Asymmetrical effects in a 2D flow, application to Pharyngeal fluid flow in obstructive sleep apnea.

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Introduction

Obstructing sleep apnea (OSA) is defined as the intermittent cessation of breathing during sleep. It has adverse consequences on patients (3 % of the adults) daily life (from cardiovascular diseases to increased risk of public traffic accidents…). The present work aims to contribute toward the complex interaction determining the OSA syndrome. From physical point of view it is a fluid structure interaction: airway collapse is due to the collapse of the pharyngeal airway during inspiration. Computing the flow in locally constricted pipe is important in numerous other applications in biomechanics. The flow is simplified as a laminar flow in a 2D non symmetrical pipe (Fig. 1). Of course this can be achieved accurately through Navier-Stokes (NS) solvers. Nevertheless, it takes a long computational time. So, we propose here a simplified model based on Interactive Boundary Layer theory (Kallse et al., Sychev et al, Lagrée et al.) and compare with the code Castem.

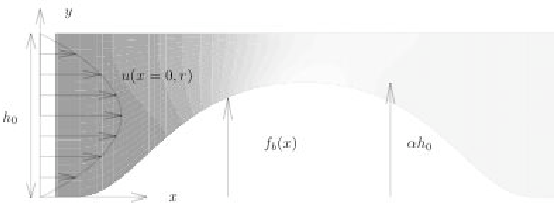

Fig. 1. The 2D non symmetrical flow configuration.

Method and Results

The flow is solved using strong Interacting Boundary layer theory. Equations are deduced from Navier Stokes supposing that the Reynolds number ($Re=U_0h_0/\nu$) is large. Viscous effects are restricted in two thin layers near the upper and lower walls. The final Karman equation for each displacement thickness in each boundary layer is:

$$\frac{d}{dx}\left(\frac{\delta_1}{H}\right)+\frac{\delta_1}{u_e}\left(1+\frac{2}{H}\right)\frac{du_e}{dx}=\frac{f_2 H}{\delta_1 u_e},$$

with either $\delta_1=\delta_1^{upper}$ and $u_e=u_e^{upper}$ either $\delta_1=\delta_1^{lower}$ and $u_e=u_e^{lower}$. The values of f_2 and H are given by the closure relation (Lagrée et al. 2000).

A second order ideal fluid theory leads to complicated expressions for the pressure on the lower and upper walls. We just write here the pressure jump across the channel due to the non symmetrical geometry and to the boundary layer displacements:

$$\Delta p_0=\mathrm{Re}^{-2}\left(\frac{(\delta_1^{upper})^2-(f_b'+\delta'^{upper}_1)^2}{1-\delta_1^{upper}-f_b'+\delta_1^{upper}}+\frac{\delta''^{upper}_1-f_b''-\delta''^{upper}_1}{2}\right)$$

The boundary layer and the ideal fluid are coupled with an iterative process. Skin friction, pressure on the walls are computed. It is compared to a NS solution with Castem (Fig 2.).

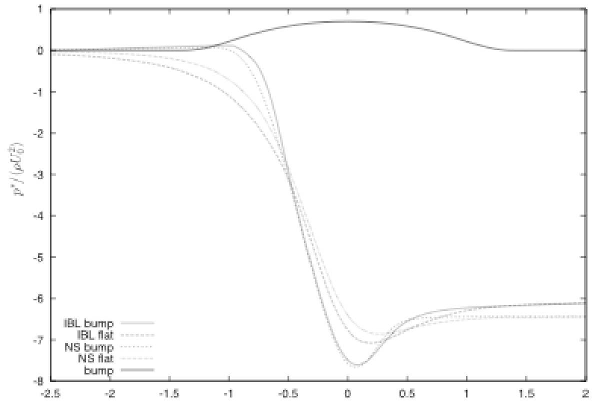

Fig. 2. Comparison of integral (IBL) and Navier Stokes pressures. The IBL well predicts the effect of asymmetry in the flow.

Conclusion

The IBL solution and NS solution show nearly the same asymmetry in the location of the minima of pressure. The upstream compression is well predicted. The values of skin friction are similar, separation of the boundary layer is solved before and after the constriction.
Comparison with an experimental set up is under work.
Having confidence in the IBL approach, we have now to introduce the fluid structure interaction.

References

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