Comparing different numerical methods for solving arterial 1D flows in networks

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1 Introduction

Simulating the flow in an arterial tree is a challenging problem. Many authors (since for example [7]) have succeeded in simulating an arterial tree with a monodimensional (1D) approach or integral approach, among them [5] and recently [1,6]. The two later [1] and [6] compared successfully an experimental realistic visco elastic model of tubes and such 1D models. Here, we emphasize on the numerical methods and we will simulate results of [6] as test case.

2 Methods

The monodimensional integral model of mass and momentum conservation is used for the arterial network:

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=0,\ \rho\left(\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)\right)=-A\frac{\partial}{\partial x}p-f\frac{Q}{A};$$

where A is the cross section area of the vessel (R is the radius), Q the velocity flux, f the friction coefficient. The pressure-section relation is:

$$p=K((R-R_0)+\varepsilon_p(R-R_0)^2)+\eta\frac{\partial R}{\partial t}$$

where K is the stiffness of the artery, a nonlinear square effect is added, and we take into account a Kelvin-Voigt visco-elastic effect. Boundary condition at the entrance is a half sinusoidal pulse. Continuity of pressure and conservation of fluxes at the nodes are preserved. To mimic the resistance of the peripheral vessels, appropriate reflection coefficients of pressure are imposed at the outflows.

We turn now to the numerical part. We test two classes of resolution. First, we use finite element with a Taylor-Galerkin scheme as explained in [3]. Second, a finite volume discretization is performed. To do the latter the system is written in a conservative form, for example the .elastic part is:

$$-A\frac{\partial}{\partial x}K(R-R_0)=\frac{1}{3\sqrt{\pi}}\frac{\partial}{\partial x}(K(A^{3/2}-A_0^{3/2}))$$

3 Results and Discussion

Our aim here is to carefully compare those numerical methods, finite volumes of first and second order, and finite elements.

We first test the schemes on simple configurations (often linearized, with or without damping…) where some analytical solutions are available. For example, on figure 1 we present the propagation (without any viscous effect) of a pulse and its reflection at a bifurcation. This is done for several schemes. The reflection and transmission coefficients of the waves through the bifurcation (for each scheme) agree with the analytical analysis. We observe that there is virtually no numerical diffusion for the FE, but a very small, almost un-noticeable dispersion. The FV methods are more diffusive. Although the accuracy of the second order FV is comparable with FE, it presents a steeper slope and a squared off effect due to the slope limiter..

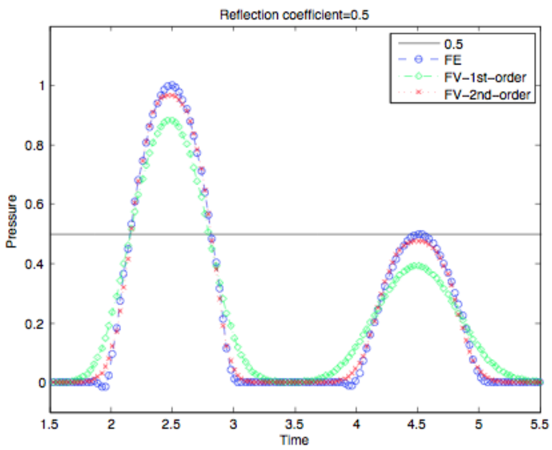

Figure 1 Example of simulation of a signal in a straight tube with a bifurcation (configuration 7-8 9) of figure 2 (same mesh for all the methods).

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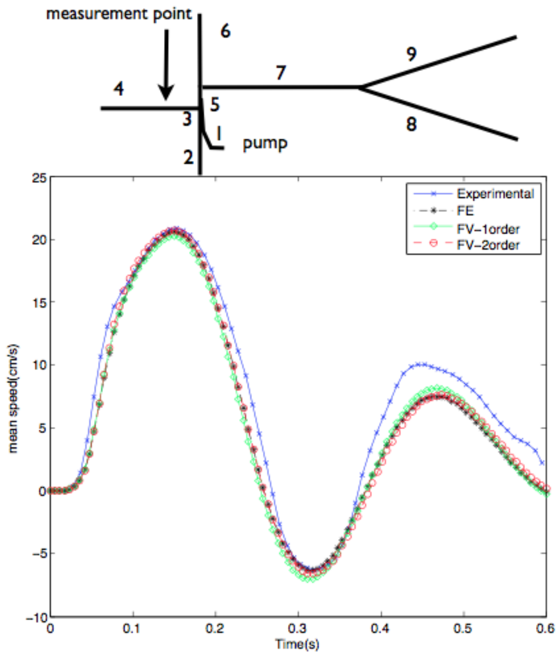

Figure 2 Example of simulation of the experimental results of [6], a pump produces a pulse of water which propagates in a 9 arteries network. The measurement of velocity by Doppler is done at "the neck" of the schematic human network. The values of parameters are those of [6].

As we observe that the numerical schemes are sensible to changes in the artery section (tapering/ aneurism) which is a source term in the equations.

A robust method (called "well-balanced" method) issued from Shallow Water equations by [2] is used. His main advantage is to properly redistribute in the numerical flux the changes of section. It preserves well the flux and avoids errors in the flow, for example there are no spurious flow due to those section changes.
Those spurious numerical flows appear even if there is no flow: in the case of Shallow Water, the "equilibrium of the lake at rest" [2] is preserved, in the Biomecanical context, the "equilibrium of the man at eternal rest" is preserved

4 Conclusions

We have presented a 1D artery flow model with finite elements and finite volume numerical resolutions. The numerical models appear to be reliable as we checked them against analytical and experimental results. The differences between the models correspond to the errors of methods and numerical accuracy. We are now able to plan to present a more realistic arterial network and we want to couple it to a venous network such as the venous network of [4].

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