Stability of laboratory scale rivers

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Fluid Dynamics provides a sound insight to the geomorphologist interested in the ubiquitous formation of regular sedimentary patterns by rivers (like bars, braids and meanders). Many theoretical advances as well as laboratory experiments tend to prove that those patterns do not simply reflect a general turbulence pattern. Instead, their formation results from the interaction between a surface flow and an erodible substrate.
The interface separating the sediment layer from water is found to be unstable in many cases. In particular, small laboratory flumes are able to generate regular sediment patterns, at Reynolds number of the order of, or below 100.
This suggests that turbulence is not essential to bars, braids and maybe meanders formation. Laminar flumes then become simple models of their natural turbulent counterparts.
This poster presents the linear stability analysis of a laminar flow confined in a slowly erodible channel. The basic state is an infinite straight river, which profile is to be determined.

Rivers never go straight in Nature.
May this behavior be interpreted in terms of simple linear stability ?
Are two-dimensionnal effects sufficient to describe the initiation of main erosion patterns ?

Above: "mode 1" instability.

Below: "mode *n*" instability.

Assuming a Poiseuille velocity profile, and an erosion time much larger than the flow inertial time :

Saint-Venant equations for the water flow.

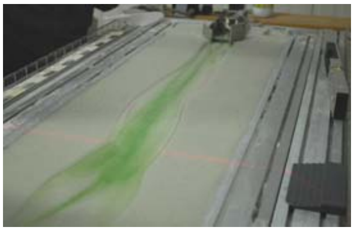
$$\begin{cases} \frac{8}{15}\left(\vec{u}\cdot\vec{\nabla}\right)\vec{u} = -g\left(\vec{\nabla}\eta + \sin(\theta)\vec{e}_x\right) - \frac{3\nu\vec{u}}{(\eta-h)^2} \\ \mathrm{div}\left(\vec{u}\,(\eta-h)\right) = 0 \end{cases}$$

$$\vec{q}_H = E\left\|\vec{\tau}\right\|^{\beta}\left(\frac{\vec{\tau}}{\left\|\vec{\tau}\right\|} - \gamma\vec{\nabla}h\right), \qquad \frac{\partial h}{\partial t} = -\mathrm{div}\,\vec{q}_H$$

Sediment mass conservation and erosion law. The sediment flux is deviated downward by the bottom slope.

Boundary conditions in the approximation of infinitely quick avalanches.
a and *b* are respectively the inner and outer boundary of the river bank.
The velocity *da/dt* does not appear explicitly.

$$\begin{cases} \left.\frac{\partial h}{\partial y}\right|_{y=a} = \alpha \\ \left.h\right|_{y=a} = \alpha\,(a-b) \\ \left(-\left.h\right|_{y=a}\right)^{\beta} = \frac{db}{dt}(b-a) \end{cases}$$



Experiment performed at the Laboratoire de Dynamique des Systèmes Géologiques, IPGP, University of Paris 7.

Width : ~ 5 cm

Water height : ~ 0.5 cm

Reynolds number : ~ 500

The influence of the erosion law on straight river cross-section is illustrated on the right.

In one dimension, the evolution equation is $\frac{\partial h}{\partial t} = -\frac{\partial q}{\partial y}$

Where *q* takes both erosion and avalanches into account :

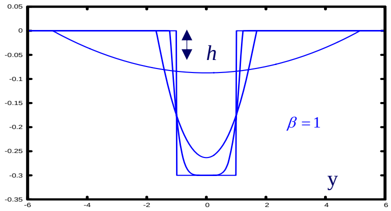
$$q = q_H + q_A$$

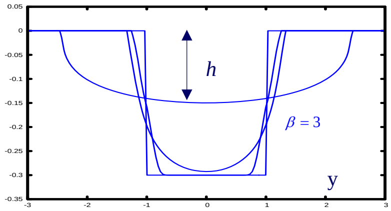
$$q_H = -(-h)^{\beta}\frac{\partial h}{\partial y}; \quad q_A = -\frac{1}{\varepsilon}H\left(\left|\frac{\partial h}{\partial y}\right| - \alpha\right)\left(\left|\frac{\partial h}{\partial y}\right| - \alpha\right)$$

H is the Heavyside function and α is the critical angle above which avalanches occur. ε is a small parameter.

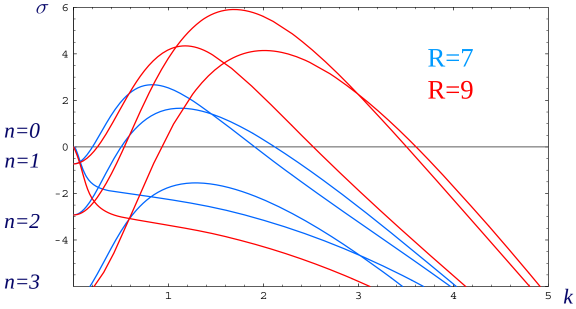
When $q_A = 0$ and $\beta = 1$, a parabolic self-similar solution exists (above), which aspect ratio is proportional to $t^{2/3}$.

The general profile (below) is the basic state of the stability analysis.





Evolution of the cross-section of a laminar laboratory flume for two sets of parameters.



Linear stability results

$$h^*(x,y,t) = F_{n,k}(y)e^{\sigma t}e^{i(kx-\omega t)}$$

Most rivers cross-section are unstable in two dimensions, provided $\beta > 1$.
As the aspect ratio increases, the most instable mode switches from *n* to *n+1*.

References
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