Testing the continuum μ(I) rheology for 2D granular flows on avalanches and collapse of columns.

Pierre-Yves Lagrée*, Lydie Staron*, Stéphane Popinet *o

*Institut Jean le Rond d'Alembert, CNRS, Université Pierre & Marie Curie, 4 place Jussieu, Paris, France

o National Institute of Water and Atmospheric Research, PO Box 14-901 Kilbirnie, Wellington, New Zealand

There is a large amount of experimental work dealing with dry granular flows (such as sand, glass beads, small rocks…) supporting the so called *μ(I)* rheology [Jop et al. 06]. This rheology states that *μ* the ratio of the tangential to the normal constraints behaves as a Coulomb like friction depending on the Inertial number *I* (this number is the product of the grain size by the shear of the velocity divided by the square root of pressure divided by the grain density). The proposed dependance of *μ* is:

$$\mu(I) = \mu_s + \frac{\Delta\mu}{\frac{I_0}{I}+1} \qquad \mu_s = 0.3 \;\; \Delta\mu = 0.26 \;\; I_0 = 0.3$$

We propose the implementation of this non newtonian rheology in a Navier Stokes Solver (the Gerris Flow Solver which uses a finite-volume approach with the Volume-of-Fluid (VOF) method to describe variable-density two-phase flows). We redefine *I*, and the kinematic viscosity

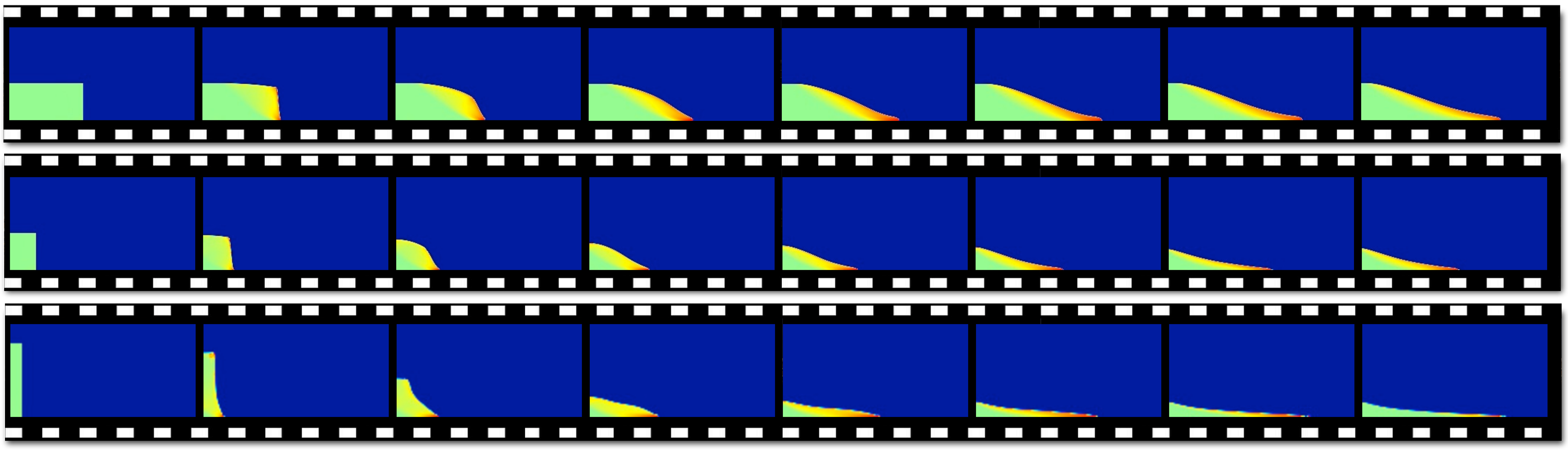
$$I = d\frac{\sqrt{2D_{ij}D_{ij}}}{\sqrt{(|p|/\rho)}} \qquad \eta = \max\left(\frac{\mu(I)}{\sqrt{2}D_2}p\ ,\ 0\right)$$

A possible experimental set up is a container filled by sand (left), the aspect ratio (height/length) is *a*. At initial time, the gate is opened quickly. After the avalanche, the grains stop, the final configuration is at rest (right). This poster compares results from Discrete Contact Method Simulations (simulation of the displacement of each grain) to a continuum Navier Stokes simulation with the μ(I) rheology.

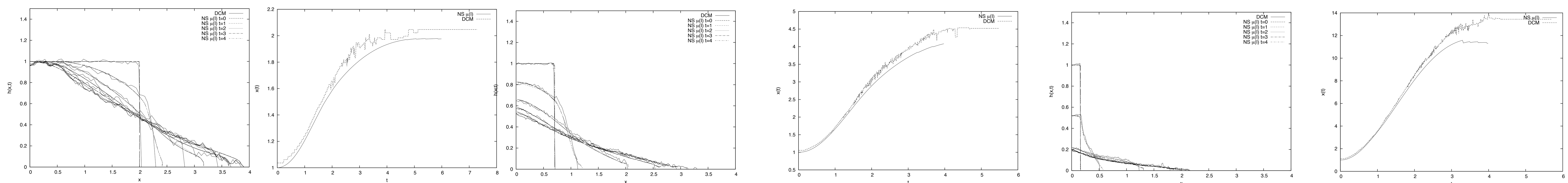
$$\nabla\cdot\mathbf{u} = 0, \;\; \frac{\partial c}{\partial t} + \nabla\cdot(c\mathbf{u}) = 0,$$
$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \nabla\cdot(2\eta\mathbf{D}),$$

The granular fluid is covered by a passive light fluid (it allows for a zero pressure boundary condition at the surface, bypassing an up to now difficulty which was to impose this condition on a unknown moving boundary).

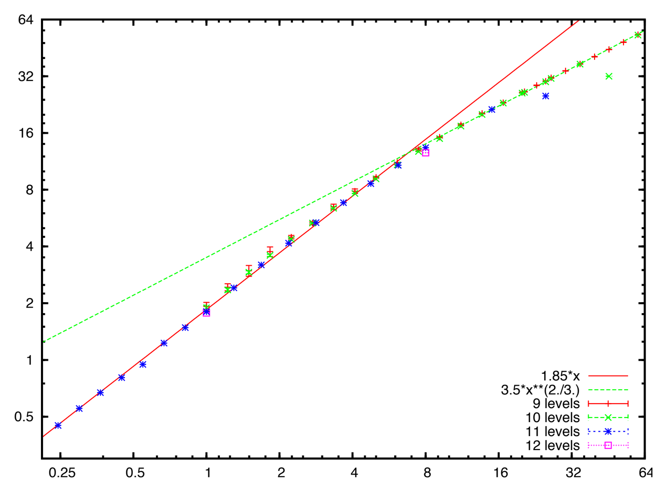
The rheology is tested on the collapse of granular columns and quantitative comparisons with numerical simulations from Contact Dynamics are done.



Snapshots of collapse of three columns of aspect ration 0.5 1.42 and 6.26 (top to bottom)



Collapse of columns of aspect ration 0.5 1.42 and 6.26 (left to right), comparison of Discrete Simulation Contact Method and Navier Stokes gerris, shape at time 0, 1, 2, 3, 4 and position of the front of the avalanche as function of time (time measured with $\sqrt{H_0/g}$ and space with aH_0)



Evolution of the normalised final deposit extent as a function of aspect ratio *a*. Well-defined power law dependencies are observed with exponents of 1 and 2/3 respectively.

We recover the experimental scaling [Lajeunesse et al. 04] and [Staron et al. 05]. Differences between the values of the prefactors are due to the difficulties to obtain the run out length: friction in the Navier Stokes code tends to underestimate it, whereas direct simulation shows that the tip is very gazeous, it can no longer explained by a continuum mechanic description.

conclusion

The μ(I) has been obtained from experimental flows of dry granular flows [Jop et al. 06], we have implemented it in a Navier Stokes solver.
To test the solver, we first recovered the classical analytical solution of steady avalanche, known as Bagnold solution (not presented on this poster [Lagrée et al.]).
As a result, we reobtain with a good precision the collapse of granular columns (shape as function of time compared to Discrete Simulations).
The experimental trends of the scaling of the run out are reobtained (however difficulties remain for the description of the front).
This opens the door to systematic studies of granular flows using this continuum approach.

references:

P. Jop, Y. Forterre, O. Pouliquen, (2006) "A rheology for dense granular flows", Nature 441, pp. 727-730 (2006)
P.-Y. Lagrée, L. Staron and S. Popinet (subm. to J.F.M.) "Flow of a granular layer along a slope: Implementing the μ(I) rheology in a Navier-Stokes solver"
E. Lajeunesse, A. Mangeney-Castelnau, and J.-P. Vilotte, (2004) "Spreading of a granular mass on an horizontal plane», Phys. Fluids, 16(7), 2371-2381.
L. Staron & E. J. Hinch (2005) "Study of the collapse of granular columns using two-dimensional discrete-grain simulation", J. Fluid Mech. (2005), vol. 545, pp. 1–27.