Modelling the flow rate dip for a silo with two openings

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Abstract. In recent experiments it was shown that a planar silo with two discharge orifices which were separated by a distance, L, displayed counter-intuitive flow rate phenomena as granular material was discharged from it. In contrast to previous studies, the flow rate did not monotonically decrease as the separation between orifices increased. Instead, a rapid decrease in flow rate was observed as the two orifices were separated until, at a critical orifice separation, a minimum flow rate was reached. Upon further separating the two orifices the flow rate steadily increased to the infinite separation limit of two openings. In this work we numerically investigate this so-called 'flow-rate dip' phenomenon. The kinematic and $\mu(I)$ models are used to examine the two opening silo, with the kinematic model failing to capture any flow rate dynamics and the $\mu(I)$ model capturing the dynamics if appropriately large (yet still physically reasonable) friction parameters are used.

Keywords: $\mu(I)$ rheology, granular silo, flow rate

1 Introduction

Granular discharge from a silo under the influence of gravity is a complex system to model. In particular, a recent experimental study [1] has shown that for a planar silo with two openings (see Figure 1), the separation distance, L, of these openings cause some unusual flow rate dynamics. For a large separation, the flow rate appears to approach the expected flow rate for two distinct silos with a single opening. When there is zero separation (i.e. the silo has one larger opening), the flow rate is much higher, as predicted by the Beverloo relation. In between these extremes, the flow rate does not monotonically decrease, it instead dips sharply to a local minimum for a small separation, then slowly increases back to the large separation value. This behaviour had not previously been observed in similar experiments [14, 13] and the new phenomenon was attributed to the larger values of inter-particle friction of the particles used in the study [1].

While it has been shown that discrete element models (DEM) with appropriately large values of inter-particle friction are able to reproduce the observed

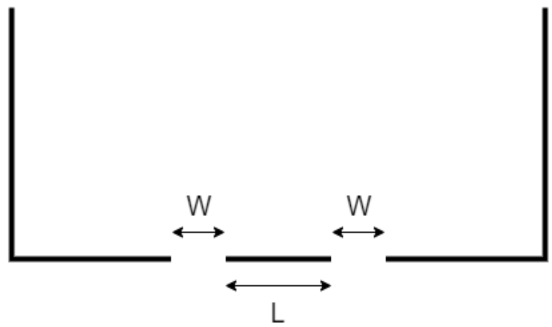

Fig. 1. Diagram of the two opening silo.

flow-rate dip [1], as yet, no continuum model of dense granular flow has successfully predicted such a dynamic. As such, the two opening silo is an excellent test of continuum mathematical models of granular flow and may potentially offer insight into the dynamics that are difficult to observe using DEM or experimental methods.

In this work we study two continuum models - the kinematic model [9] and $\mu(I)$ model [8, 4], and use these to examine the flow rate from a dual orifice silo.

2 Numerical Models

2.1 Kinematic model

The kinematic model [9] can be formulated with the assumption that a gradient in vertical velocity causes horizontal motion i.e. $u = -B\frac{\partial v}{\partial x}$ where u and v are the horizontal and vertical velocity components respectively, and B is some constant parameter. This assumption combined with that of incompressibility gives a relatively simple model of granular flow, taking the form of the heat equation [7],

$$\frac{\partial v}{\partial y} = B\frac{\partial^2 v}{\partial x^2}. \quad (1)$$

This partial differential equation can be solved exactly across the infinite half-plane with a Dirac delta $v = \delta(x)$ boundary condition at $y = 0$, corresponding to an infinitesimal orifice. The solution of this boundary value problem is

$$v = -\frac{Q}{\sqrt{4\pi By}}\exp\left(-\frac{r^2}{4By}\right), \quad (2)$$

where Q is the flow rate and $r = \sqrt{x^2 + y^2}$. For a silo with two openings separated by some distance L, the boundary condition at $y = 0$ is $v = \delta(x - L/2) + \delta(x + L/2)$, resulting in

$$v = \frac{Q}{\sqrt{4\pi By}}\exp\left(\frac{(x - L/2)^2}{4By} + \frac{(x + L/2)^2}{4By}\right). \quad (3)$$

Note that this model requires scaling by the flow rate, by using the Beverloo relation for example. For the two opening case, this means that the flow rate for different separations is constant, since the flow rate is effectively determined for each orifice individually. This common model of granular flow from a silo is unable to capture the flow-rate dip phenomenon.

2.2 $\mu(I)$ model

The $\mu(I)$ model defines an effective viscosity η_{eff} which can be used in the incompressible Navier-Stokes equations to model the flow of granular material. In a standard incompressible fluid, shear stress τ can be related to shear rate $\dot{\gamma}$ by the relation $\tau = \eta\dot{\gamma}$, where the viscosity of the fluid η is assumed to be constant. For granular material, the viscosity is determined by the bulk friction μ, which changes with the dimensionless 'inertial number', I, which is given by

$$I = \frac{\dot{\gamma} d}{\sqrt{P/\rho}}, \tag{4}$$

where $\dot{\gamma}$ is the shear rate, d is the particle diameter, P is the confining pressure, and ρ is the grain density.

The friction, μ, is the ratio between the shear stress and the pressure, i.e. $\mu = \frac{\tau}{P}$, where τ is the shear stress. It has been found that μ increases with I for granular material in the dense regime [8], with a common form of the relation between μ and I given as

$$\mu(I) = \mu_s + \frac{\Delta\mu I}{I_0 + I}, \tag{5}$$

where μ_s, I_0, $\Delta\mu$ are all constant parameters. From this relation, the effective viscosity can be defined as

$$\eta_{\text{eff}} = \frac{\mu(I)P}{|\dot{\gamma}|}, \tag{6}$$

with a maximum value, η_{max}, imposed to prevent numerical issues in stationary zones [6]. The effective viscosity together with the incompressible Navier-Stokes equations

$$\frac{\partial \boldsymbol{u}}{\partial t} = -(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} + \nabla \cdot (\eta \nabla \boldsymbol{u}) - \frac{\nabla P}{\rho} + \rho \boldsymbol{g}, \tag{7}$$

defines the velocity given initial and boundary conditions. At the side walls and base, no-slip boundary conditions are used, with a free surface at the top and at the orifices. Note that this model is a planar analog, and does not take into account friction at the front and back walls.

This model has previously been used to model a single opening planar silo [10, 11] and the collapse of a granular column [6]. Recently, a selection of conical silos were modelled using the $\mu(I)$ model and compared to experimental data gathered from Magnetic Resonance Imaging experiments [3]. It was shown that the $\mu(I)$ model was able to qualitatively predict the shape of velocity contours in the silo, but was not able to accurately quantify the flow rate, likely at least in part due to the incompressible assumption.

3 Results

The $\mu(I)$ model was implemented in Basilisk [12] (a computational fluid dynamics software) for a two opening silo for various orifice separations, L. The case where the separation is zero ($L = 0$) is equivalent to a silo with a single orifice twice the orifice width. For each simulation the predicted flow rate is normalised by the flow rate for the zero separation case. To gauge the effect of model constants the simulation is also repeated for various different parameters for the $\mu(I)$ rheology, shown in Table 1. The results are shown in Figure 2. Basilisk is designed to solve fluid dynamics problems in non-dimensional variables hence we scale our system such that $W_s/\hat{l} = 10$, where W_s is the silo width and $\hat{l}$ is some reference length. The non-dimensional particle diameter used was $\hat{d} = 0.033$, the scaled orifice width was $\hat{W} = 0.625$ and the initial height of the bed of grains was $\hat{H} = 4.5$. A scaled maximum viscosity of $\hat{\eta}_{max} = 100$ was used to regularise the viscosity at low shear rates.

Low friction	$0.32 + 0.28I/(0.4 + I)$
Medium friction	$0.47 + 0.38I/(0.5 + I)$
High friction	$0.62 + 0.48I/(0.6 + I)$
Extra high friction	$0.77 + 0.58I/(0.7 + I)$

Table 1. Different parameters for $\mu(I)$ implementation in Basilisk

For low friction values, the model predicts a monotonic smooth decrease from zero separation distance to large separations, as observed in some past studies [14, 13]. However, for larger friction values the flow rate decreases more sharply as the distance between orifices increases, and for large enough friction values the flow rate dips below the large separation limit at a critical value of L, before rising back up to a steady rate. The behaviour for high friction coefficients is qualitatively similar to that observed experimentally.

4 Conclusion

In this work we numerically studied whether continuum models of granular flow are able to capture the 'flow rate dip' dynamic in a two opening silo, as observed experimentally. It was shown that kinetic models, such as the kinematic model [9], are unable to produce flow rate interference between the two openings. However, for large enough values of the friction parameters, the continuum model with the $\mu(I)$ granular rheology is able to capture this interference and a type of flow rate dip. For lower values of friction, a monotonic decrease in flow rate was observed as the spacing between openings was increased. This work reports the first time a flow rate dip has been observed in a continuum model of granular flow. Since the $\mu(I)$ model is purely a local-model (as compared with non-local

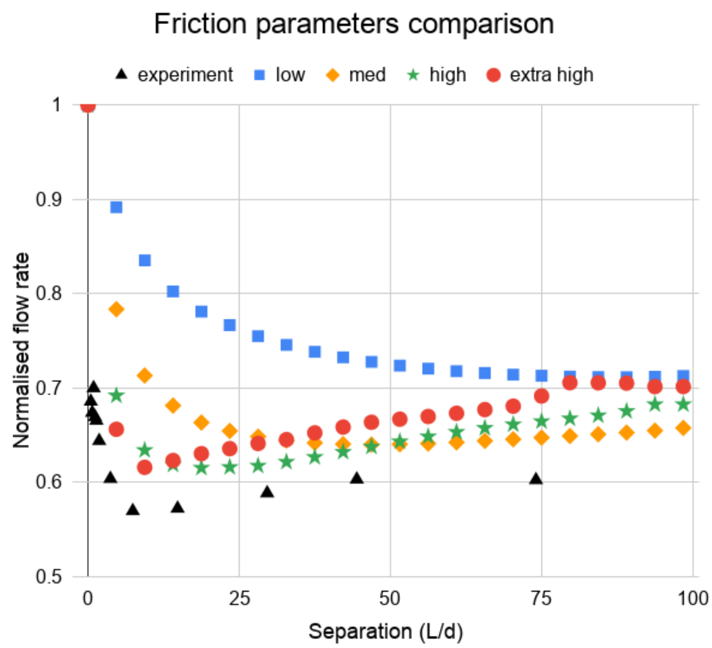

Fig. 2. Normalised flow rate for various orifice separations and $\mu(I)$ parameters.

models of granular friction [5]), this suggests that the flow rate dip is a local effect. The flow rate dip can not be associated with a finite size effect, since, in our continuum model, the particle size enters only into the inertial number definition. Furthermore, since the model is of incompressible type, the dip in flow rate appears to be mainly caused by pressure or shear-rate dynamic interaction. Fully explaining the observed phenomenon is ongoing work.

We note that the values chosen for the $\mu(I)$ parameters greatly affect the dynamics, with high bulk friction being necessary to capture the dip in flow rate. The choice of material in experiments is also important, as experiments with relatively smooth non-frictional granules such as glass beads have lower bulk friction [8] may miss some granular dynamics. It is unclear whether any other phenomena contributes to this flow rate dip, with further work needing to be done to see if accounting for non-local effects [5], dilatancy [2], and wall friction [15] improves the prediction of two opening silo dynamics.

There are still outstanding problems with the $\mu(I)$ model. In particular, the $\mu(I)$ rheology can capture the shape of the velocity field in a silo, and effect of changing certain factors such as the distance between two orifices, but it gives poor predictions for the actual flow rate [3]. Further work may reveal that accounting for additional factors could provide a more powerful model for predicting the dynamics of granular material discharging from a silo.

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