# INFLUENCE OF THE ENTROPY LAYER ON VISCOUS TRIPLE DECK HYPERSONIC SCALES.

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ABSTRACT. Inviscid- viscous interaction on a flat blunted plate in weak hypersonic régime (*i.e.* when wall perfect fluid pressure non dimensionalized by free stream pressure, say $\omega$, is much greater than the classical hypersonic viscous interaction parameter knowed as $\chi_\infty$) is studied on the triple deck scales (Stewartson (1974), Neiland (1970)). We seek to delineate the influence of an asymptotically small nose bluntness (which create a thin layer of perfect fluid called entropy layer: Guiraud, Vallée & Zolver (1965)) on the flow structure near a laminar separation. We outline some limiting cases depending on the relative sizes of the upper deck and the entropy layer. As a result, in the framework of triple deck, we first establish numerically that increasing bluntness increases the separated bulb, and second, that further increase lowers the scales of the interactive region.

## 1. THE ENTROPY LAYER IS SMALLER THAN THE UPPER DECK.

### 1.1 The fourth deck

The entropy layer is characterized by its small density (say gauged by $r\rho_\infty^*$ where r is small) and by its small thickness (gauged in Von Mises transverse variable by $d^*\rho_\infty^* U_\infty^*$, we note $d=d^*/L^*$ the ratio of tip bluntness versus the longitudinal scale. Notations are standard, see figure (1)). The non dimentionalized disturbances arising from the lower deck are transmitted through the main deck and emerge from it in the two layered perfect fluid:

$$\frac{\partial v}{\partial \xi} = -\frac{\partial p}{\partial \psi}, \quad \frac{\partial v}{\partial \psi} = -\frac{1}{\rho}\frac{\partial p}{\partial \xi}.$$

For those relations, either we are in the entropy layer where

$$\psi = (d/\Psi)\ \hat{\psi}, \quad \rho = r\ \hat{\rho},$$

($\Psi$ being the classical upper deck stream function gauge $(d/\Psi)<<1$), either we are in the upper deck where

$$\psi = \bar{\psi}, \quad \rho = \bar{\rho}.$$

The entropy layer is deamed infinitely small, so it is a fourth deck lying between the main and the upper deck. We may found the transverse perturbed speed solution through the entropy layer as:

$$v(\xi,\hat{\psi}) = v(\xi,\hat{\psi}=0) - \frac{\partial p}{\partial \xi}\int_0^{\hat{\psi}} \frac{1}{\rho} d\ \hat{\psi},$$

with the matching with the main deck:

$$v(\xi,\hat{\psi}=0) = -\frac{dA(x)}{dx}, \quad p(\xi,\hat{\psi}=0) = p(x), x \equiv \xi.$$

and in the upper deck of constant density we recover:

$$p(\xi,\bar\psi) = v(\xi,\bar\psi) = F(\xi - \bar\psi), \Rightarrow p(\xi,\bar\psi=0) = v(\xi,\bar\psi=0).$$

Thus, writing the integral in the sense of finite part shows that the entropy layer transmits perturbations such that:

$$p(\xi,\bar\psi=0) + \eta\frac{\partial p(\xi,\bar\psi=0)}{\partial\xi} = v(\xi,\bar\psi=0).$$

The infinitely small parameter η (directly proportional to the nose blunting by the thickness of the entropy layer, and inversly proportional to the upper deck's scale) gauges the departure from classical theory:

$$\eta = (d/\Psi)Fp\left\{\int_0^\infty \frac{1}{r\ \hat\rho(\hat\psi)} d\hat\psi\right\},\ \text{and}\, O(\eta) = O(d\, r^{-1}\chi_\infty^{3/4}\,\omega^{-1/2}).$$

1.2 FUNDAMENTAL EQUATION

As a result, we obtain a new fundamental equation of the triple deck written in standard scales. The reduced lower deck equations are identical to those of classical theory:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,\ u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial^2 u}{\partial x^2}.$$

As boundary conditions, as usual, we have no slip velocity and:

$$u(x,y\to\infty) \to y + A(x).$$

However, the pressure deflection relation is different:

$$p(x) + \eta\frac{dp(x)}{dx} = -\frac{dA(x)}{dx}.$$

This may be compared with an other entropy effect in Sokolov (1983), and to the wall temperature effect in Brown, Cheng & Lee (1990).

1.3 NUMERICAL RESOLUTION

This problem has to be solved numerically. To achieve it, we choose an iterative method based on standard inverse Keller Box method for Prandt'l equations plus revisited Le Balleur (1978) semi- inverse relaxation method. This permits strong coupling by means of the relaxation:

$$(-A^{n+1}) - (-A^n) = \lambda\left(\frac{dp}{dx}^n - \frac{d\tilde p}{dx}^n\right) + K(p^n - \tilde p^n).$$

Where, at stage n, p is the pression deduced directly from the pressure deflection relation, and p tilda is the pression deduced from the Prandt'l equations, both with the same deflection -A given.
The coefficients λ and K are chosen in order to stabilise the iterations: the complex gain modulus is imposed to be smaller than one for all spatial frequencies smaller than $k_{max}=\pi/\Delta x$ ($\Delta x$ is the longitudinal discretization step). The gain may be explicited in the viscinity of the null solution:

$$G = 1 + (\lambda i k + K)\left(ik(1+\eta i k)^{-1} - \left(\frac{(ik)^{1/3}}{-3Ai'(0)}\right)^{-1}\right).$$

Results are close to those of Cheng et al. (1990). Figure (2) depicts pressure and wall shear obtained in the classical ramp- induced interaction test case of reduced angle α=2.5 for η=0.25 and η=1 (values of order one are presented to magnify the entropy effect, η is strictly infinitely small). The linear solution predicts that the Lighthill eigenvalue *k* increases with η: so the curve toe stiffens with η. The separation bubble size appears to icrease with η.

2. THICKER ENTROPY LAYER, TWO PARTICULAR CASES:

2.1 Entropy layer and upper deck are the same

When the upper deck *is* the entropy layer the complete equationsof perturbation have to be solved in the upper deck, the density is given by the density profile of the entropy layer (Guiraud et al. (1965)). This resolution has not yet been performed. The longitudinal gauge is imposed by the size of the upper deck.

2.2 Entropy layer is thicker than upper deck

When the upper deck is smaller than the entropy layer we find again the classical case with but differnt scales because the propagation takes place in a layer of very small constant density (r), for example, the longitudinal gauge is:

$$x_3 = \lambda^{-5/4}(\gamma - 1)^{3/2} s_w^{3/2} (\chi_\infty/\omega)^{3/4} r^{3/8}.$$

3 CONCLUSION

To summarize, a rough sketch of small nose bluntness influence may be drawn. For $\eta$<<1 the study of section 1 may apply: raising $\eta$ increases separation. For $\eta$ of order one, present study fails and complete calculation of inviscid perturbation trough a thick entropy layer has to be performed.For bigger blunting (but always d very small) section 2 suggests new scales. So increasing $\eta$ first promotes growth of separated region, reduces *k* and diminish appearent interacting region, further increase lowers the scale of separated region. This is qualitatively comparable with the experimental data of Holden (1971). Incipient angle separation is correlated with:

$$M_\infty \alpha (M_\infty^3 d)/\chi_\infty^2 \propto (\chi_\infty/((M_\infty^3 d)^{3/2}))^a (d)^b.$$

Holden (1971), whith combinaison of parameters and experiment, found a=-7/5 and b=0, we propose, deduced from triple deck scales, a=-3/2 and b=-1/6$\gamma$, those coefficients reflect the locality of the interaction.

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