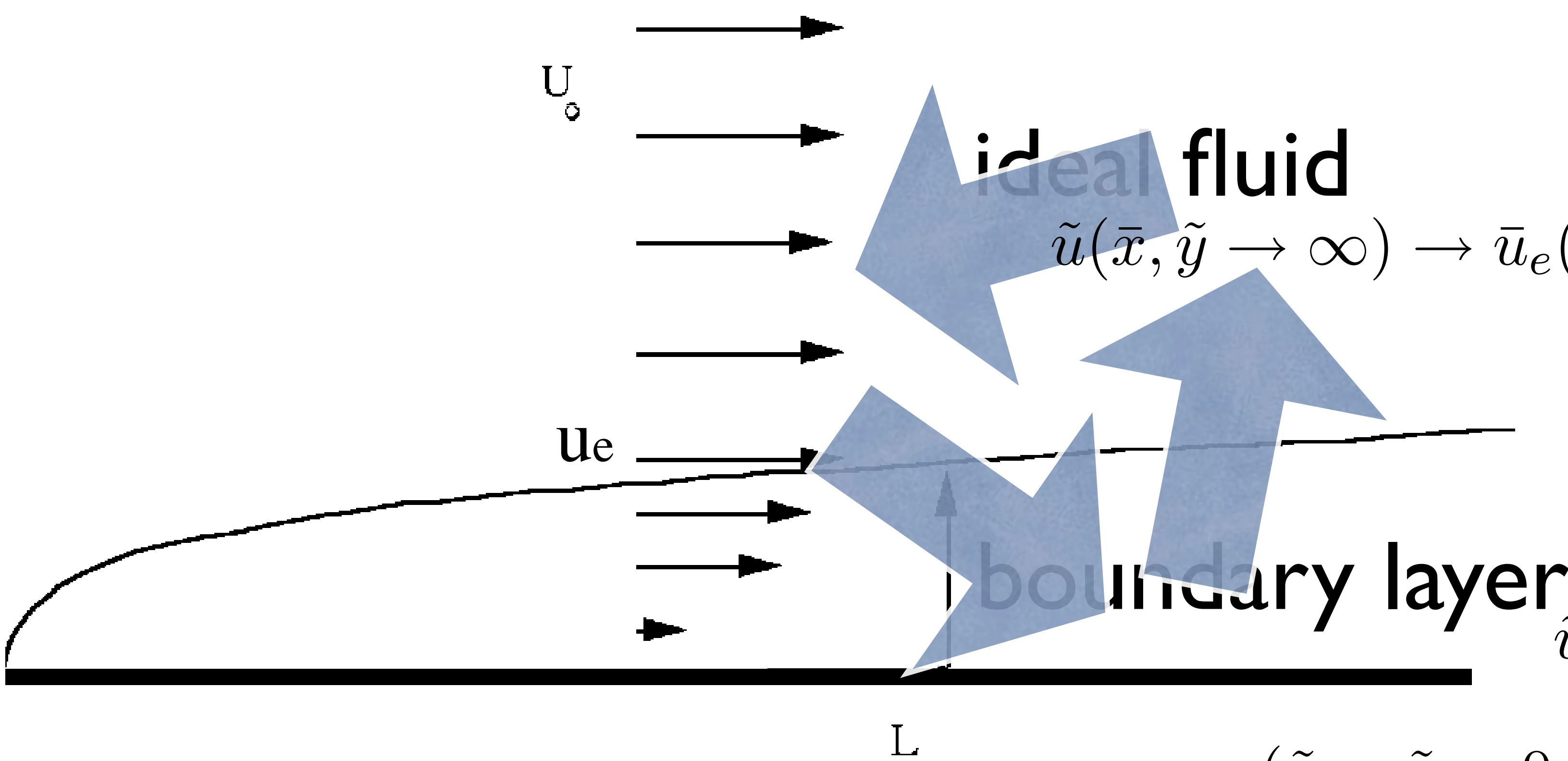
$$\frac{\partial \tilde{u}}{\partial \bar{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0,$$

$$\tilde{u}\frac{\partial \tilde{u}}{\partial \bar{x}} + \tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}} = \bar{u}_e \frac{d\bar{u}_e}{d\bar{x}} + \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2},$$

$$(\tilde{u} = \tilde{v} = 0 \text{ on the body } \bar{f}(\bar{x})).$$

INTERACTING BOUNDARY LAYER

$$\bar{u}_e = 1 + \frac{1}{\pi}\int \frac{\bar{f}'(\bar{x}) + Re^{-1/2}\frac{d(\tilde{\delta}_1 \bar{u}_e)}{d\bar{x}}}{x - \xi} d\xi$$



$$\tilde{u}(\bar{x}, \tilde{y} \to \infty) \to \bar{u}_e(\bar{x}) \qquad \bar{v}_e = Re^{-1/2}\frac{d(\tilde{\delta}_1 \bar{u}_e)}{d\bar{x}}$$

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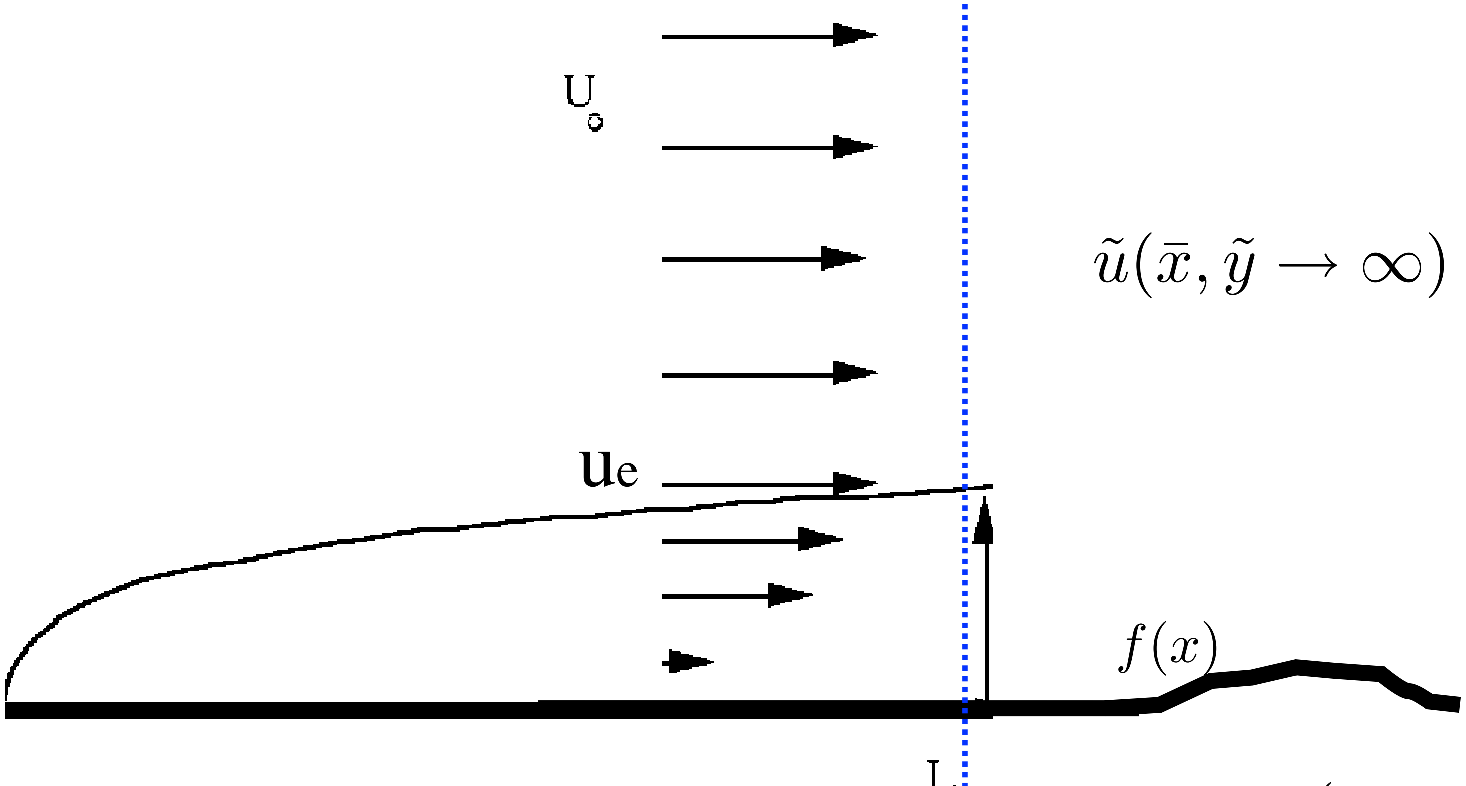
$(\tilde{u} = \tilde{v} = 0$ on the body $\bar{f}(\bar{x}))$

$$\bar{u}_e = 1 + \frac{1}{\pi}\int \frac{\bar{f}'(\bar{x}) + Re^{-1/2}\frac{d(\tilde{\delta}_1 \bar{u}_e)}{d\bar{x}}}{x-\xi}\,d\xi$$



$$\tilde{u}(\bar{x}, \tilde{y} \to \infty) \to \bar{u}_e(\bar{x}) \qquad \bar{v}_e = Re^{-1/2}\frac{d(\tilde{\delta}_1 \bar{u}_e)}{d\bar{x}}$$

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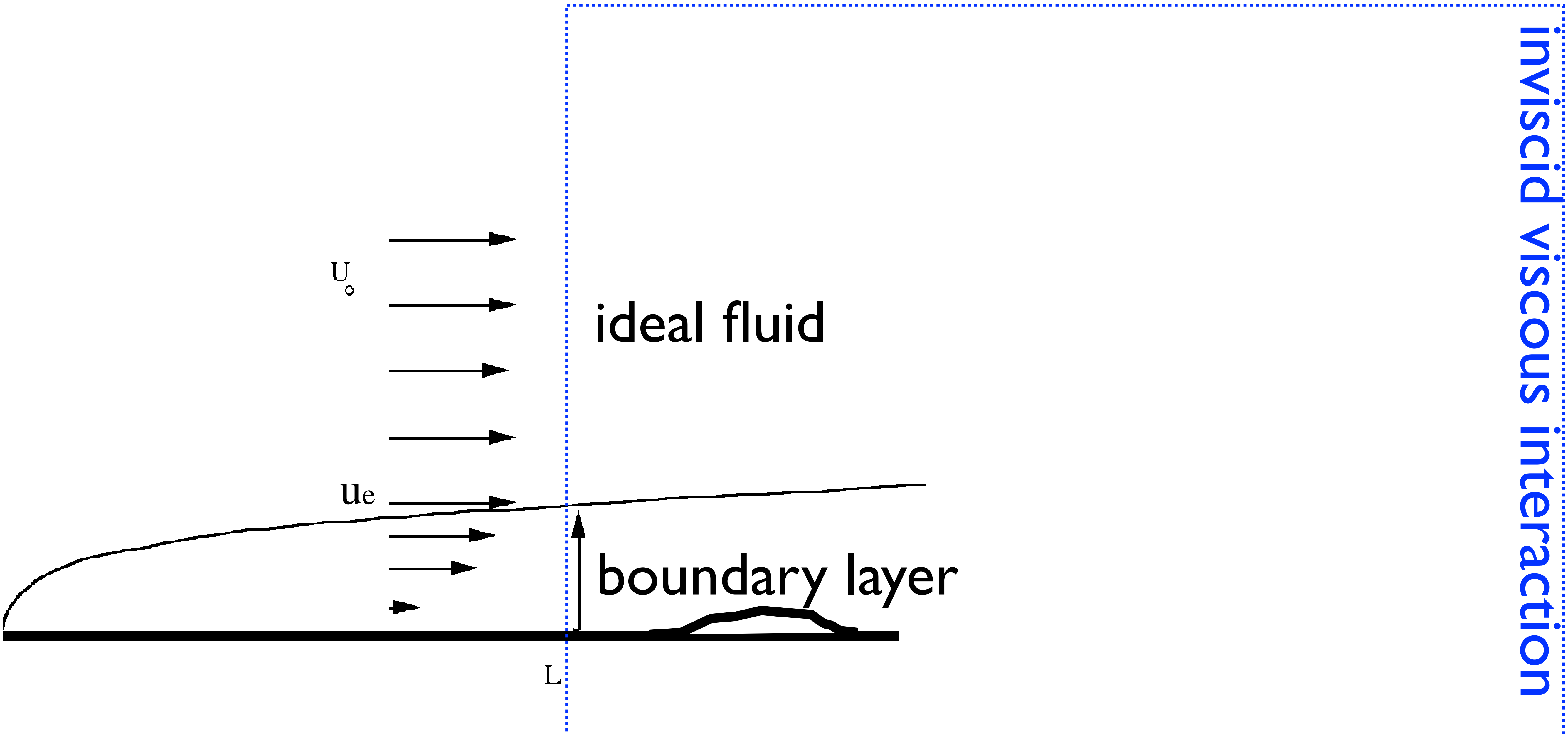
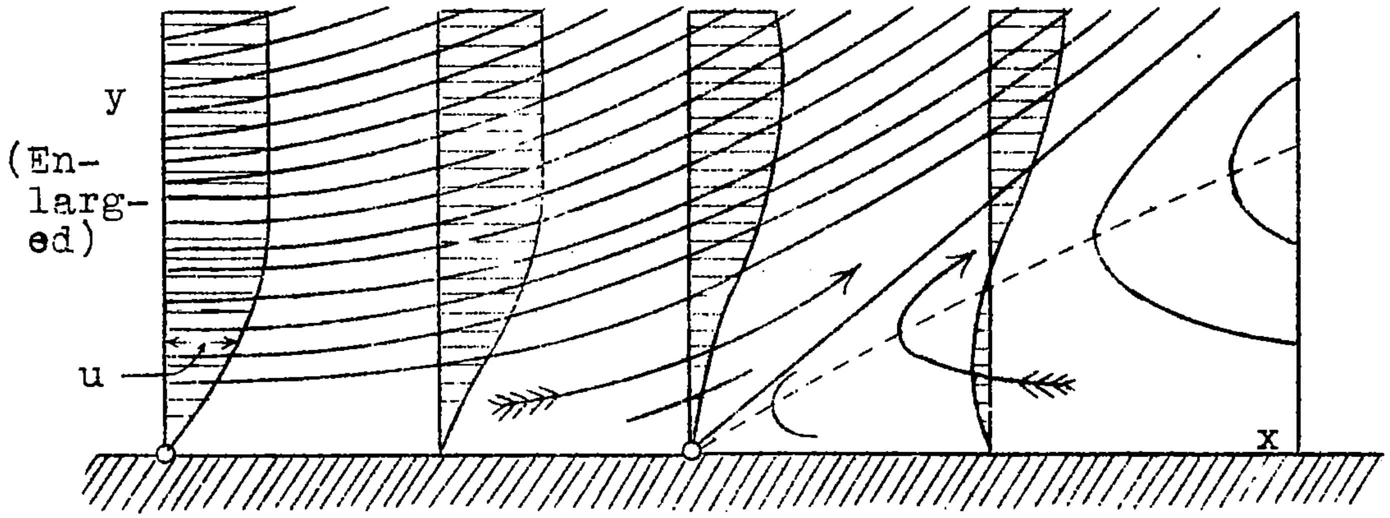
$(\tilde{u} = \tilde{v} = 0$ on the body $\bar{f}(\bar{x}))$

inviscid viscous interaction

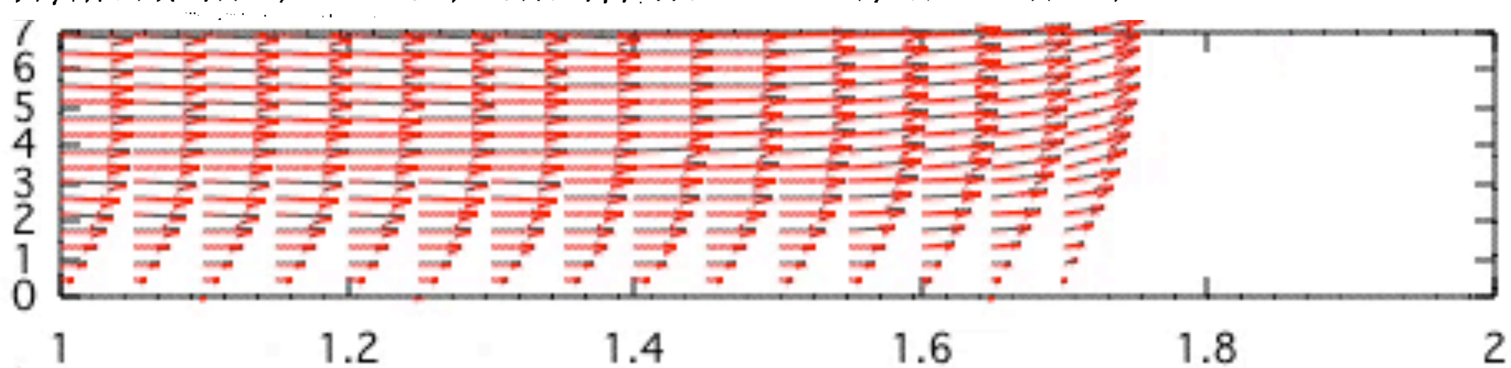


U_o
u_e
ideal fluid
boundary layer
L
inviscid viscous interaction

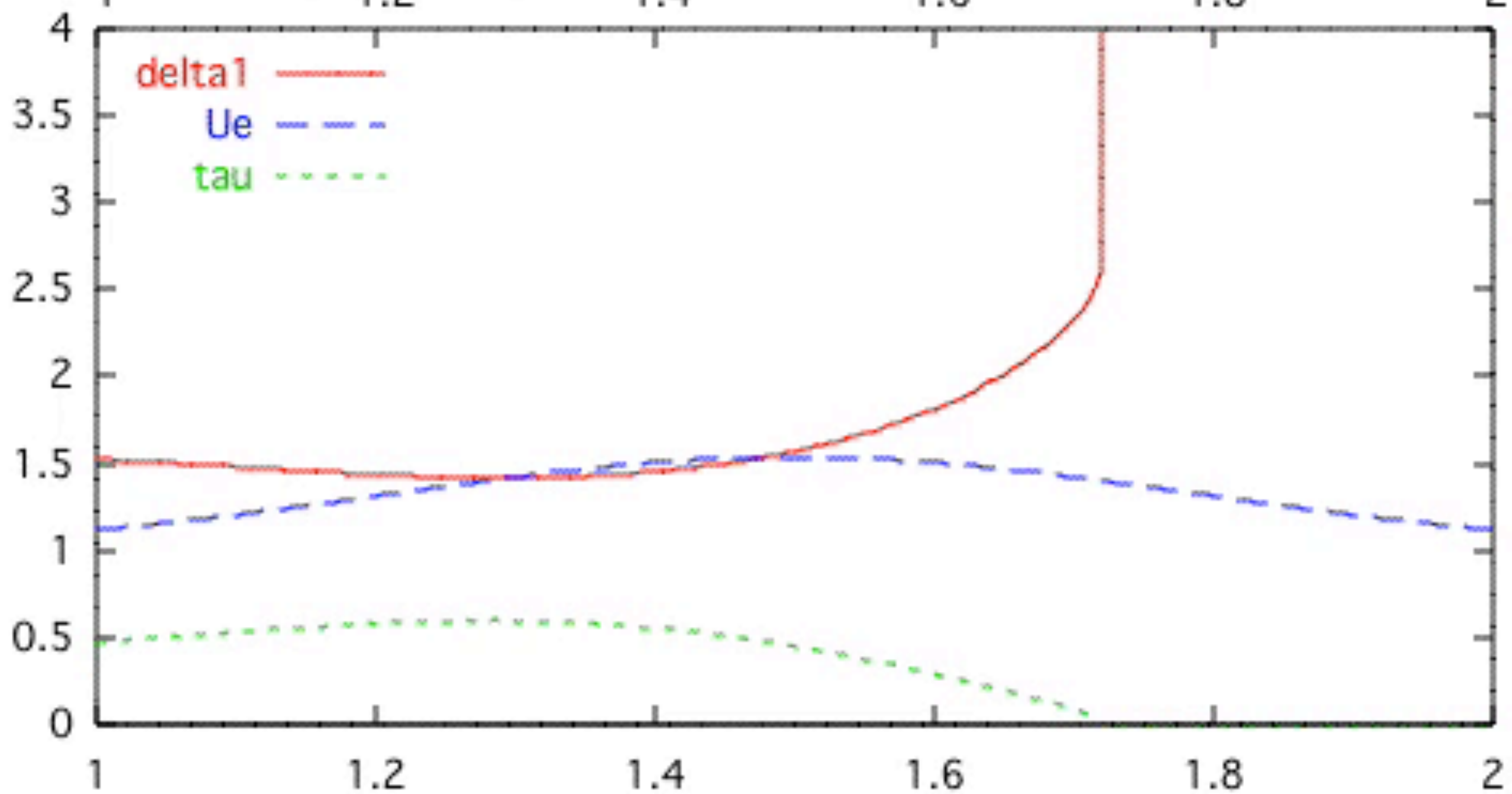
some problems: separation



prescribed $u_e(x)$



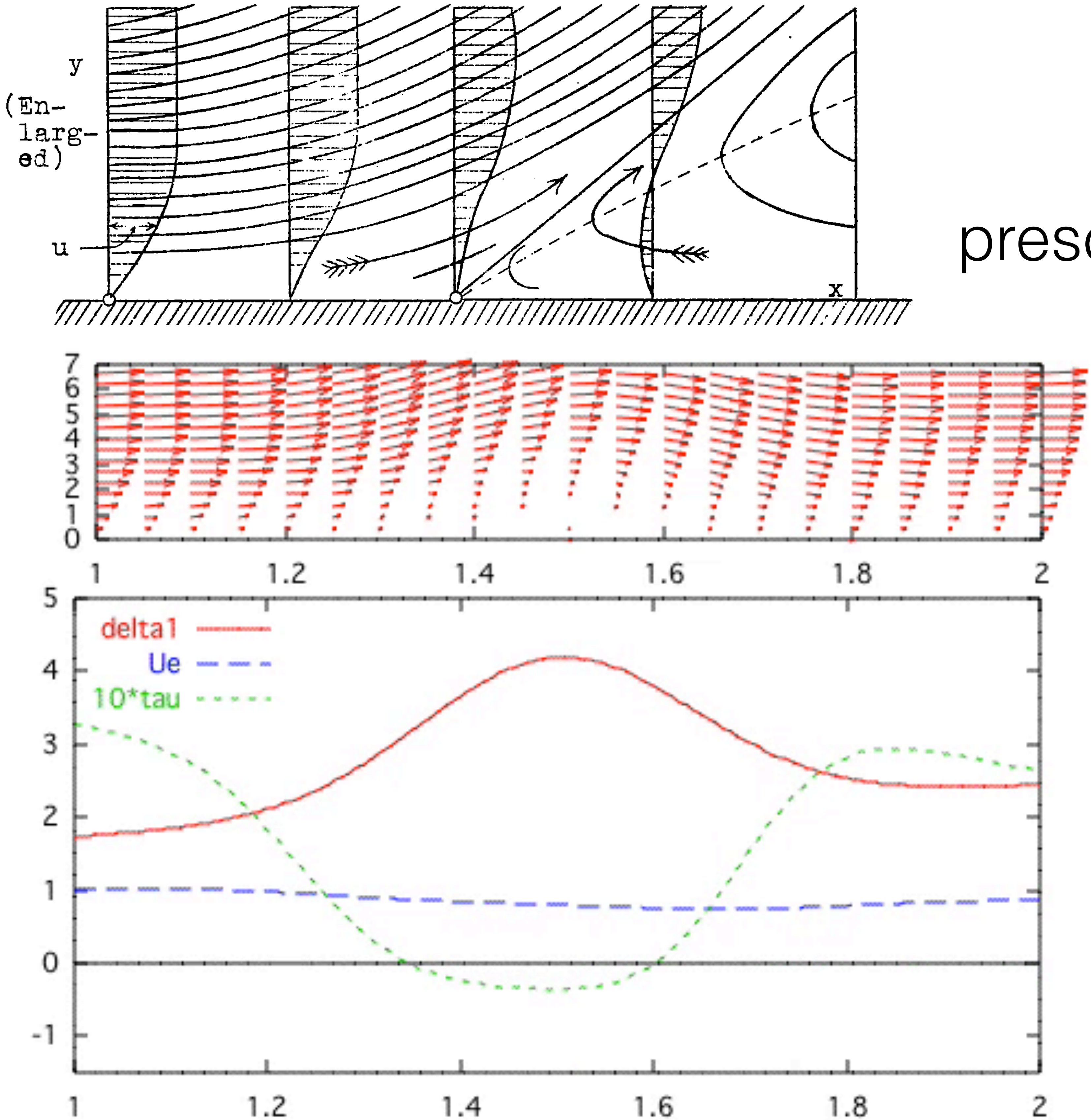
direct resolution



$$\frac{\partial u}{\partial y} \sim \sqrt{x_s - x}$$

singularity

some problems: separation



prescribed $\tilde{\delta}_1$

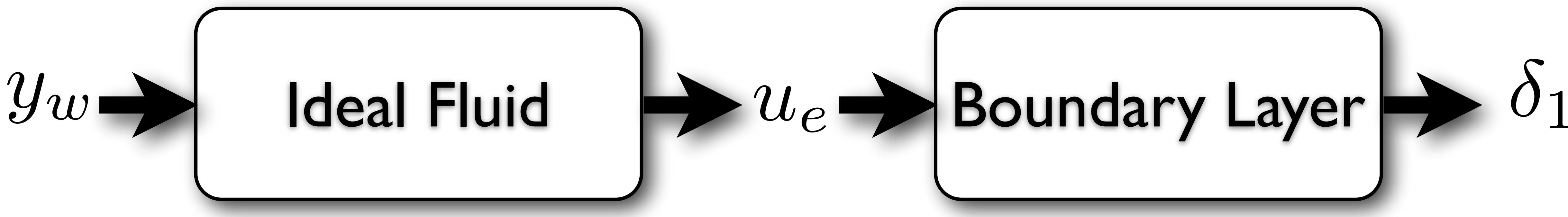
inverse resolution

no problem!

Catherall, Mangler, 1966

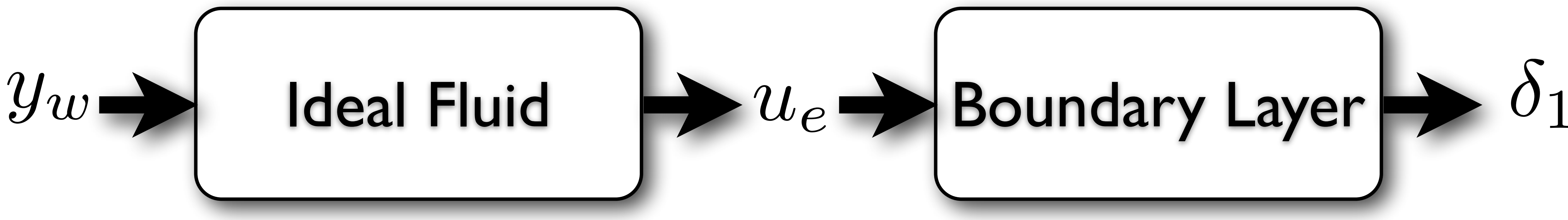
Reyhner, Flügge-Lotz 1968

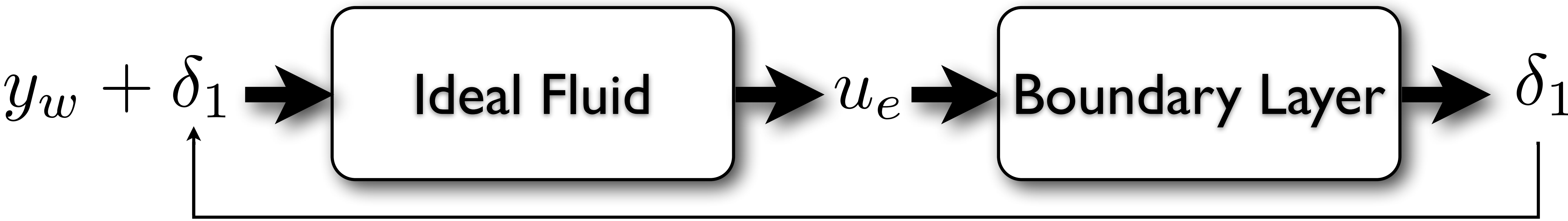
pressure- blowing velocity
characteristics/
panel methods/
finite diff...



Keller Box,
Finite differences....

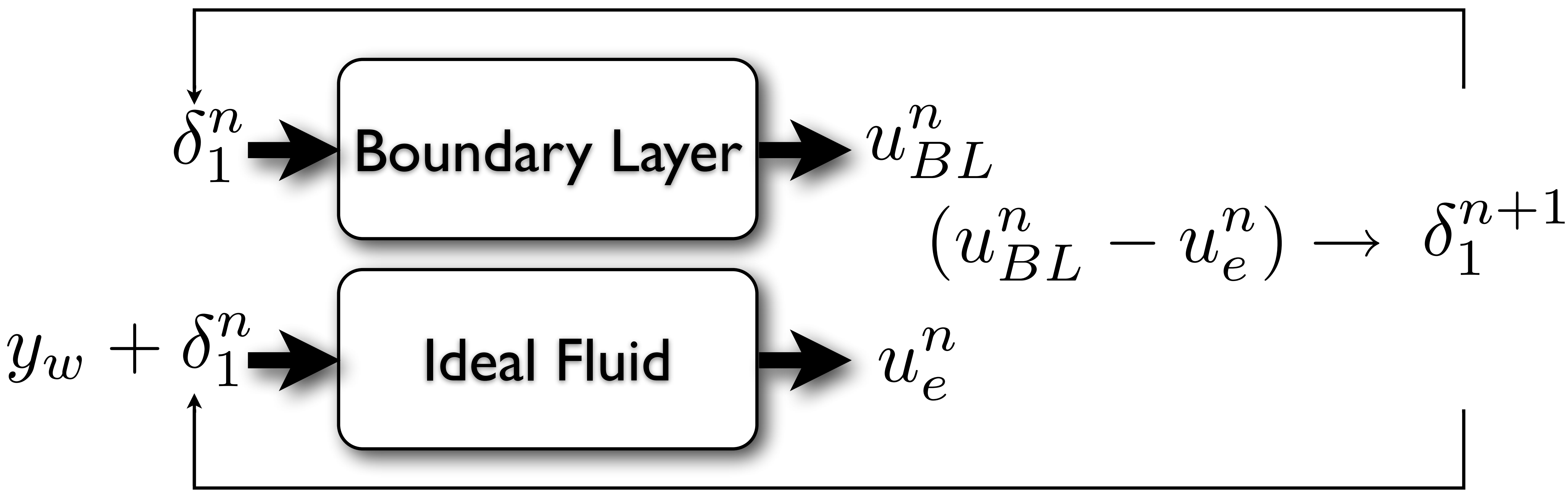
Finite elements


y_w
Ideal Fluid
u_e
Boundary Layer
δ_1



$y_w + \delta_1$
Ideal Fluid
u_e
Boundary Layer
δ_1

Semi Inverse Leballeur 78 coupling



Relaxation:

$$\delta^{n+1} = \delta^n + \lambda(u_{BL}^n - u_e^n)$$

an optimal parameter of relaxation can be evaluated

A word about the numerics in BL

Finite differences marching in x

$$u_{ij}\frac{u_{i+1j}-u_{ij}}{\Delta x}+...=-\frac{p_{i+1}-p_i}{\Delta x}+\frac{u_{i+1j+1}-2u_{i+1j}+u_{i+1j+1}}{\Delta y^2}$$

at each station find pressure by Newton
such that is the prescribed one

more robust variation : Keller Box

Finite differences marching in x

$$u_{ij}\frac{u_{i+1j}-u_{ij}}{\Delta x}+...=-\frac{p_{i+1}-p_i}{\Delta x}+\frac{u_{i+1j+1}-2u_{i+1j}+u_{i+1j+1}}{\Delta y^2}$$

at each station find pressure by Newton
such that is the prescribed one

more robust variation : Keller Box

problem: remove when $u_{ij}<0$ to ensure stability
Flare Approximation

Finite differences marching in x

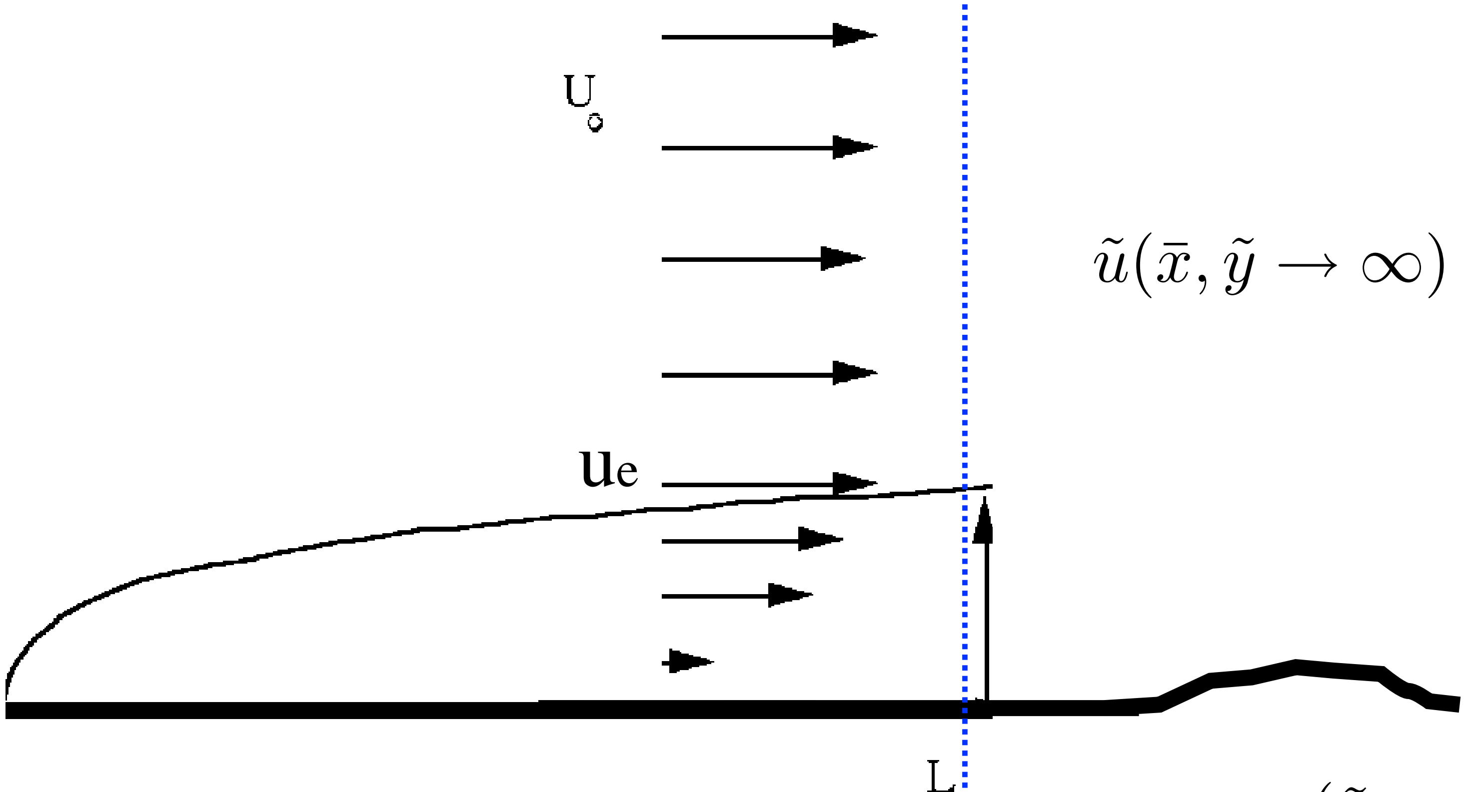
$$u_{ij}\frac{u_{i+1j}-u_{ij}}{\Delta x} + ... = -\frac{p_{i+1}-p_i}{\Delta x} + \frac{u_{i+1j+1}-2u_{i+1j}+u_{i+1j+1}}{\Delta y^2}$$

at each station find pressure by Newton
such that is the prescribed one

more robust variation : Keller Box

problem: remove when $u_{ij} < 0$ to ensure stability
Flare Approximation

far more robust Finite Elements not implemented yet
(see the end Double Deck)



$$\bar{u}_e = 1 + \frac{1}{\pi} \int \frac{\bar{f}'(\bar{x}) + Re^{-1/2} \frac{d(\tilde{\delta}_1 \bar{u}_e)}{d\bar{x}}}{x - \xi} d\xi$$

$$\tilde{u}(\bar{x}, \tilde{y} \to \infty) \to \bar{u}_e(\bar{x}) \qquad \bar{v}_e = Re^{-1/2} \frac{d(\tilde{\delta}_1 \bar{u}_e)}{d\bar{x}}$$

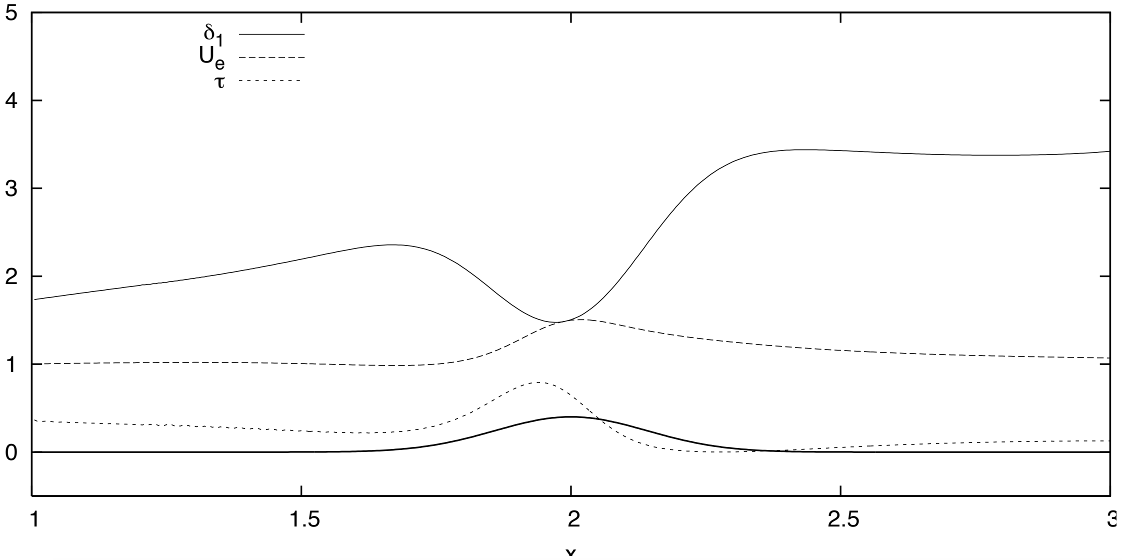
$$\frac{\partial \tilde{u}}{\partial \bar{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0,$$

$$\tilde{u} \frac{\partial \tilde{u}}{\partial \bar{x}} + \tilde{v} \frac{\partial \tilde{u}}{\partial \tilde{y}} = \bar{u}_e \frac{d\bar{u}_e}{d\bar{x}} + \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2},$$

$(\tilde{u} = \tilde{v} = 0$ on the body $\bar{f}(\bar{x}))$.

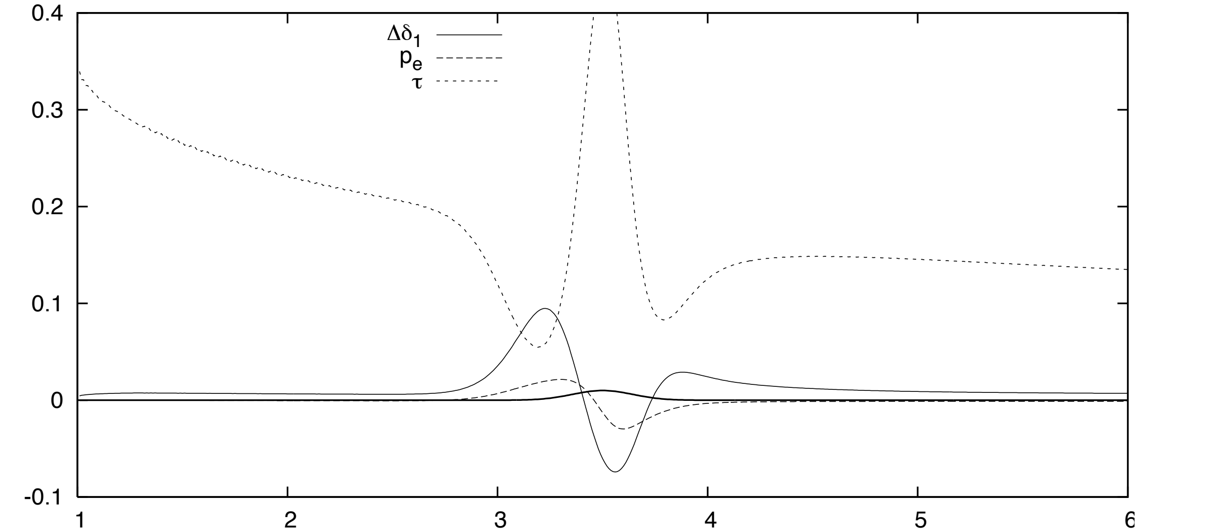
inviscid viscous interaction

Numerical Non Linear Examples



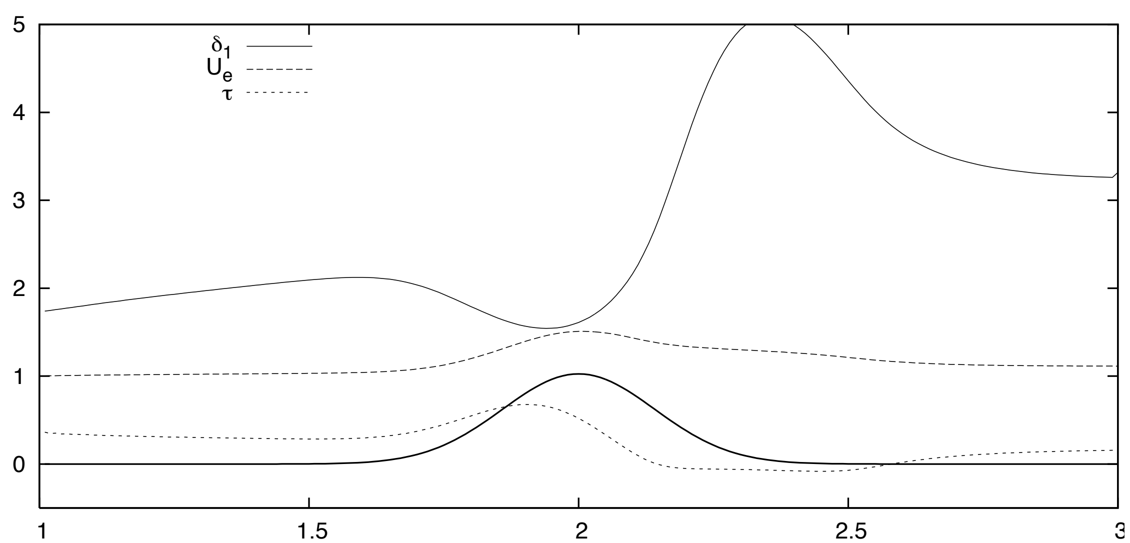
subsonic $\bar{u}_e = 1 + \frac{1}{\pi}\int \frac{\bar{f}'(\bar{x}) + Re^{-1/2}\frac{d(\tilde{\delta}_1 \bar{u}_e)}{d\bar{x}}}{x - \xi} d\xi$

small upstream infuence



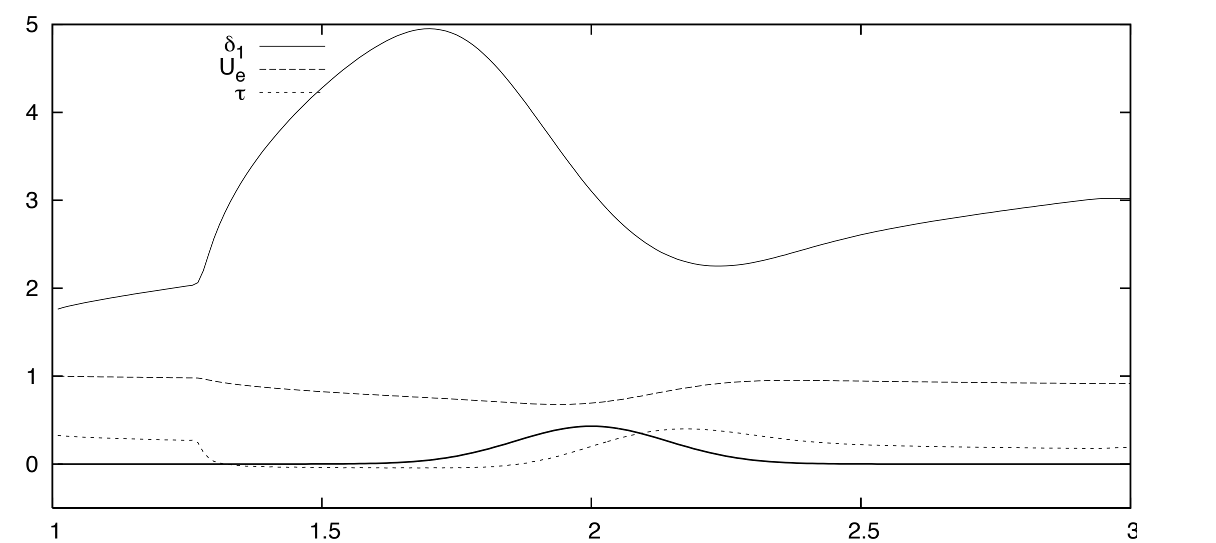
supersonic $\bar{u}_e = 1 - \frac{M^2}{\sqrt{M^2 - 1}}[\frac{d}{d\bar{x}}\bar{f}(\bar{x}) + Re^{-1/2}\frac{d(\tilde{\delta}_1 \bar{u}_e)}{d\bar{x}}].$

upstream infuence



subcritical $\bar{u}_e = 1 + [\bar{f}(\bar{x}) + \tilde{\delta}_1 Re^{-1/2}]$

no upstream infuence

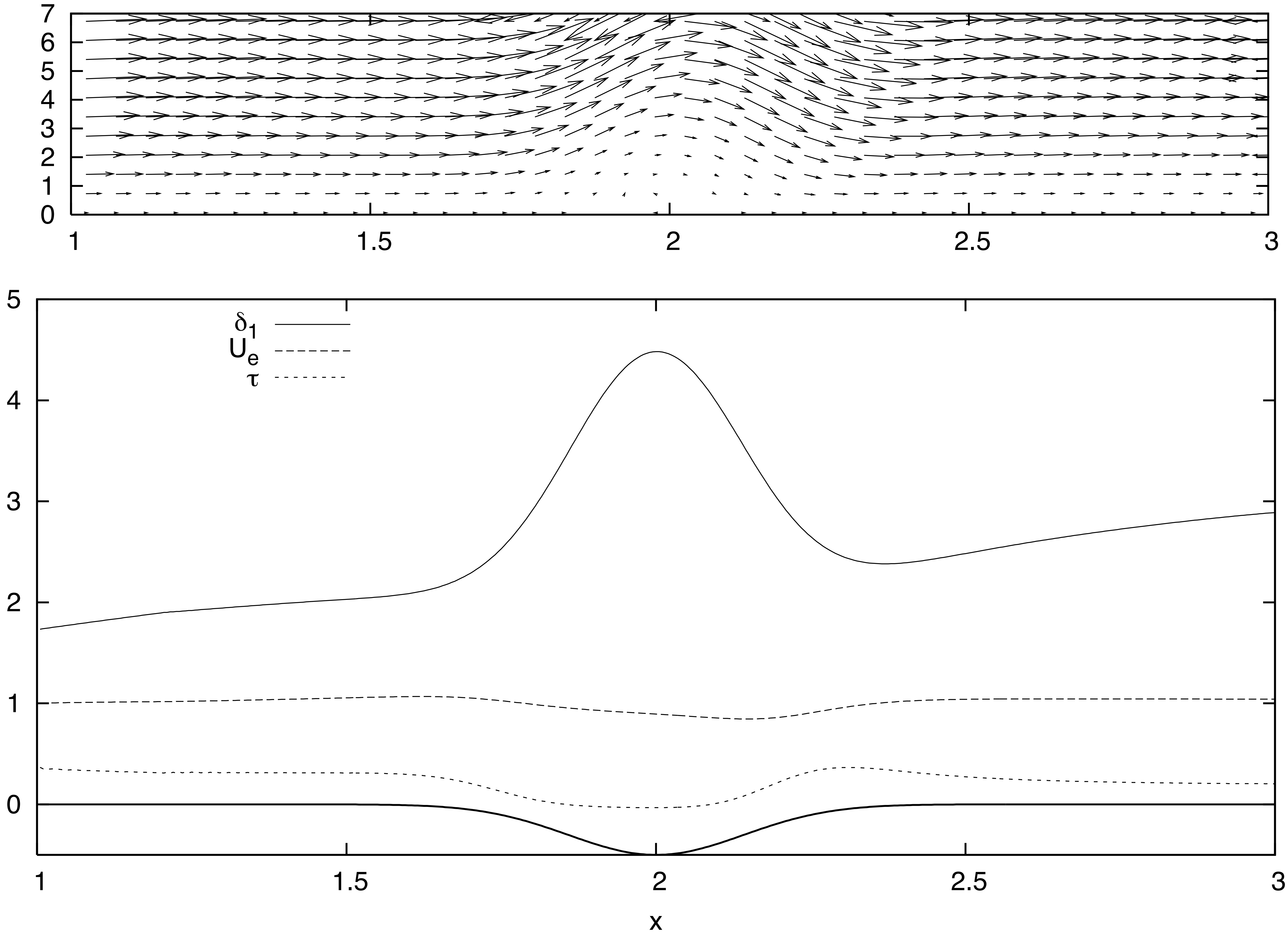


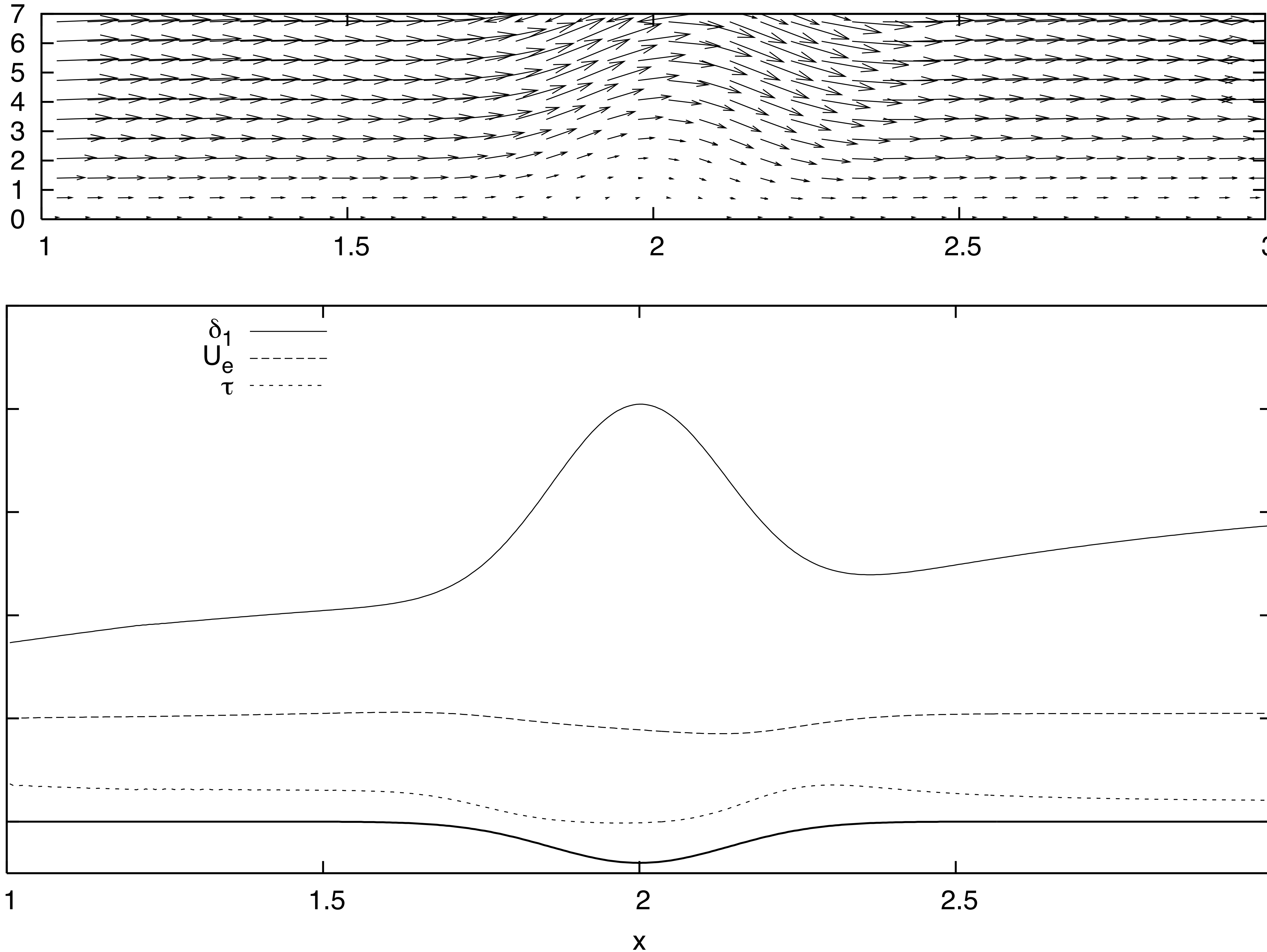
supercritical $\bar{u}_e = 1 + \frac{1}{1 - F}[\bar{f}(\bar{x}) + \tilde{\delta}_1 Re^{-1/2}]$

upstream infuence

next

subsonic





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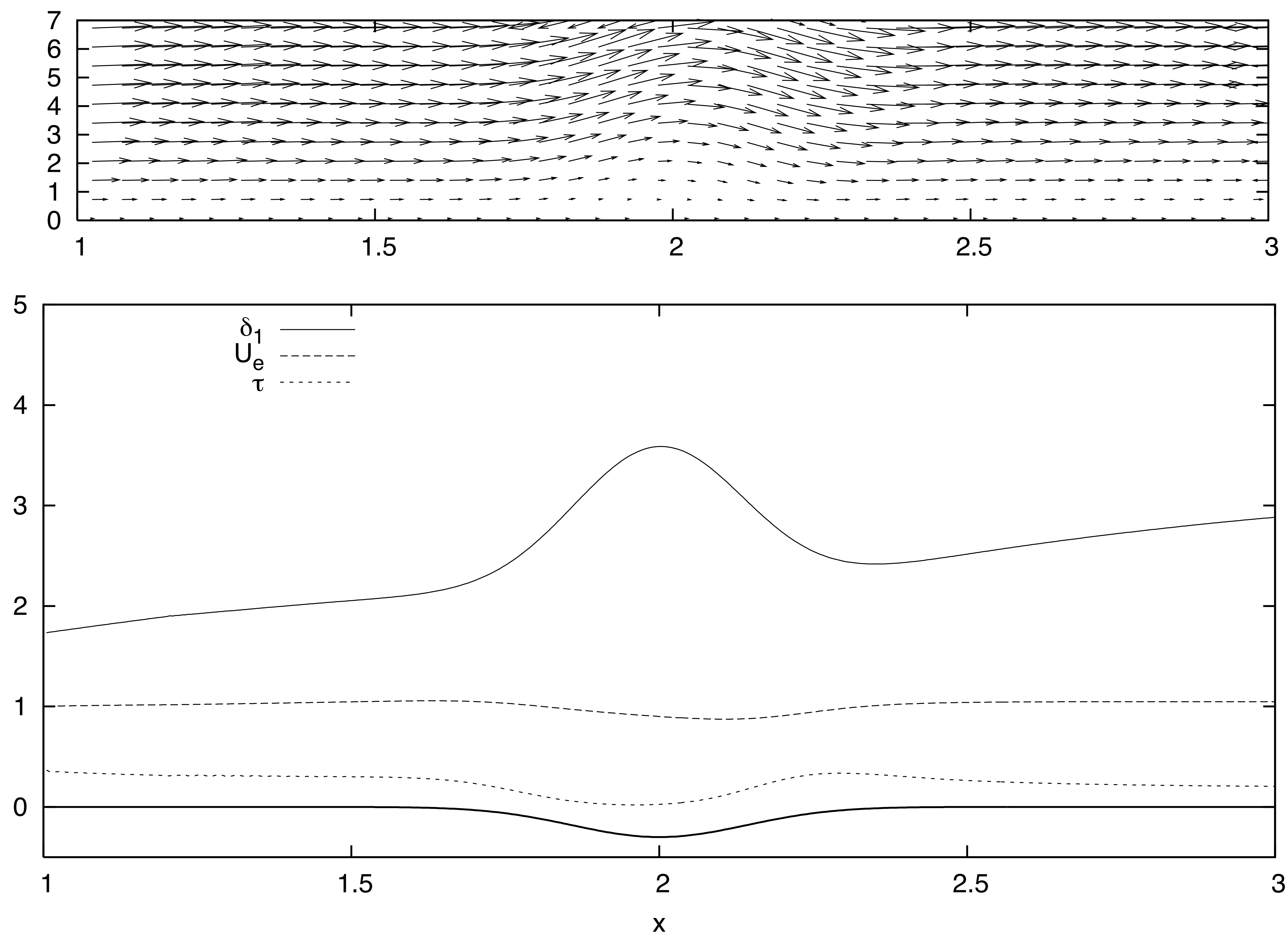

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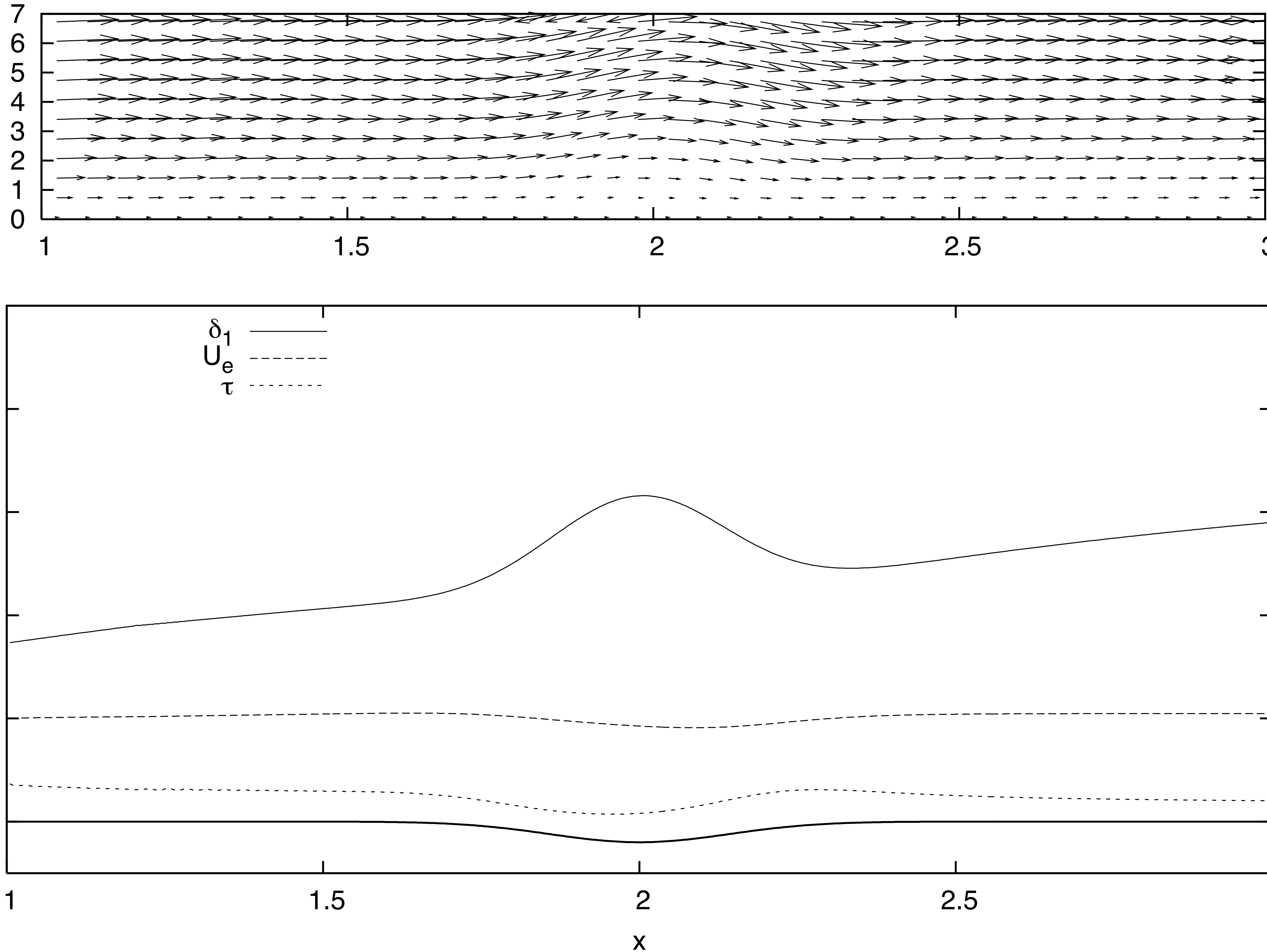

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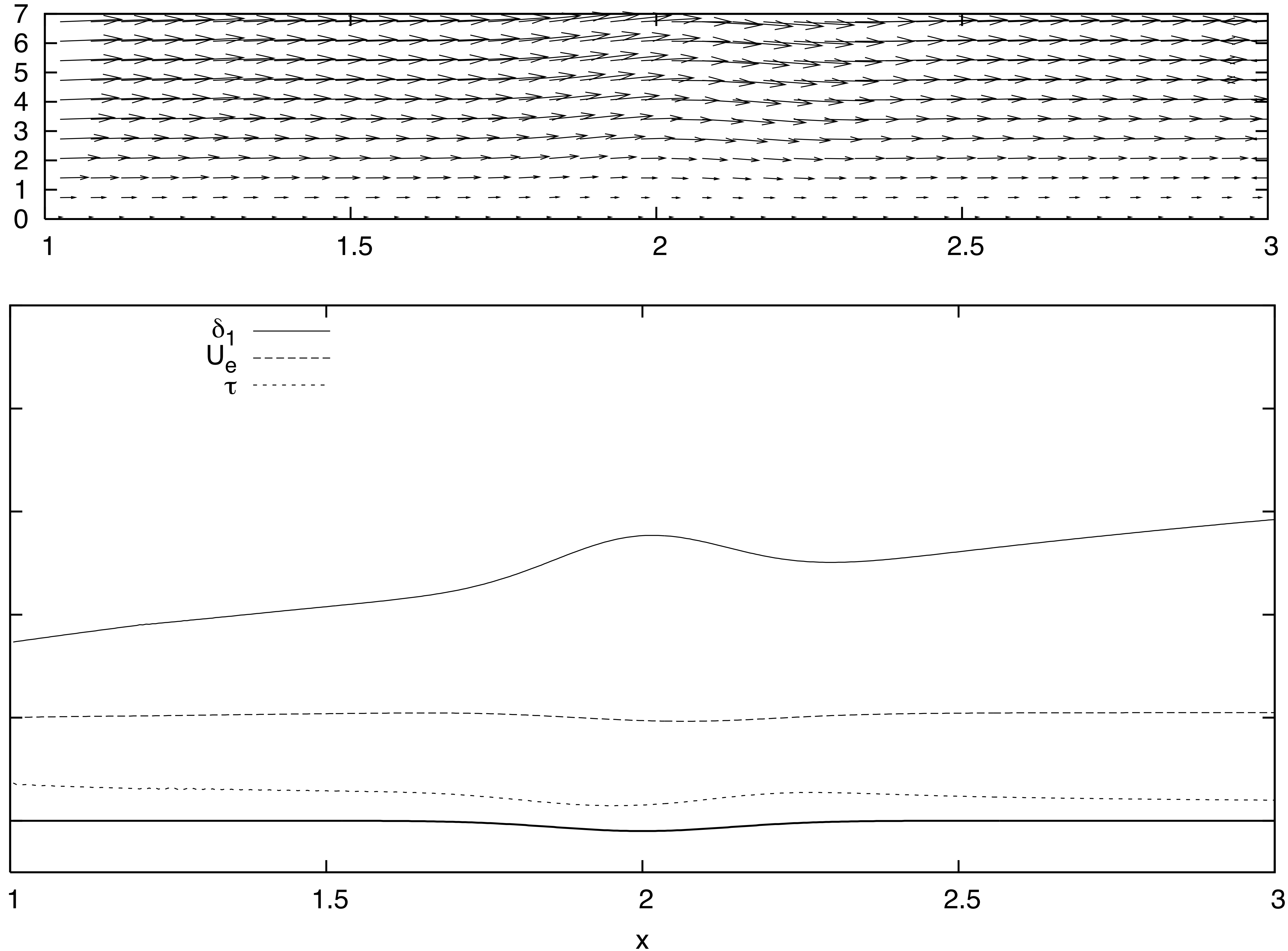
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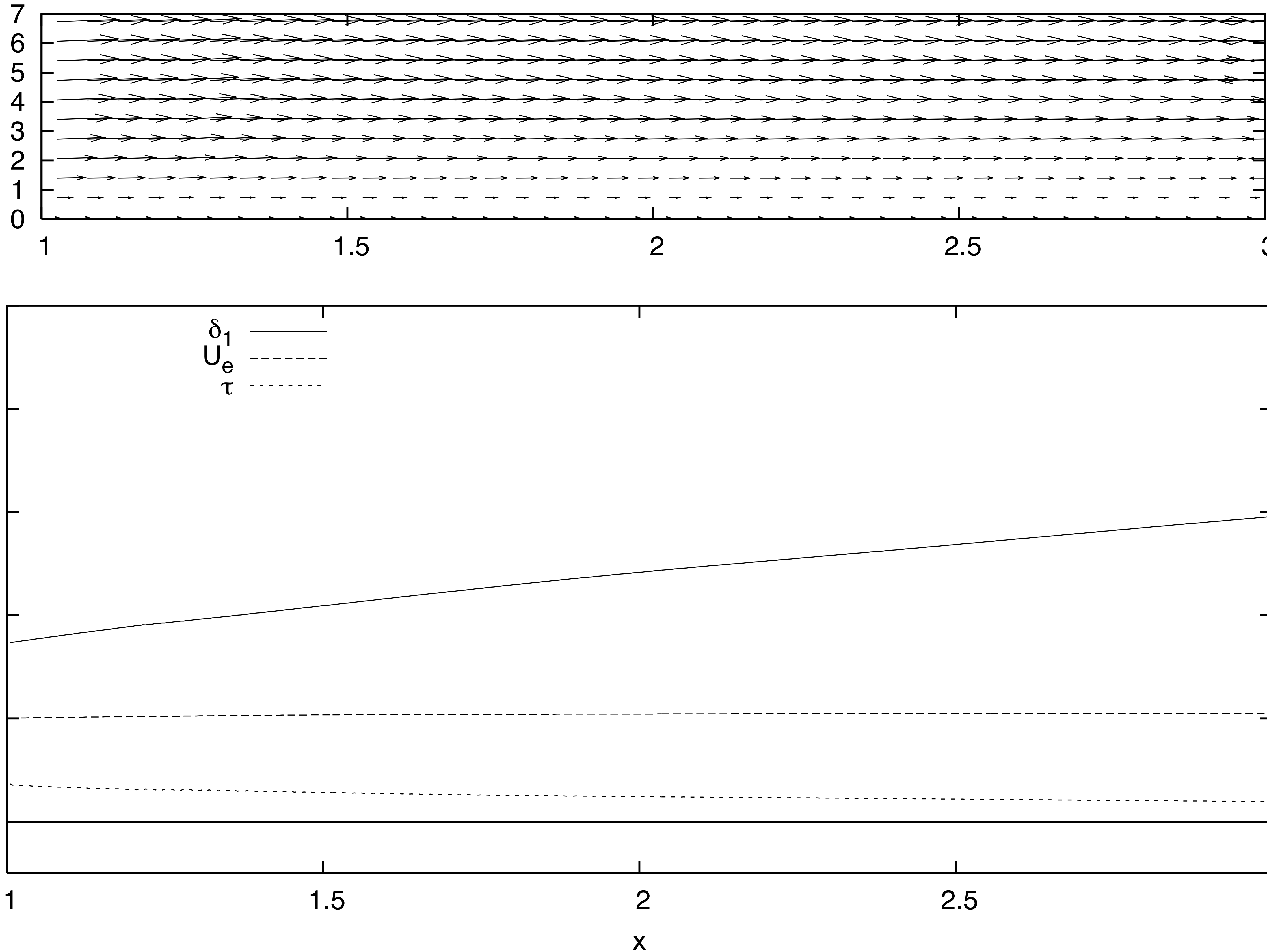

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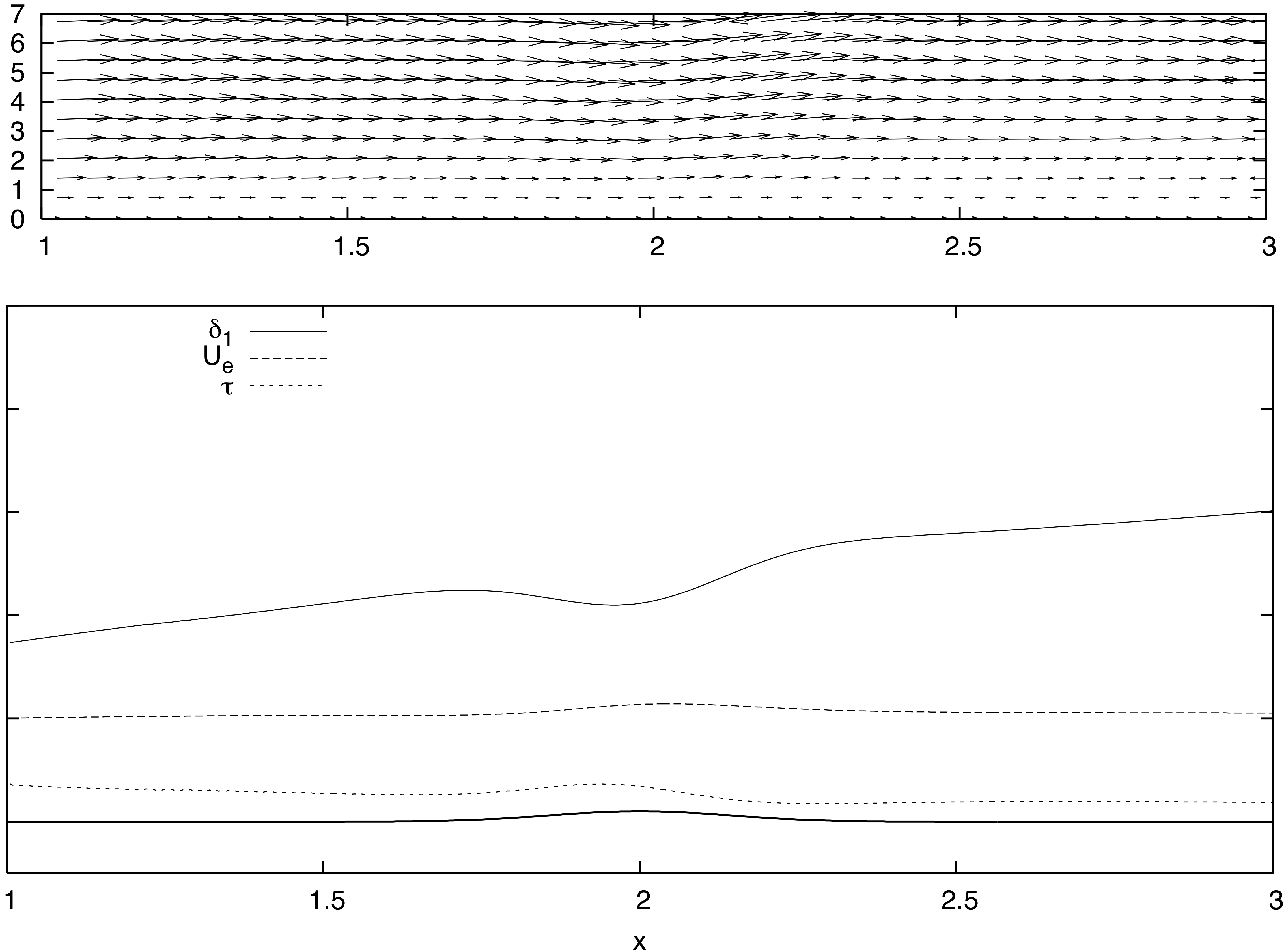

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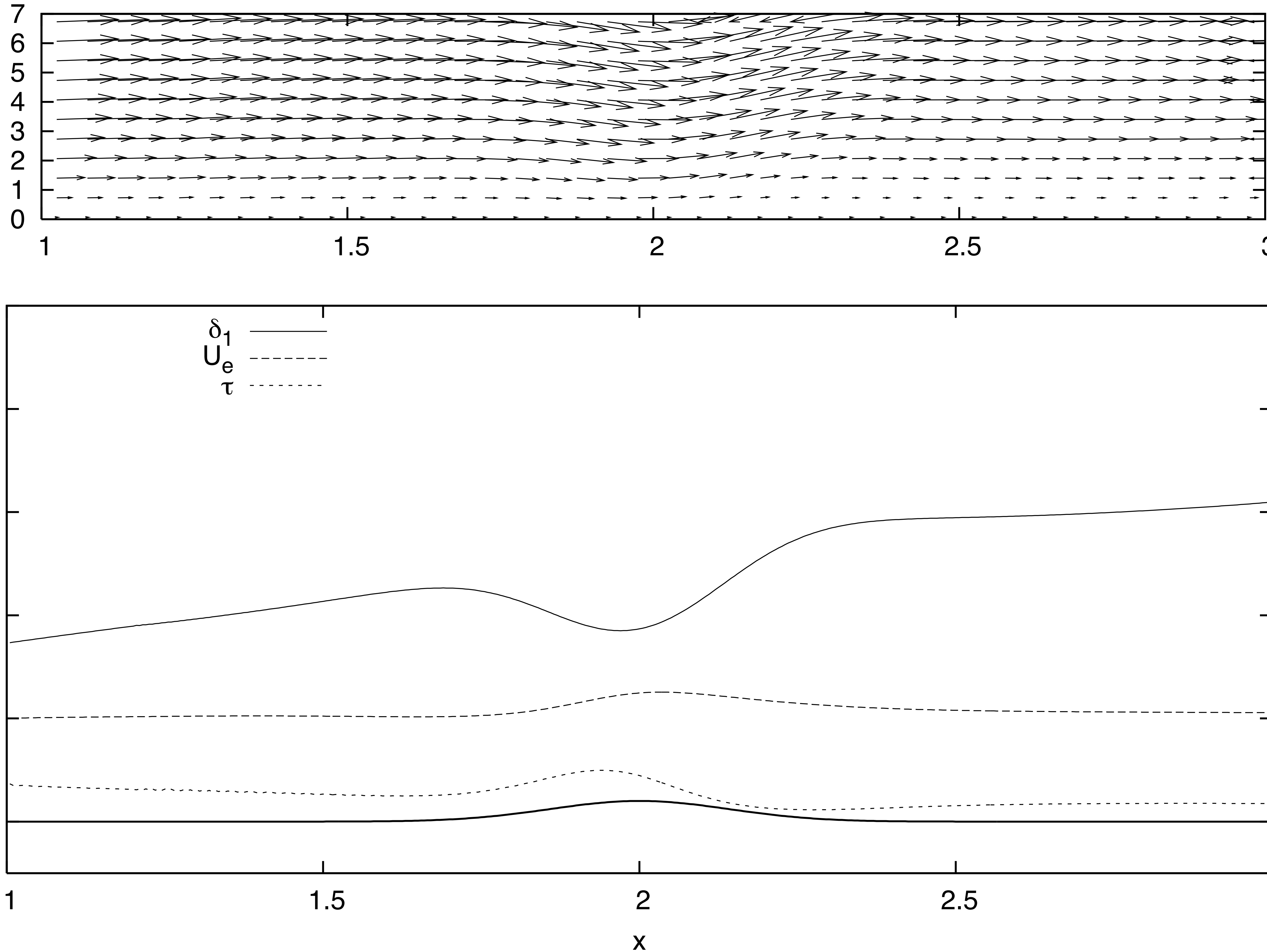

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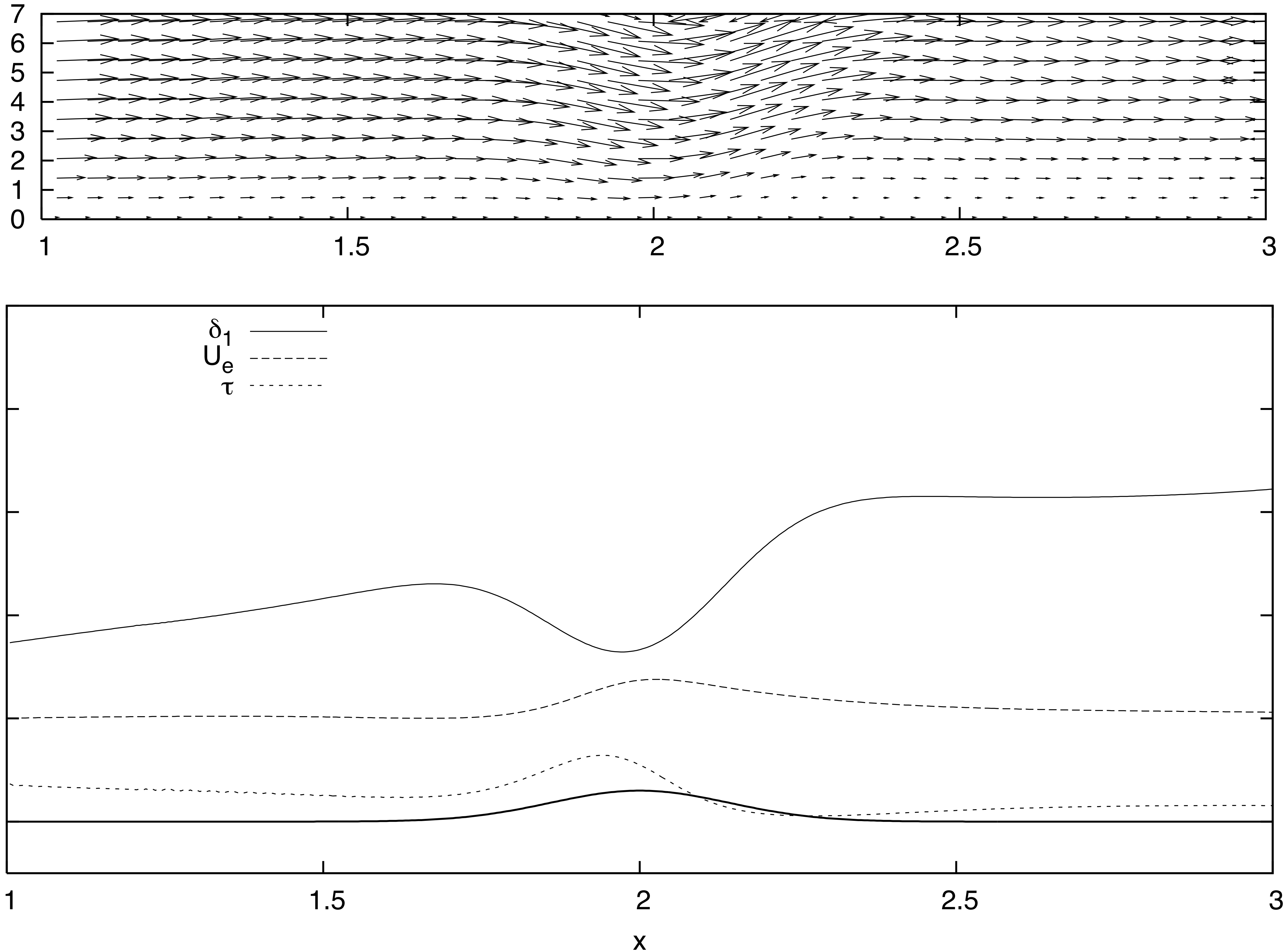

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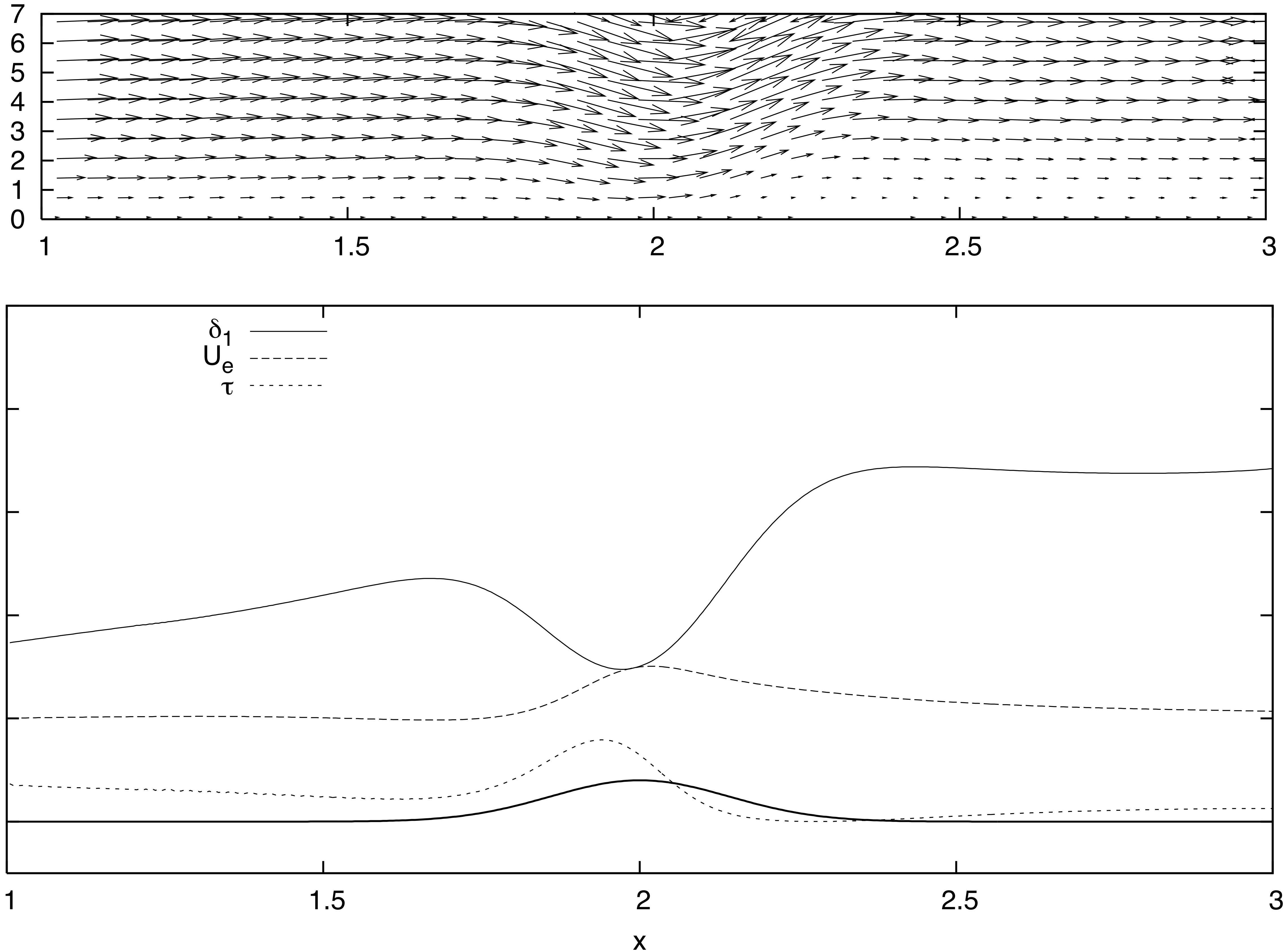

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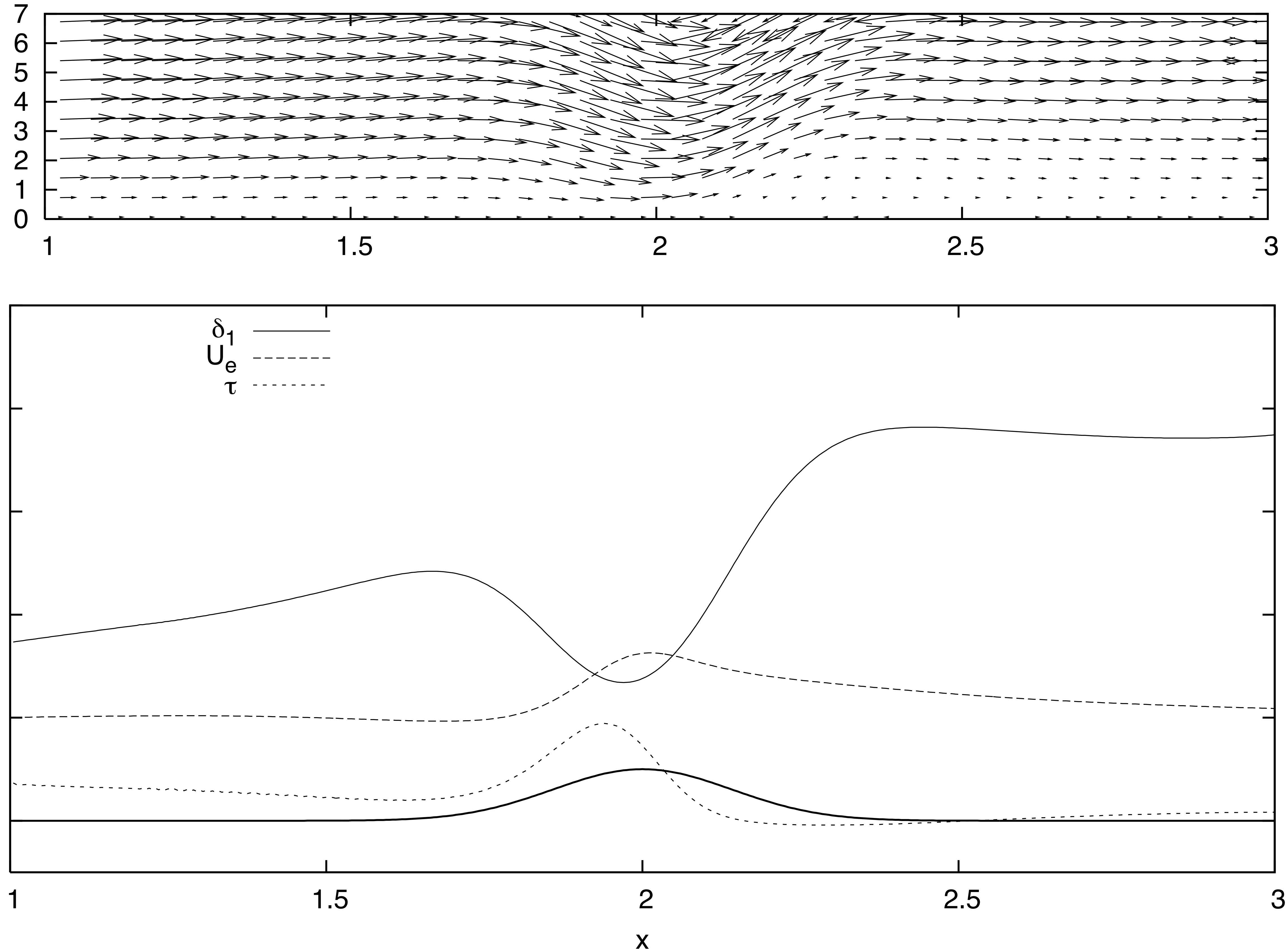
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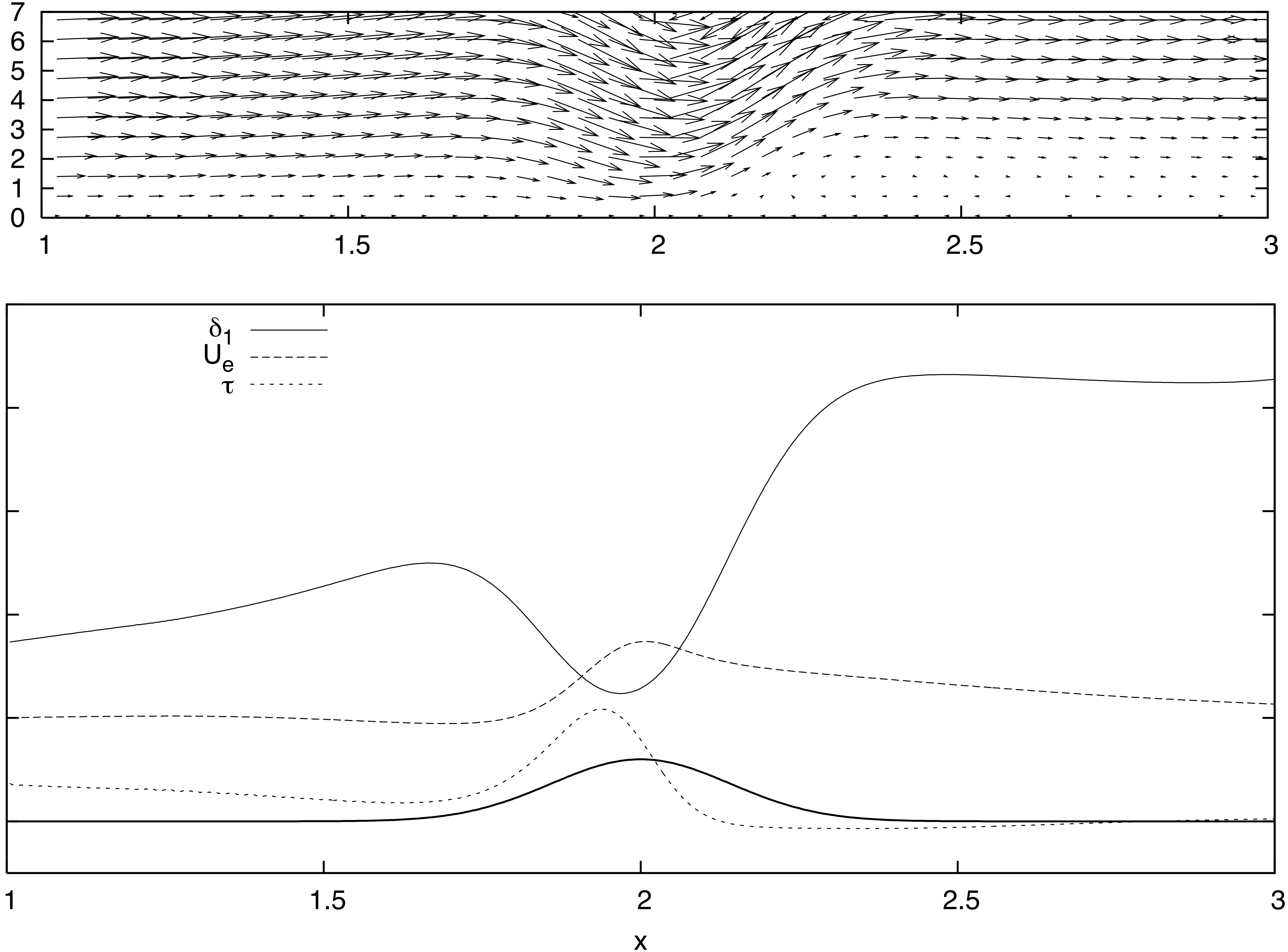
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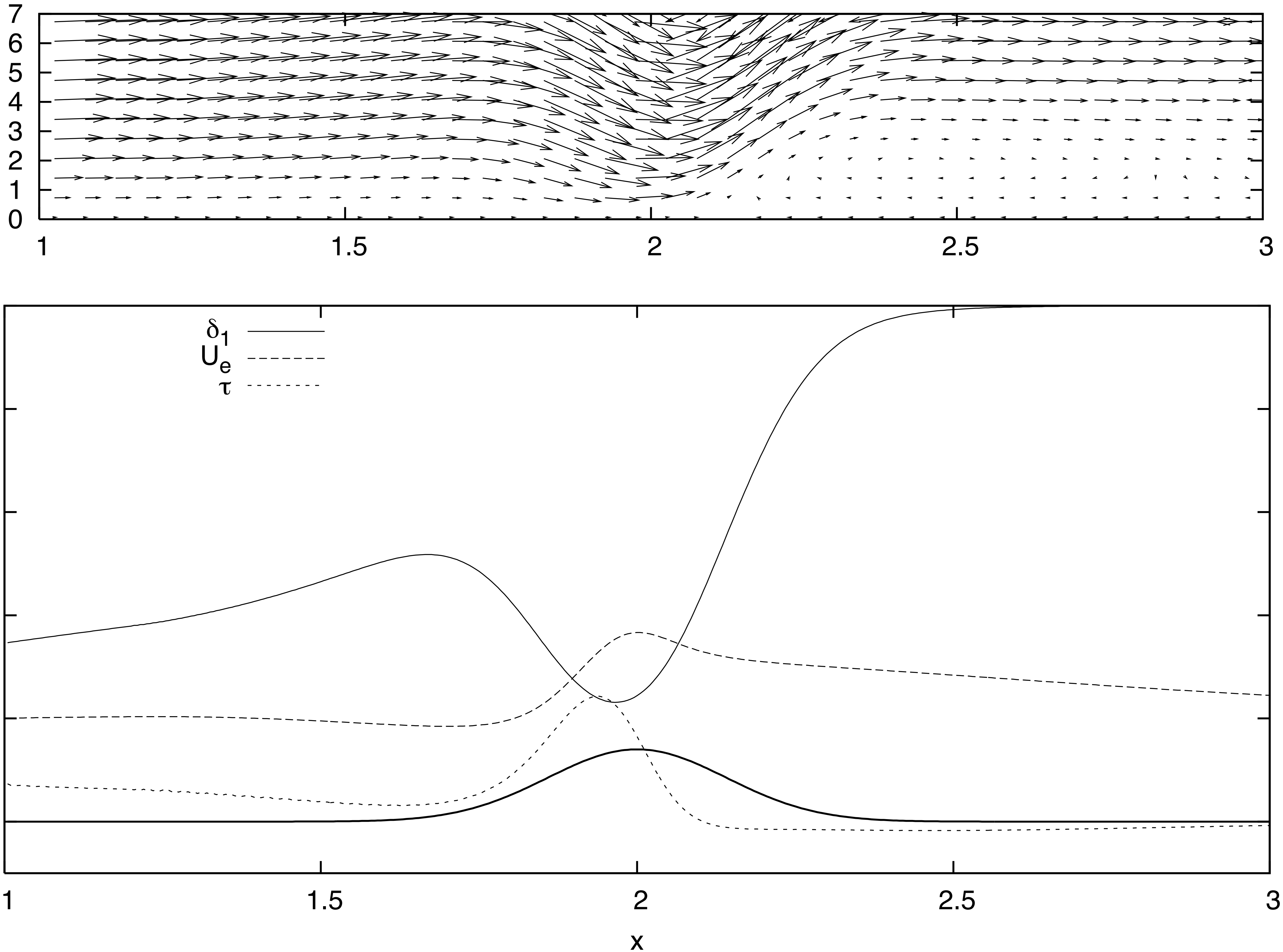

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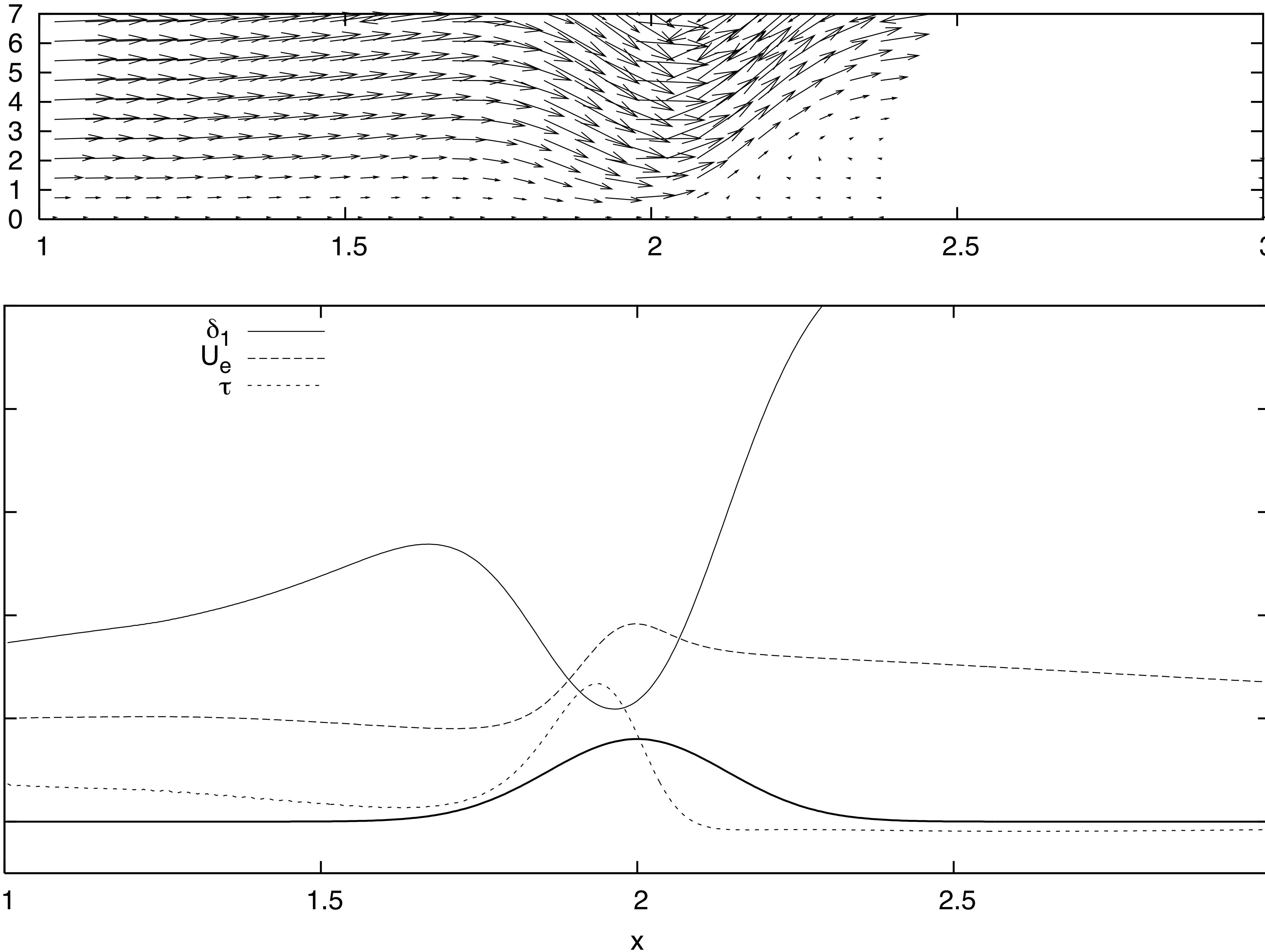

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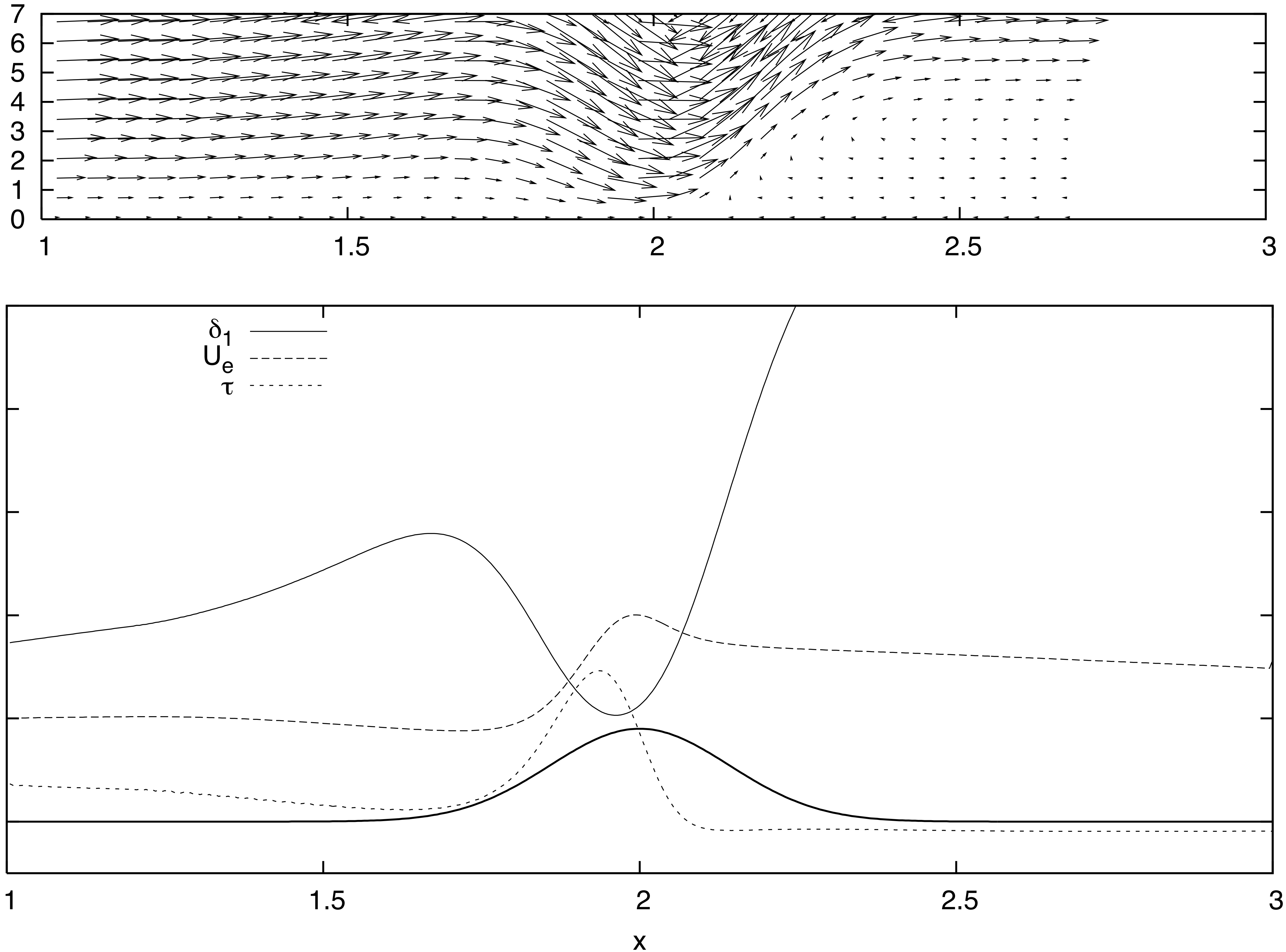

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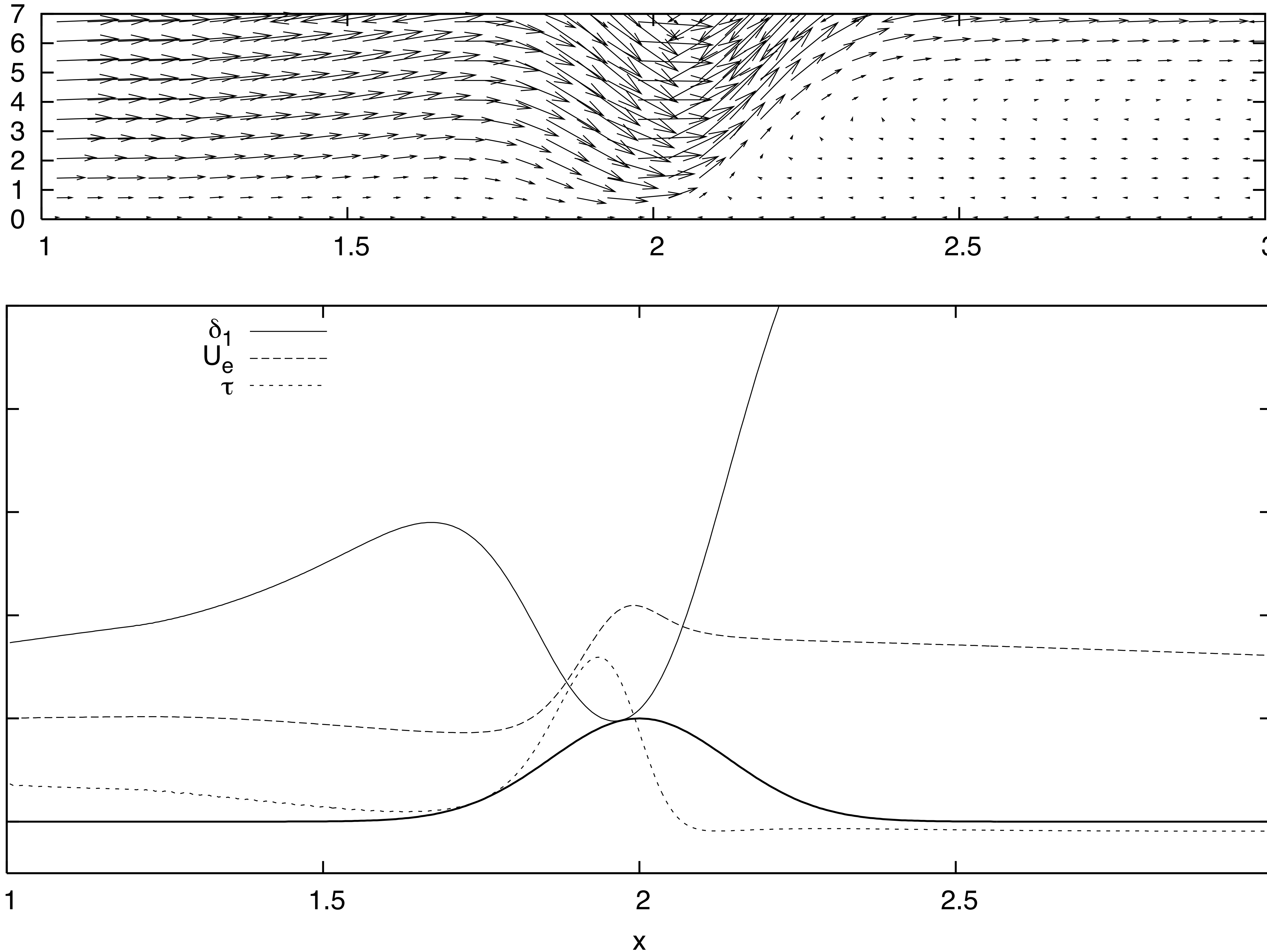

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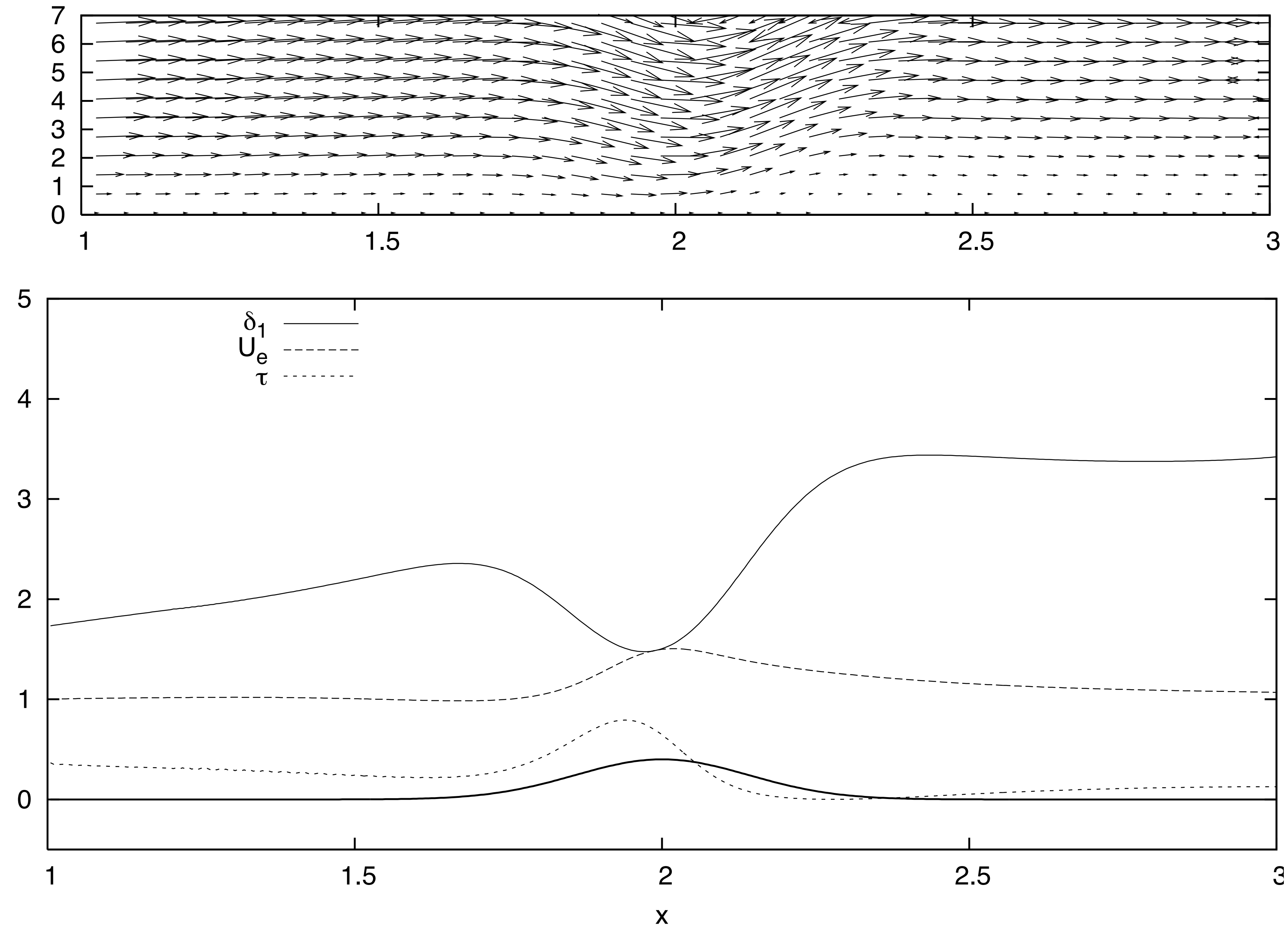
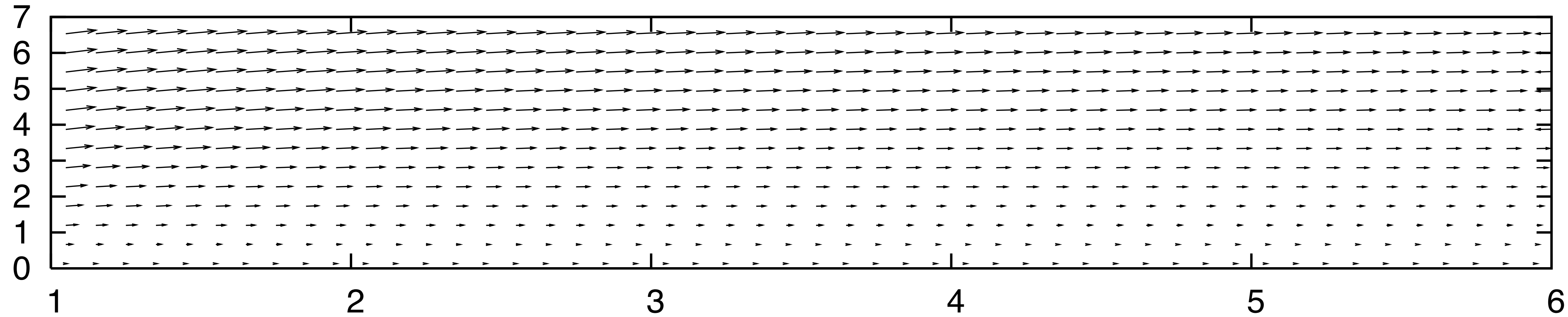
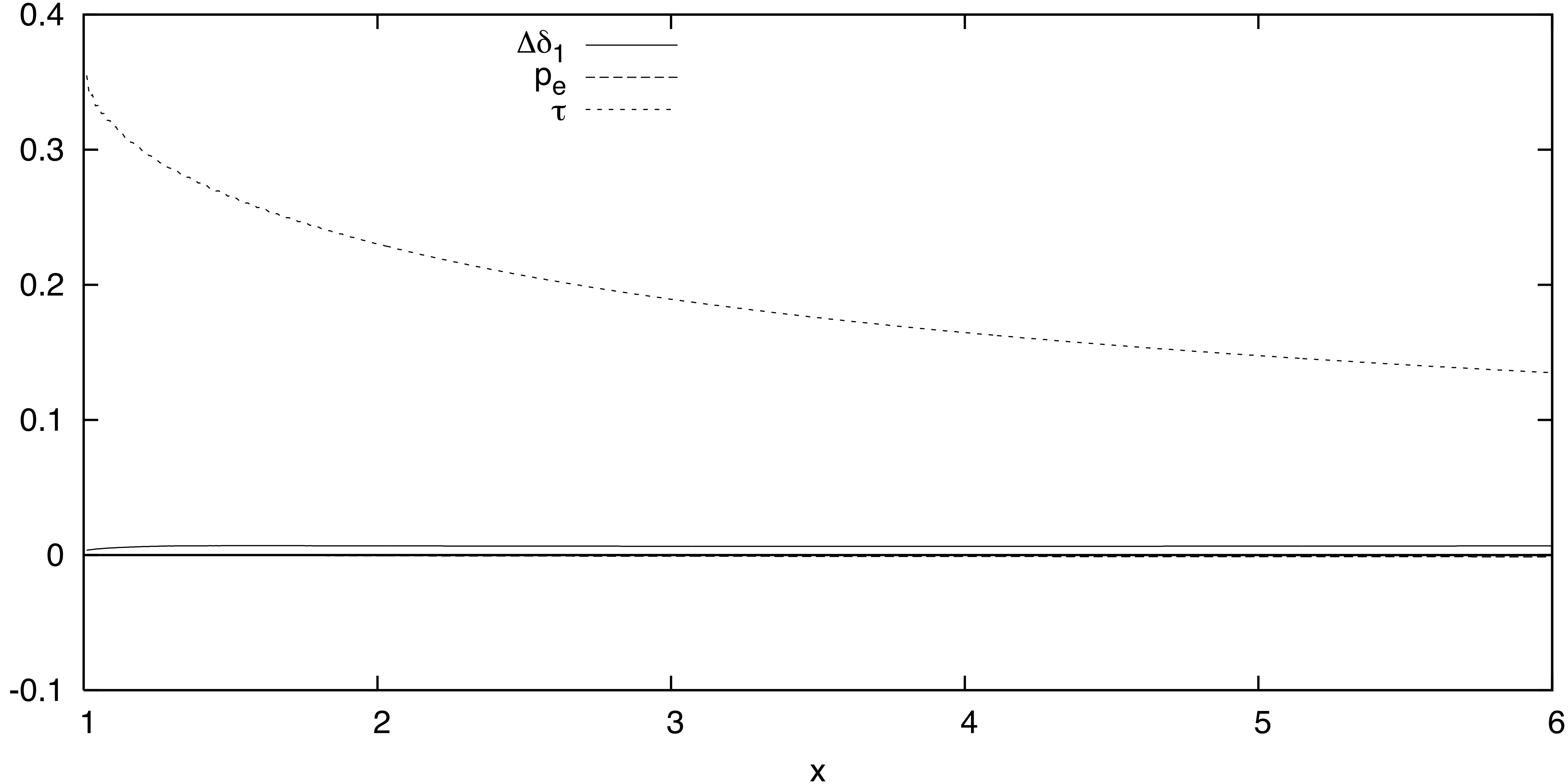


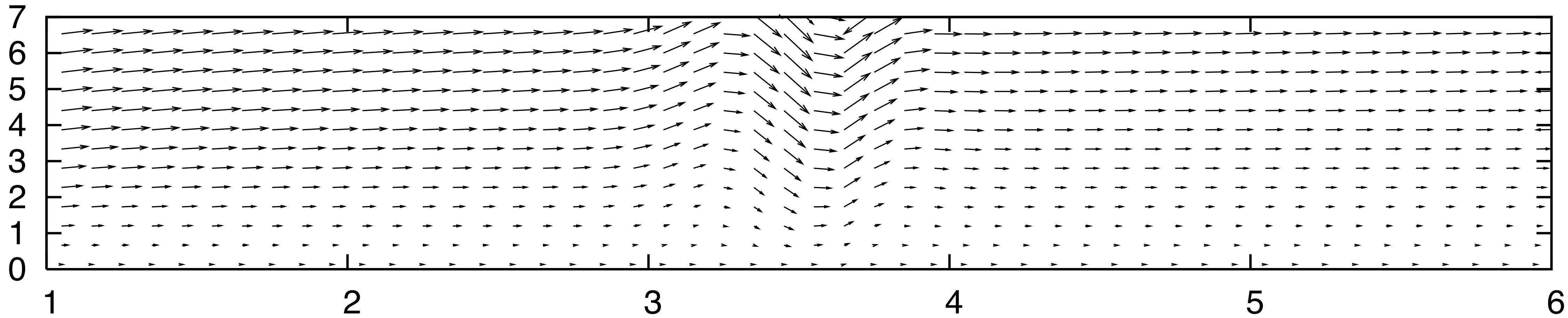
Figure 16: Incompressible flow [click to launch the movie, Adobe Reader required]. Top the velocity field $\tilde{u}, \tilde{v}$ (Prandtl transform), bottom the wall, here a bump, the displacement thickness $\tilde{\delta}_1$ (starting from Blasius value 1.7 in $\bar{x} = 1$), the skin friction (starting from Blasius value 0.3 in $\bar{x} = 1$) and the outer velocity starting from Ideal Fluid value 1 in $\bar{x} = 1$. A positive disturbance of the wall increases the velocity and decreases the displacement. Separation may occur after the bump, or before the tough.

back

supersonic







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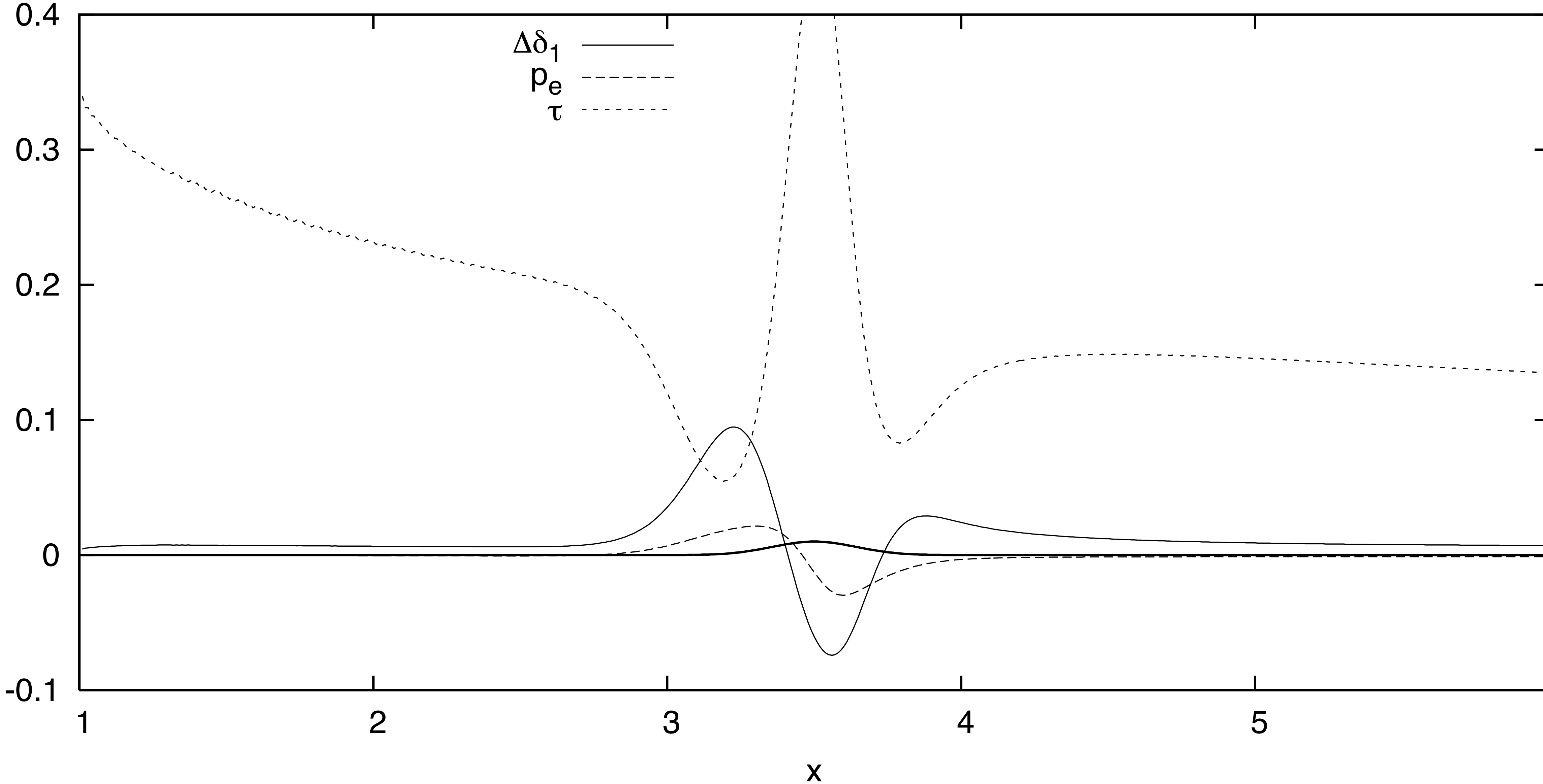

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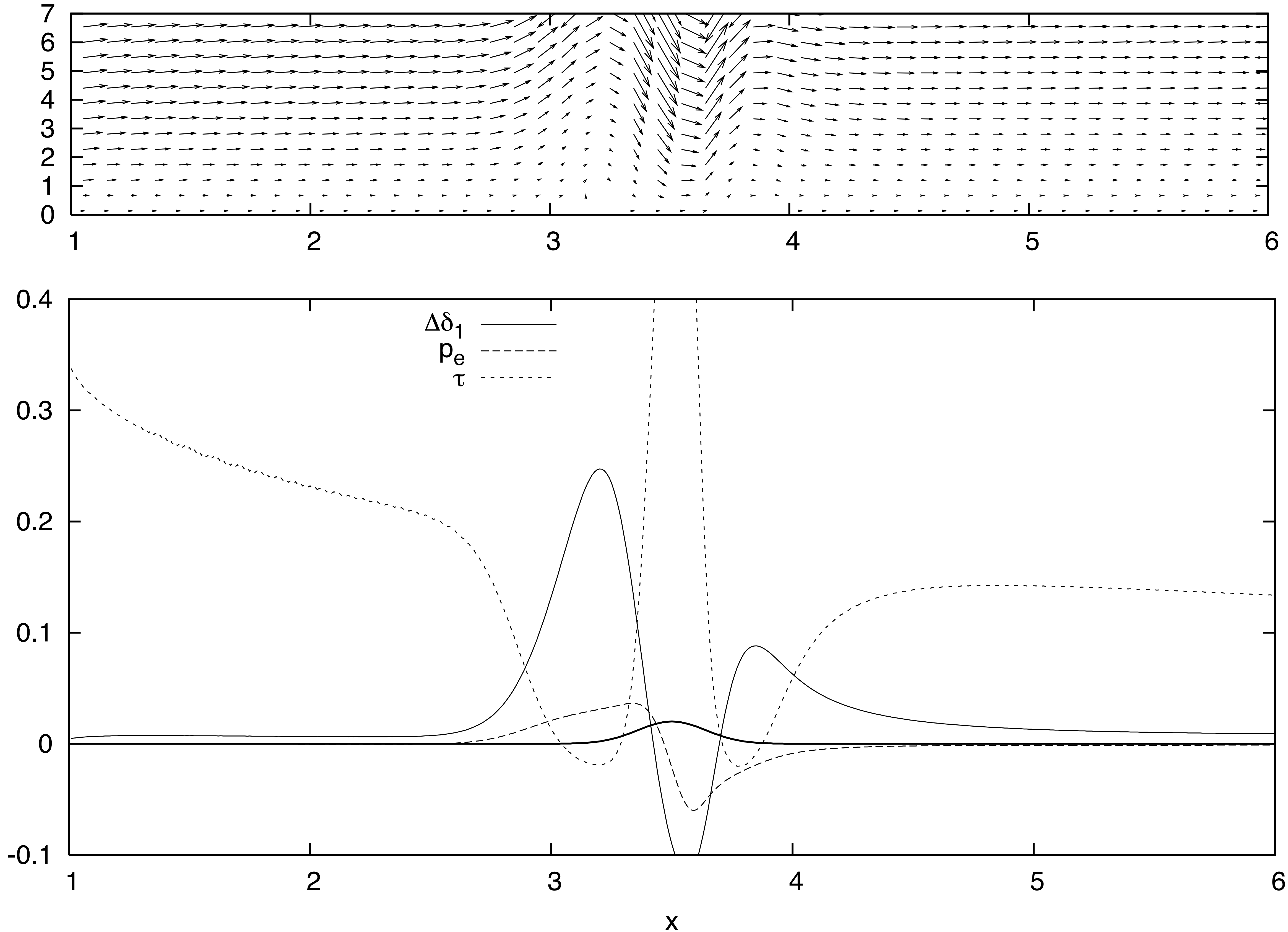

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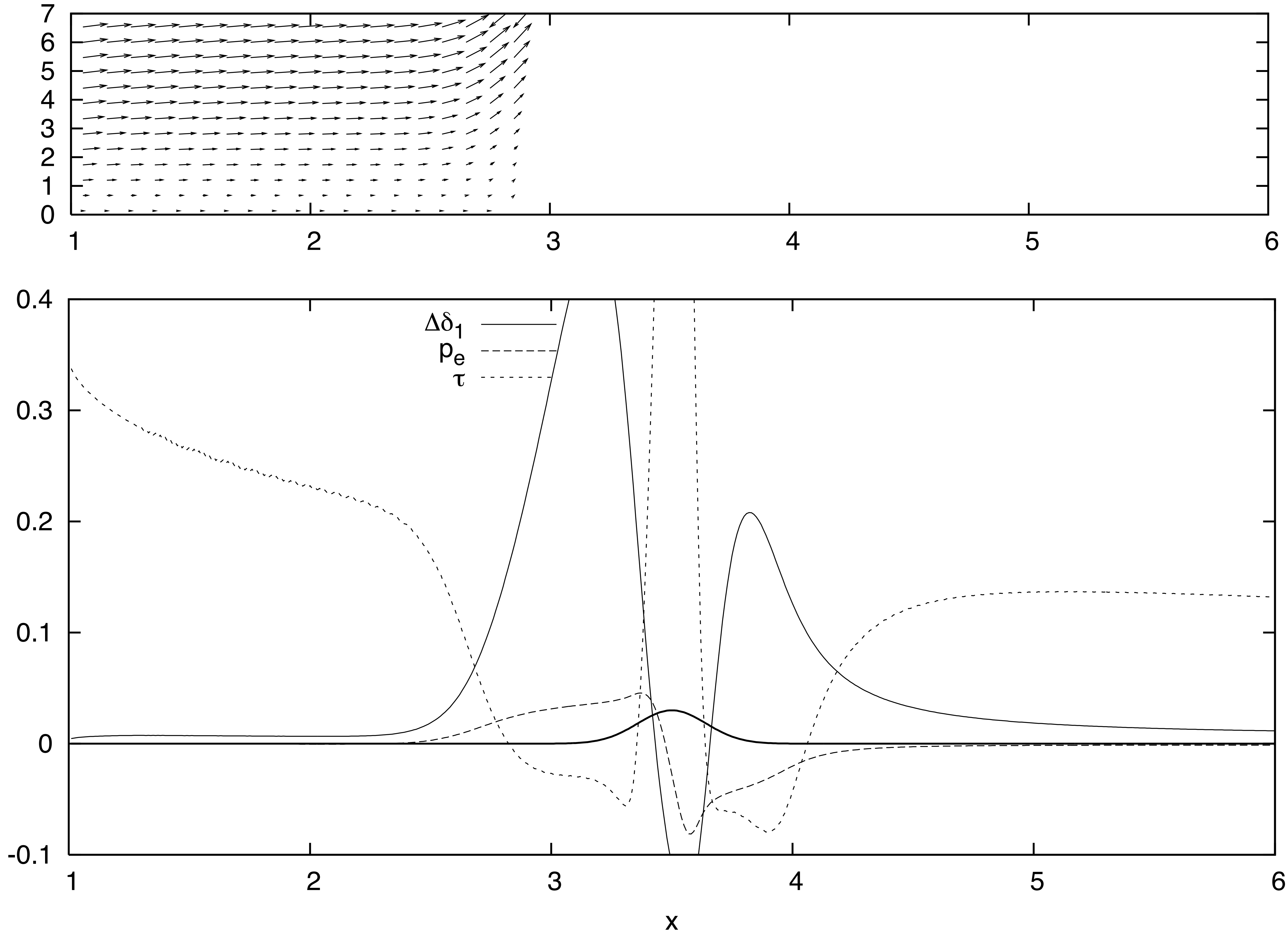

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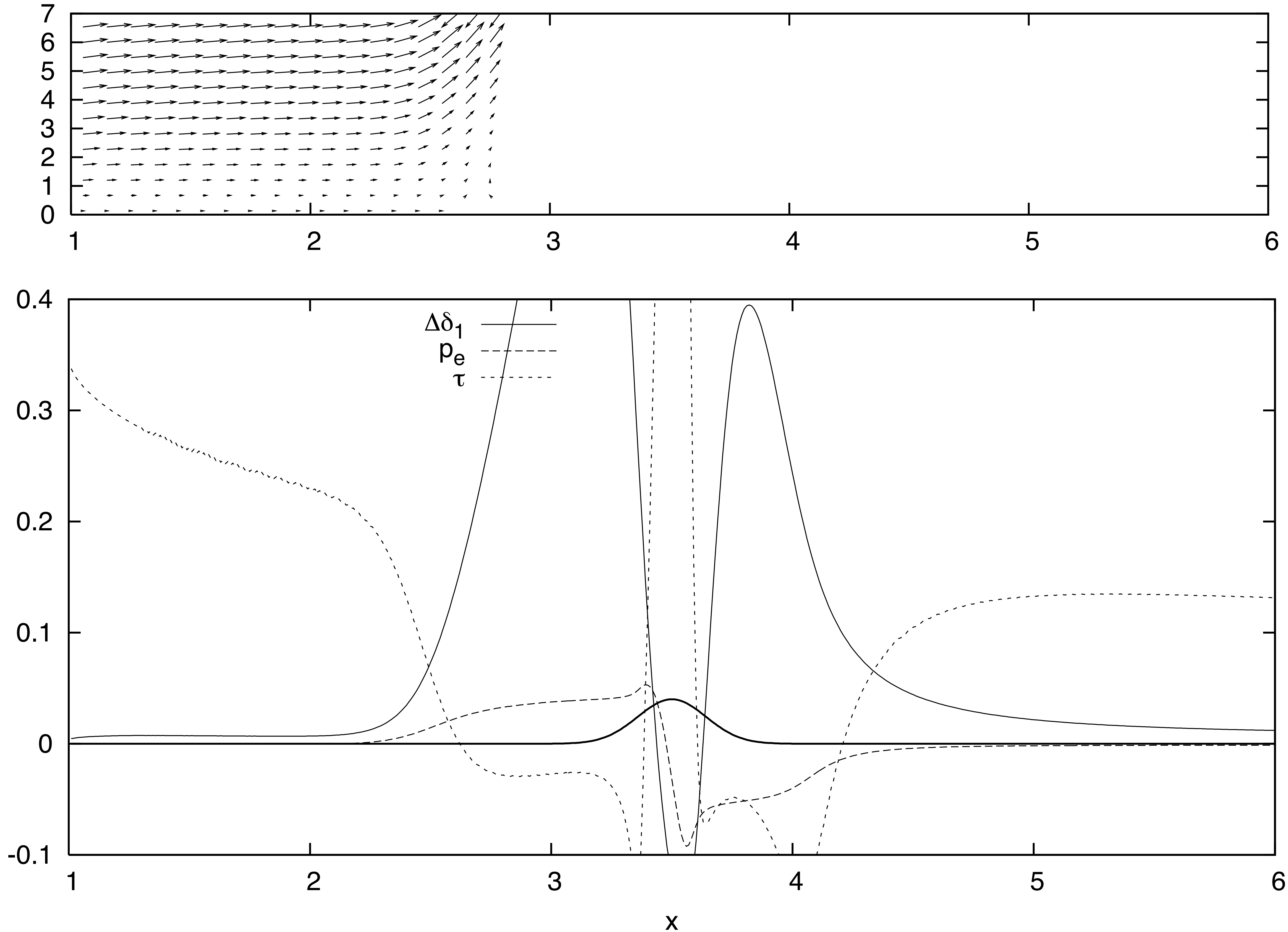

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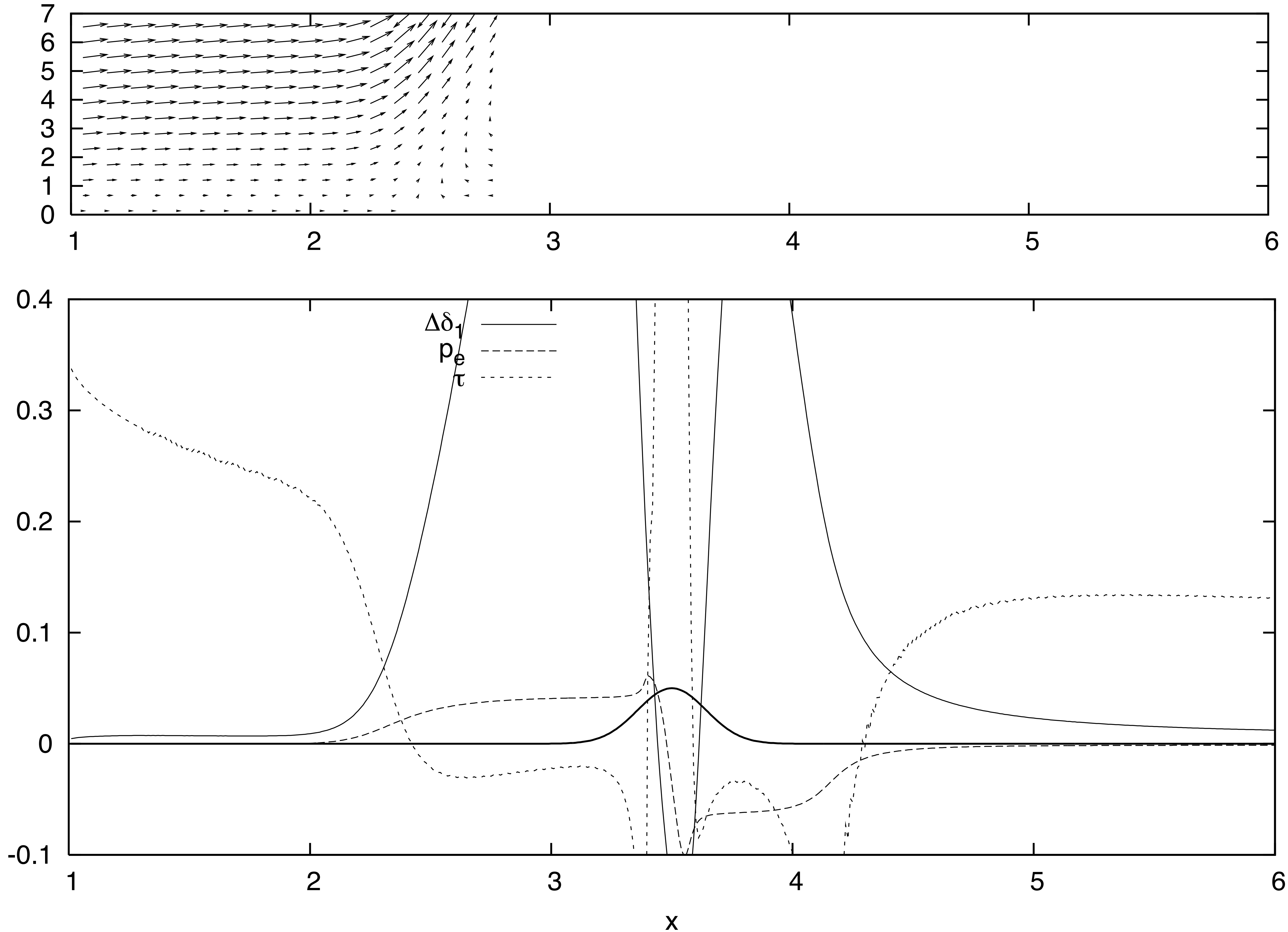

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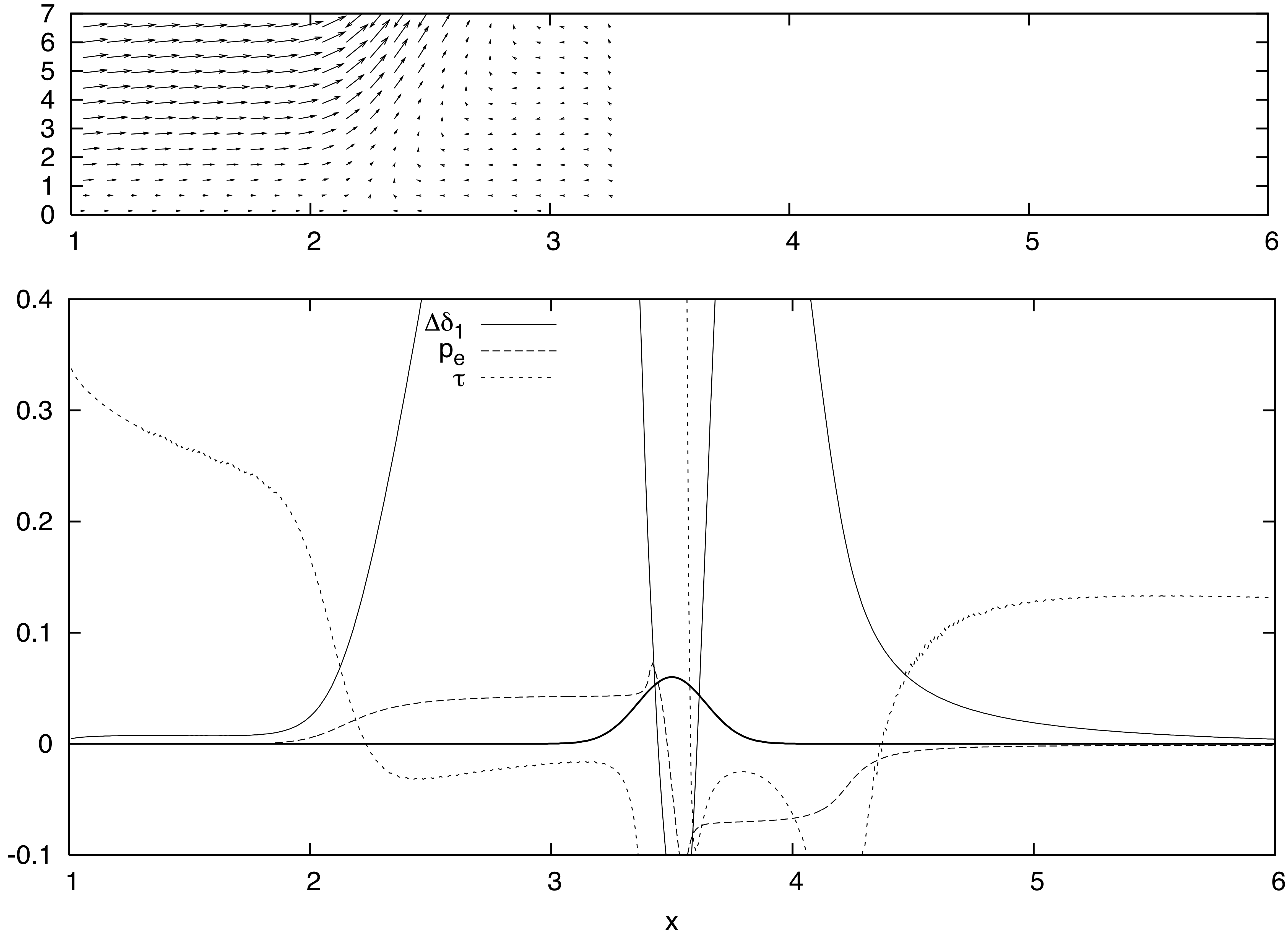

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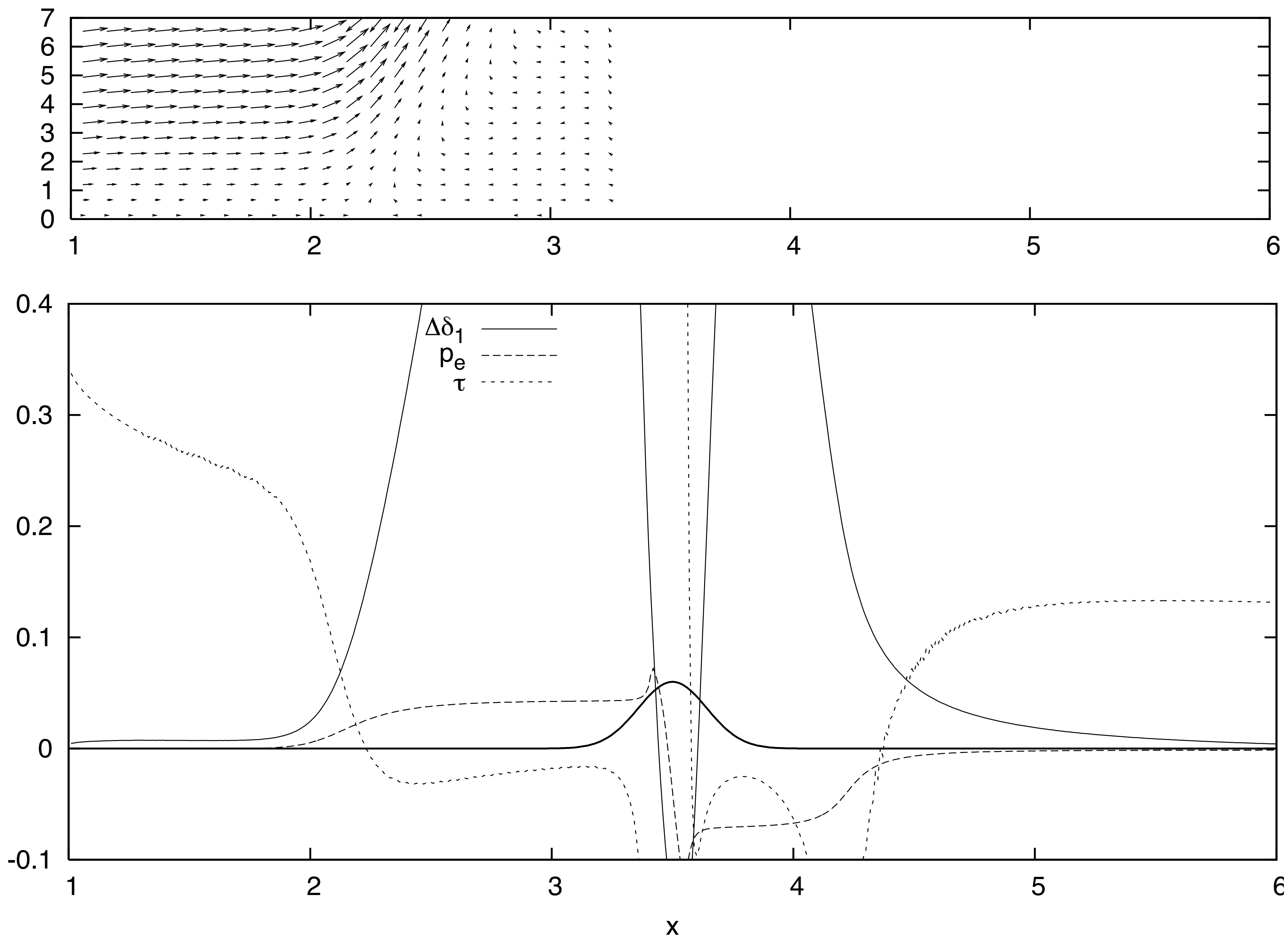
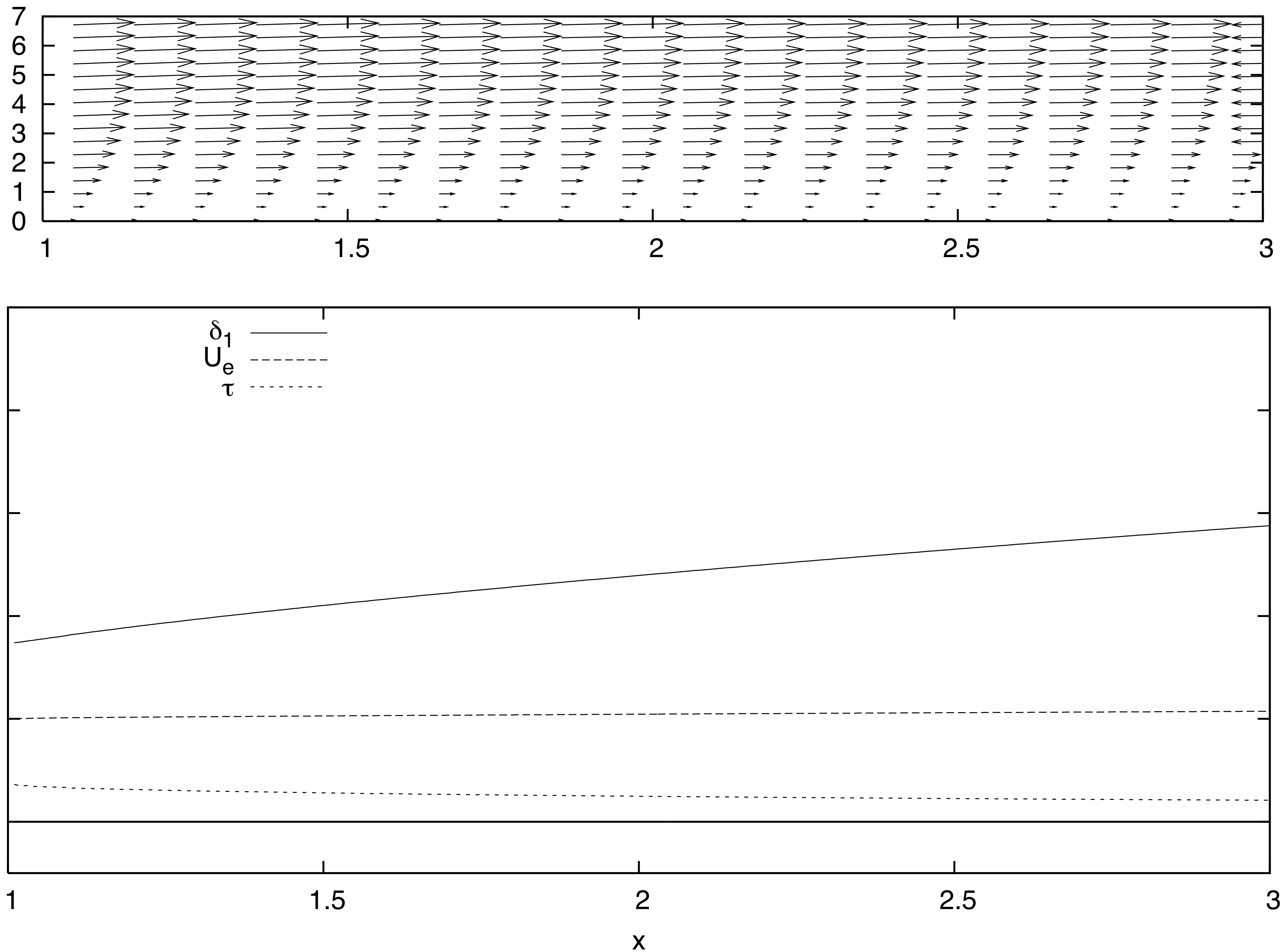
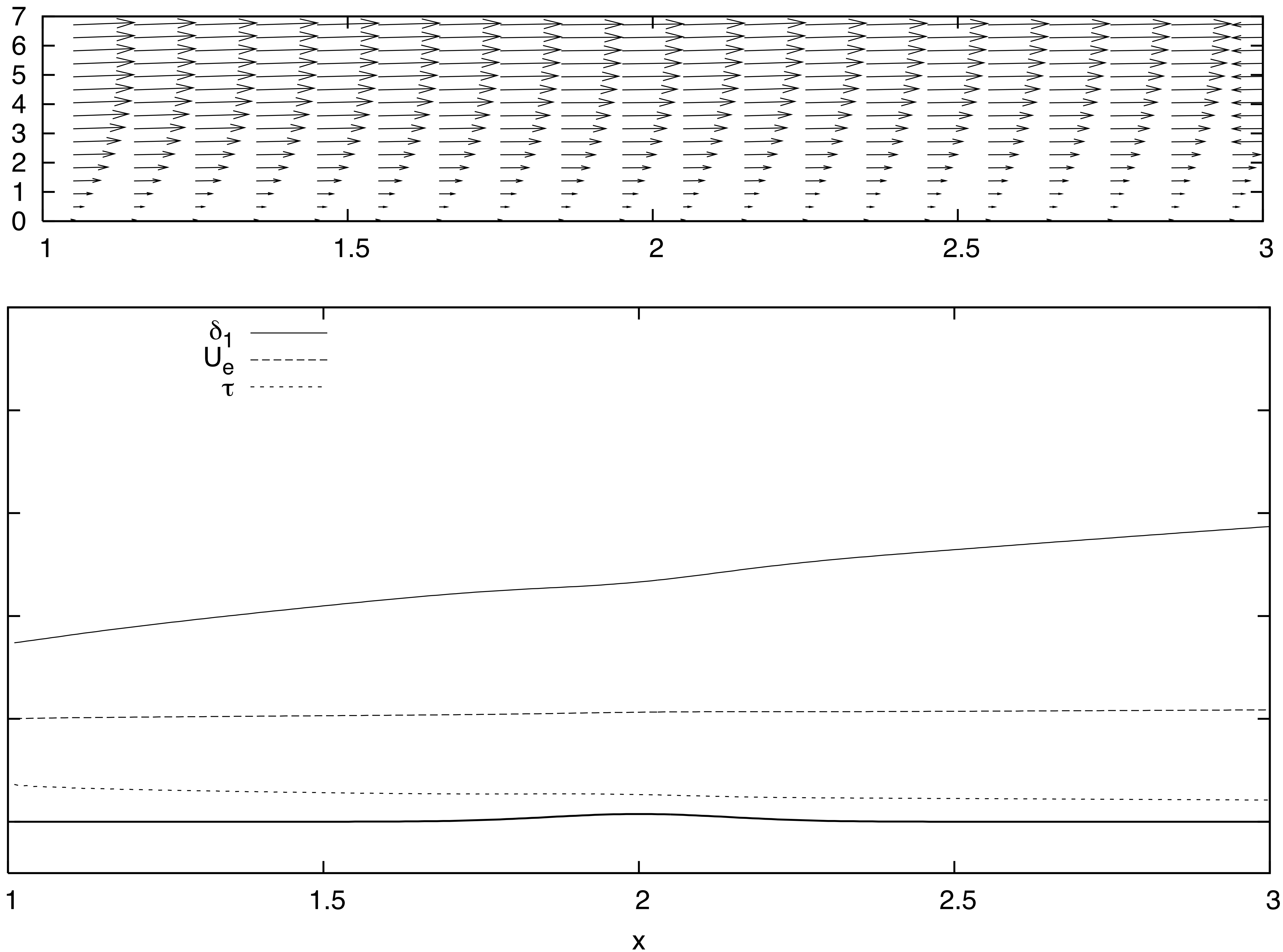


Figure 17: Supersonic flow on a flat plate with a bump [click to launch the movie, Adobe Reader required]. Top the velocity field $\tilde{u}, \tilde{v}$ (Prandtl transform), bottom the wall, here a bump, the perturbation of displacement thickness from Blasius $\Delta\tilde{\delta}_1$ (starting from 0 in $\bar{x} = 1$), the skin friction (starting from Blasius value 0.3 in $\bar{x} = 1$) and the outer pressure starting from Ideal Fluid value 0 in $\bar{x} = 1$. Note the pressure plateau associated to separation.

back

subcritique F<1





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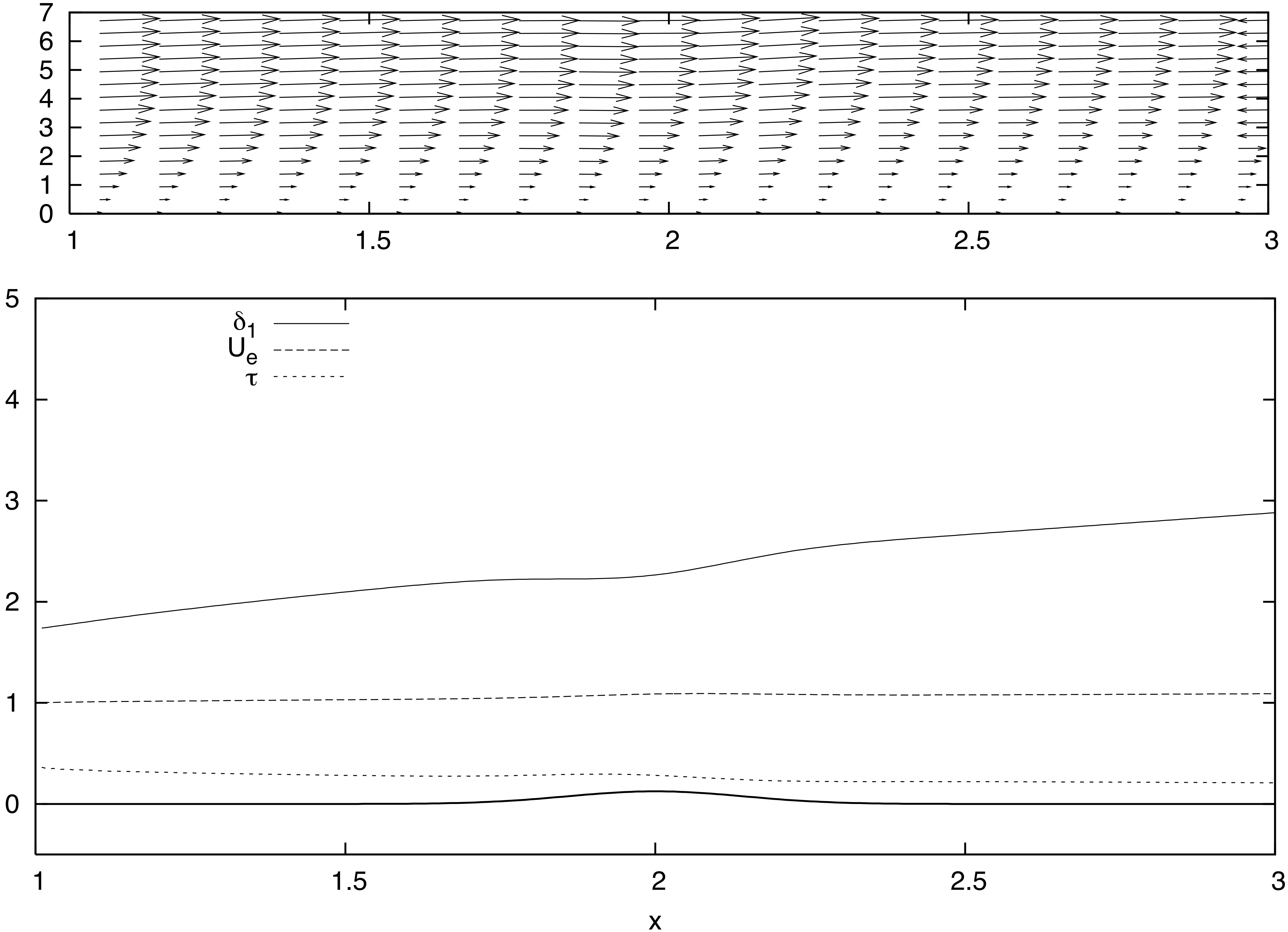

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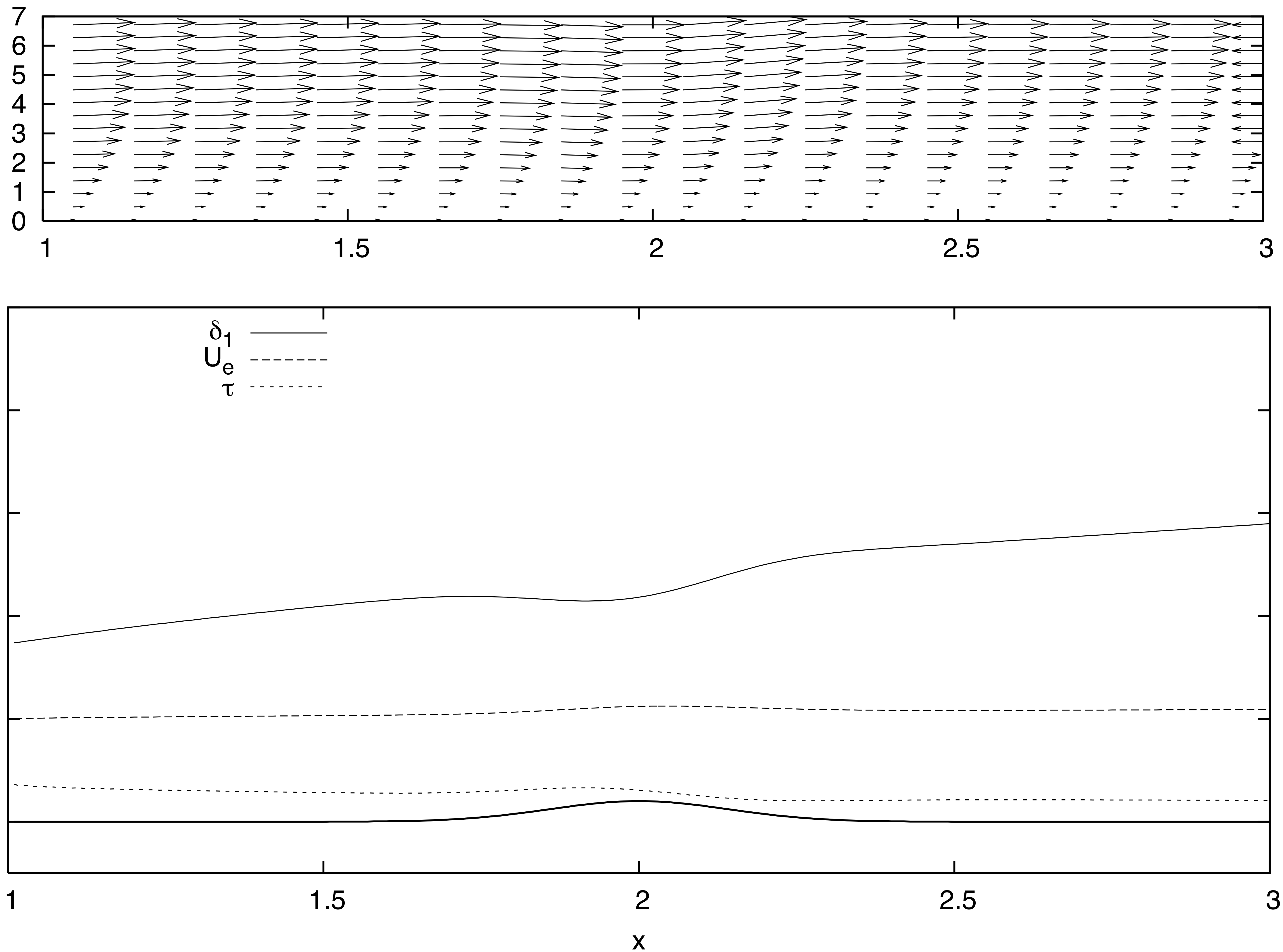

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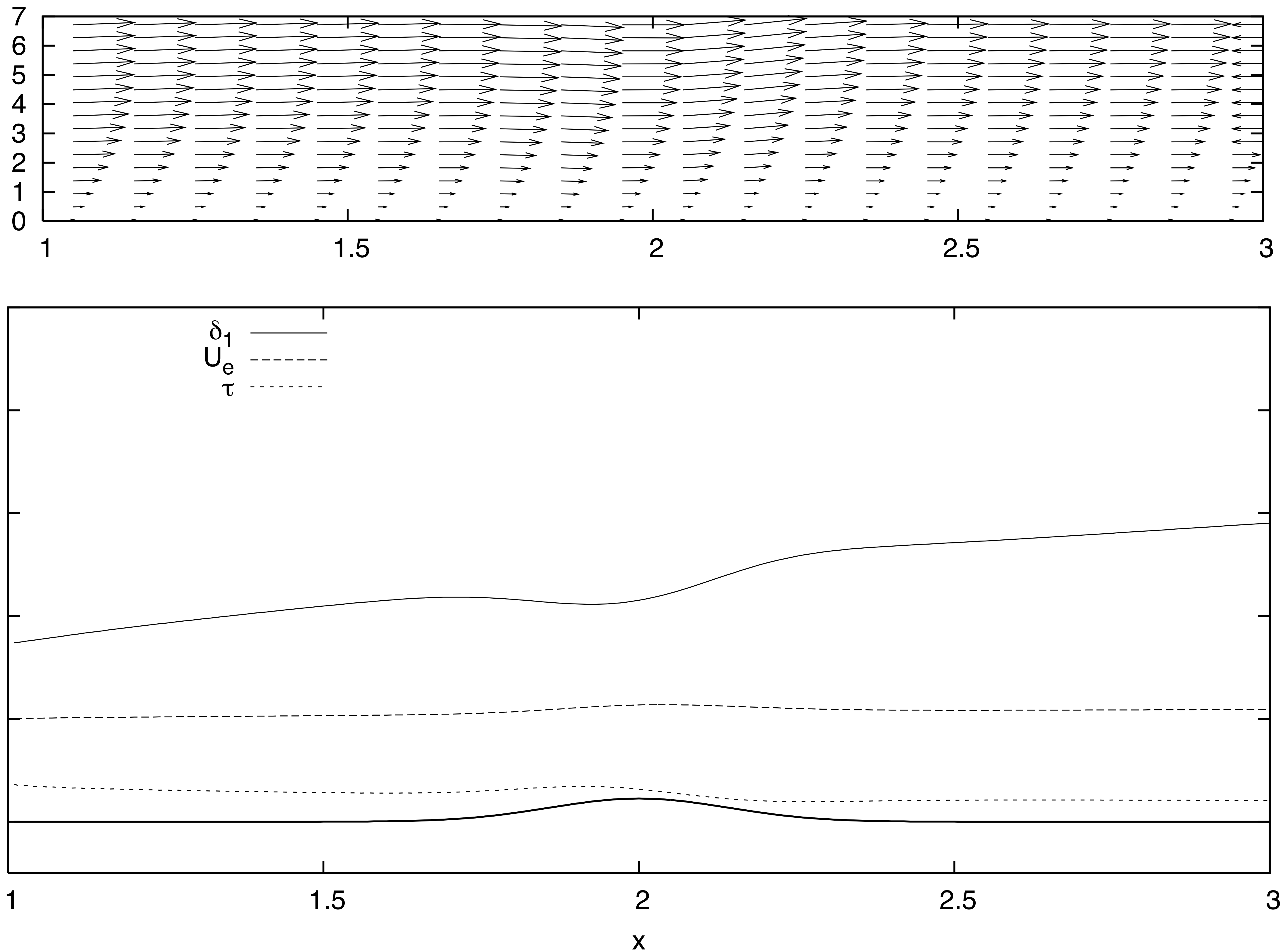

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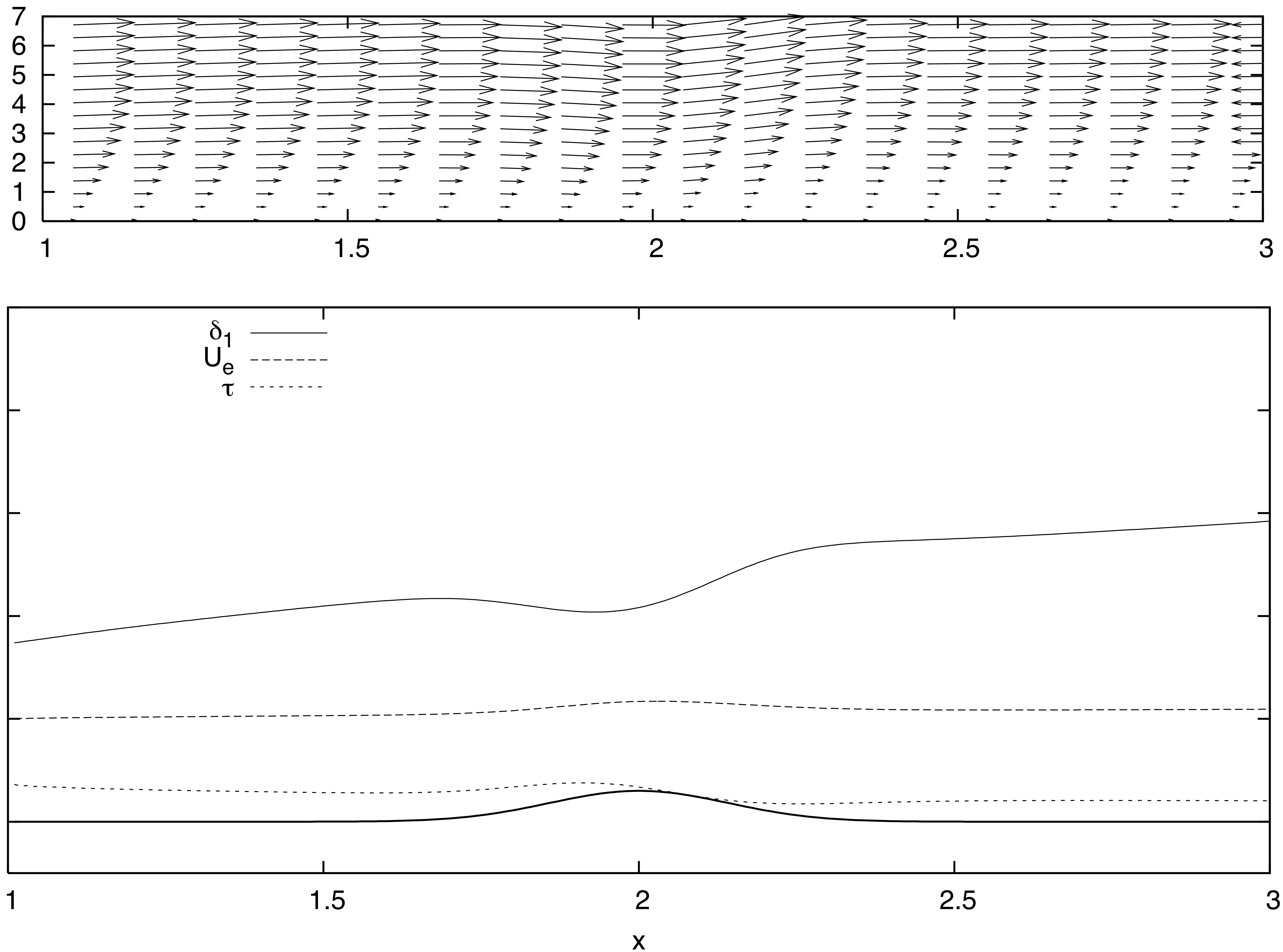

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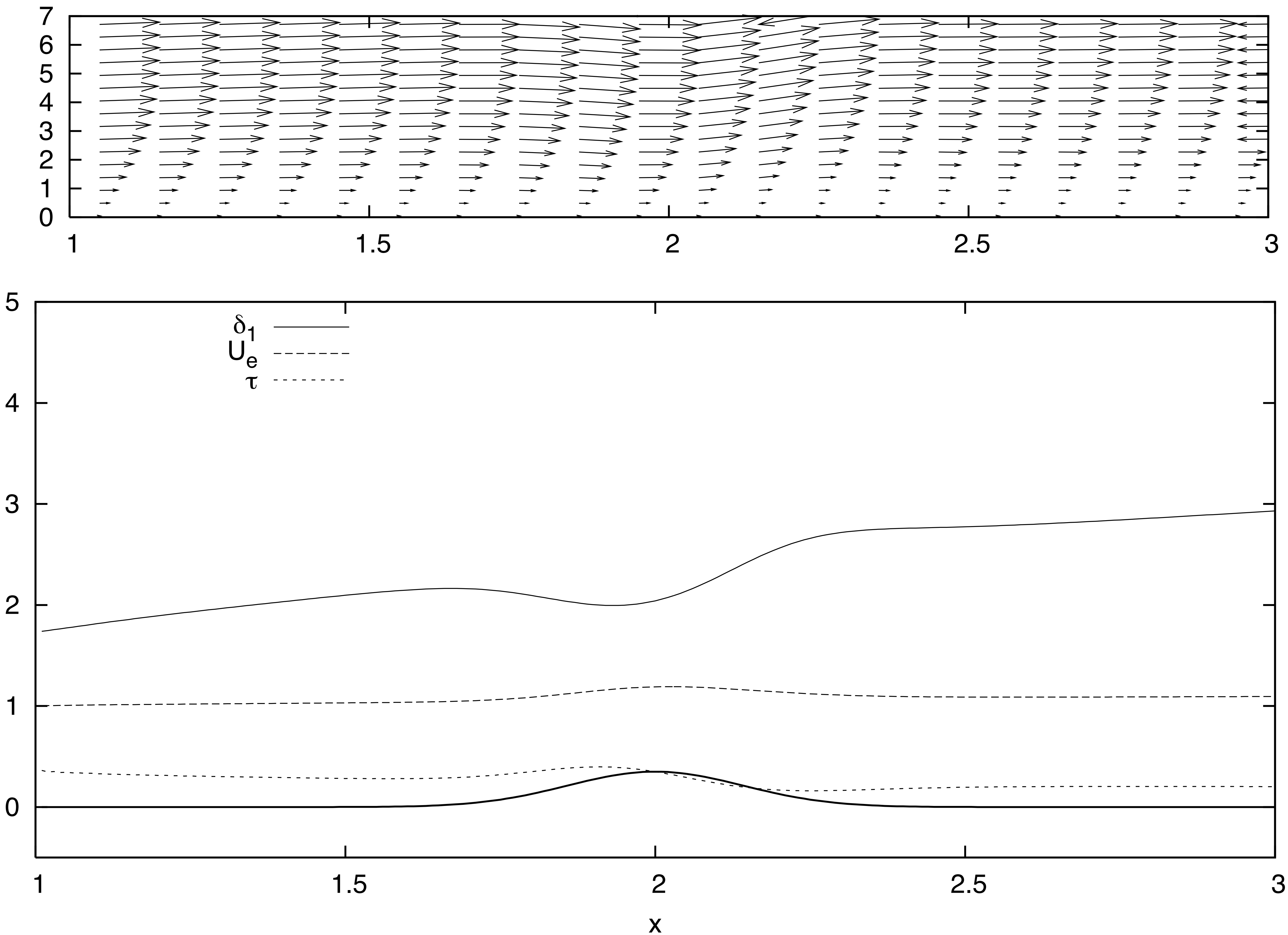

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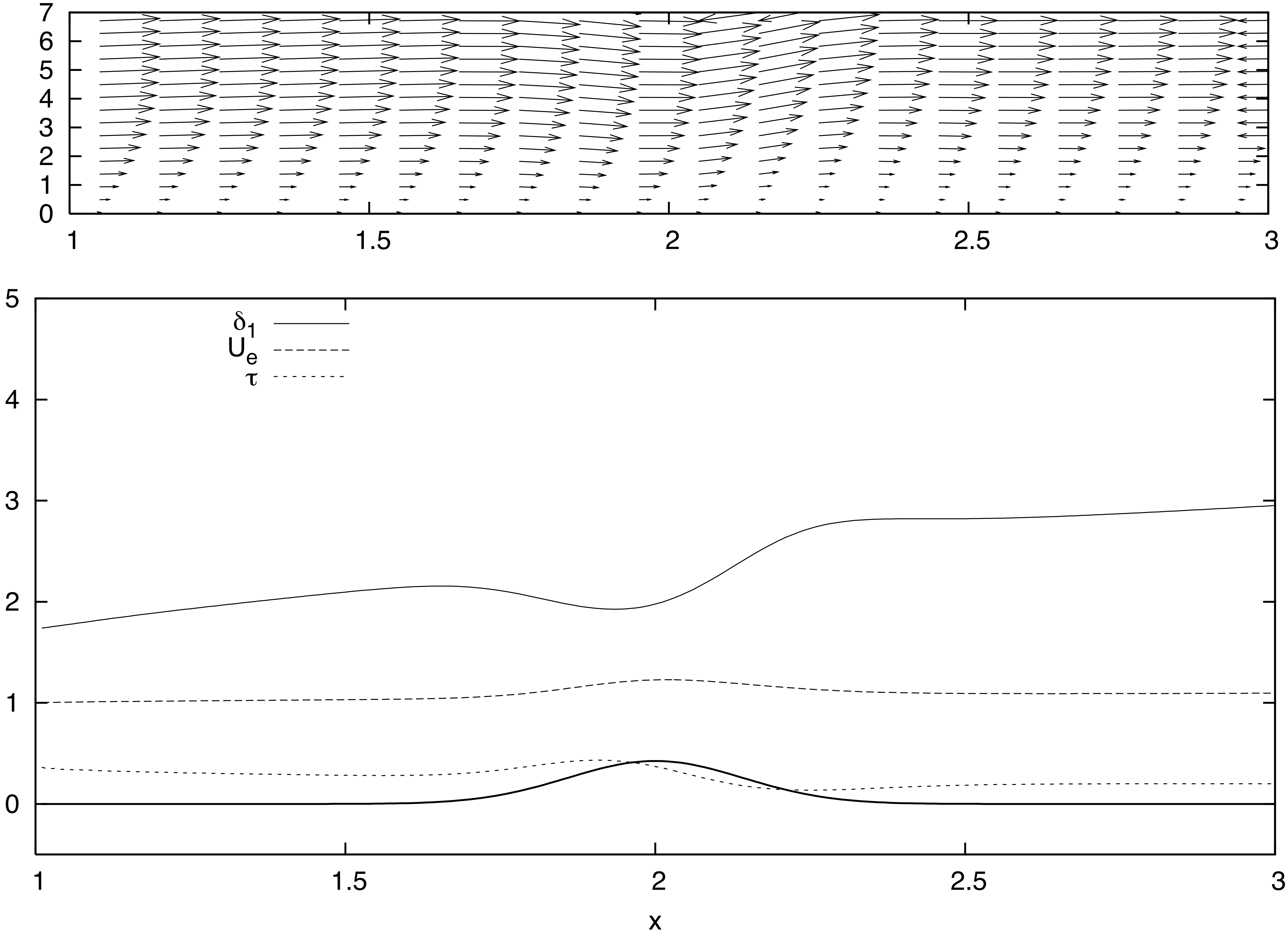

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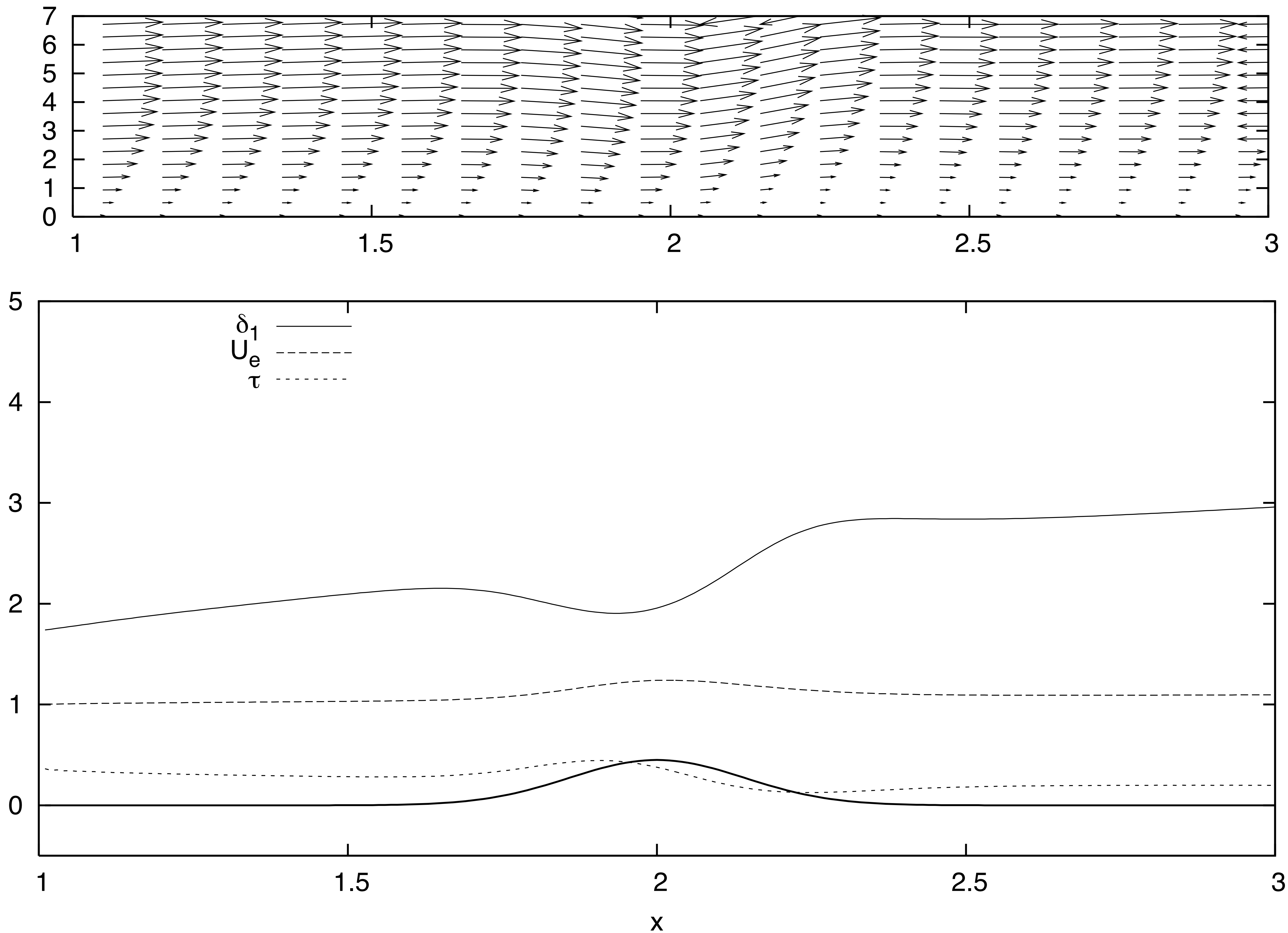
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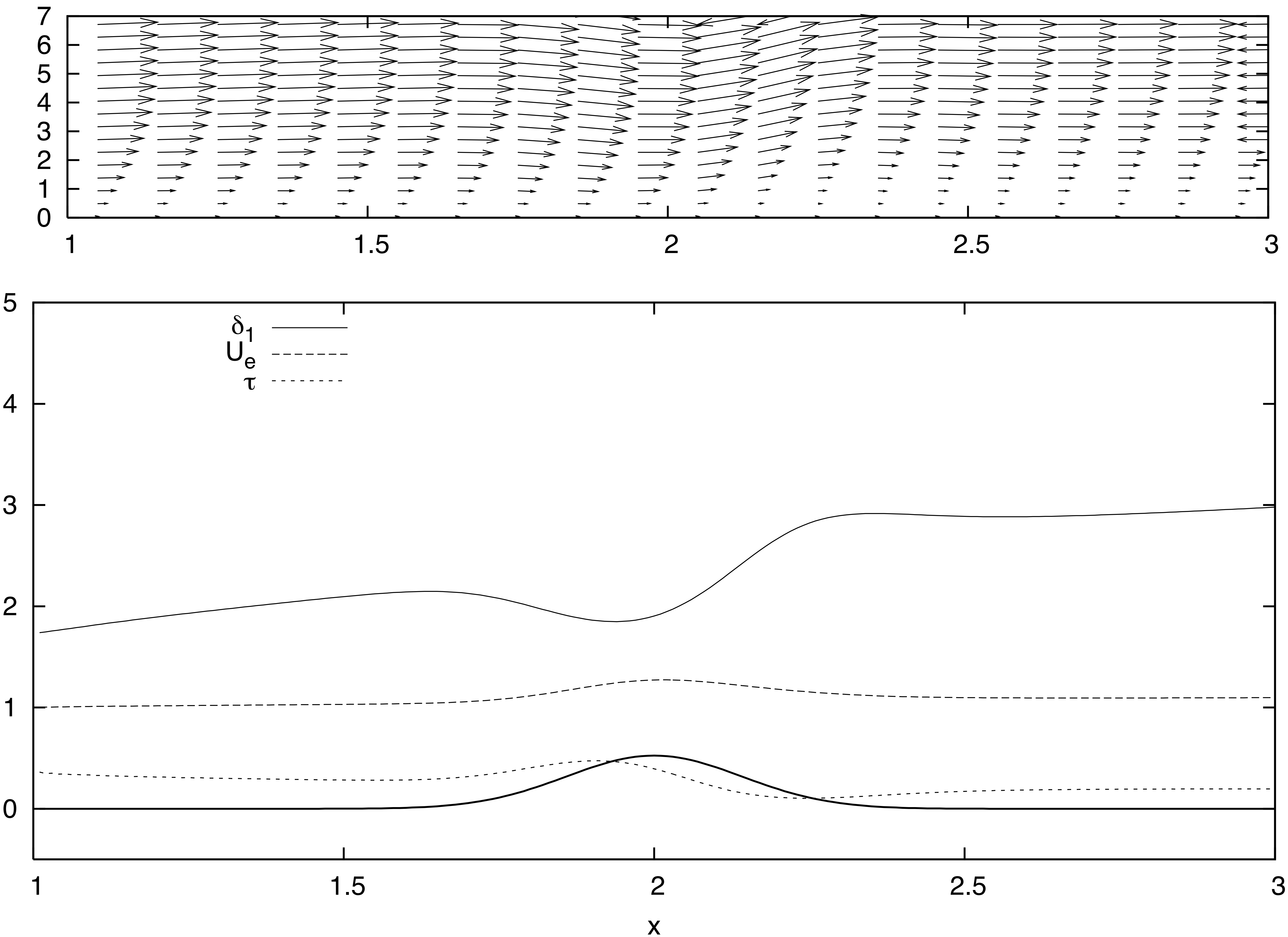

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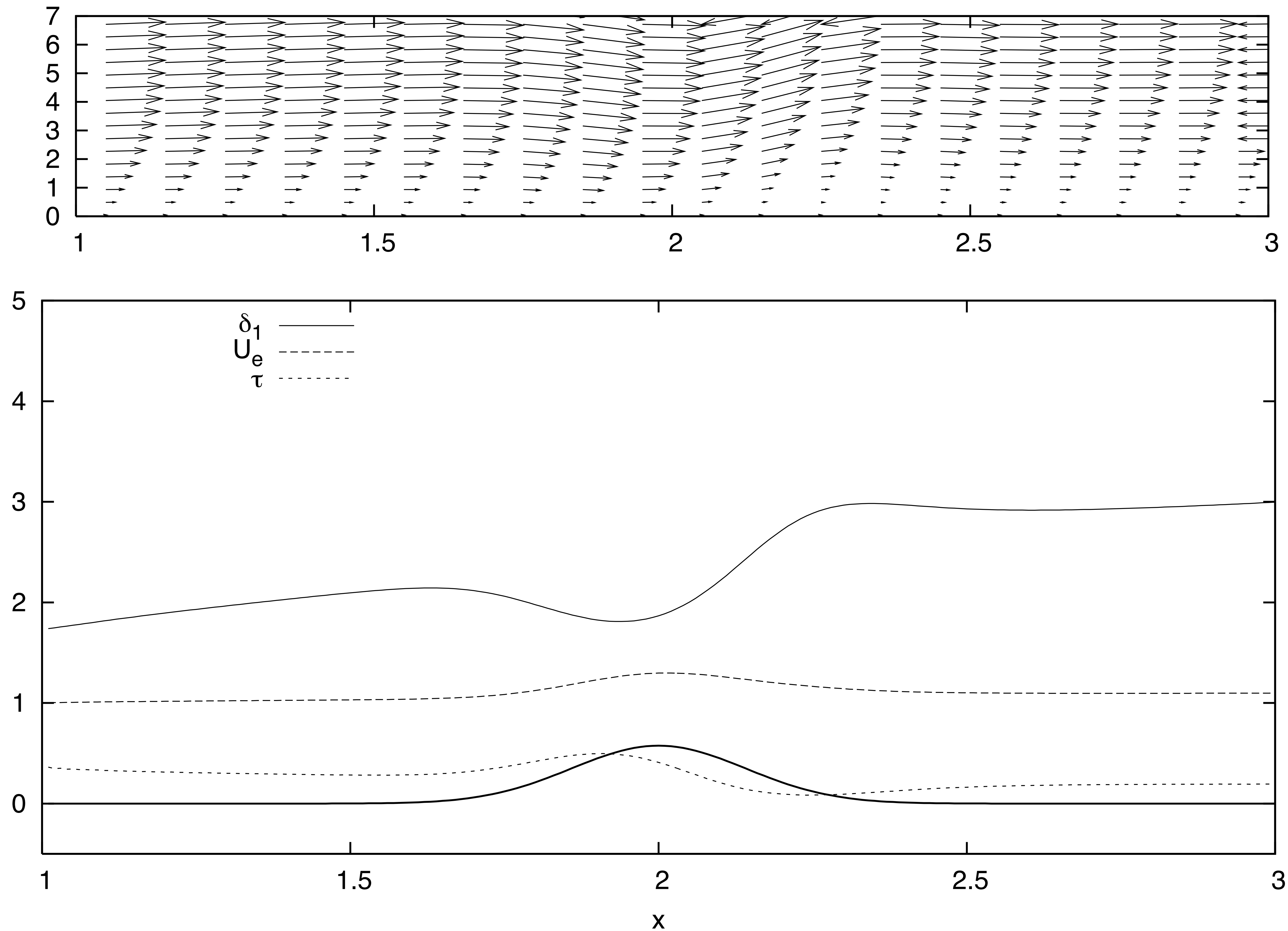

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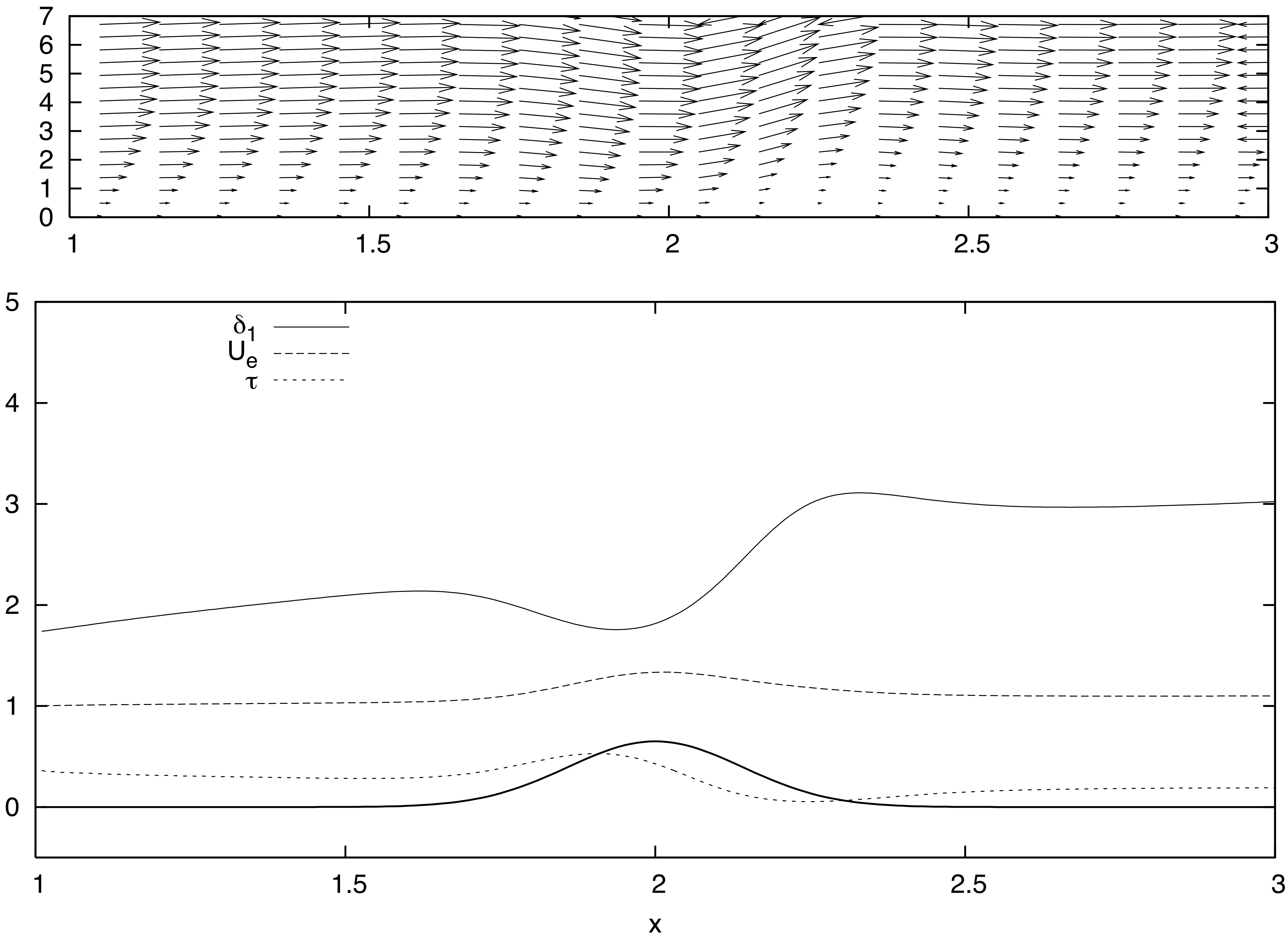

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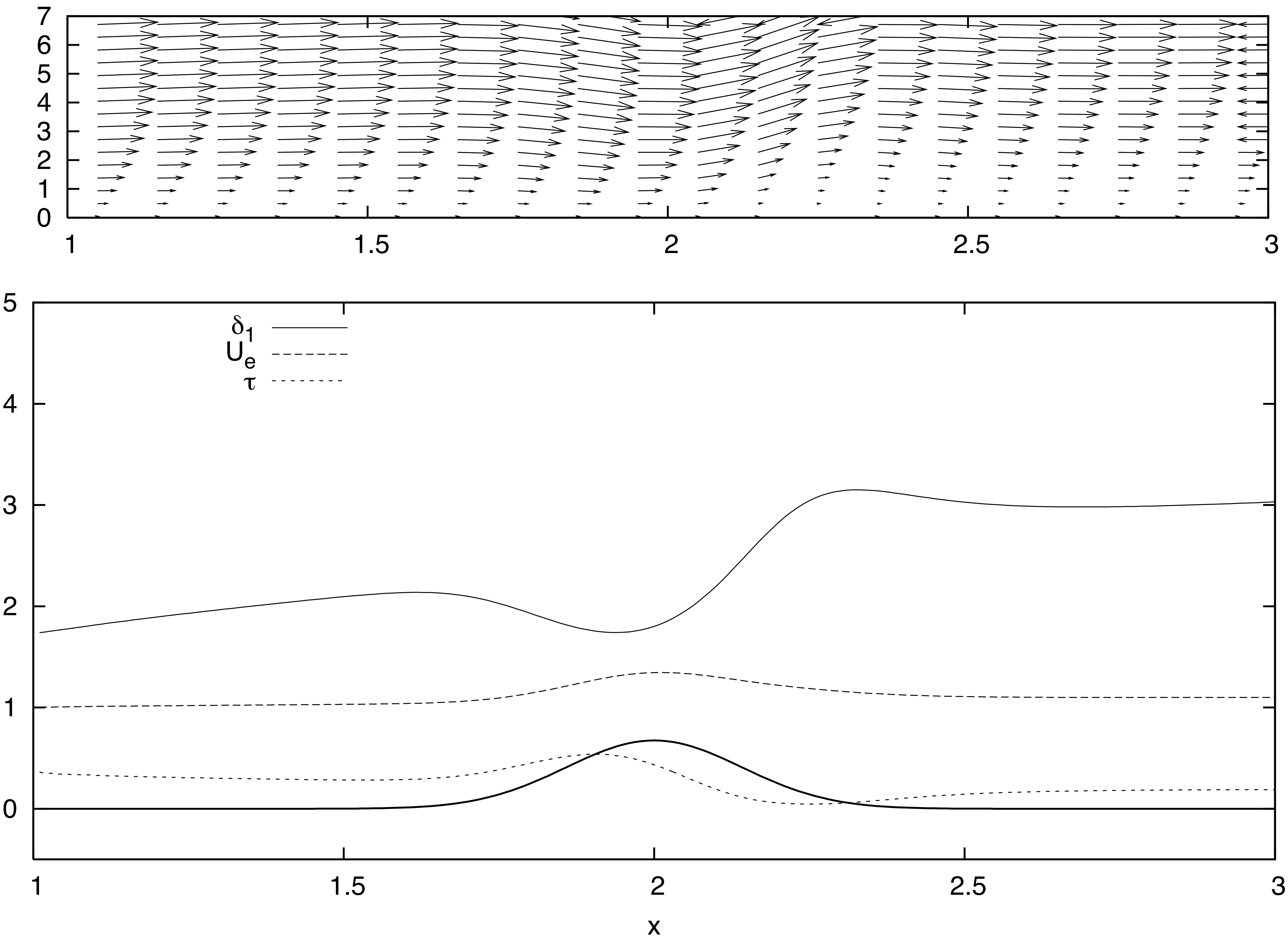

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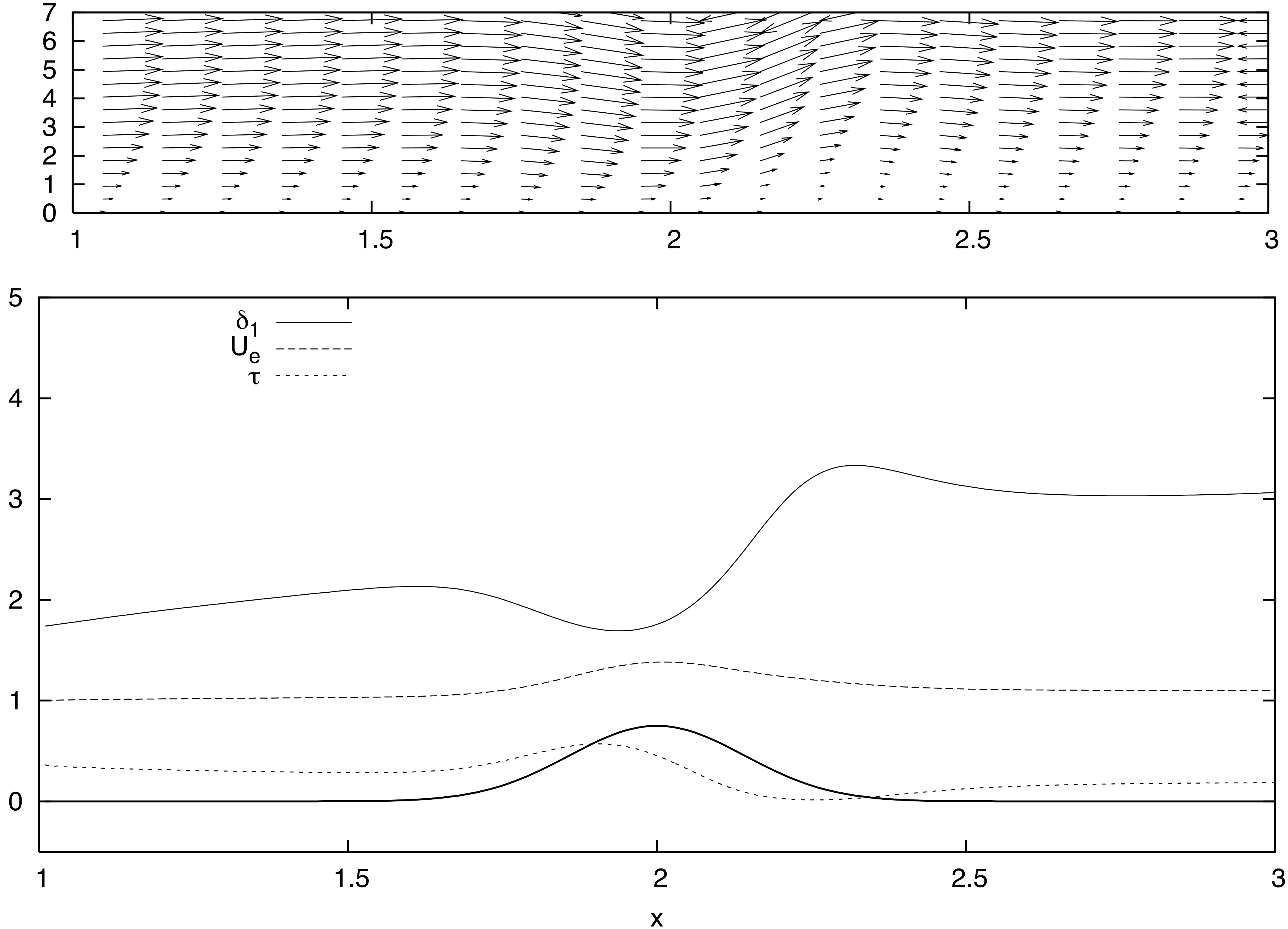

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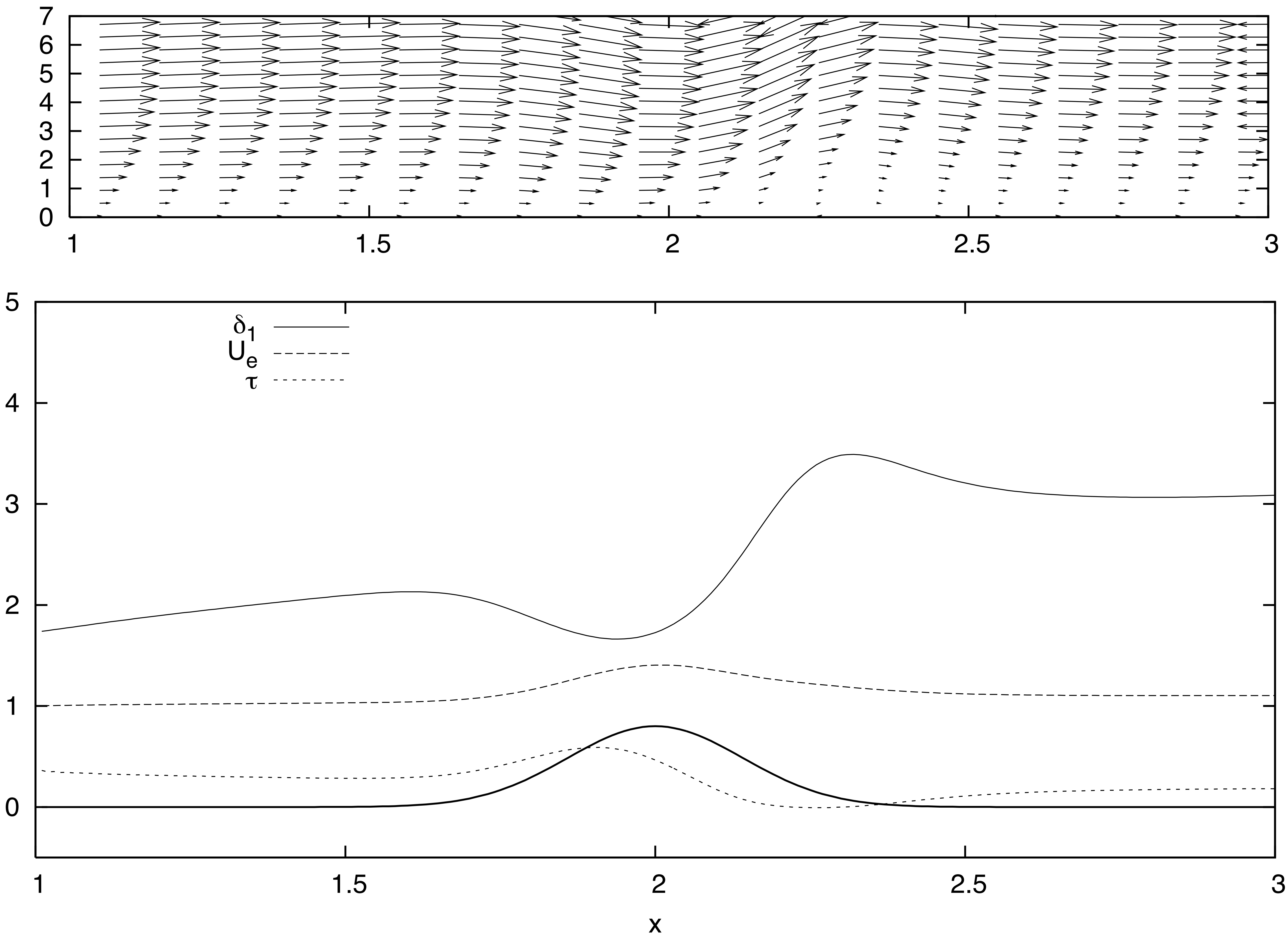

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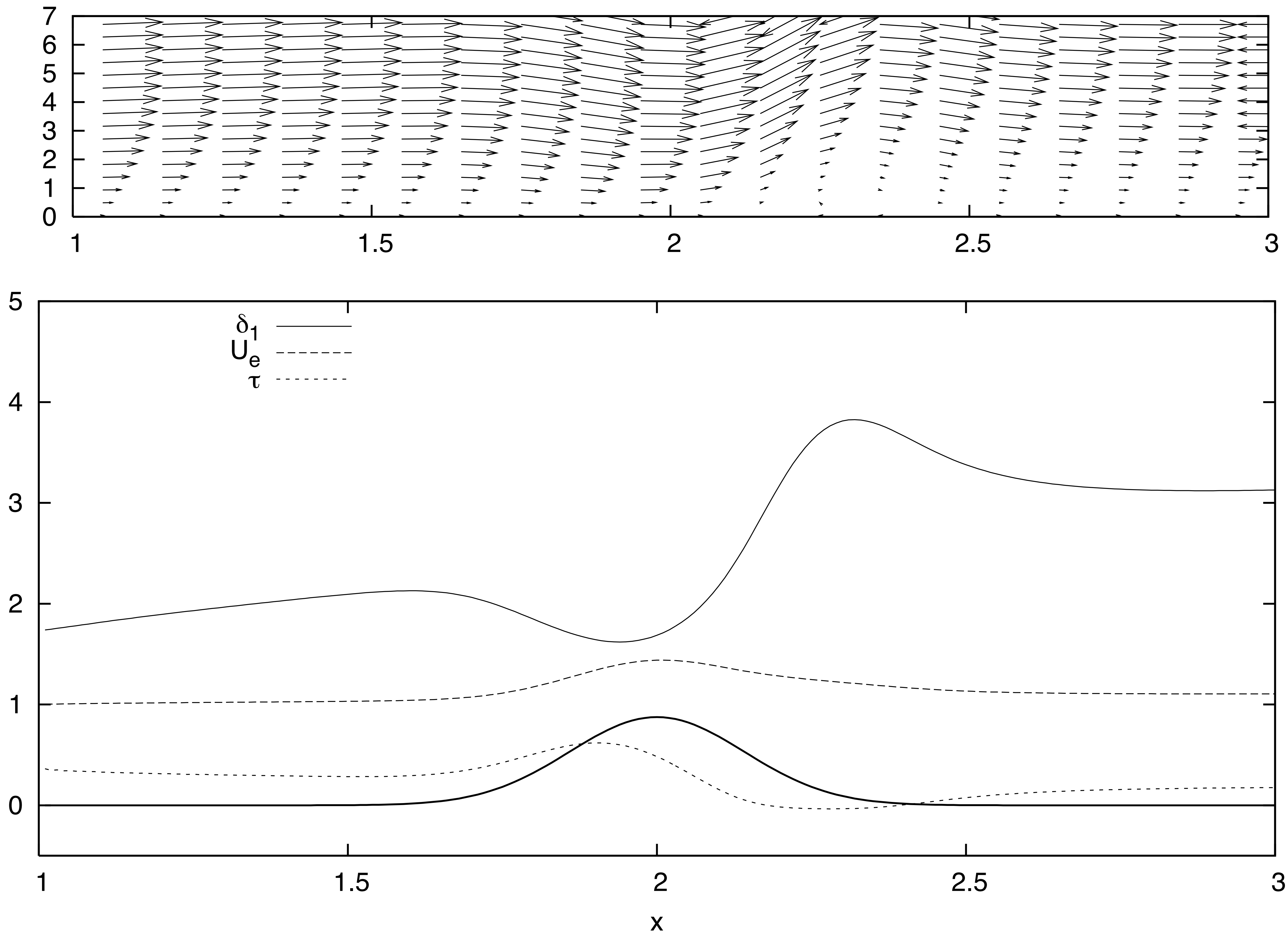

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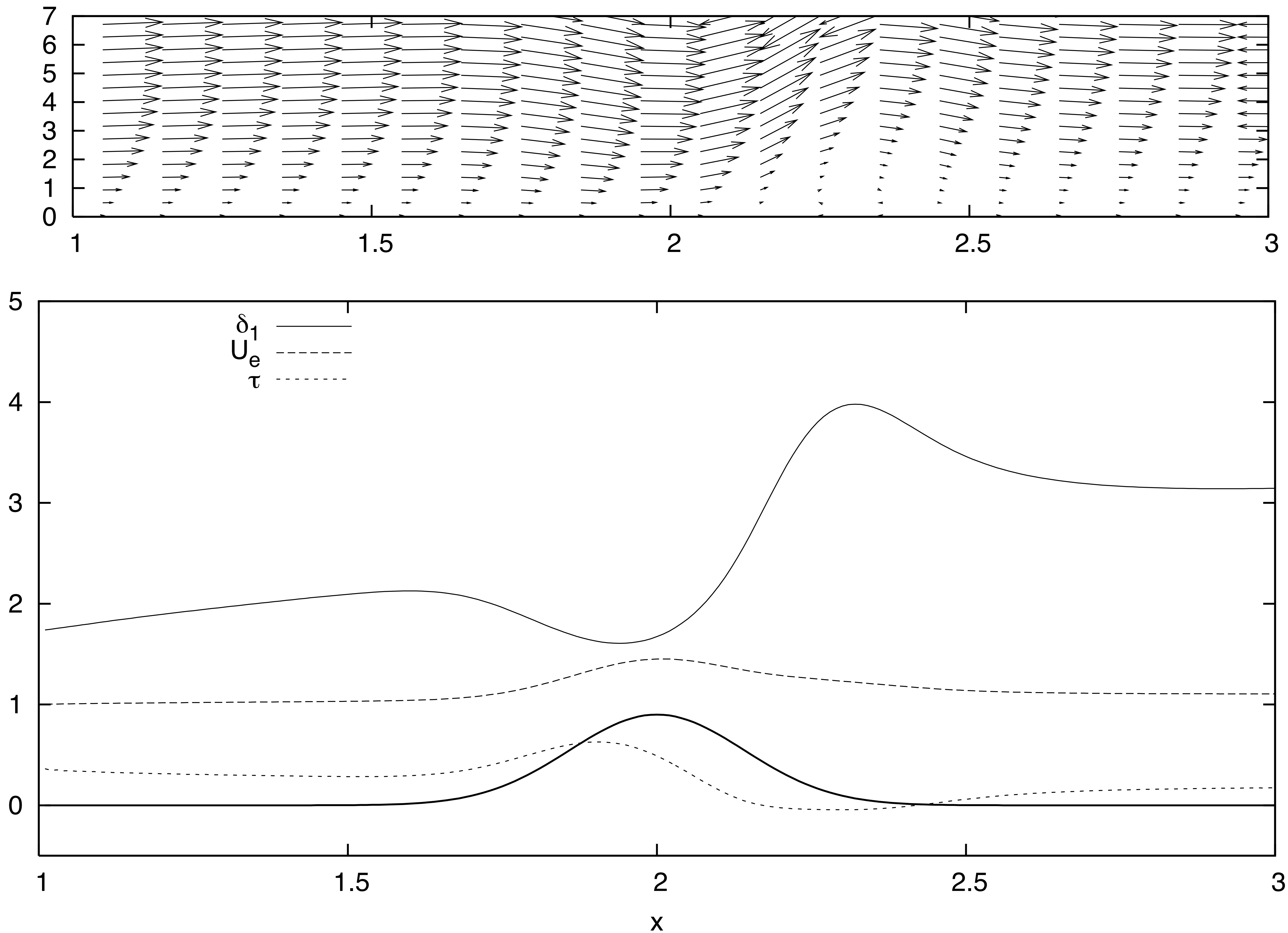

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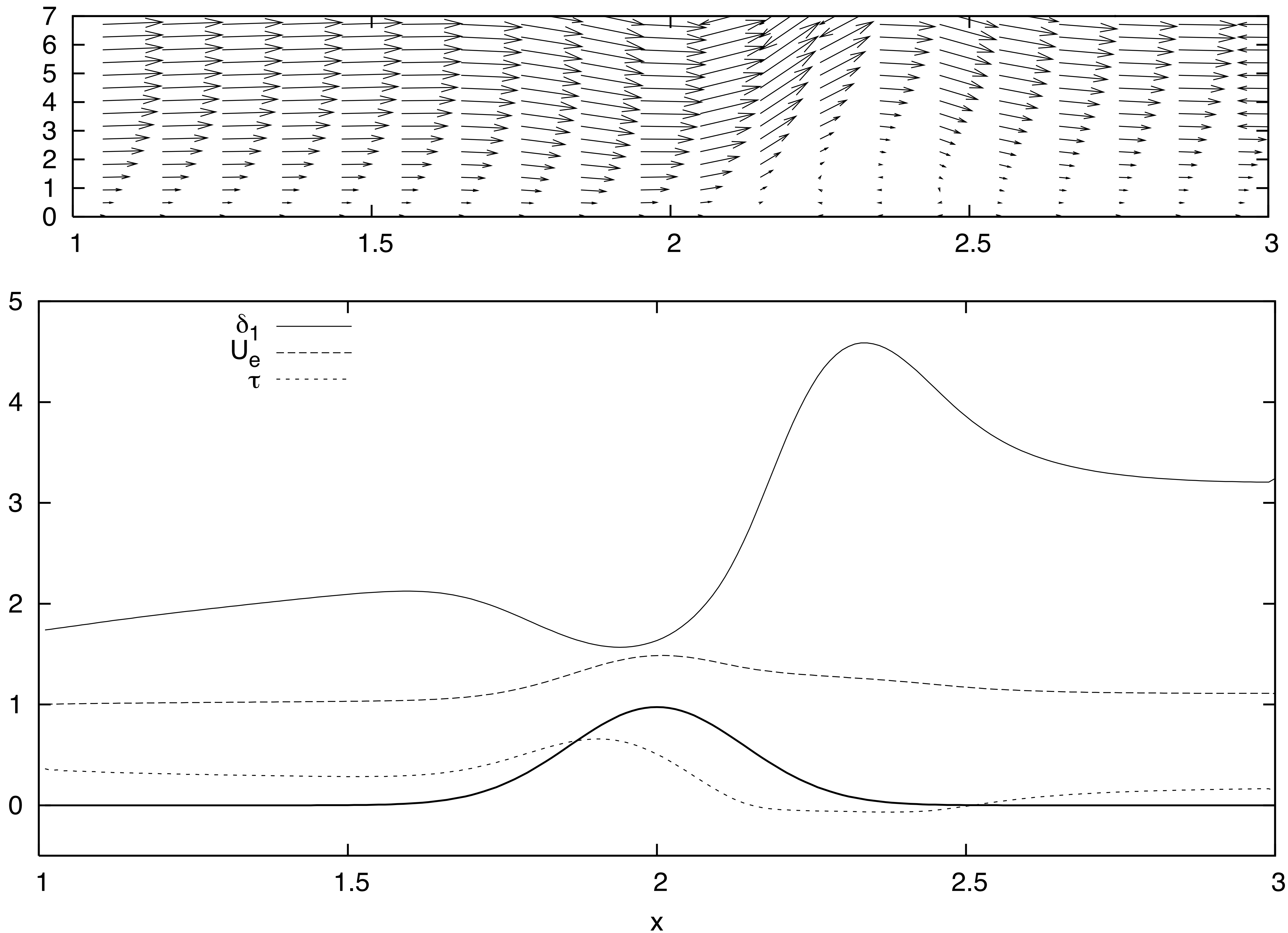

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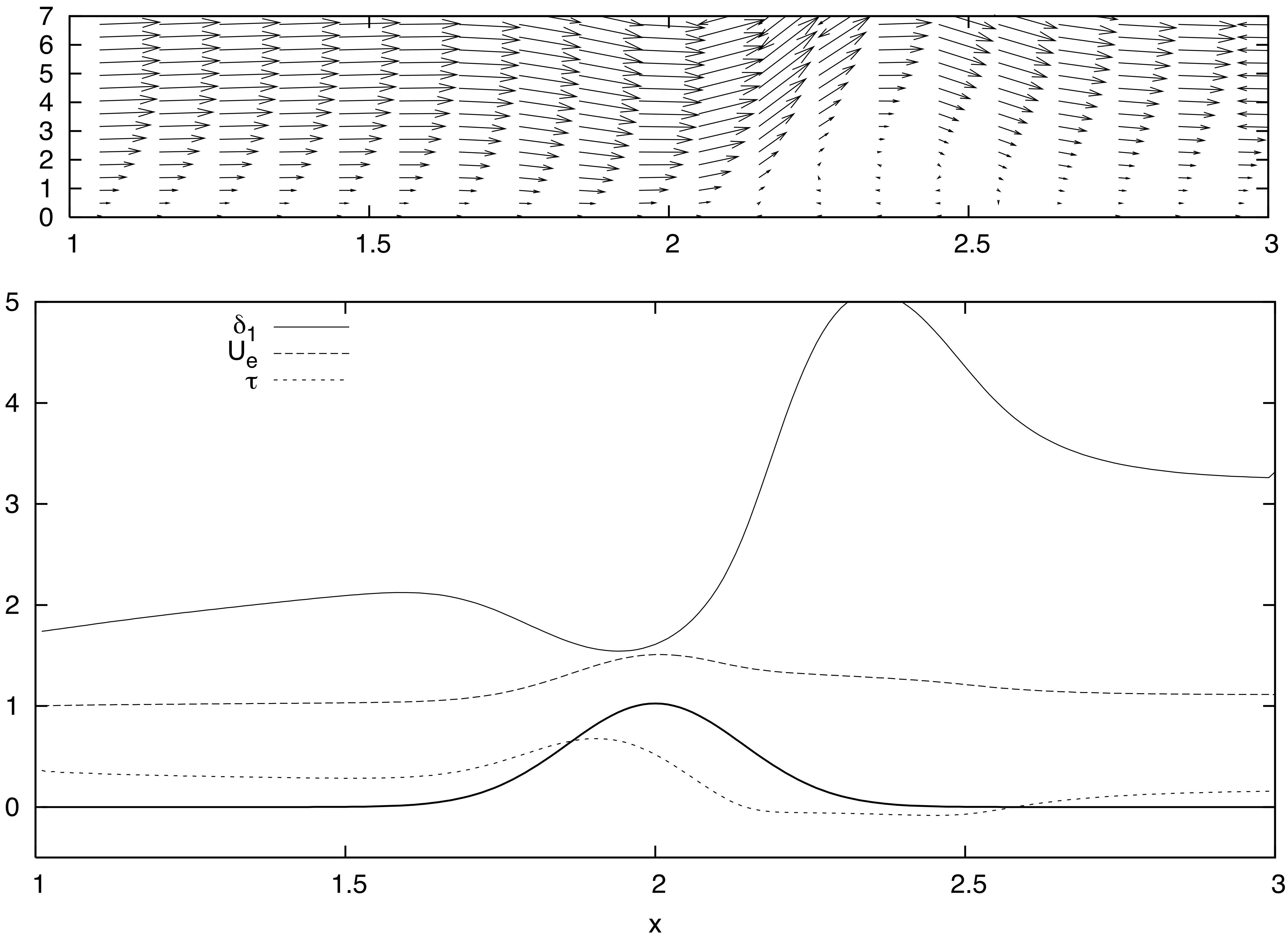

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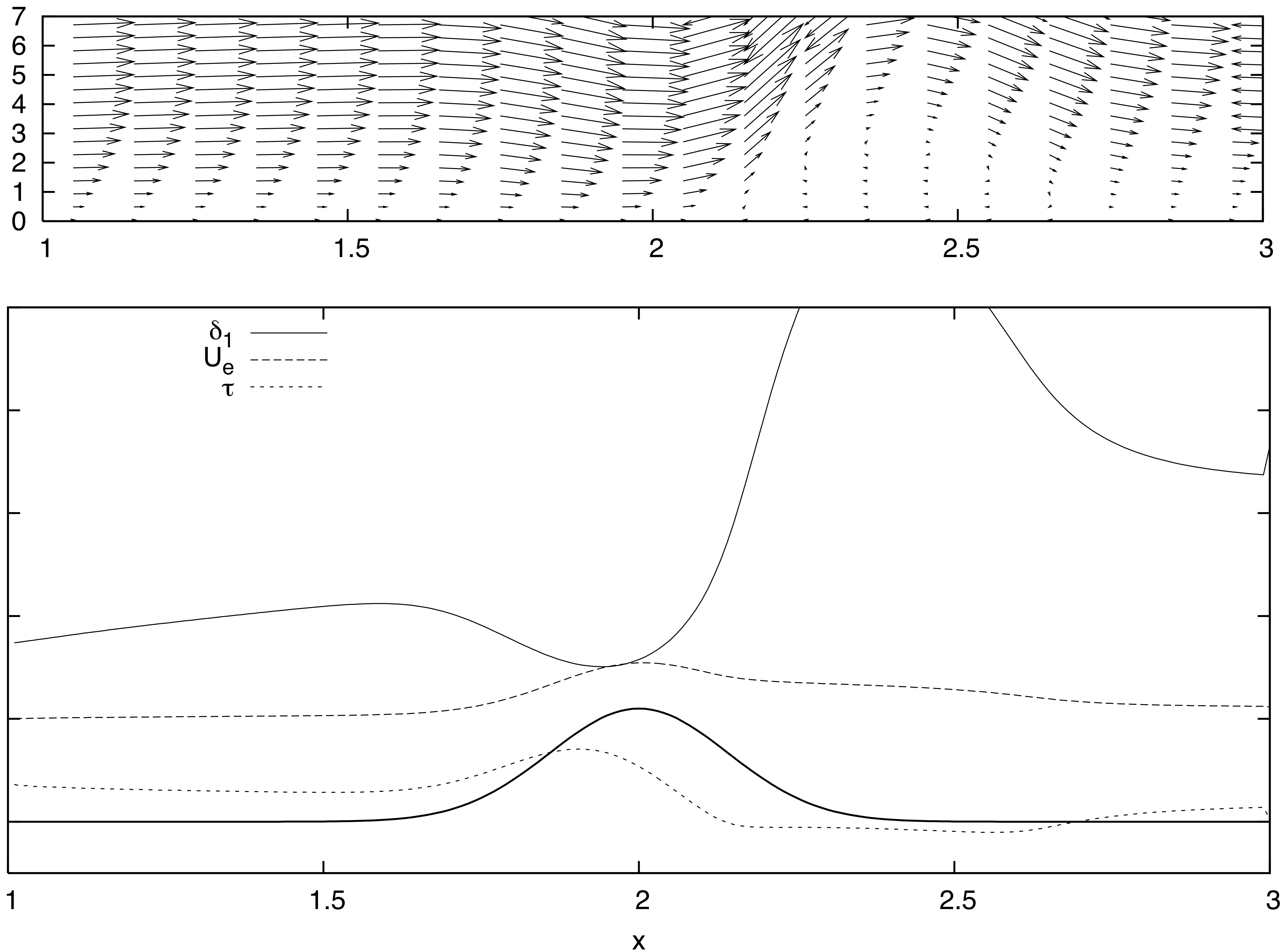

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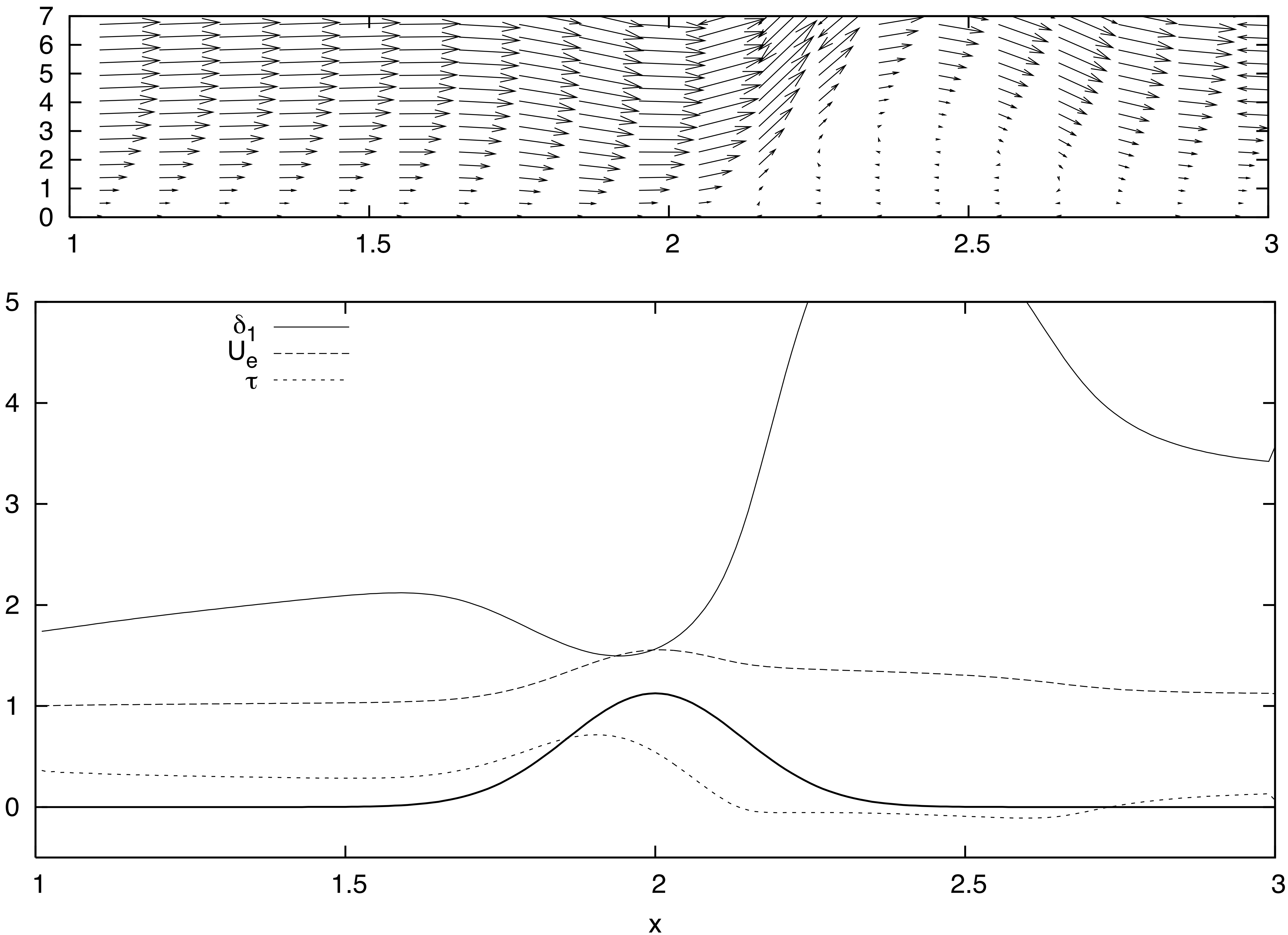

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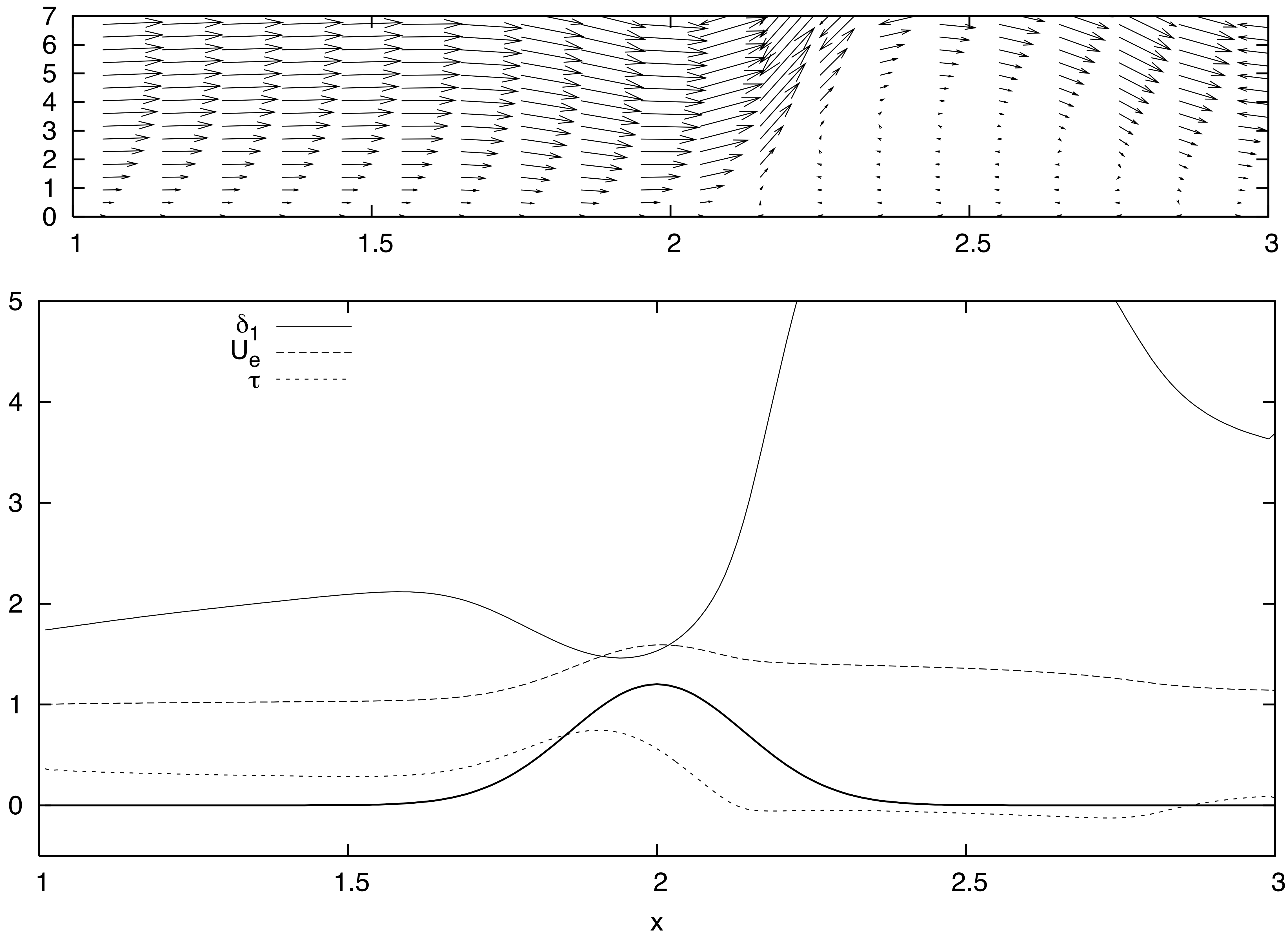

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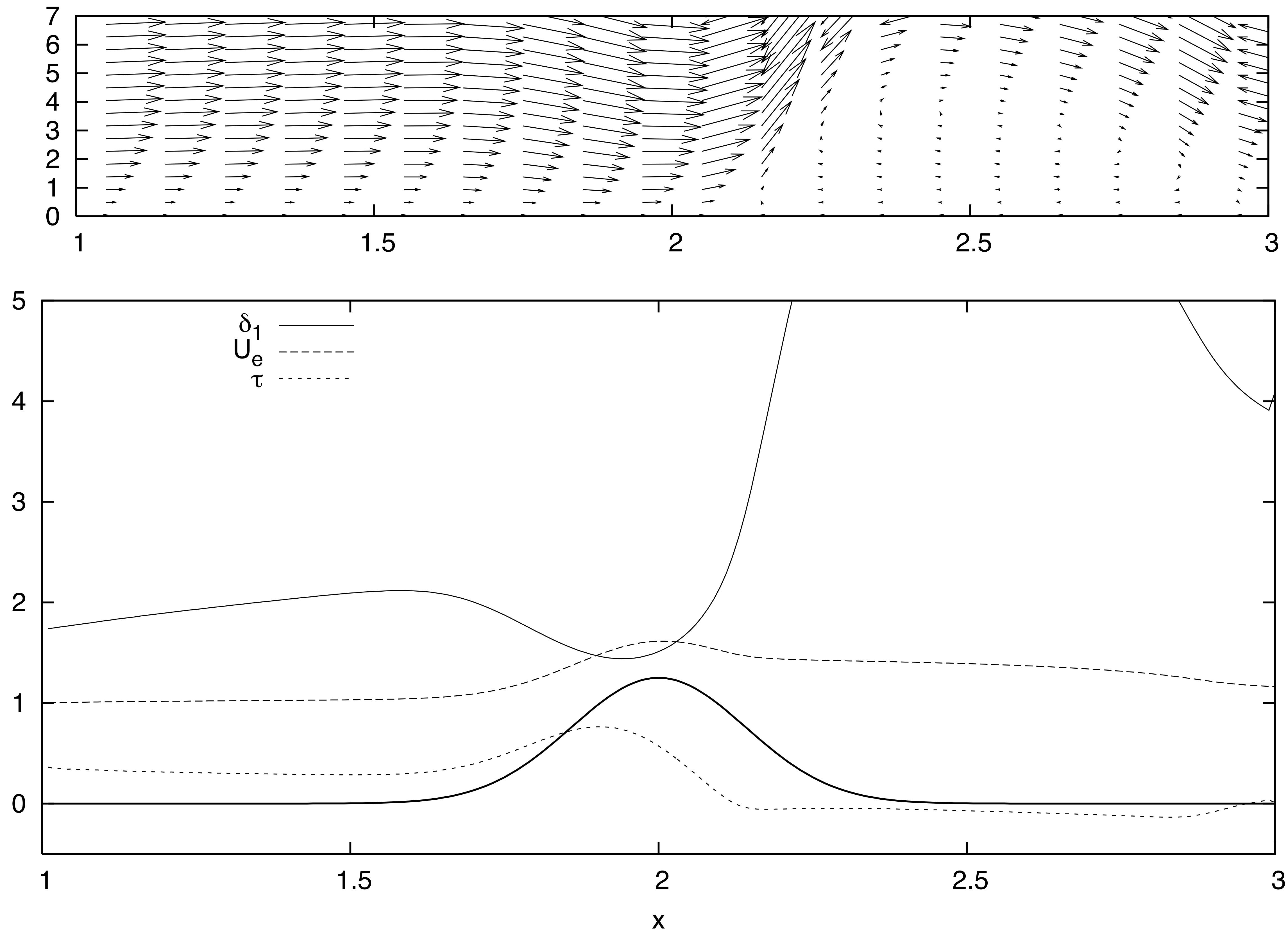

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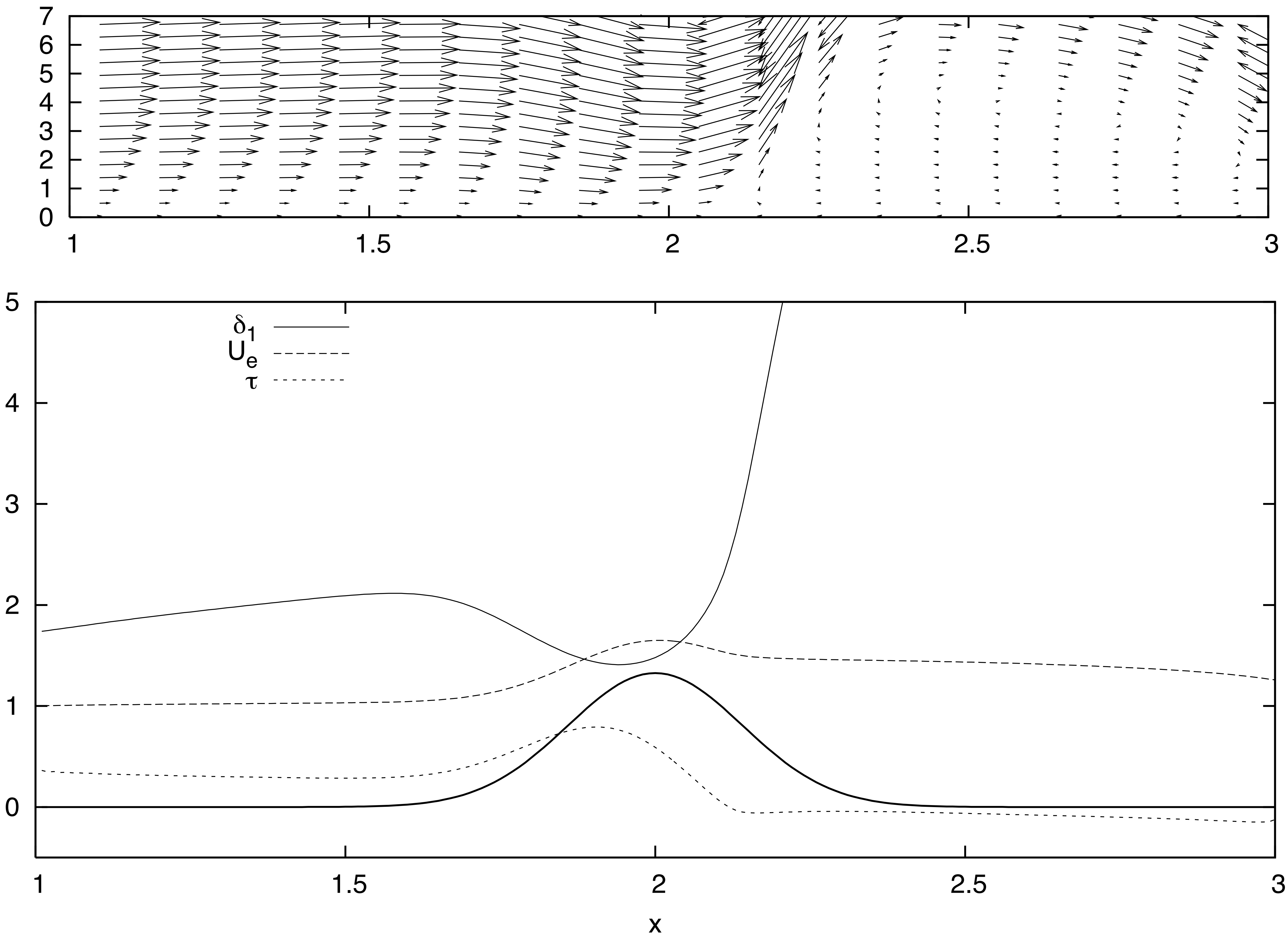

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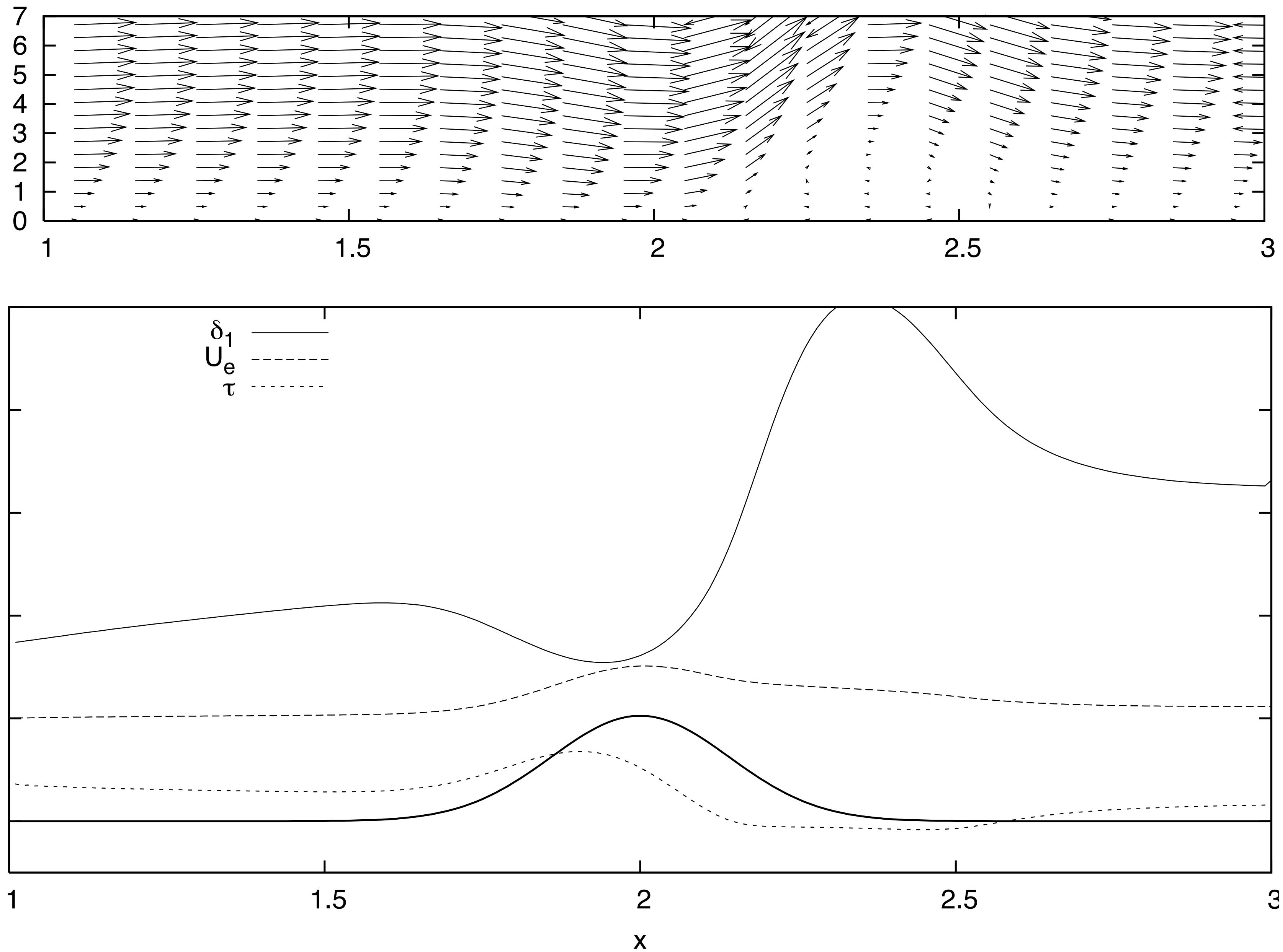
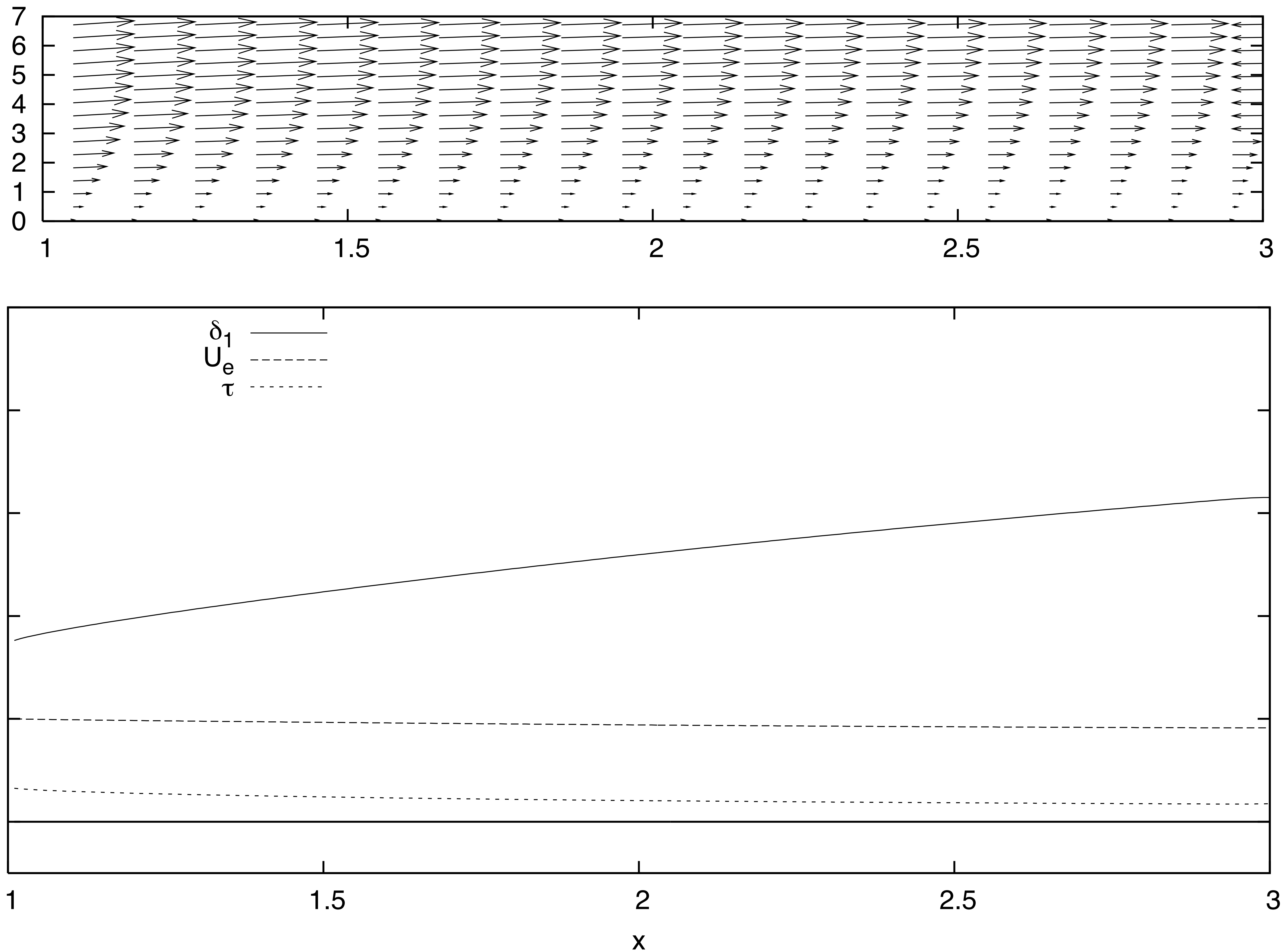
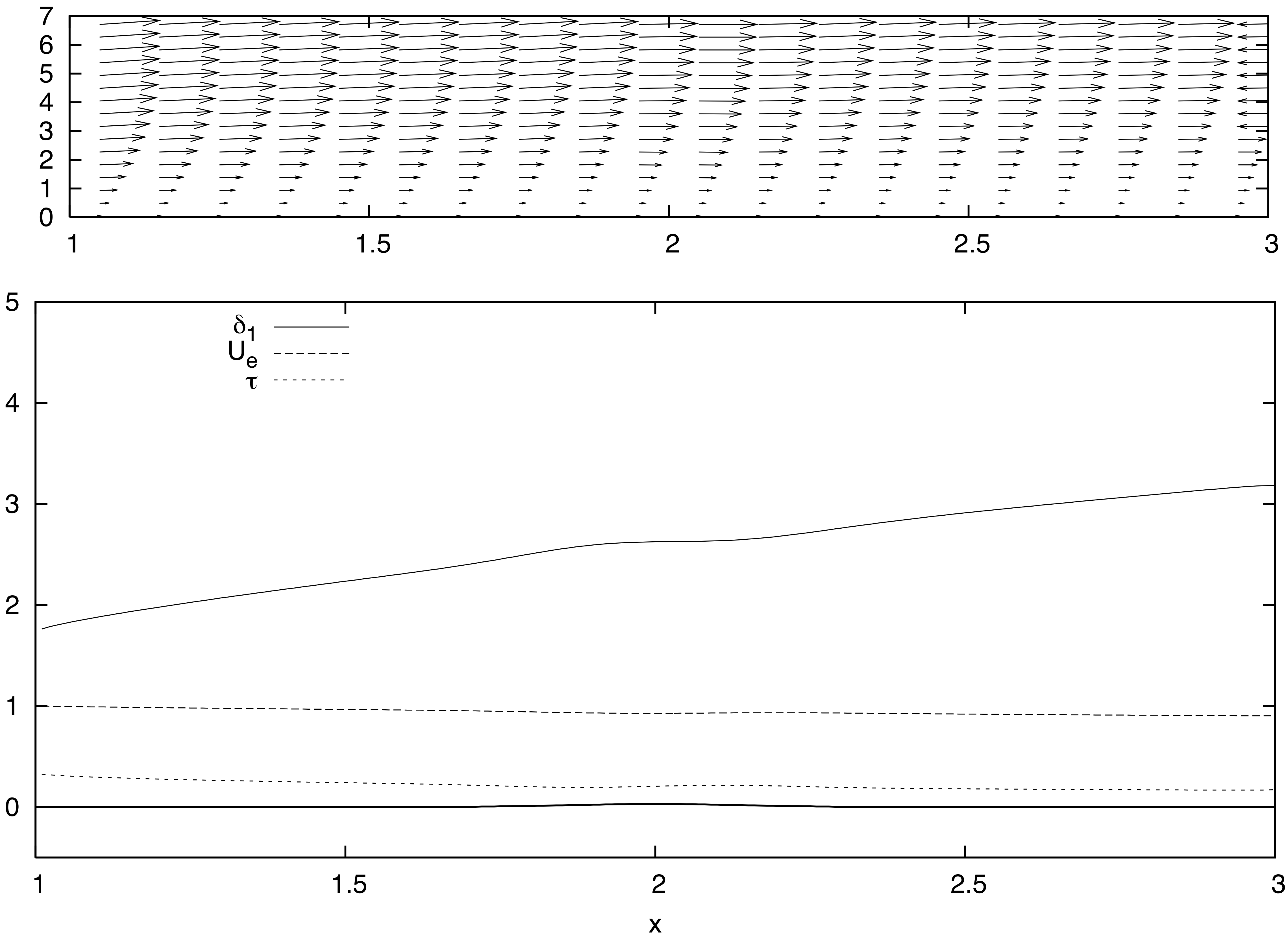


Figure 18: Subcritical flow on a flat plate[click to launch the movie, Adobe Reader required]. Top the velocity field $\tilde{u}, \tilde{v}$ (Prandtl transform), bottom the wall, here a bump, the displacement thickness $\tilde{\delta}_1$ (starting from Blasius value 1.7 in $\bar{x} = 1$), the skin friction (starting from Blasius value 0.3 in $\bar{x} = 1$) and the outer velocity starting from Ideal Fluid value 1 in $\bar{x} = 1$. A positive disturbance of the wall increases the velocity and decreases the displacement. Separation may occur after the bump.

back

supercritical F>1





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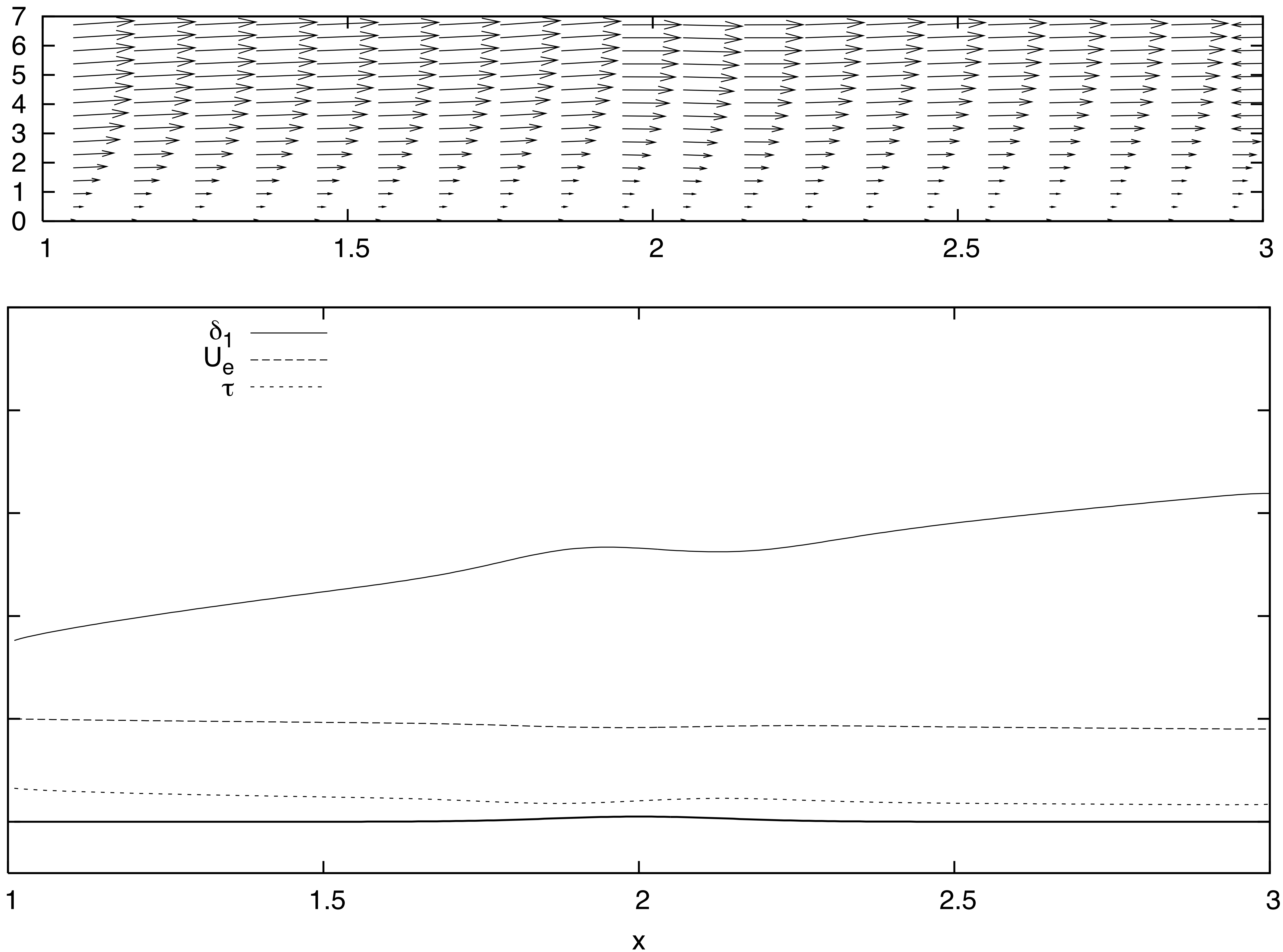
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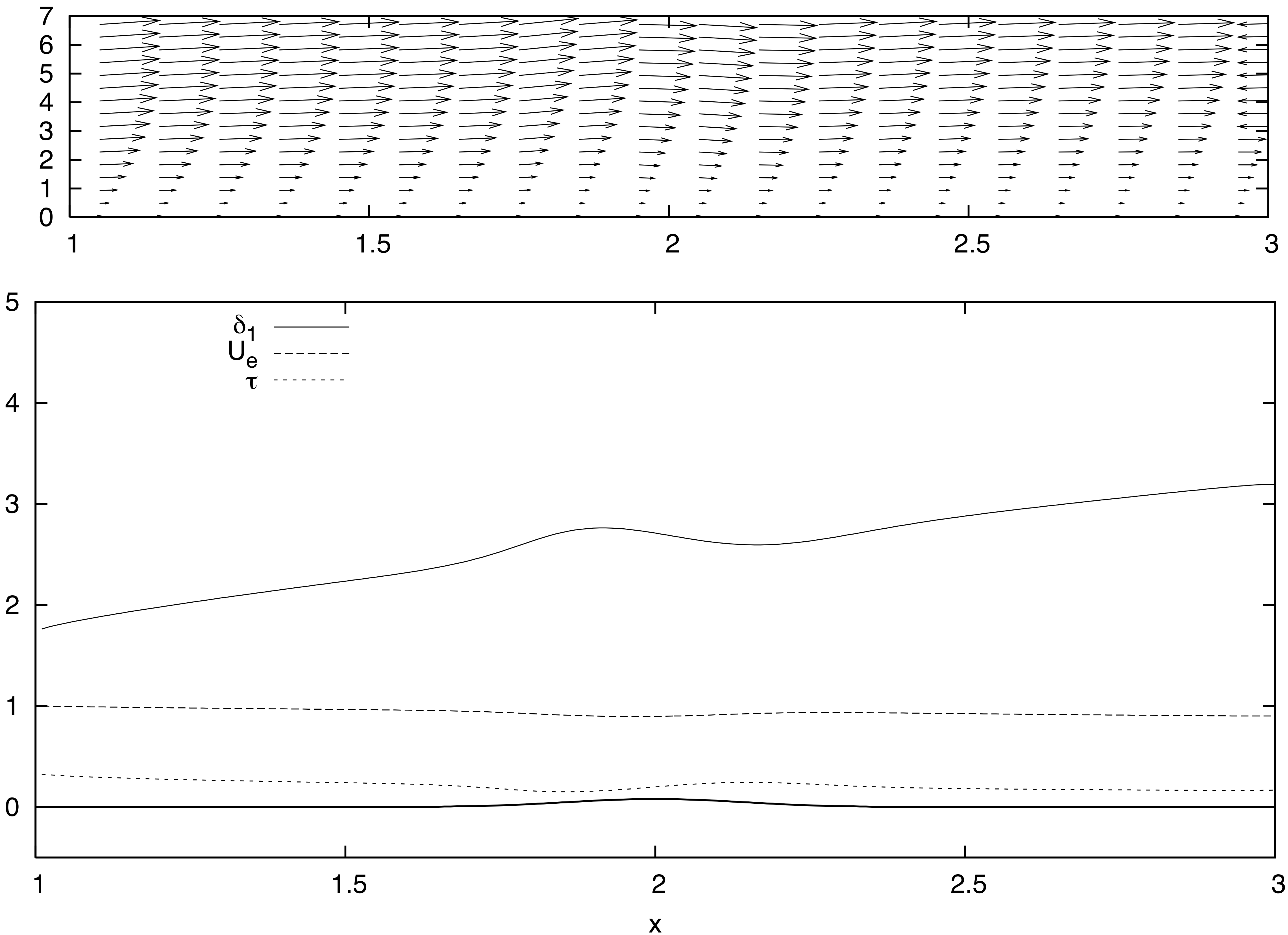

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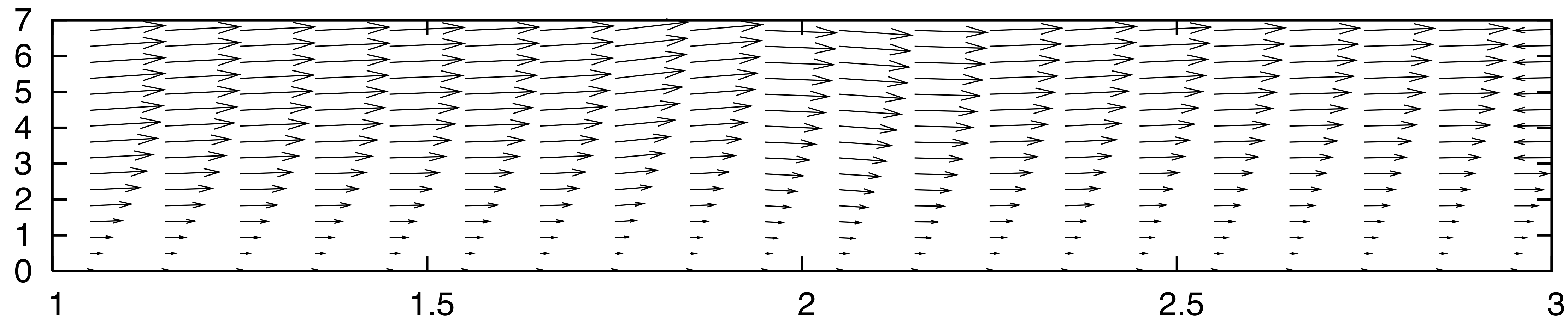

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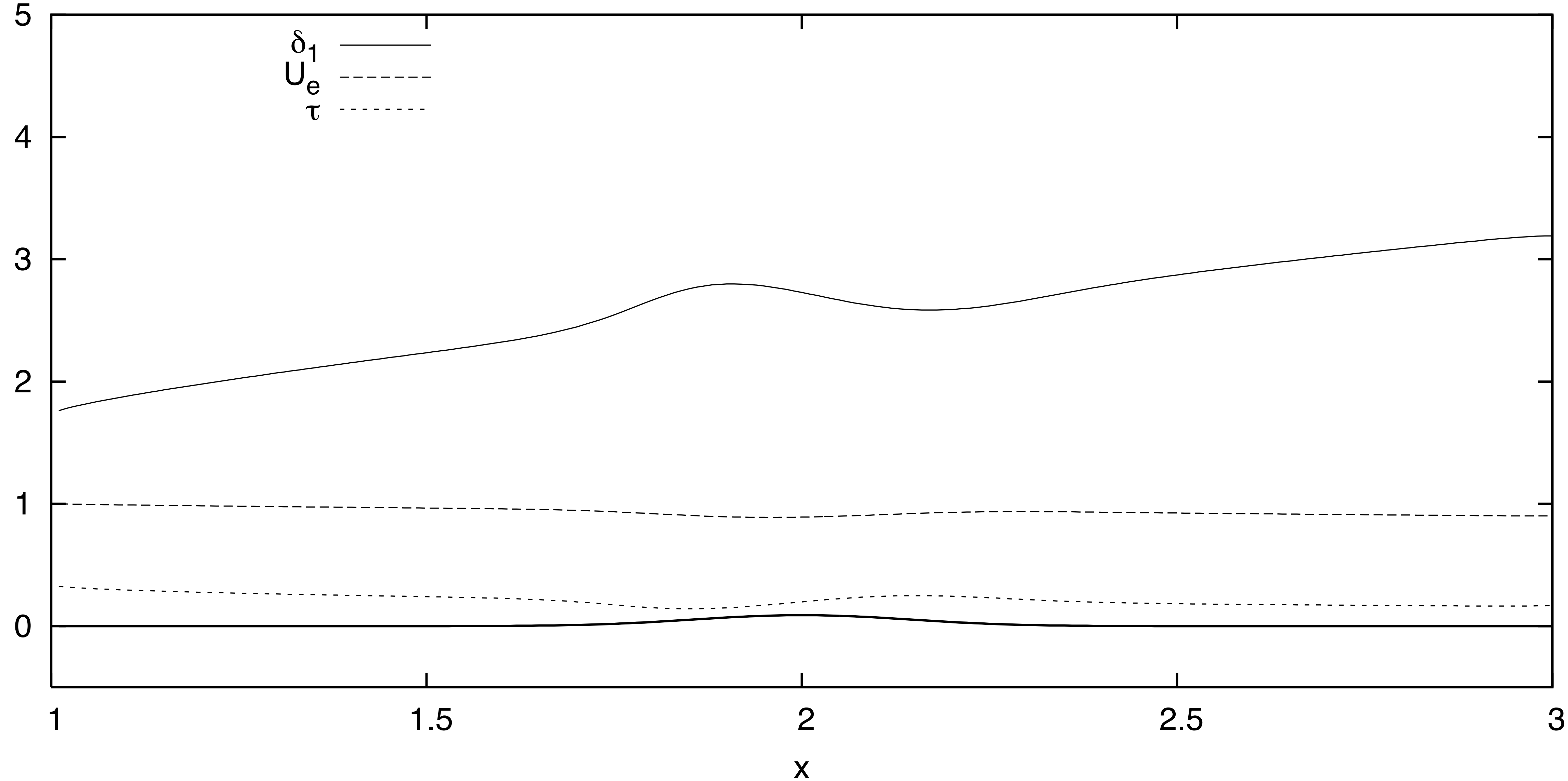

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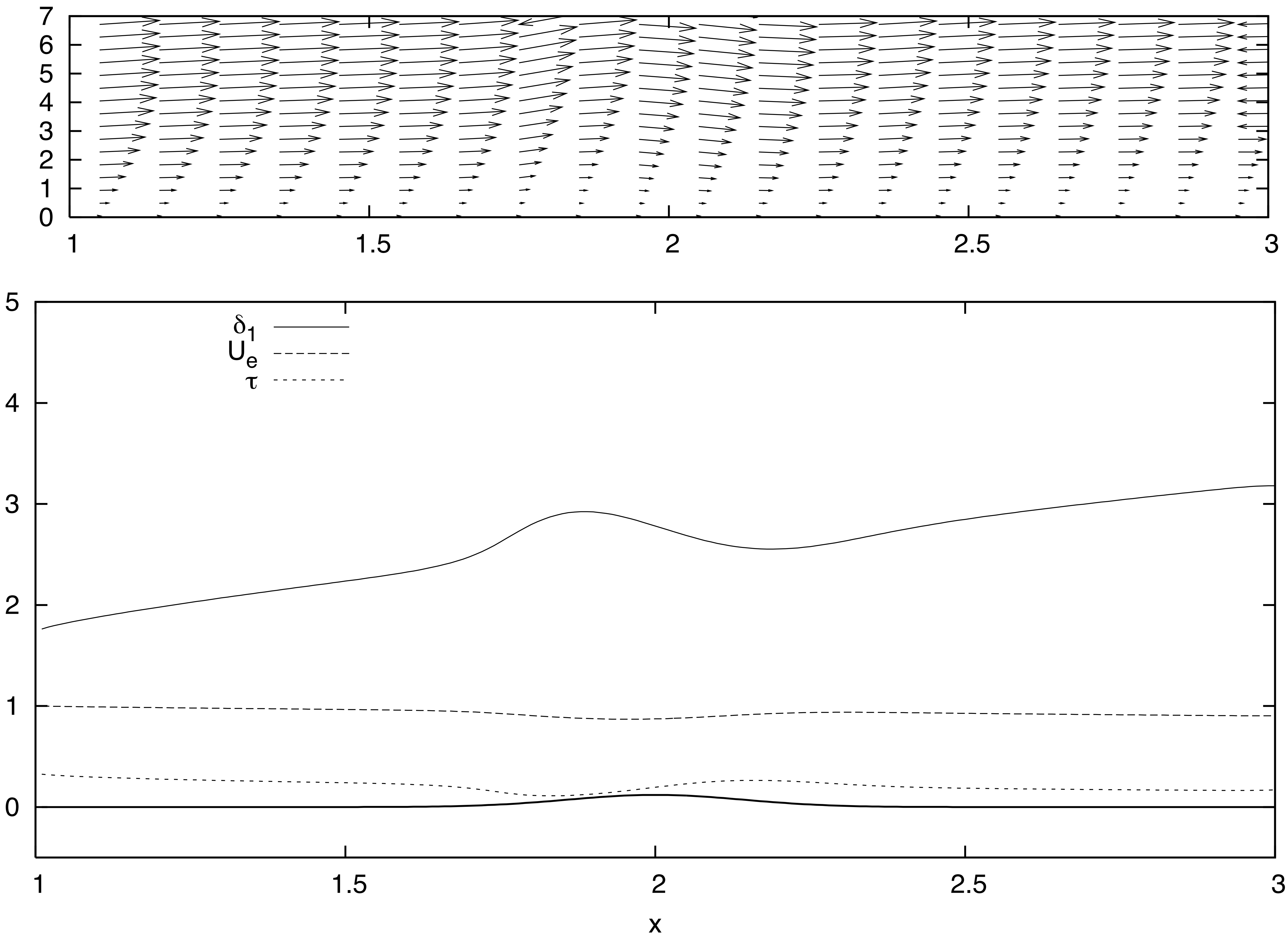
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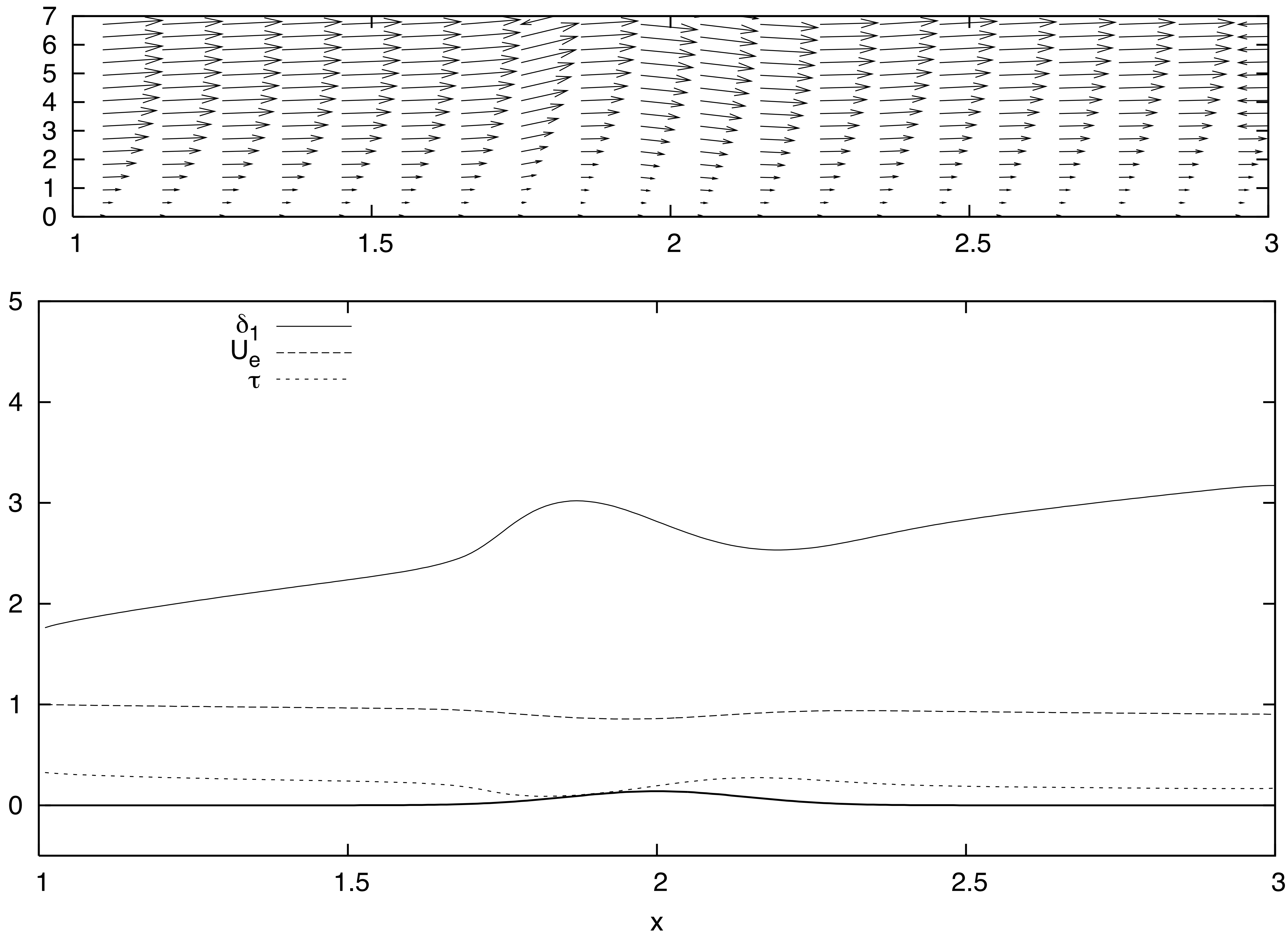

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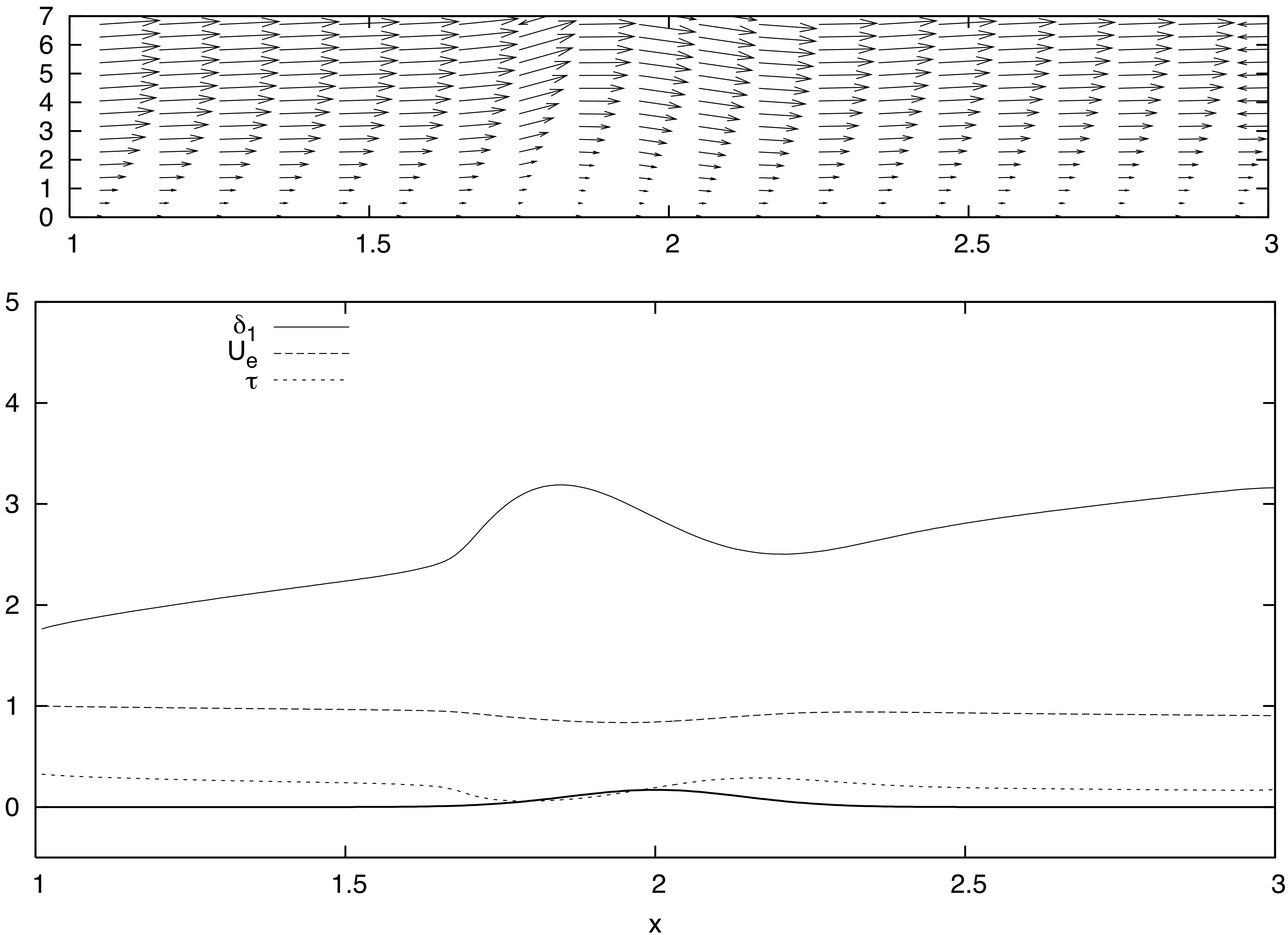

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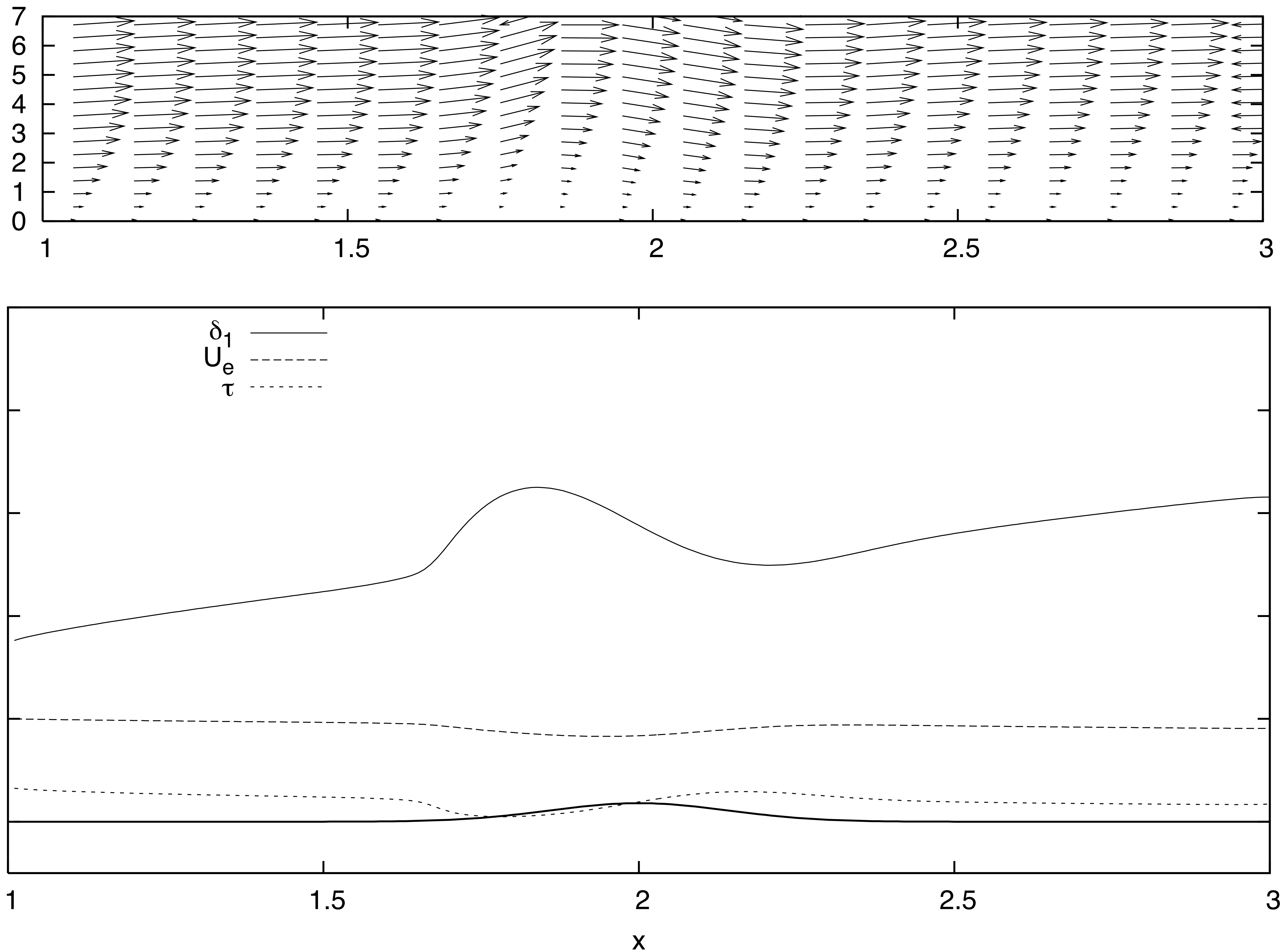
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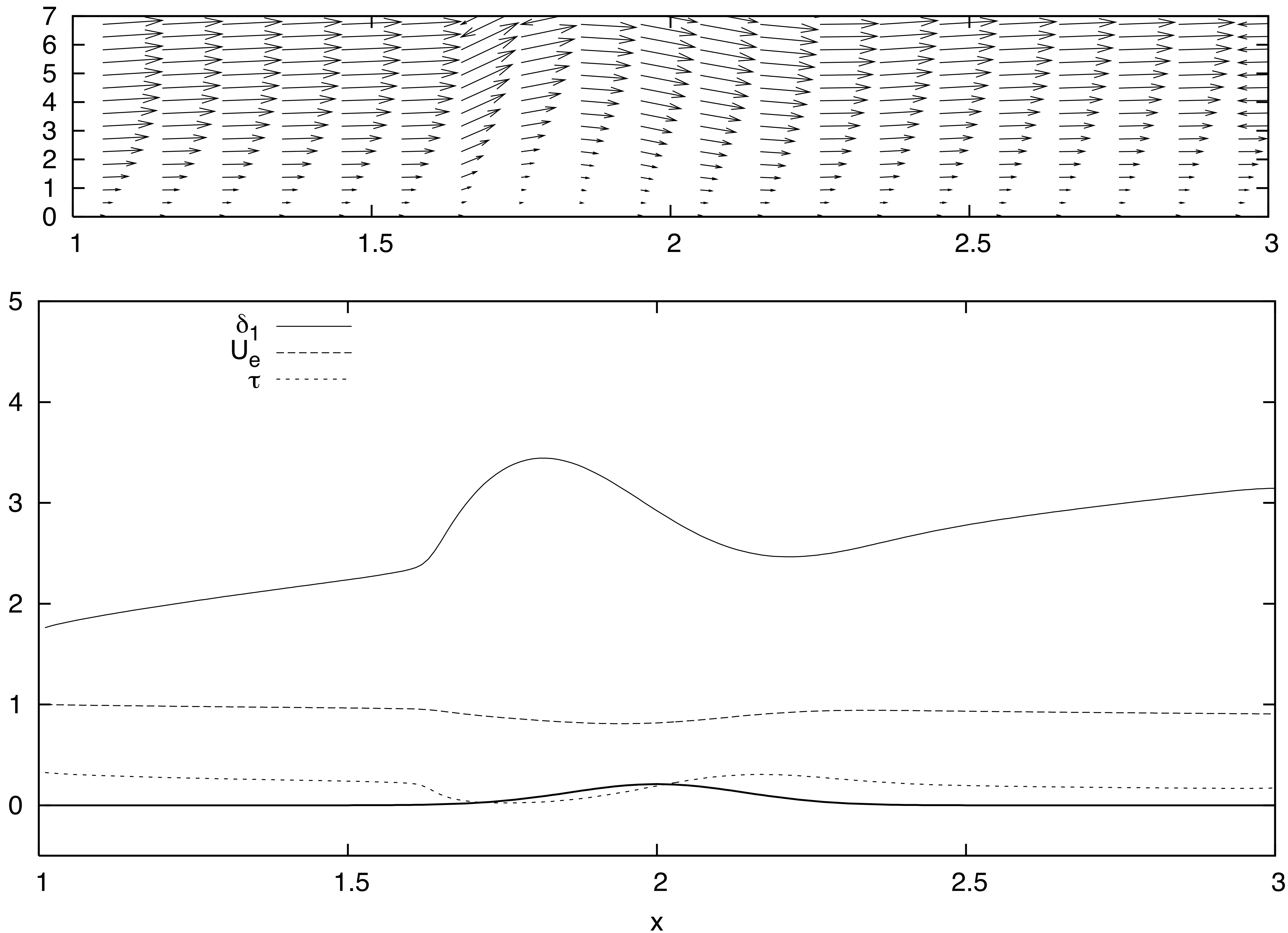

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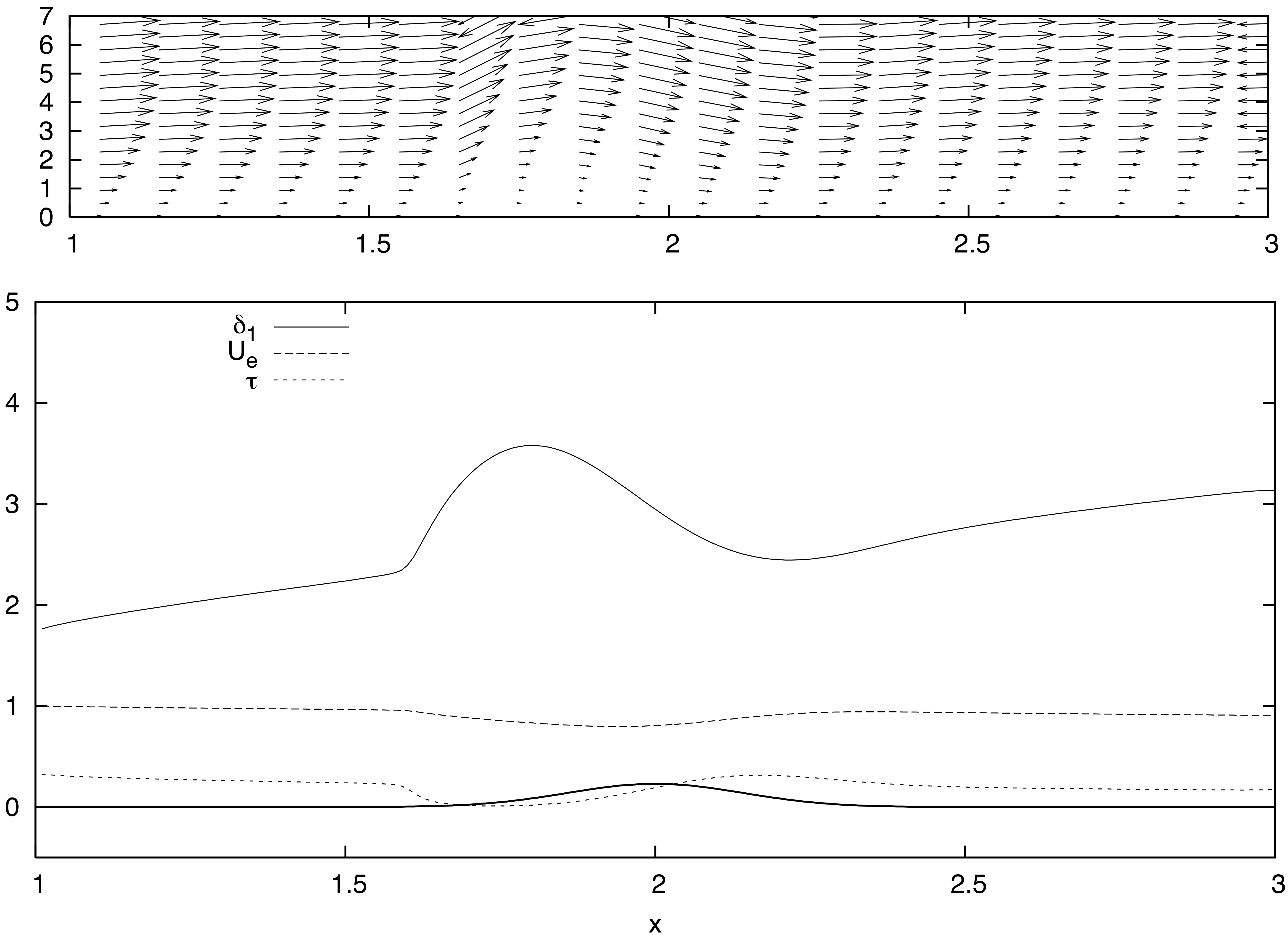

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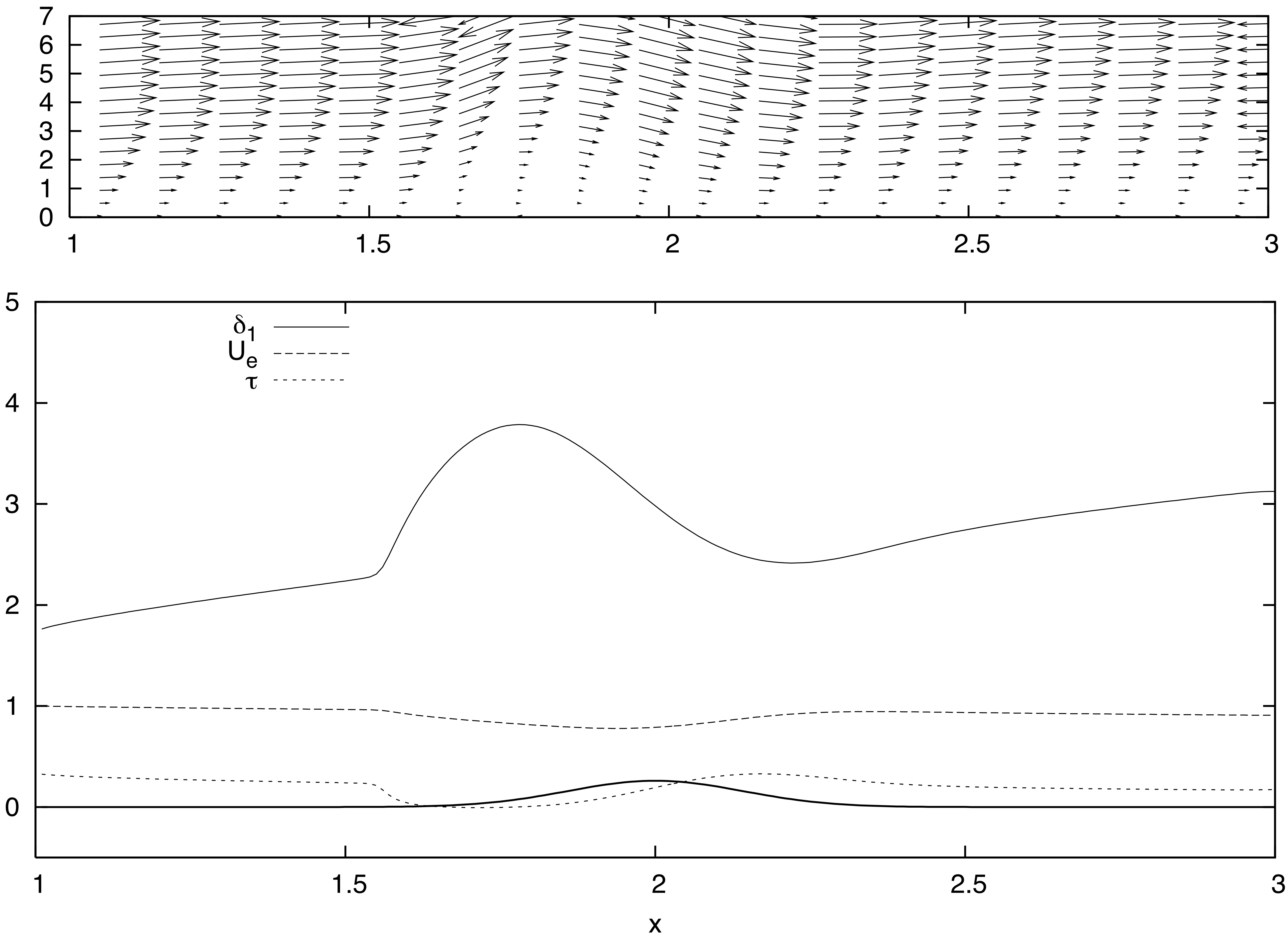

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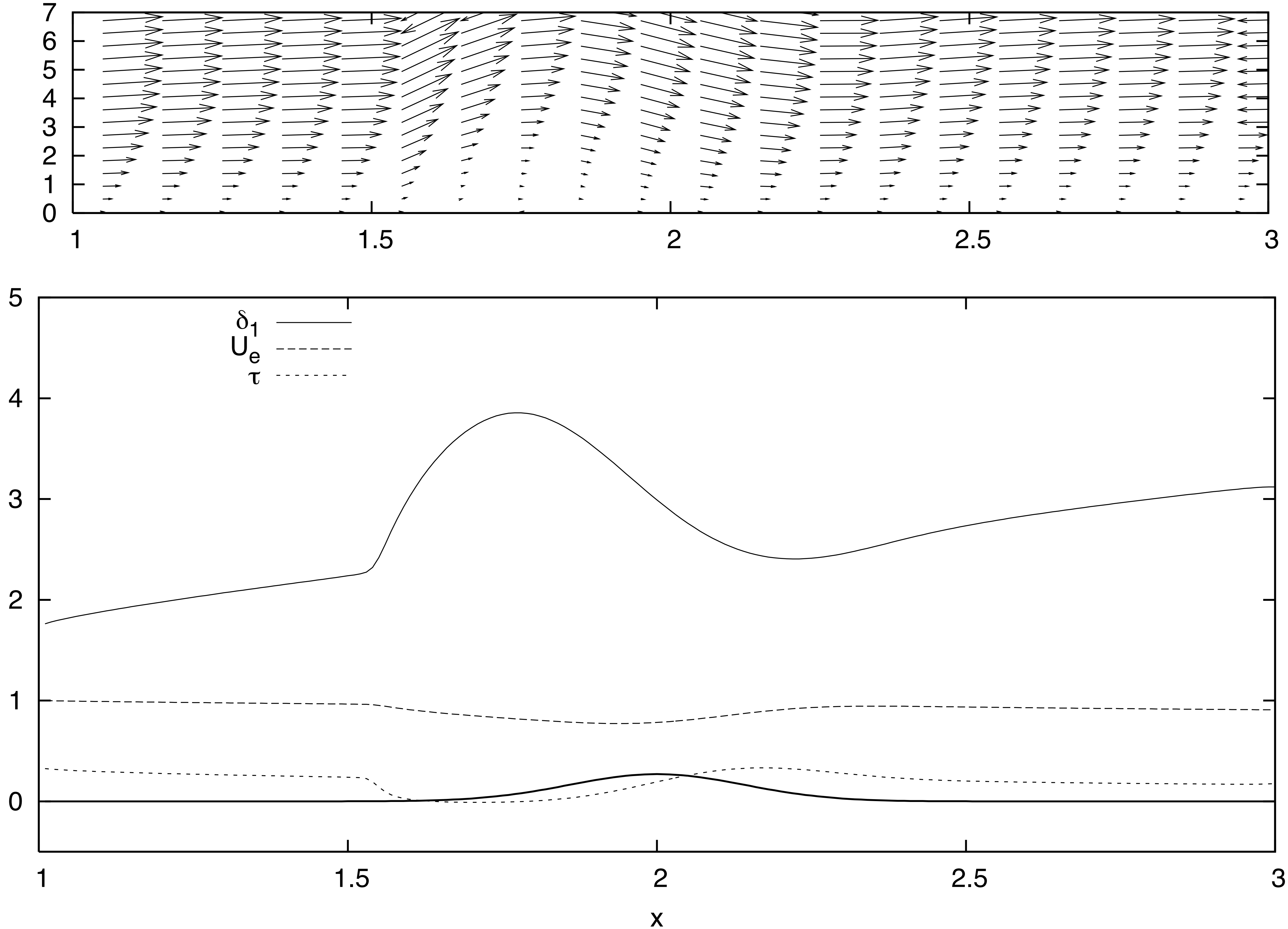

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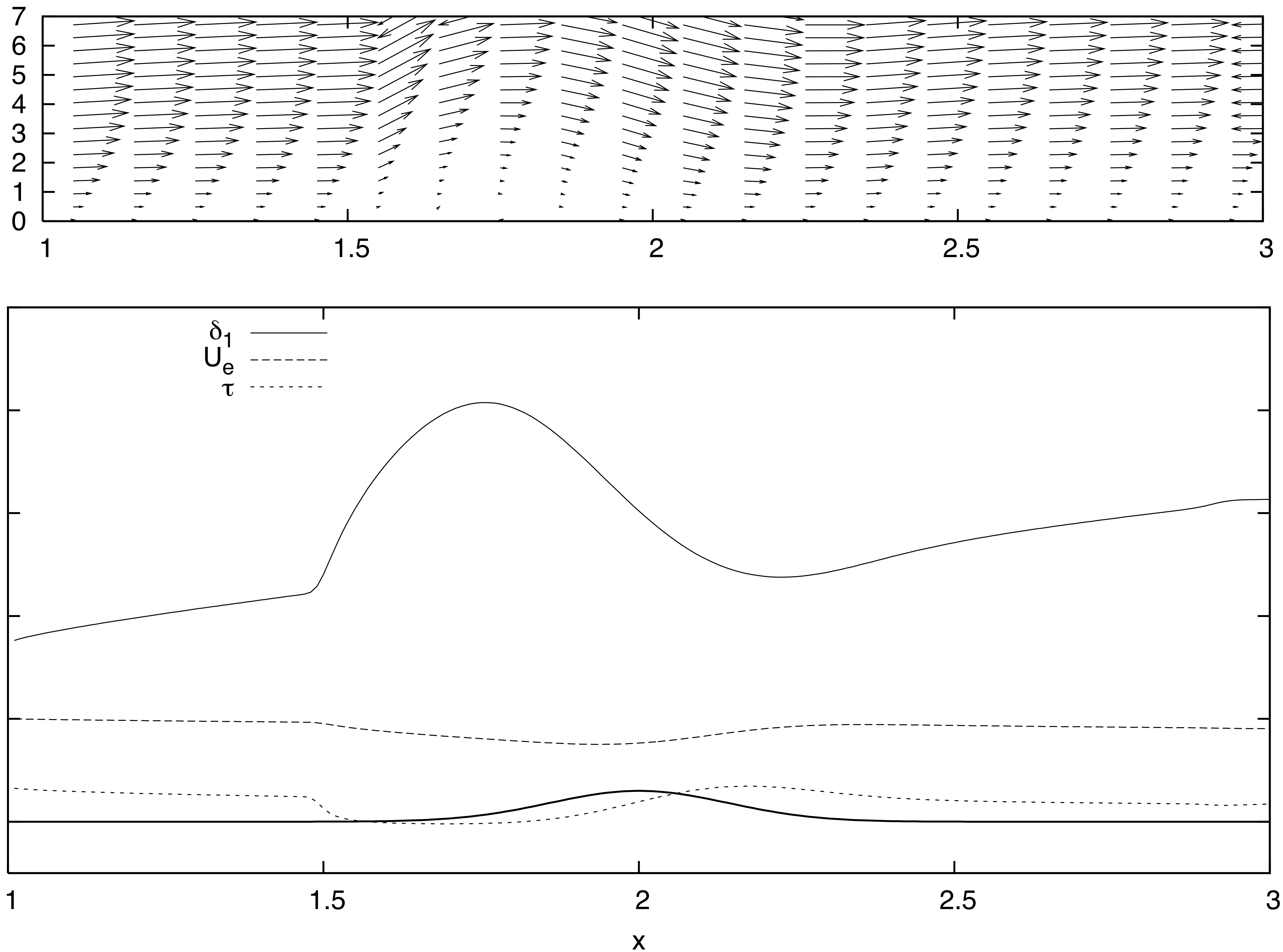

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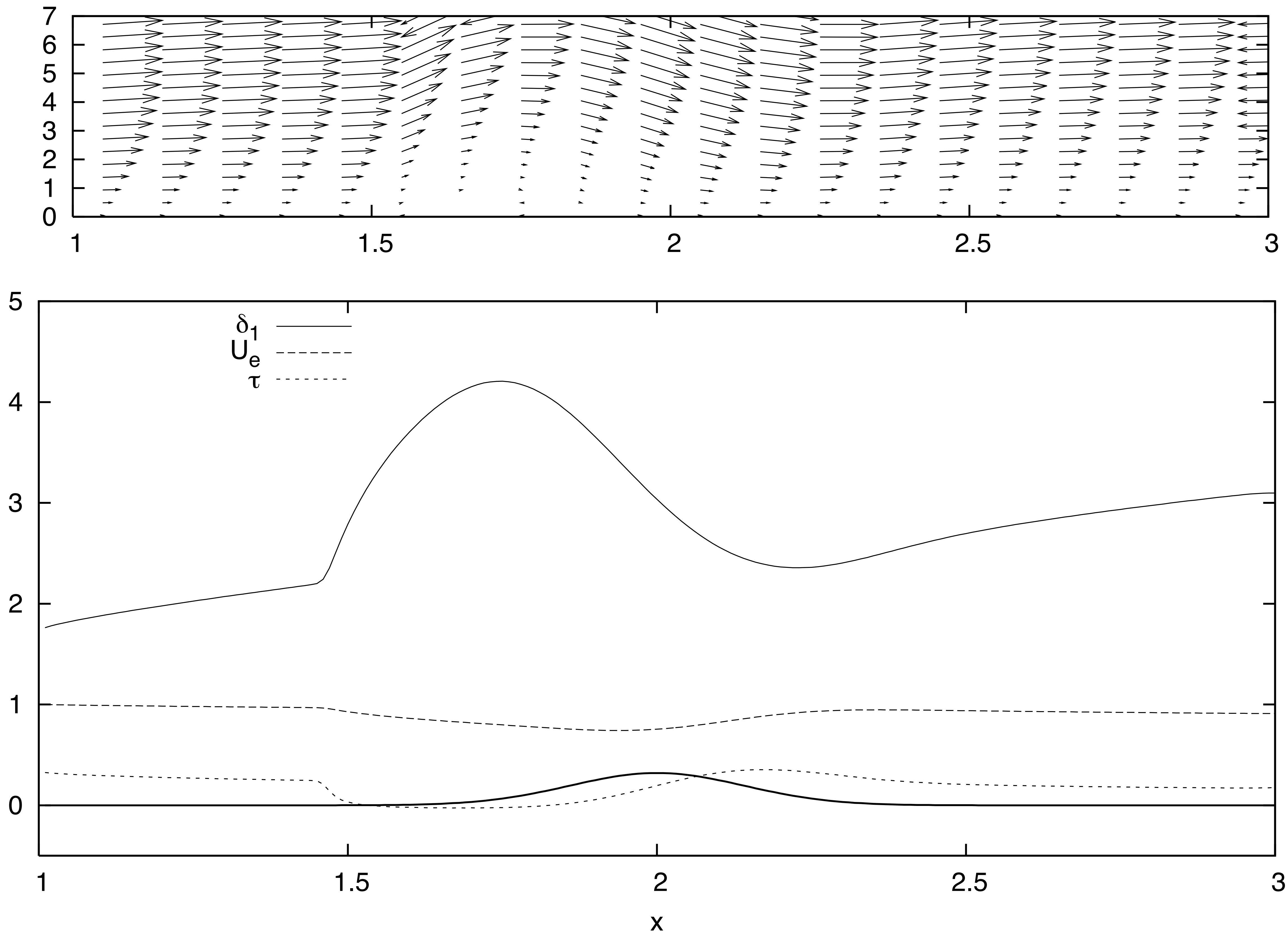

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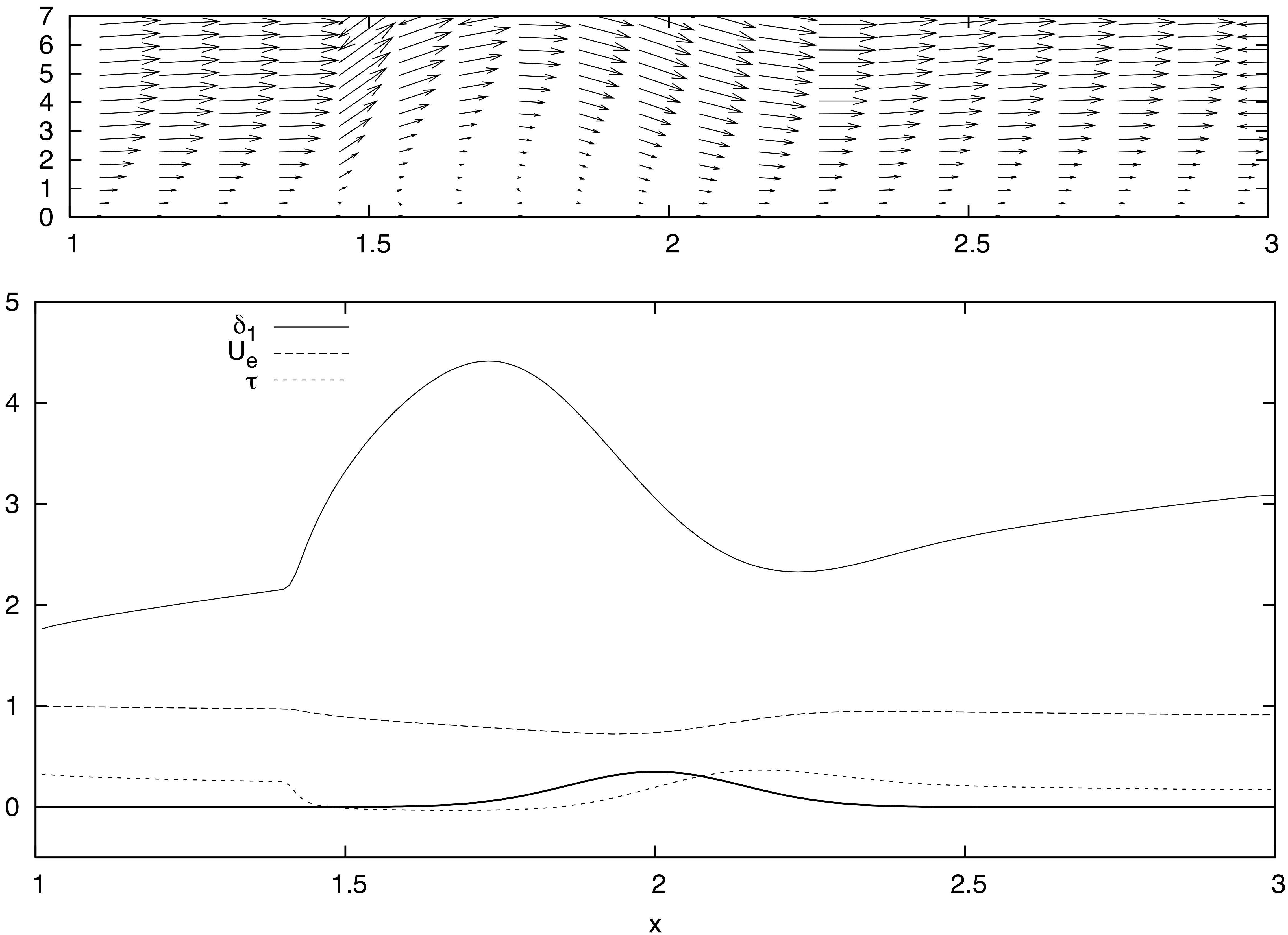

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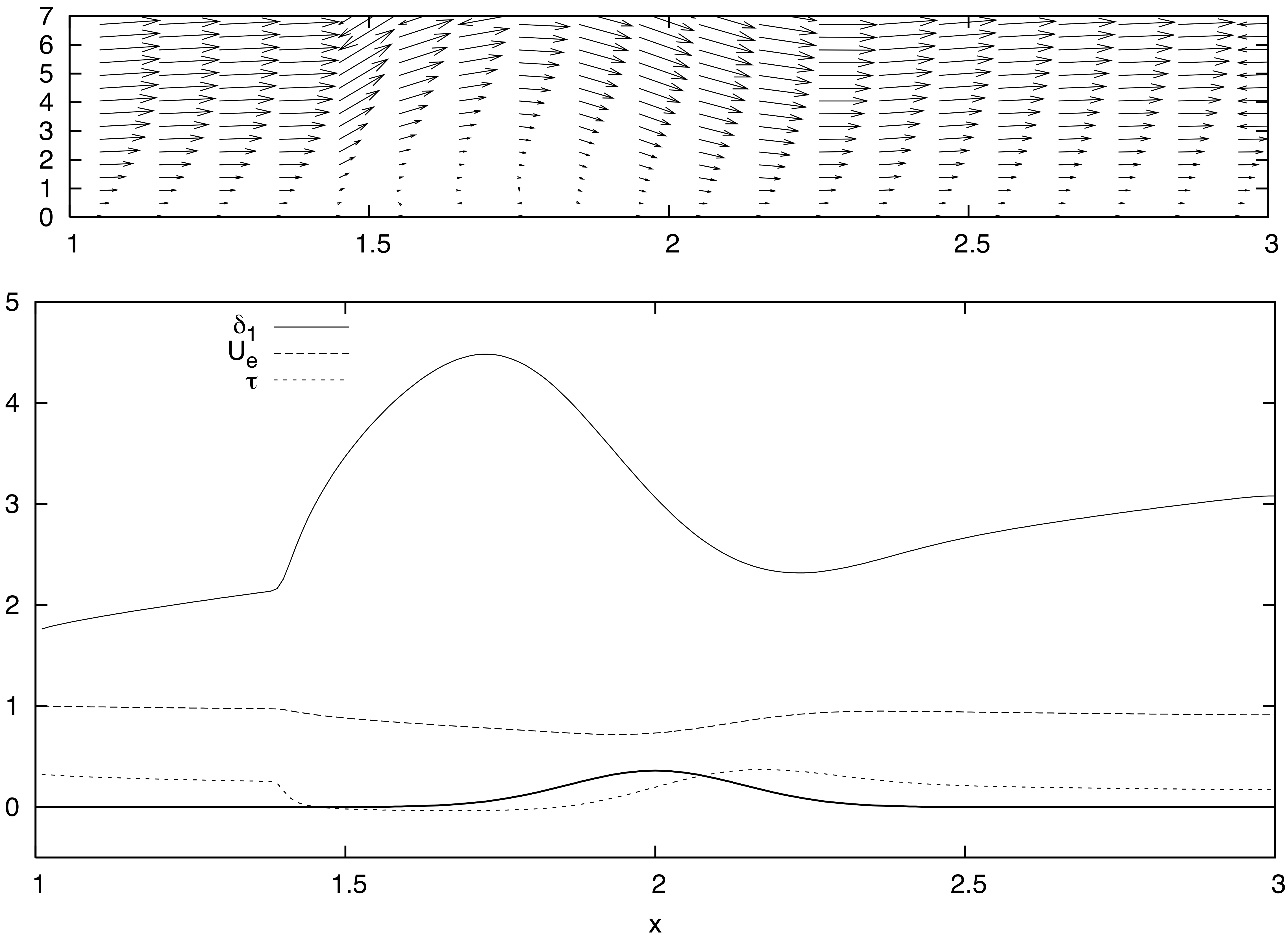
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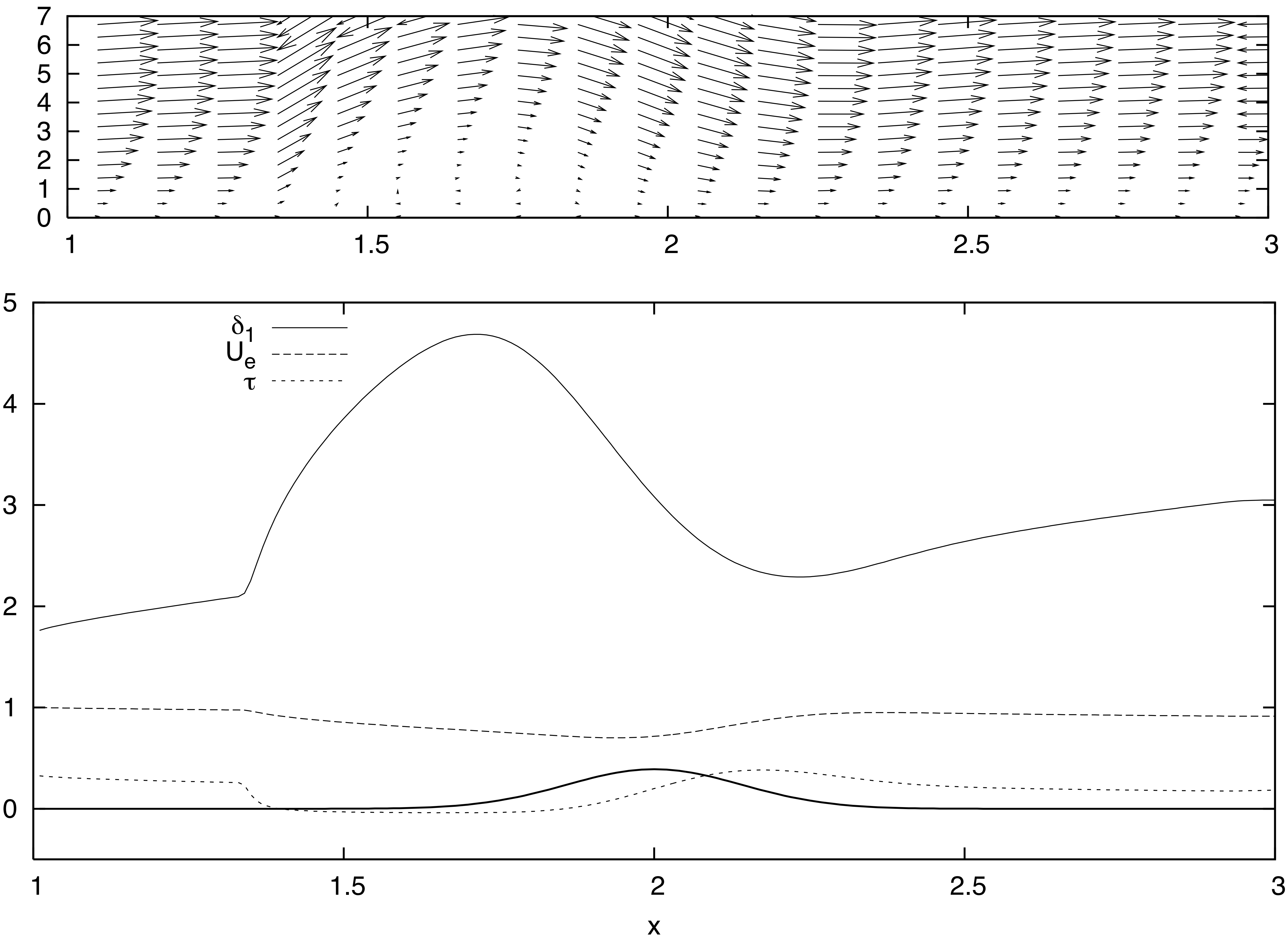

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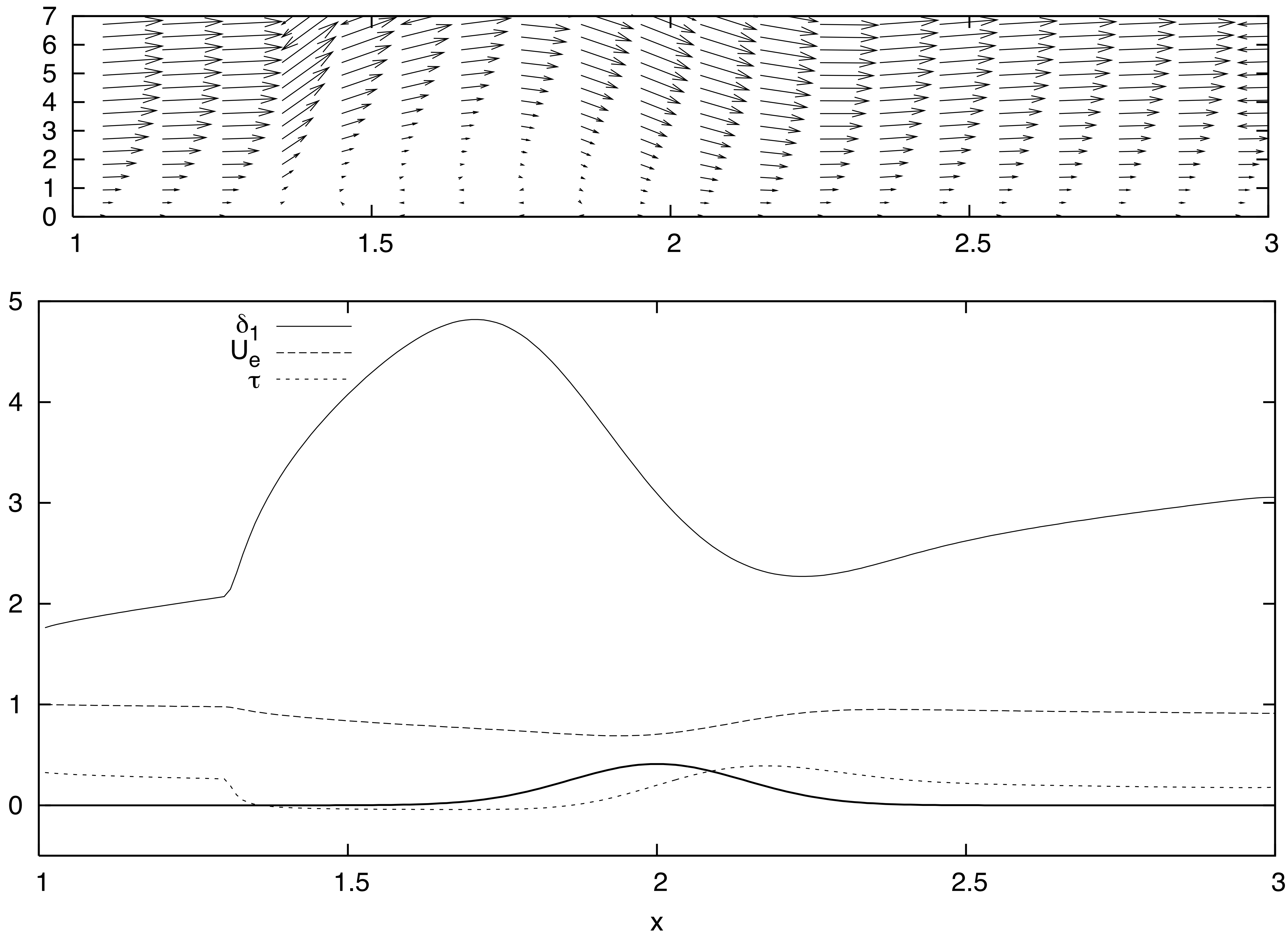

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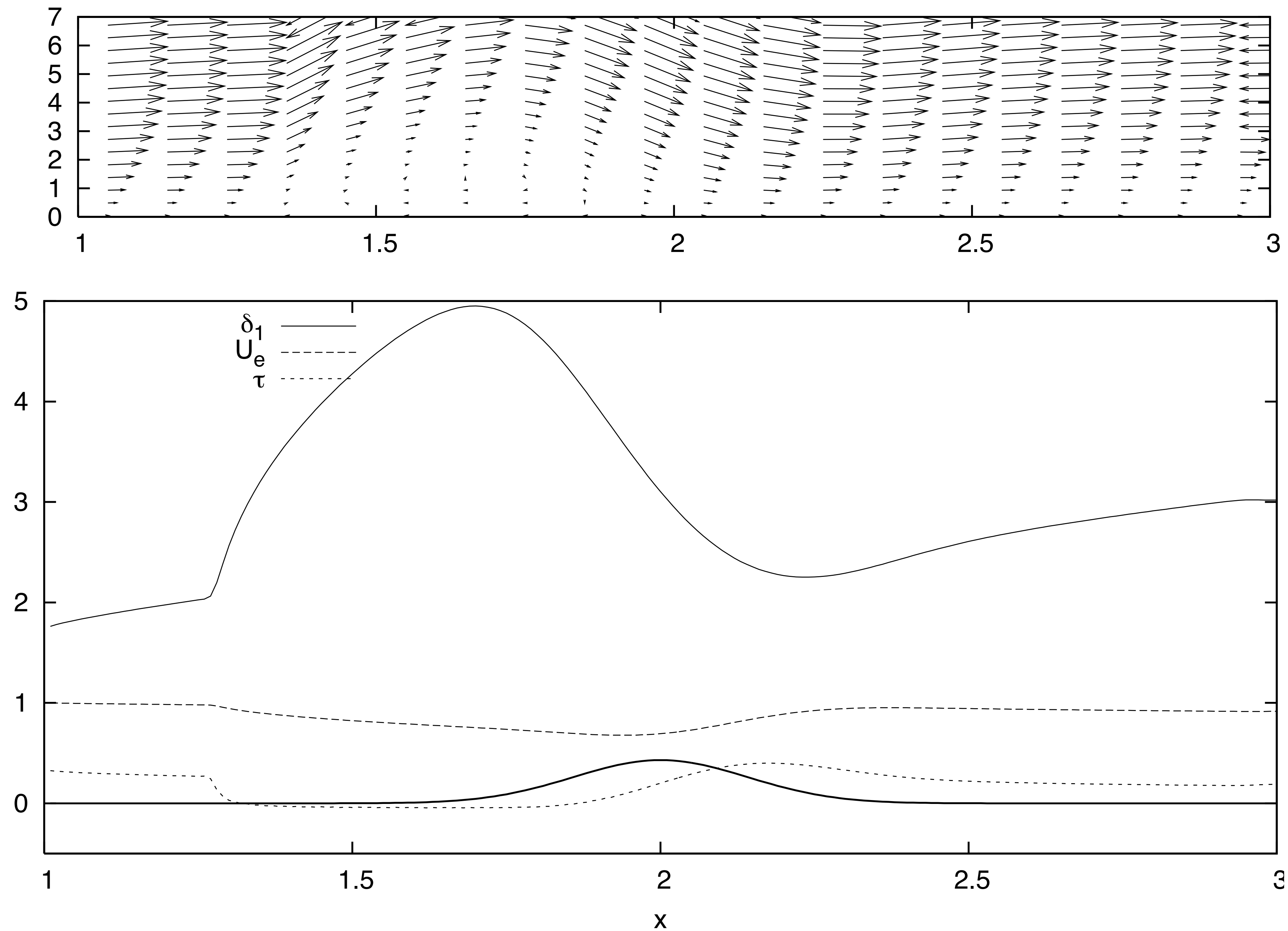
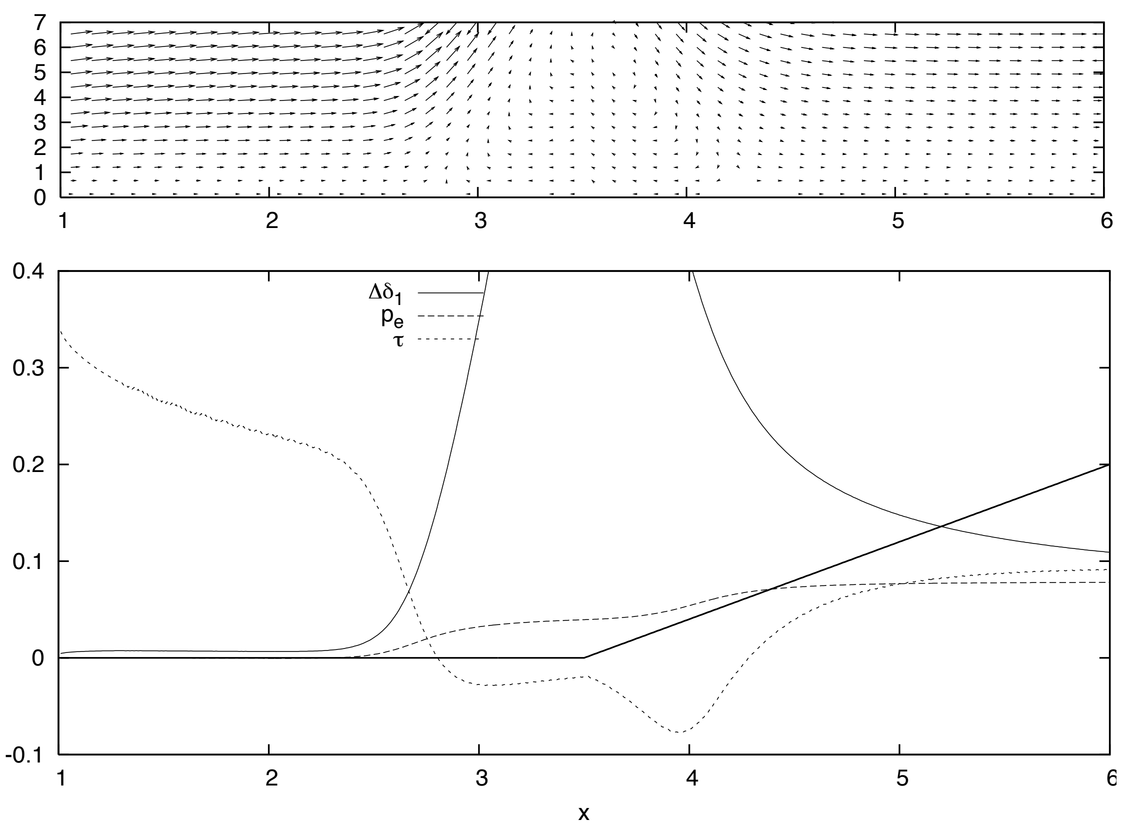


Figure 19: Supercritical flow on a flat plate [click to launch the movie, Adobe Reader required]. Top the velocity field $\tilde{u}, \tilde{v}$ (Prandtl transform), bottom the wall, here a bump, the displacement thickness $\tilde{\delta}_1$ (starting from Blasius value 1.7 in $\bar{x} = 1$), the skin friction (starting from Blasius value 0.3 in $\bar{x} = 1$) and the outer velocity starting from Ideal Fluid value 1 in $\bar{x} = 1$. A positive disturbance of the wall decreases the velocity and decreases the displacement. Separation may occur before the bump, note the long upstream influence.

back



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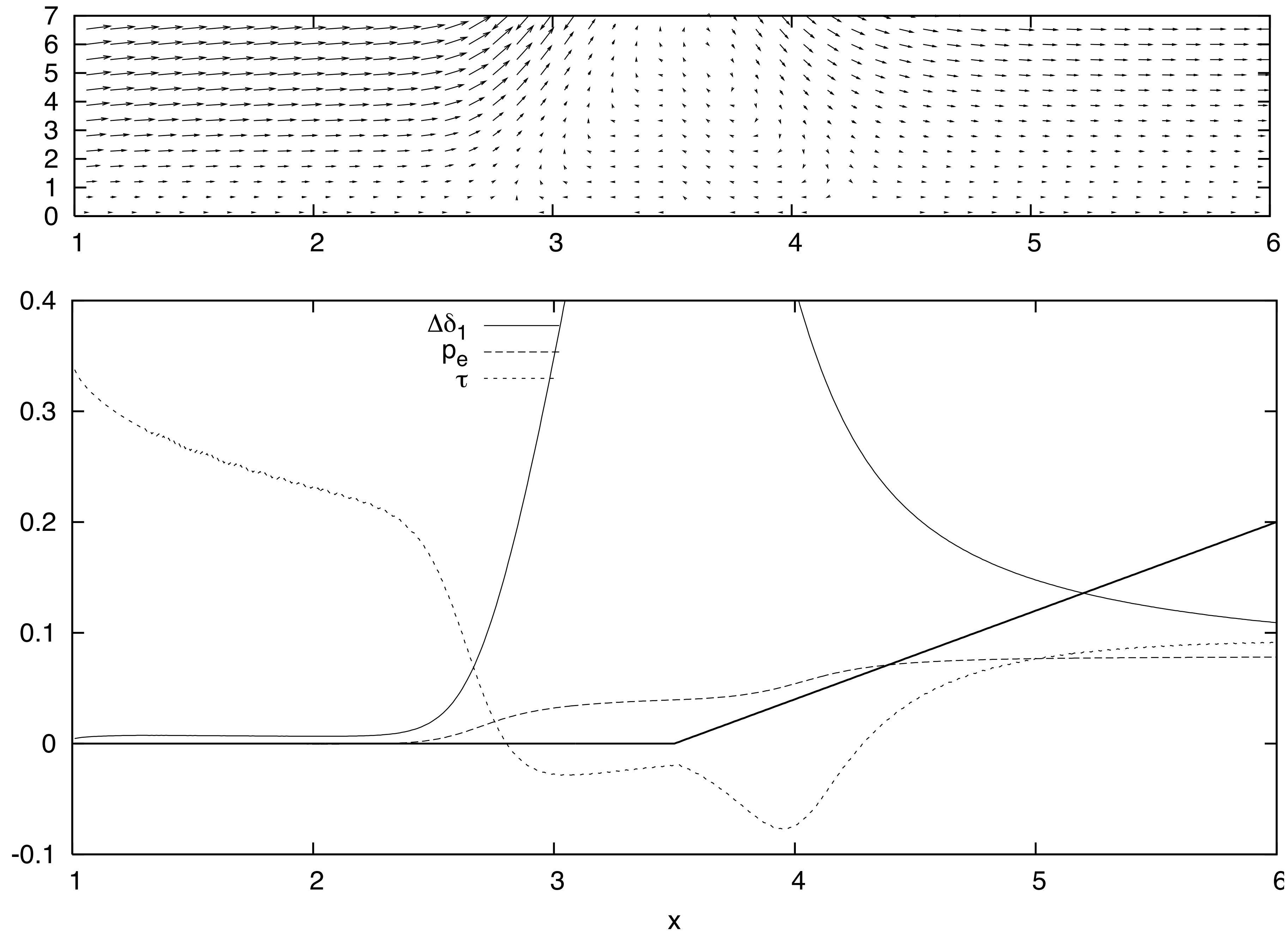
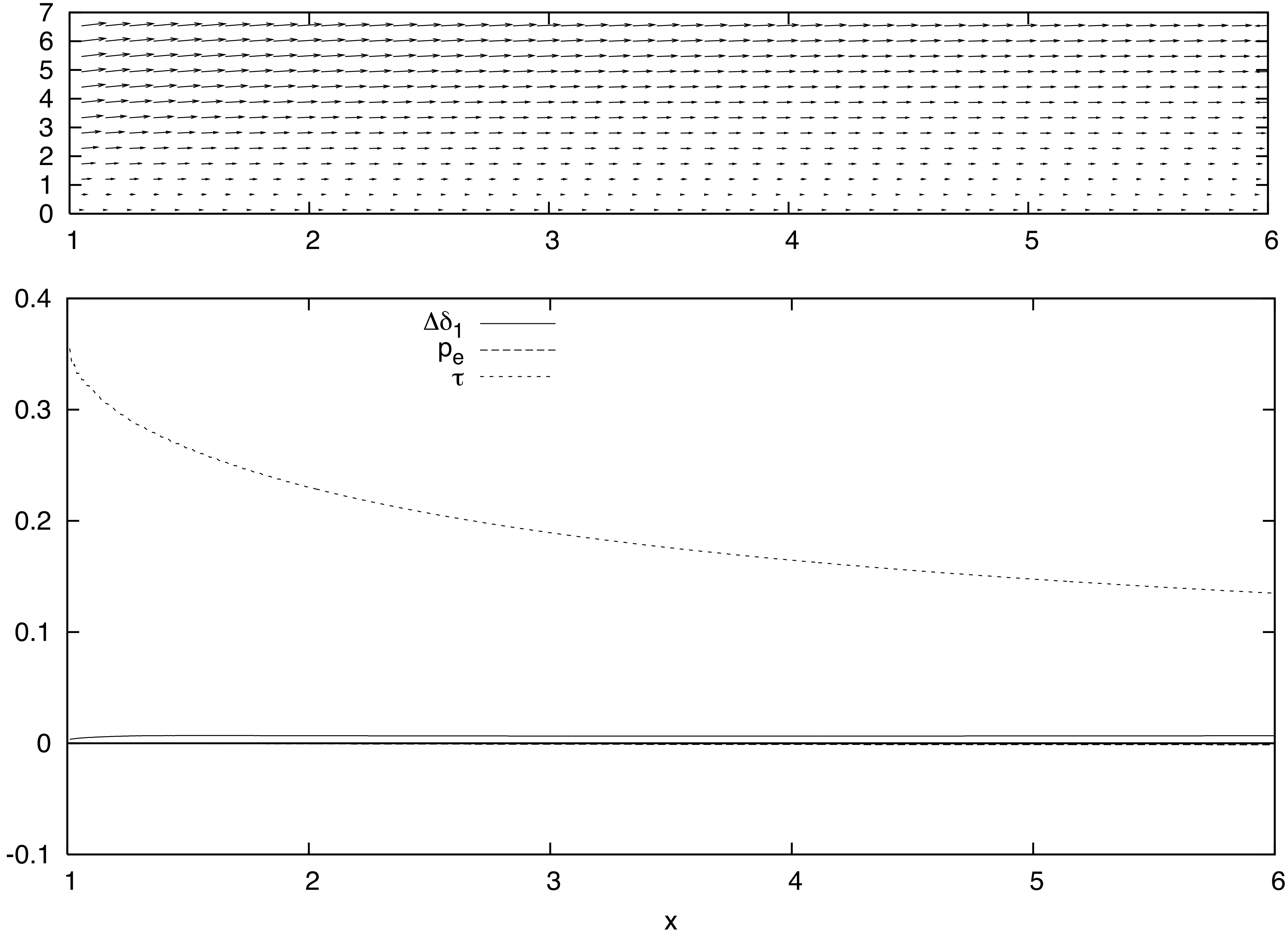


Figure 20: Supersonic flow on a flat plate with a wedge [click to launch the movie, Adobe Reader required]. Top the velocity field $\tilde{u}, \tilde{v}$ (Prandtl transform), bottom the wall, here a wedge in $\bar{x} = 3.5$, the perturbation of displacement thickness $\Delta\tilde{\delta}_1$ (starting from 0 in $\bar{x} = 1$), the skin friction (starting from Blasius value 0.3 in $\bar{x} = 1$) and the outer pressure starting from Ideal Fluid value 0 in $\bar{x} = 1$. Note the plateau pressure and the separation far upstream of the wedge.



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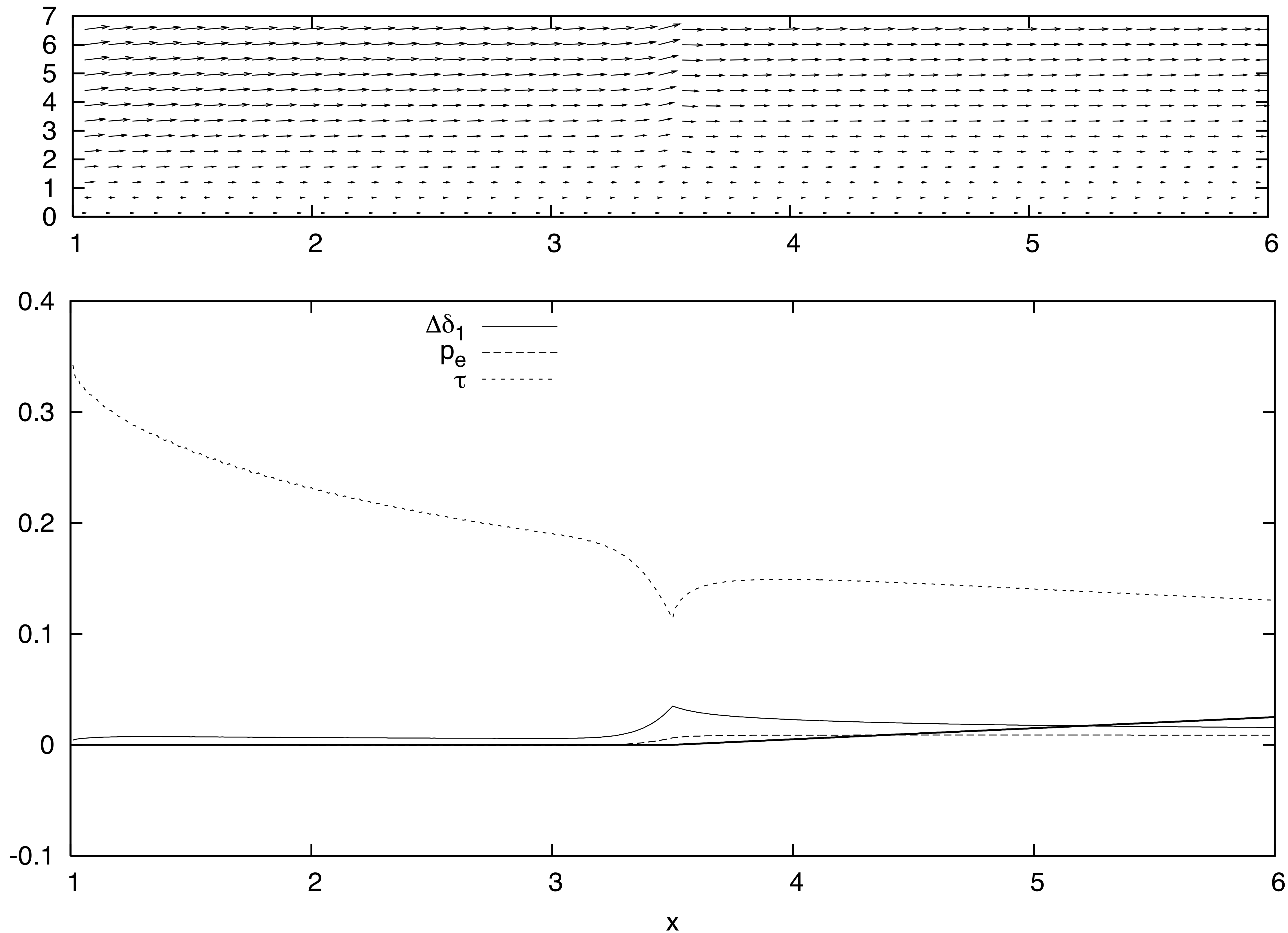

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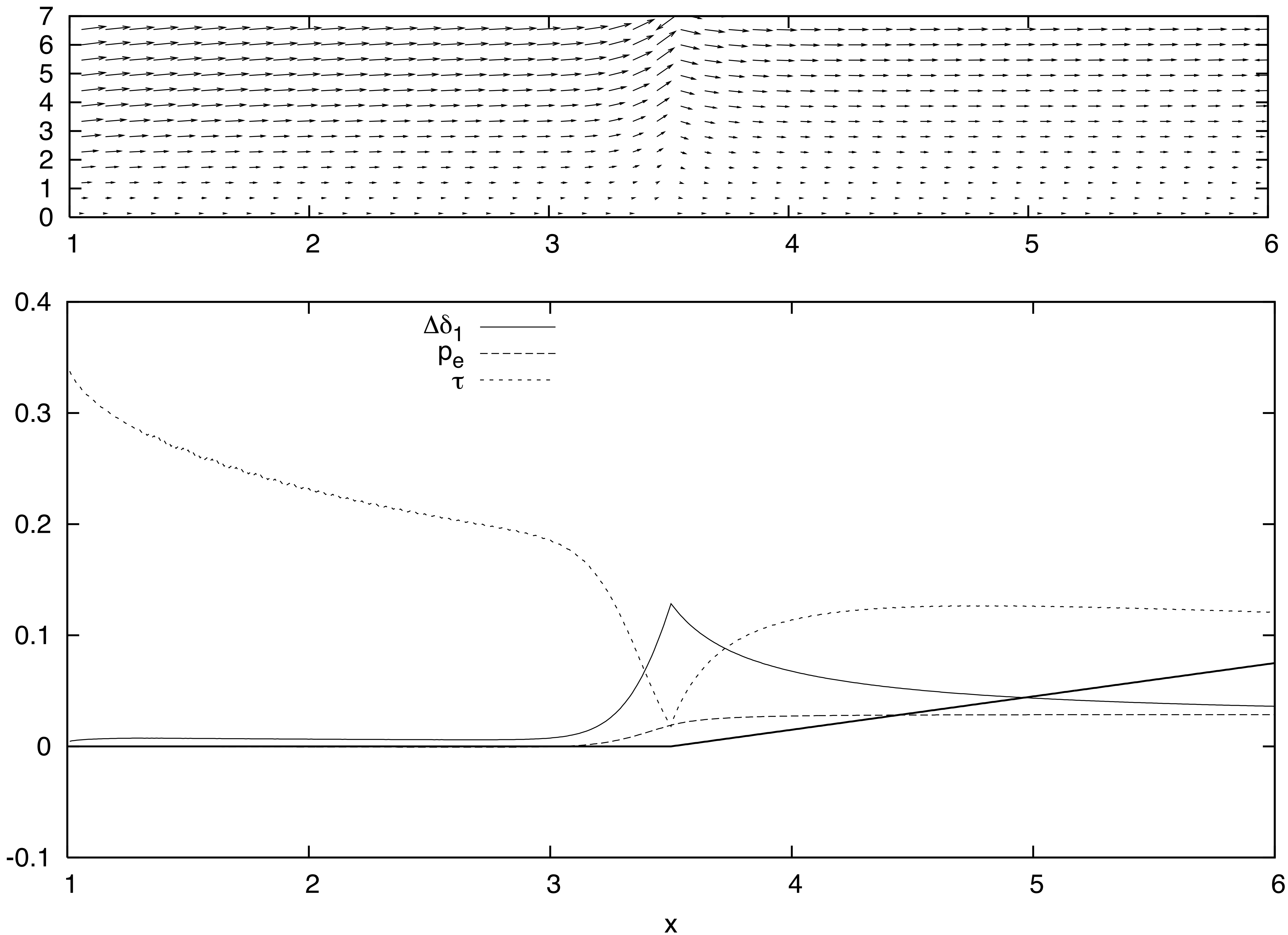

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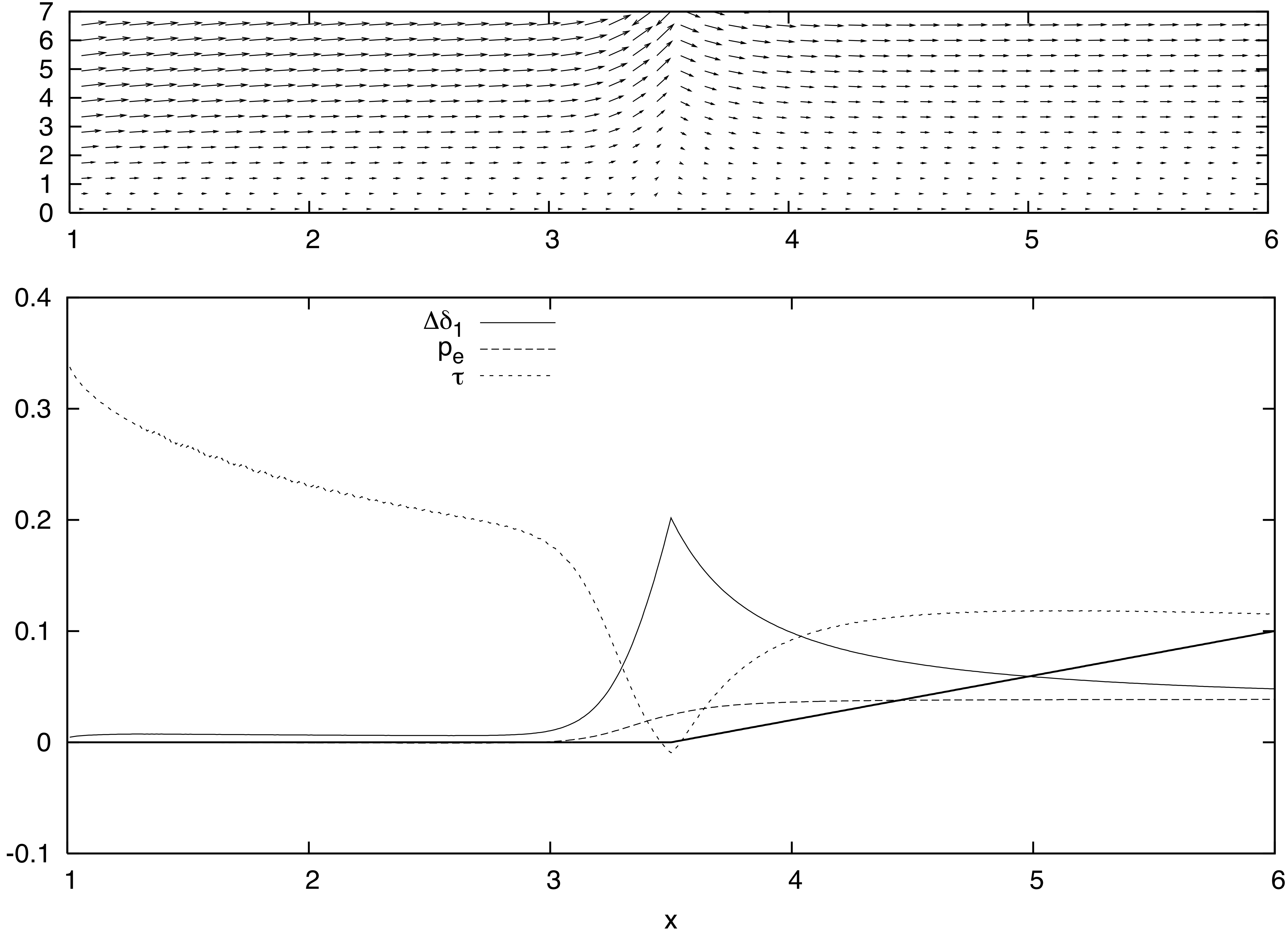

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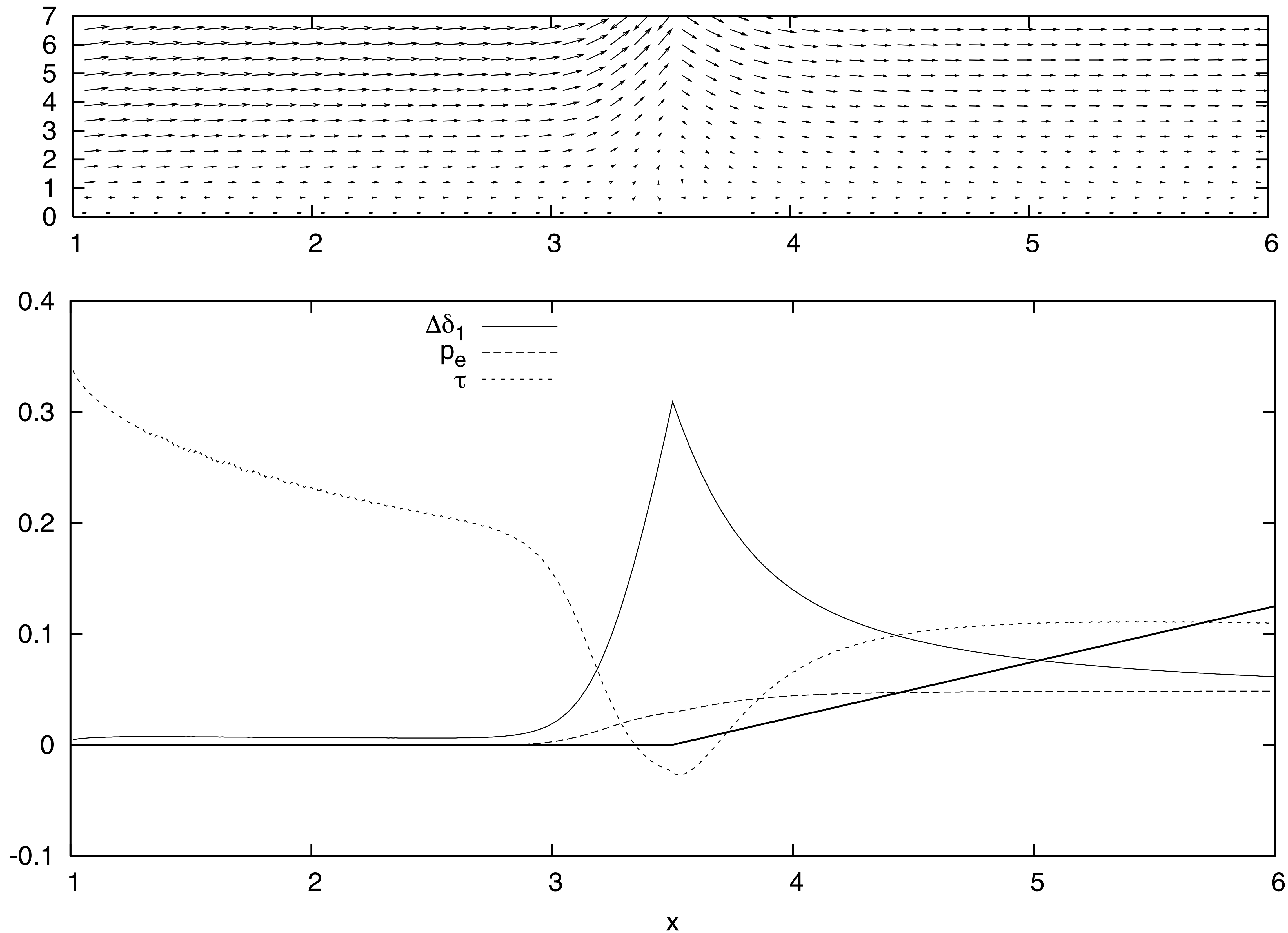

7
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5
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3
2
1
0
1
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0.4
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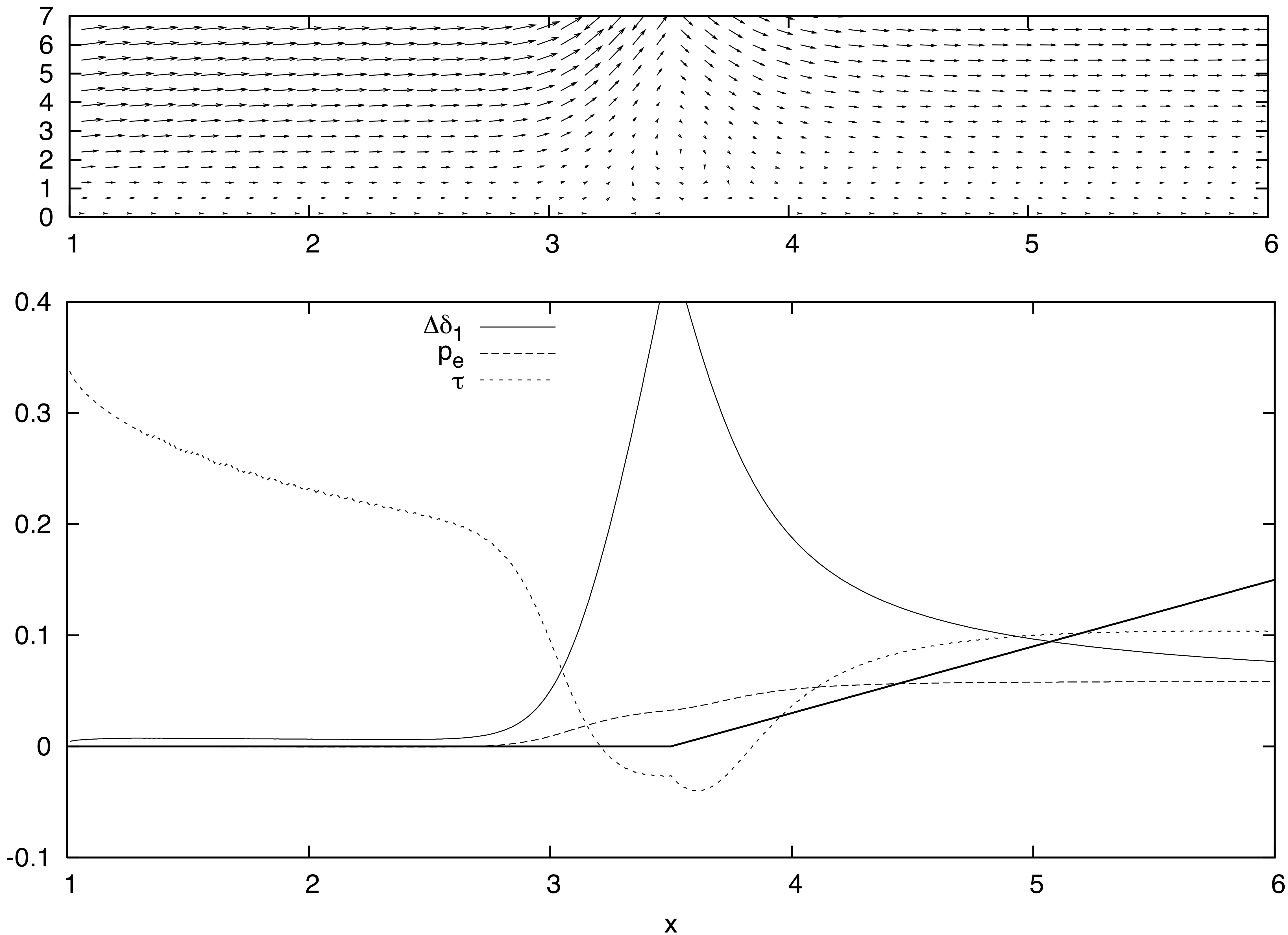
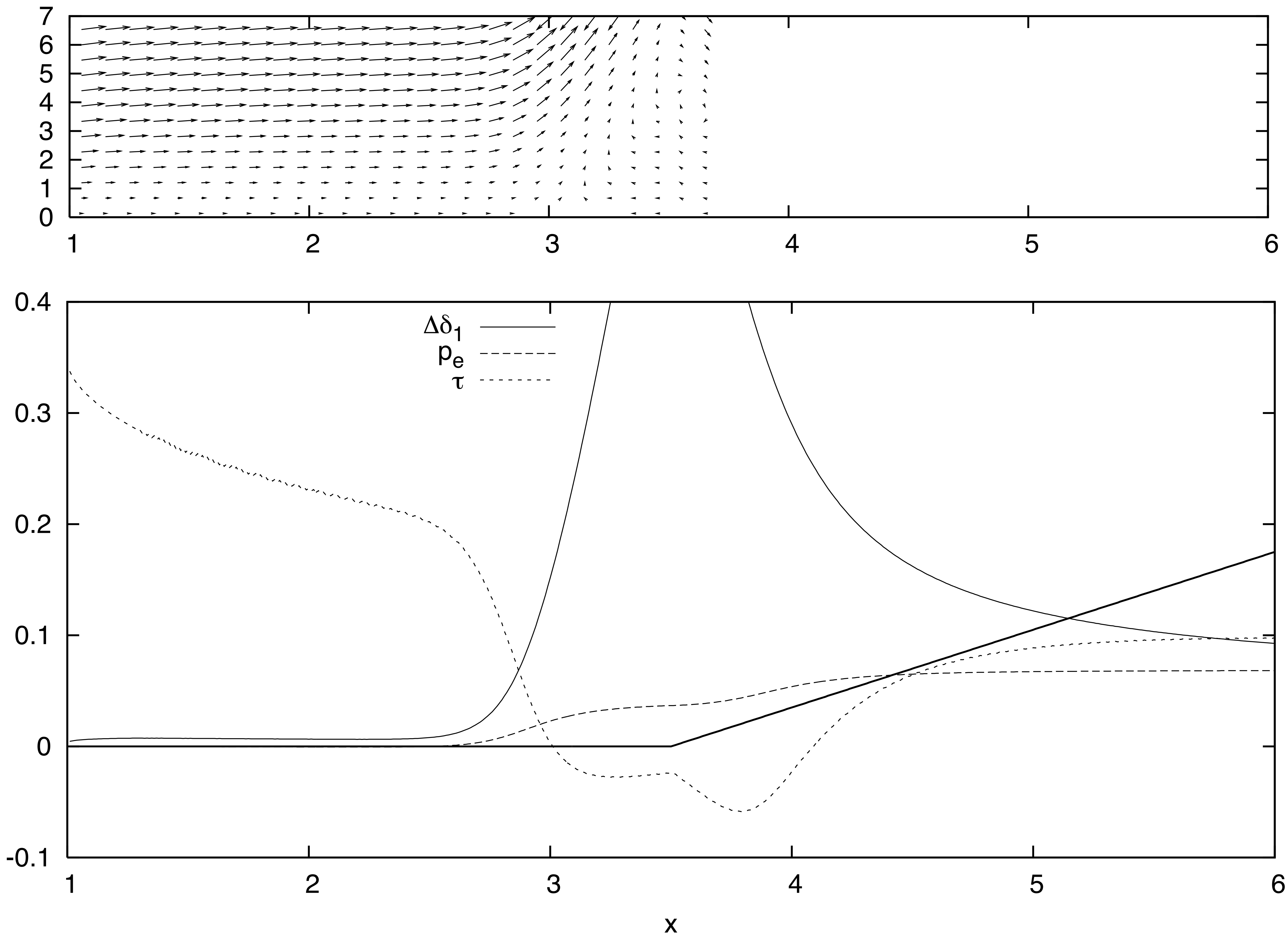
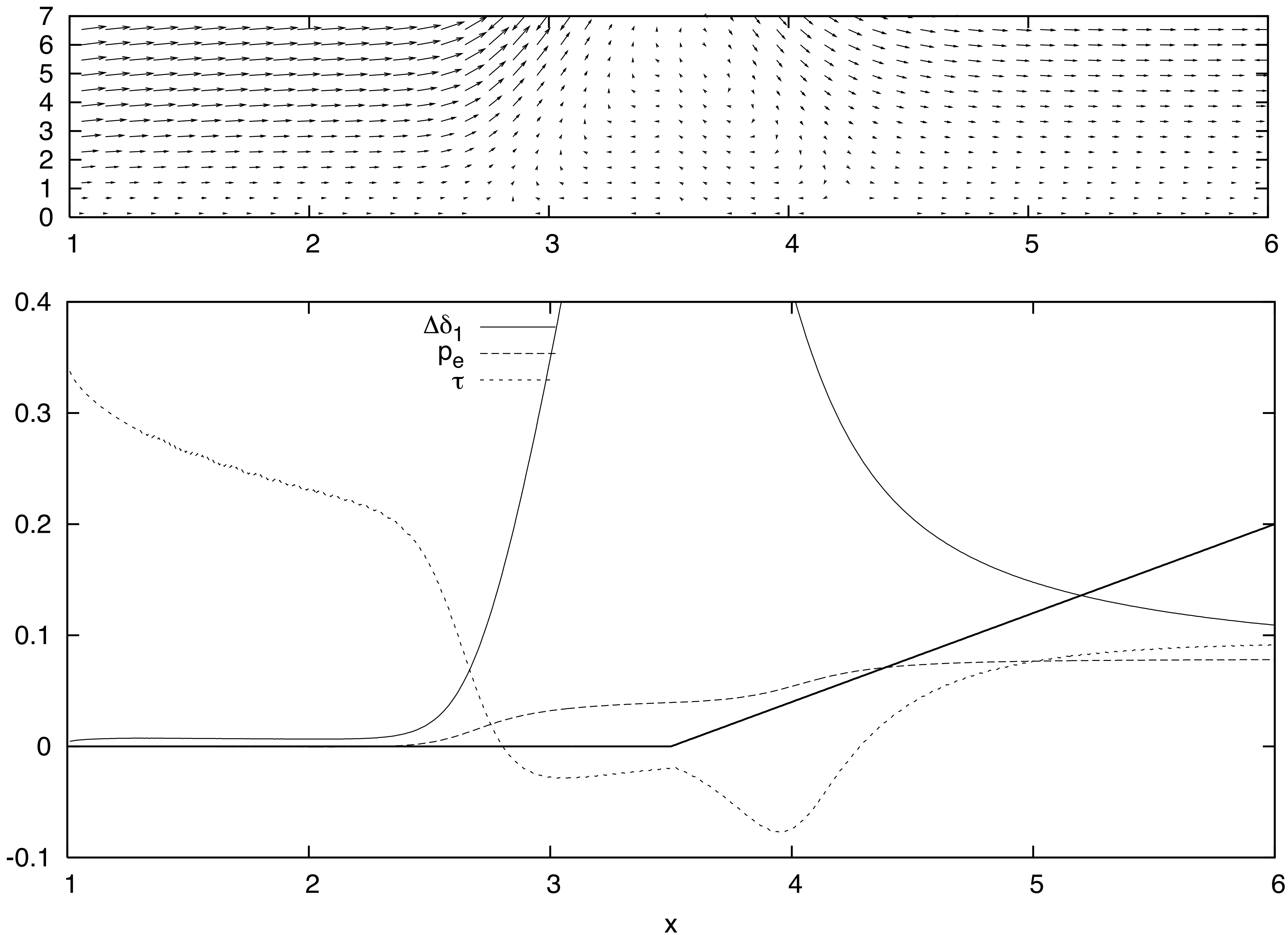
0.2
0.1
0
-0.1
$\Delta\delta_1$
p_e
τ
x



Δδ1
pe
τ
x



Δδ1
pe
τ
x



7
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0.1
0
-0.1
$\Delta\delta_1$
p_e
τ
x

outline

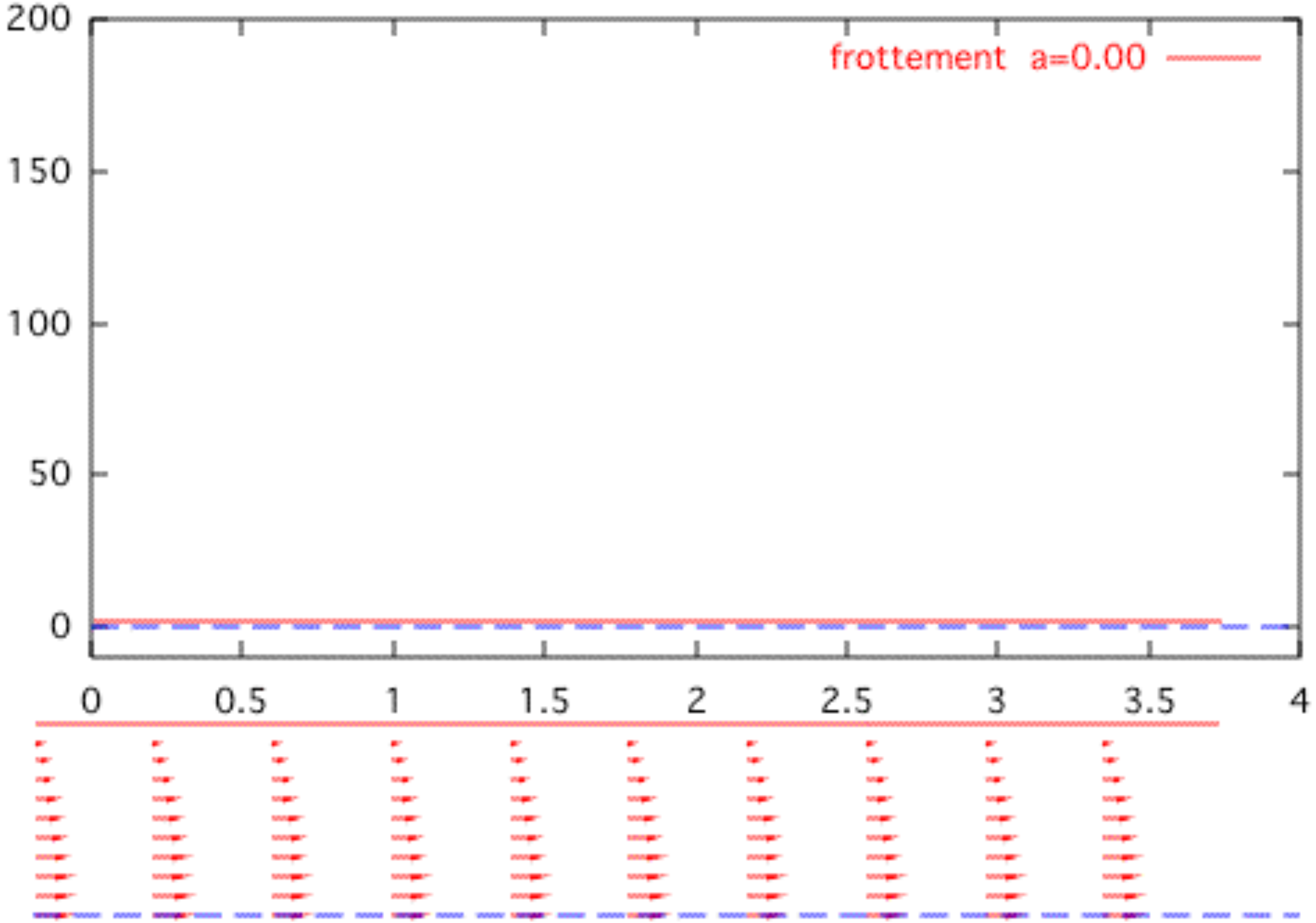
- the classical Boundary Layer
- second order Boundary Layer
- Interactive Boundary Layer
- some examples of numerical resolution with some comparaisons with Navier Stokes
- the Triple Deck, example of numerical solution
- the Double Deck, example of numerical solution FD FE
- summary

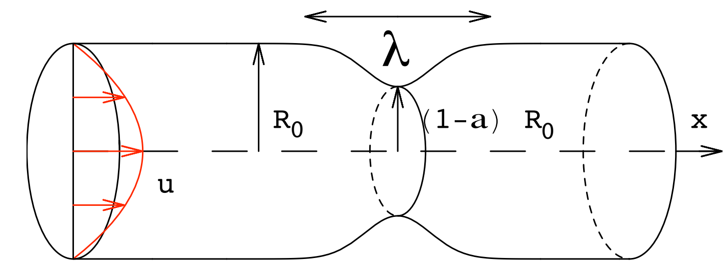
Does it work?

some comparisons with steady 2D NS

- flow in an axi constriction (stenosis)
- flow over a 2D bump
- entrance axi flow
- flow in a 2D channel with a constriction

Exemple: flow in a stenosis





steady flow
increase the degree
of closure of the
stenosis

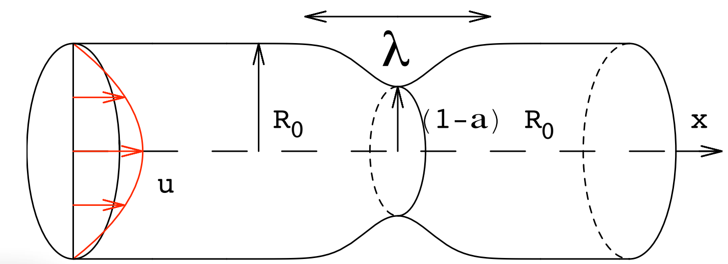
- variation of velocity (flux conservation) $U_0 \to U_0/(1-\alpha-\delta_1)^2$

- acceleration: boundary layer $\delta_1 \simeq \frac{\lambda}{\sqrt{Re_\lambda}}$, with $Re_\lambda = \frac{\lambda U_0}{(1-\alpha)^2\nu} = \frac{Re\lambda}{(1-\alpha)^2}$

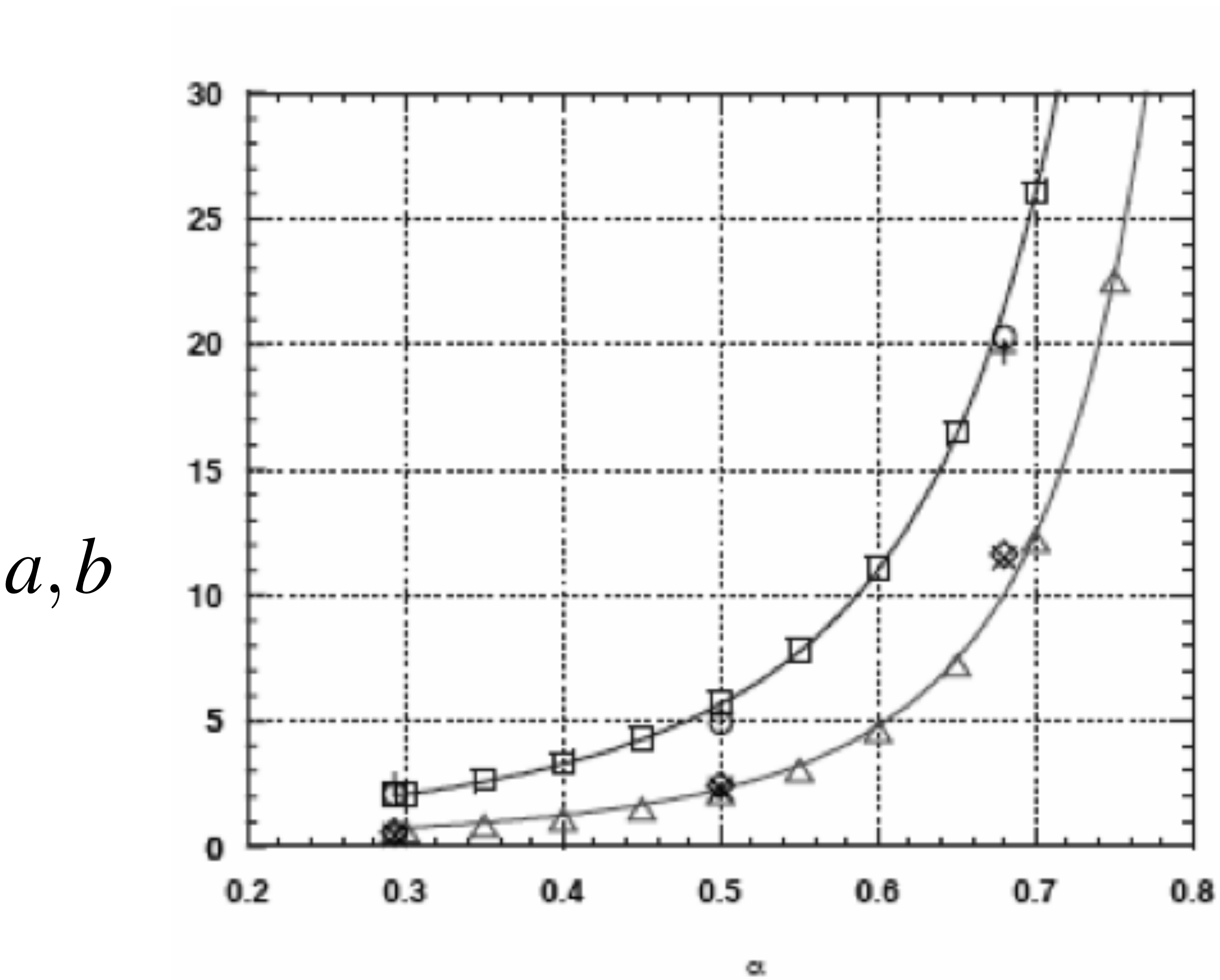
- WSS = (variation of velocity)/(boundary layer thickness) $= \frac{(Re/\lambda)^{1/2}}{(1-\alpha)^3}$

Exemple: flow in a stenosis

Siegel et al 94



$$WSS = aRe^{1/2} + b$$



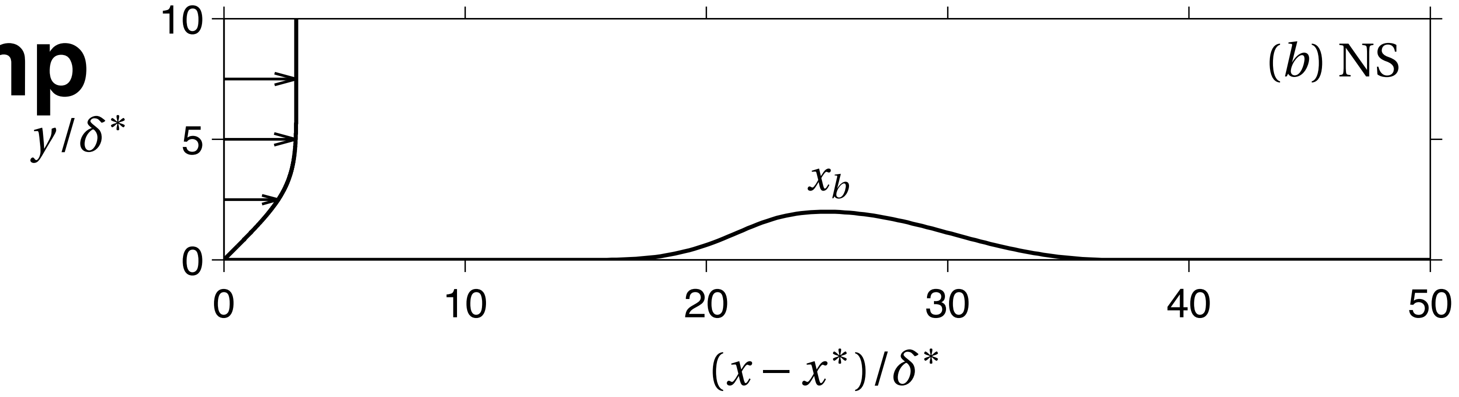
Coefficient a and b for the maximum WSS.
solid lines with △ and "square" : coefficient a and b obtained using the IBL integral method ;

◇ : coefficient a derived from Siegel for $\lambda = 3$;
× : coefficient a derived from Siegel for $\lambda = 6$;
○ : coefficient b derived from Siegel for $\lambda = 3$;
+ : coefficient b derived from Siegel for $\lambda = 6$.

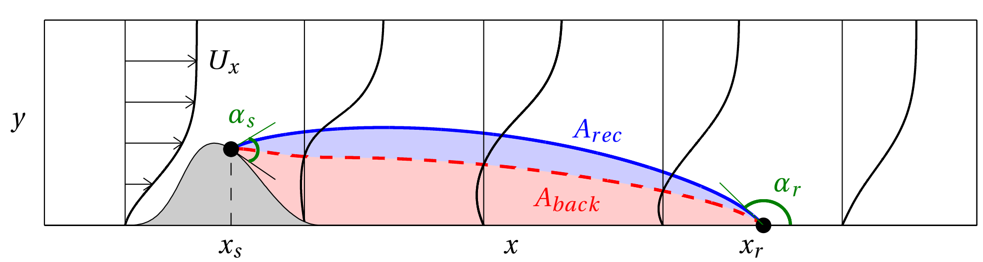
$$WSS = (\mu \frac{\partial u}{\partial y})/(\mu \frac{4U_0}{R}) \simeq 0.22 \frac{(Re/\lambda)^{1/2} + 3}{(1-\alpha)^3}$$

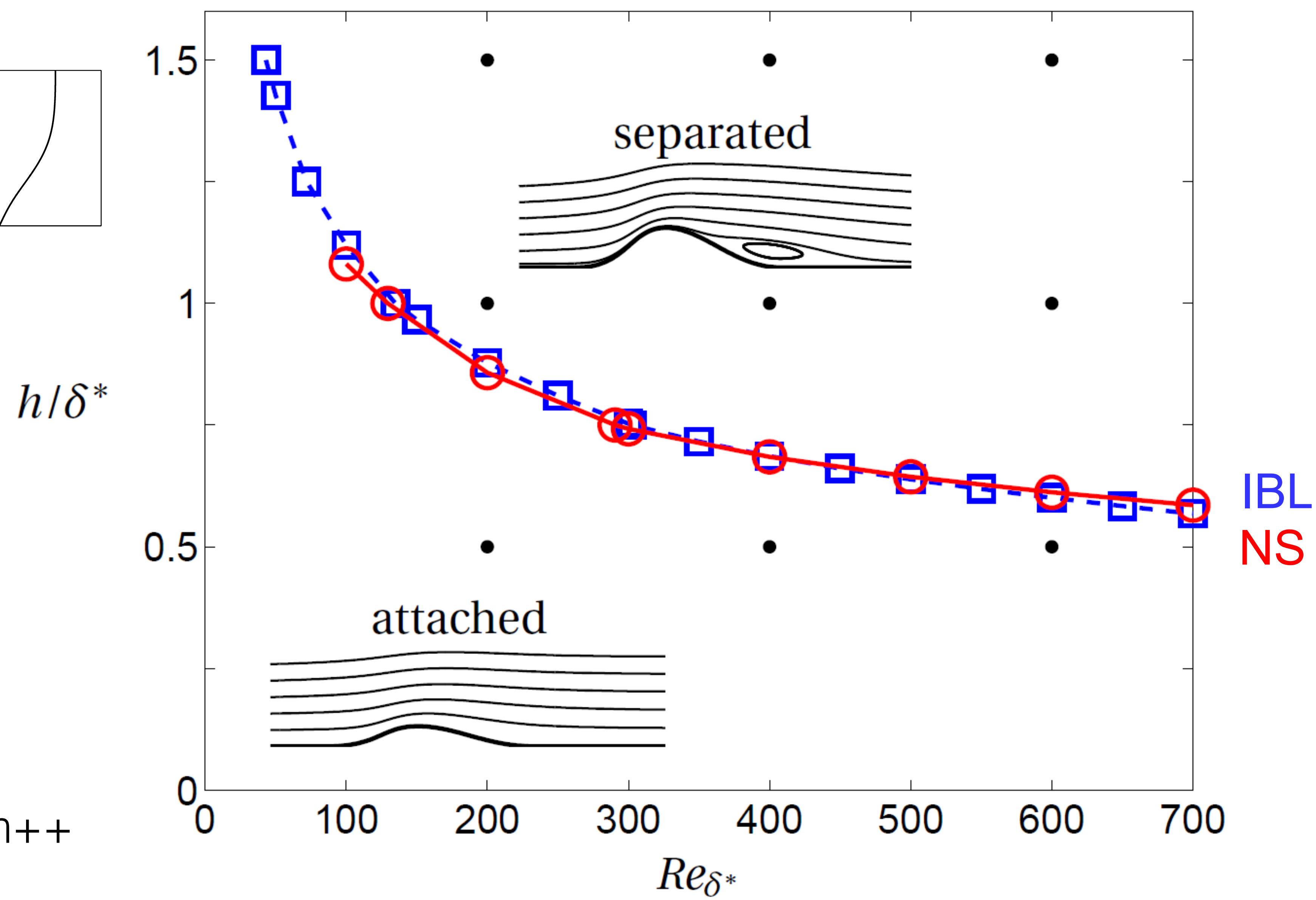
Lagrée Lorthois 05, Lorthois et al 00

Exemple: flow over a bump



Incipient separation



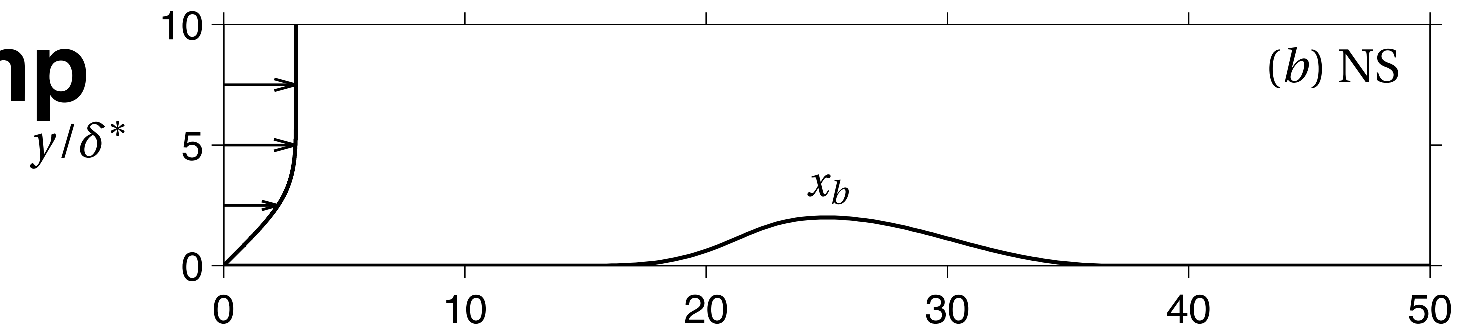


$$Re = Re_{\delta^*}^2$$

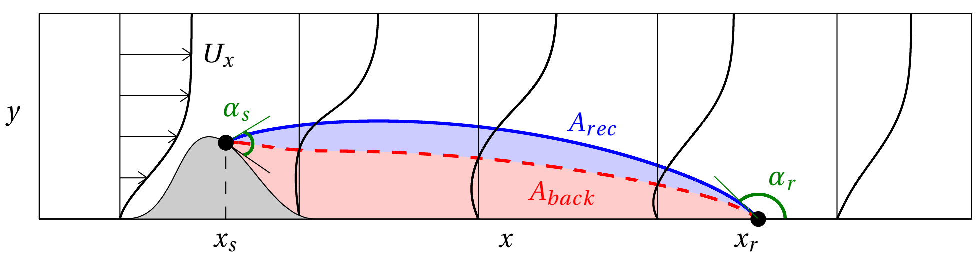
Boujo 14

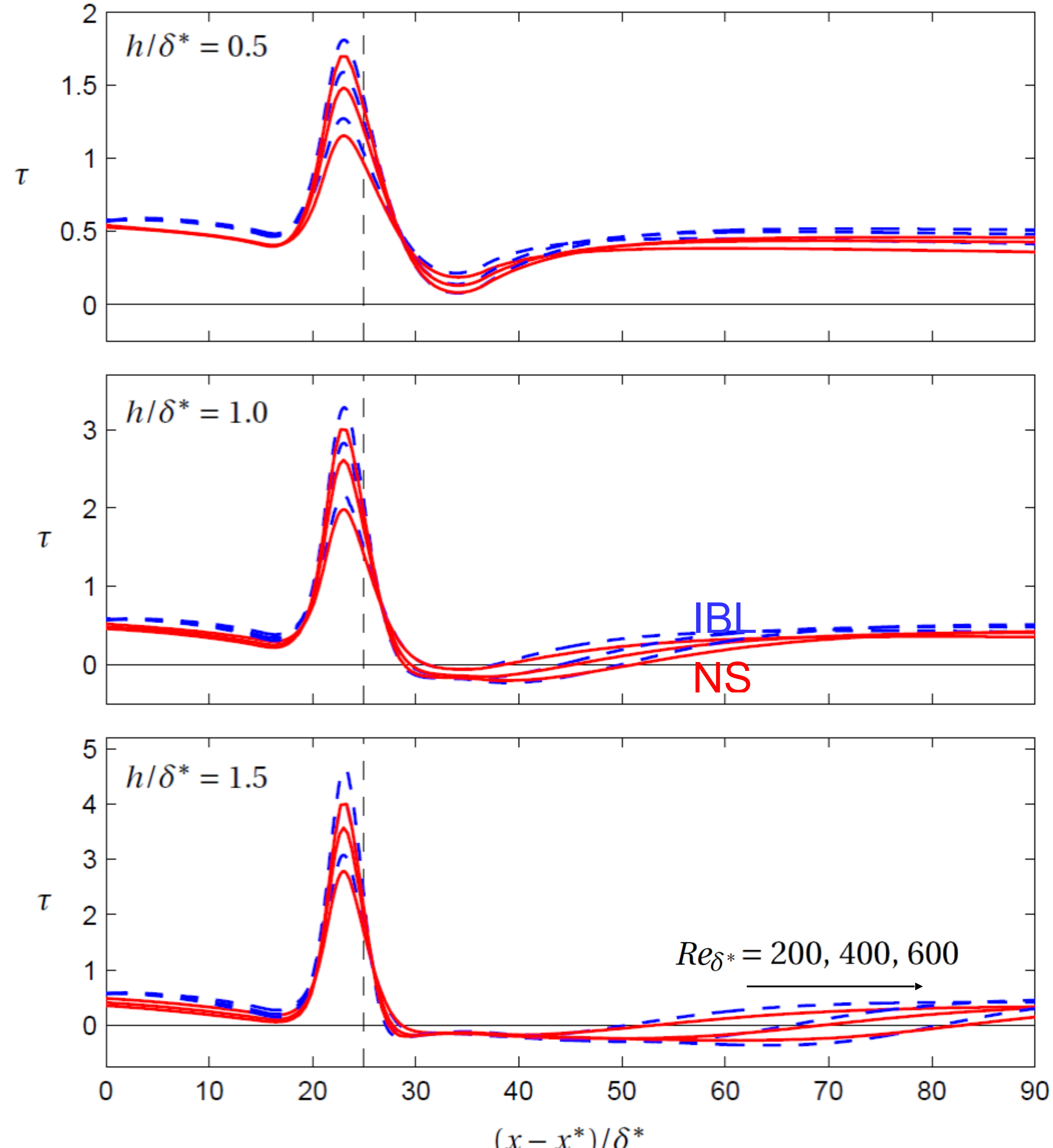
freefem++

Exemple: flow over a bump



Wall shear stress



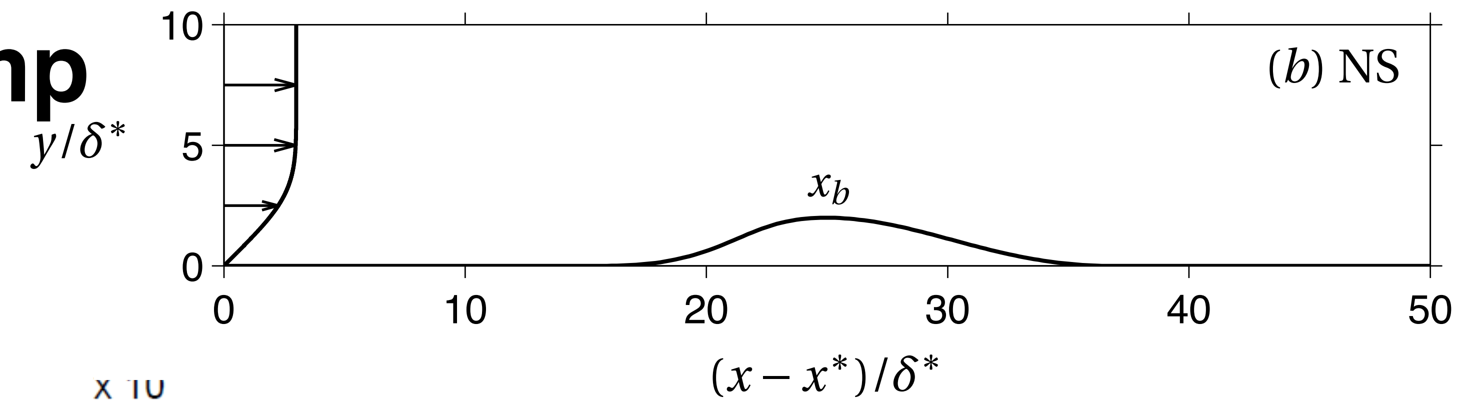


$$Re = Re_{\delta^*}^2$$

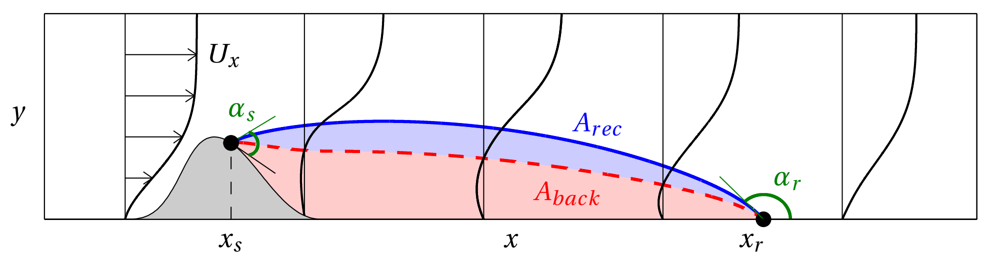
Boujo 14

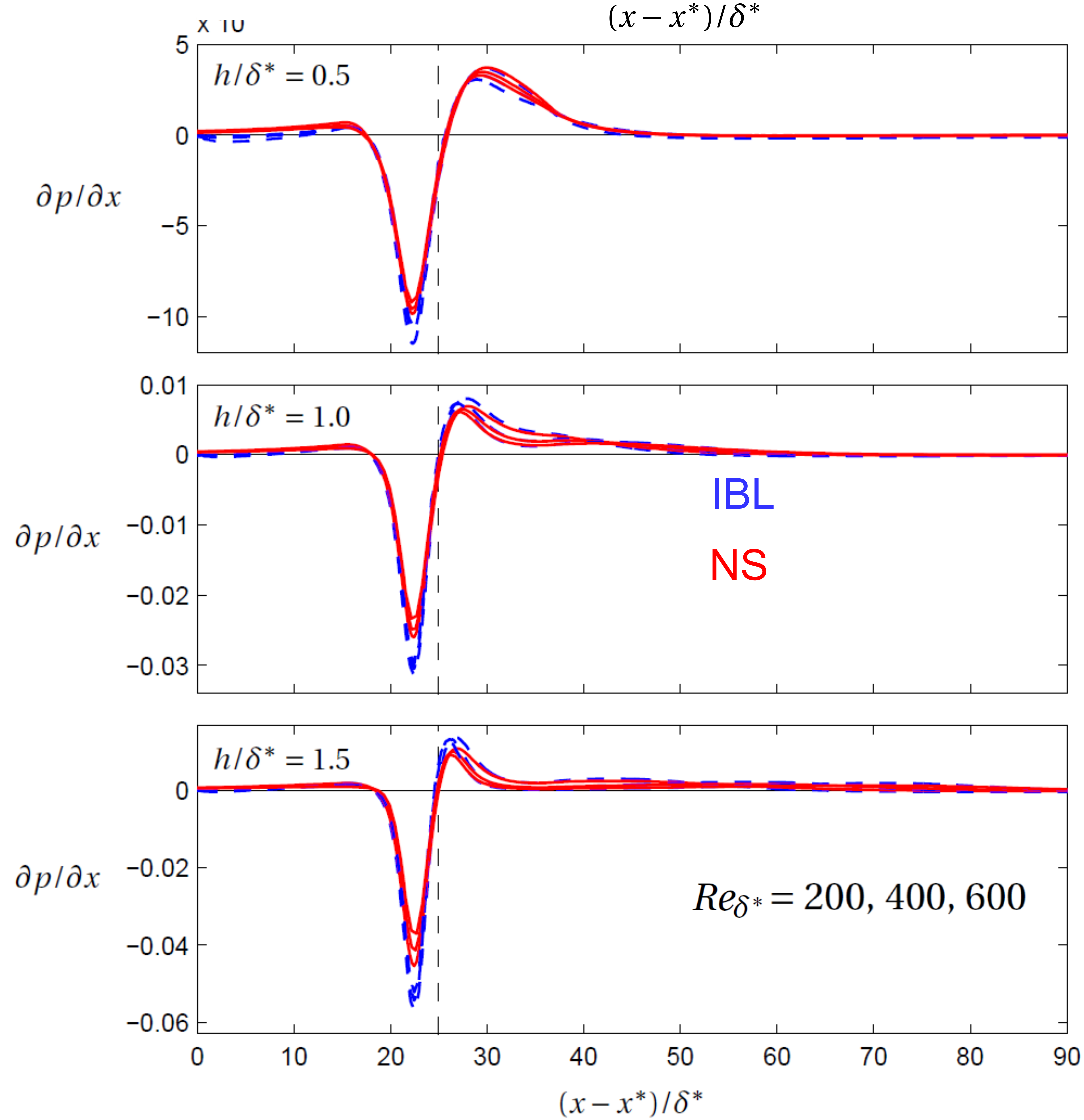
freefem++

Exemple: flow over a bump



Wall pressure gradient





$$Re = Re_{\delta^*}^2$$

Boujo 14

freefem++

IBL is not so bad, it allows boundary layer separation, qualitative and quantitative comparisons with NS

Turn now to Triple Deck, the sound asymptotic framework for flow separation

outline

- the classical Boundary Layer
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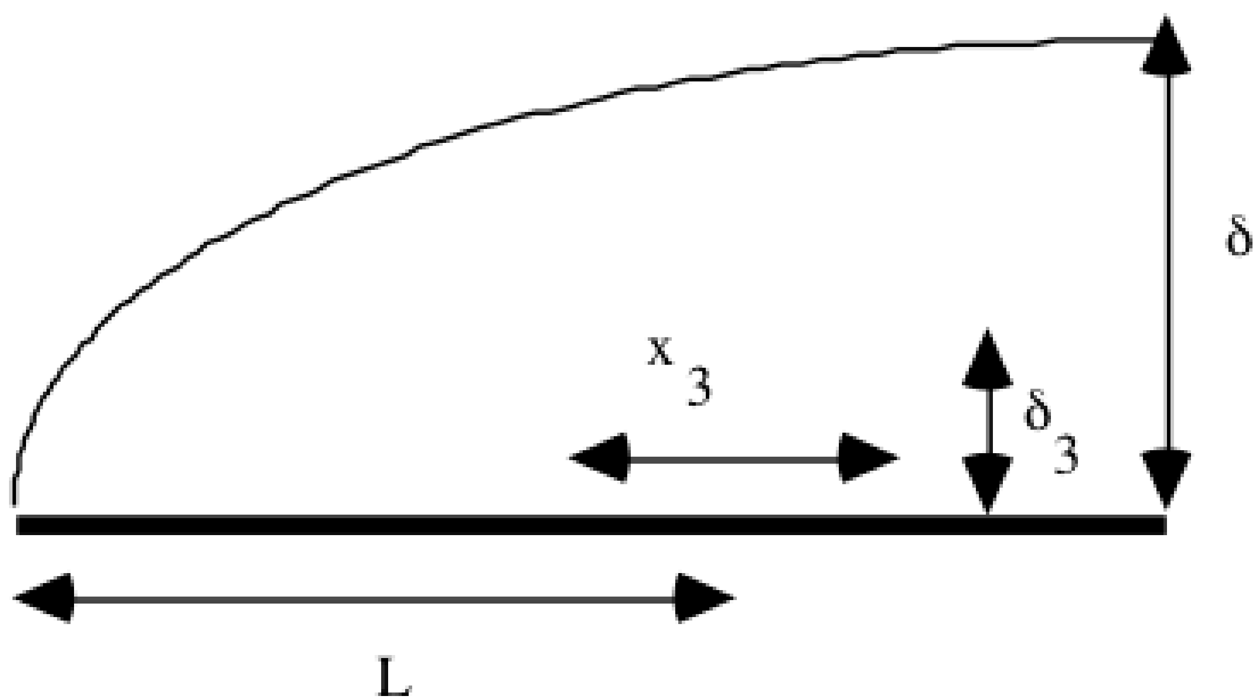
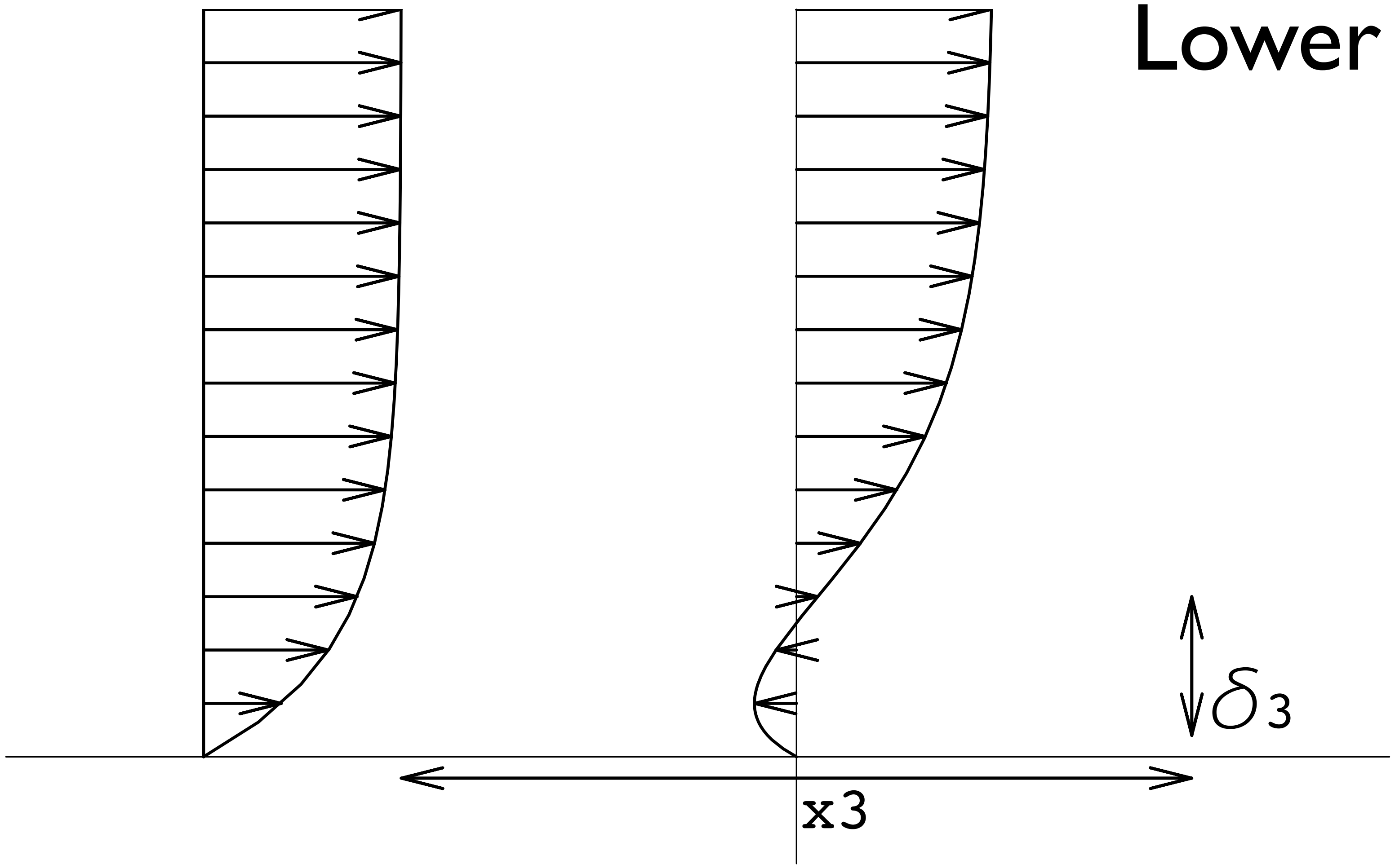
Triple Deck

new scales with balance between inertia and viscosity:
rational asymptotic framework for boundary layer separation



boundary layer in the boundary layer

Brown Stewartson Williams 69
Neiland 69
Messiter 70
Sychev 72
Smith 77…

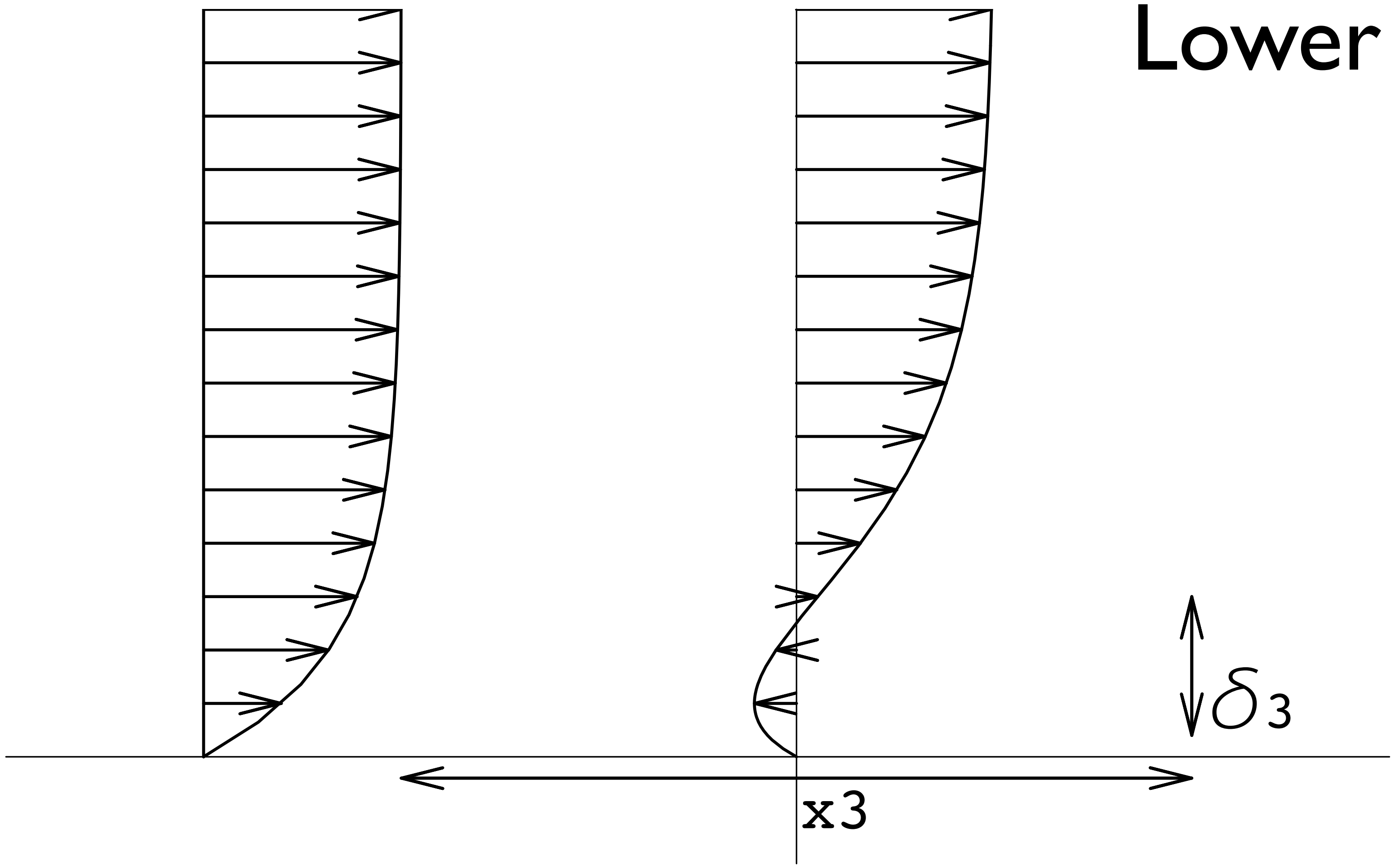


Lower Deck

$$\delta_3 = \varepsilon\delta$$

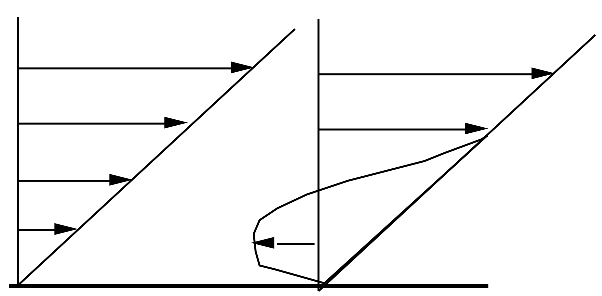
introduce new scale longitudinally and transversally as we look at vanishingly small perturbation of the Blasius boundary layer

Lower Deck



$$\delta_3 = \varepsilon\delta$$

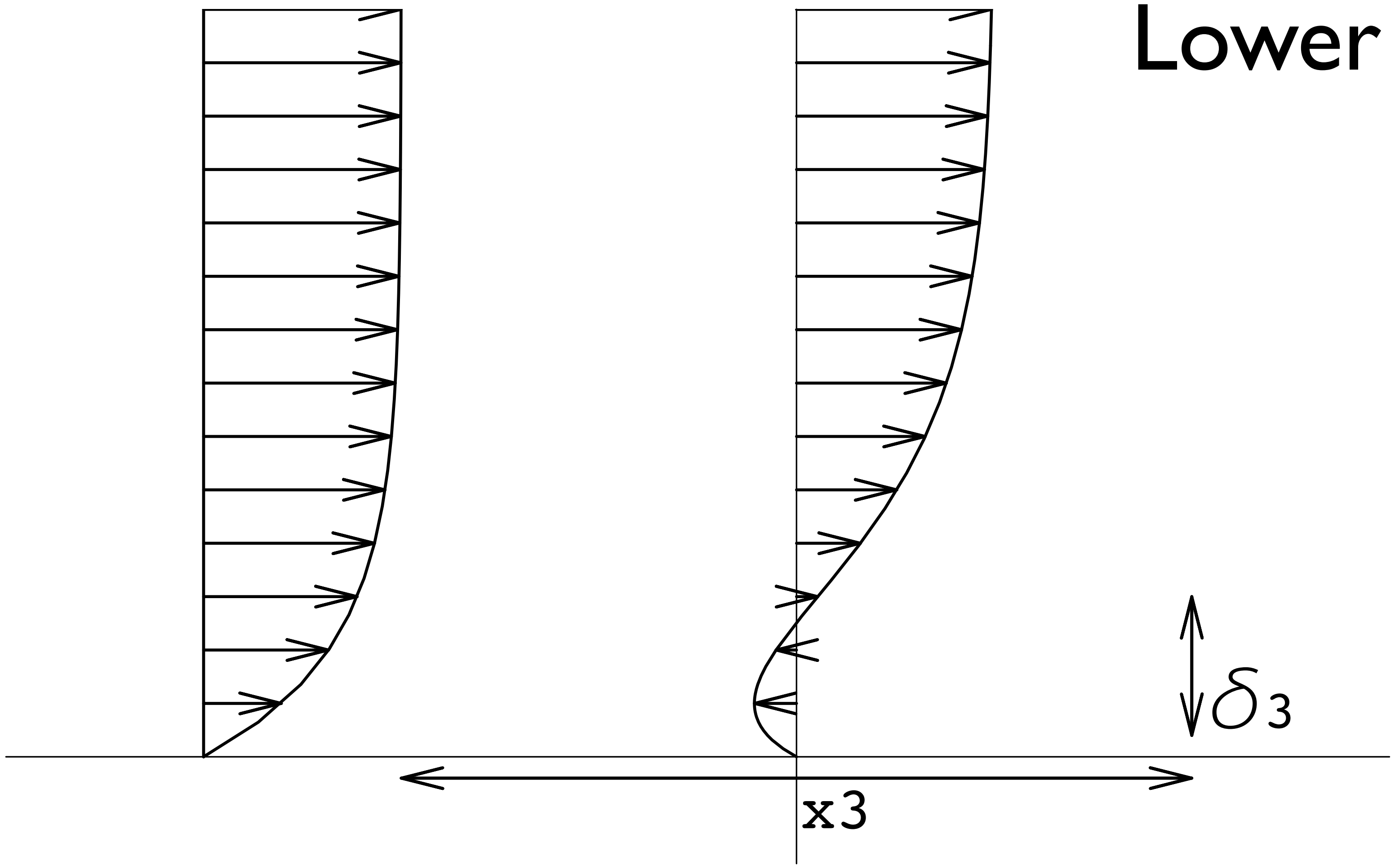
$u \sim \varepsilon$



$\Delta u \sim \varepsilon$

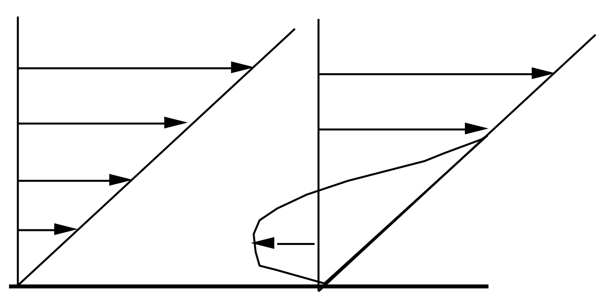
Neiland 69

Lower Deck



$$\delta_3 = \varepsilon\delta$$

$$u \sim \varepsilon$$

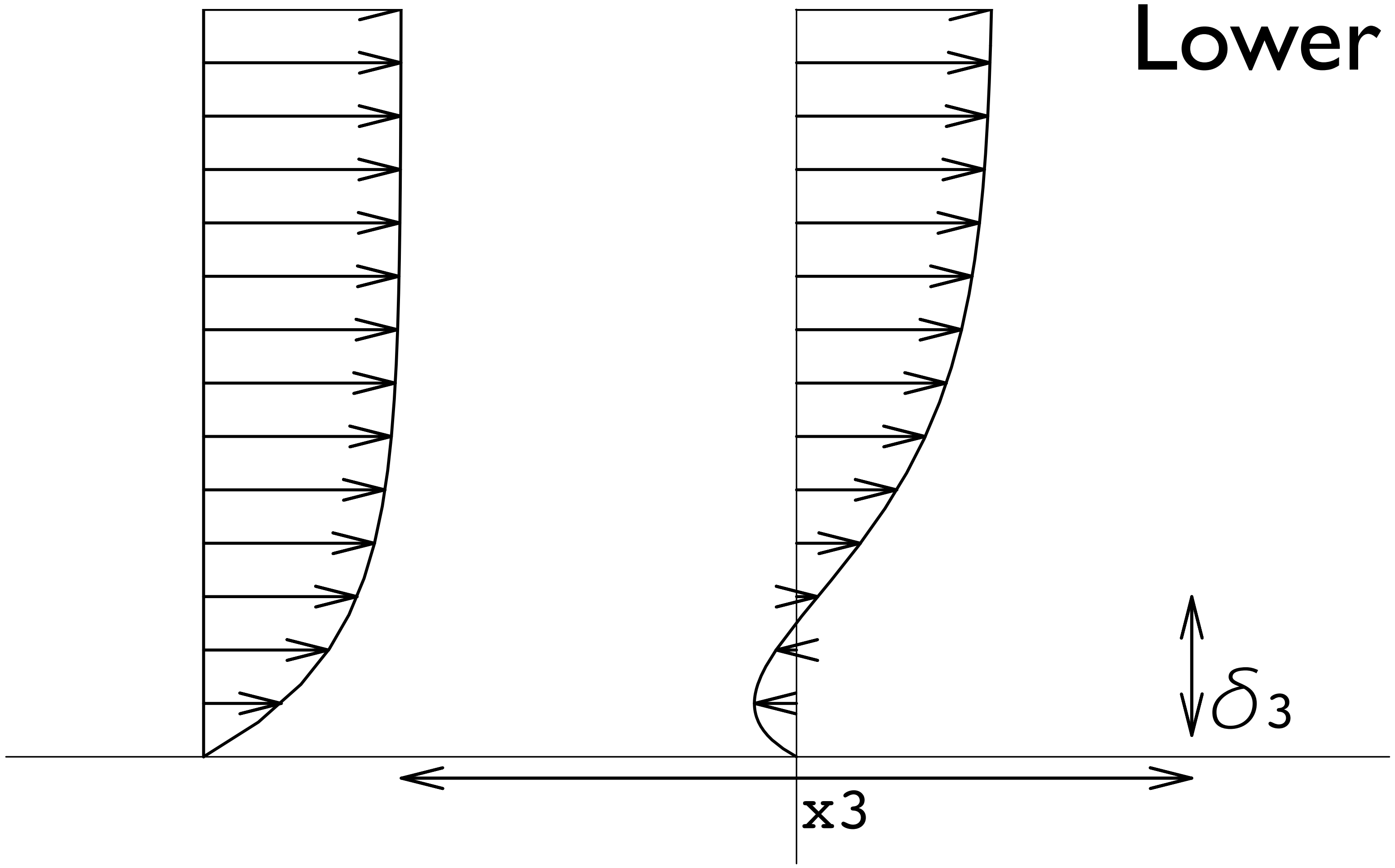


$$\Delta u \sim \varepsilon$$

$$u\frac{\partial u}{\partial x} \sim \frac{1}{Re}\frac{\partial u}{\partial y^2}$$

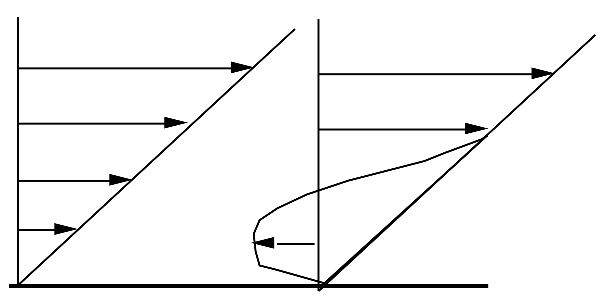
$$\frac{\varepsilon}{x_3} \sim Re^{-1}\frac{1}{(\varepsilon Re^{-1/2})^2}$$

Lower Deck



$$\delta_3 = \varepsilon\delta$$

$$u \sim \varepsilon$$



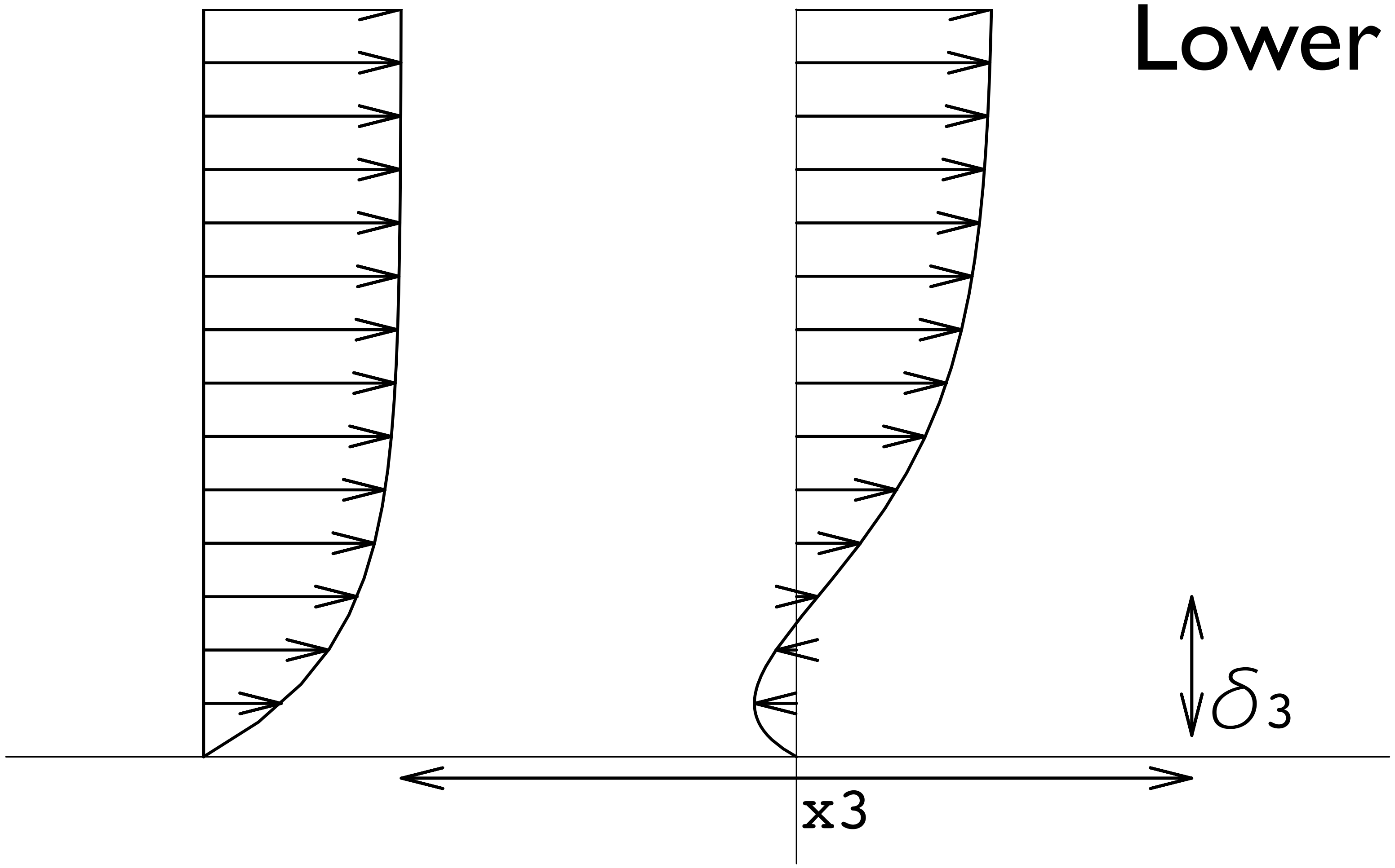
$$\Delta u \sim \varepsilon$$

$$u\frac{\partial u}{\partial x} \sim \frac{1}{Re}\frac{\partial u}{\partial y^2}$$

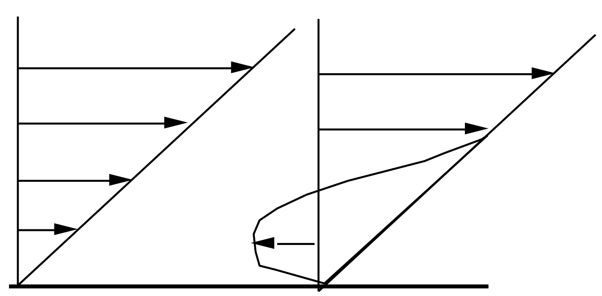
$$\frac{\varepsilon}{x_3} \sim Re^{-1}\frac{1}{(\varepsilon Re^{-1/2})^2}$$

the new small longitudinal scale is $x_3 = \varepsilon^3$

Lower Deck



$$\delta_3 = \varepsilon\delta$$



after dominant balance the equations are

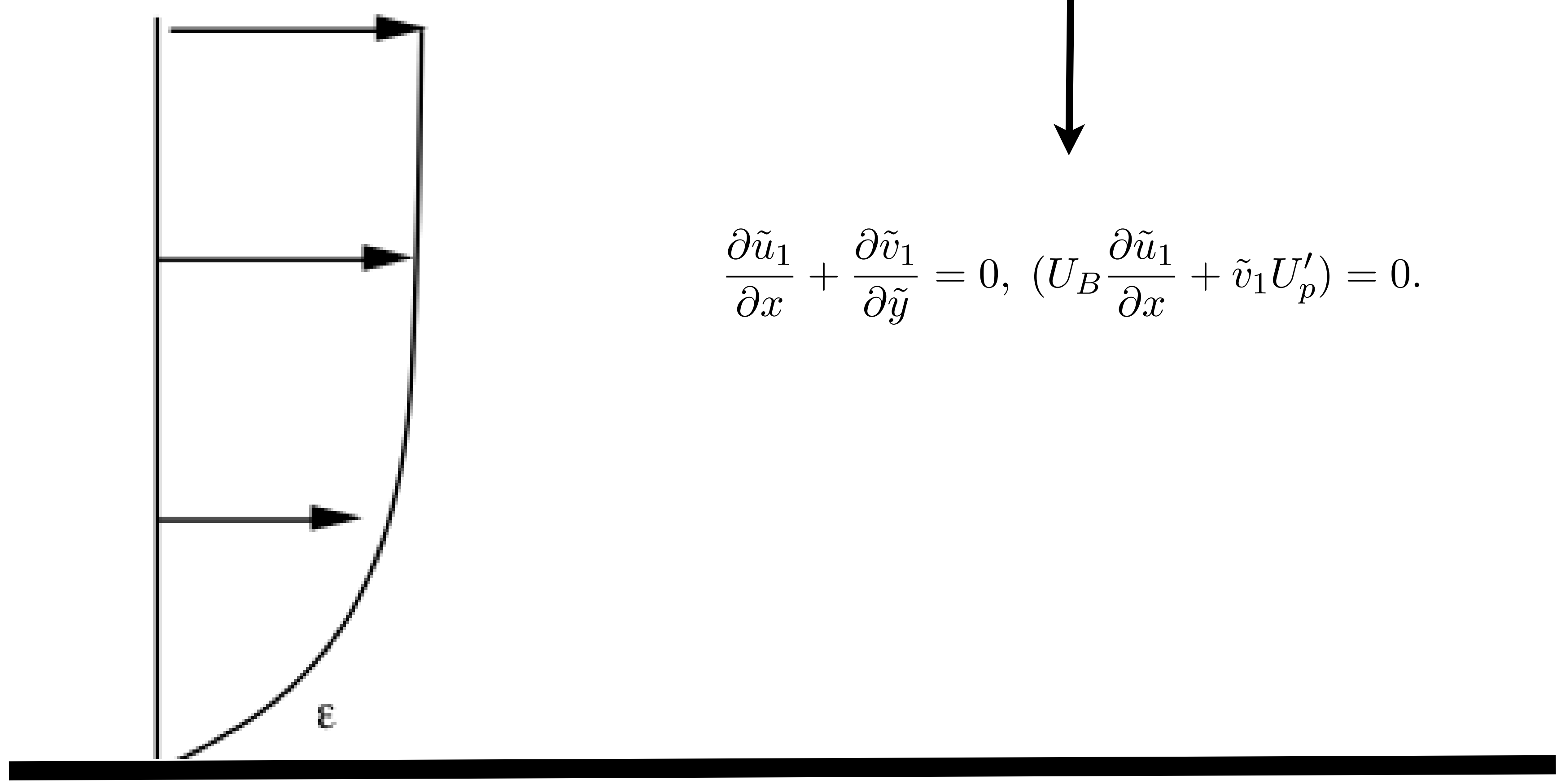
$$
\begin{aligned}
\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v &= 0, \\
u\frac{\partial}{\partial x}u + v\frac{\partial}{\partial y}u &= -\frac{d}{dx}p + \frac{\partial^2}{\partial y^2}u. \\
u(x, y = f(x)) &= 0, \qquad v(x, y = f(x)) = 0 \\
\& \qquad \lim_{y\to\infty} u(x, y) &= y + A.
\end{aligned}
$$

anticipating matching

which are again Prandtl with different scales !

Main Deck

$$\tilde{u} = U_B(\tilde{y}) + \varepsilon \tilde{u}_1, \ \ \tilde{v} = \frac{\varepsilon \sqrt{Re}^{-1}}{x_3} \tilde{v}_1$$



$$\frac{\partial \tilde{u}_1}{\partial x} + \frac{\partial \tilde{v}_1}{\partial \tilde{y}} = 0, \ (U_B \frac{\partial \tilde{u}_1}{\partial x} + \tilde{v}_1 U_p') = 0.$$

no pressure

Brown Stewartson Williams 69

The displacement function appears as a perturbation of the boundary layer at a small scale: the Main Deck

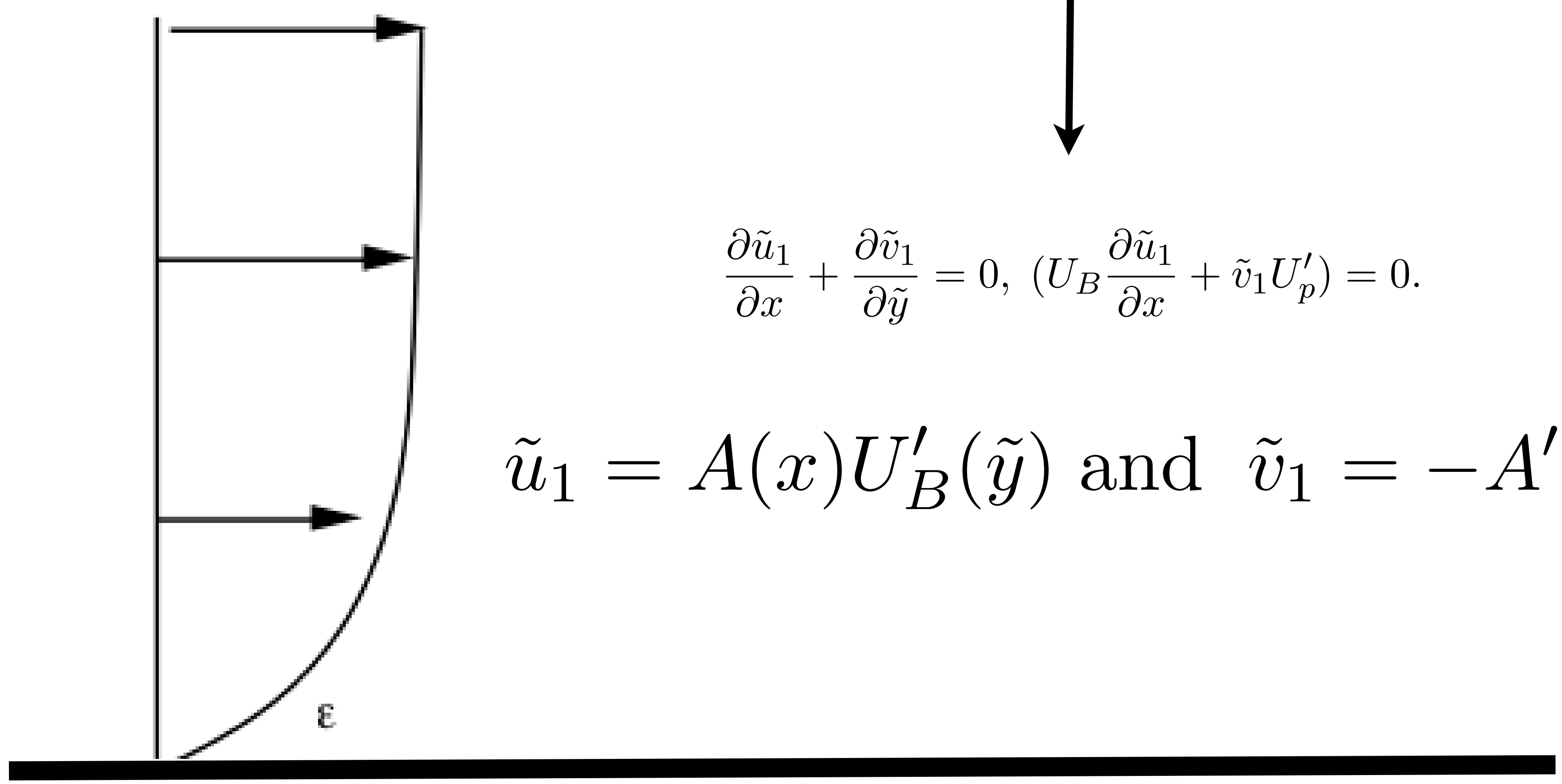
Main Deck

$$\tilde{u} = U_B(\tilde{y}) + \varepsilon \tilde{u}_1, \quad \tilde{v} = \frac{\varepsilon \sqrt{Re}^{-1}}{x_3} \tilde{v}_1$$

$$\frac{\partial \tilde{u}_1}{\partial x} + \frac{\partial \tilde{v}_1}{\partial \tilde{y}} = 0, \; (U_B \frac{\partial \tilde{u}_1}{\partial x} + \tilde{v}_1 U_p') = 0.$$

no pressure

$$\tilde{u}_1 = A(x) U_B'(\tilde{y}) \text{ and } \tilde{v}_1 = -A'(x) U_B(\tilde{y})$$



The displacement function appears as a perturbation of the boundary layer at a small scale: the Main Deck

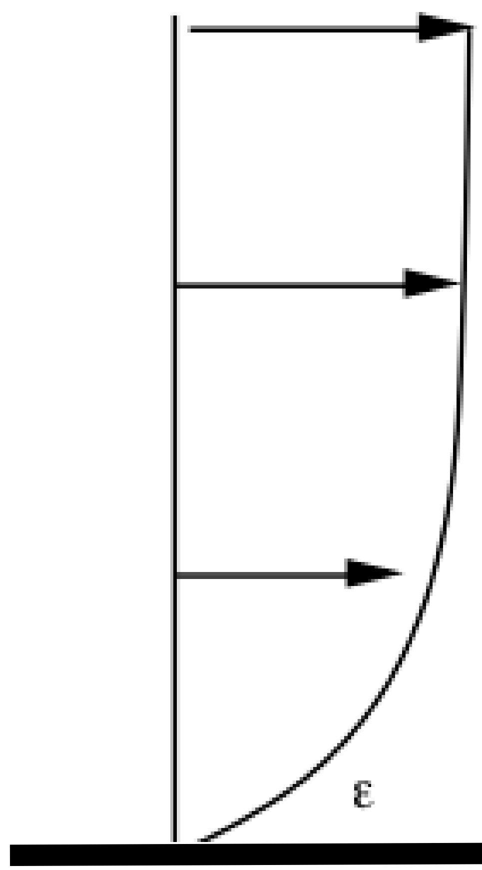
Brown Stewartson Williams 69

Main Deck

$$\tilde{u} = U_B(\tilde{y}) + \varepsilon \tilde{u}_1, \quad \tilde{v} = \frac{\varepsilon \sqrt{Re}^{-1}}{x_3} \tilde{v}_1$$

$$\frac{\partial \tilde{u}_1}{\partial x} + \frac{\partial \tilde{v}_1}{\partial \tilde{y}} = 0, \ (U_B \frac{\partial \tilde{u}_1}{\partial x} + \tilde{v}_1 U_p') = 0.$$

no pressure

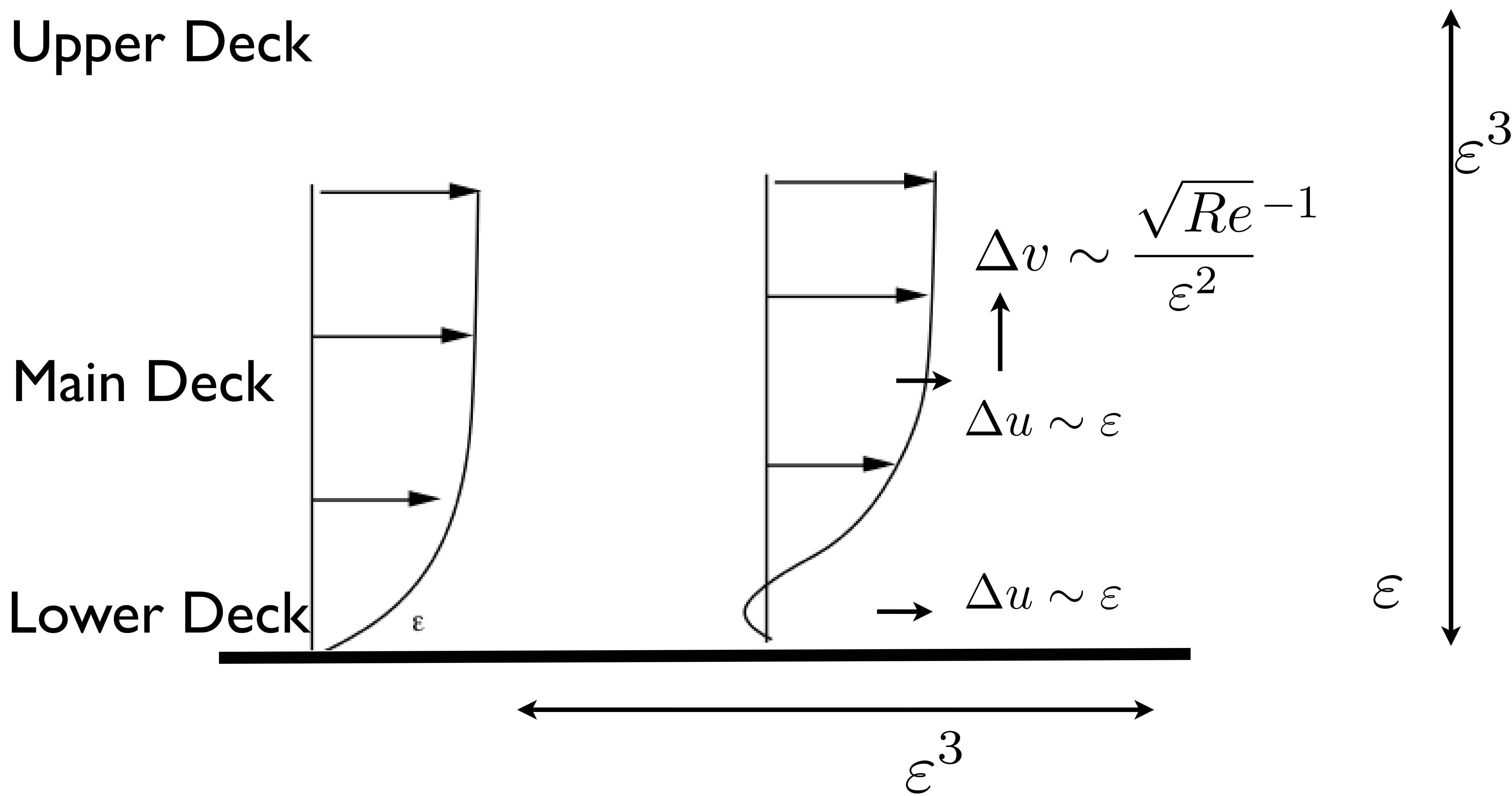


$$\tilde{u}_1 = A(x) U_B'(\tilde{y}) \text{ and } \tilde{v}_1 = -A'(x) U_B(\tilde{y})$$

matching

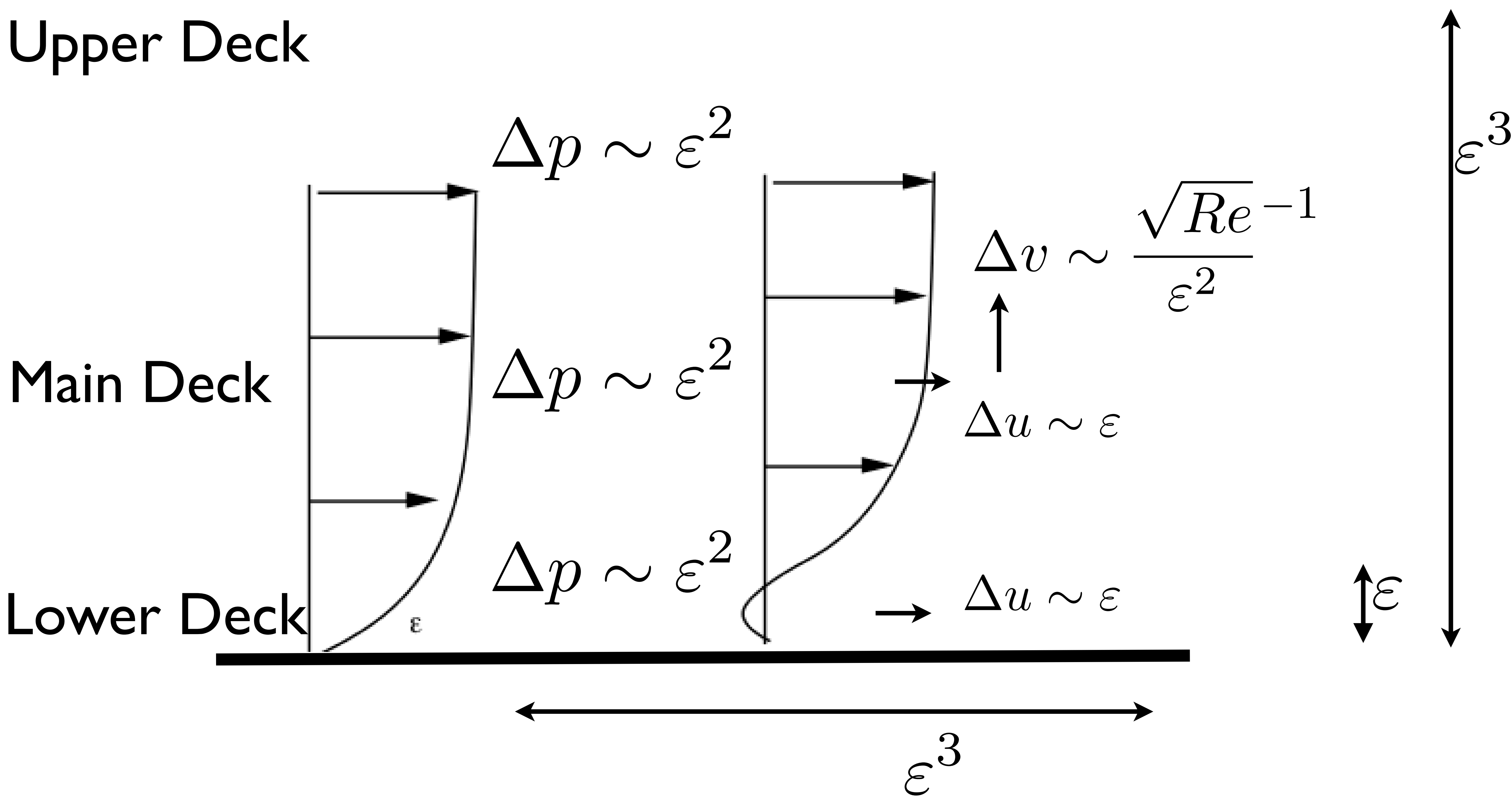
The displacement function appears as a perturbation of the boundary layer at a small scale: the Main Deck

Brown Stewartson Williams 69



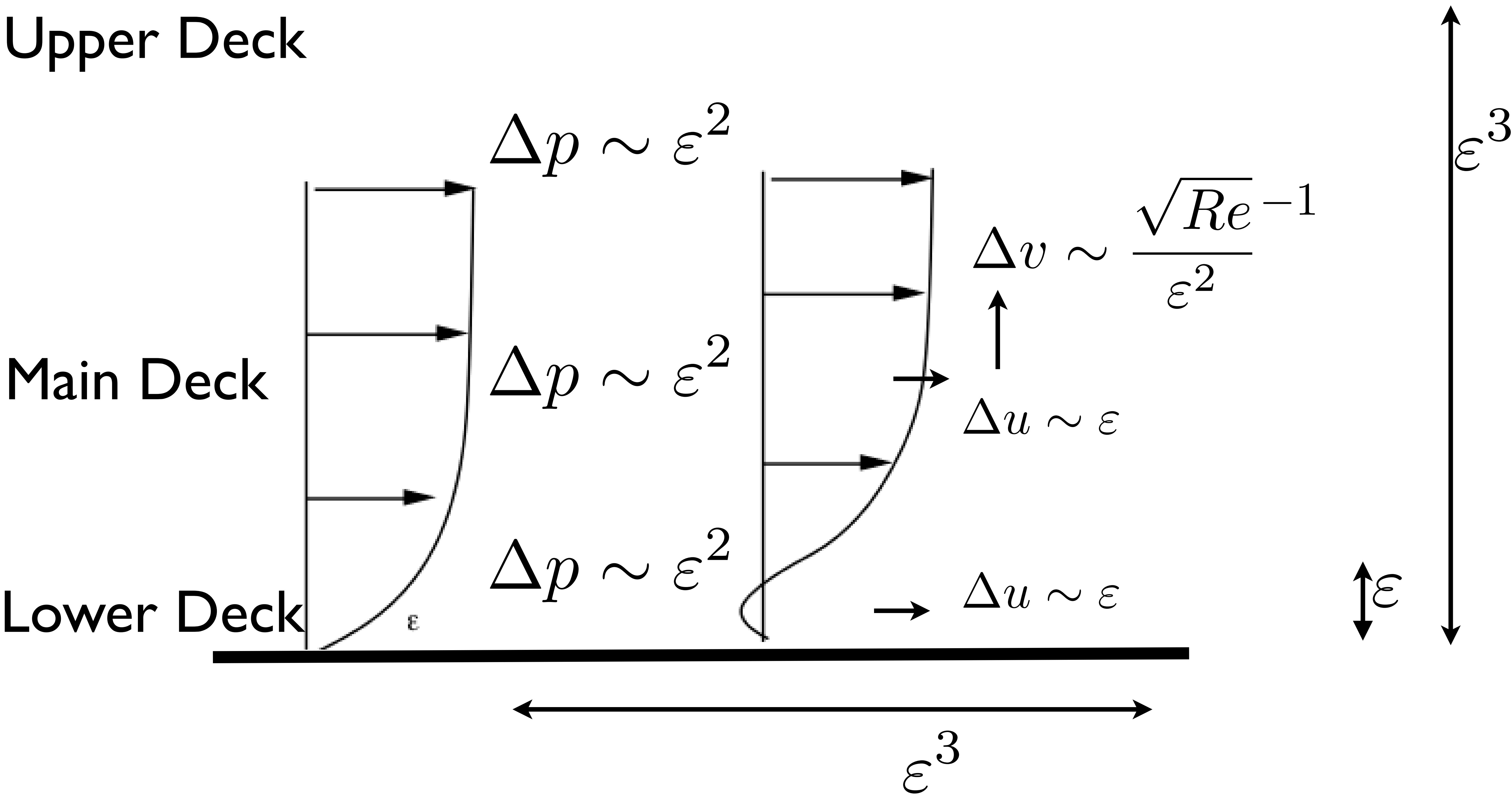
Upper Deck
Main Deck
Lower Deck
ε
$\Delta v \sim \frac{\sqrt{Re}^{-1}}{\varepsilon^2}$
$\Delta u \sim \varepsilon$
$\Delta u \sim \varepsilon$
ε^3
ε
ε^3

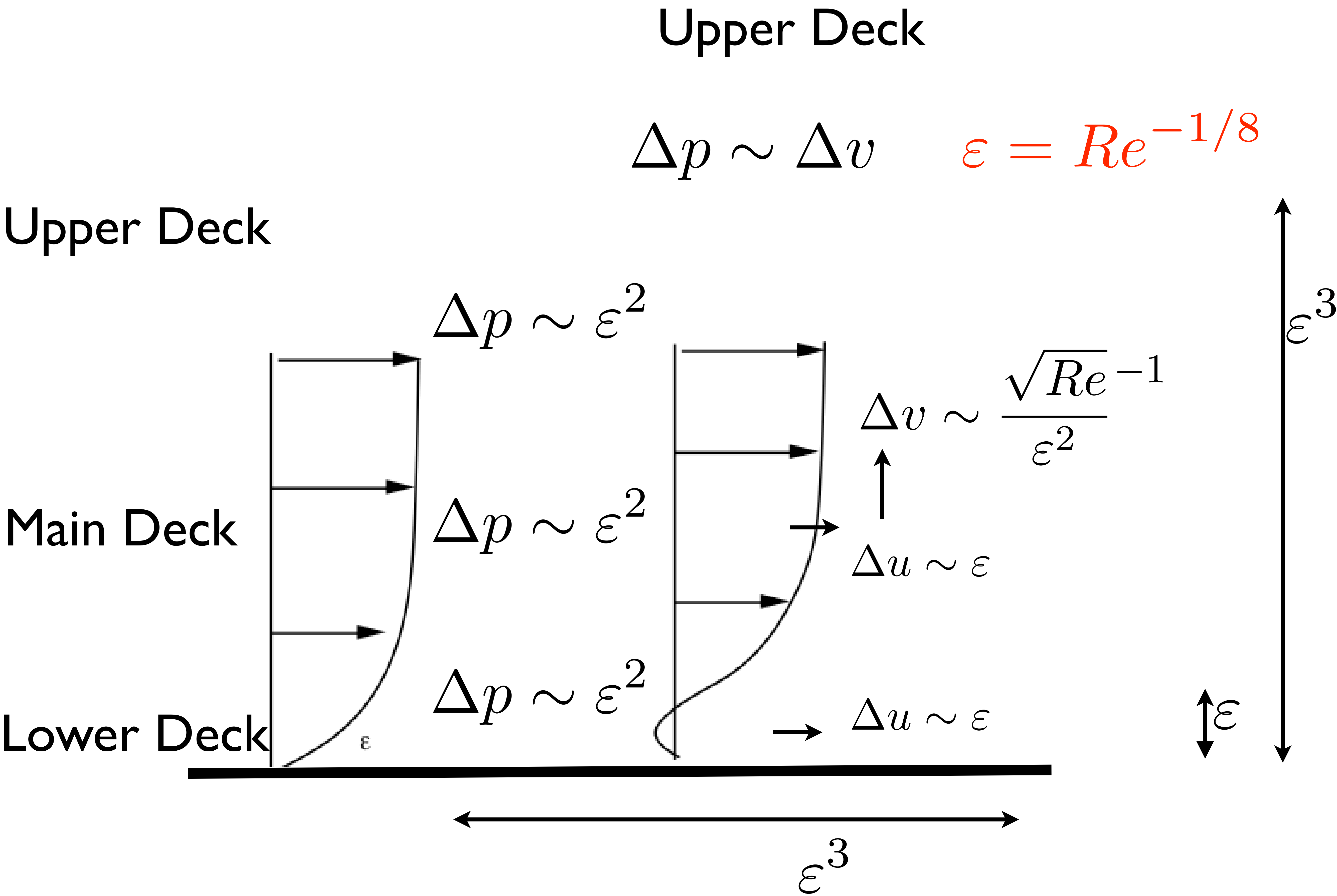
Upper Deck


Upper Deck
$\Delta p \sim \varepsilon^2$
$\Delta v \sim \frac{\sqrt{Re}^{-1}}{\varepsilon^2}$
ε^3
Main Deck
$\Delta p \sim \varepsilon^2$
$\Delta u \sim \varepsilon$
$\Delta p \sim \varepsilon^2$
Lower Deck
ε
$\Delta u \sim \varepsilon$
ε
ε^3

Upper Deck

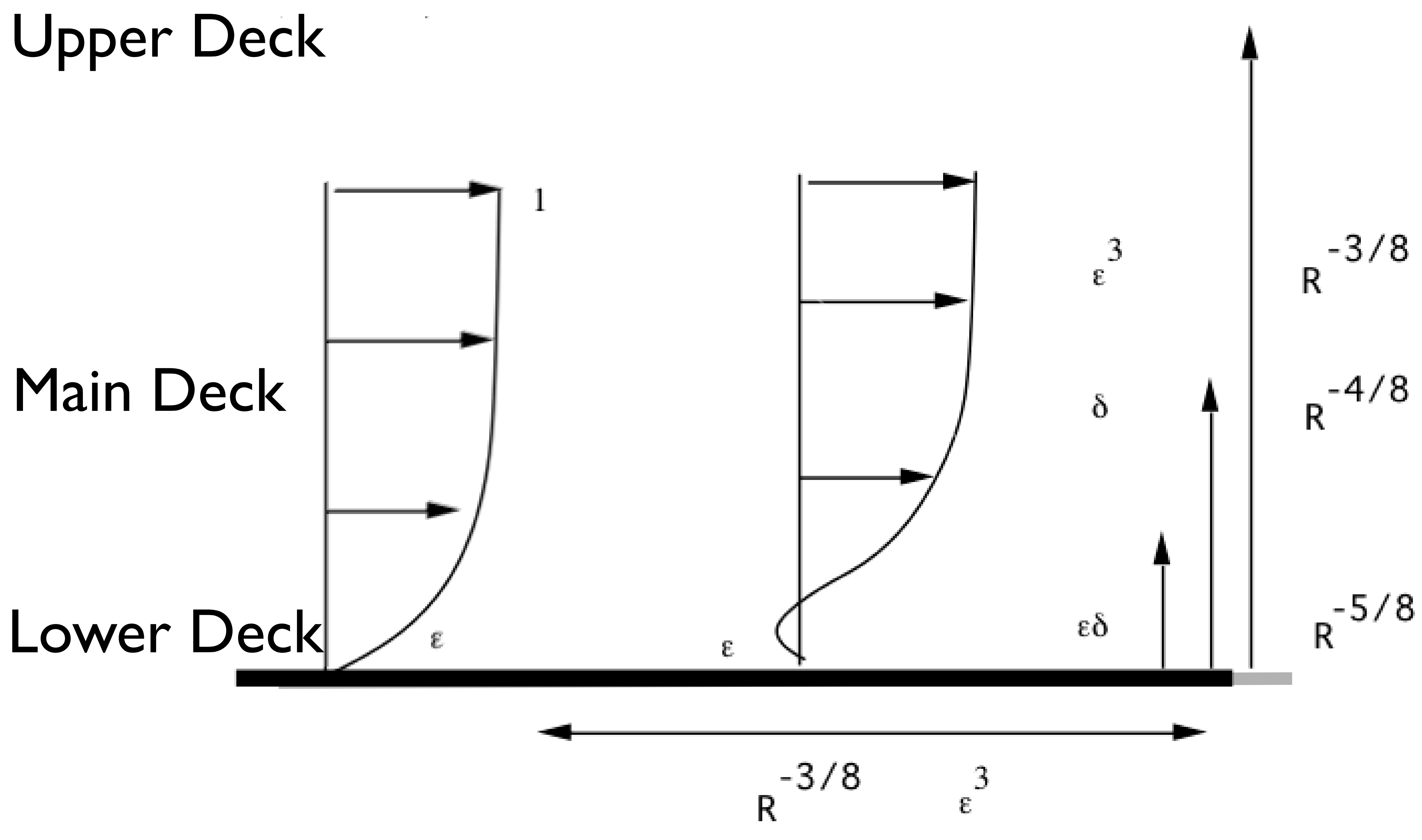
$$\Delta p \sim \Delta v$$




Upper Deck
$\Delta p \sim \Delta v$
$\varepsilon = Re^{-1/8}$
Upper Deck
$\Delta p \sim \varepsilon^2$
ε^3
$\Delta v \sim \frac{\sqrt{Re}^{-1}}{\varepsilon^2}$
Main Deck
$\Delta p \sim \varepsilon^2$
$\Delta u \sim \varepsilon$
$\Delta p \sim \varepsilon^2$
Lower Deck
ε
$\Delta u \sim \varepsilon$
ε
ε^3

triple deck scales

$\varepsilon = Re^{-1/8}$

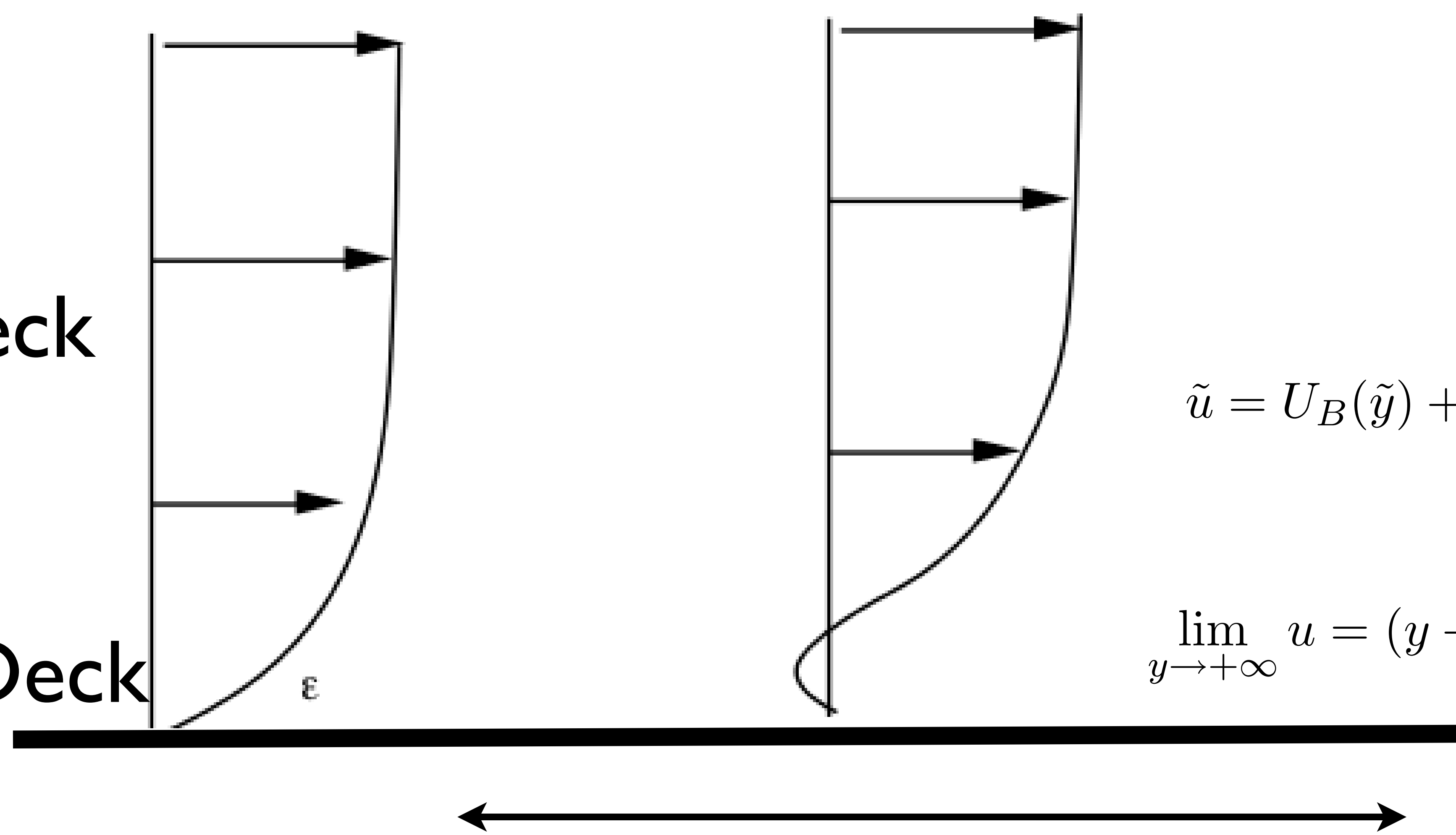


matching

Upper Deck

Main Deck

Lower Deck



$$\tilde{u} = U_B(\tilde{y}) + \varepsilon A(x) U_B'(\tilde{y}).$$

$$\lim_{y \to +\infty} u = (y + A(x)) U_B'(0)$$

ε^3

The displacement function appears as a perturbation of the boundary layer at a small scale: the Main Deck

matching

$$p = \frac{1}{\pi} fp \int_{-\infty}^{\infty} \frac{A'}{x-\xi} d\xi$$

the Hilbert integral solves
the Laplacian in the ideal fluid
with a Neumann BC at the wall

Upper Deck

$$Re^{-1/4}\tilde{p}$$

$$\tilde{v} = -Re^{-1/4} A'(x)$$

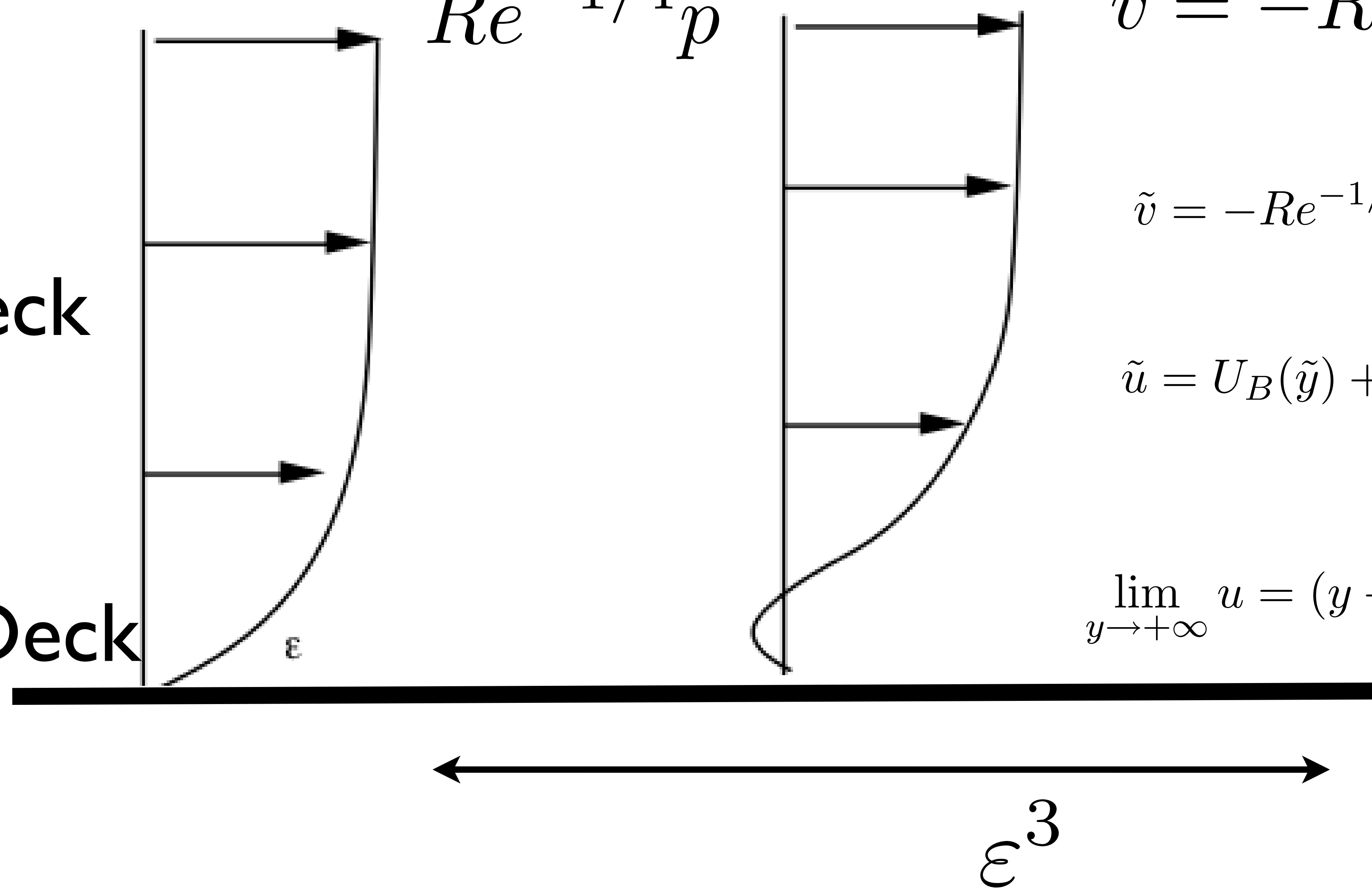
Main Deck

$$\tilde{v} = -Re^{-1/4} A'(x) U_0(\tilde{y})$$

$$\tilde{u} = U_B(\tilde{y}) + \varepsilon A(x) U_B'(\tilde{y}).$$

Lower Deck

$$\lim_{y \to +\infty} u = (y + A(x)) U_B'(0)$$



coupled problem

$$p = \frac{1}{\pi} fp \int_{-\infty}^{\infty} \frac{A'}{x - \xi} d\xi$$

pressure displacement
in incompressible

Upper Deck

Lower Deck

$$\begin{aligned} \frac{\partial}{\partial x} u + \frac{\partial}{\partial y} v &= 0, \\ u \frac{\partial}{\partial x} u + v \frac{\partial}{\partial y} u &= -\frac{d}{dx} p + \frac{\partial^2}{\partial y^2} u. \\ u(x, y = f(x)) &= 0, \qquad v(x, y = f(x)) = 0 \\ \& \qquad \lim_{y \to \infty} u(x, y) &= y + A. \end{aligned}$$

“Prandtl” equations with
different boundary conditions

coupled problem

Upper Deck

$$p = \frac{1}{\pi} fp \int_{-\infty}^{\infty} \frac{A'}{x-\xi} d\xi$$

pressure displacement in incompressible

or $p = \pm A$ — super sub critical

or $A = 0$ — in long tube (Double Deck)

or $p = -\frac{dA}{dx}$ — super sonic

or ...

Lower Deck

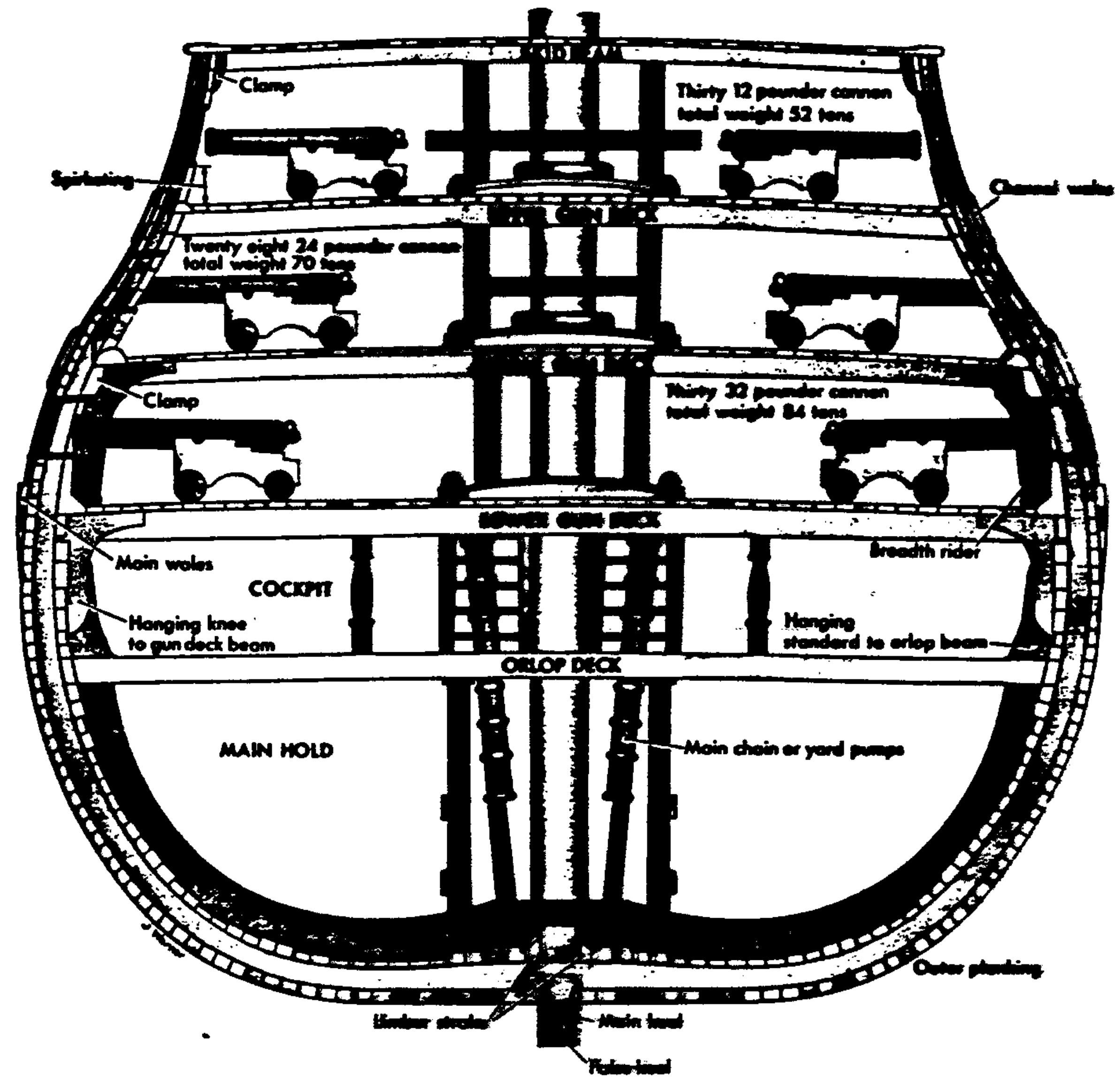
$$\begin{aligned} \frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v &= 0, \\ u\frac{\partial}{\partial x}u + v\frac{\partial}{\partial y}u &= -\frac{d}{dx}p + \frac{\partial^2}{\partial y^2}u. \\ u(x, y = f(x)) &= 0, \qquad v(x, y = f(x)) = 0 \\ \& \qquad \lim_{y \to \infty} u(x, y) &= y + A. \end{aligned}$$

"Prandtl" equations with different boundary conditions

upper deck

main deck

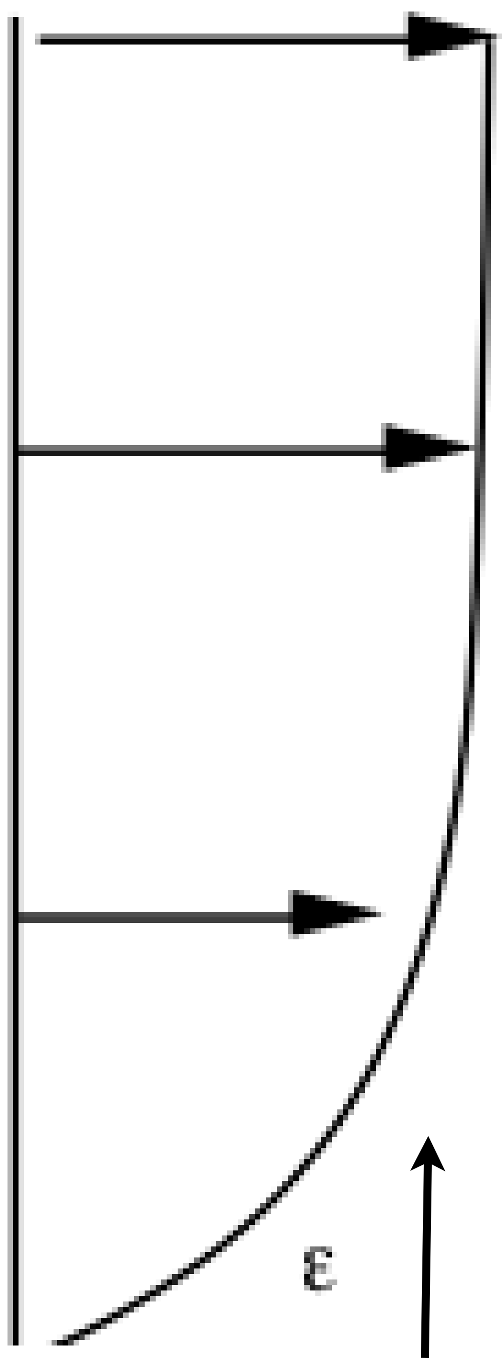
lower deck.



layer *couche*

Triple Deck *Triple Pont* (*~~triple couche~~*)

Link with IBL



definition of displacement thickness

$$\delta_1 = (Re^{-1/2})\int_0^\infty (1-u(x,\tilde{y}))d\tilde{y}$$

$$\bar{v}(\bar{x},0) = Re^{-1/2}\frac{\partial}{\partial \bar{x}}(\bar{u}_e\tilde{\delta}_1)$$

shape of perturbation in Main Deck

$$\tilde{u} = U_B(\tilde{y}) + \varepsilon A(x)U_B'(\tilde{y}).$$

$$\tilde{v} = -Re^{-1/4}A'(x)$$

by substitution

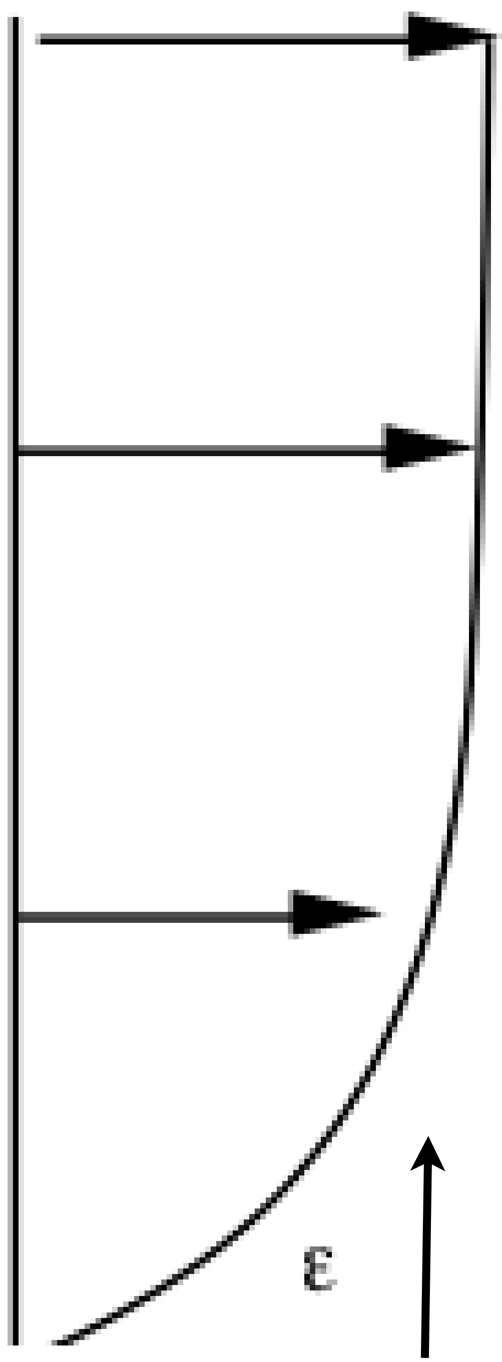
$$\delta_1 = (Re^{-1/2})\{\int_0^\infty (1-U_B(\tilde{y}))d\tilde{y} - \varepsilon A(x) - O(\varepsilon^2)\}.$$

the -*A* function is the perturbation of the displacement thickness

Link with IBL

$$\delta_1 = (Re^{-1/2})\{\int_0^{\infty}(1 - U_B(\tilde{y}))d\tilde{y} - \varepsilon A(x) - O(\varepsilon^2)\}.$$

the -*A* function is the perturbation of the displacement thickness



remember the coupling with velocity in IBL

$$\bar{u}_e = 1 + \frac{1}{\pi}\int \frac{\bar{f}'(\bar{x}) + Re^{-1/2}\frac{d(\tilde{\delta}_1 \bar{u}_e)}{d\bar{x}}}{x - \xi} d\xi$$

this is exactly the same than the Triple Deck
(variation of velocity are the opposite of variation of pressure)

$$p = \frac{1}{\pi}\int \frac{\frac{dA(\xi)}{dx}}{x - \xi} d\xi$$

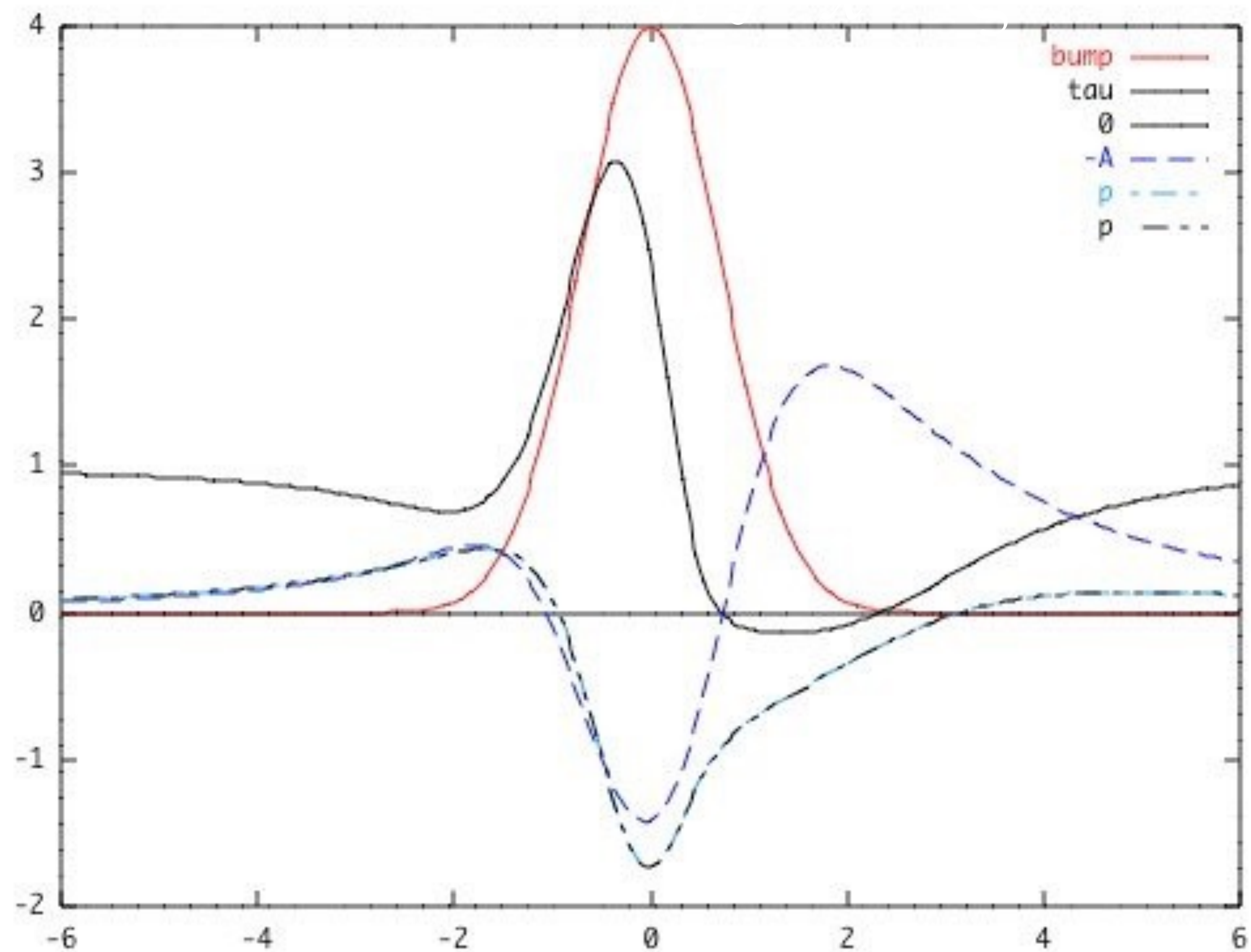
exemple: incompressible

pressure displacement
in incompressible

$$p = \frac{1}{\pi}\int \frac{\frac{dA(\xi)}{dx}}{x-\xi}d\xi$$

coupled to lower deck

$$\begin{aligned}
\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v &= 0,\\
u\frac{\partial}{\partial x}u + v\frac{\partial}{\partial y}u &= -\frac{d}{dx}p + \frac{\partial^2}{\partial y^2}u.\\
u(x, y = f(x)) &= 0, \qquad v(x, y = f(x)) = 0\\
\lim_{y\to\infty} u(x,y) &= y + A.
\end{aligned}$$



non linear simulation, note the shear max before the summit, the recirculation after the bump, the pressure drop

linear solution

$$\begin{cases} -ik\hat{u}_1 + \dfrac{\partial \hat{v}_1}{\partial y} = 0, \\ -iky\hat{u}_1 + \hat{v}_1 = ik\hat{p}_1 \quad + \dfrac{\partial^2 \hat{u}_1}{\partial y^2}, \end{cases}$$

$$\downarrow$$

$$-iky\hat{\tau}_1 = \frac{\partial^2 \hat{\tau}_1}{\partial y^2} \longrightarrow Ai((-ik)^{1/3} y)$$

linear solution

$$\begin{aligned}
\beta^* &= (3Ai'(0))^{-1}(-ik)^{1/3} \\
\beta_{pf} &= 1/\left|k\right|, 0, 1, -1, ik \\
FT[\tau] &= \frac{(-ik)^{2/3}}{Ai'(0)} Ai(0) \frac{FT[f]}{\beta^* - \beta_{pf}}
\end{aligned}$$

3.10 Plots of linearised solutions

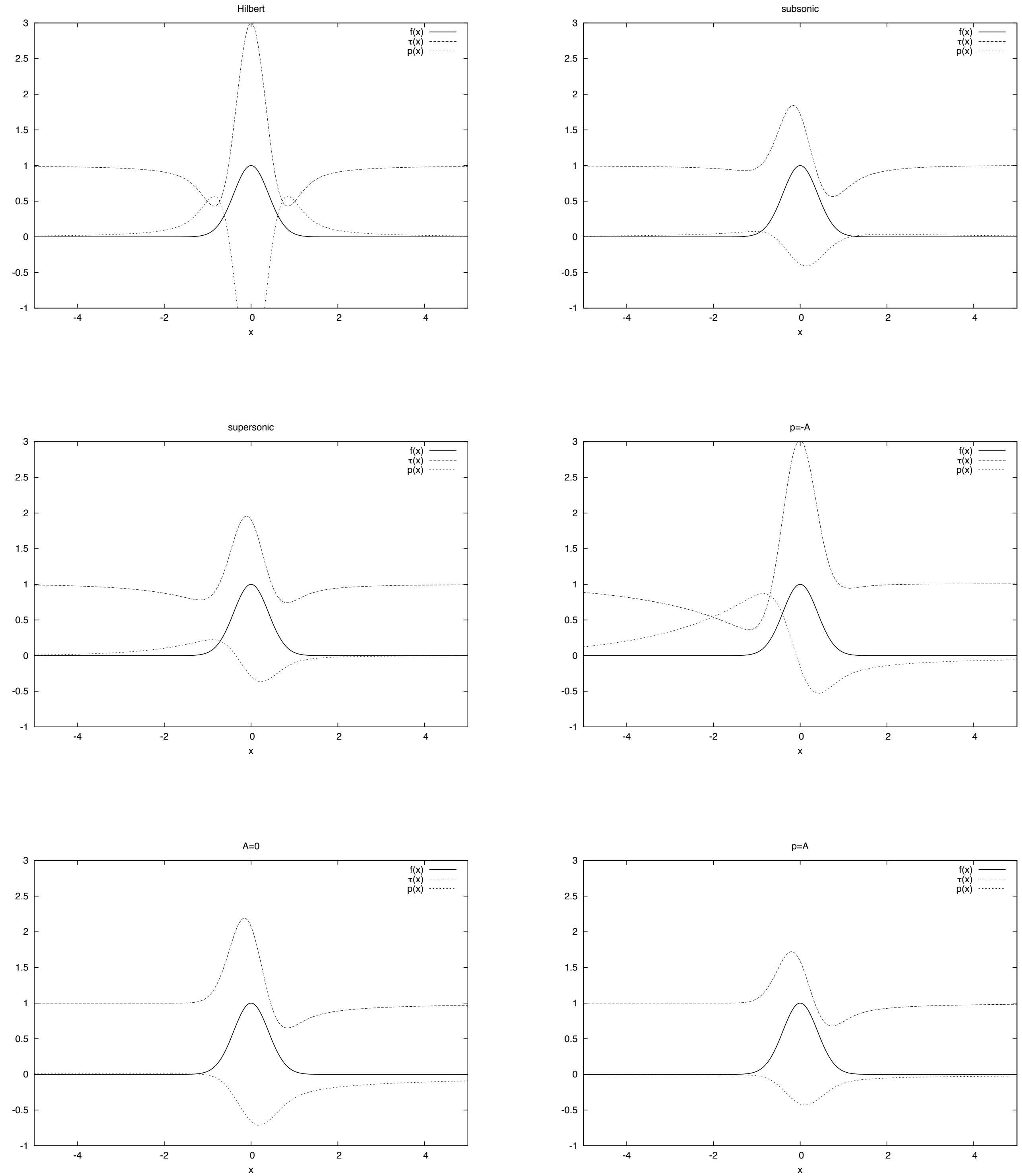
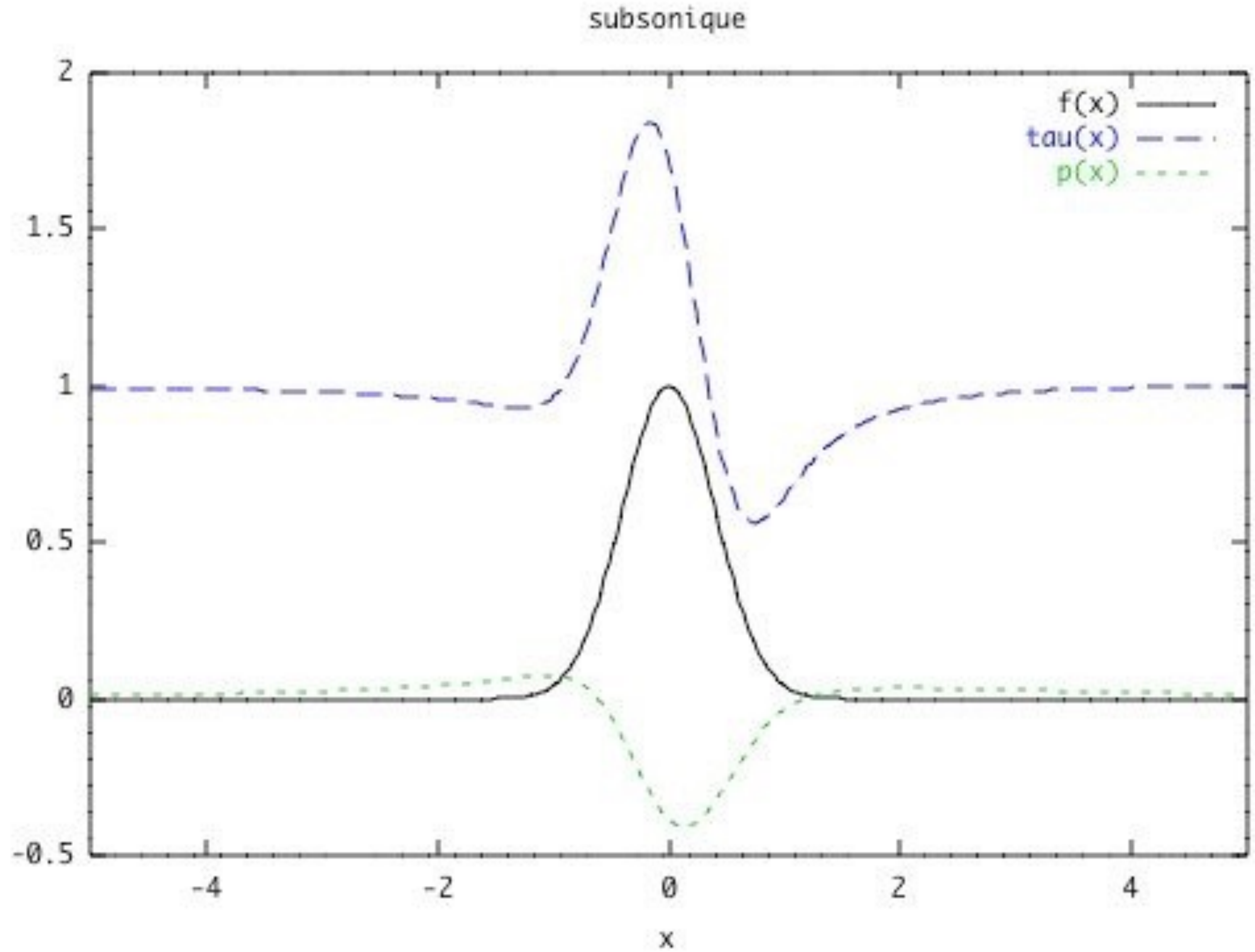


Figure 3: Friction distribution and pressure over a bump in 6 cases, linear solution. Top left the Hilbert case, just to compare. Top right the subsonic case $p = \frac{-1}{\pi}\int - \frac{\frac{dA}{dx}}{x-\xi}d\xi$. Middle left, the supersonic $p = -A'$ case. Middle right, $p = -A$ case. Bottom left, the $A = 0$ case. Bottom right, the $p = A$

incompressible

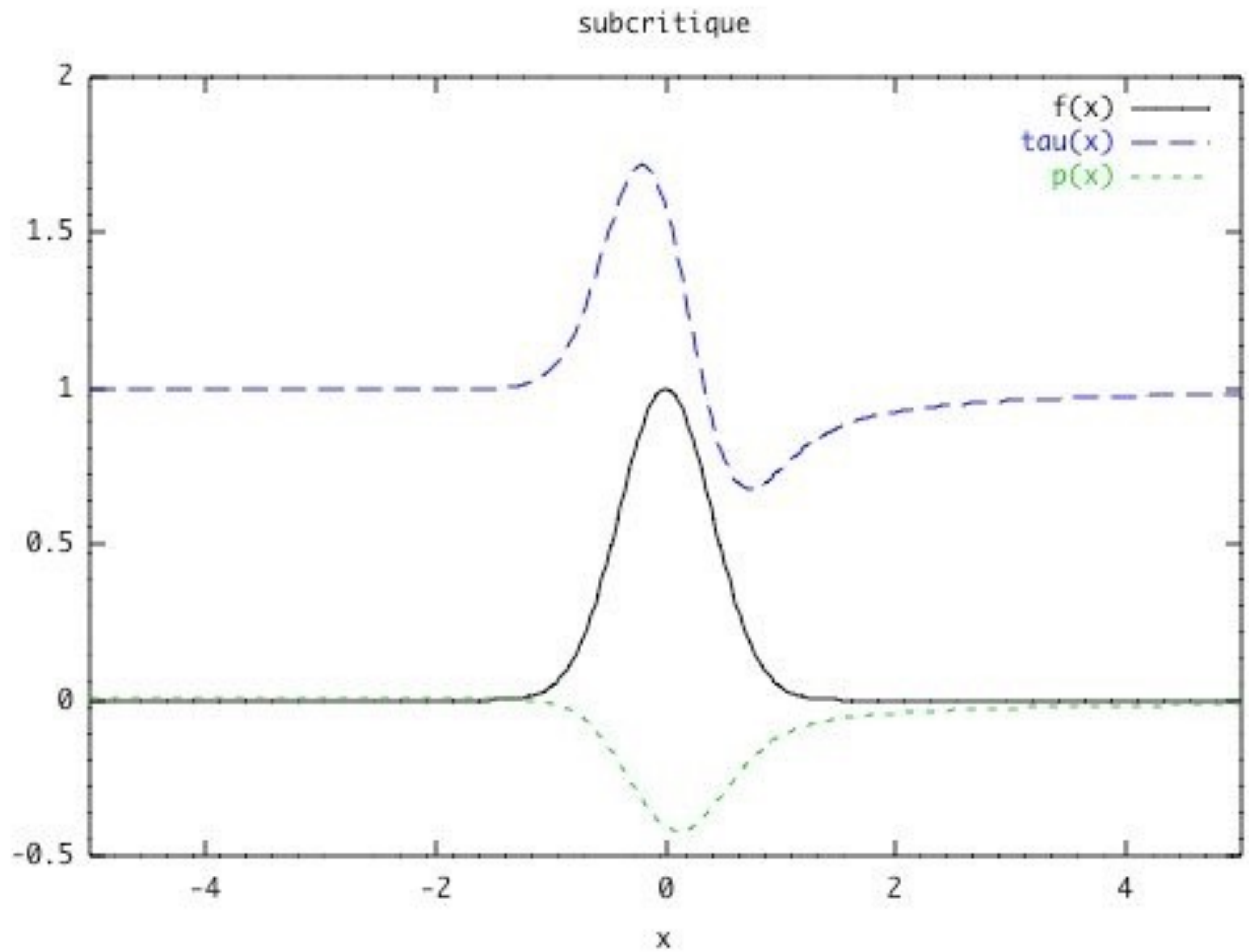
$$p = \frac{1}{\pi} fp \int_{-\infty}^{\infty} \frac{A'}{x-\xi} d\xi$$



linear

pipe/ subcritical

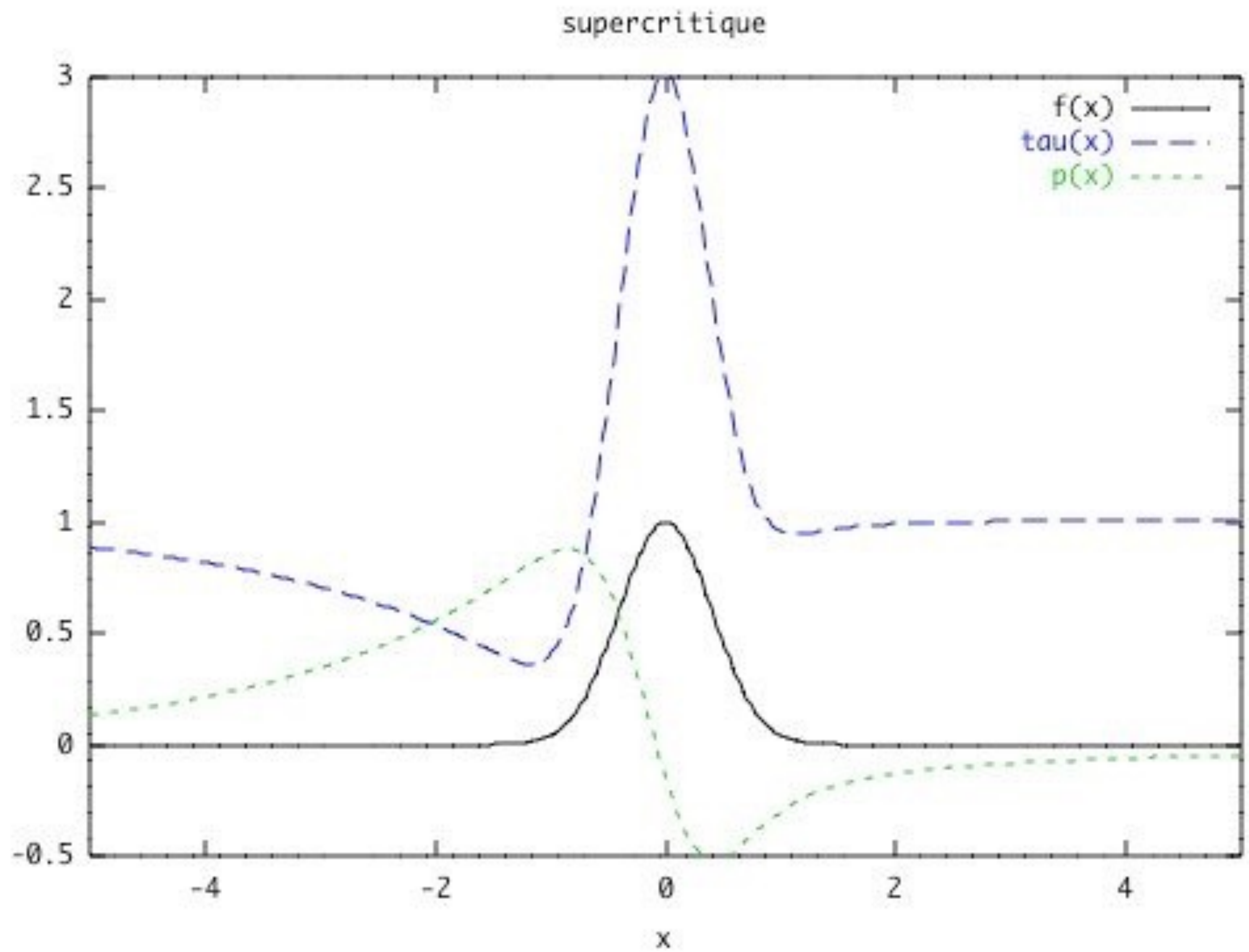
$$p = A$$



linear

supercritical

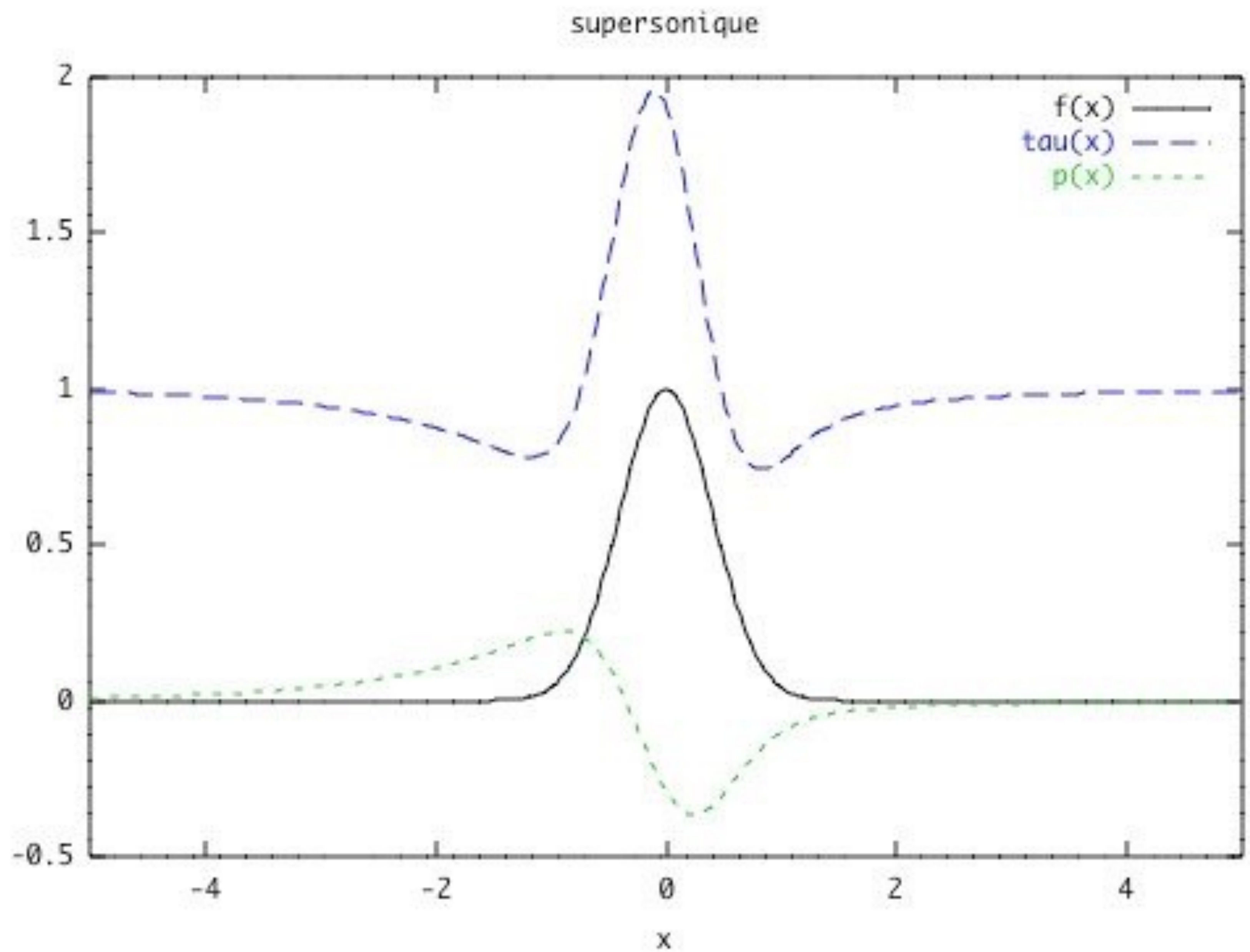
$$p = -A$$



linear

supersonic

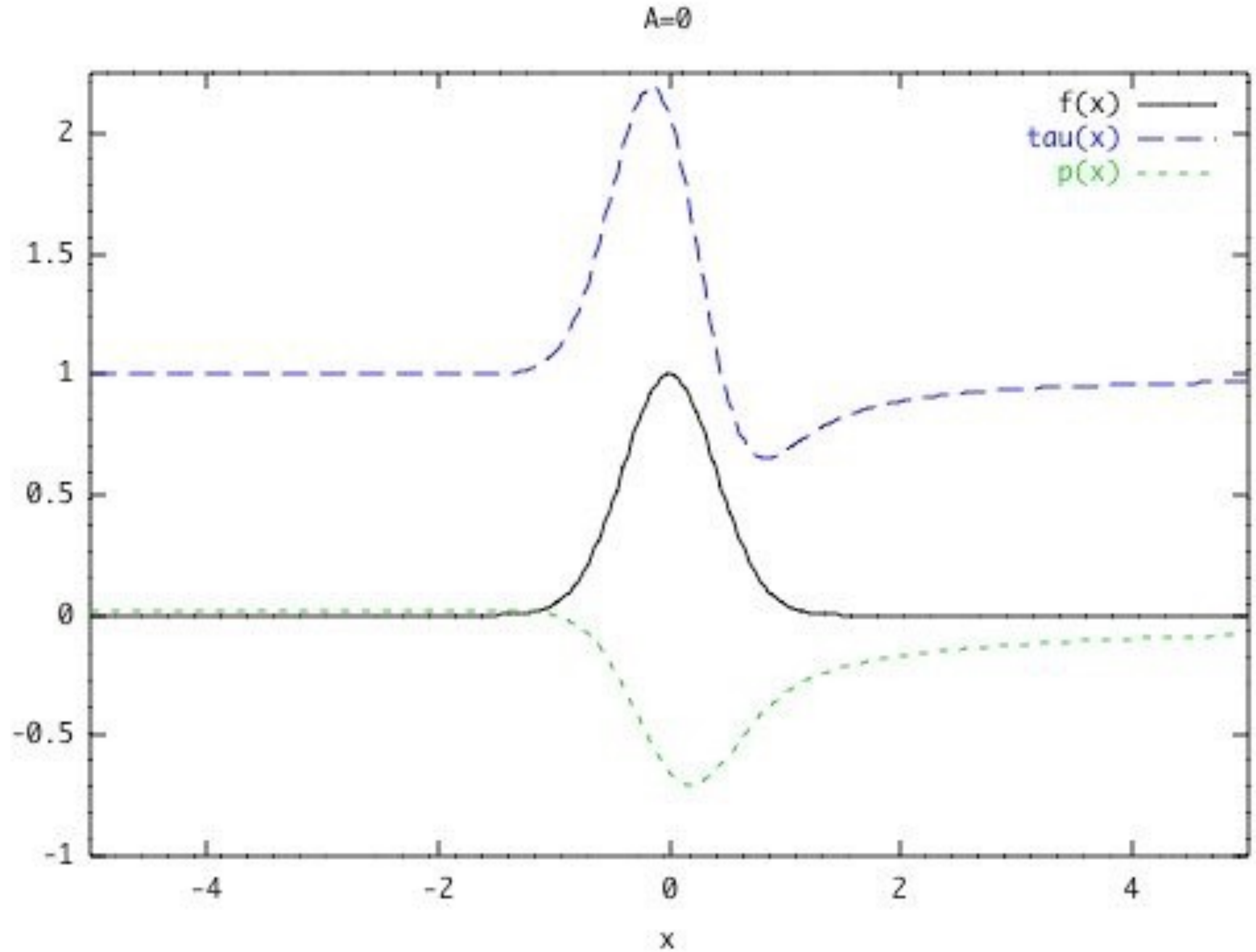
$$p = -A'$$



linear

shear flow

$$A = 0$$

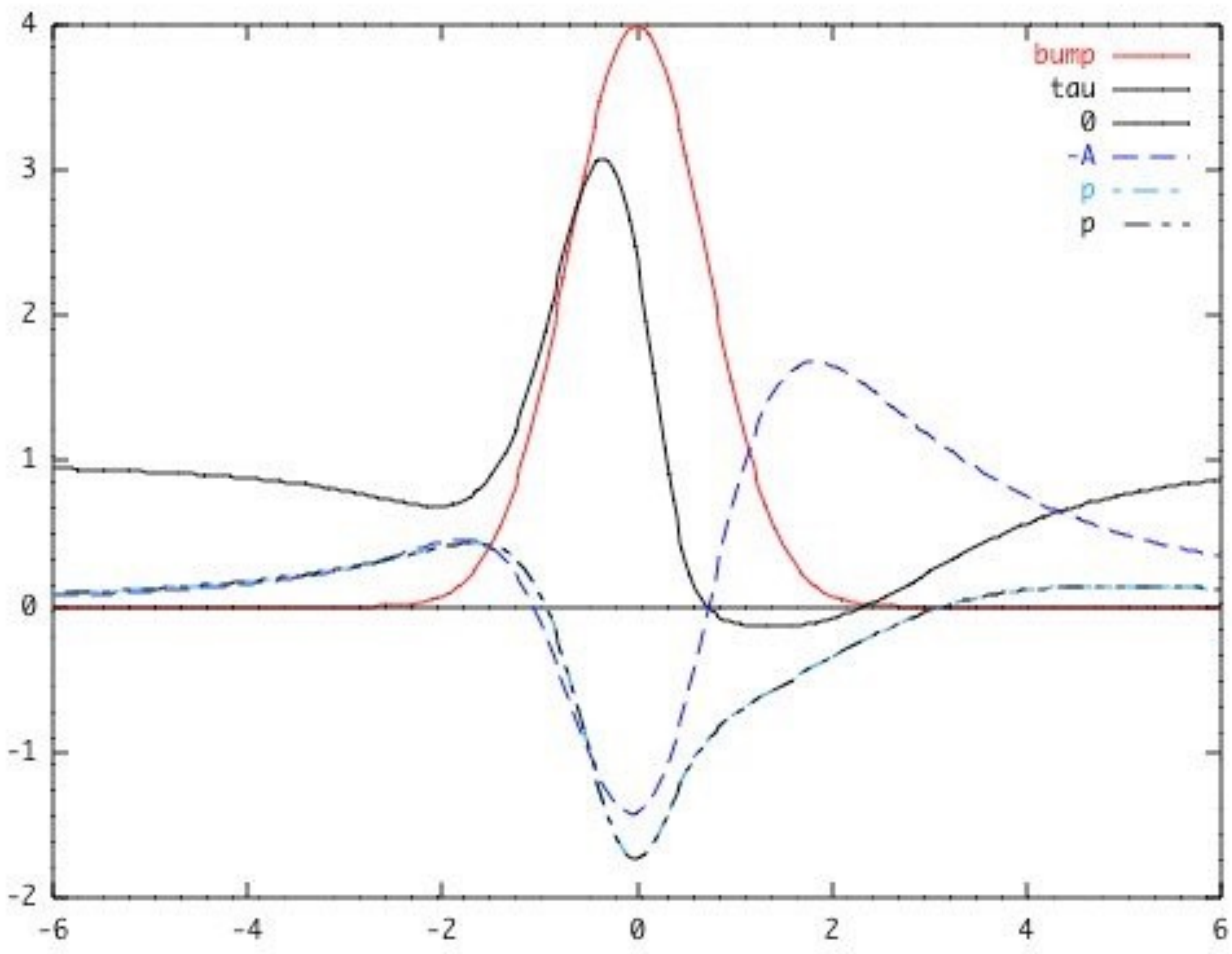


linear

Exemples with Boundary layer separation

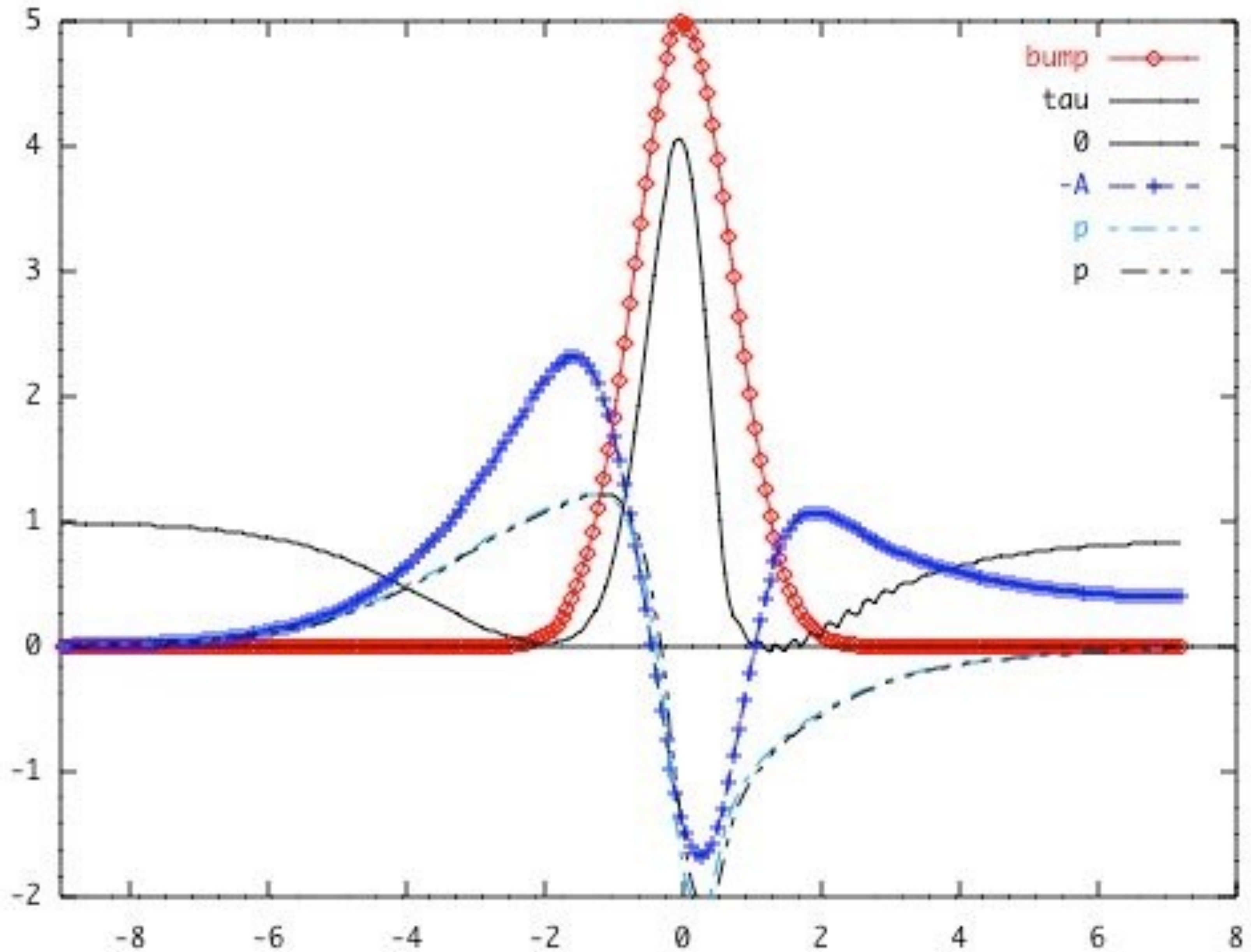
small separation bubble

incompressible



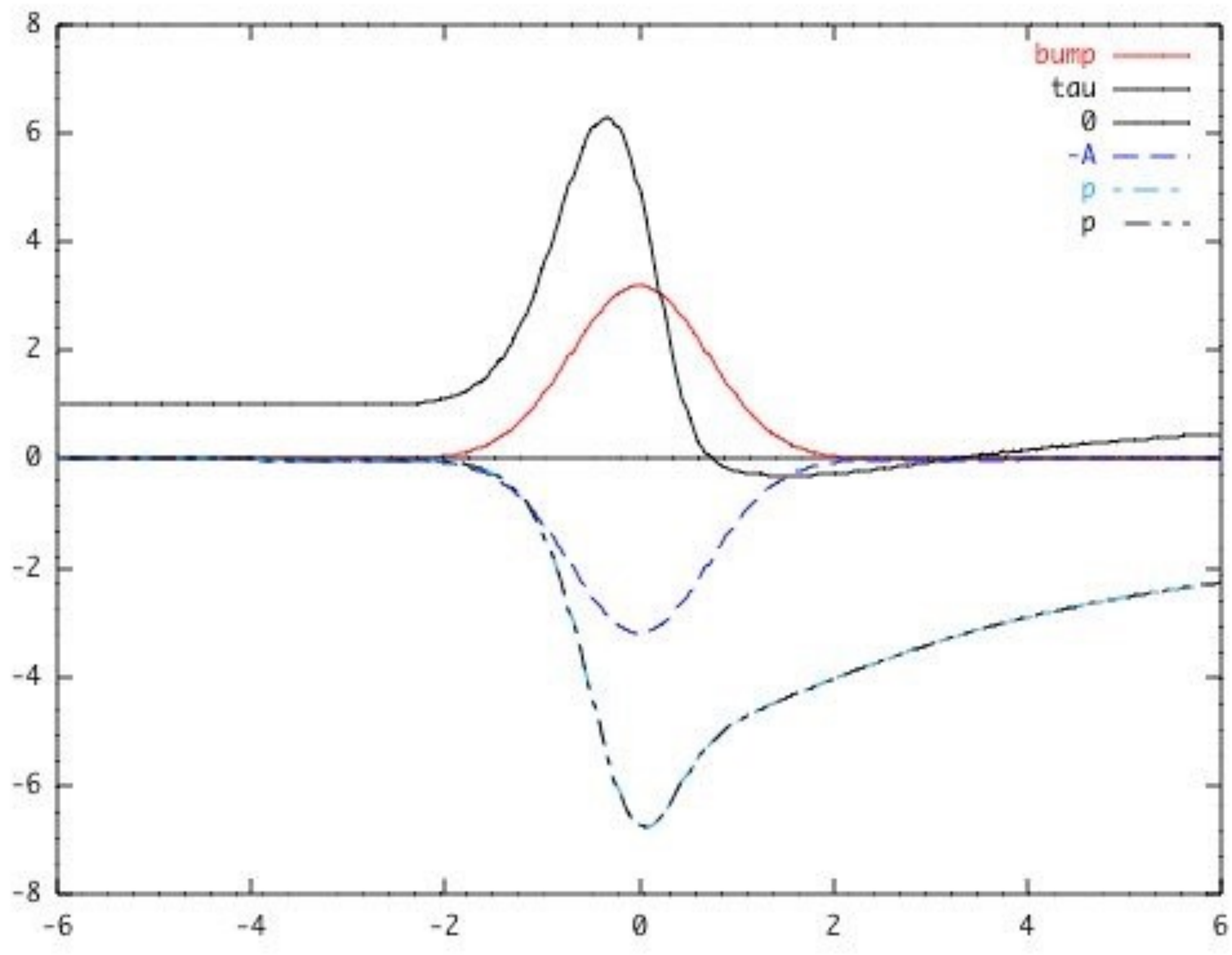
non linear

supersonic



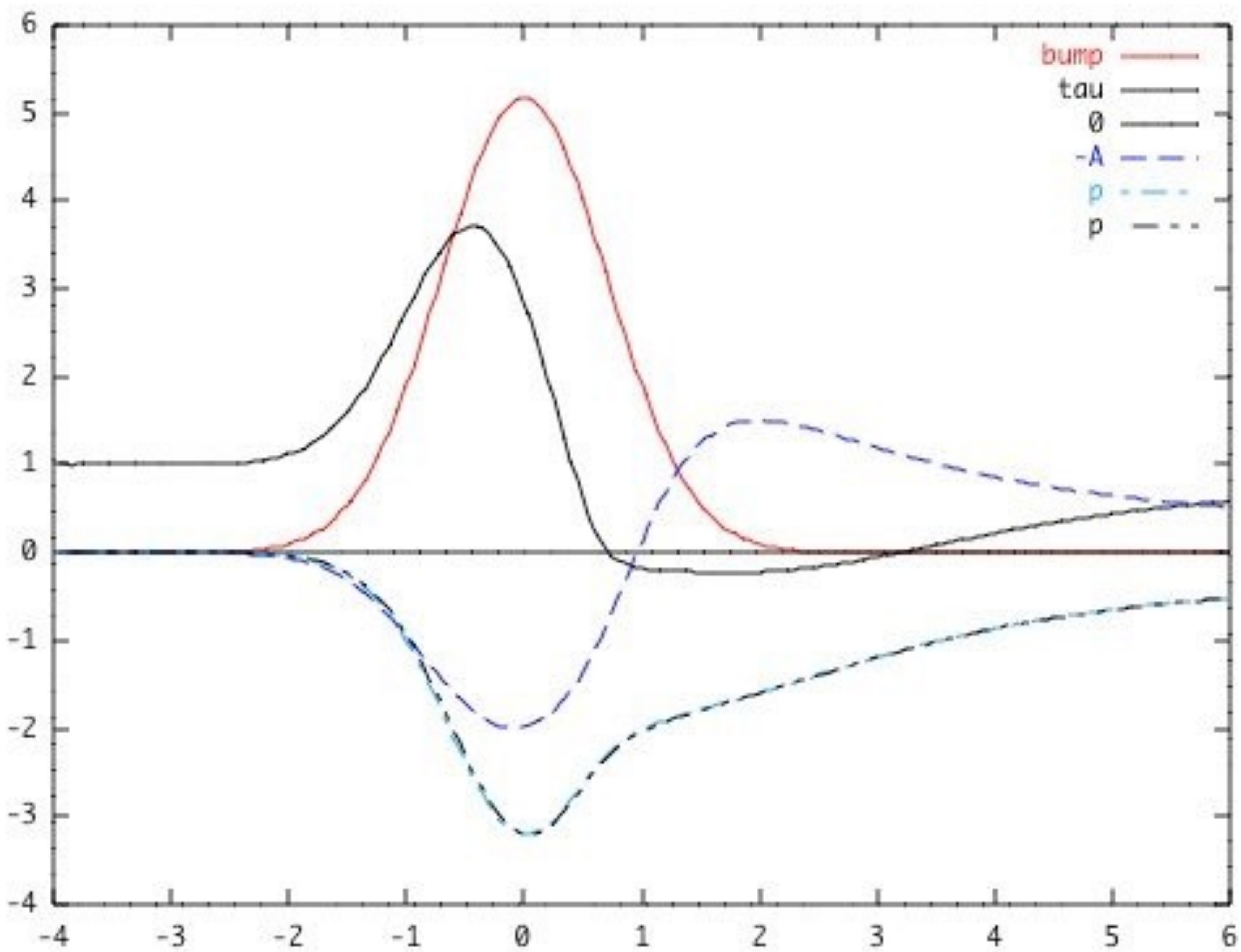
non linear

shear flow



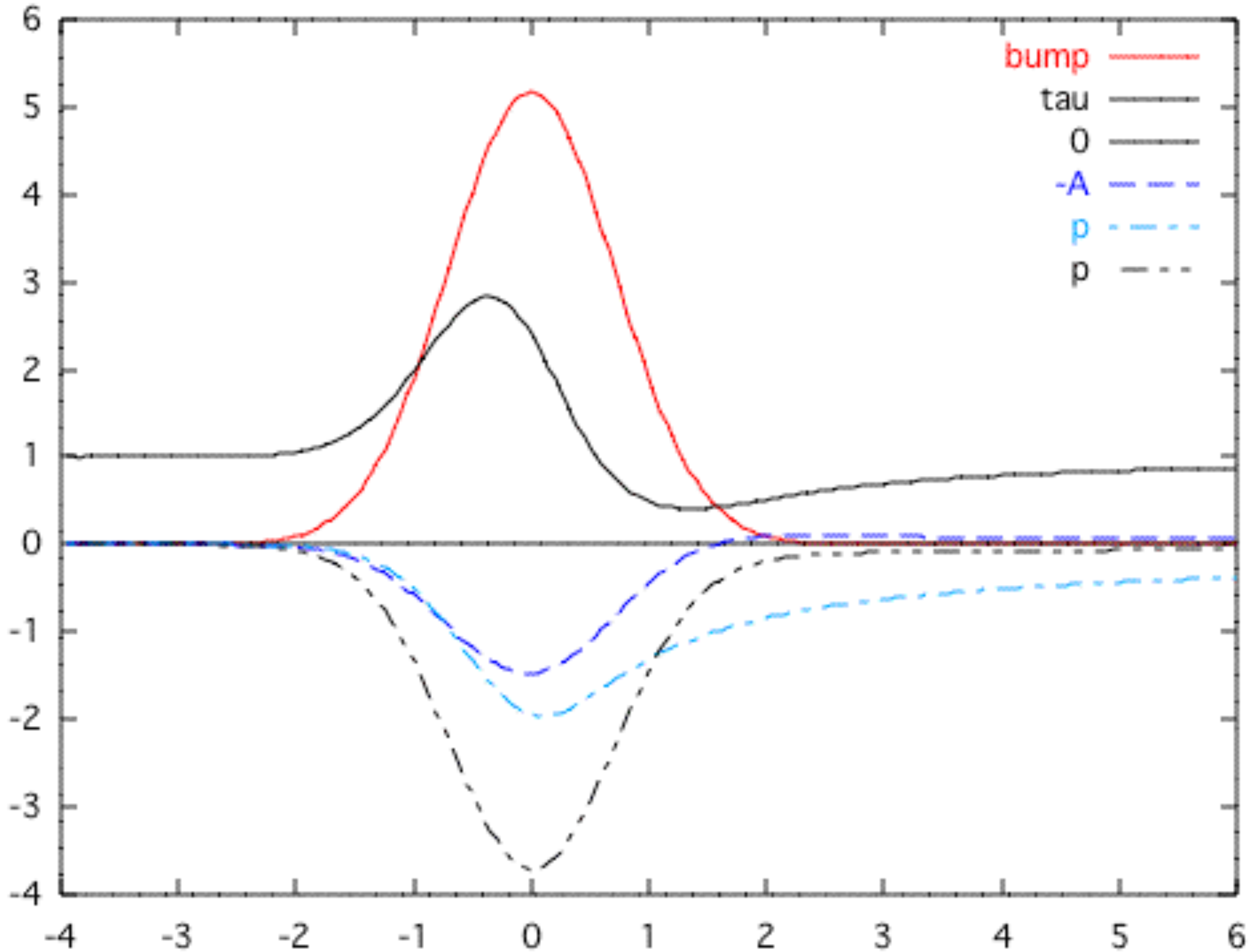
non linear

subcritical



non linear

subcritical



non linear

Trailing Edge

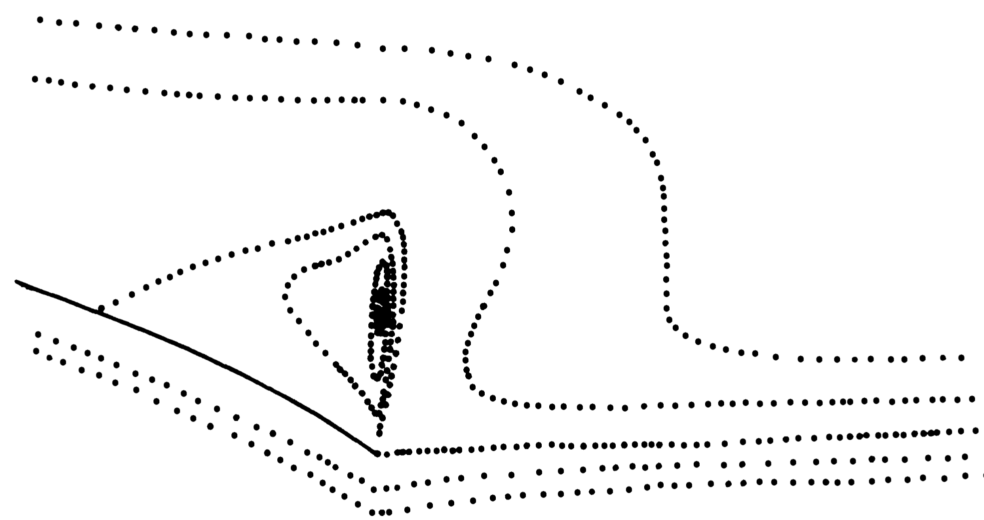


Figure 12. Triple-deck solutions for subsonic/incompressible fluid flowing past nonaligned trailing edges with separation [141].

[141] F. T. Smith: IMA J. of App. Math. 28, 207 (1982).

[T._Cebeci,__K._Stewartson,__J._H._Whitelaw__(auth(BookZa.org).pdf

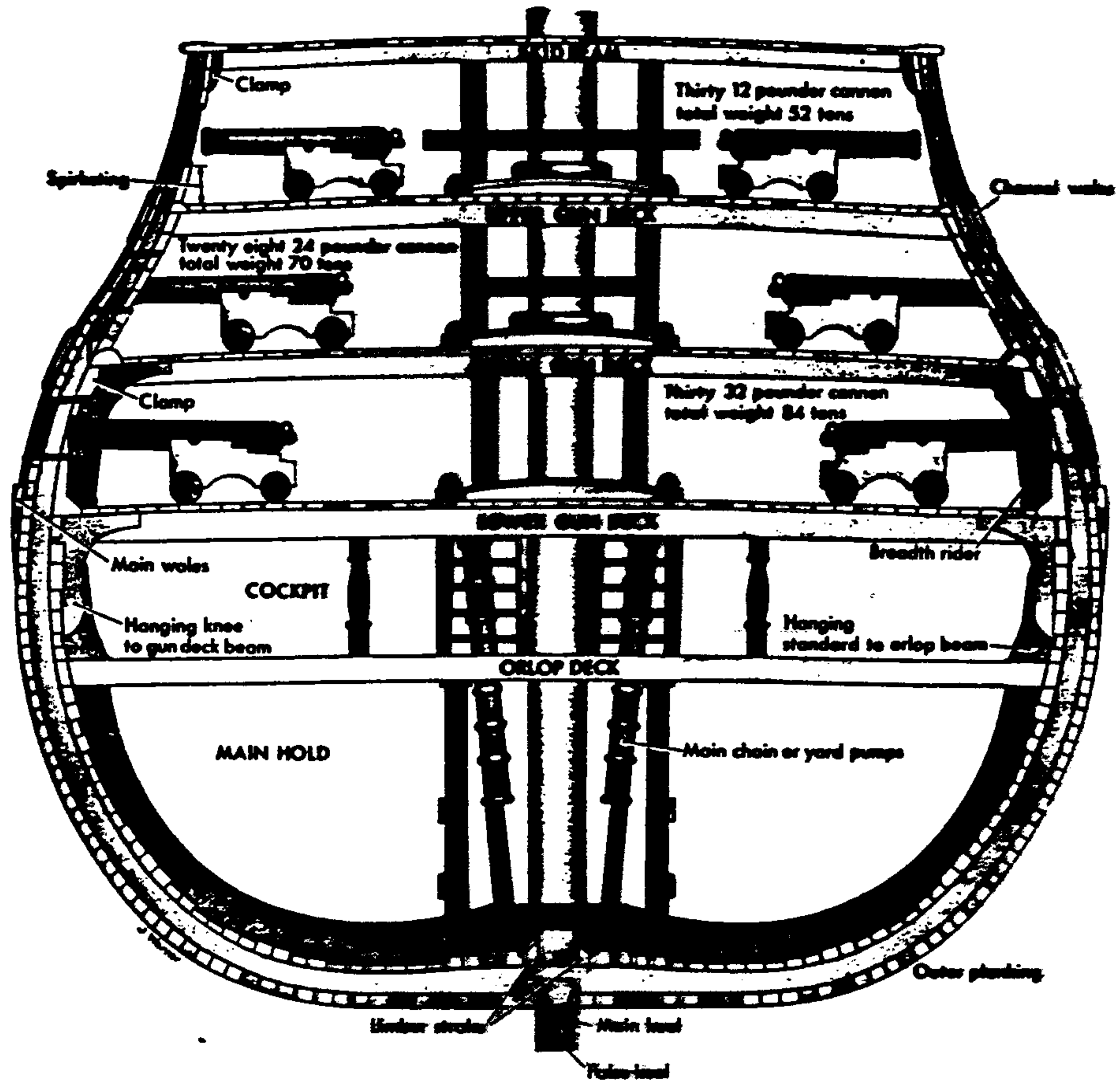
p144

Why Triple Deck?

upper deck

main deck

lower deck.

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?

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110 **AMERICAN** Ham, bologna, turkey, american cheese, lettuce & tomato 7.95
111 **CHEF** Roast beef, turkey, swiss cheese, tomato, onion, oil & vinegar 7.95
112 **ITALIAN** Cappicola ham, genoa slami, pepperoni, provolone cheese, lettuce, tomato, pepperoni, oil & vinegar 7.95
113 **VIP** Hot pastrami, corned beef, swiss cheese, coleslaw & russian dressing 7.95
114 **SUPER MELT** Hot salami, cappicola ham, swiss cheese, lettuce & tomato 7.95
115 **COLUMBUS** Hot smoked turkey, melted provolone cheese, green pepper and honey muenster, toasted hero 7.95
116 **ARISTOCRAT** Hot roast beef, melted swiss cheese, coleslaw & russian dressing, toasted hero 7.95

TRIPLE DECKERS

CALORIES 510-820

117 **TURKEY** 7.95
with bacon, lettuce & tomato

118 **TUNA FISH SALAD** 7.95
with tomato, lettuce, sliced egg & onion

119 **HOT PASTRAMI** 7.95
Corned beef, swiss russian dressing & coleslaw

120 **CORNED BEEF** 7.95
with pastrami, swiss and coleslaw

121 **CHICKEN SALAD** 7.95
with bacon, lettuce and tomato

122 **TURKEY BREAST** 7.95
Corned beef, swiis cheese and coleslaw

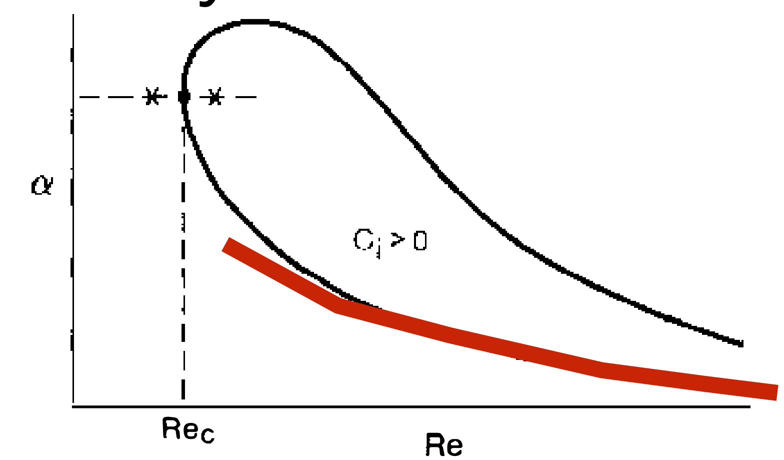
123 **ROAST BEEF** 7.95
Pastrami and coleslaw

124 **GRILLED CHICKEN** 7.95
with bacon, lettuce & tomato

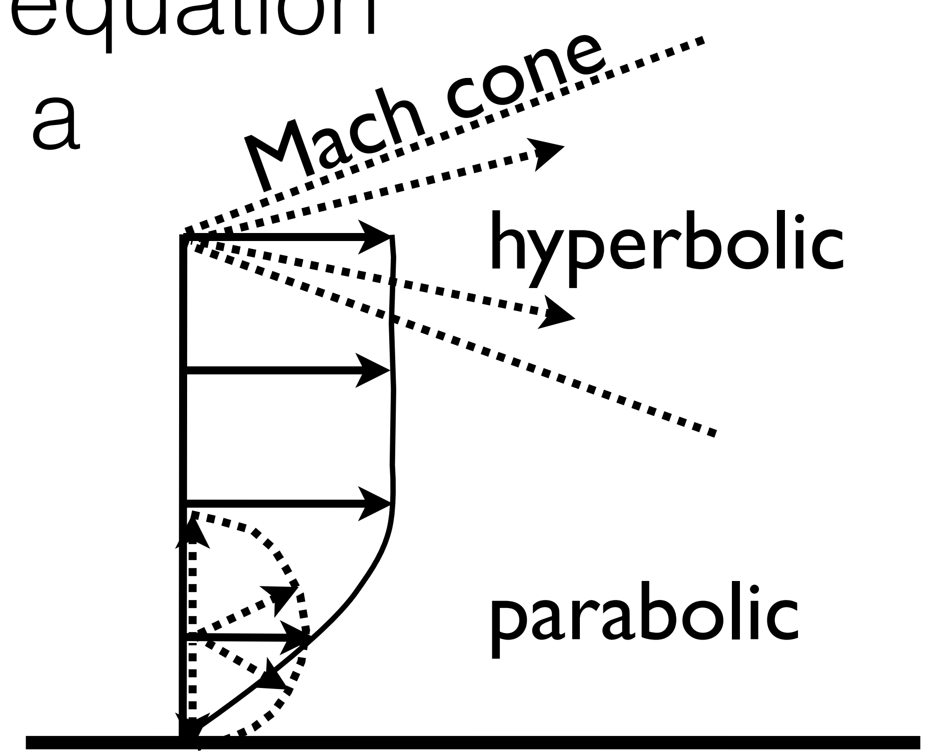
Served with Pickle and Coleslaw

Remarks

- unsteady triple deck is the lower branch of the Tollmien-Schlichting branch in the theory of stability of the boundary layer (linearly unstable)

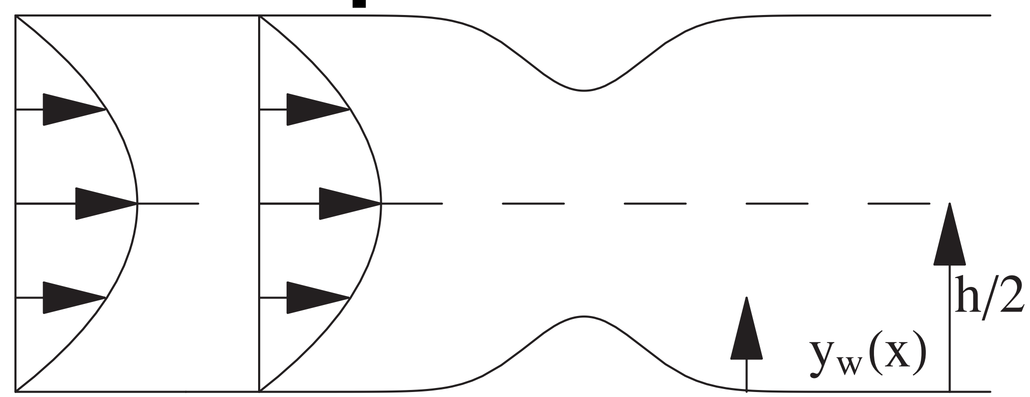


- through the coupling of the Lower Deck and Main Deck perturbation appear far upstream. Coupling a supersonic hyperbolic equation and a parabolic Lower Deck gives a solution to the “upstream influence paradox”



- and lot more

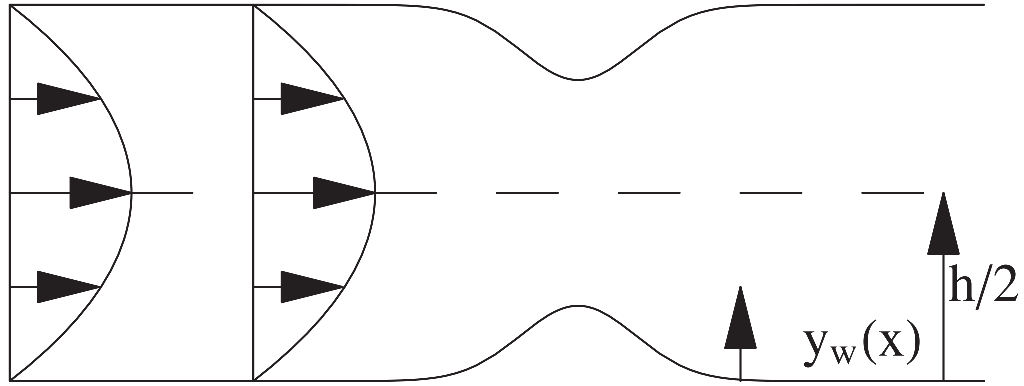
Case with no displacement *A=0*   "Double Deck"



In the case of pipe flow for a bump length of scale *h* and height $hRe^{-1/3}$ we are with no displacement *A=0* it is the "Double Deck" (Smith 77)

$$
\begin{aligned}
\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v &= 0,\\
u\frac{\partial}{\partial x}u + v\frac{\partial}{\partial y}u &= -\frac{d}{dx}p + \frac{\partial^2}{\partial y^2}u.\\
u(x, y = f(x)) &= 0, \qquad v(x, y = f(x)) = 0\\
\& \qquad \lim_{y\to\infty} u(x, y) &= y.
\end{aligned}
$$

Case with no displacement *A=0* “Double Deck”



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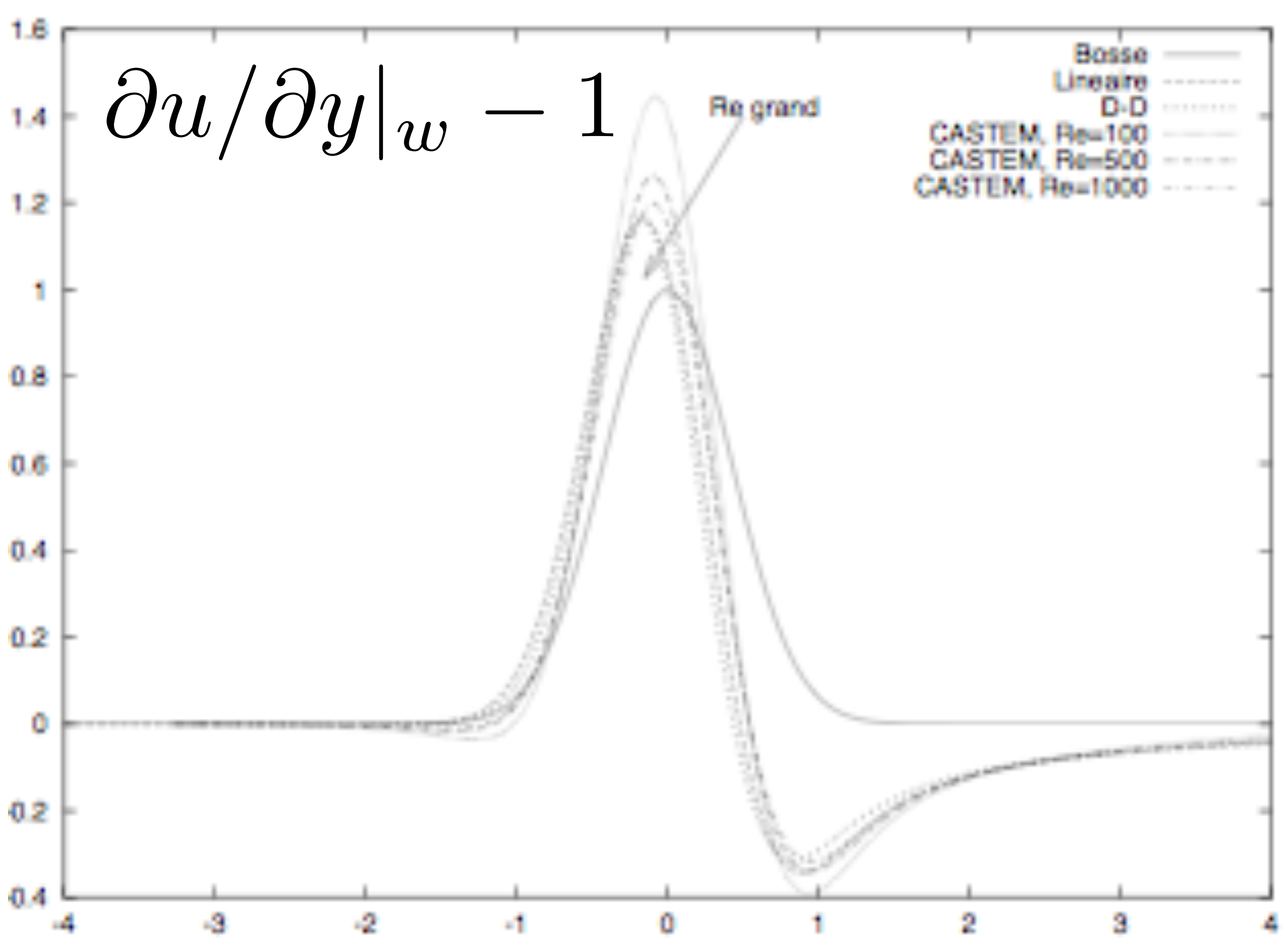
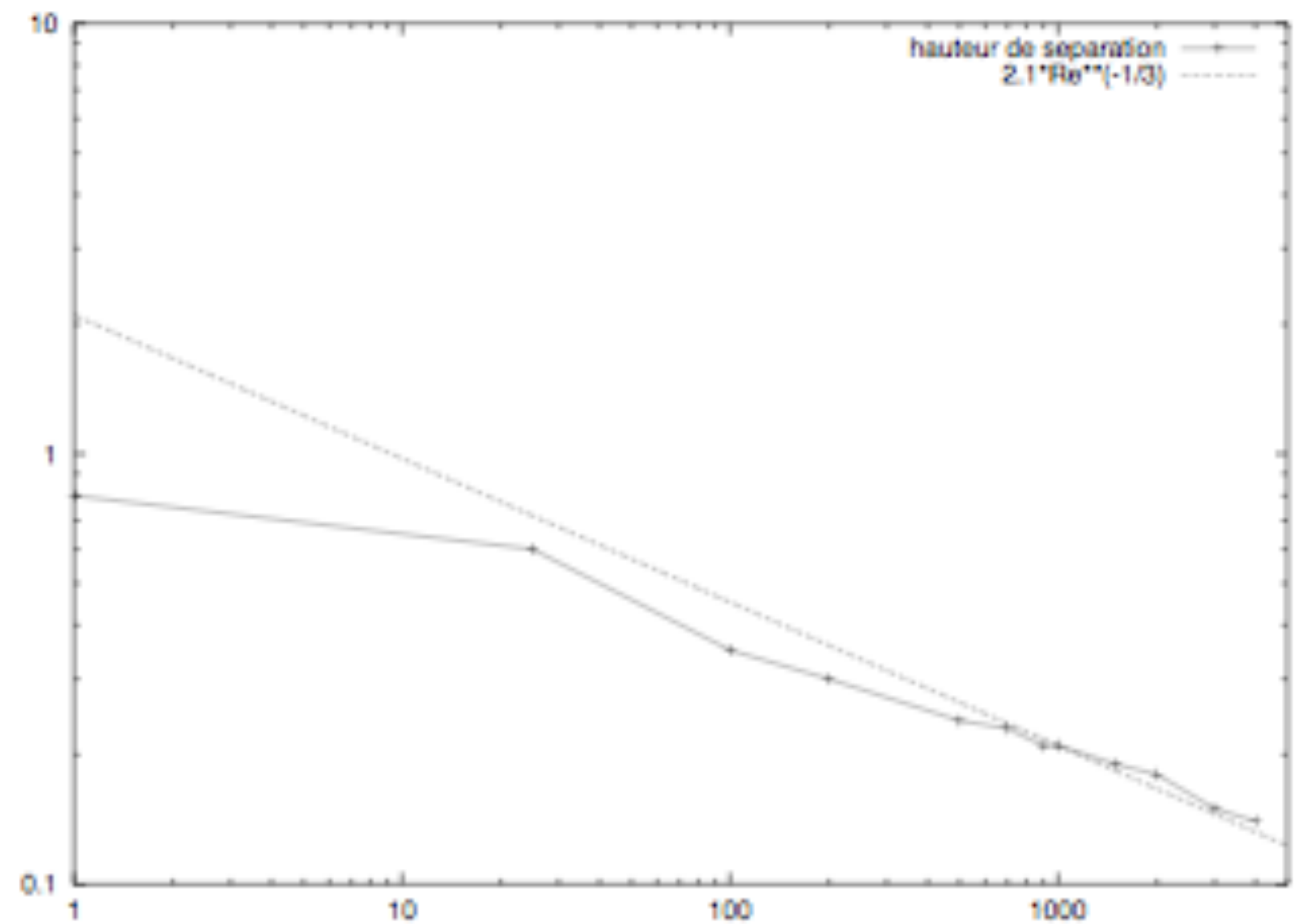




Double Deck equations

$$
\begin{aligned}
\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v &= 0, \\
u\frac{\partial}{\partial x}u + v\frac{\partial}{\partial y}u &= -\frac{d}{dx}p + \frac{\partial^2}{\partial y^2}u. \\
u(x, y = f(x)) &= 0, \qquad v(x, y = f(x)) = 0 \\
\& \qquad \lim_{y\to\infty} u(x, y) &= y.
\end{aligned}
$$

remember these equations are solved with marching in space in finite differences

Chouly Lagrée 09 proposed a variational formulation

$$
\begin{cases}
\int_\Omega \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)\zeta + \frac{1}{Re}\int_\Omega \frac{\partial u}{\partial y}\frac{\partial \zeta}{\partial y} + \int_\Omega \lambda\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\left(\frac{\partial \zeta}{\partial x} + \frac{\partial \xi}{\partial y}\right) \\
-\int_\Omega p\left(\frac{\partial \zeta}{\partial x} + \frac{\partial \xi}{\partial y}\right) + \int_\Omega q\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = \mathbf{0},
\end{cases}
$$

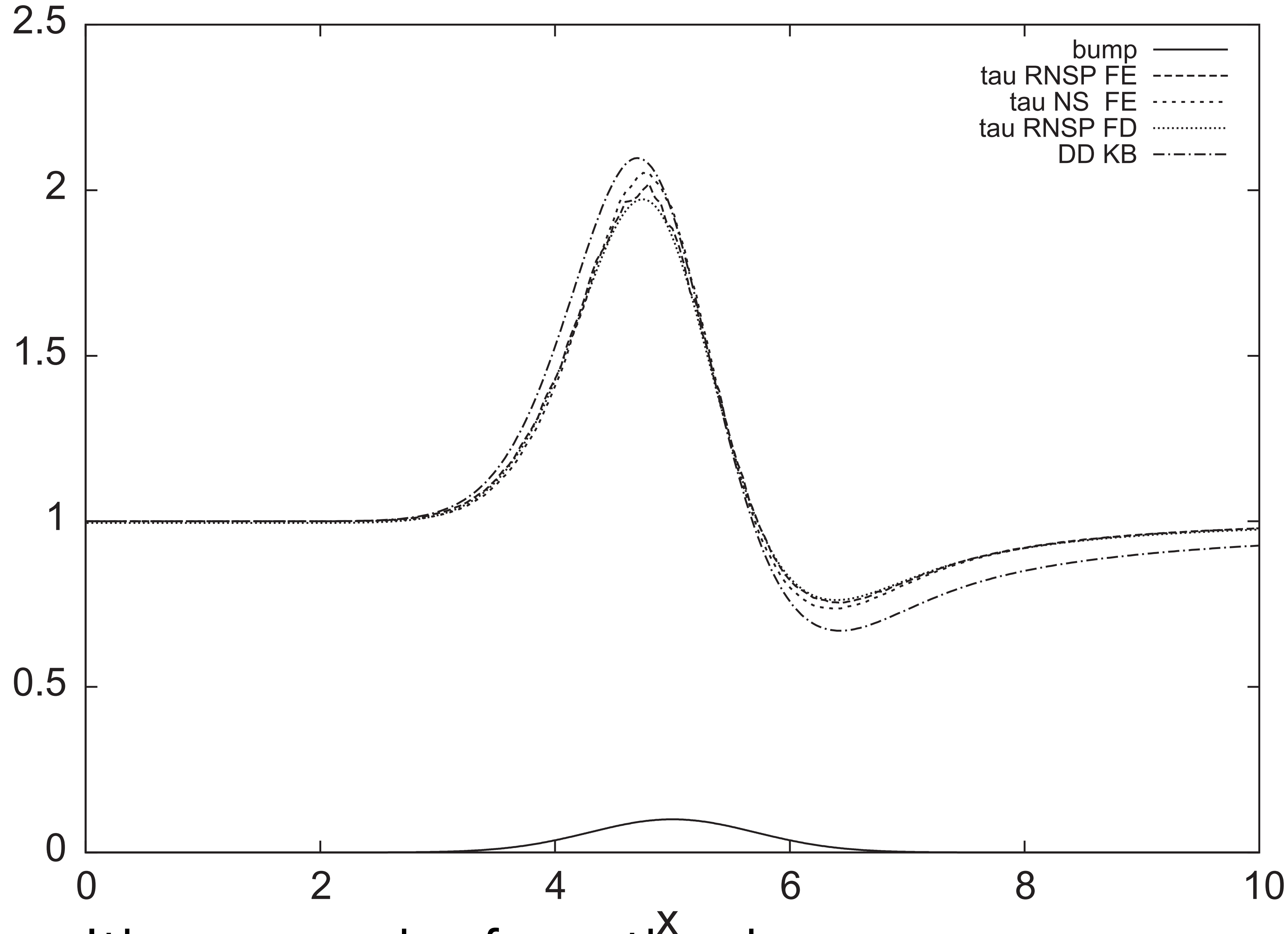
elements P2 P1 P0 Barrenechea Chouly 09

compare with Keller Box, Finite Differences

Double Deck (and Triple Deck) have a simple analytical solution in Fourier space

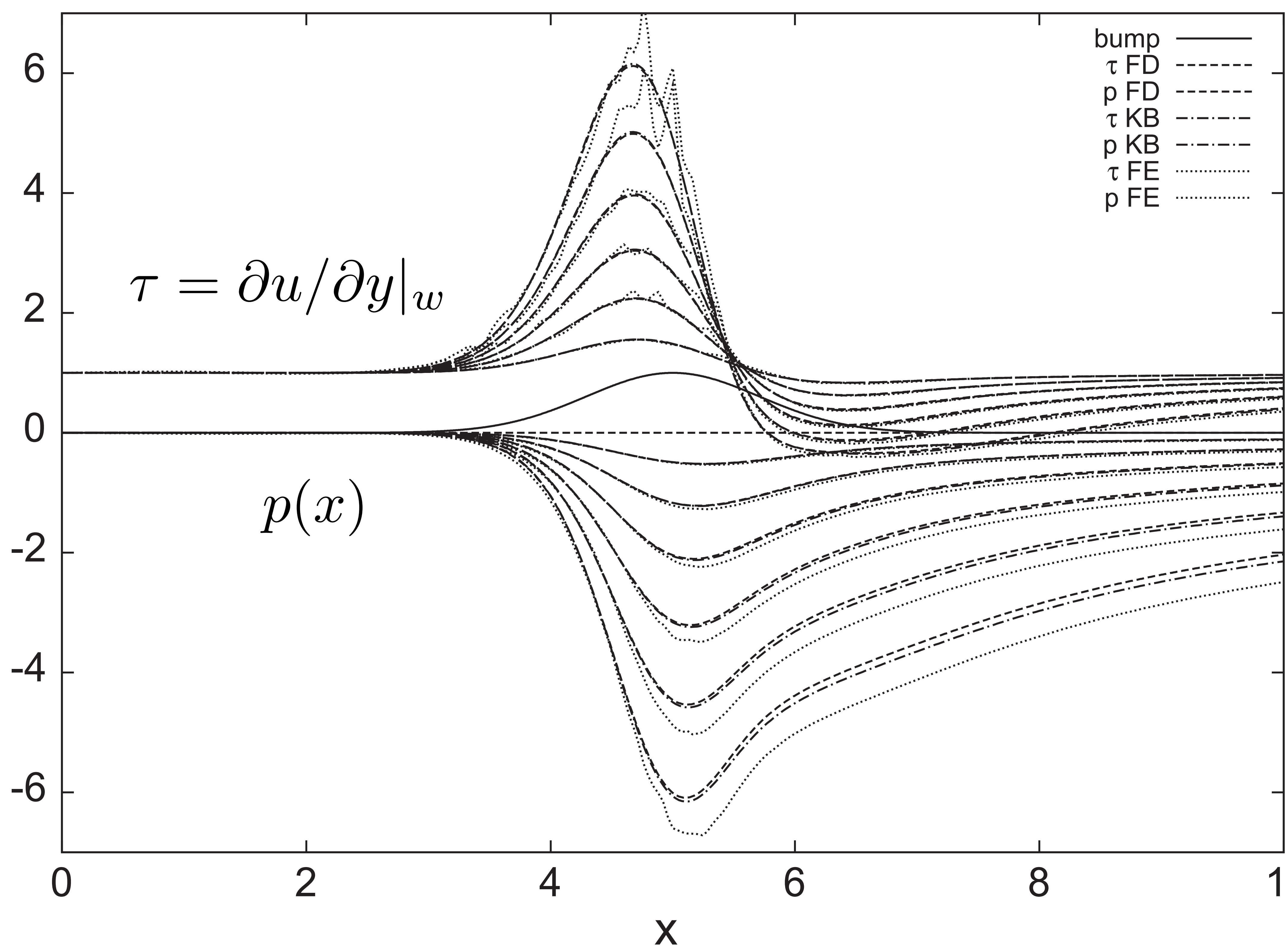
$$\tau = U_0' + U_0'(3Ai(0))(U_0')^{1/3}TF^{-1}[(-ik)^{1/3}TF[y_w]]$$

$$p = (U_0')^2(3Ai'(0))(U_0')^{-1/3}TF^{-1}[(-ik)^{-1/3}TF[y_w]].$$



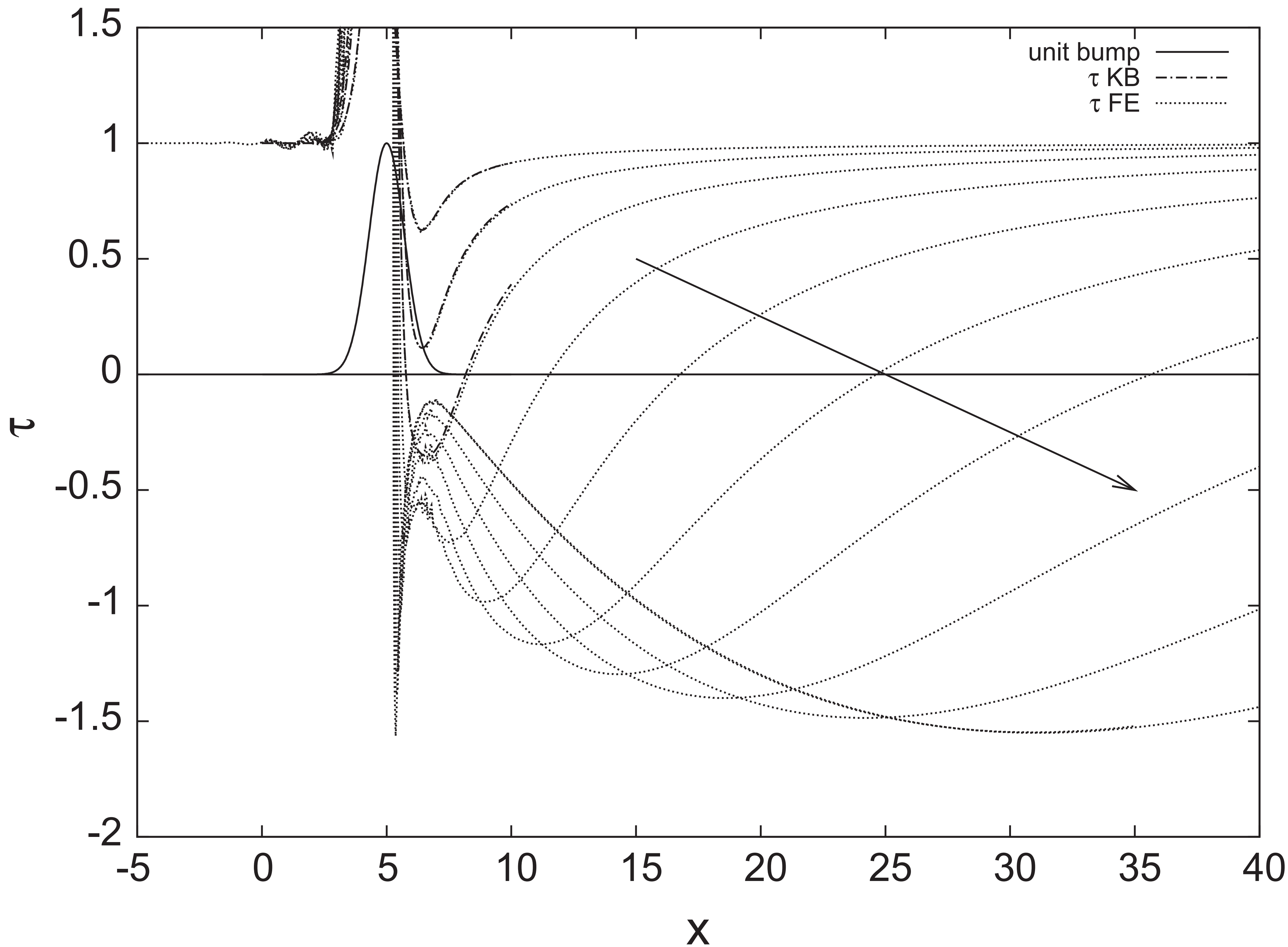
Skin friction with a max before the bump,

Double Deck non linear numerical solution



Skin friction with a max before the bump, decrease of pressure

Double Deck non linear numerical solution,
shear at the wall $\tau = \partial u/\partial y|_w$

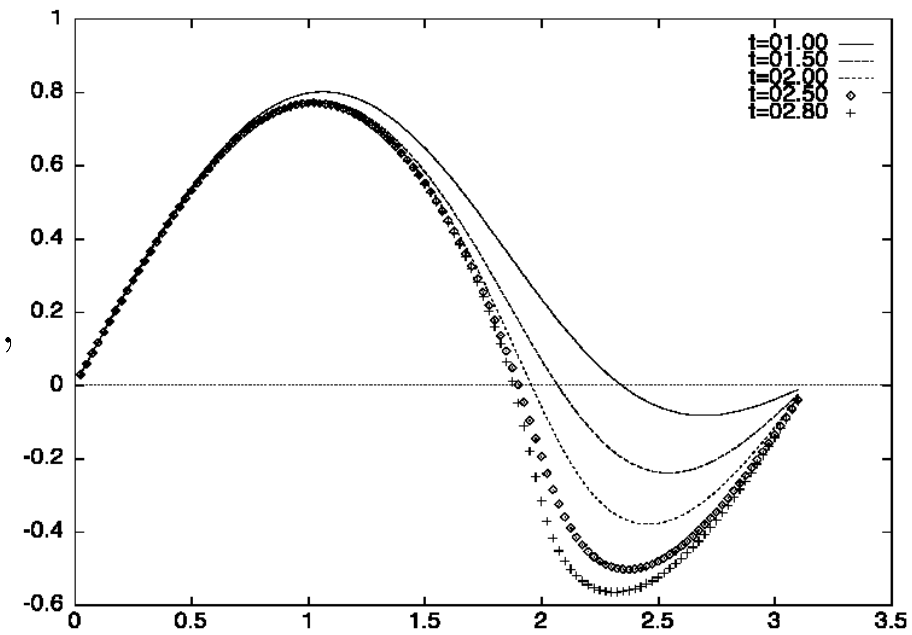


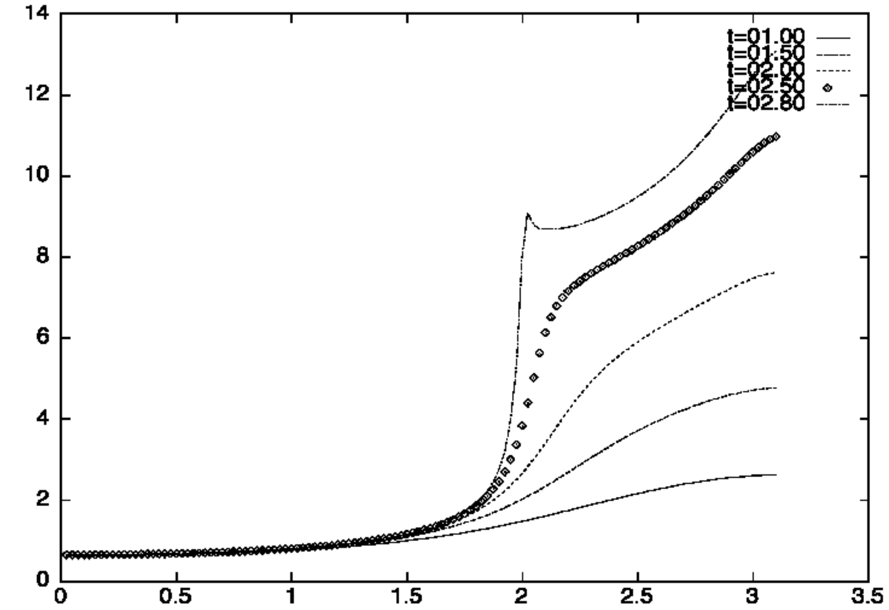
with finite elements we can compute very large recirculations everything perfect ?

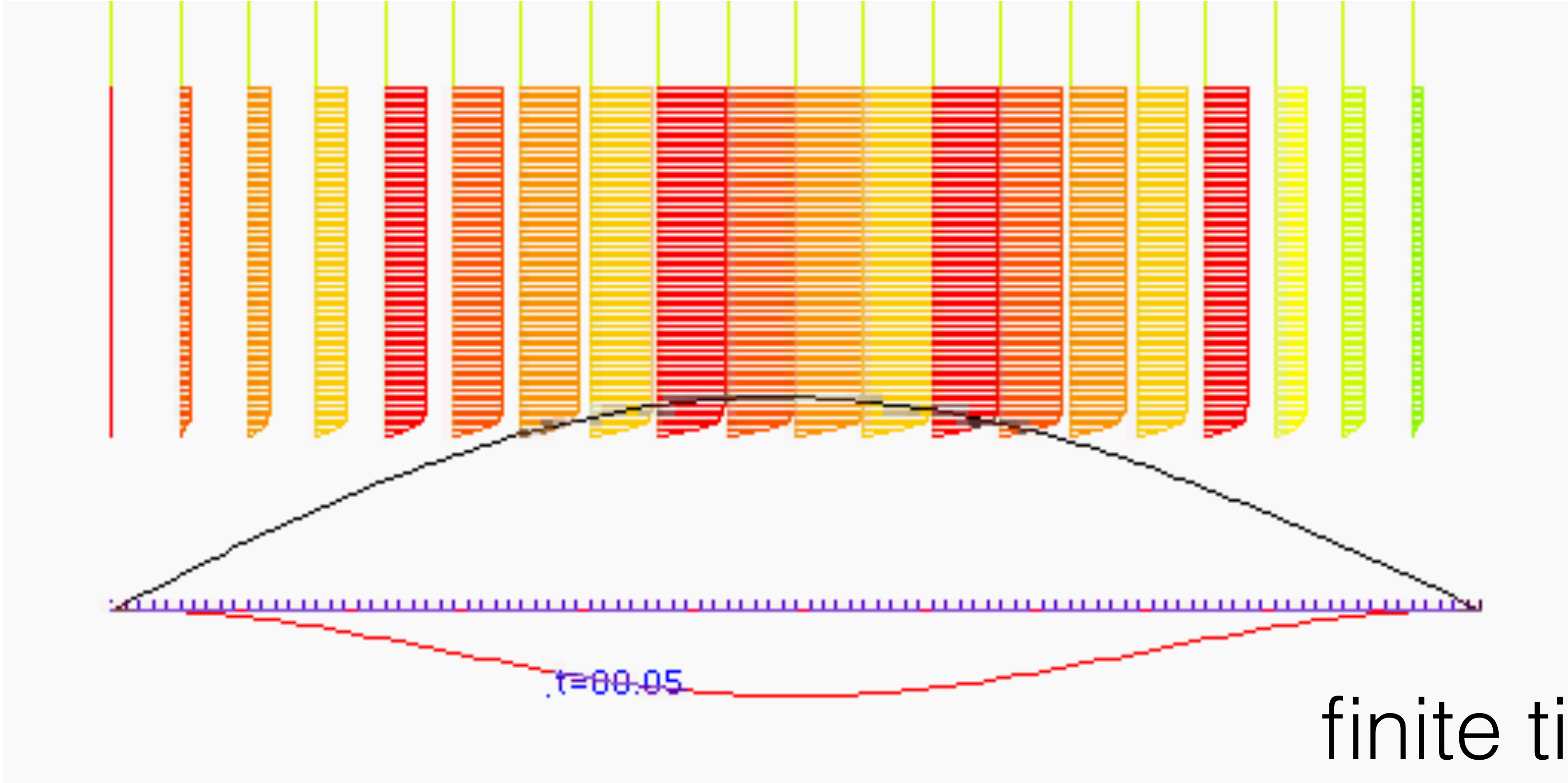
no!

some problems: unsteady boundary layer

$$\begin{cases} \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0, \\ \dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} = u_e\dfrac{du_e}{dx} + \dfrac{\partial^2 u}{\partial y^2}, \\ u(x,0,t) = v(x,0,t) = 0, \\ u(x, y>0, t=0) = u_e(x) \\ v(x, y>0, t=0) = 0 \\ \text{and } u(x,\infty,t>0) = u_e(x), \quad \text{with } \; u_e(x) = sin(x). \end{cases}$$







finite time singularity

outline

- the classical Boundary Layer
- second order Boundary Layer
- Interactive Boundary Layer
- some examples of numerical resolution with some comparaisons with Navier Stokes
- the Triple Deck, example of numerical solution
- the Double Deck, example of numerical solution FD FE
- summary

summary

with various scales

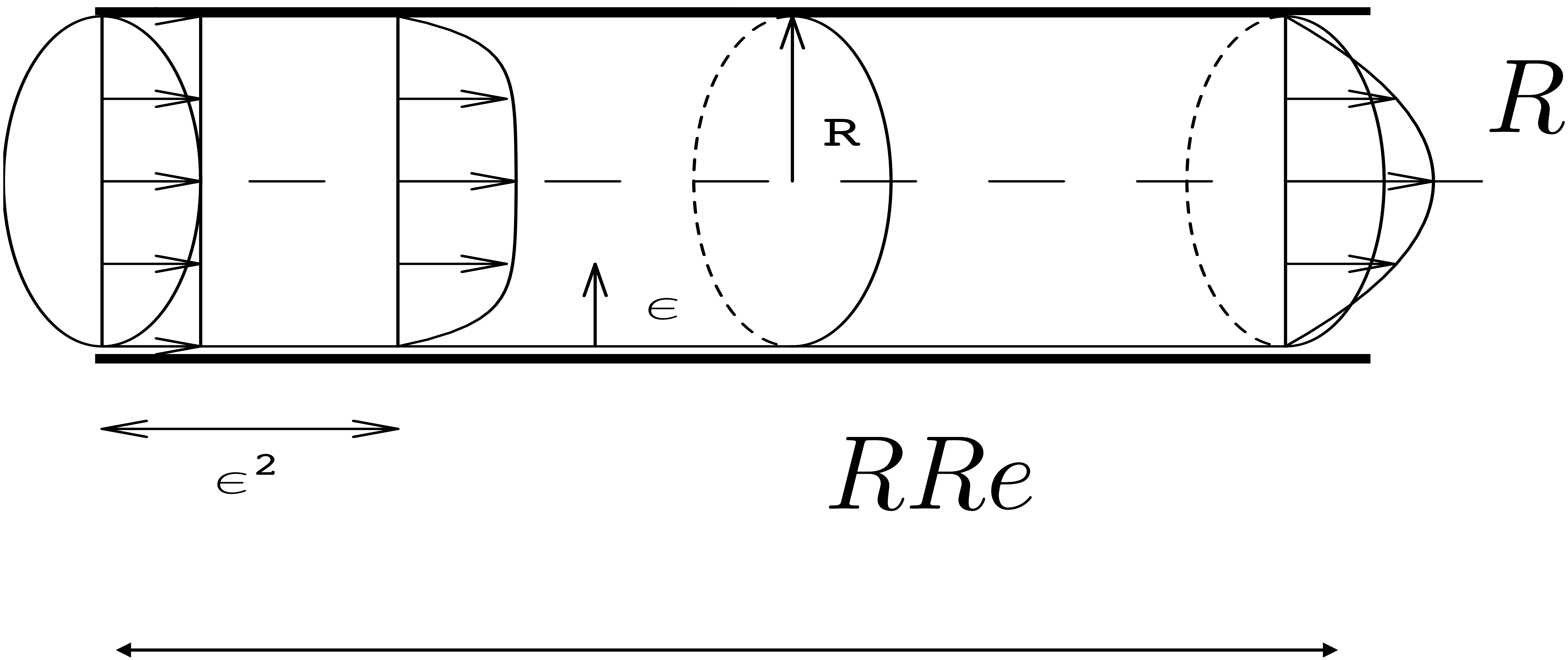
dominant equations are “Prandtl” equations

$$\frac{\partial}{\partial x}u+\frac{\partial}{\partial y}v=0 \qquad u\frac{\partial}{\partial x}u+v\frac{\partial}{\partial y}u=-\frac{\partial}{\partial x}p+\frac{\partial^2}{\partial y^2}u \qquad 0=-\frac{\partial}{\partial y}p$$

with no slip conditions
with various boundary conditions at the top

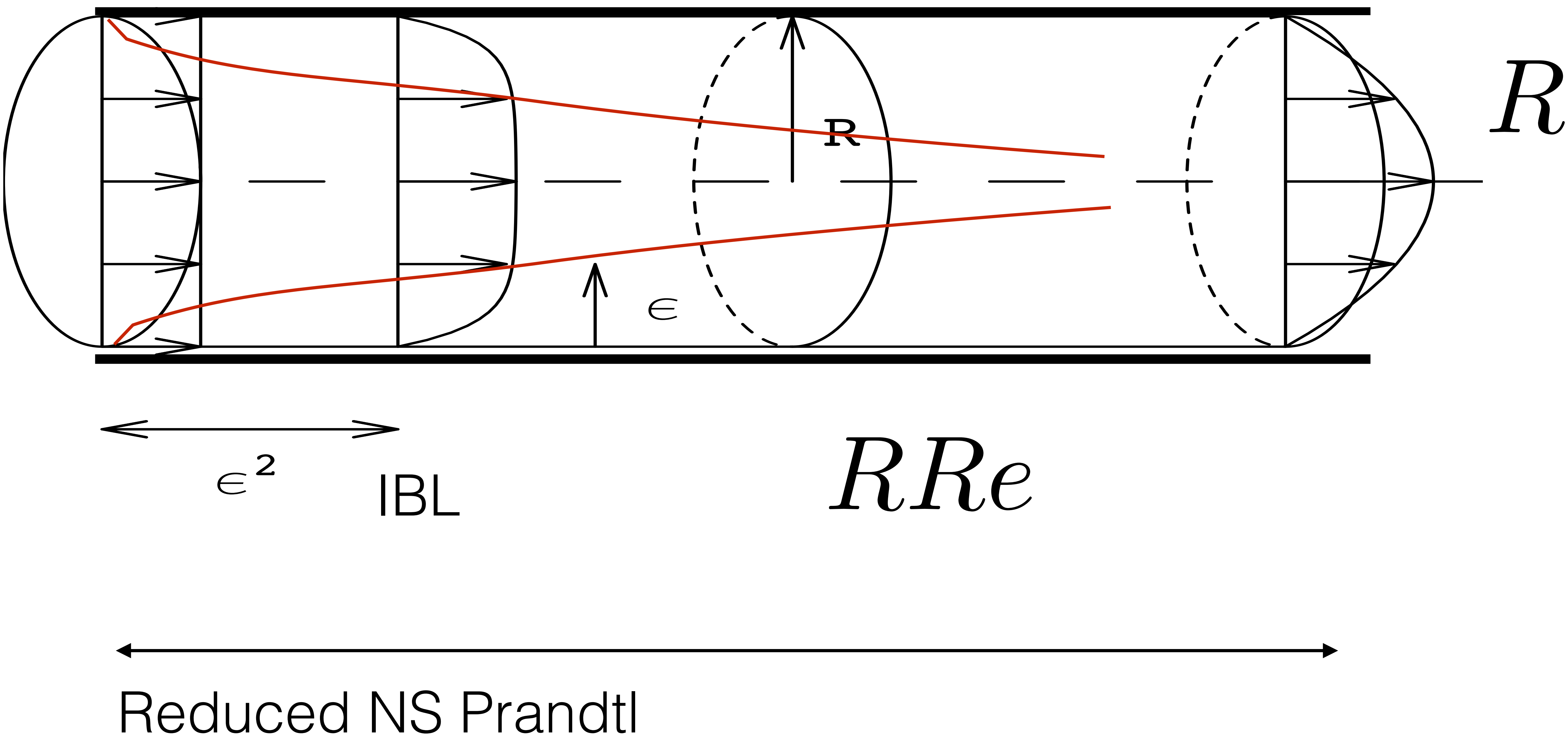
parabolic
sometimes coupled with an external ideal fluid
which makes a global retroaction

summary

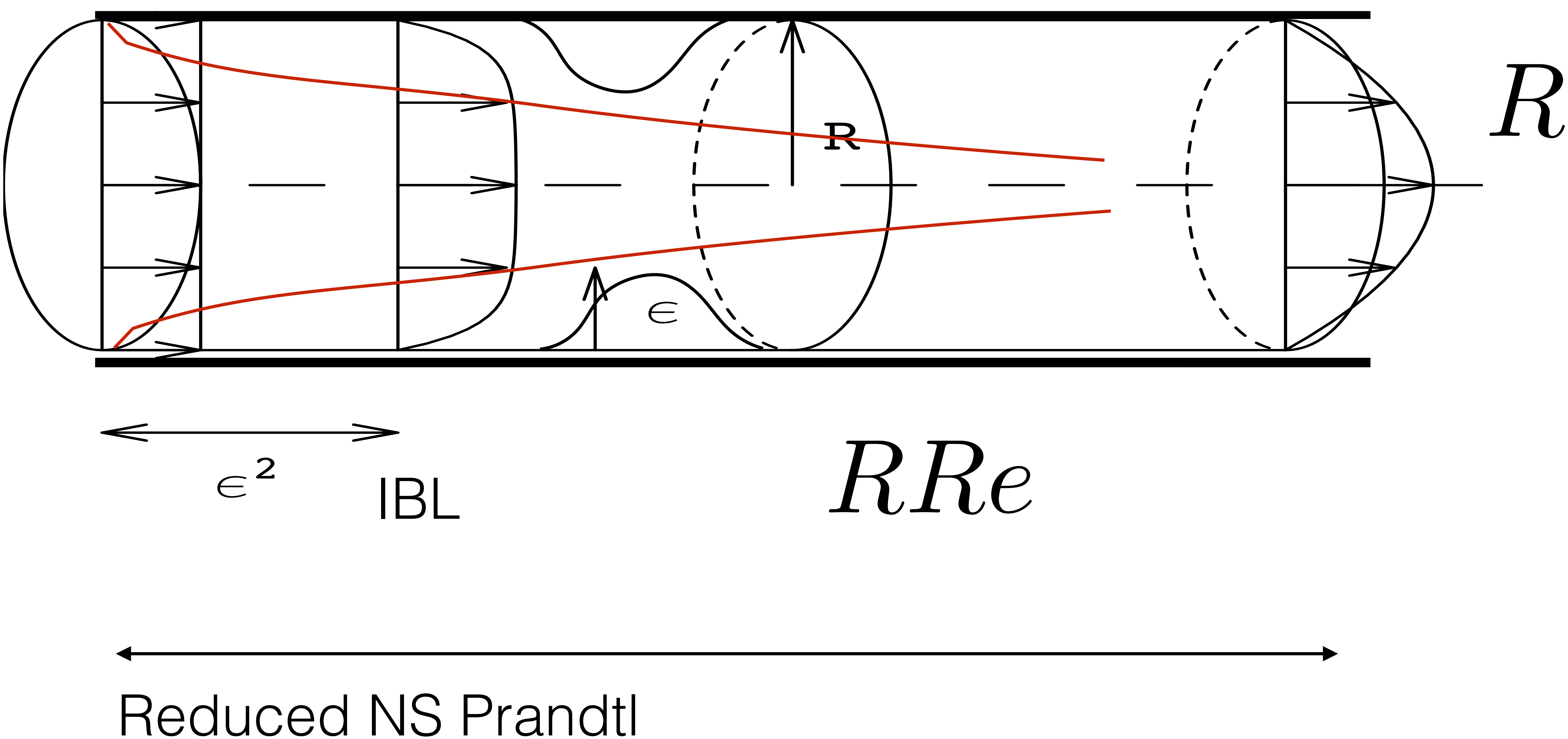


Reduced NS Prandtl

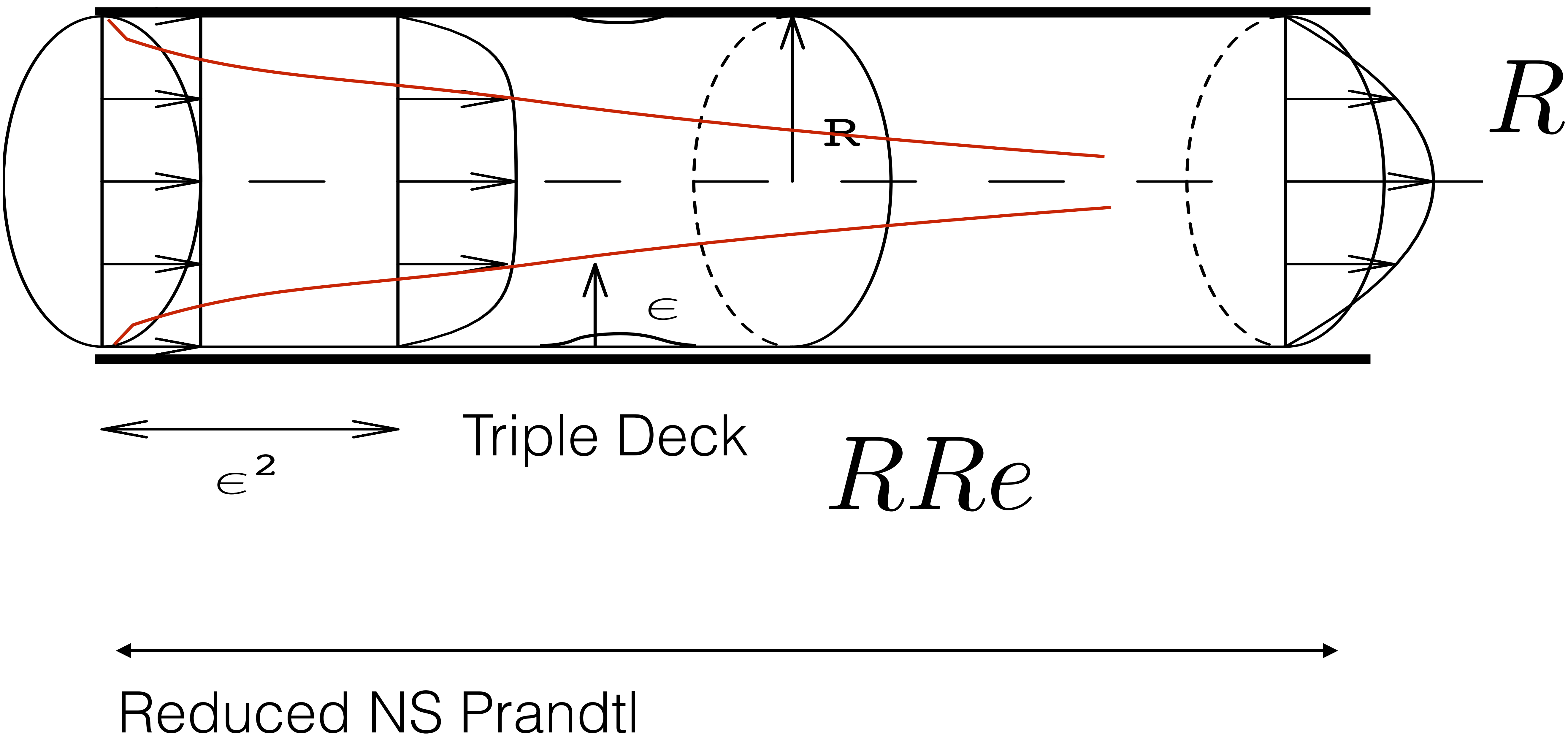
summary



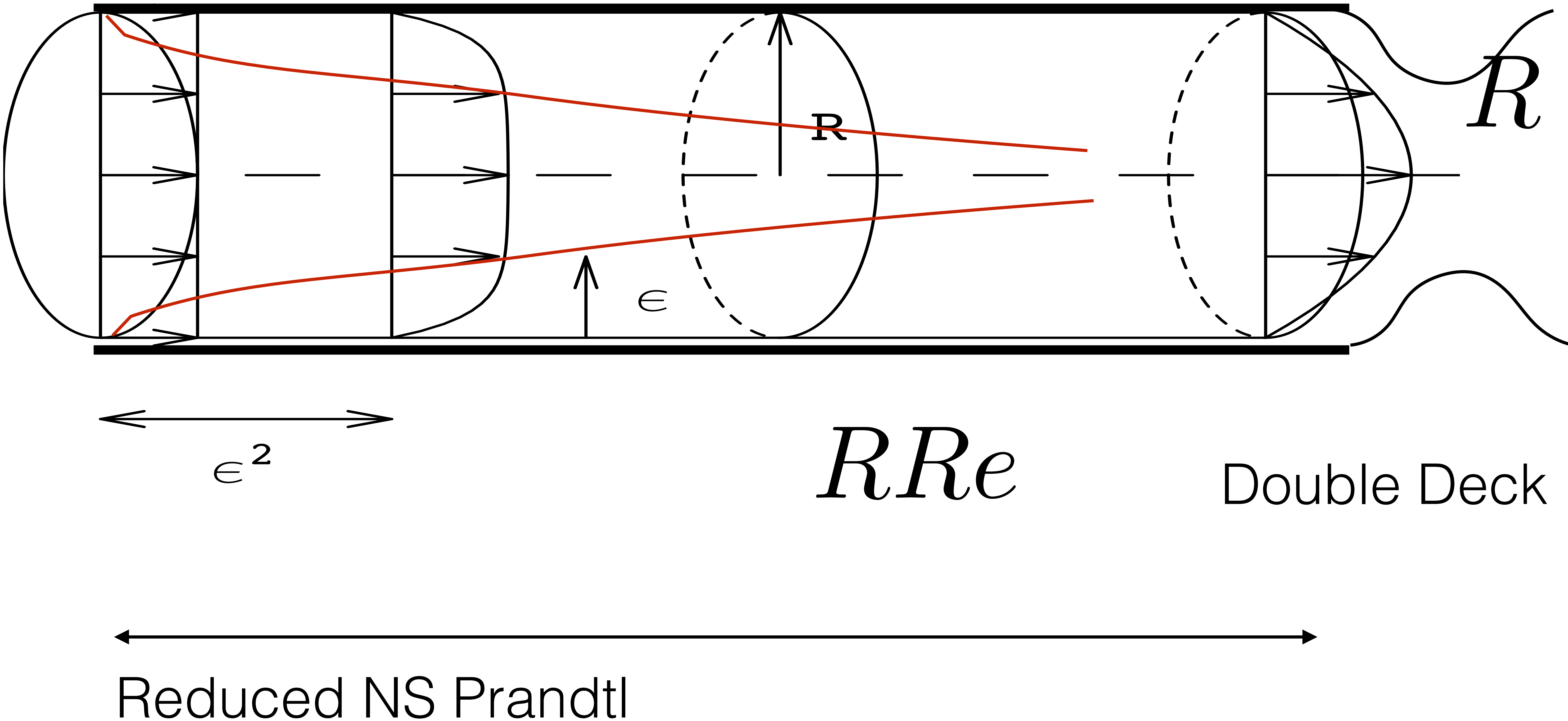
summary



summary



summary



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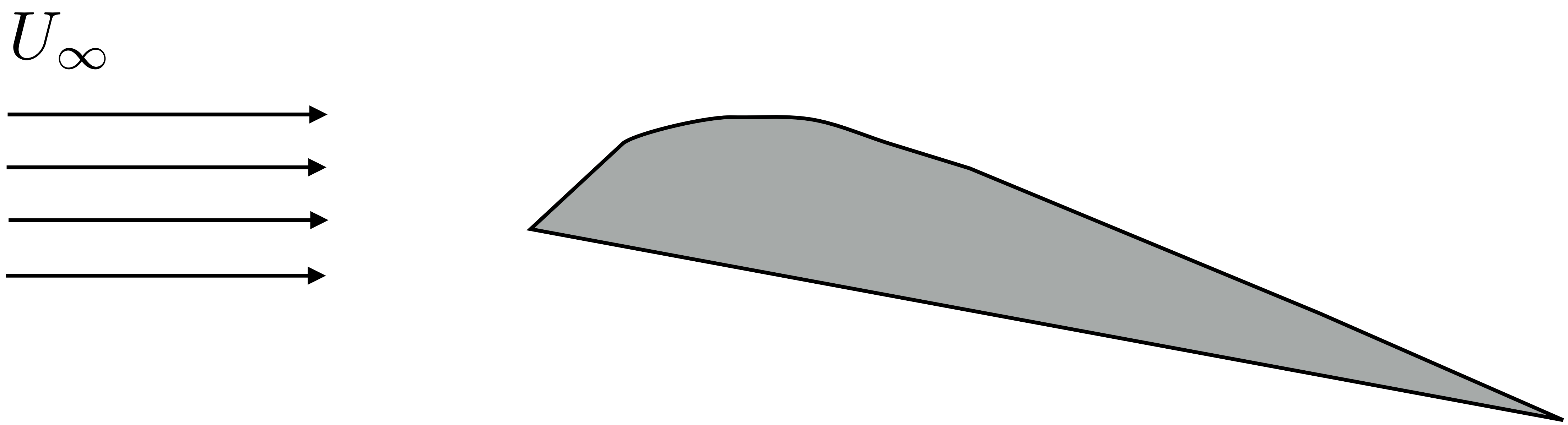

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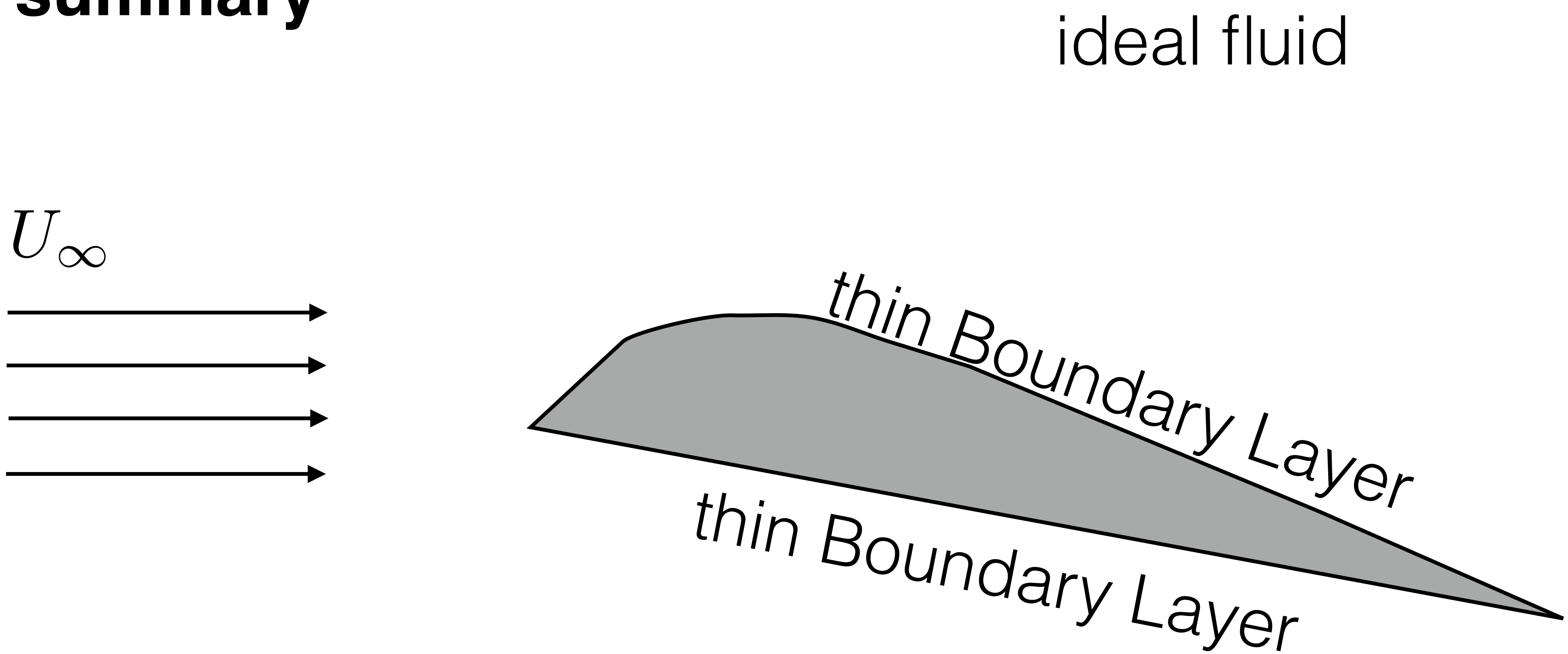
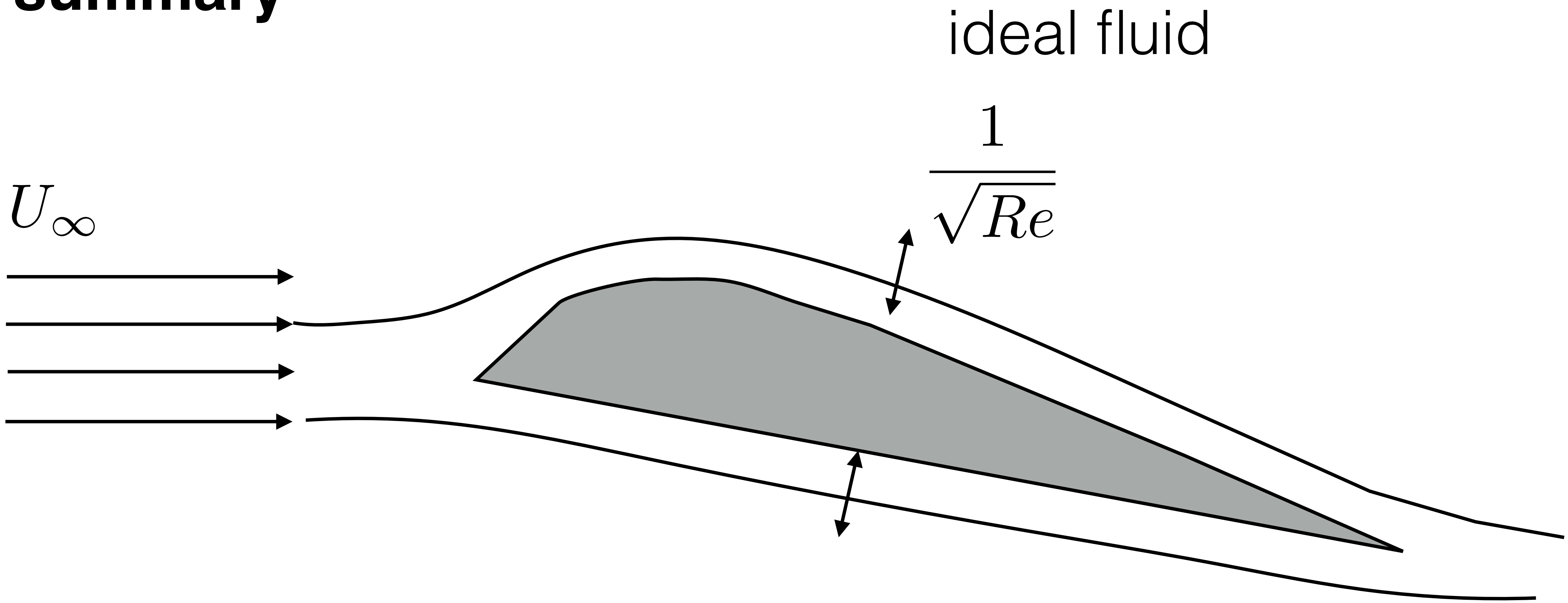


ideal fluid
U_∞
thin Boundary Layer
thin Boundary Layer

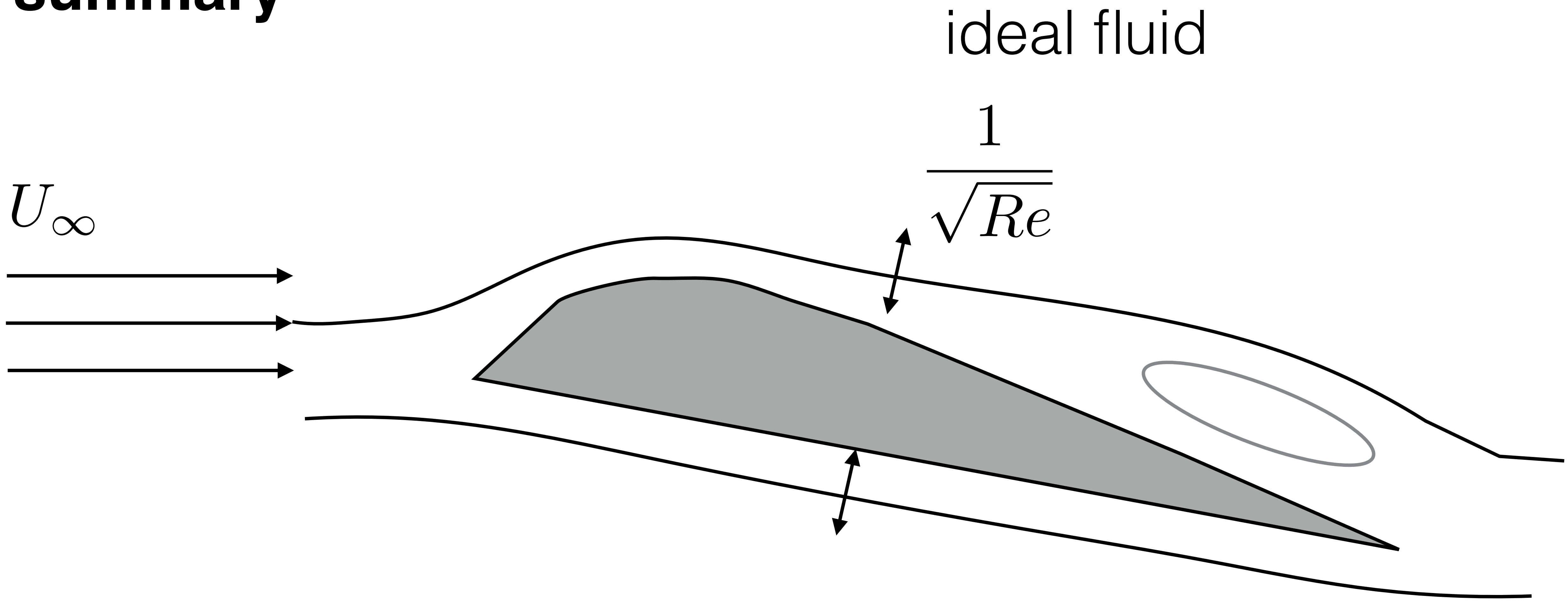
summary



small pressure gradient every thing OK

Ideal Fluid drives Boundary Layer

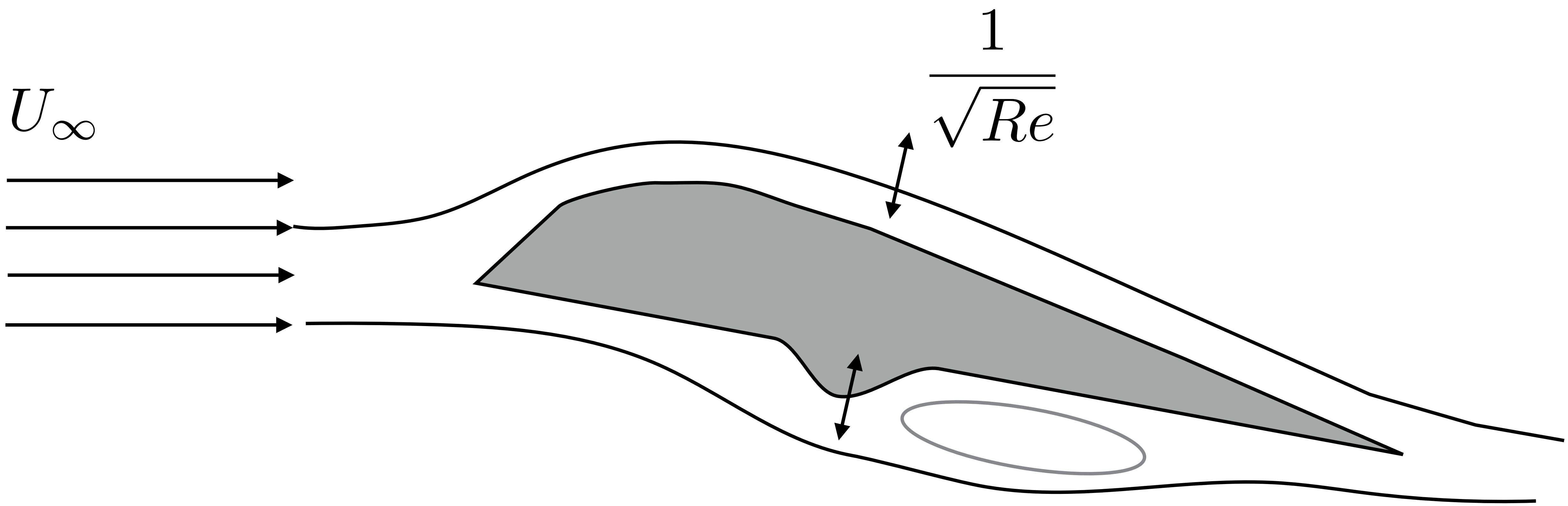
summary



larger pressure gradient IBL

Ideal Fluid interacts with Boundary Layer

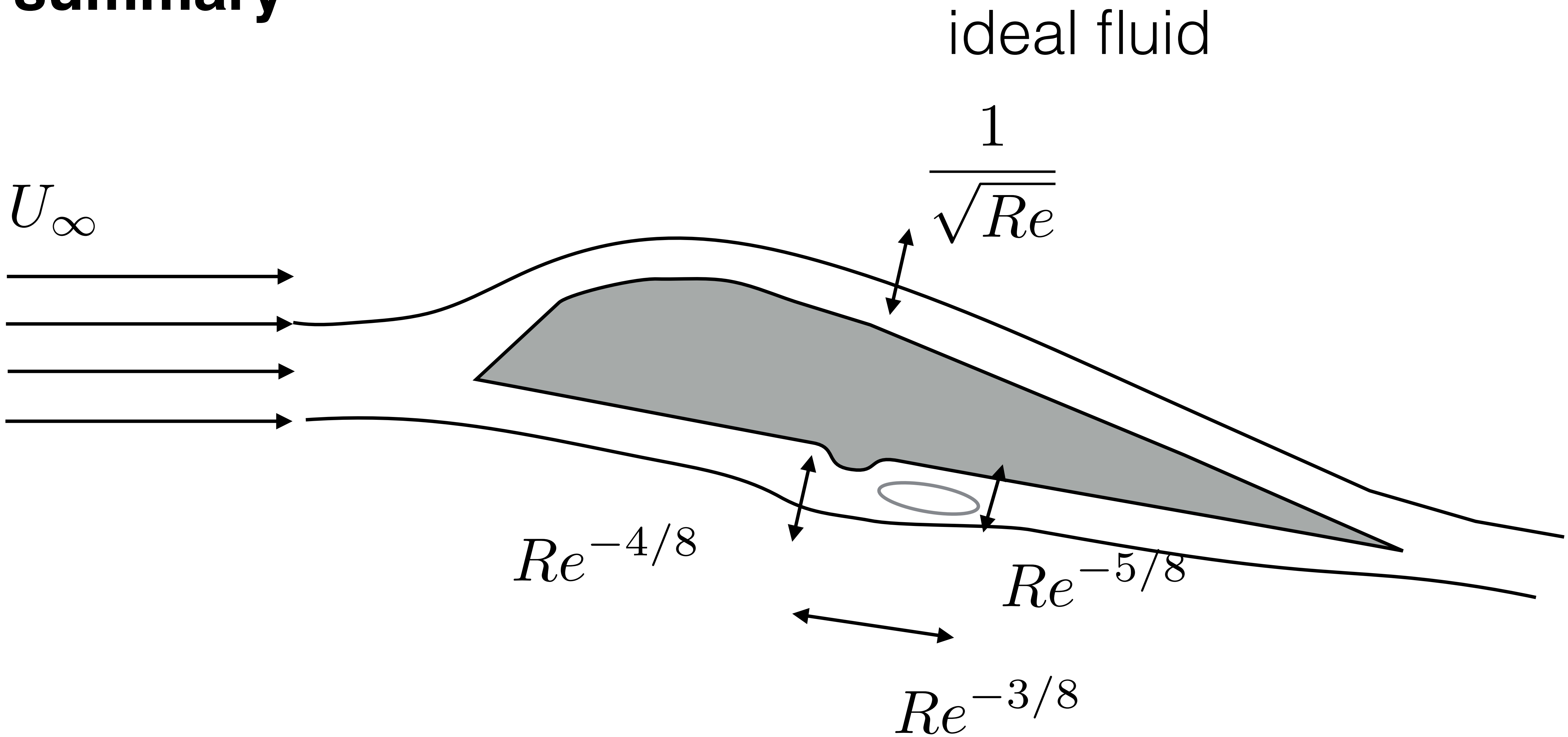
summary



larger pressure gradient IBL

Ideal Fluid interacts with Boundary Layer

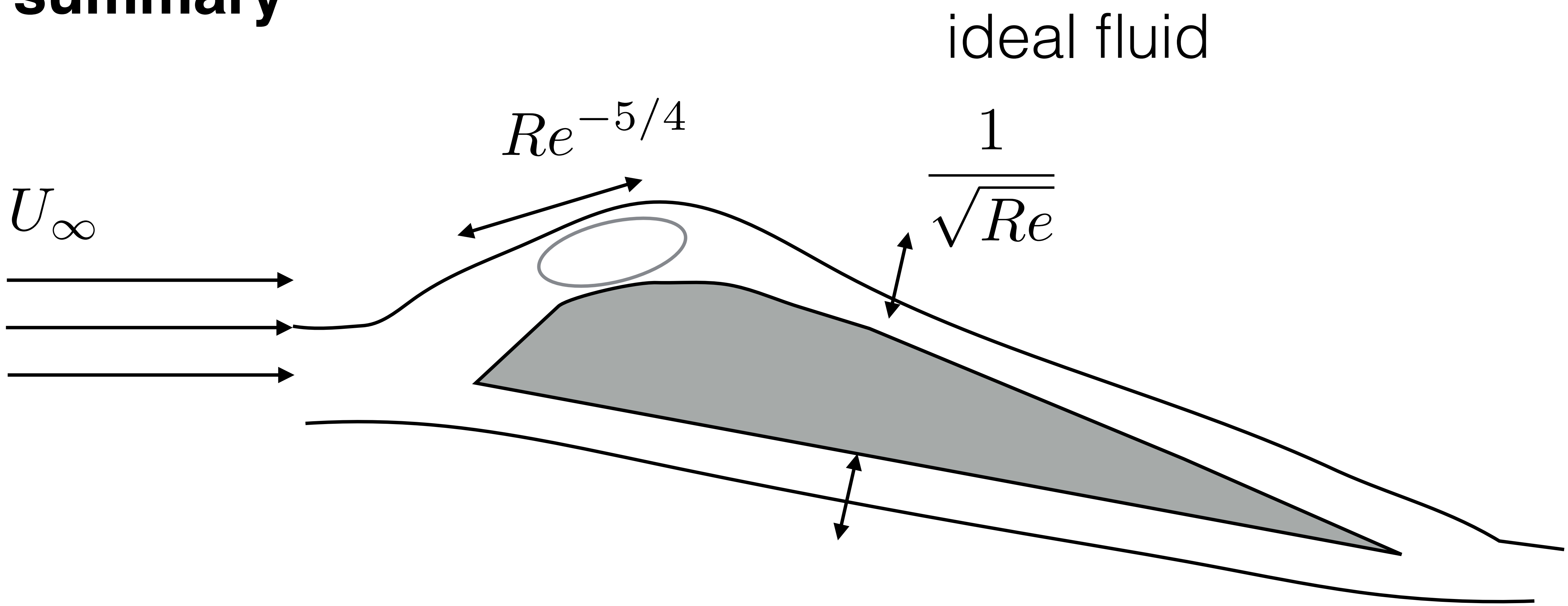
summary



small size structure Triple Deck

Ideal Fluid interacts with Boundary Layer

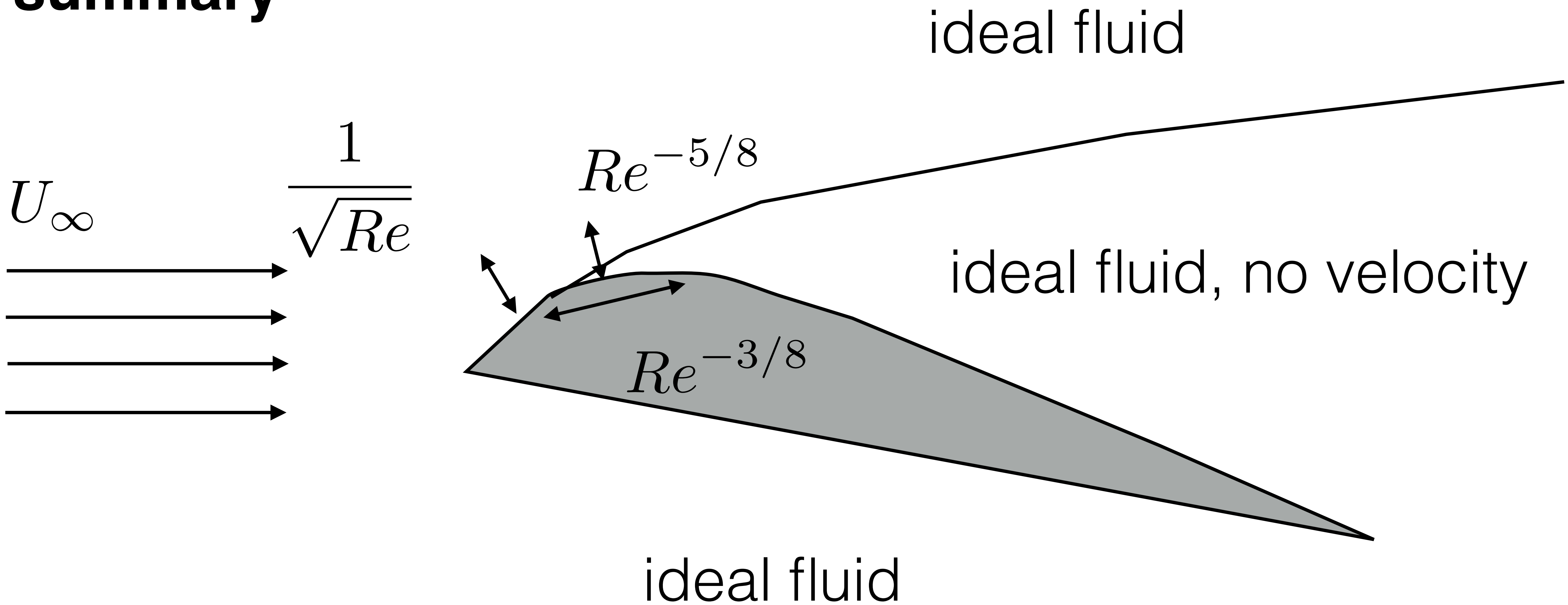
summary



weak “short bubble”

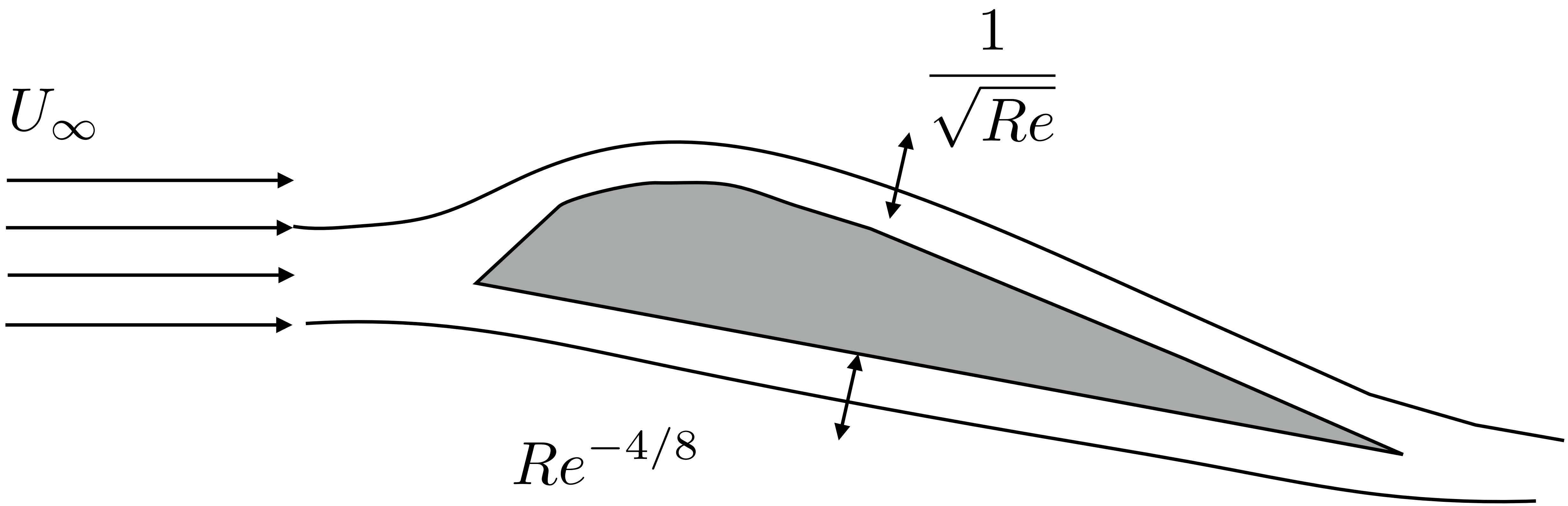
Marginal Separation Theory Ruban 82

summary



Triple Deck and large size separation
coupling with Kirchhof-Helmholtz wake

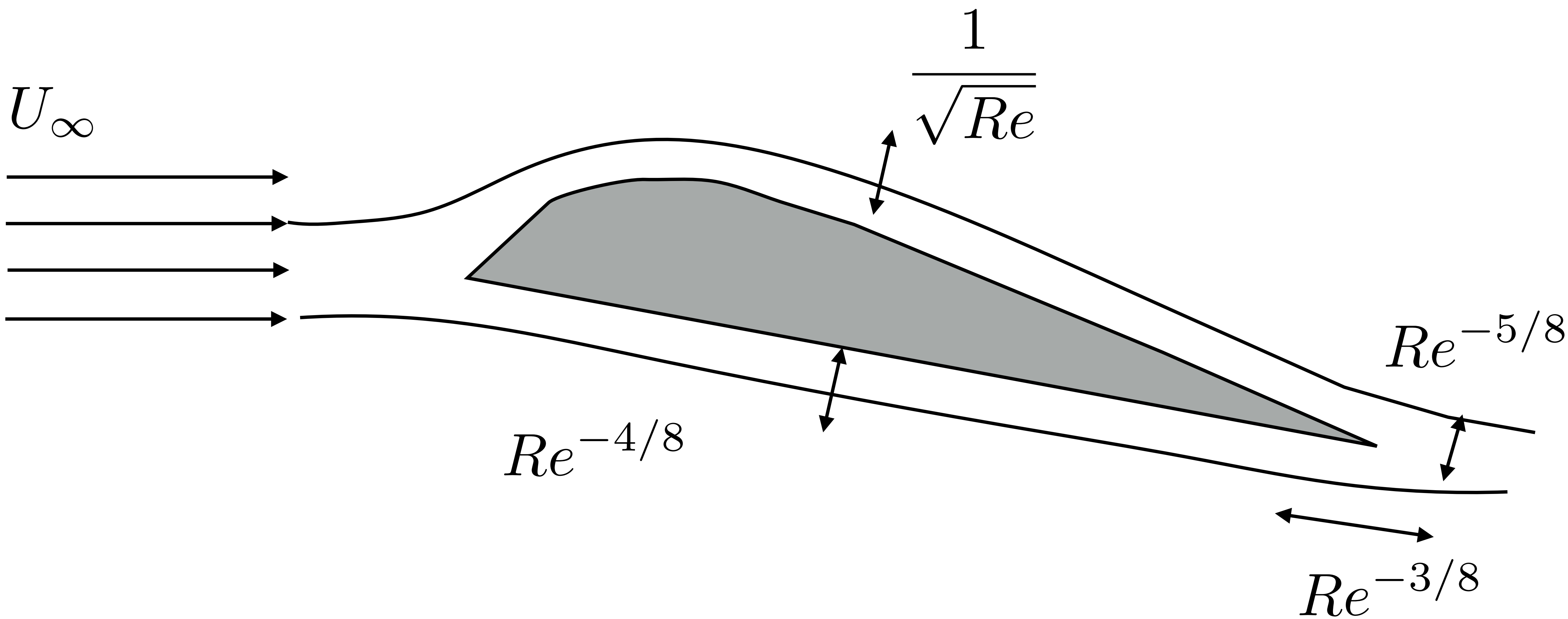
summary



small pressure gradient every thing OK

Ideal Fluid drives Boundary Layer

summary



small pressure gradient every thing OK

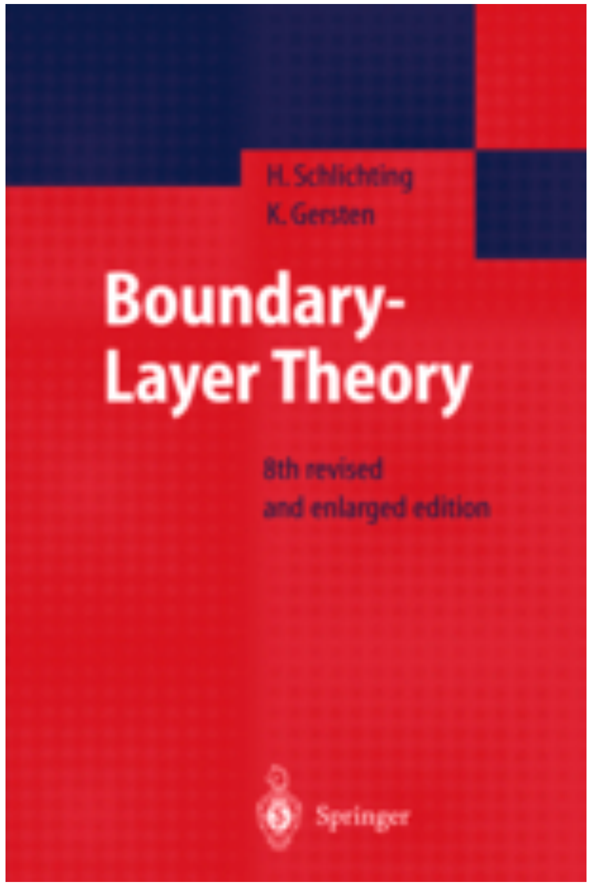
d'Alembert is solved by Kutta-Joukowski
But Kutta-Joukowski is maybe solved by Triple Deck

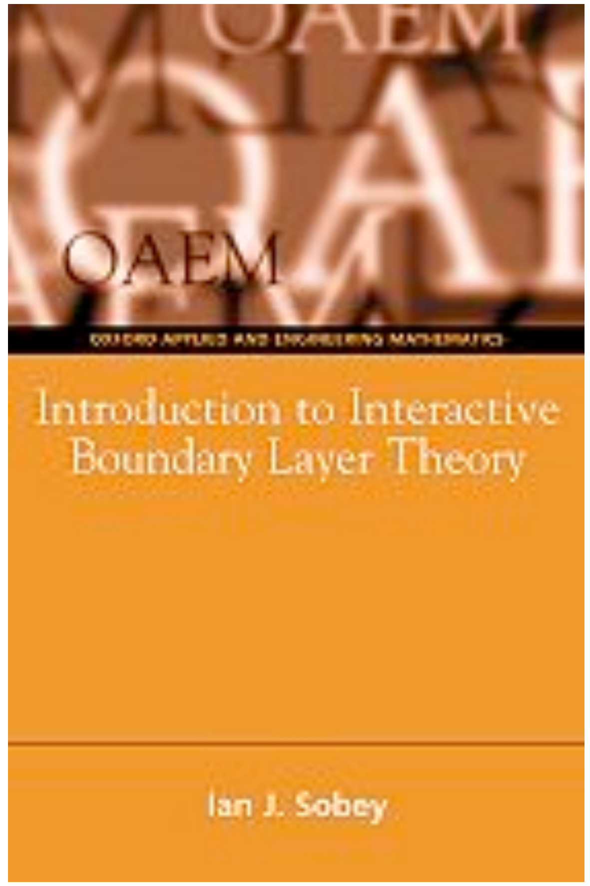
Conclusion

- Interactive Boundary Layer allows separation
- Triple Deck is in IBL, this is the rational framework for boundary layer separation
- longitudinal transverse equilibrium
- Prandtl balance is very strong
- strong viscous-inviscid interaction
- allows to “understand” the key feature of the flow

Open problems

- unsteady : finite time singularity (Van Dommeln)
- Vortex Breakdown
- better numerics for large separated bulbs
- adapt this for better modelization in Shallow Water
- need for numerical help


H. Schlichting
K. Gersten
Boundary-Layer Theory
8th revised and enlarged edition
Springer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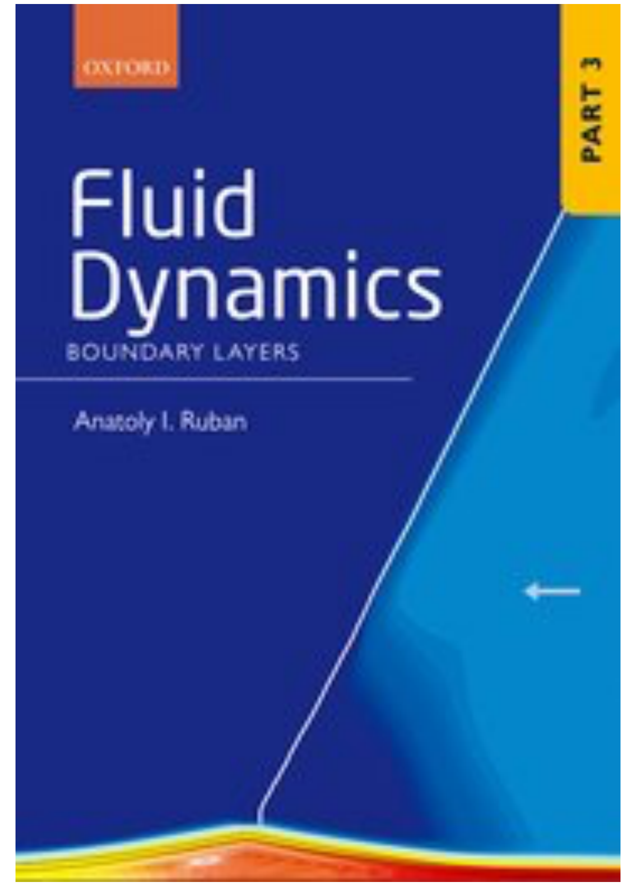
Introduction to Interactive Boundary Layer Theory
Ian J. Sobey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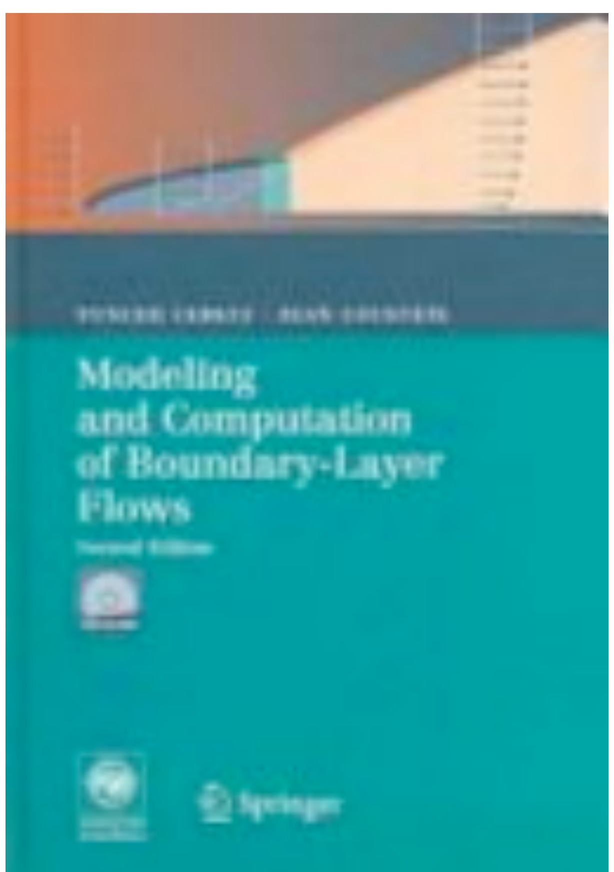
Modeling and Computation of Boundary-Layer Flows
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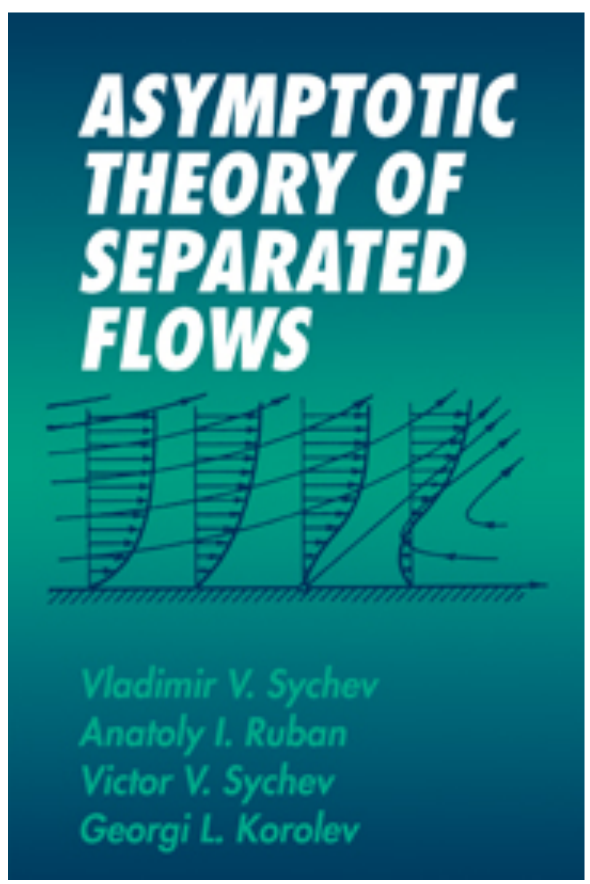
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Vladimir V. Sychev
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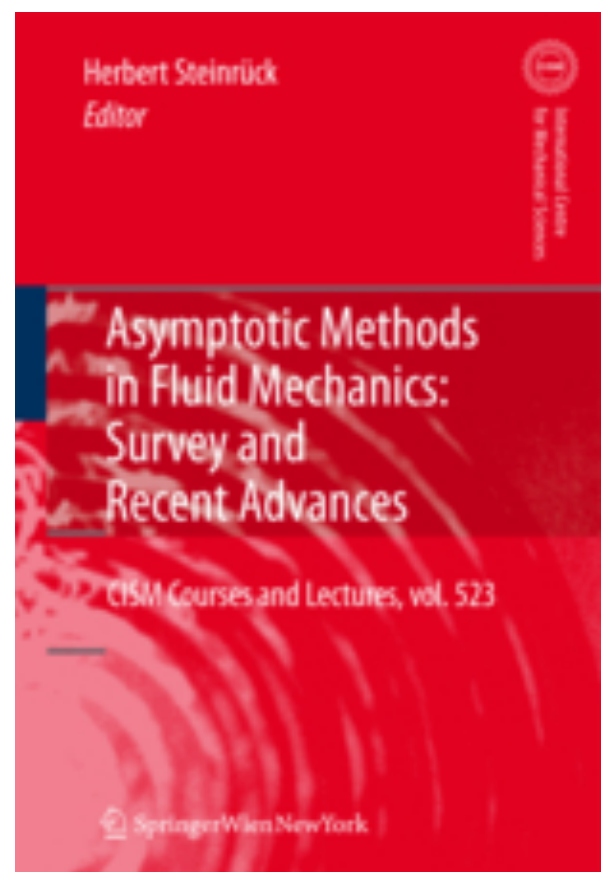
Georgi L. Korolev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Asymptotic Methods in Fluid Mechanics: Survey and Recent Advances
CISM Courses and Lectures, vol. 523
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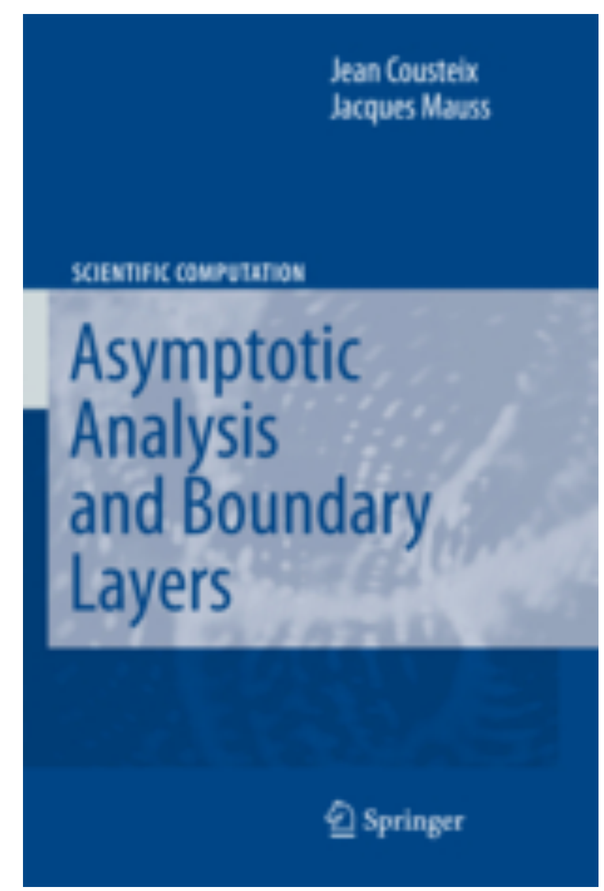
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PYL with Frank T. Smith (2012)