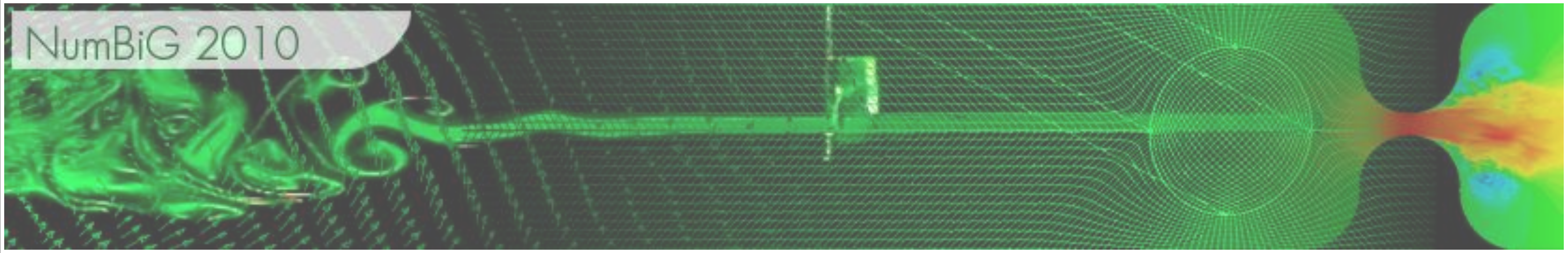

from Classical Boundary Layer to Interacting Boundary Layer Application to stenosed flows

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Collaborations
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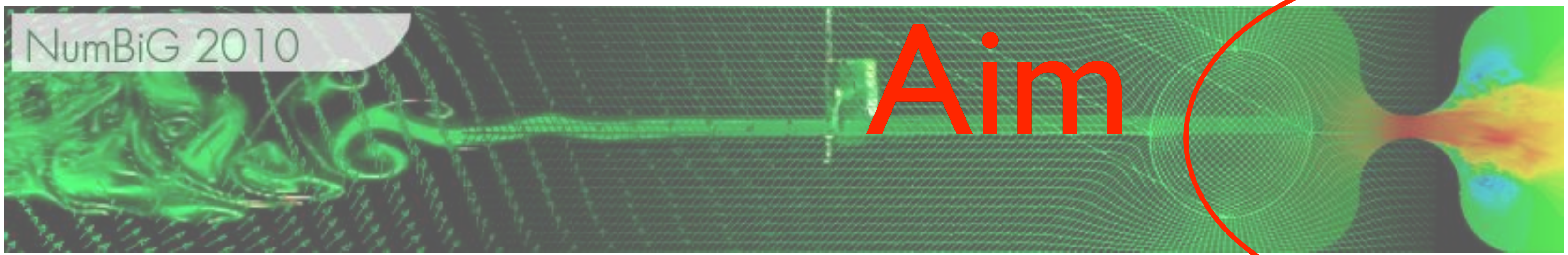


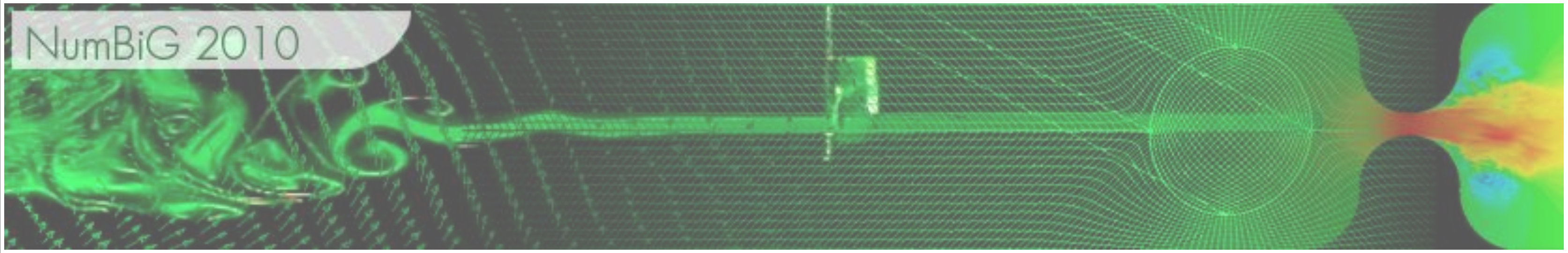
Aim

- simplification of Navier Stokes equations
- thanks to asymptotic theory:

  "Boundary Layer"

Starting from Navier Stokes

- we simplify NS to a Reduced set of equations
  - which contains the physical scales,
  - the most important phenomena
- much more simple set of equations: Integral equations (1D)
- cross comparisons in some cases of NS/ RNSP/ Integral



Prandtl 04
Golstein 48
paradox of upstream influence
- *Triple Deck*

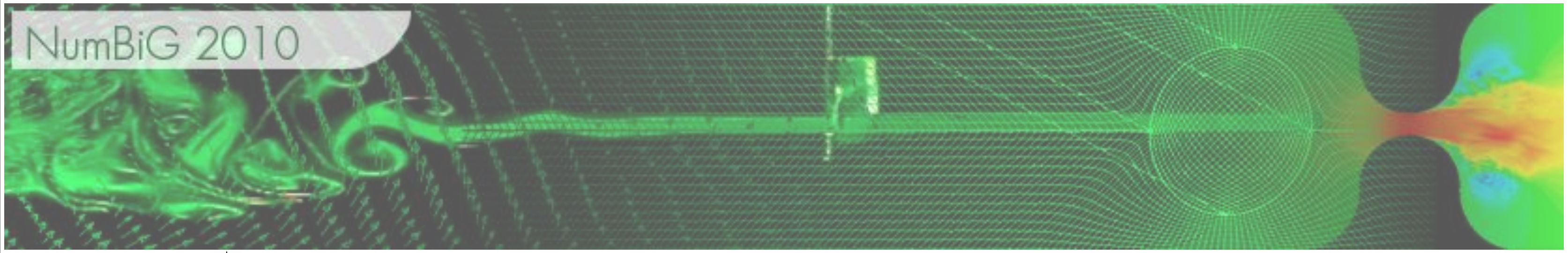
Lighthill
Stewartson Neiland Messiter 69
Smith

- *Interactive Boundary Layer / Viscous Inviscid Interactions*

Le Balleur 78, Carter 79, Cebeci 70s
Veldman 81

- *Boundary layer Asymptotics*

Sychev, Ruban, Sychev, Korelev, 98
Sobey 00
Cebeci Cousteix 01
Mauss Cousteix 07 (SCEM)



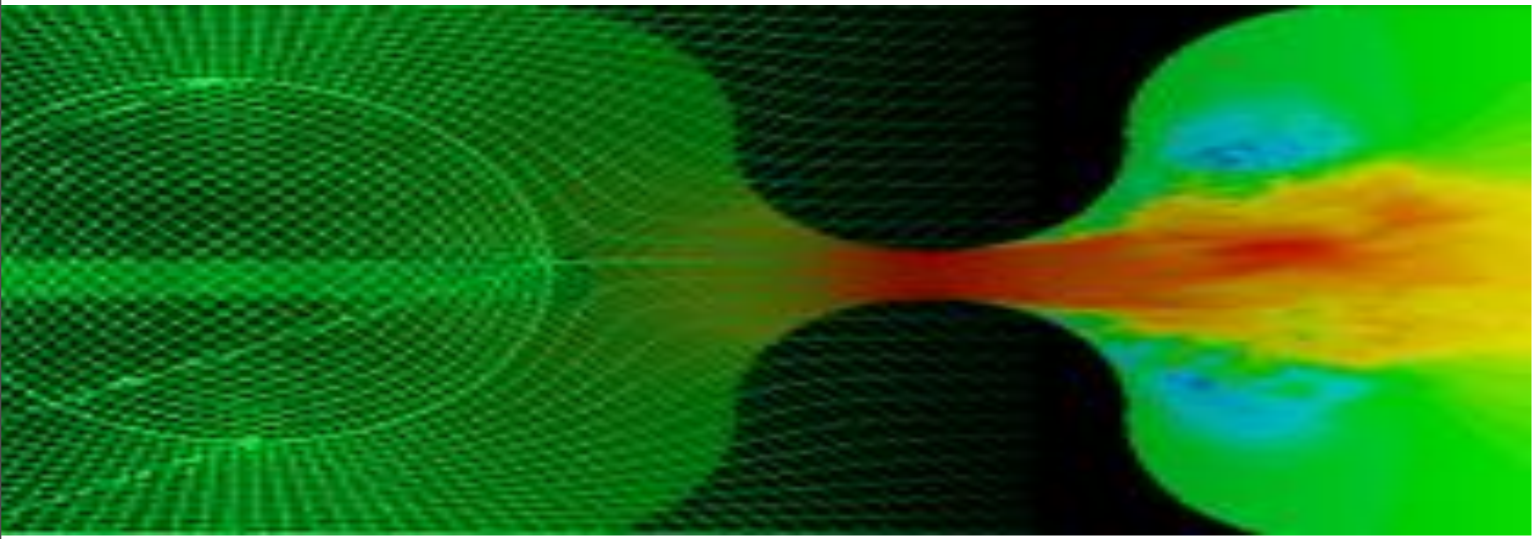
3 full NS 3D

2 NS 2D/Axi

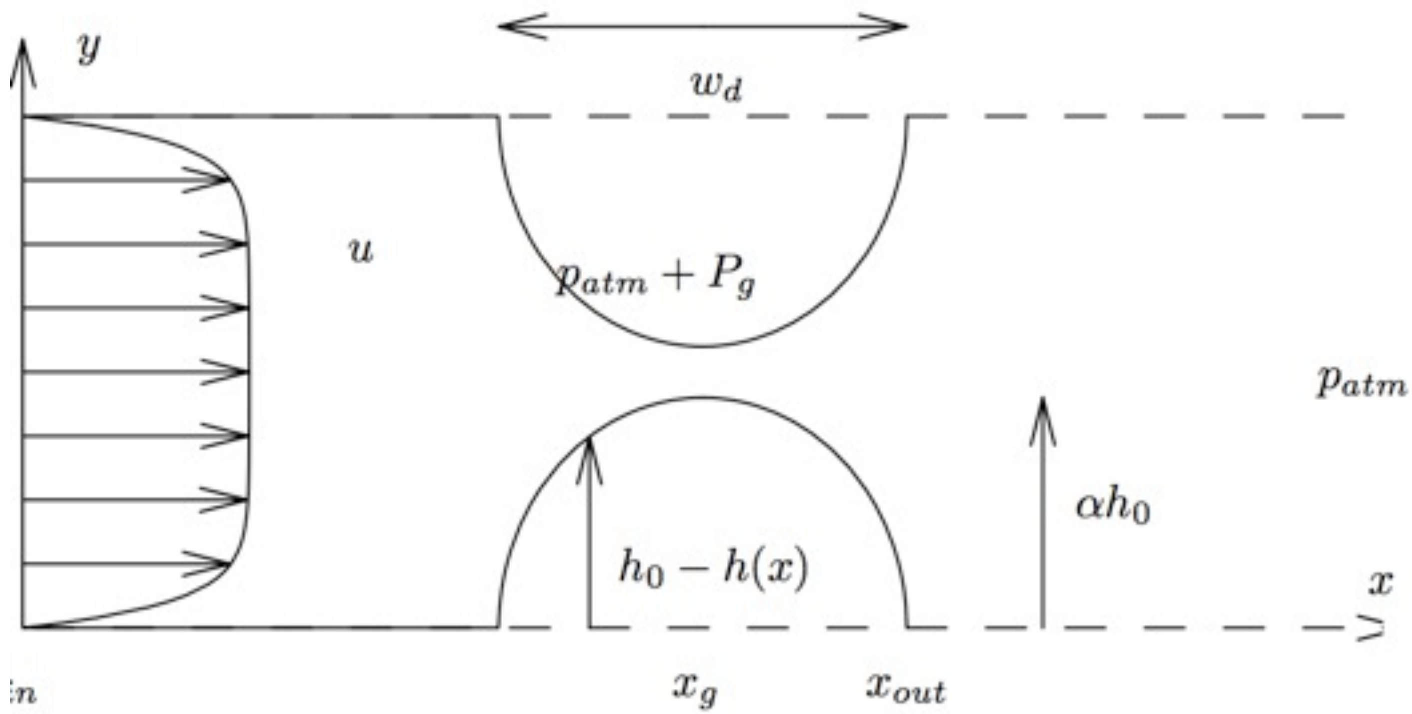
1 1D

0 0D lumped model

Our model equations

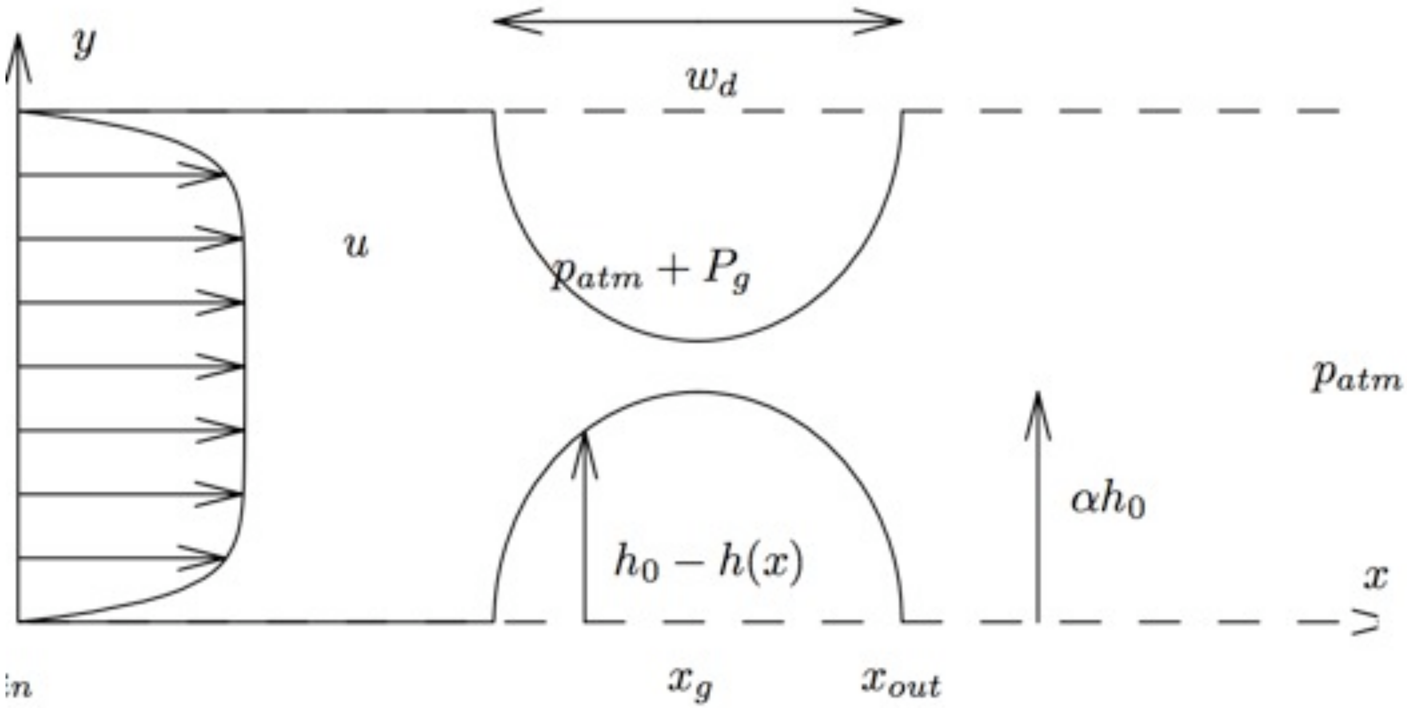


Navier Stokes Problem



$$
\begin{cases}
\dfrac{\partial \bar{u}}{\partial \bar{x}} + \dfrac{\partial \bar{v}}{\partial \bar{y}} = 0, \\
\bar{u}\dfrac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\dfrac{\partial \bar{u}}{\partial \bar{y}} = -\dfrac{\partial \bar{p}}{\partial \bar{x}} + \dfrac{1}{Re}\left(\dfrac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \dfrac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right), \\
\bar{u}\dfrac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\dfrac{\partial \bar{v}}{\partial \bar{y}} = -\dfrac{\partial \bar{p}}{\partial \bar{y}} + \dfrac{1}{Re}\left(\dfrac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \dfrac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right).
\end{cases}
$$

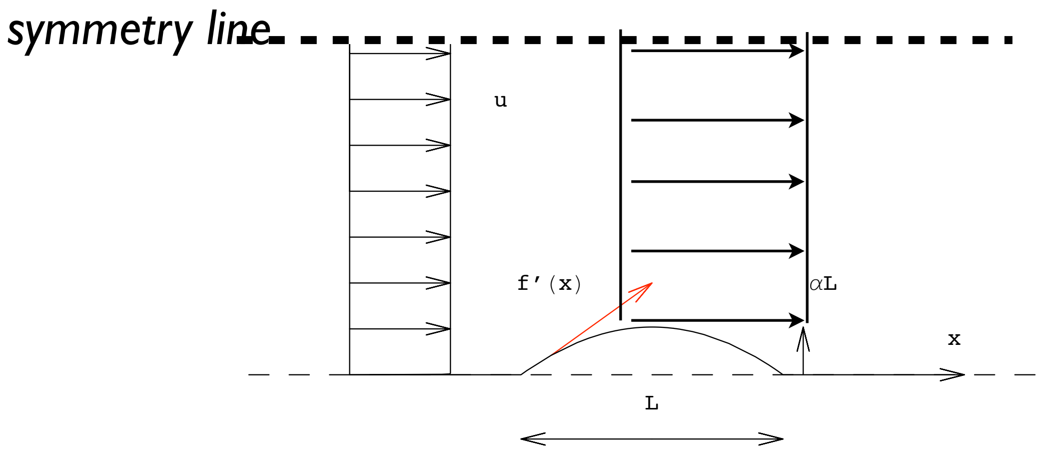
Euler Problem



$$\begin{cases} \dfrac{\partial \bar{u}}{\partial \bar{x}} + \dfrac{\partial \bar{v}}{\partial \bar{y}} = 0, \\ \bar{u}\dfrac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\dfrac{\partial \bar{u}}{\partial \bar{y}} = -\dfrac{\partial \bar{p}}{\partial \bar{x}} \\ \bar{u}\dfrac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\dfrac{\partial \bar{v}}{\partial \bar{y}} = -\dfrac{\partial \bar{p}}{\partial \bar{y}} \end{cases} \qquad 1/Re = 0.$$

Euler Problem

$\bar{y}_w(\bar{x}) = \alpha \bar{f}(\bar{x})$



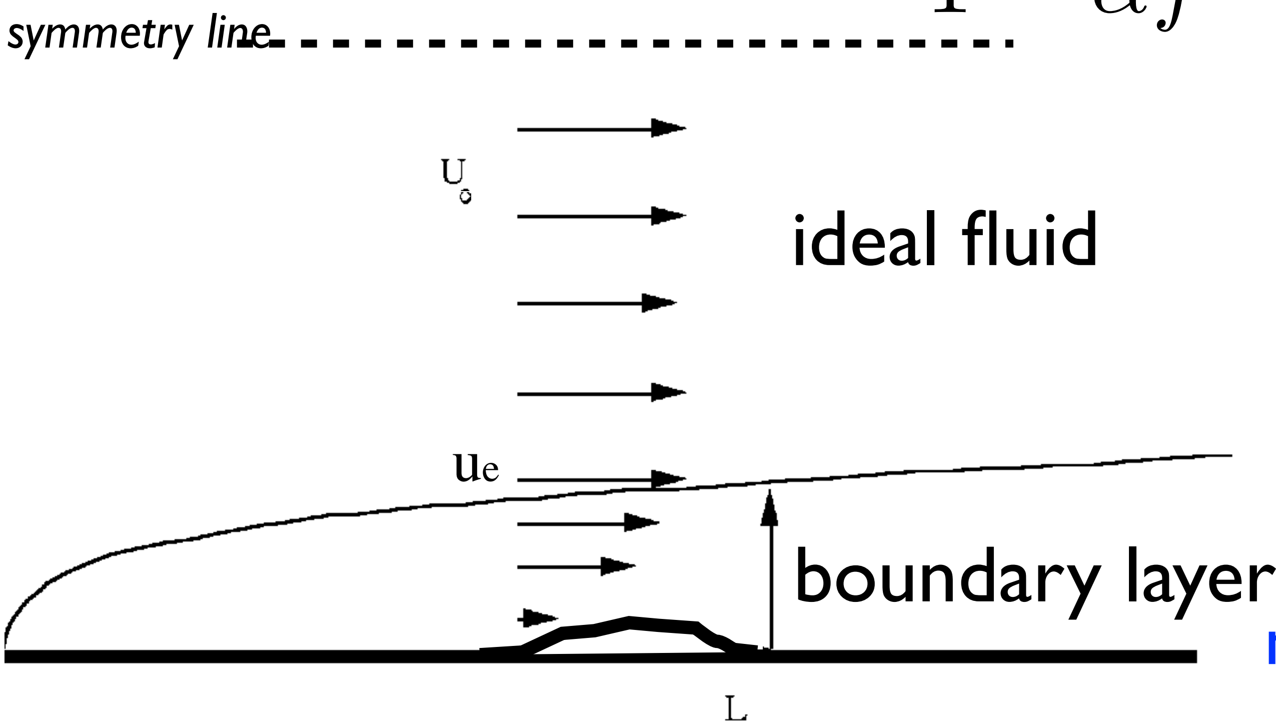
flux conservation

Section * Velocity = cste

$$\bar{u}_e = \frac{1}{1 - \alpha \bar{f}}$$

Boundary layer problem

$$\bar{u}_e = \frac{1}{1 - \alpha \bar{f}}$$



slip condition

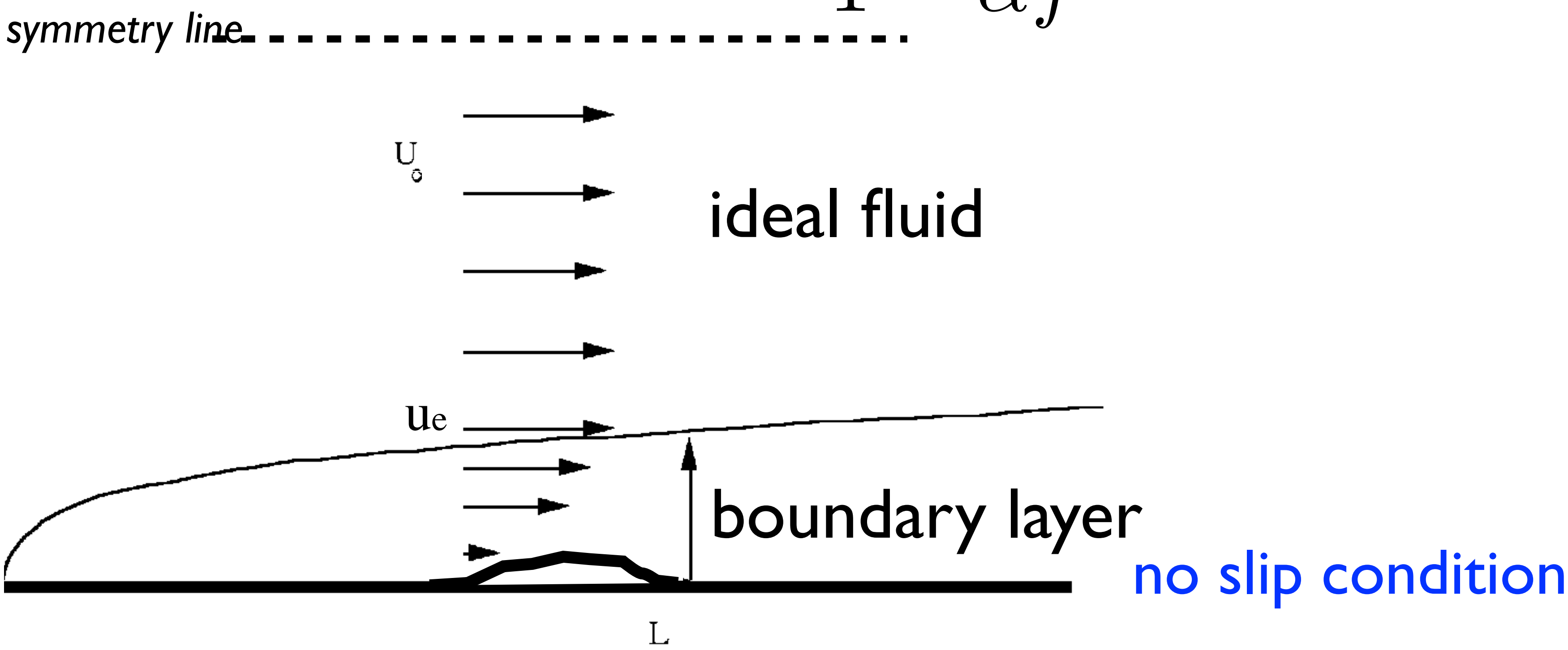
no slip condition

$$\tilde{u}\frac{\partial \tilde{u}}{\partial \bar{x}} \propto \frac{1}{Re(\delta/L)^2}\frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2},$$

$$Re^{-1/2}$$

Boundary layer problem

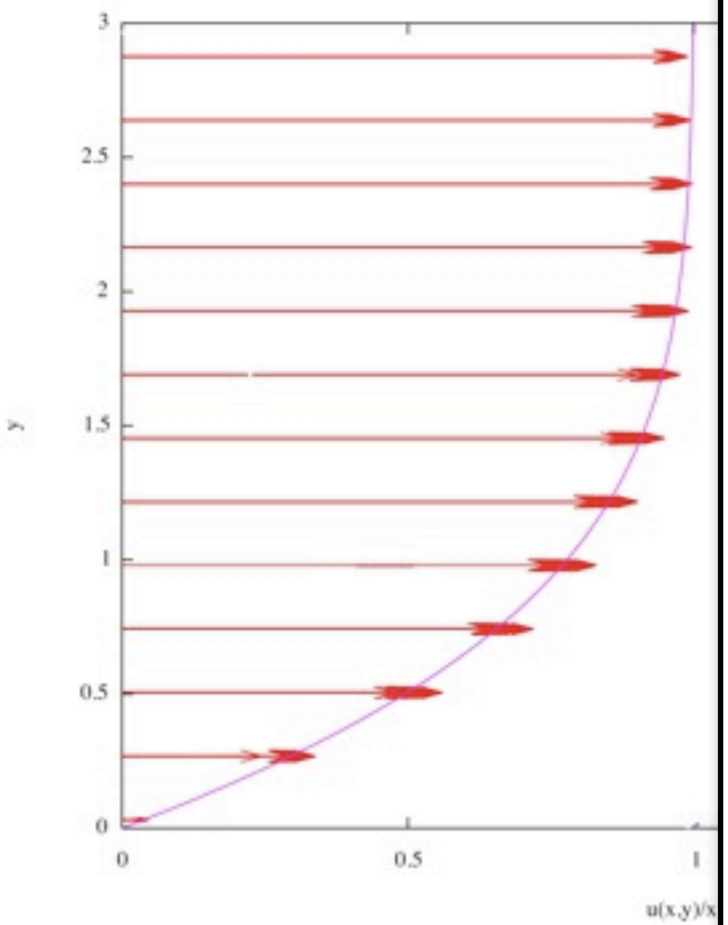
$$\bar{u}_e = \frac{1}{1 - \alpha \bar{f}}$$



$$\begin{cases} \dfrac{\partial \tilde{u}}{\partial \tilde{x}} + \dfrac{\partial \tilde{v}}{\partial \tilde{y}} = 0, \\ u\dfrac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v}\dfrac{\partial \tilde{u}}{\partial \tilde{y}} = -\dfrac{\partial \tilde{p}}{\partial \tilde{x}} + \dfrac{\partial^2 \tilde{u}}{\partial \tilde{y}^2}, \\ 0 = -\dfrac{\partial \tilde{p}}{\partial \tilde{y}}. \end{cases}$$

$$Re^{-1/2}$$

Boundary Layer Equations



matching at infinity

$$\tilde{u}(\bar{x}, \infty) = \bar{u}_e(\bar{x})$$
$$\tilde{p}(\bar{x}, \infty) = \bar{p}_e(\bar{x})$$

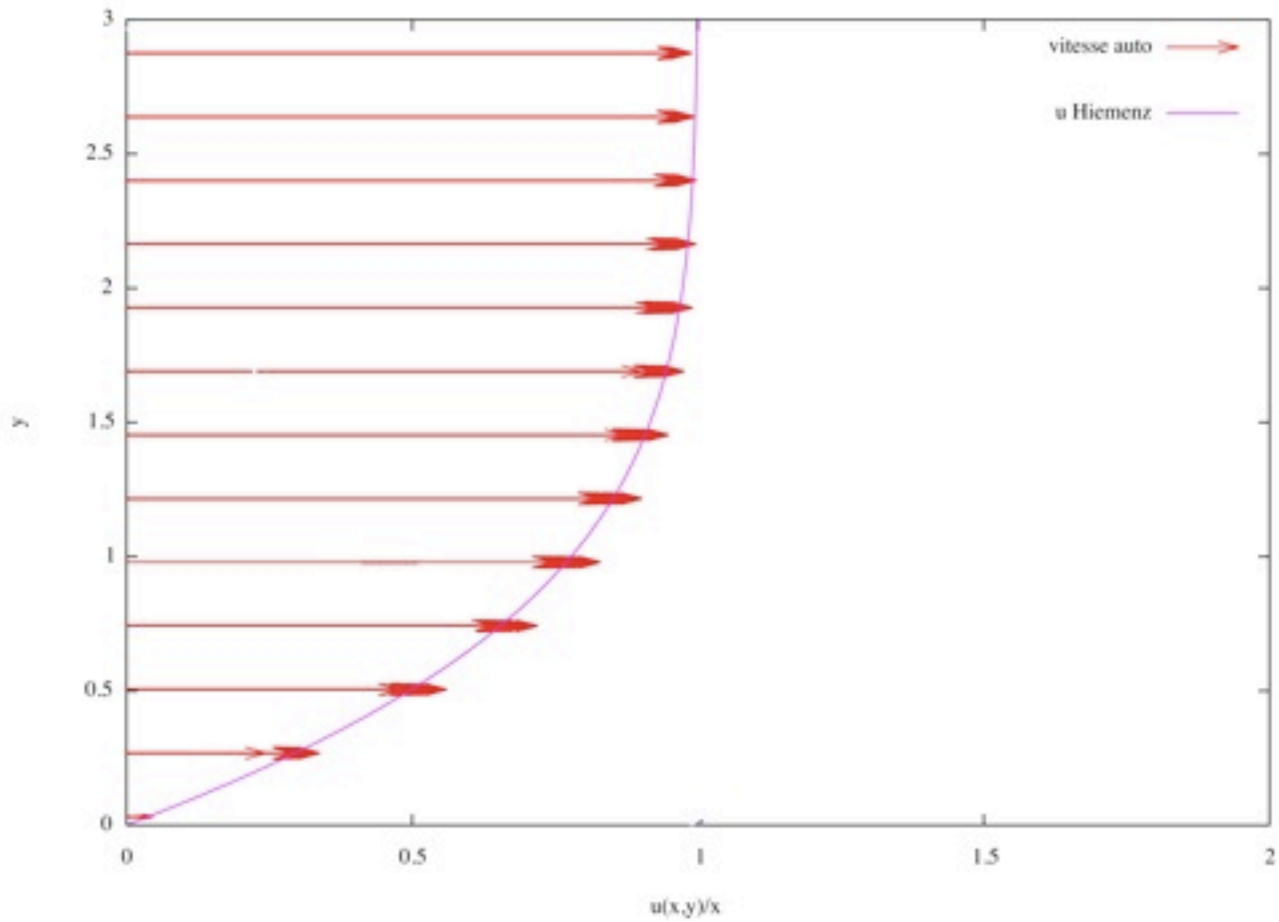
$$\left\{ \begin{aligned} \frac{\partial \tilde{u}}{\partial \bar{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} &= 0, \\ u\frac{\partial \tilde{u}}{\partial \bar{x}} + \tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}} &= -\frac{\partial \tilde{p}}{\partial \bar{x}} + \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2}, \\ 0 &= -\frac{\partial \tilde{p}}{\partial \tilde{y}}. \end{aligned} \right.$$

no slip condition at the wall

$$\tilde{u}(\bar{x}, 0) = \tilde{v}(\bar{x}, 0) = 0$$

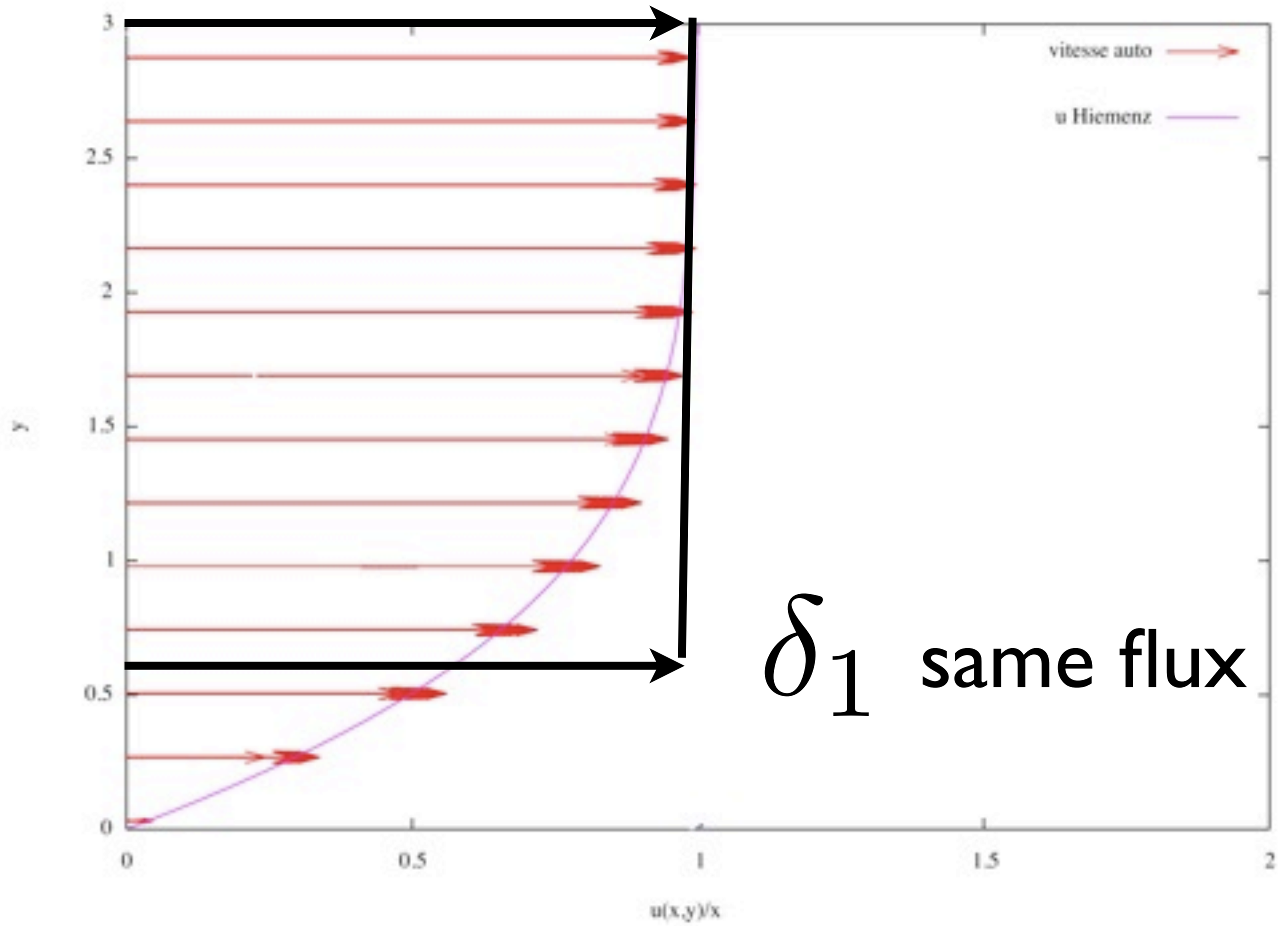
$$Re^{-1/2}$$

Boundary layer problem



displacement thickness $Re^{-1/2}$

Boundary layer problem



$$\tilde{\delta}_1 = \int_0^\infty (1 - \frac{\tilde{u}}{\bar{u}_e}) d\tilde{y}, \quad \tilde{\delta}_2 = \int_0^\infty \frac{\tilde{u}}{\bar{u}_e}(1 - \frac{\tilde{u}}{\bar{u}_e}) d\tilde{y}$$

displacement thickness

Boundary Layer Equations

$$\tilde{\delta}_1 = \int_0^{\infty} (1 - \frac{\tilde{u}}{\bar{u}_e}) d\tilde{y}, \quad \tilde{\delta}_2 = \int_0^{\infty} \frac{\tilde{u}}{\bar{u}_e} (1 - \frac{\tilde{u}}{\bar{u}_e}) d\tilde{y}$$

displacement thickness

$\int_0^\infty d\tilde{y}$ Boundary Layer Equations

$$\tilde{\delta}_1 = \int_0^\infty (1 - \frac{\tilde{u}}{\bar{u}_e}) d\tilde{y}, \quad \tilde{\delta}_2 = \int_0^\infty \frac{\tilde{u}}{\bar{u}_e}(1 - \frac{\tilde{u}}{\bar{u}_e}) d\tilde{y}$$

displacement thickness

$$\frac{d}{d\bar{x}}(\tilde{\delta}_2 \bar{u}_e^2) + \tilde{\delta}_1 \bar{u}_e \frac{d\bar{u}_e}{d\bar{x}} = \frac{\partial \tilde{u}}{\partial \tilde{y}}\Big|_{\tilde{y}=0}$$

$\int_0^\infty d\tilde{y}$ Boundary Layer Equations

$$\tilde{\delta}_1 = \int_0^\infty (1 - \frac{\tilde{u}}{\bar{u}_e}) d\tilde{y}, \quad \tilde{\delta}_2 = \int_0^\infty \frac{\tilde{u}}{\bar{u}_e}(1 - \frac{\tilde{u}}{\bar{u}_e}) d\tilde{y}$$

displacement thickness

$$\frac{d}{d\bar{x}}(\frac{\tilde{\delta}_1}{H}) + \frac{\tilde{\delta}_1}{\bar{u}_e}(1 + \frac{2}{H})\frac{d\bar{u}_e}{d\bar{x}} = \frac{f_2 H}{\tilde{\delta}_1 \bar{u}_e},$$

$$H = \frac{\tilde{\delta}_1}{\tilde{\delta}_2},$$

$$\frac{\partial \tilde{u}}{\partial \tilde{y}} = f_2 \frac{H \bar{u}_e}{\delta_1}$$

$$\tilde{\delta}_1 = \int_0^\infty (1 - \frac{\tilde{u}}{\bar{u}_e}) d\tilde{y}, \quad \tilde{\delta}_2 = \int_0^\infty \frac{\tilde{u}}{\bar{u}_e}(1 - \frac{\tilde{u}}{\bar{u}_e}) d\tilde{y}$$

displacement thickness

$$\frac{d}{d\bar{x}}\left(\frac{\tilde{\delta}_1}{H}\right) + \frac{\tilde{\delta}_1}{\bar{u}_e}\left(1 + \frac{2}{H}\right)\frac{d\bar{u}_e}{d\bar{x}} = \frac{f_2 H}{\tilde{\delta}_1 \bar{u}_e},$$

unknown (depends on the solution)

$$H = \frac{\tilde{\delta}_1}{\tilde{\delta}_2},$$

$$\frac{\partial u}{\partial \tilde{y}} = f_2 \frac{H \bar{u}_e}{\delta_1}$$

$$\tilde{\delta}_1 = \int_0^\infty \left(1 - \frac{\tilde{u}}{\bar{u}_e}\right) d\tilde{y}, \quad \tilde{\delta}_2 = \int_0^\infty \frac{\tilde{u}}{\bar{u}_e}\left(1 - \frac{\tilde{u}}{\bar{u}_e}\right) d\tilde{y}$$

displacement thickness

$$\frac{d}{d\bar{x}}\left(\frac{\tilde{\delta}_1}{H}\right) + \frac{\tilde{\delta}_1}{\bar{u}_e}\left(1+\frac{2}{H}\right)\frac{d\bar{u}_e}{d\bar{x}} = \frac{f_2 H}{\tilde{\delta}_1 \bar{u}_e},$$

unknown (depends on the solution)

$$H = \frac{\tilde{\delta}_1}{\tilde{\delta}_2},$$

$$\frac{\partial u}{\partial \tilde{y}} = f_2 \frac{H \bar{u}_e}{\tilde{\delta}_1}$$

depends on the PROFILE of VELOCITY

$$\tilde{\delta}_1 = \int_0^{\infty} \left(1 - \frac{\tilde{u}}{\bar{u}_e}\right) d\tilde{y}, \quad \tilde{\delta}_2 = \int_0^{\infty} \frac{\tilde{u}}{\bar{u}_e}\left(1 - \frac{\tilde{u}}{\bar{u}_e}\right) d\tilde{y}$$

displacement thickness

$$\frac{d}{d\bar{x}}\left(\frac{\tilde{\delta}_1}{H}\right) + \frac{\tilde{\delta}_1}{\bar{u}_e}\left(1 + \frac{2}{H}\right)\frac{d\bar{u}_e}{d\bar{x}} = \frac{f_2 H}{\tilde{\delta}_1 \bar{u}_e},$$

ODE!

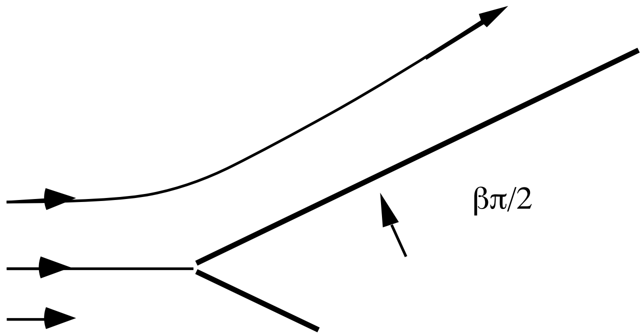
equation between:
the displacement thickness and the external velocity

$$\left\{ \begin{array}{l} \tilde{\delta}_1 = \int_0^\infty (1 - \frac{\tilde{u}}{\bar{u}_e}) d\tilde{y}, \\ \\ \bar{u}_e \end{array} \right.$$

H f_2 functions of $\tilde{\delta}_1$ $\bar{u}_e$

need for a Closure Falkner Skan

n=β/(2-β); β=(2n)/(n+1).

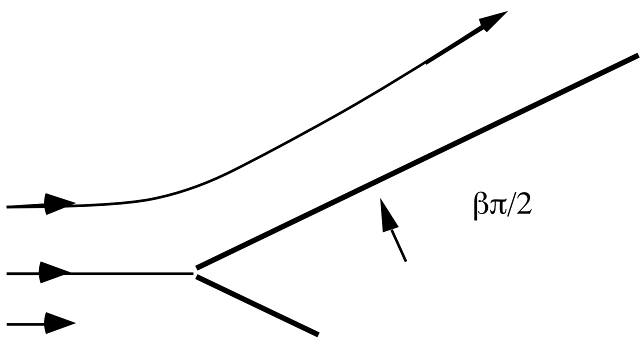


$$\psi = r^{\frac{2}{2-\beta}} sin(\frac{2}{2-\beta}\theta).$$

need for a Closure

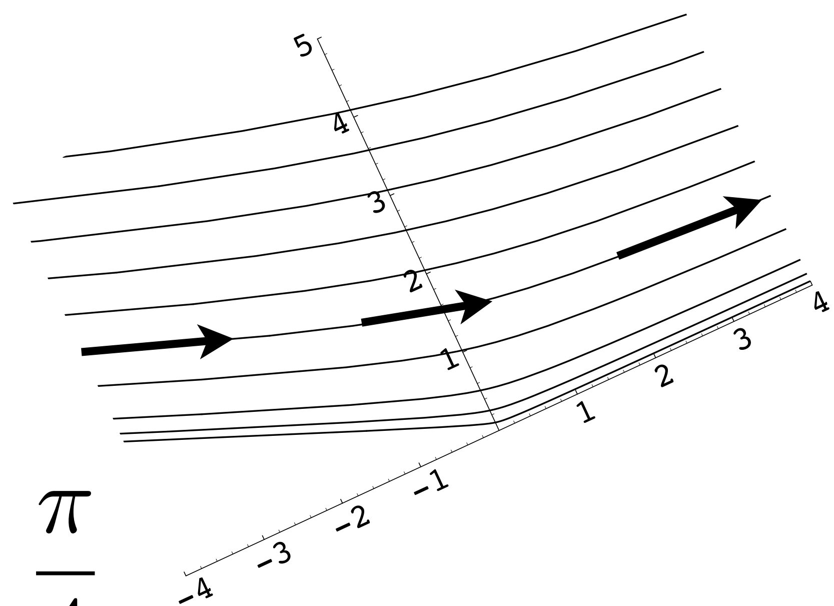
Falkner Skan

n=β/(2-β); β=(2n)/(n+1).



$$\psi = r^{\frac{2}{2-\beta}} sin(\frac{2}{2-\beta}\theta).$$

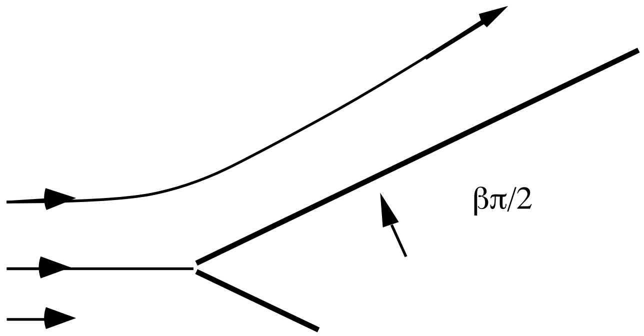
$$\psi = r^n sin(n\theta)$$



$$\frac{\pi}{4}$$

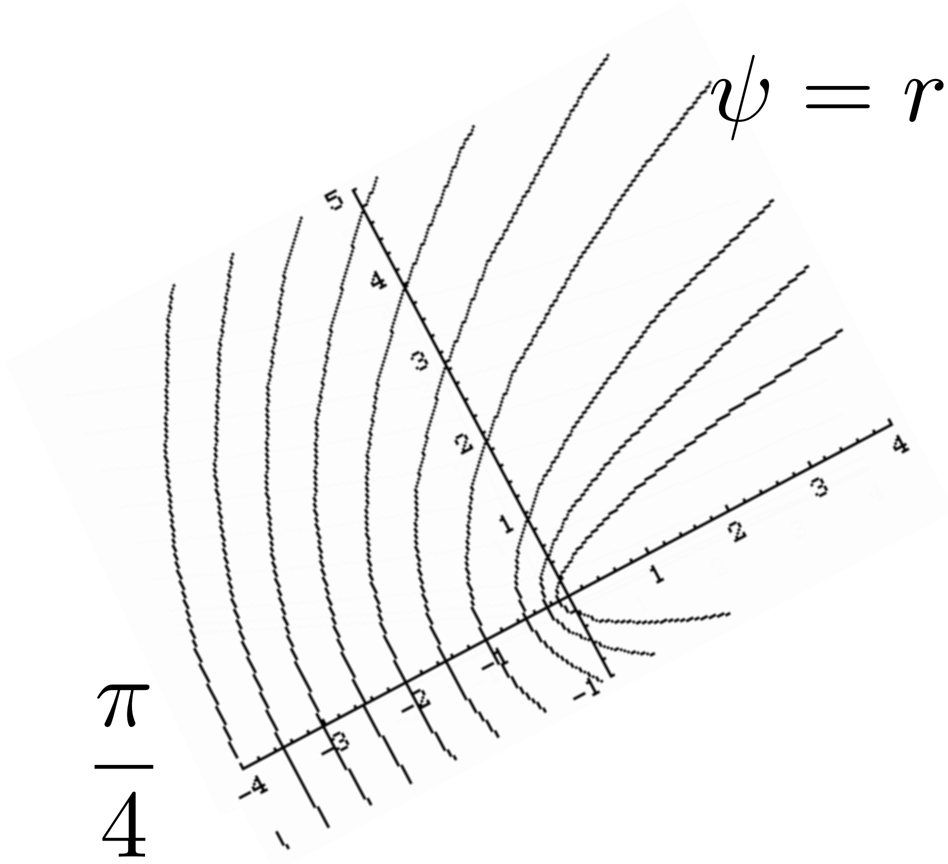
need for a Closure Falkner Skan

n=β/(2-β); β=(2n)/(n+1).



$$\psi = r^{\frac{2}{2-\beta}} sin(\frac{2}{2-\beta}\theta).$$

$$\psi = r^n sin(n\theta)$$



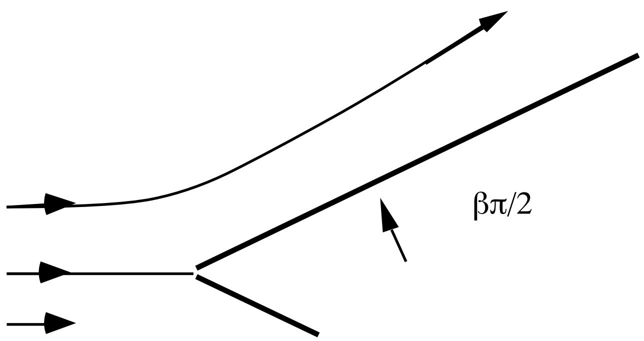
$$\frac{\pi}{4}$$

need for a Closure

Falkner Skan

n=β/(2-β); β=(2n)/(n+1).

$$u_e = x^n \text{ with } n = \frac{\beta}{2-\beta}$$



$$\xi = \bar{x}, \quad \eta = (\sqrt{\frac{n+1}{2}})\frac{\tilde{y}}{\bar{\xi}^{(1-n)/2}} \quad \psi = (\sqrt{\frac{2}{n+1}})\bar{\xi}^{(n+1)/2} f(\eta)$$

$$\tilde{u} = \xi^n f'(\eta), \quad \tilde{v} = -\sqrt{\frac{n+1}{2}\xi^{n-1}}(f + \frac{n-1}{n+1}\eta f')$$

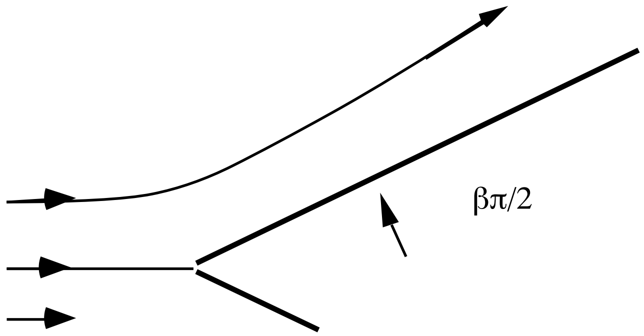
$$f'''(\eta) + f(\eta)f''(\eta) + \beta(1 - f'(\eta)^2) = 0, \quad f(0) = f'(0) = 0 \text{ and } f'(\infty) = 1.$$

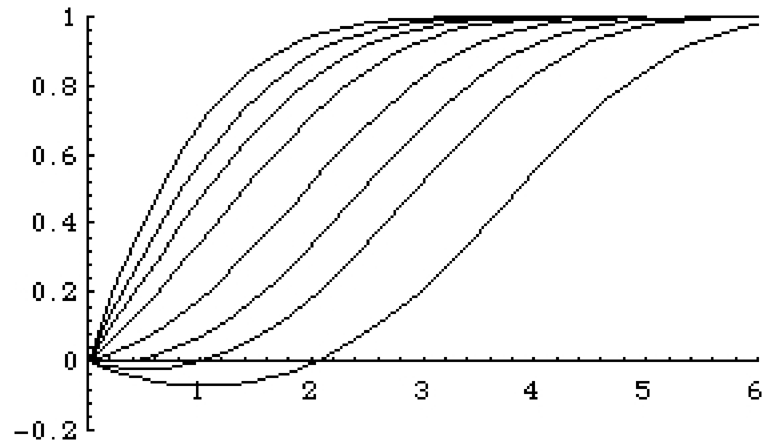
need for a Closure

Falkner Skan

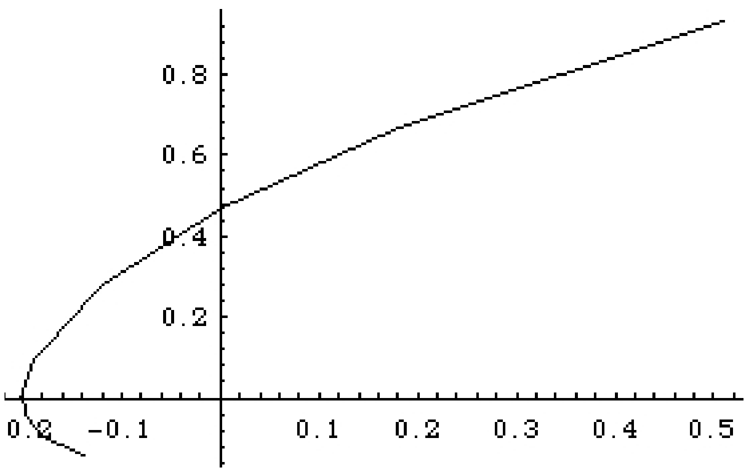
n=β/(2-β); β=(2n)/(n+1).

$$u_e = x^n \text{ with } n = \frac{\beta}{2-\beta}$$



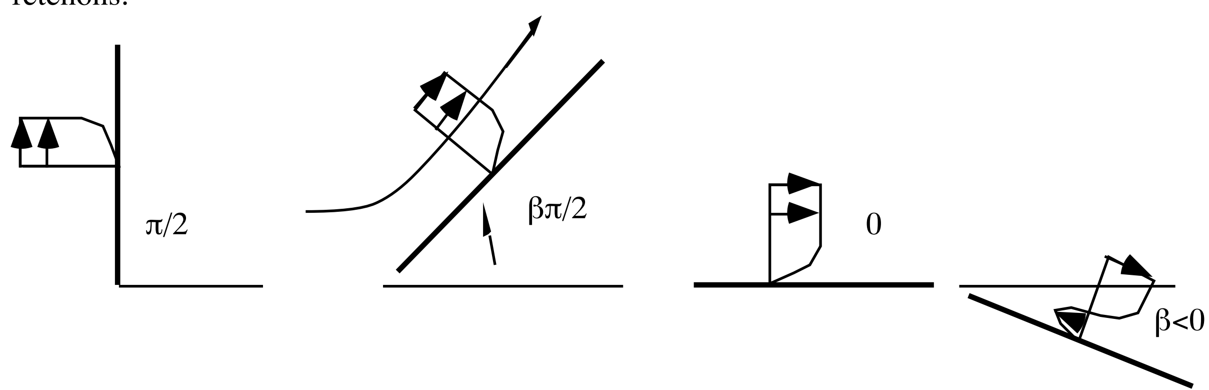


η en abscisse f'(η) en ordonnée.



f"$_0$ de β

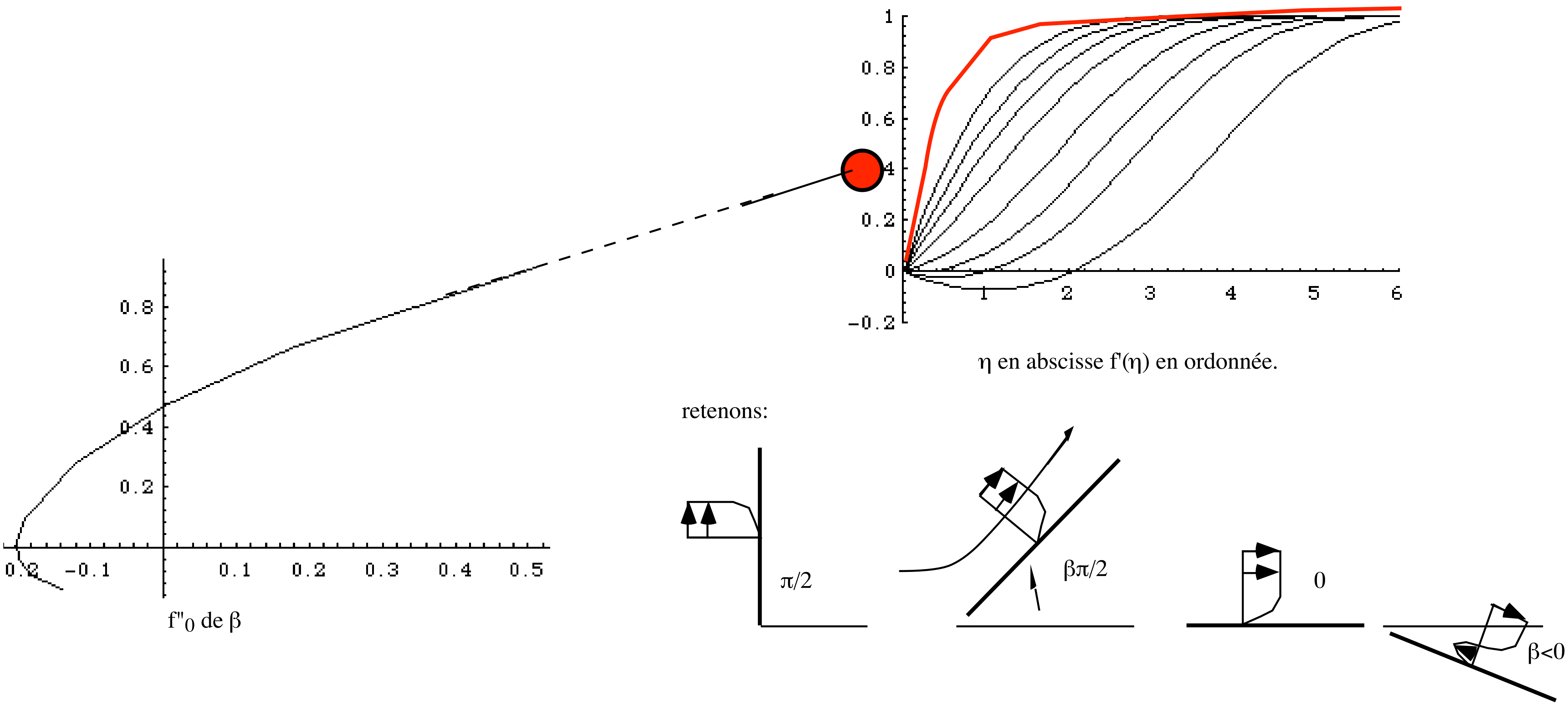
retenons:



need for a Closure

Falkner Skan

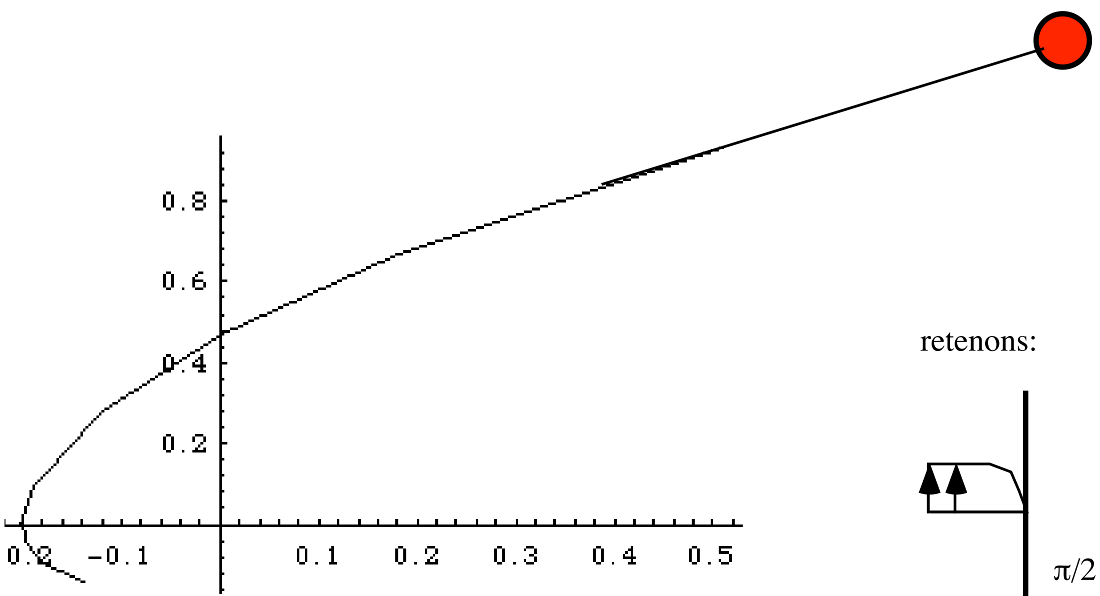
converging channel



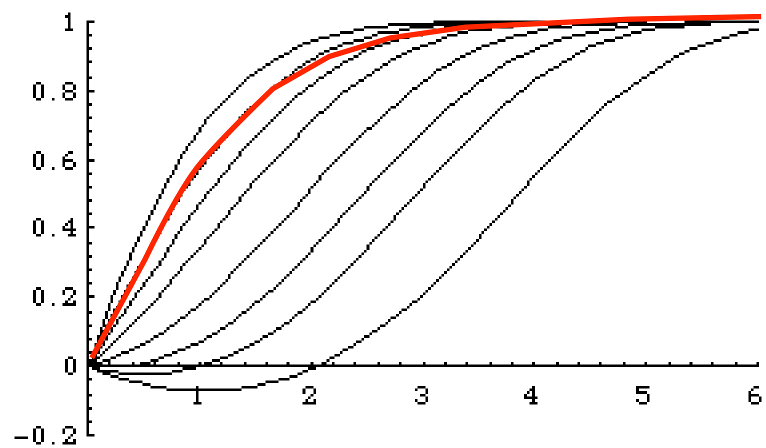
need for a Closure

Falkner Skan

Hiemenz

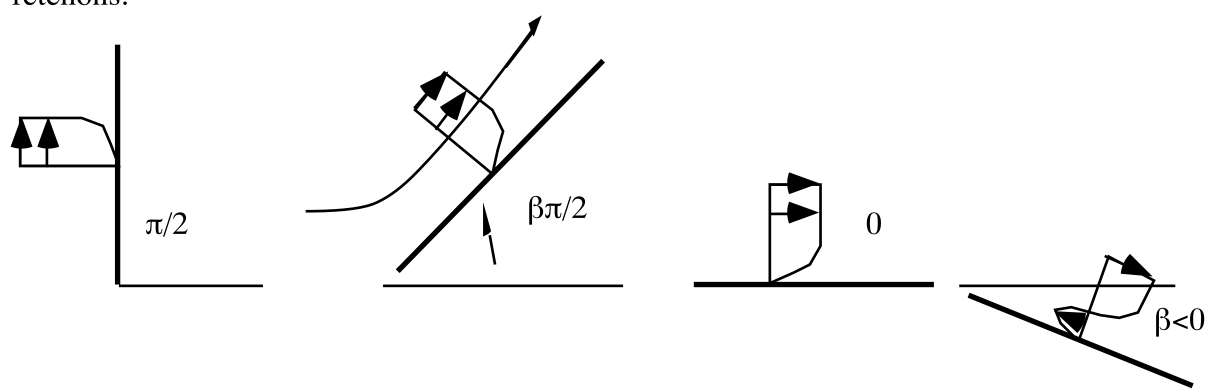


f"$_0$ de β



η en abscisse f'(η) en ordonnée.

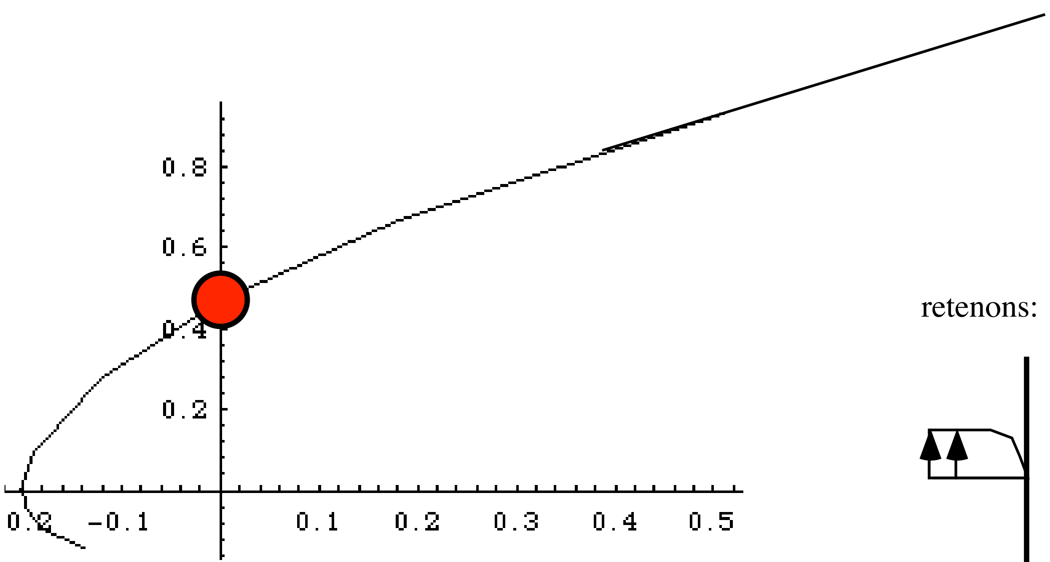
retenons:



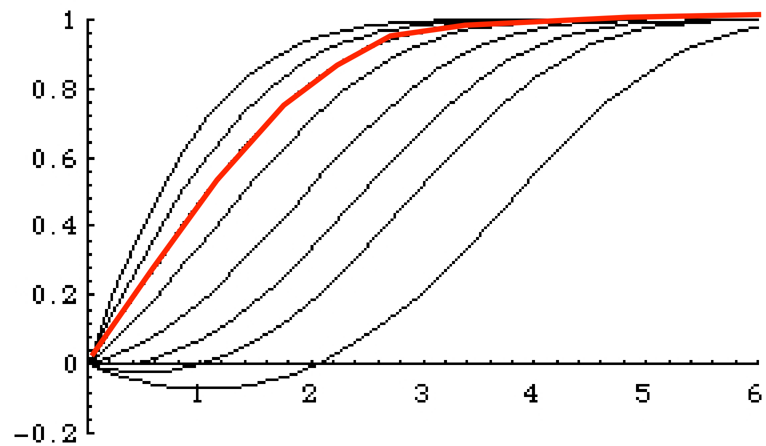
need for a Closure

Falkner Skan

Blasius

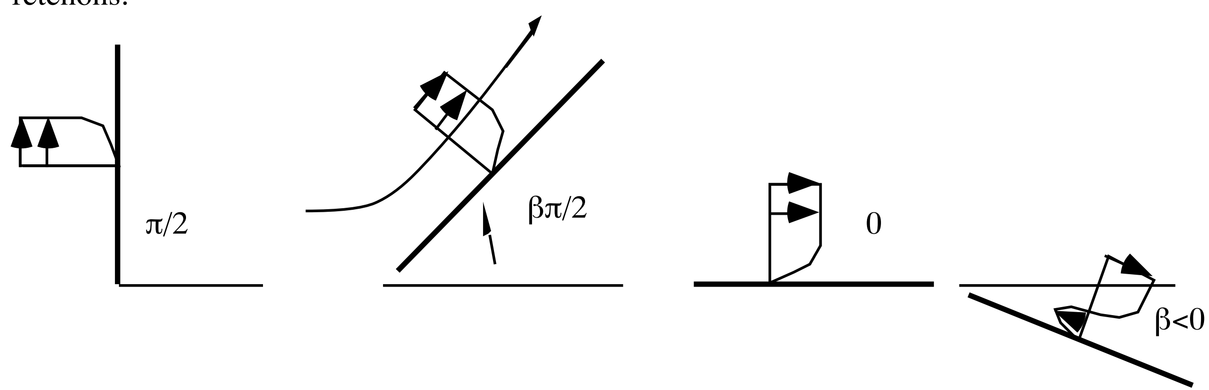


f''_0 de β



η en abscisse f'(η) en ordonnée.

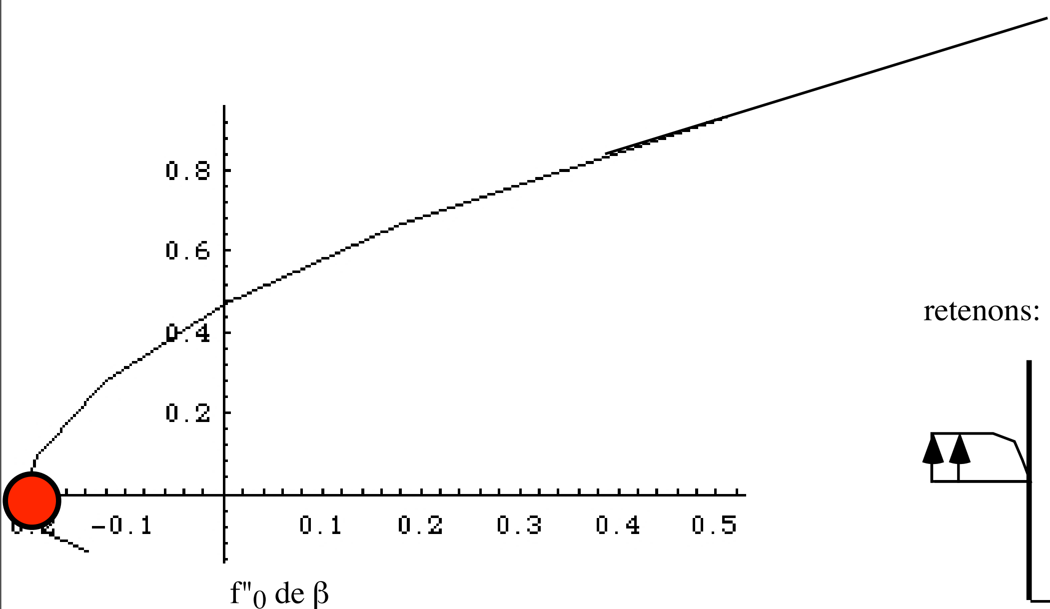
retenons:

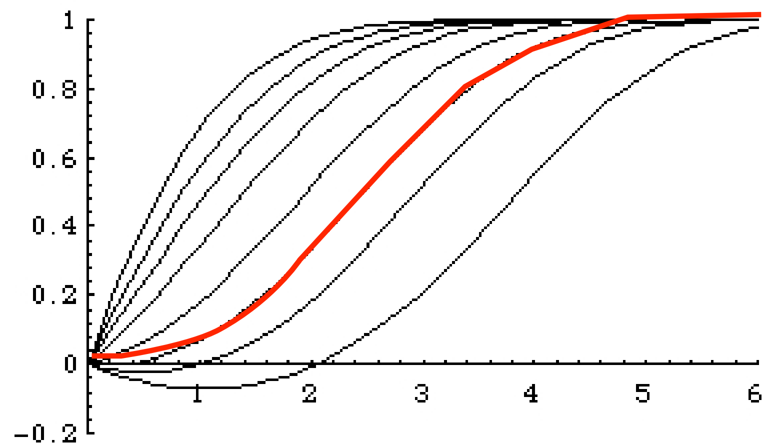


need for a Closure

Falkner Skan

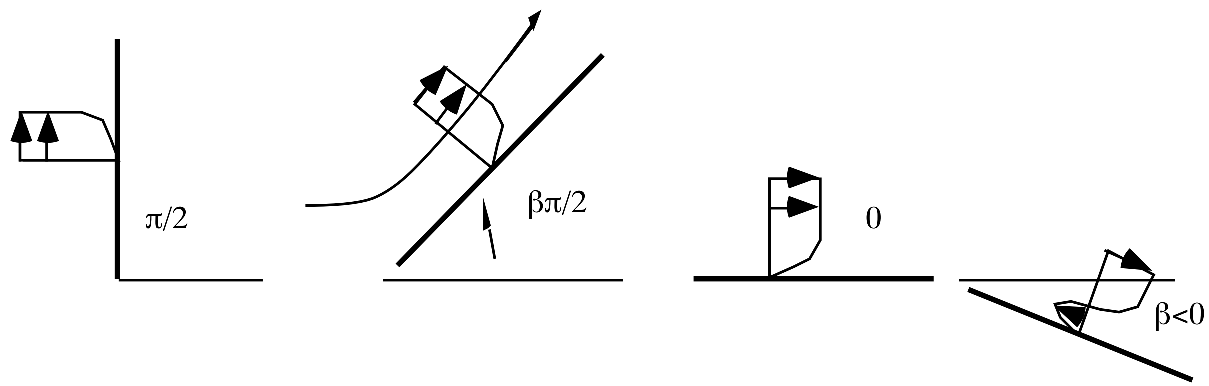
incipient separation





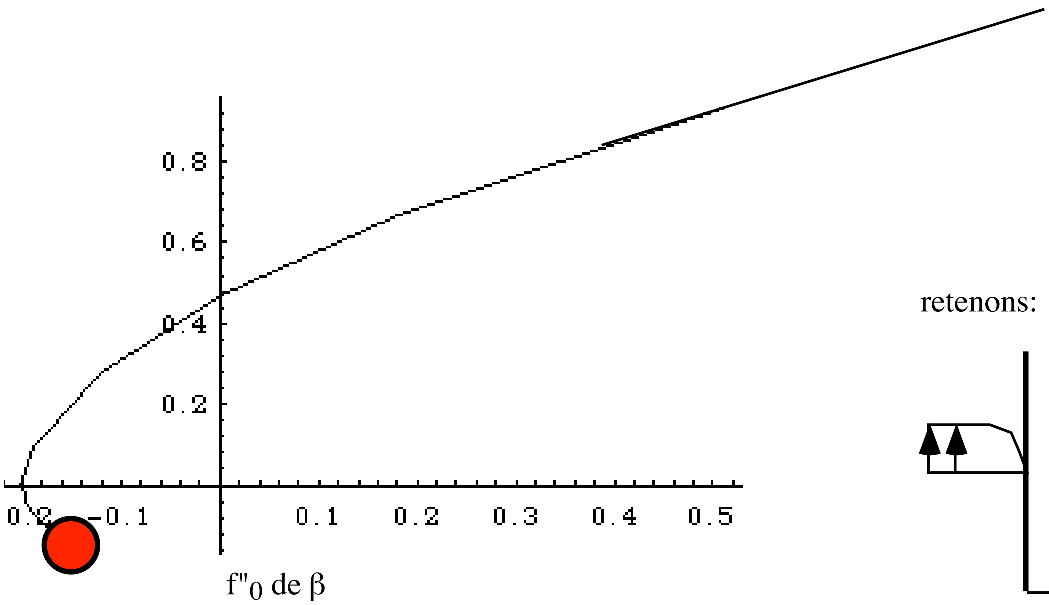
η en abscisse f'(η) en ordonnée.

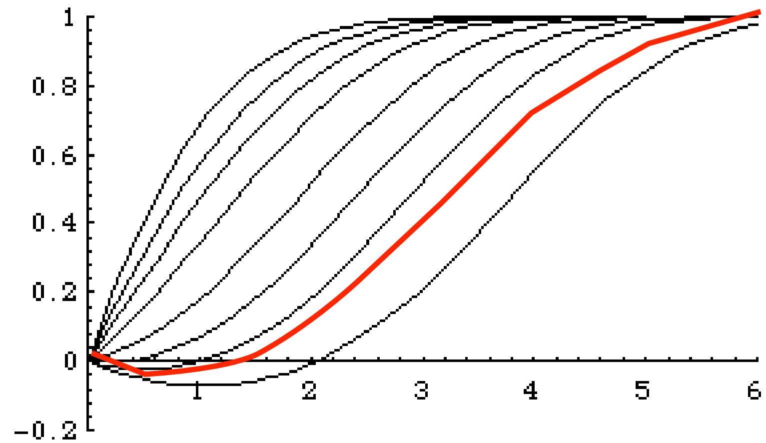
retenons:



need for a Closure

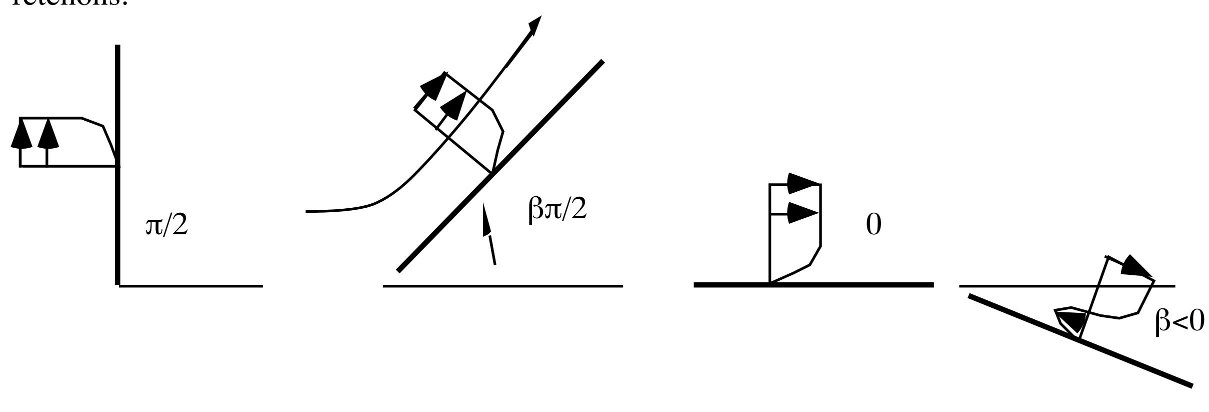
Falkner Skan



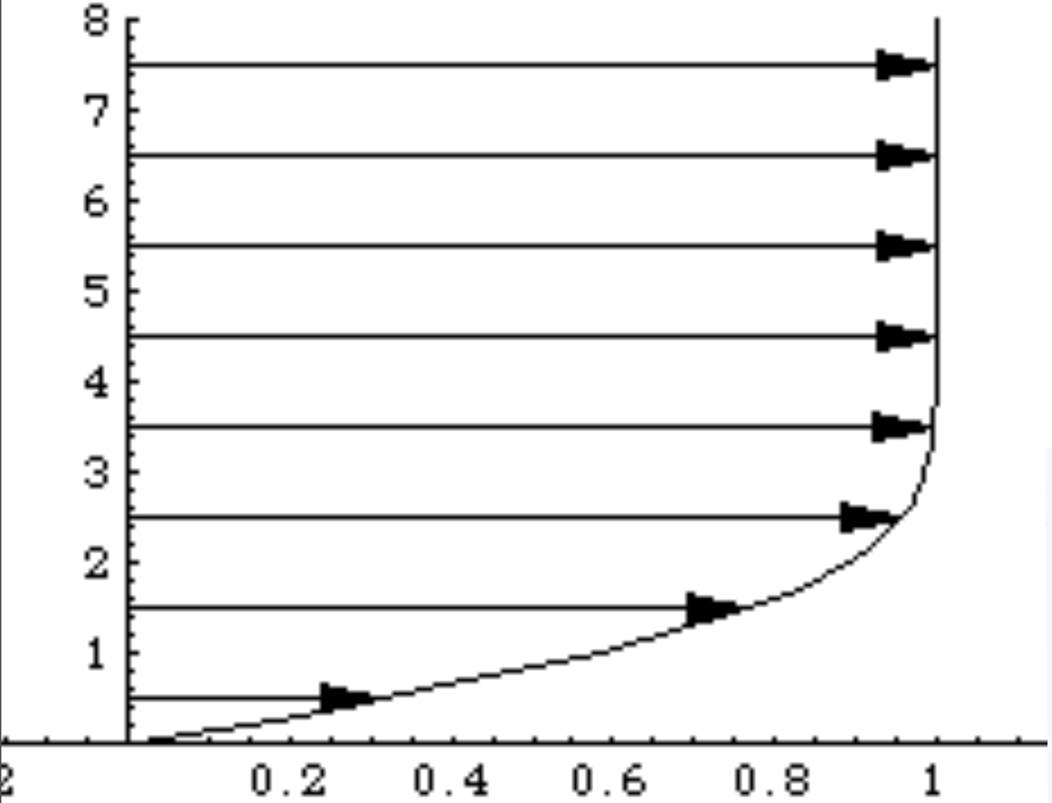


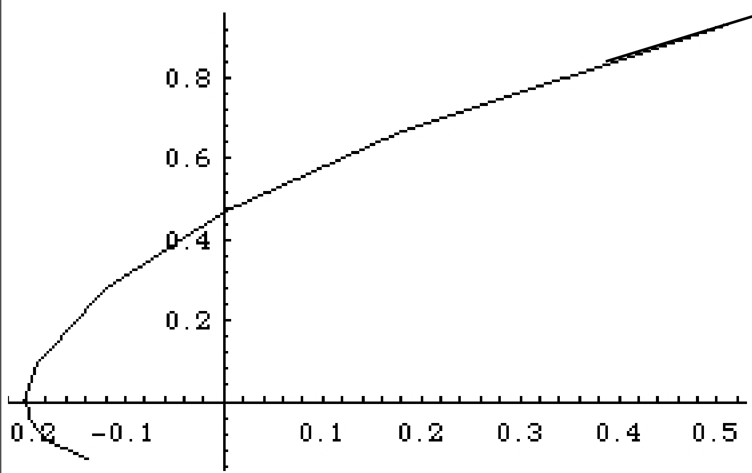
η en abscisse f'(η) en ordonnée.

retenons:

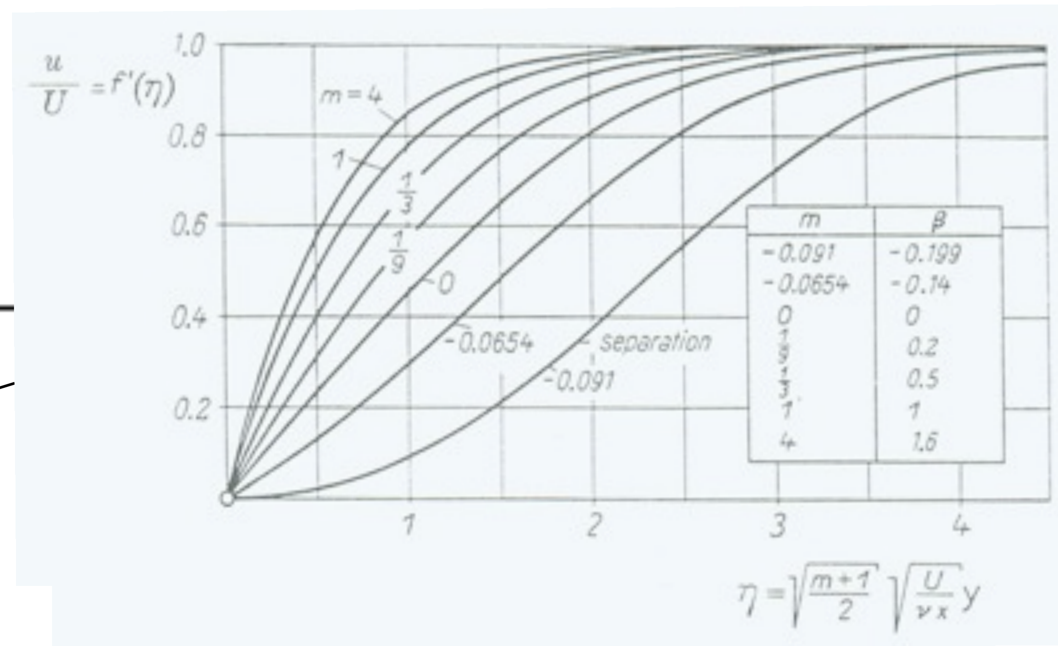


Falkner Skan

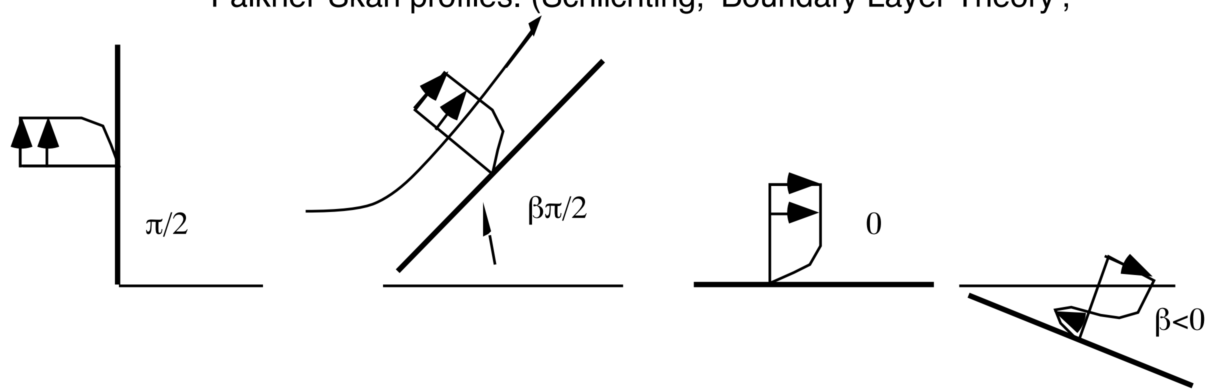




f"$_0$ de β

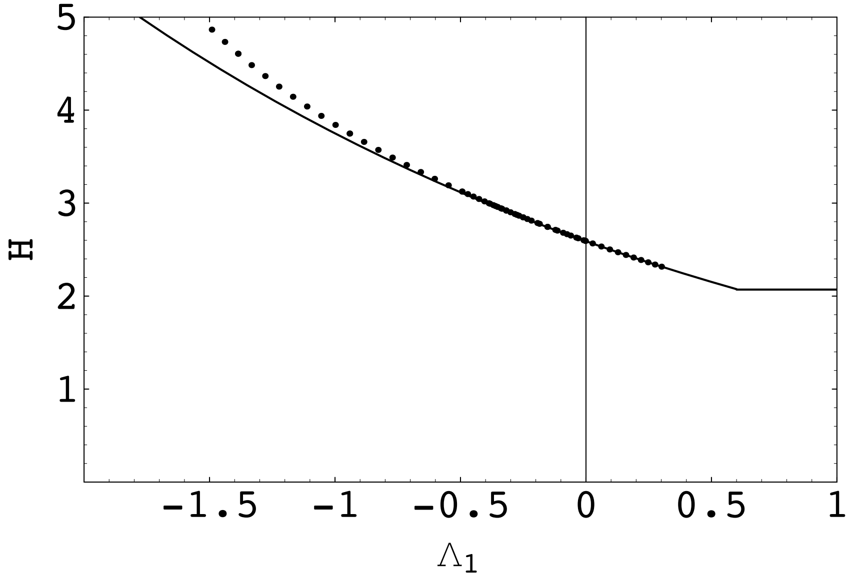


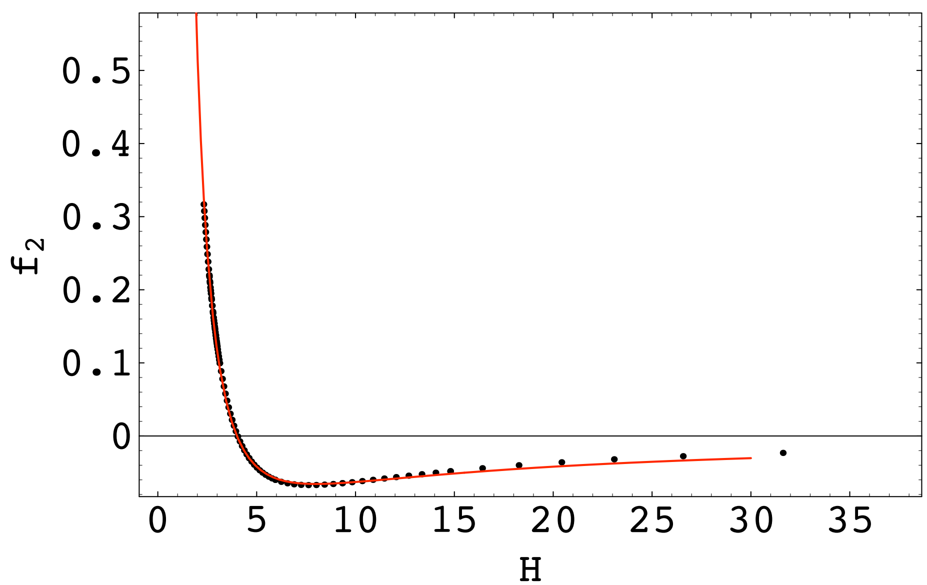
Falkner-Skan profiles. (Schlichting, 'Boundary Layer Theory',



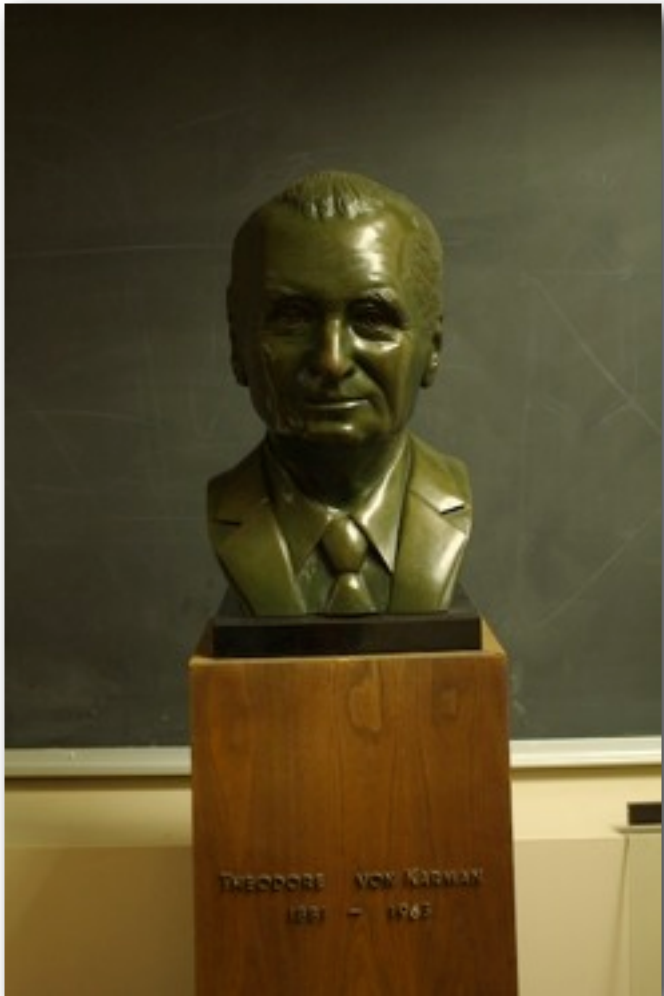
closure Falkner Skan

$$\Lambda_1 = \tilde{\delta}_1^2 \frac{d\bar{u}_e}{d\bar{x}}$$





$$H = \left\{ \begin{array}{ll} 2.5905 e^{-0.37098 \Lambda_1} & \text{if } \Lambda_1 < 0.6 \\ 2.074 & \text{if } \Lambda_1 > 0.6 \end{array} \right\}, \qquad f_2 = 1.05(-H^{-1} + 4H^{-2}).$$



Von Kármán equation integral relation

just one ODE

$$\frac{d}{d\bar{x}}\left(\frac{\tilde{\delta}_1}{H}\right) + \frac{\tilde{\delta}_1}{\bar{u}_e}\left(1 + \frac{2}{H}\right)\frac{d\bar{u}_e}{d\bar{x}} = \frac{f_2 H}{\tilde{\delta}_1 \bar{u}_e},$$

equation between:
the displacement thickness and the external velocity

$$\left\{ \begin{array}{l} \tilde{\delta}_1 = \int_0^\infty \left(1 - \frac{\tilde{u}}{\bar{u}_e}\right) d\tilde{y}, \\ \\ \bar{u}_e \end{array} \right.$$

$$\left\{ \begin{array}{l} \Lambda_1 = \tilde{\delta}_1^2 \frac{d\bar{u}_e}{d\bar{x}} \\ \\ H = \left\{ \begin{array}{ll} 2.5905 e^{-0.37098\Lambda_1} & \text{if } \Lambda_1 < 0.6 \\ 2.074 & \text{if } \Lambda_1 > 0.6 \end{array} \right\}, \qquad f_2 = 1.05(-H^{-1} + 4H^{-2}). \end{array} \right.$$

numerical resolution of Boundary Layer Equations

$$\begin{cases} u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} = -\dfrac{\partial p}{\partial x} \qquad + \dfrac{\partial^2 u}{\partial y^2}, \\ 0 = -\dfrac{\partial p}{\partial y}, \\ \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0. \end{cases}$$

numerical resolution of Boundary Layer Equations

$$u\frac{\partial u}{\partial x} + ... = \frac{\partial^2 u}{\partial y^2} + ...$$

«heat equation» finite differences

$$u(i-1,j)(\frac{u(i,j)-u(i-1,j)}{\Delta x}) + ... = \frac{u(i,j+1)-2u(i,j)-u(i,j-1)}{\Delta y^2}$$

PDE

numerical resolution of Boundary Layer Equations

$$\begin{cases} \dfrac{\partial \psi}{\partial y} = u, & \dfrac{\partial u}{\partial y} = G, \\ \dfrac{\partial G}{\partial y} = -\dfrac{\partial (W^2/2)}{\partial x} + u\dfrac{\partial u}{\partial x} - G\dfrac{\partial \psi}{\partial x}, & \dfrac{\partial W}{\partial y} = 0. \\ \end{cases}$$

centered derivatives, linearisation, Iteration

$$\frac{(\frac{u(i,j)+u(i-1,j)}{2} + \frac{u(i,j-1)+u(i,j-1)}{2})}{2} \frac{(\frac{u(i,j)-u(i-1,j)}{\Delta x} + \frac{u(i,j-1)-u(i-1,j-1)}{\Delta x})}{2},$$

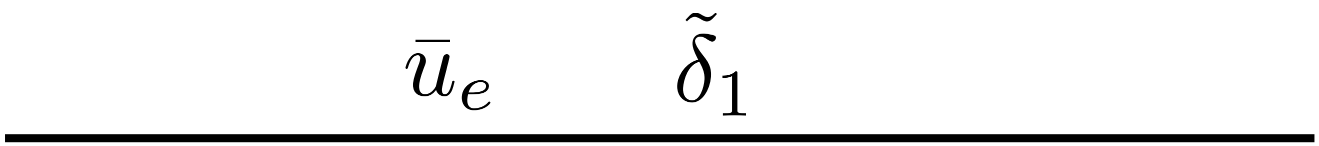
numerical resolution of Boundary Layer Equations

Finite Elements

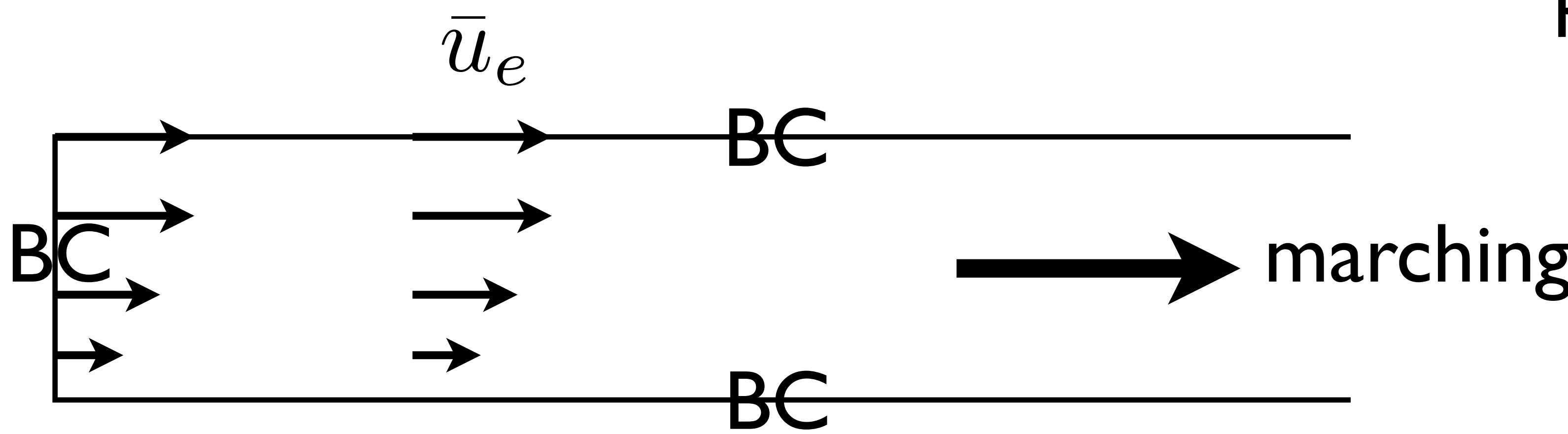
$$\begin{cases} \int_\Omega (u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y})\zeta + \frac{1}{Re}\int_\Omega \frac{\partial u}{\partial y}\frac{\partial \zeta}{\partial y} + \int_\Omega \lambda(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y})(\frac{\partial \zeta}{\partial x} + \frac{\partial \xi}{\partial y}) \\ \qquad - \int_\Omega p(\frac{\partial \zeta}{\partial x} + \frac{\partial \xi}{\partial y}) + \int_\Omega q(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) = 0, \end{cases}$$

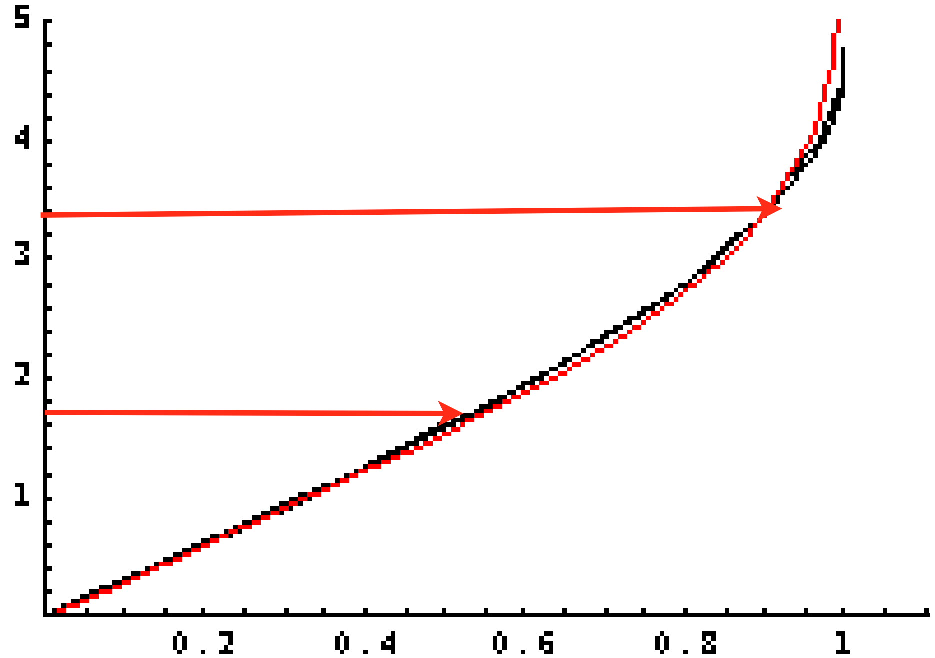
=> Several Methods to compute the Boundary Layer

ODE



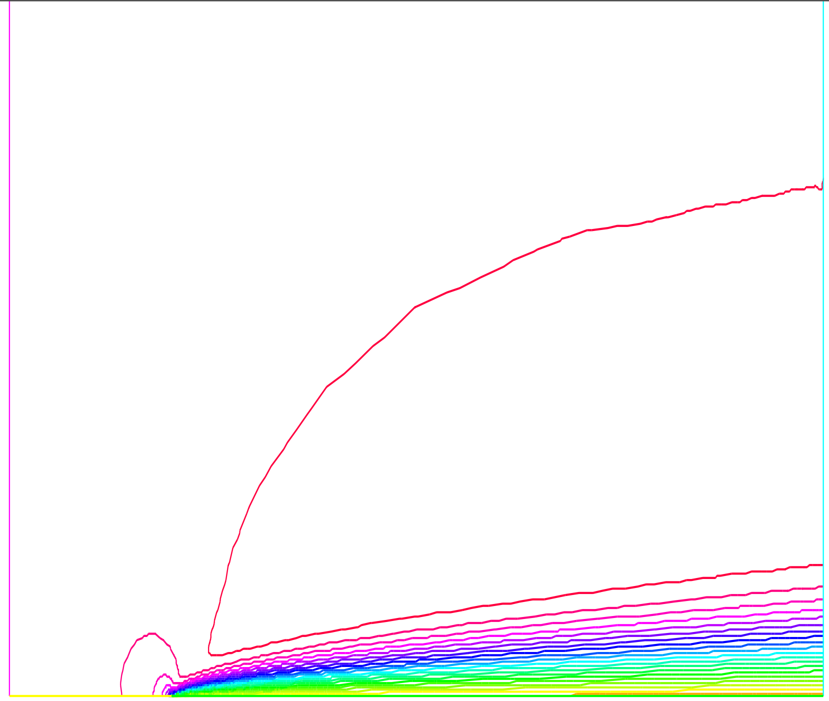
PDE

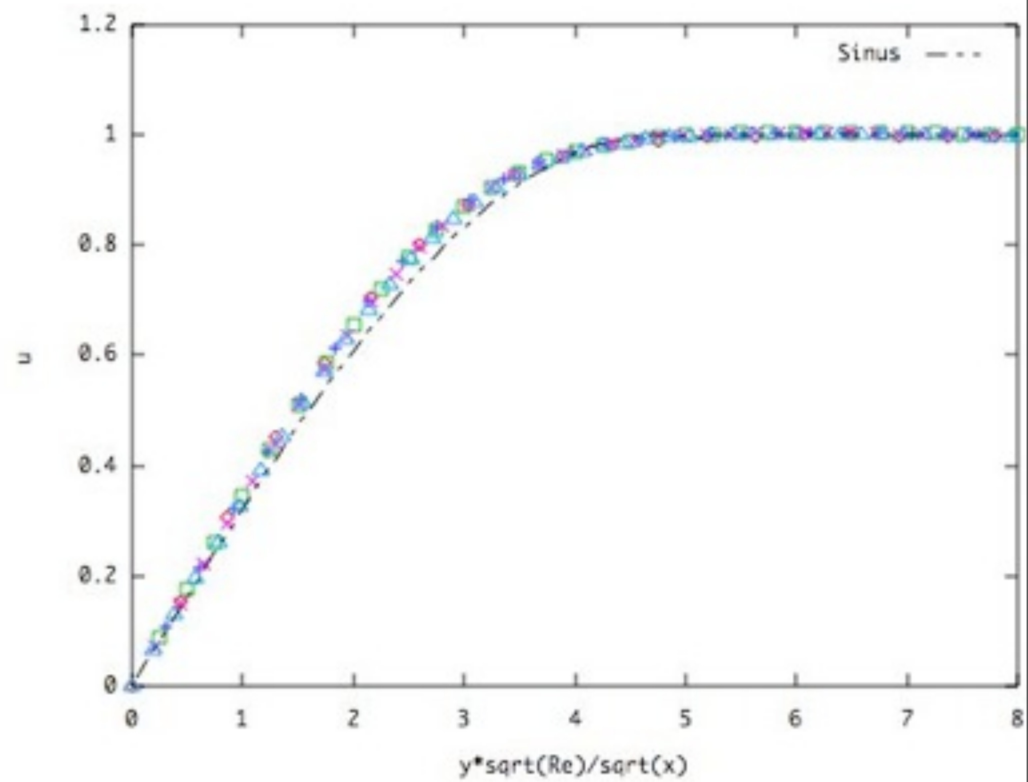




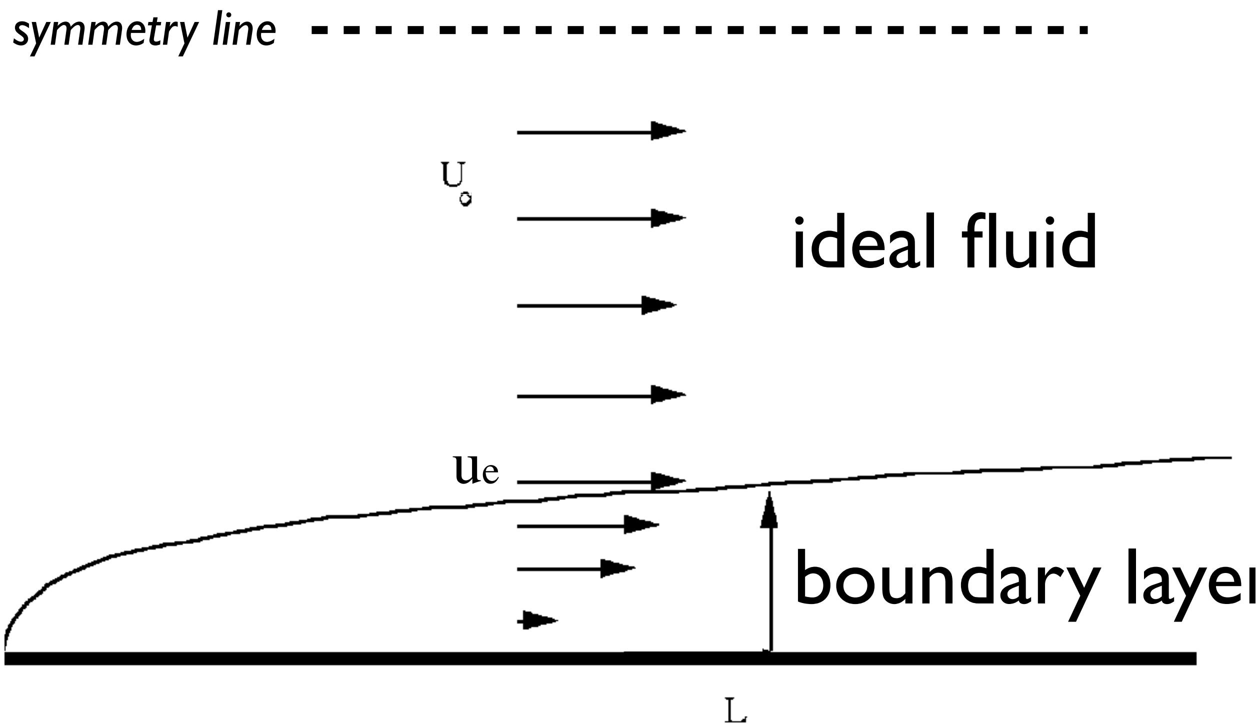
superposition

Bls, P4, Sin





2nd order BLT



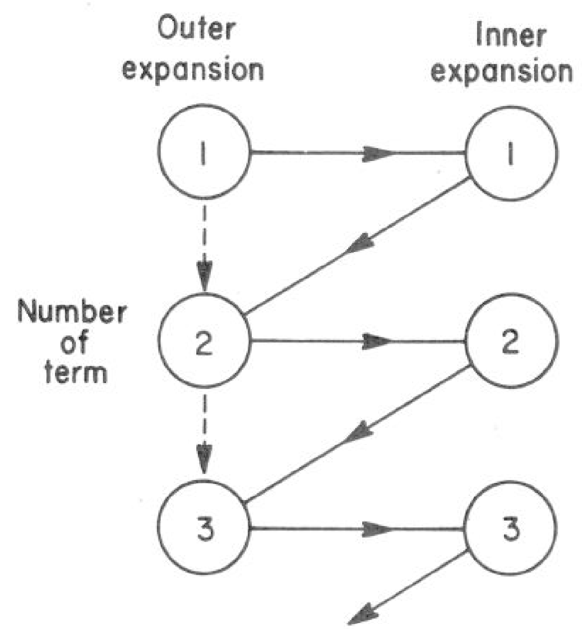
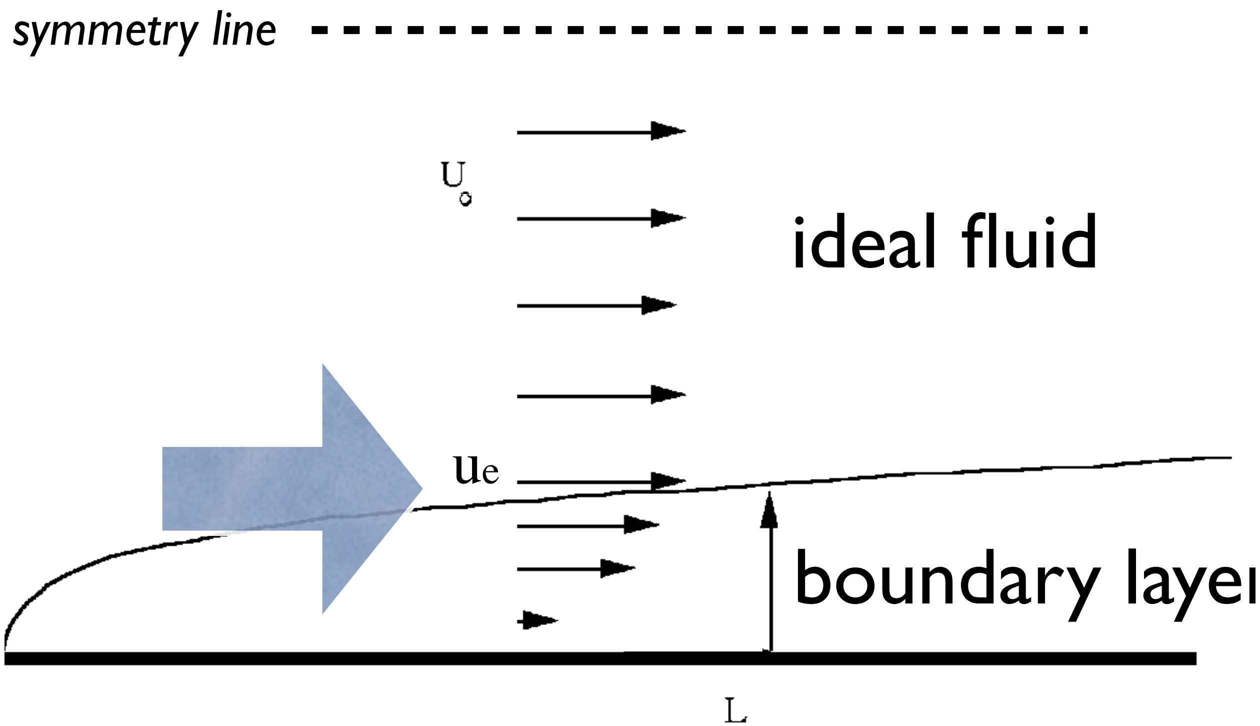


Fig. 5.6. Matching order for inner and outer expansions.

2nd order BLT



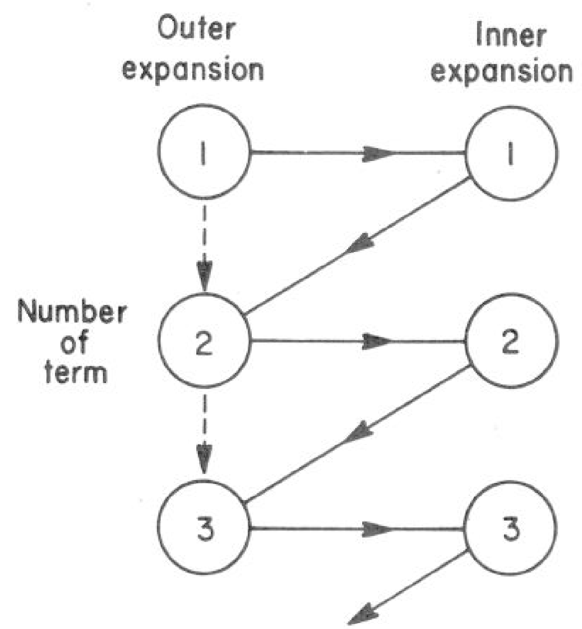
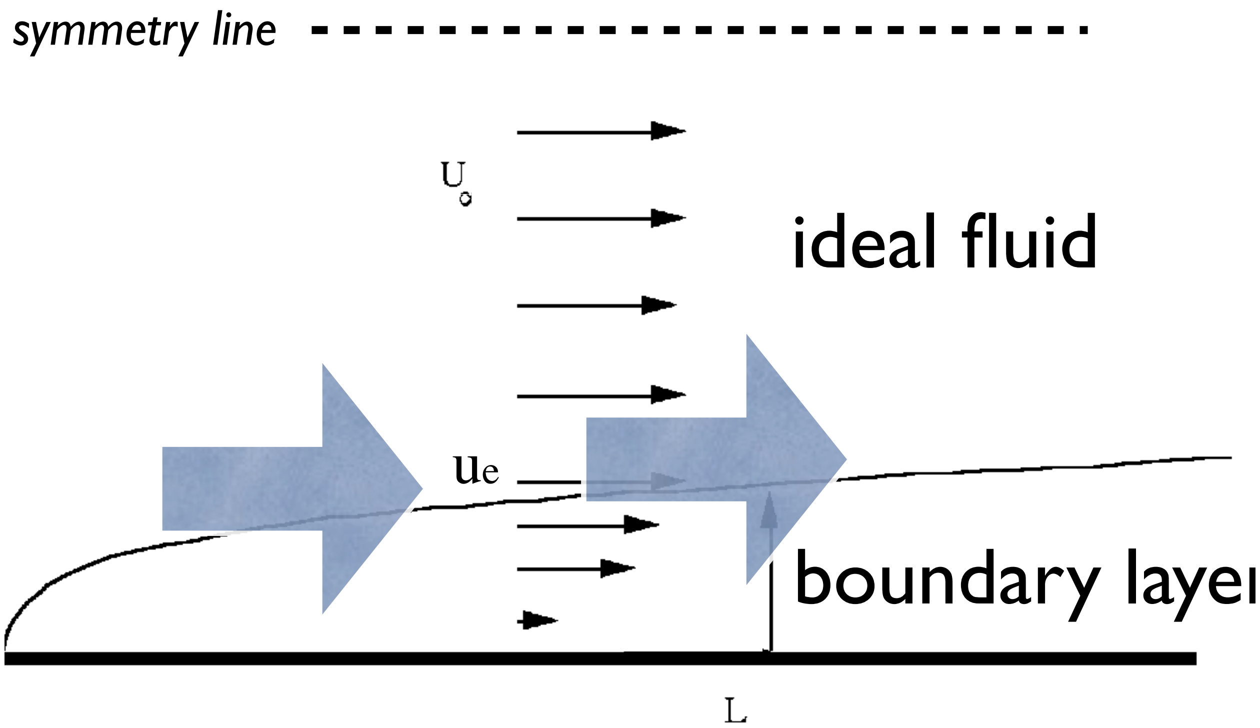


Fig. 5.6. Matching order for inner and outer expansions.

2nd order BLT



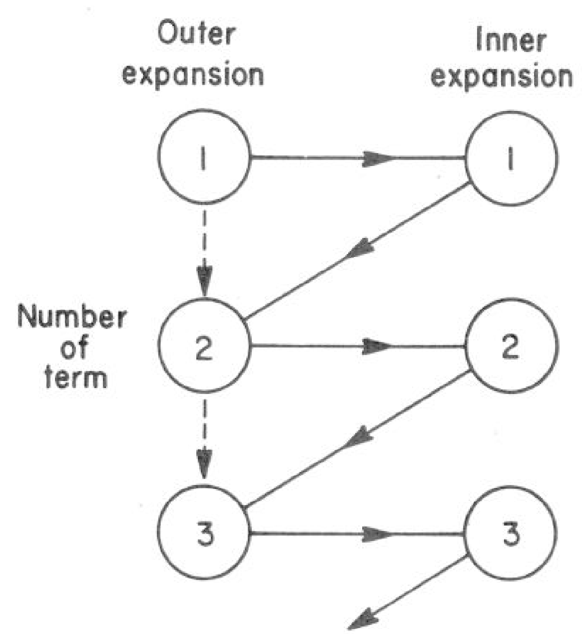
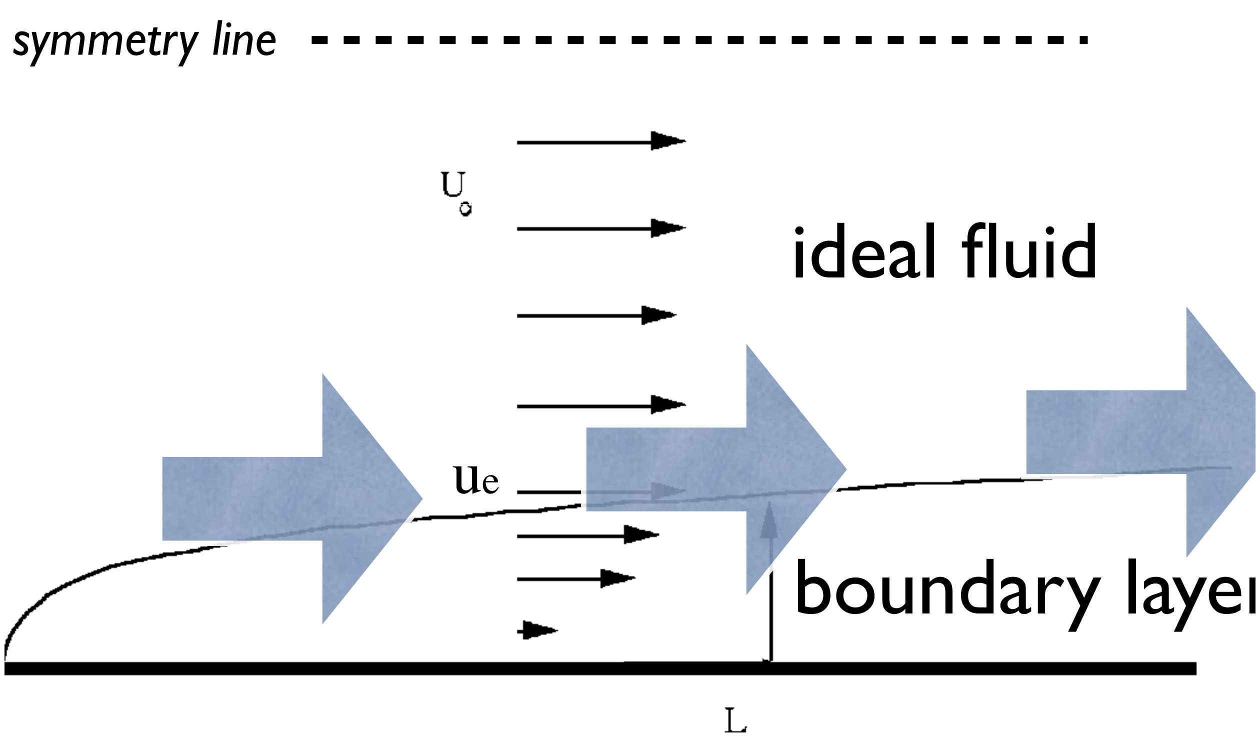


Fig. 5.6. Matching order for inner and outer expansions.

2nd order BLT



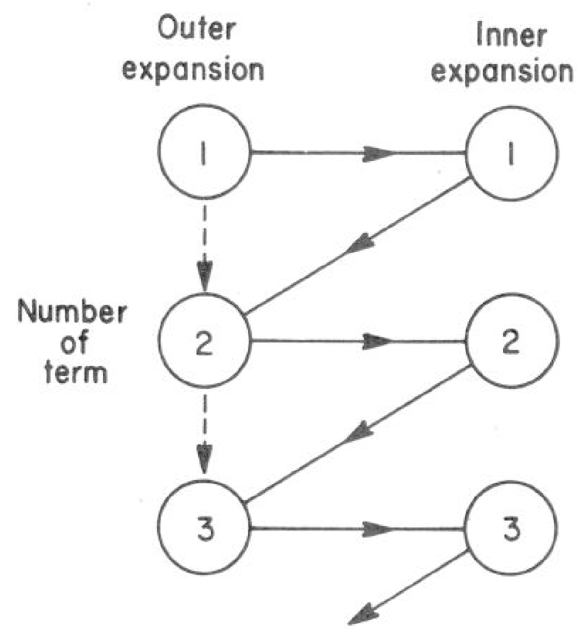
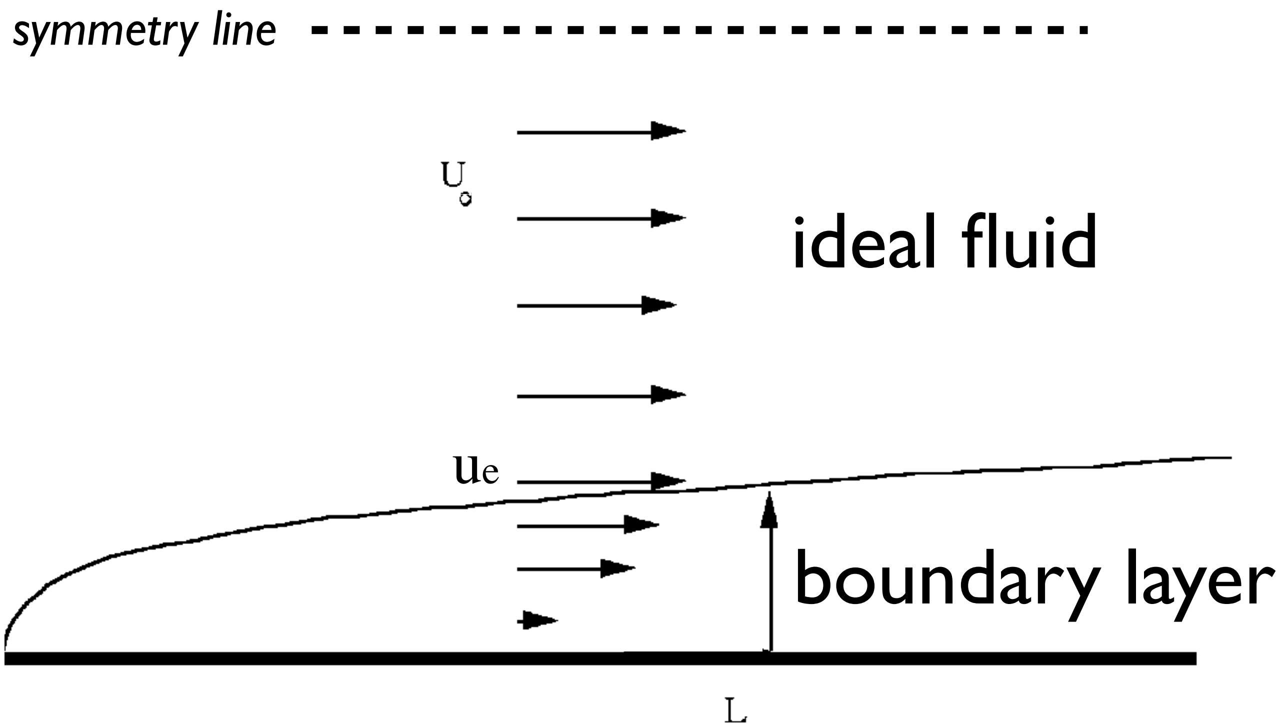


Fig. 5.6. Matching order for inner and outer expansions.

2nd order BLT

Perturbation of the Ideal fluid at the next order



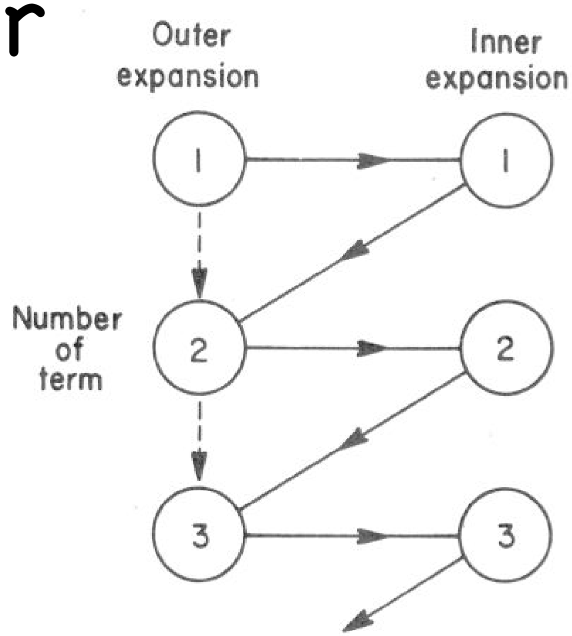
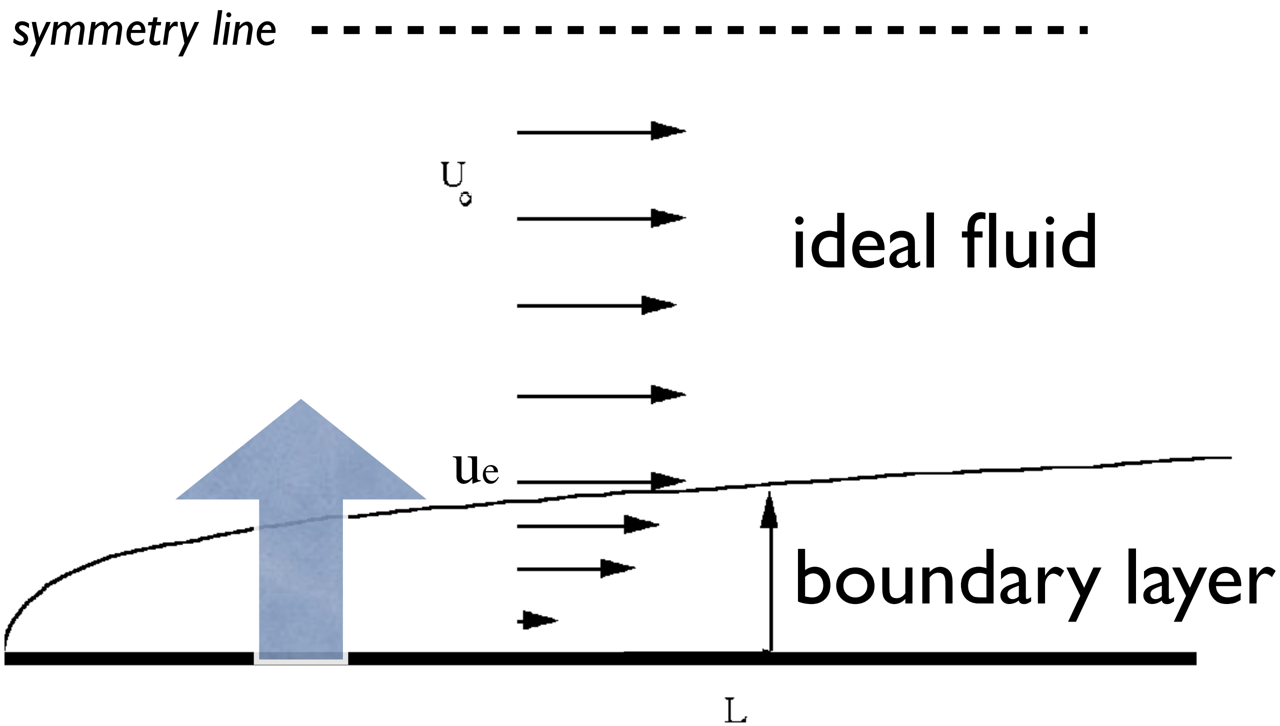


Fig. 5.6. Matching order for inner and outer expansions.

2nd order BLT

Perturbation of the Ideal fluid at the next order



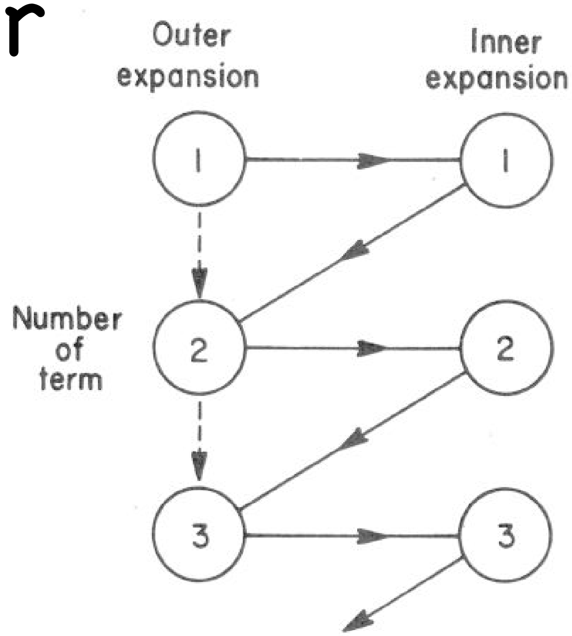
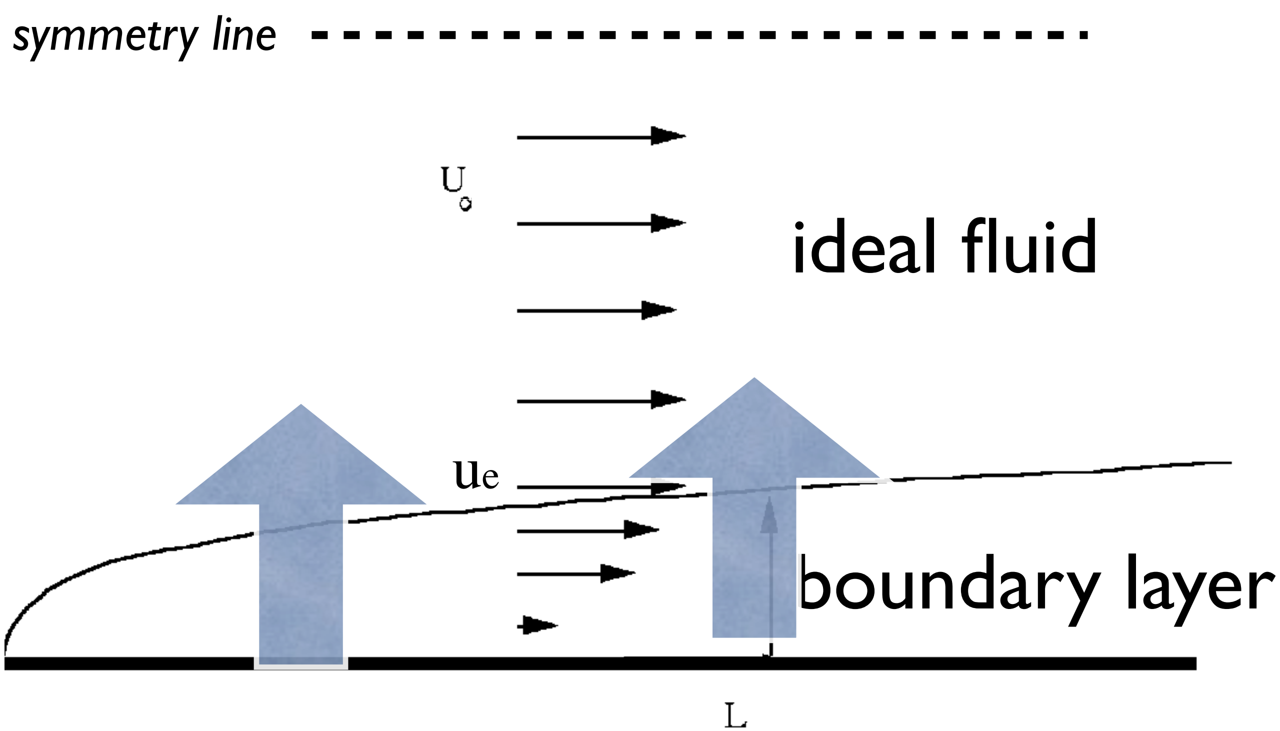


Fig. 5.6. Matching order for inner and outer expansions.

2nd order BLT

Perturbation of the Ideal fluid at the next order



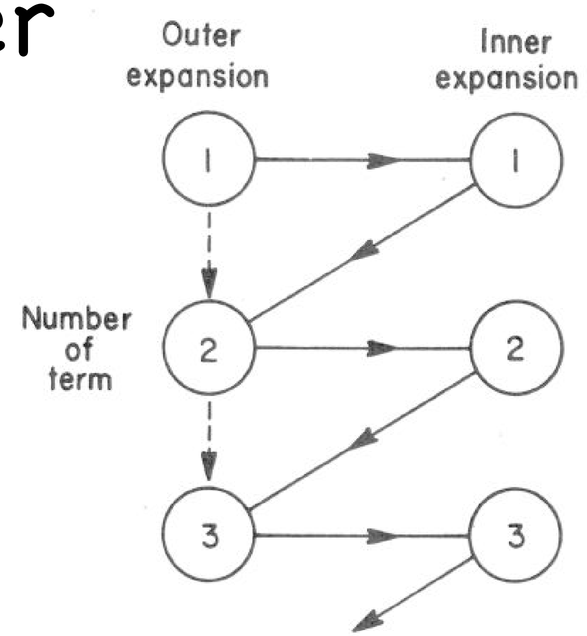
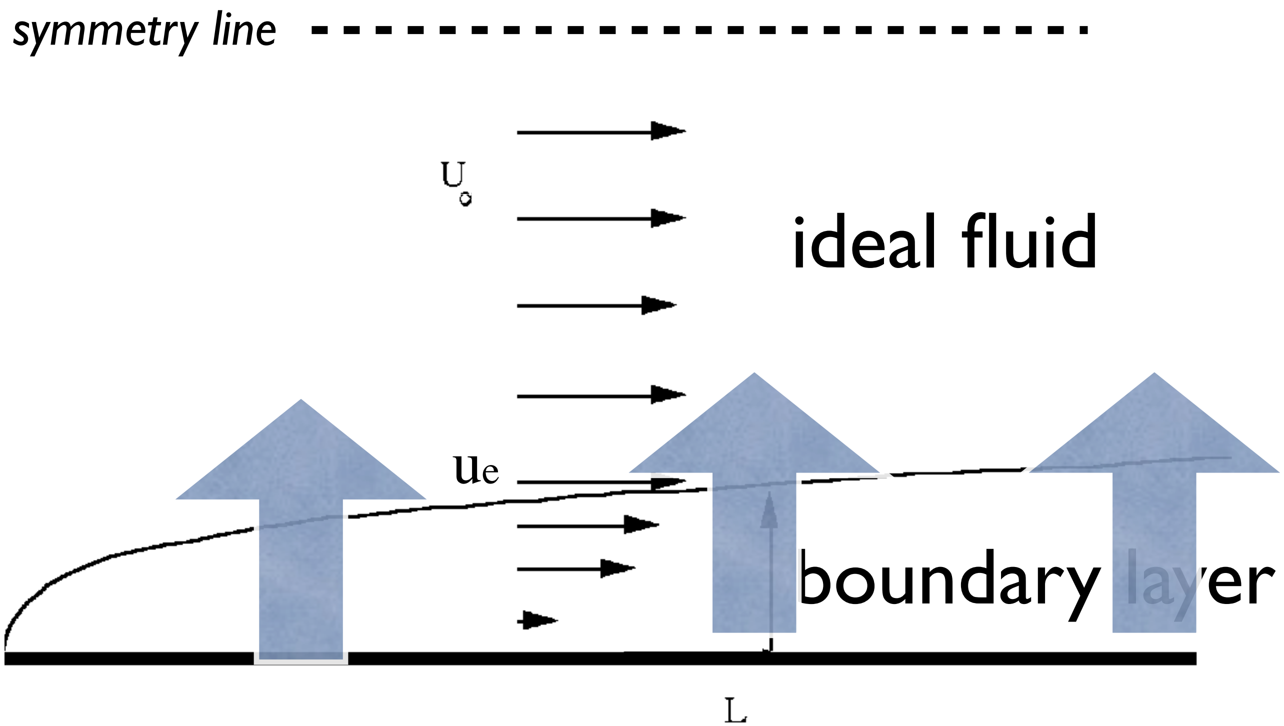


Fig. 5.6. Matching order for inner and outer expansions.

2nd order BLT

Perturbation of the Ideal fluid at the next order



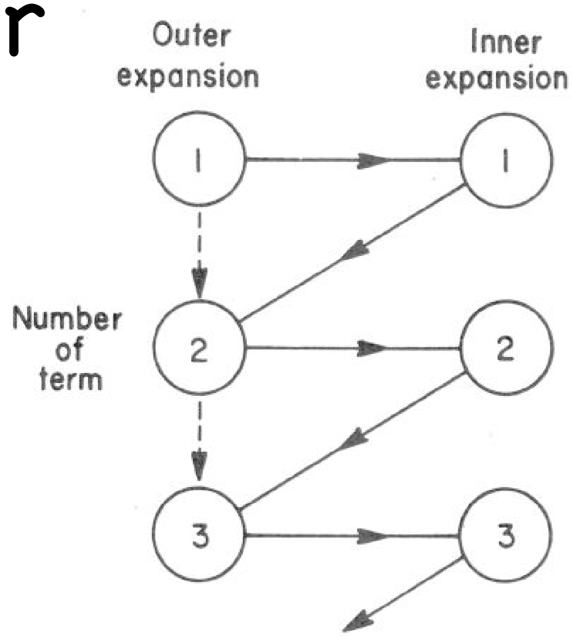
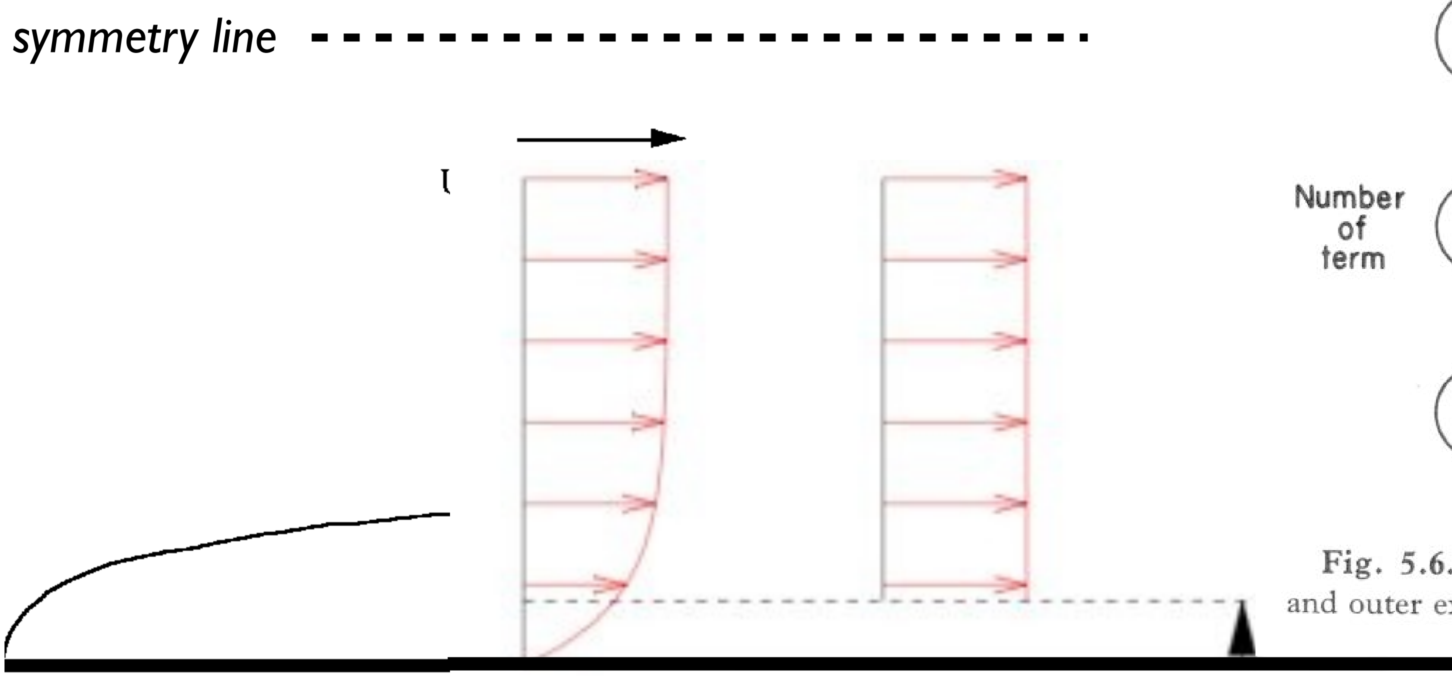


Fig. 5.6. Matching order for inner and outer expansions.

2nd order BLT

Perturbation of the Ideal fluid at the next order



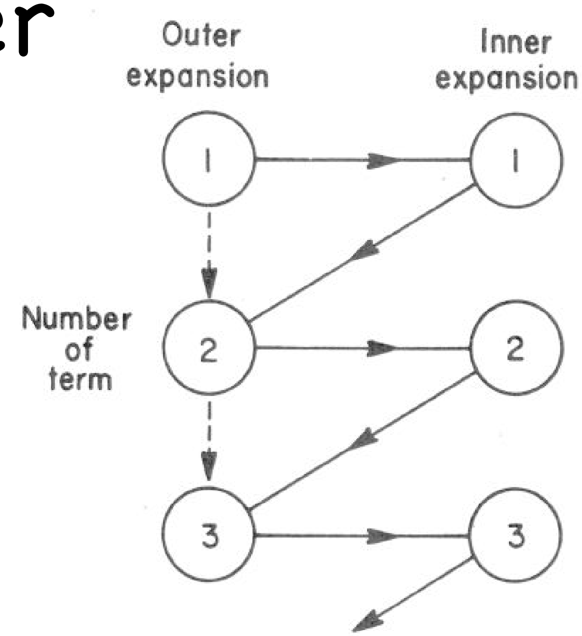
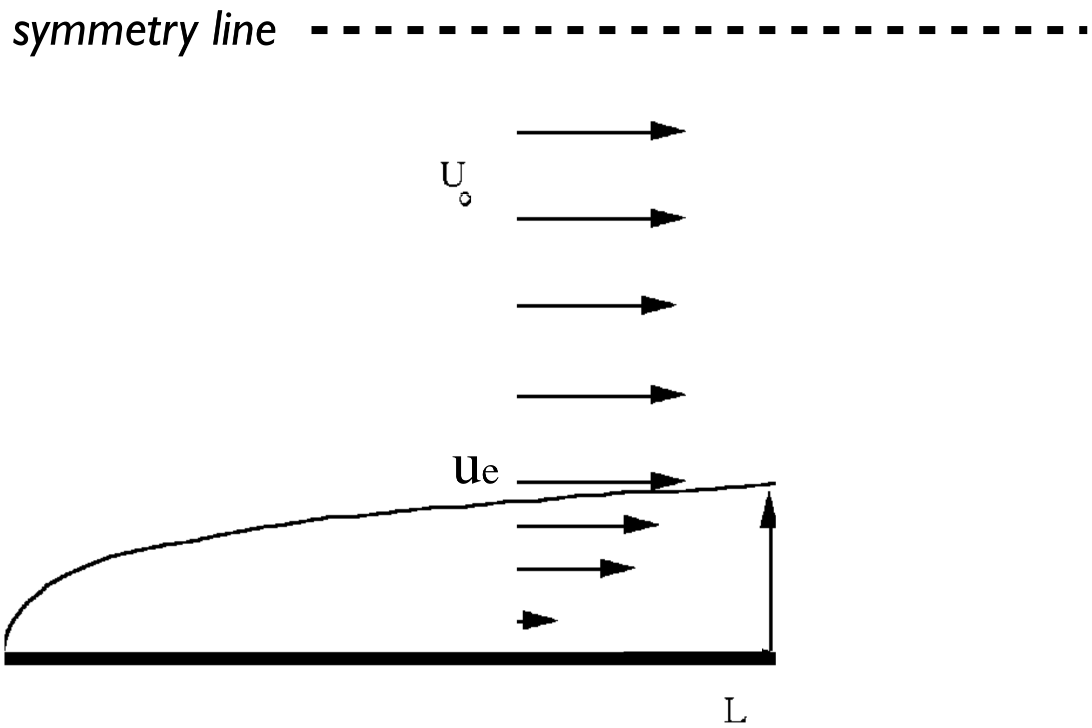


Fig. 5.6. Matching order for inner and outer expansions.

2nd order BLT



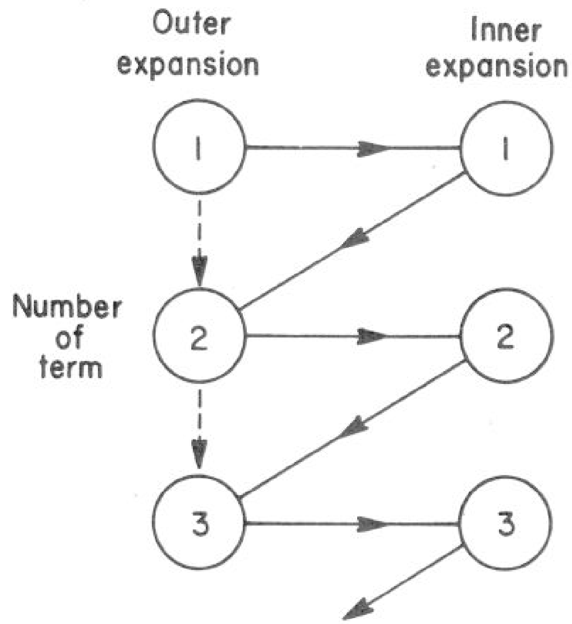
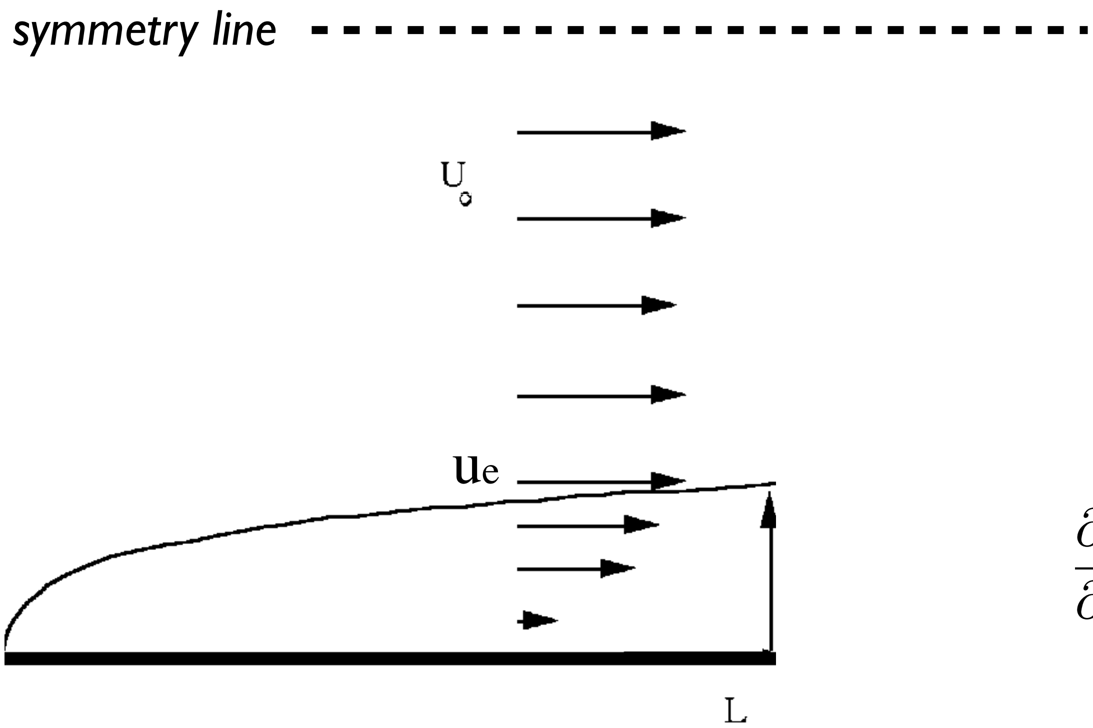


Fig. 5.6. Matching order for inner and outer expansions.

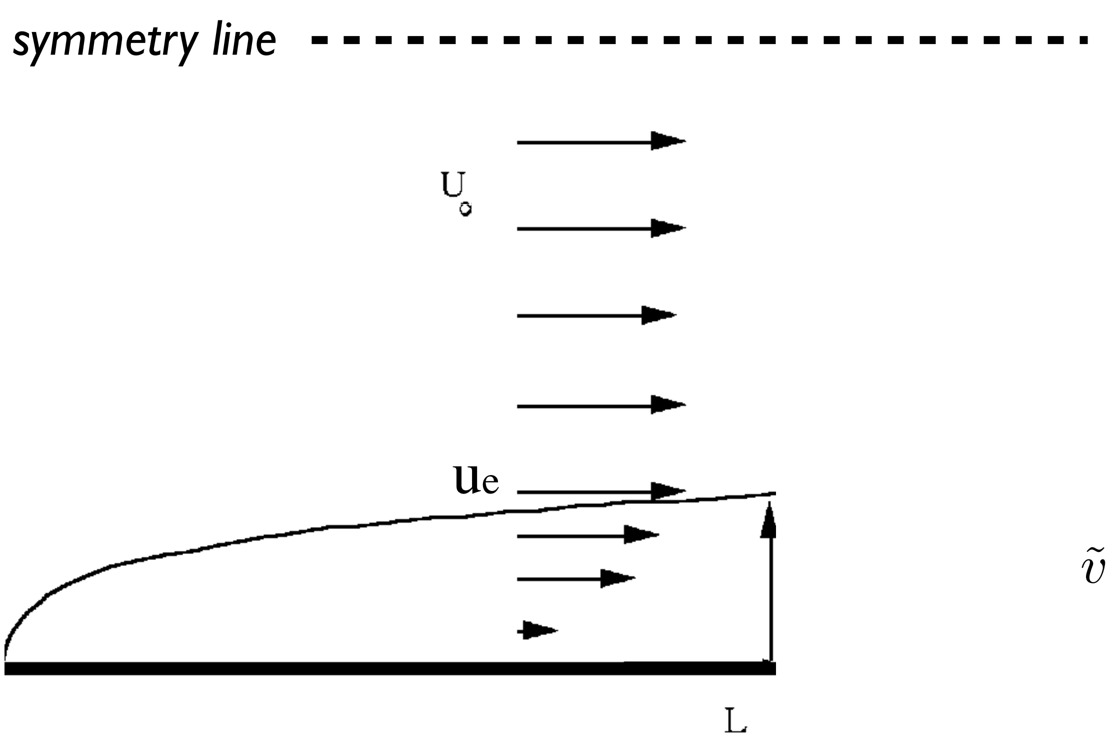
2nd order BLT



$$\frac{\partial \tilde{v}}{\partial \tilde{y}} = (-\frac{\partial \tilde{u}}{\partial x} + \frac{\partial \bar{u}_e}{\partial \bar{x}}) - \frac{\partial \bar{u}_e}{\partial \bar{x}},$$

effect of the displacement thickness

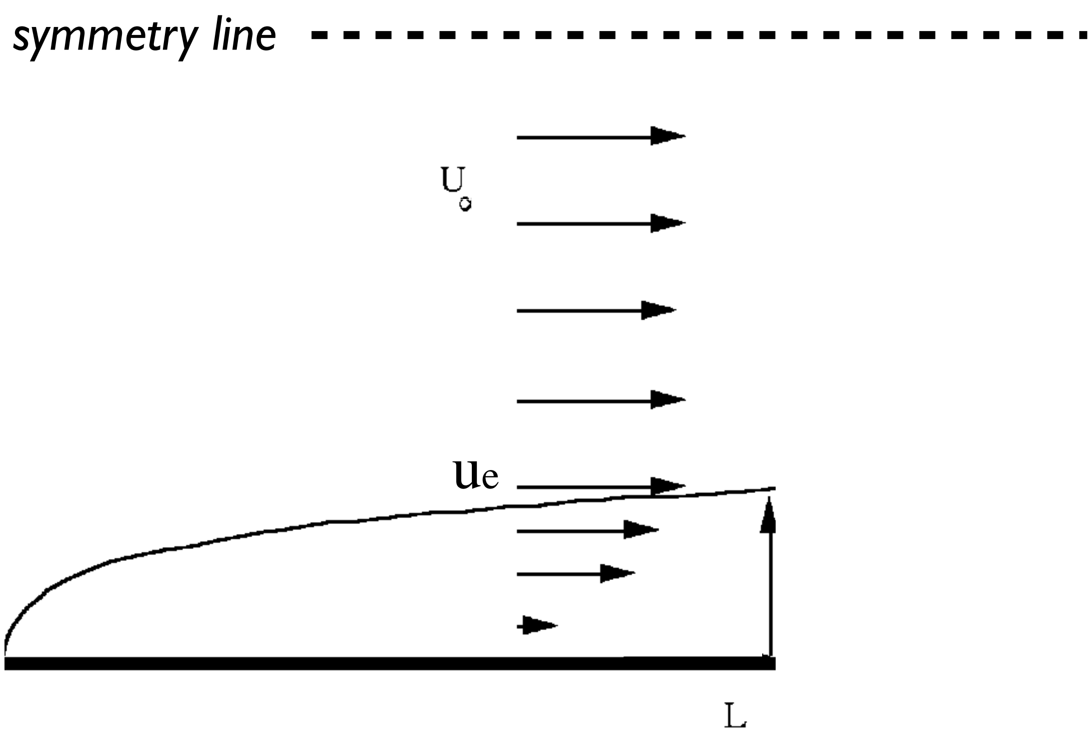
2nd order BLT



$$\tilde{v}(\tilde{y}) - \tilde{v}(0) = -\frac{\partial}{\partial \bar{x}} \int_0^{\tilde{y}} (\tilde{u} - \bar{u}_e) d\tilde{y} - \tilde{y} \frac{\partial \bar{u}_e}{\partial \bar{x}}$$

effect of the displacement thickness

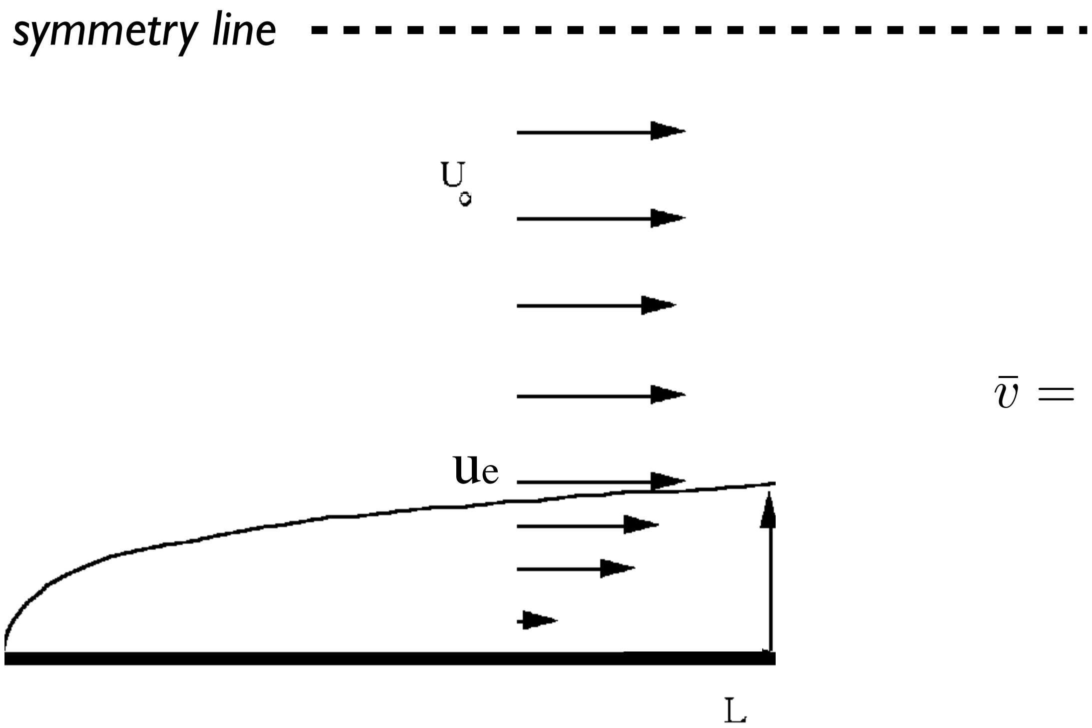
2nd order BLT



$$\tilde{v}(\tilde{y}) \simeq \frac{\partial}{\partial \bar{x}}(\bar{u}_e \tilde{\delta}_1) - \tilde{y}\frac{\partial \bar{u}_e}{\partial \bar{x}}$$

effect of the displacement thickness

2nd order BLT



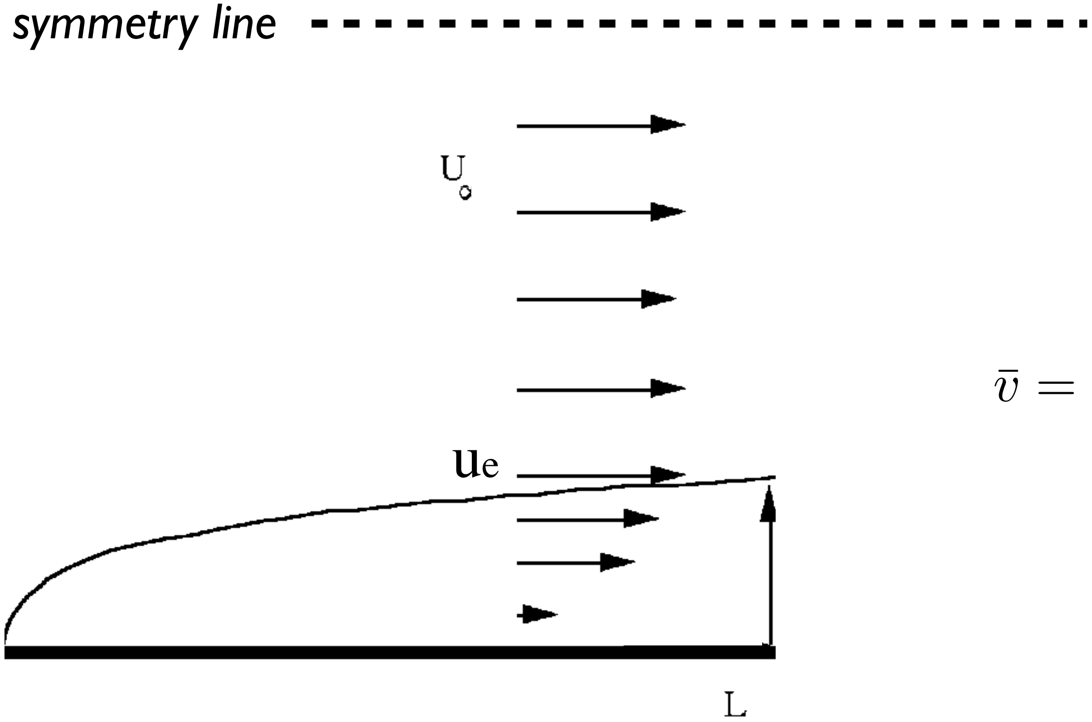
$$\bar{v} = \bar{v}(\bar{x}, 0) + \bar{y}\frac{\partial \bar{v}}{\partial \bar{y}} + ... = \bar{v}(\bar{x}, 0) - \bar{y}\frac{\partial \bar{u}_e}{\partial \bar{x}} + ...$$

$$\tilde{v}(\tilde{y}) \simeq \frac{\partial}{\partial \bar{x}}(\bar{u}_e \tilde{\delta}_1) - \tilde{y}\frac{\partial \bar{u}_e}{\partial \bar{x}}$$

effect of the displacement thickness

2nd order BLT

symmetry line



$$\bar{v} = \bar{v}(\bar{x},0) + \bar{y}\frac{\partial \bar{v}}{\partial \bar{y}} + ... = \bar{v}(\bar{x},0) - \bar{y}\frac{\partial \bar{u}_e}{\partial \bar{x}} + ...$$

$$\tilde{v}(\tilde{y}) \simeq \frac{\partial}{\partial \bar{x}}(\bar{u}_e\tilde{\delta}_1) - \tilde{y}\frac{\partial \bar{u}_e}{\partial \bar{x}}$$

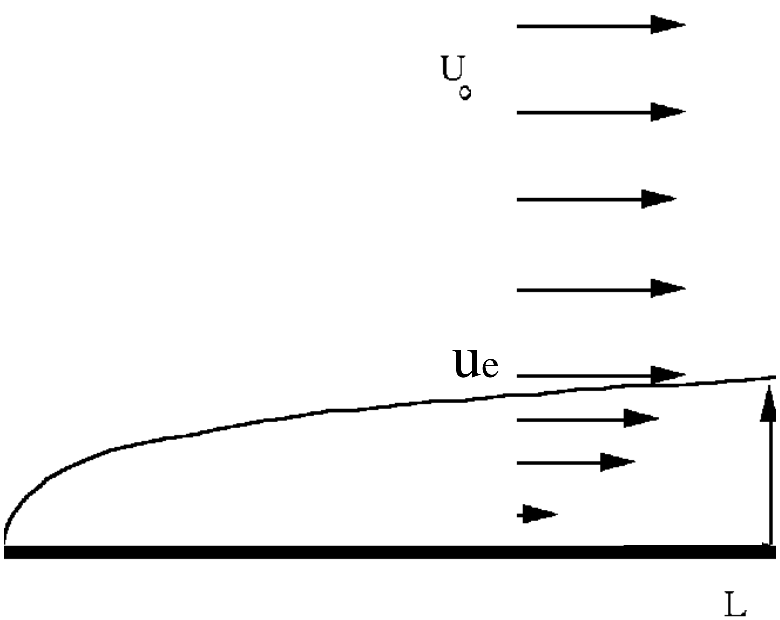
$$\bar{v}(\bar{x},0) = Re^{-1/2}\frac{\partial}{\partial \bar{x}}(\bar{u}_e\tilde{\delta}_1)$$

effect of the displacement thickness

2nd order BLT

symmetry line

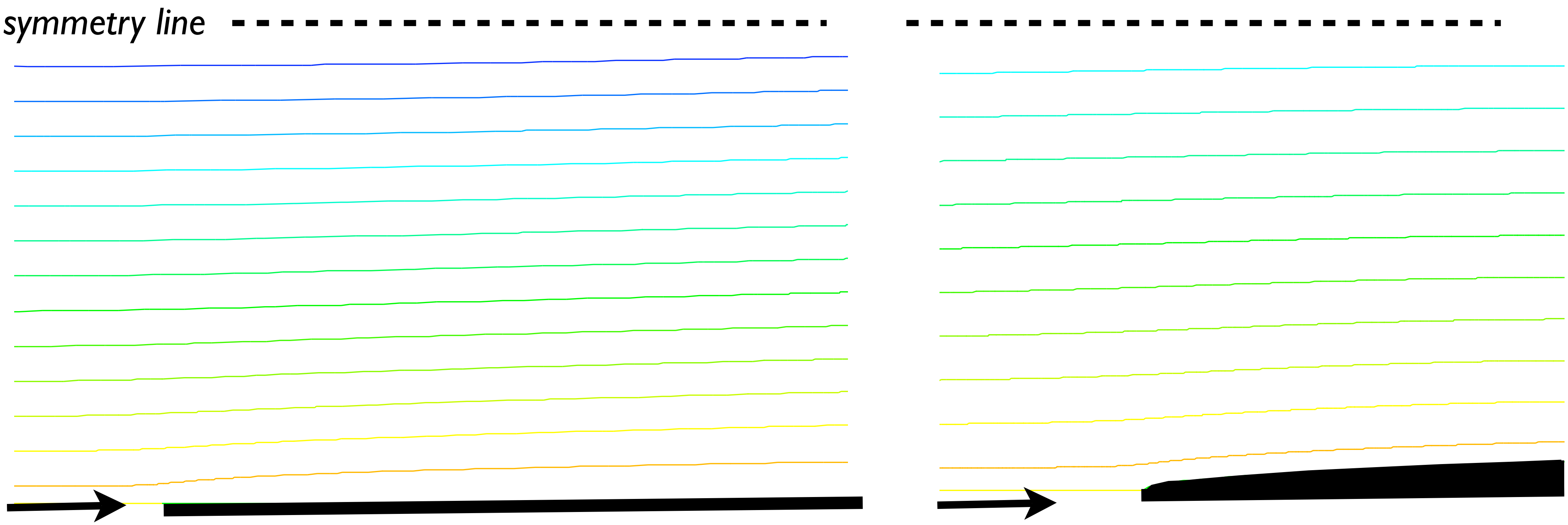
$$\bar{u} = \bar{u}_1 + Re^{-1/2}\bar{u}_2, \quad \bar{v} = \bar{v}_1 + Re^{-1/2}\bar{v}_2 \; \bar{p} = \bar{p}_1 + Re^{-1/2}\bar{p}_2 \dots.$$



$$\bar{v}(\bar{x}, 0) = Re^{-1/2}\frac{\partial}{\partial \bar{x}}(\bar{u}_e \tilde{\delta}_1)$$

effect of the displacement thickness

2nd order BLT



effect of the displacement thickness

a problem: Separation

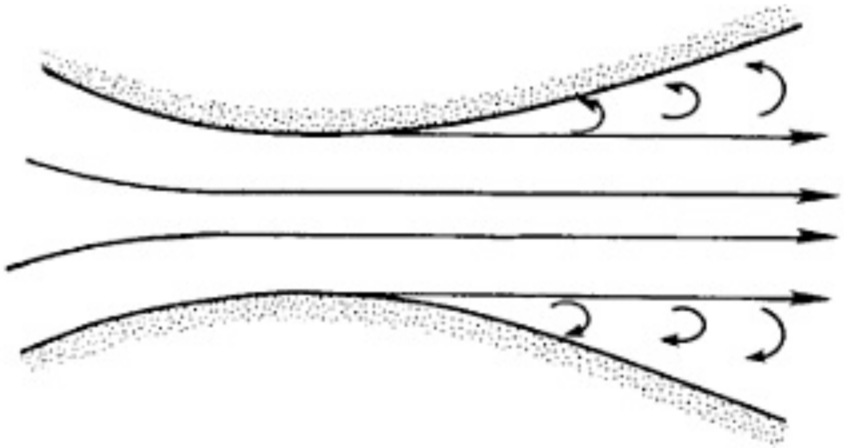
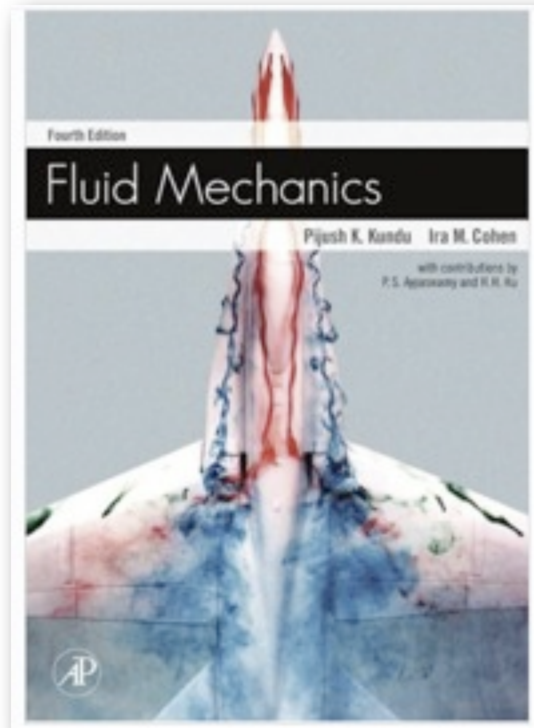


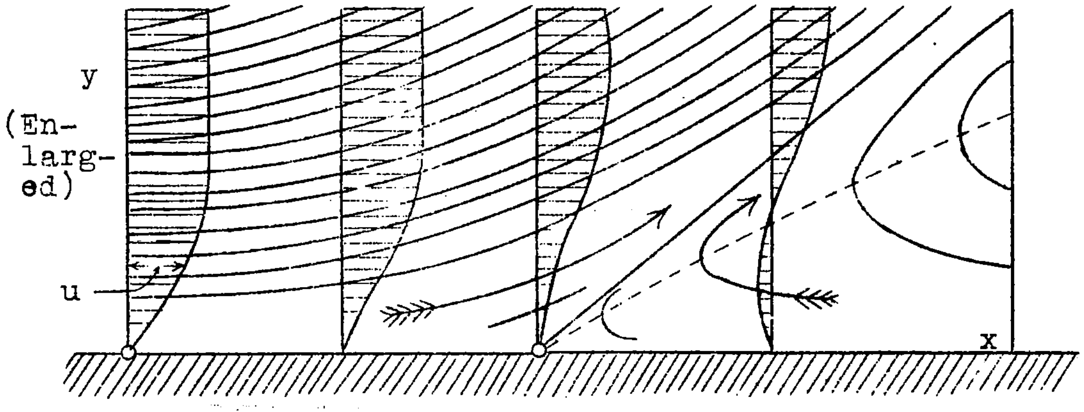
Figure 10.16 Separation of flow in a highly divergent channel.

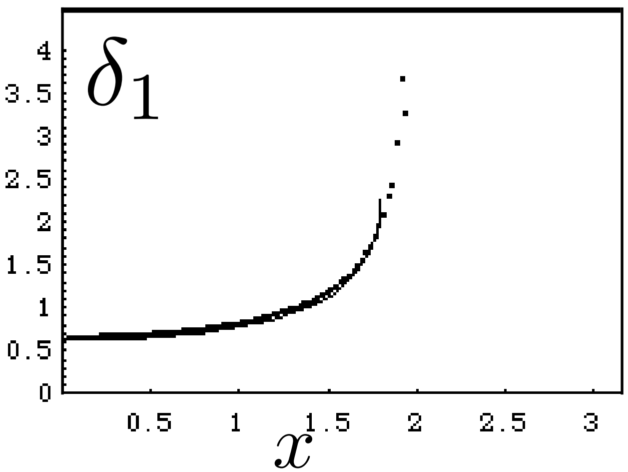
gradient is favorable and the flow adheres to the wall. Downstream of the throat a large enough adverse pressure gradient can cause separation.

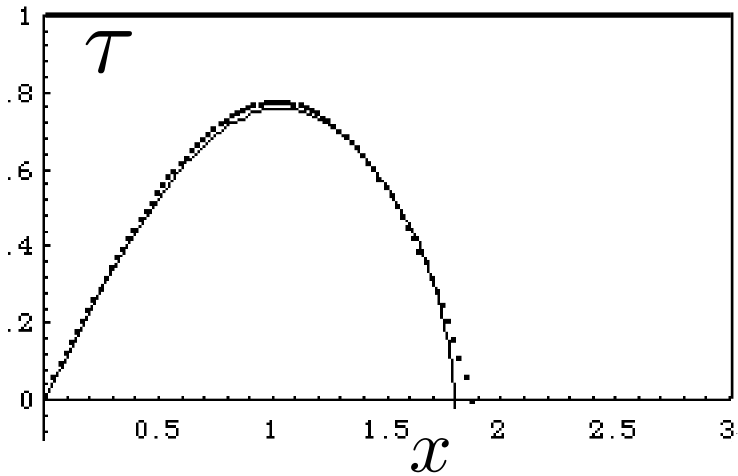
The boundary layer equations are valid only as far downstream as the point of separation. Beyond it the boundary layer becomes so thick that the basic underlying assumptions become invalid. Moreover, the parabolic character of the boundary layer equations requires that a numerical integration is possible only in the direction of advection (along which information is propagated), which is *upstream* within the reversed flow region. A forward (downstream) integration of the boundary layer equations therefore breaks down after the separation point. Last, we can no longer apply potential theory to find the pressure distribution in the separated region, as the effective boundary of the irrotational flow is no longer the solid surface but some unknown shape encompassing part of the body plus the separated region.



a problem: Separation

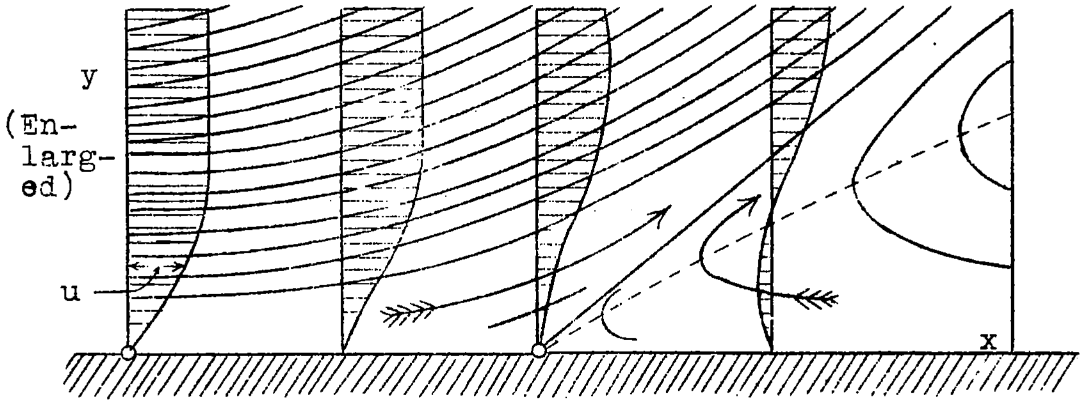


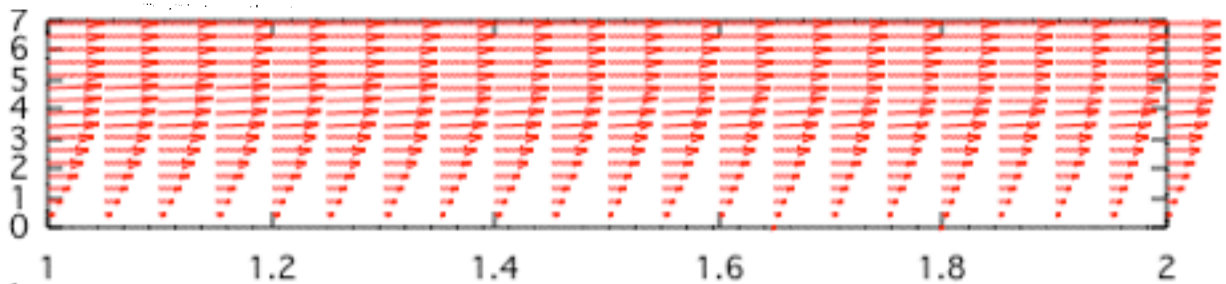


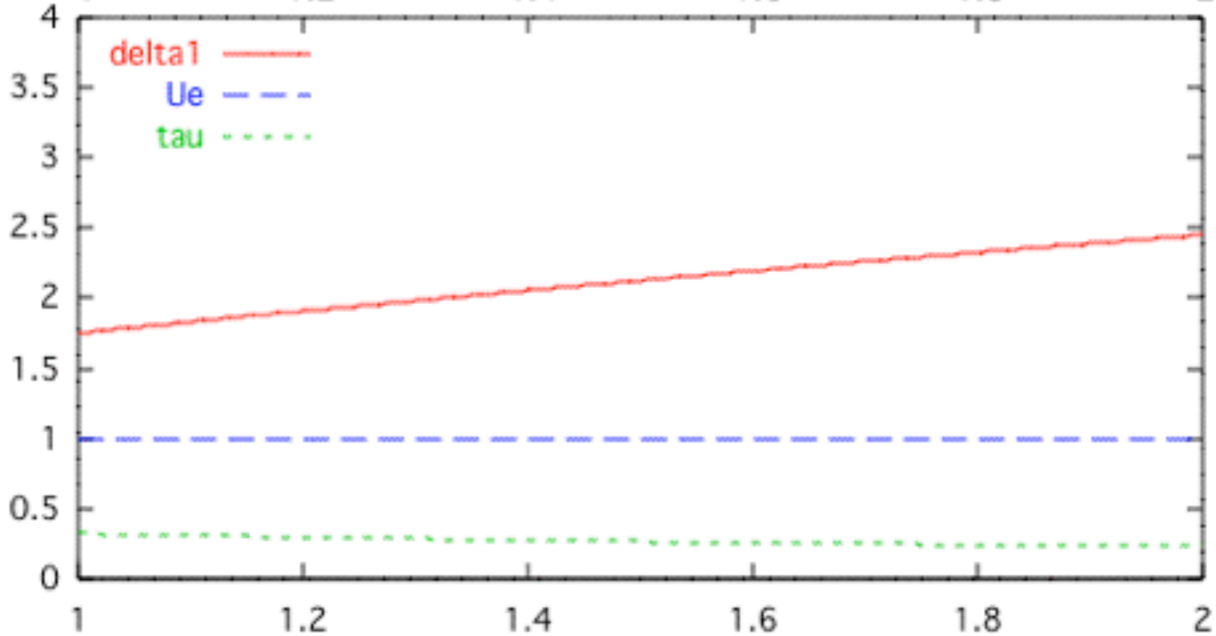


$$\frac{\partial u}{\partial y} \sim \sqrt{x_s - x}$$

a problem: Separation



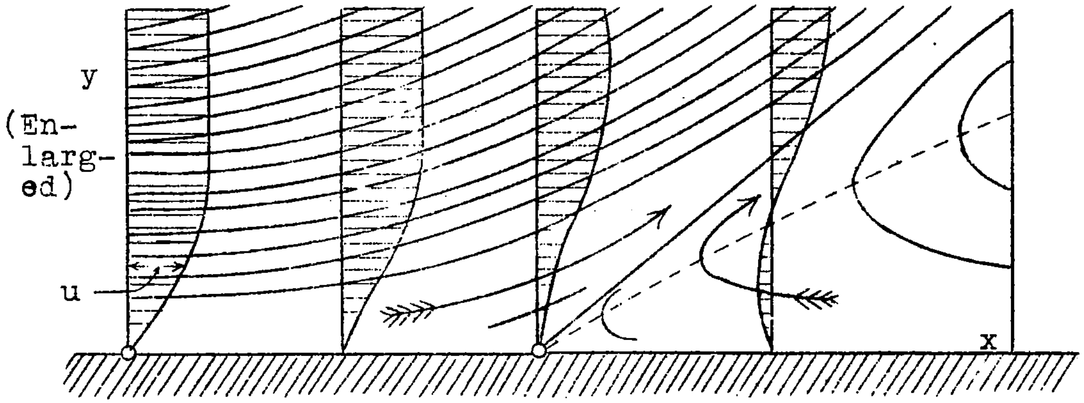


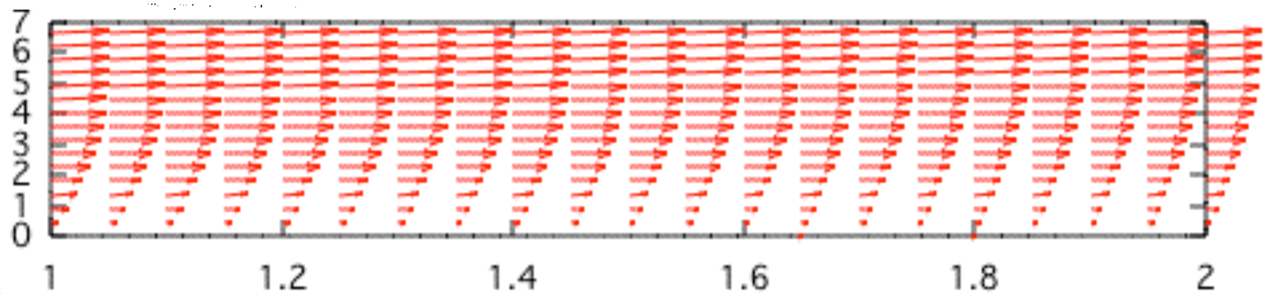


direct resolution

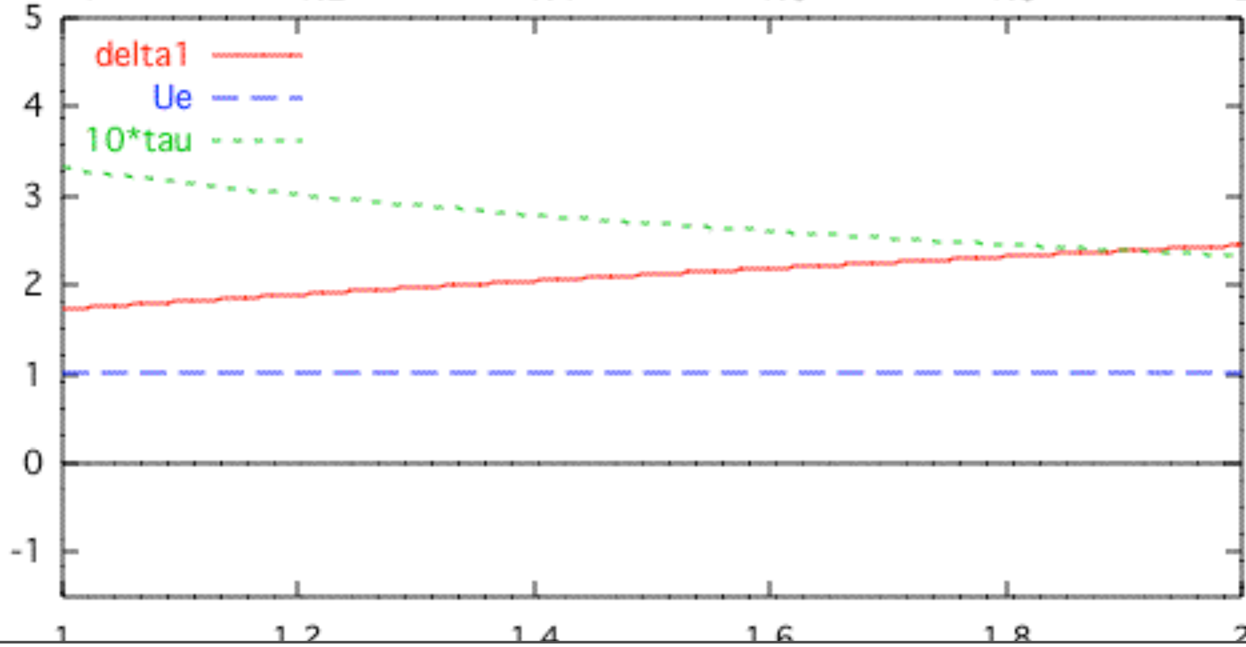
$$\frac{\partial u}{\partial y} \sim \sqrt{x_s - x}$$

no problem! Separation



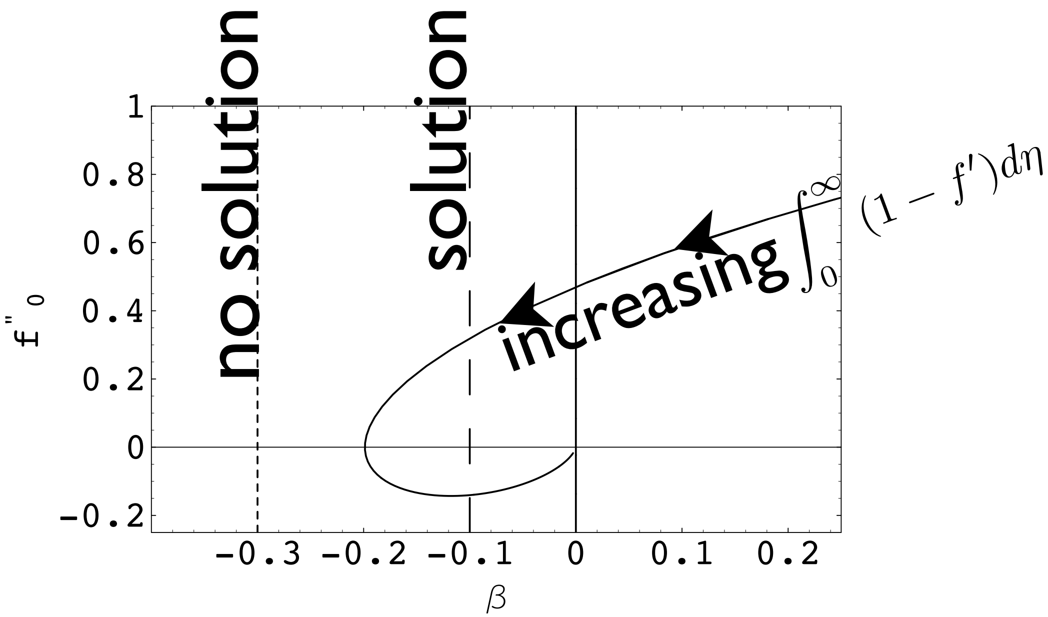


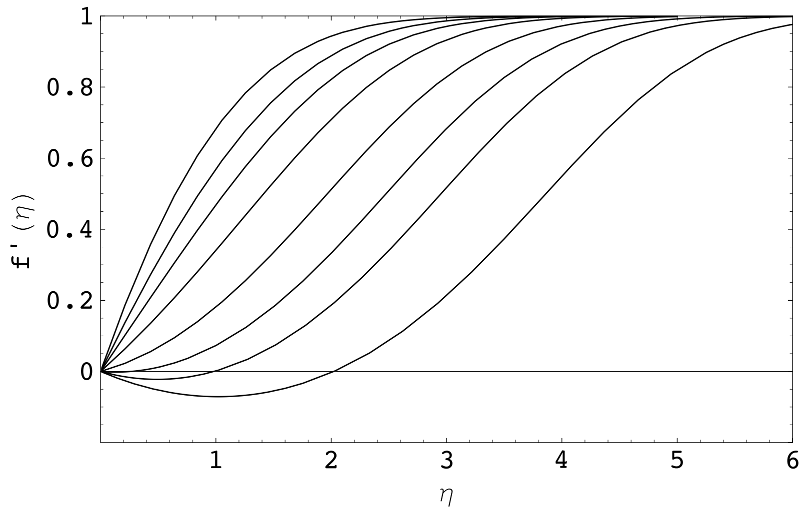
inverse resolution



no problem!

no problem! Separation





no problem! Separation

strong effect of the displacement thickness

$$\bar{v}(\bar{x},0) = Re^{-1/2}\frac{\partial}{\partial \bar{x}}(\bar{u}_e\tilde{\delta}_1)$$

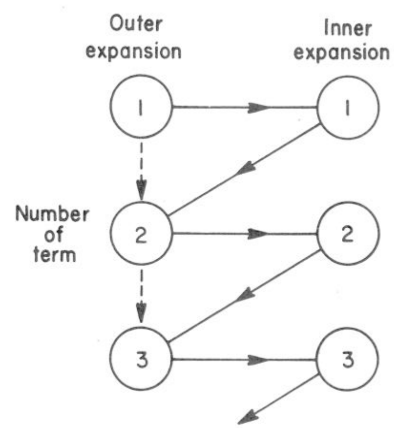
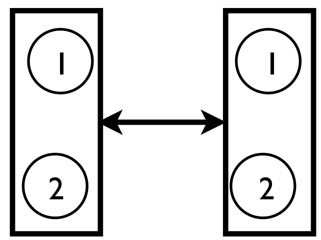


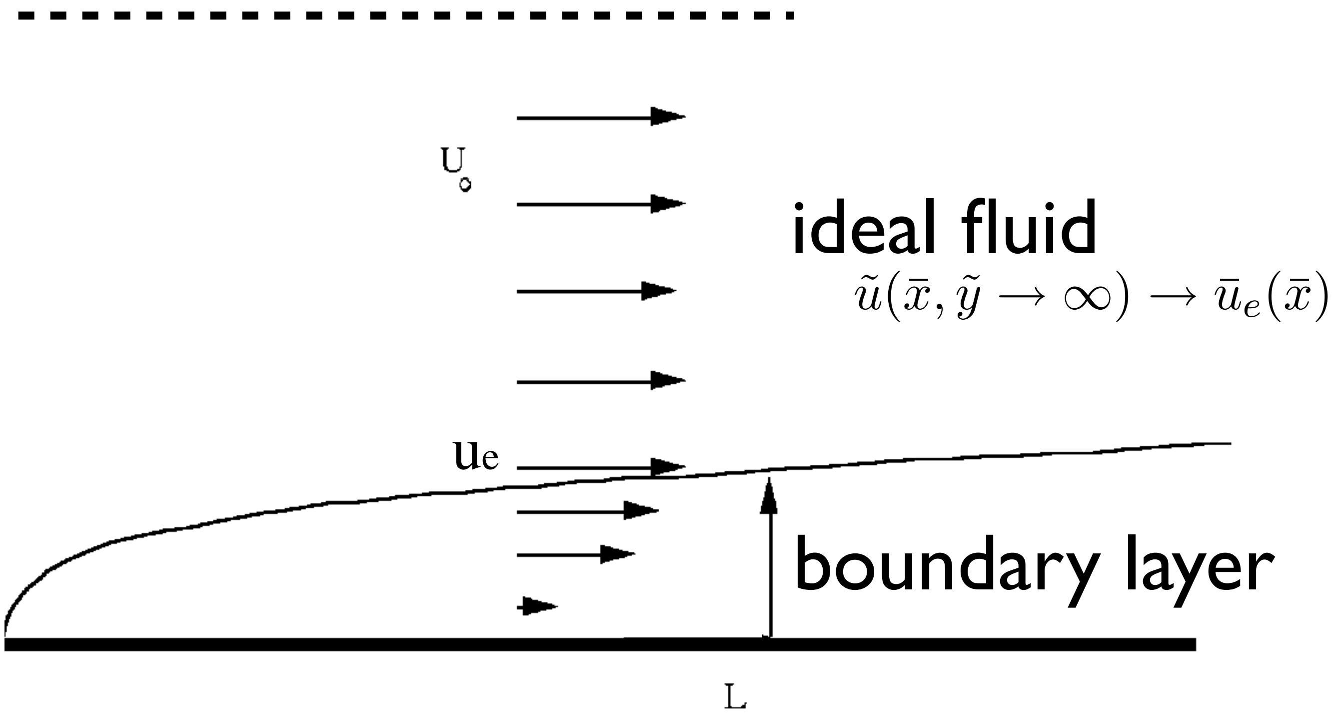
Fig. 5.6. Matching order for inner and outer expansions.



ideal fluid is modified at first order

INTERACTING BOUNDARY LAYER

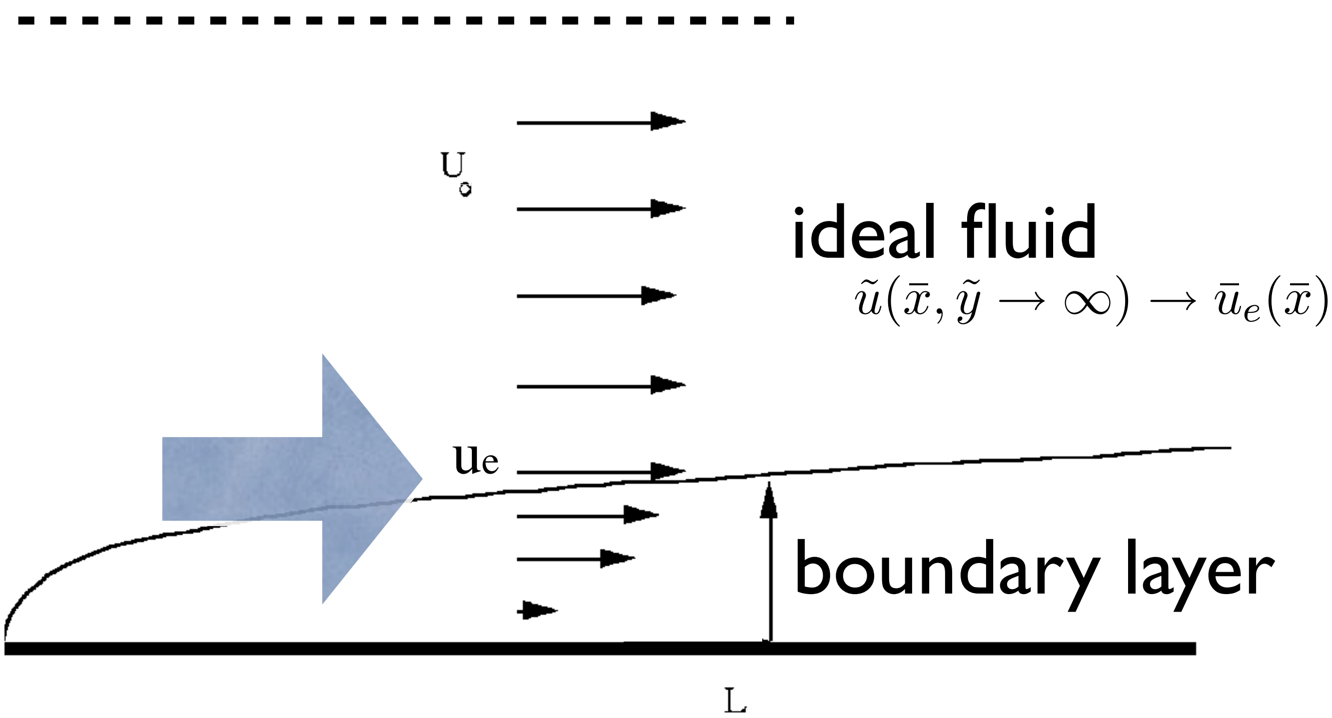
VISCOUS INVISCID INTERACTIONS



$$\tilde{u}(\bar{x}, \tilde{y} \to \infty) \to \bar{u}_e(\bar{x})$$

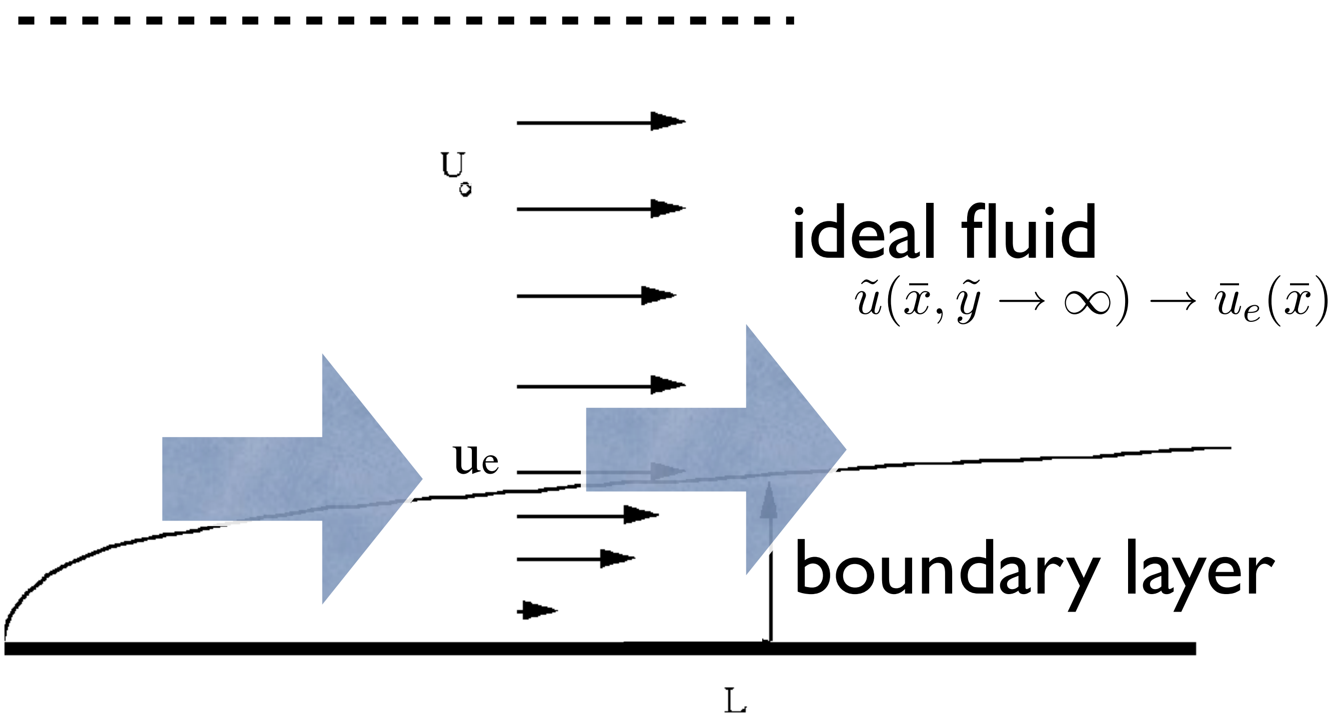
INTERACTING BOUNDARY LAYER

VISCOUS INVISCID INTERACTIONS



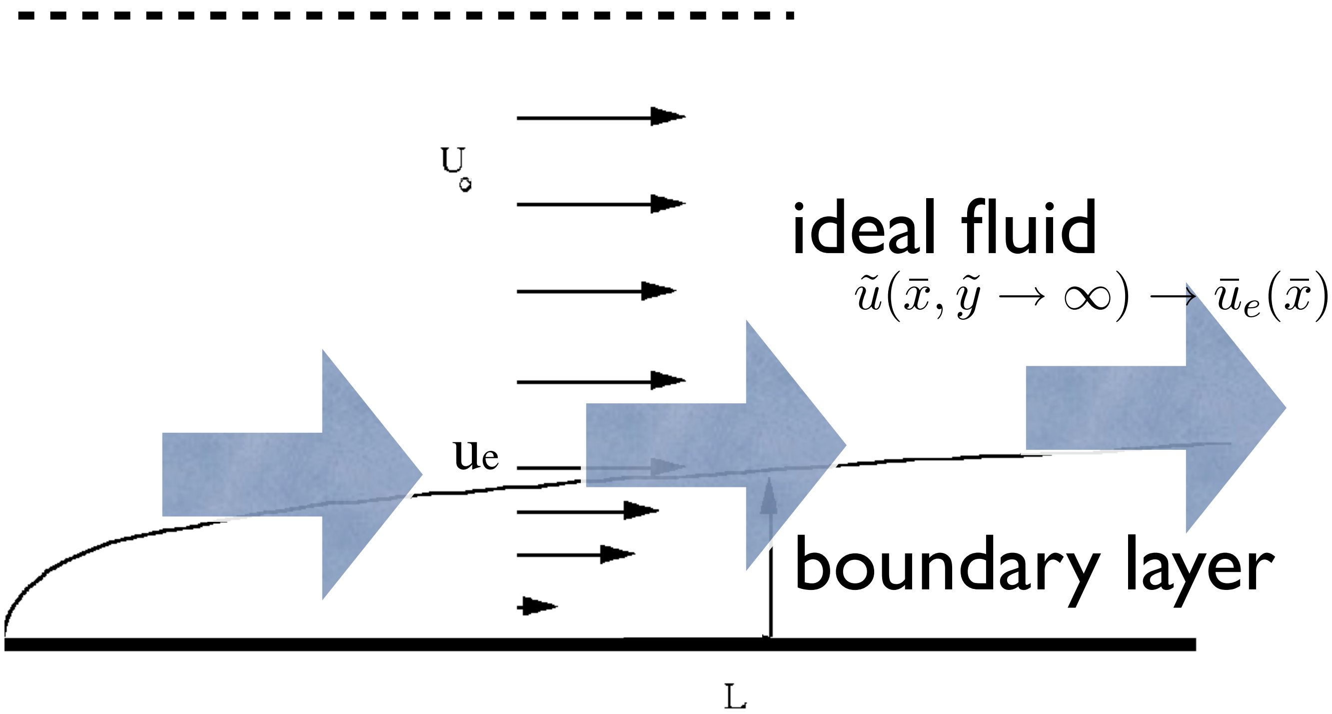
INTERACTING BOUNDARY LAYER

VISCOUS INVISCID INTERACTIONS



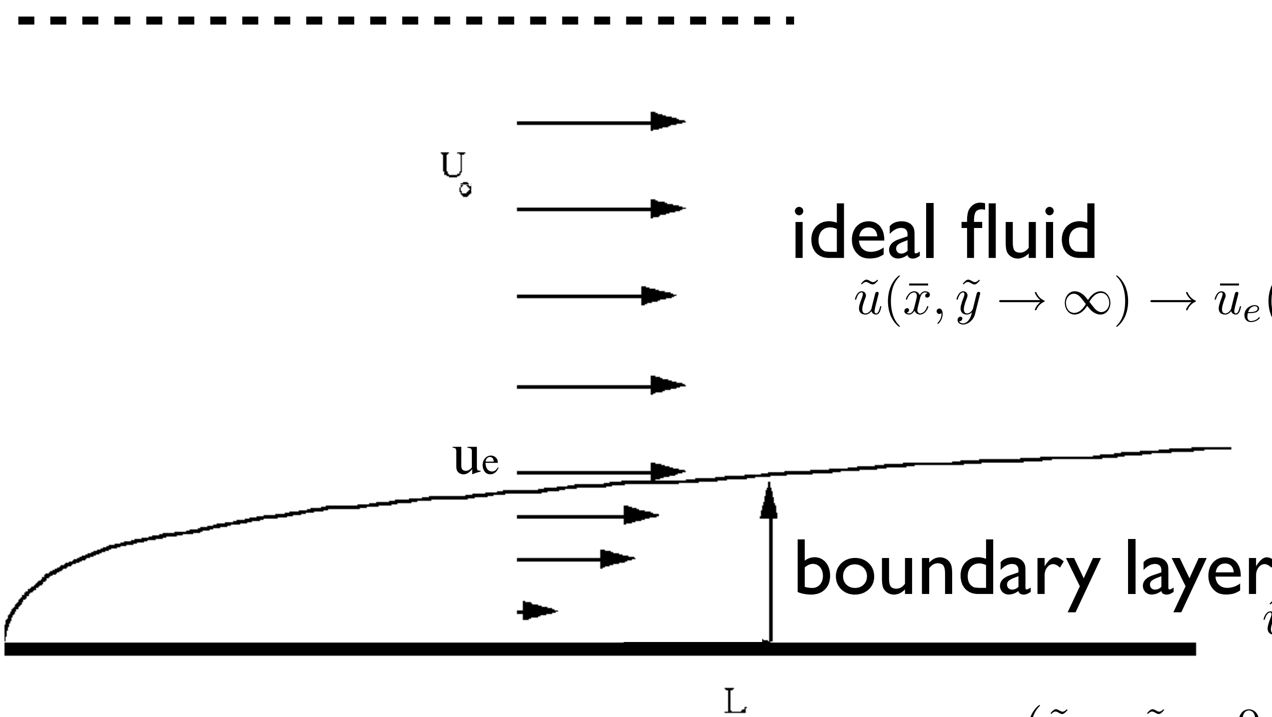
INTERACTING BOUNDARY LAYER

VISCOUS INVISCID INTERACTIONS



INTERACTING BOUNDARY LAYER

VISCOUS INVISCID INTERACTIONS



$$\tilde{u}(\bar{x}, \tilde{y} \to \infty) \to \bar{u}_e(\bar{x}) \qquad \bar{v}_e = Re^{-1/2}\frac{d(\tilde{\delta}_1 \bar{u}_e)}{d\bar{x}}$$

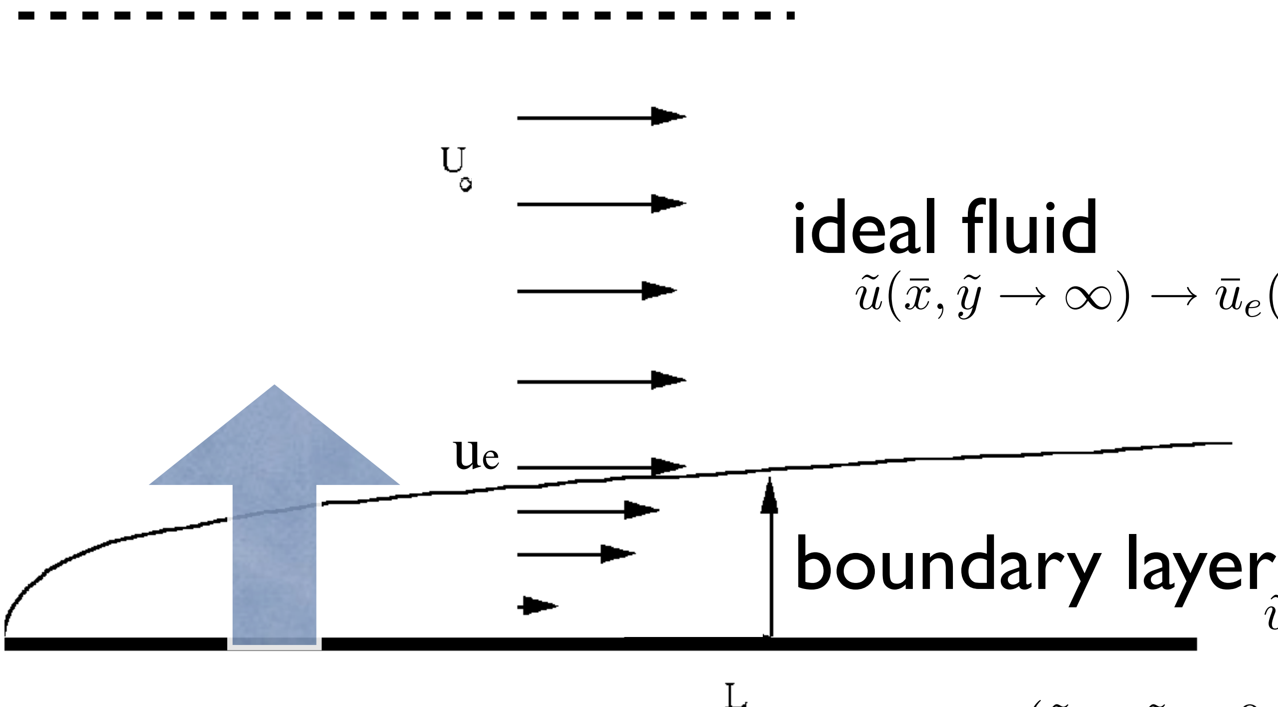
$$\frac{\partial \tilde{u}}{\partial \bar{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0,$$

$$\tilde{u}\frac{\partial \tilde{u}}{\partial \bar{x}} + \tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}} = \bar{u}_e\frac{d\bar{u}_e}{d\bar{x}} + \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2},$$

$(\tilde{u} = \tilde{v} = 0$ on the body $\bar{f}(\bar{x}))$

INTERACTING BOUNDARY LAYER

VISCOUS INVISCID INTERACTIONS



$$\tilde{u}(\bar{x}, \tilde{y} \to \infty) \to \bar{u}_e(\bar{x}) \qquad \bar{v}_e = Re^{-1/2}\frac{d(\tilde{\delta}_1 \bar{u}_e)}{d\bar{x}}$$

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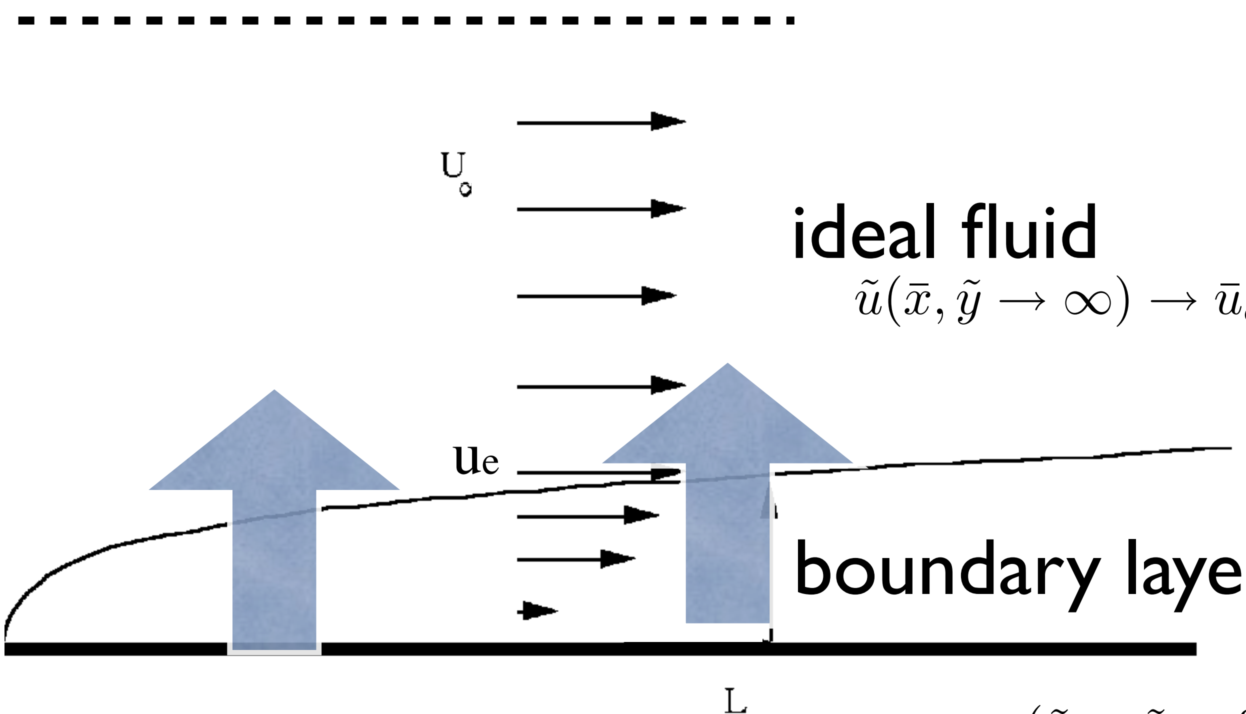
INTERACTING BOUNDARY LAYER

VISCOUS INVISCID INTERACTIONS



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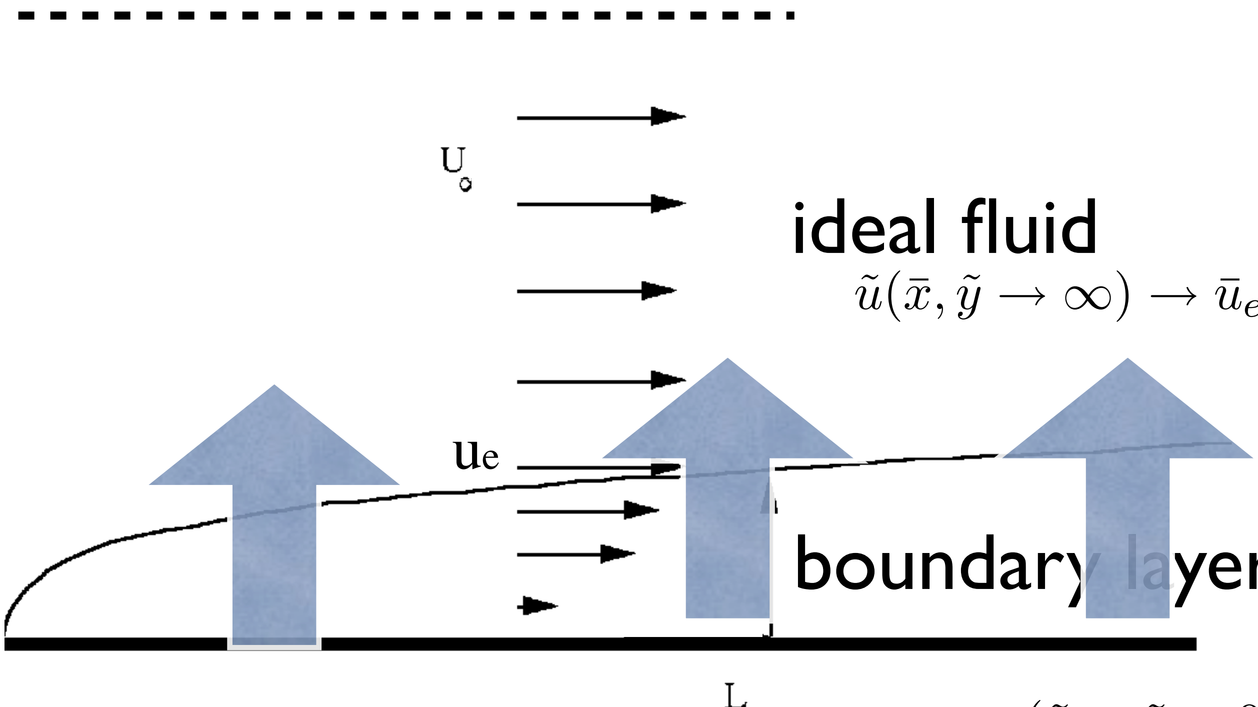
INTERACTING BOUNDARY LAYER

VISCOUS INVISCID INTERACTIONS



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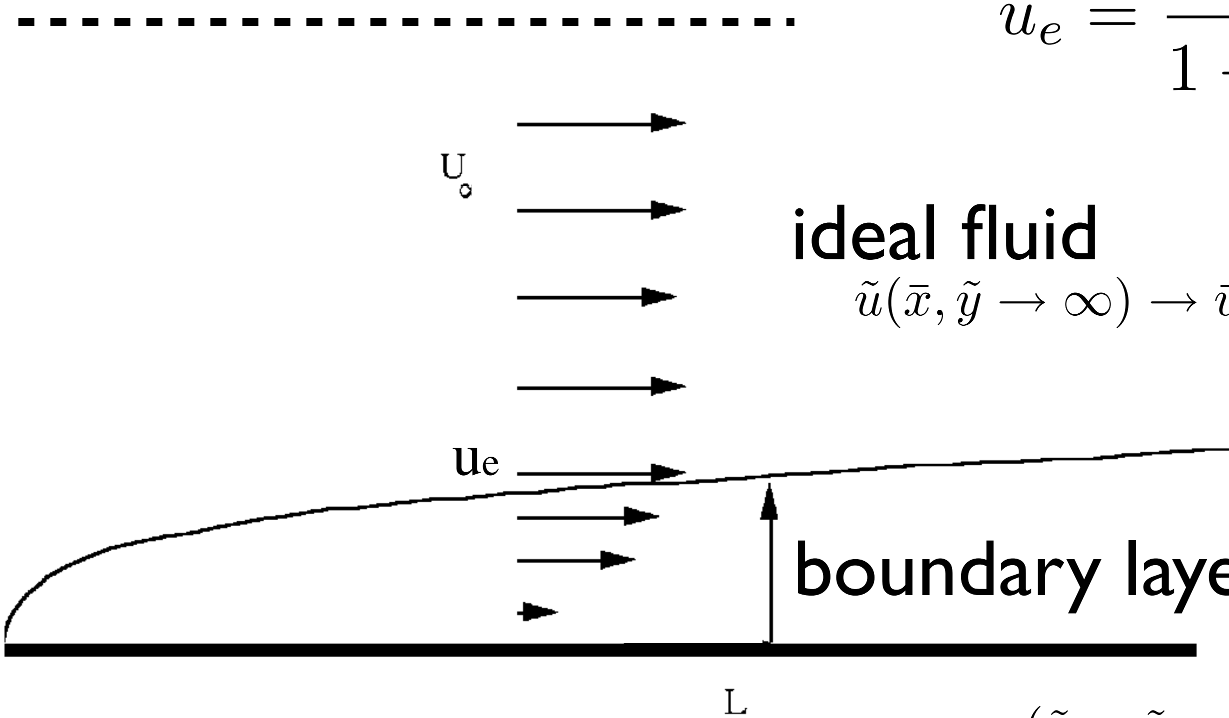
INTERACTING BOUNDARY LAYER

$$\bar{u}_e = \frac{1}{1 - (\alpha \bar{f} + \tilde{\delta}_1 Re^{-1/2})}$$



$$\tilde{u}(\bar{x}, \tilde{y} \to \infty) \to \bar{u}_e(\bar{x}) \qquad \bar{v}_e = Re^{-1/2} \frac{d(\tilde{\delta}_1 \bar{u}_e)}{d\bar{x}}$$

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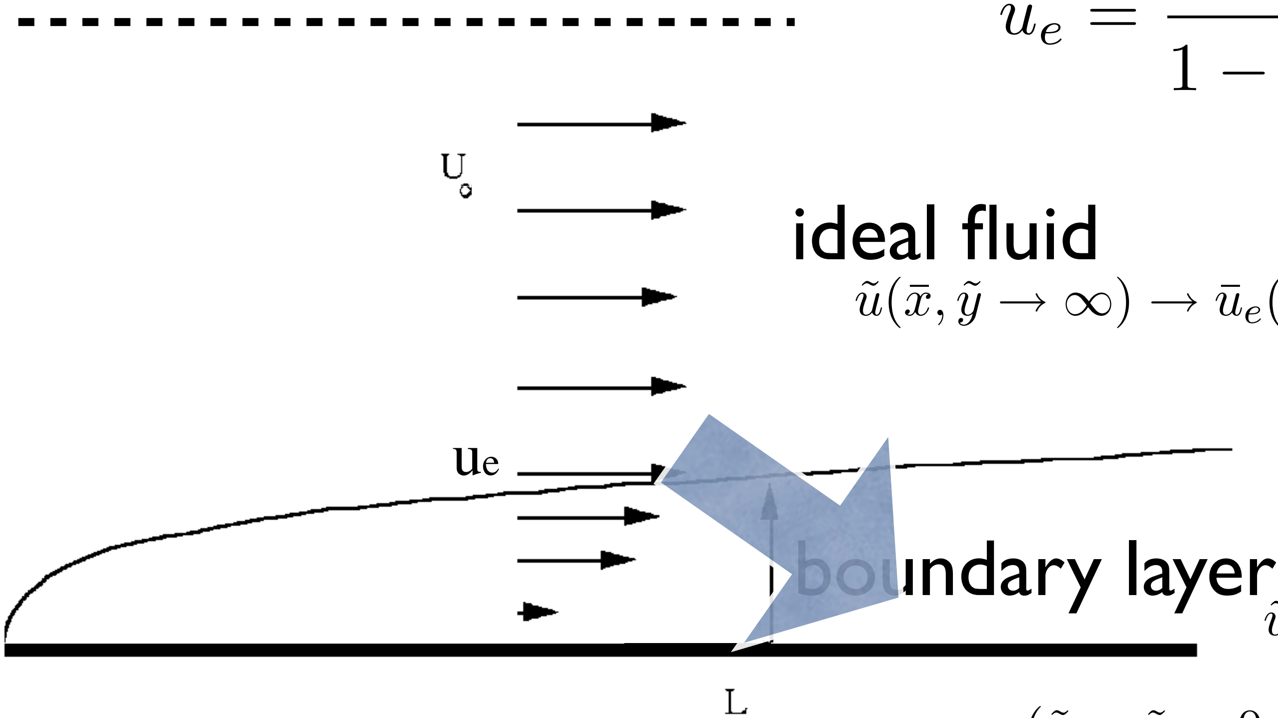

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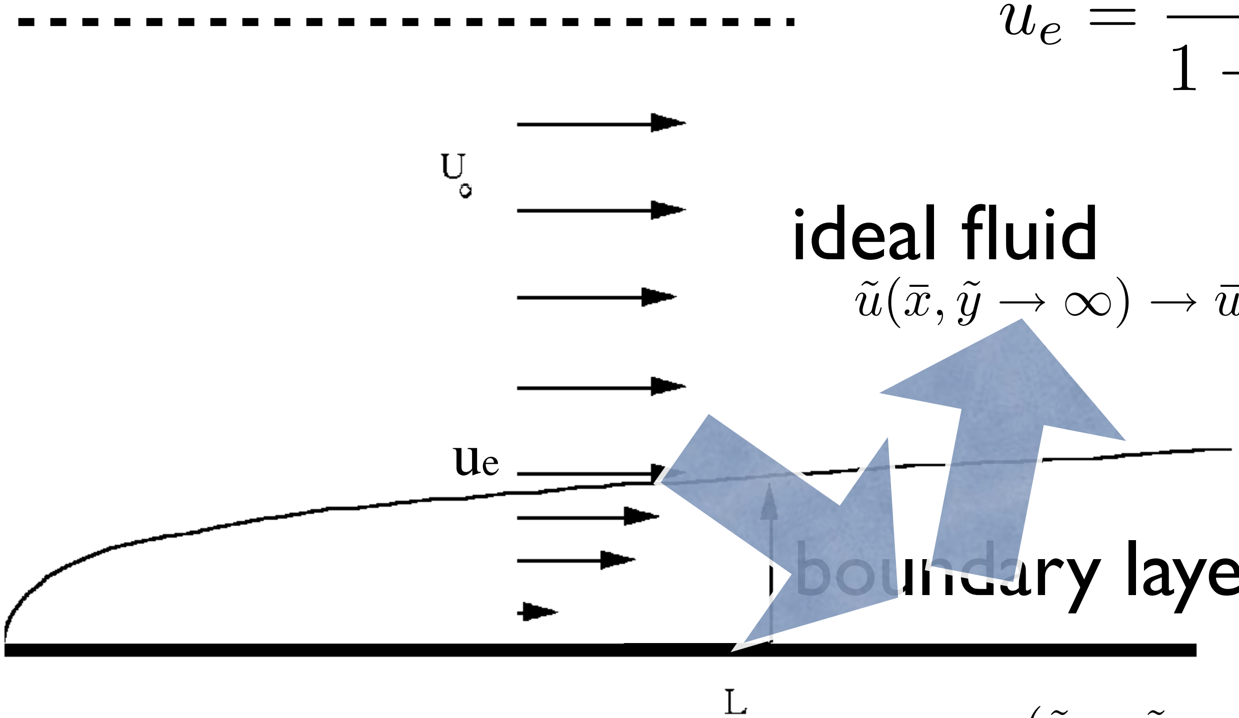
INTERACTING BOUNDARY LAYER



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INTERACTING BOUNDARY LAYER

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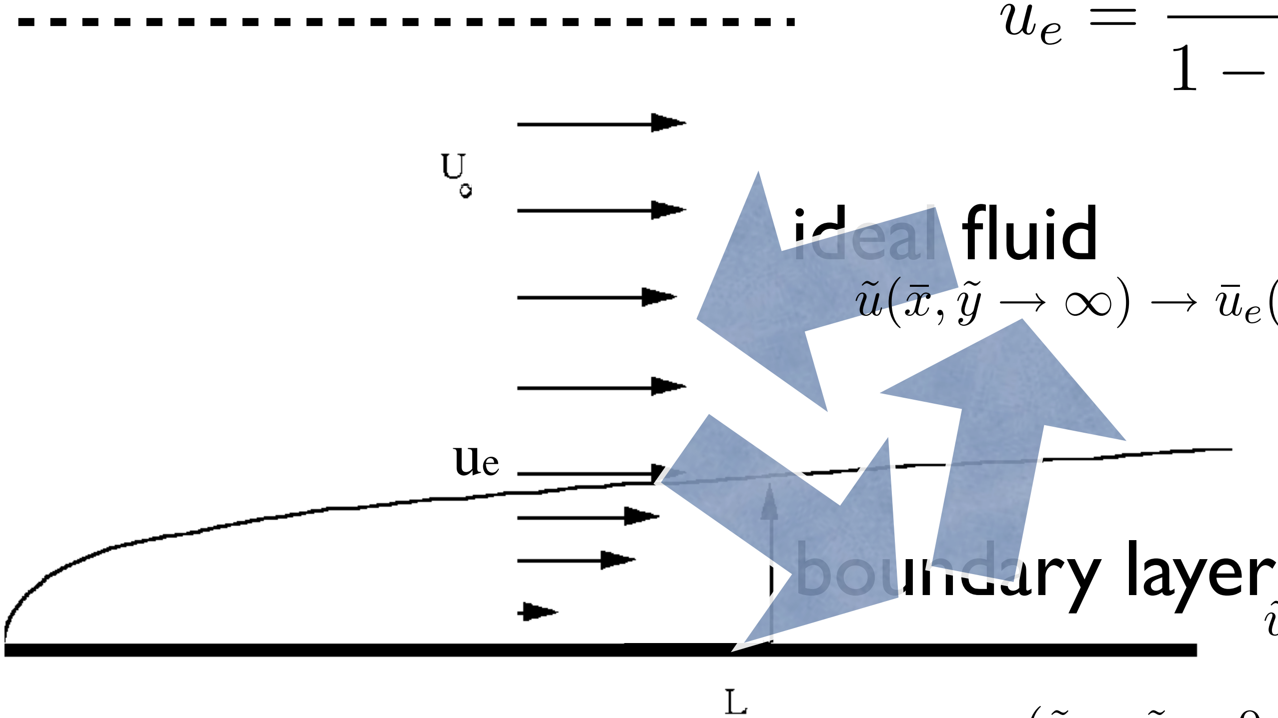

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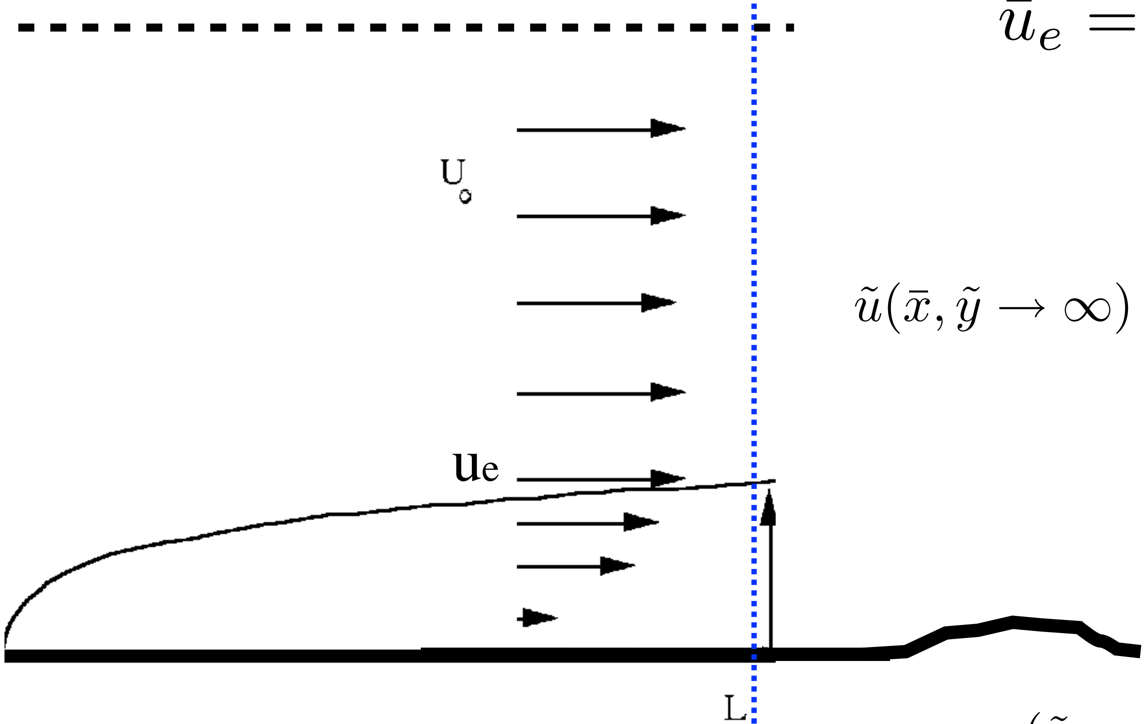

inviscid viscous interaction

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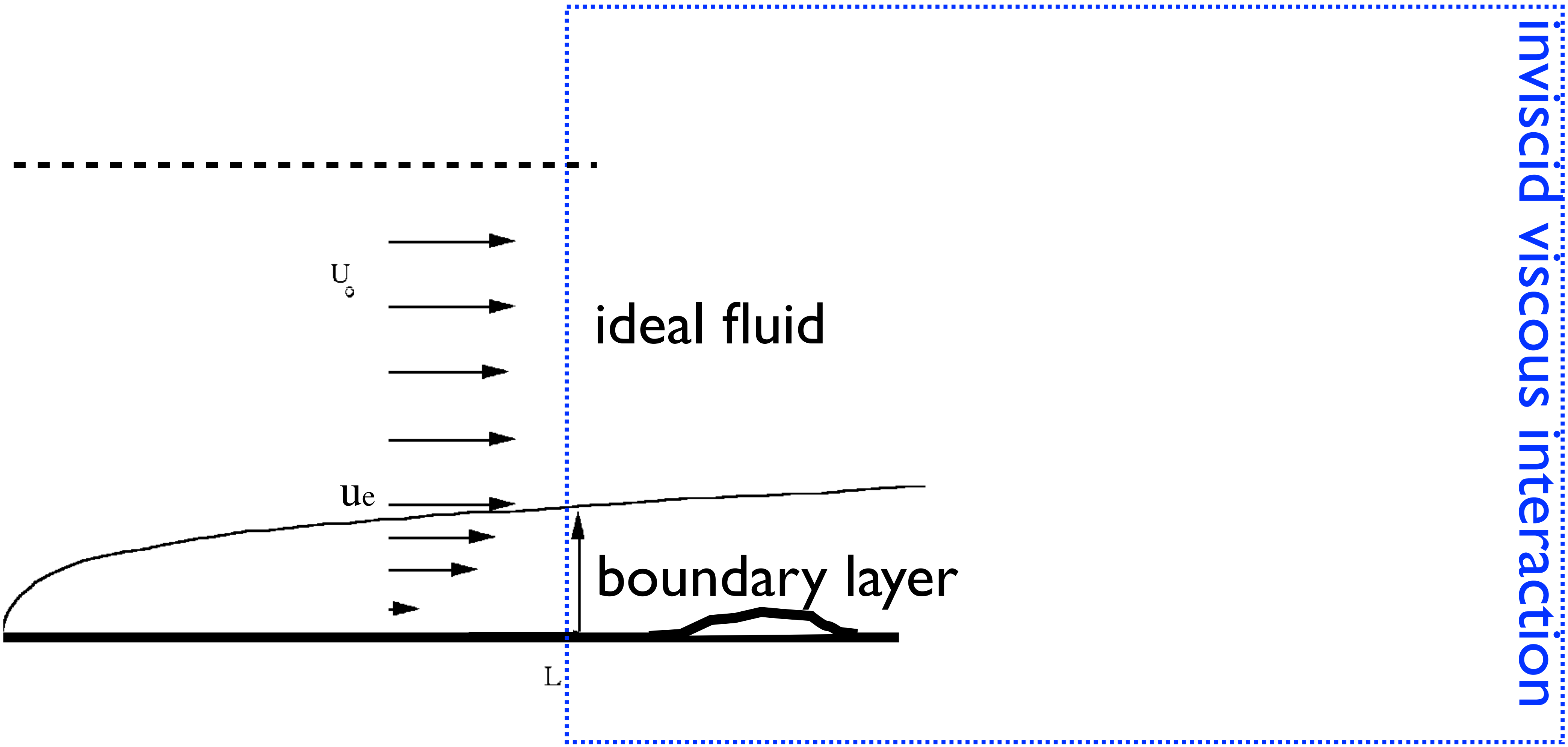

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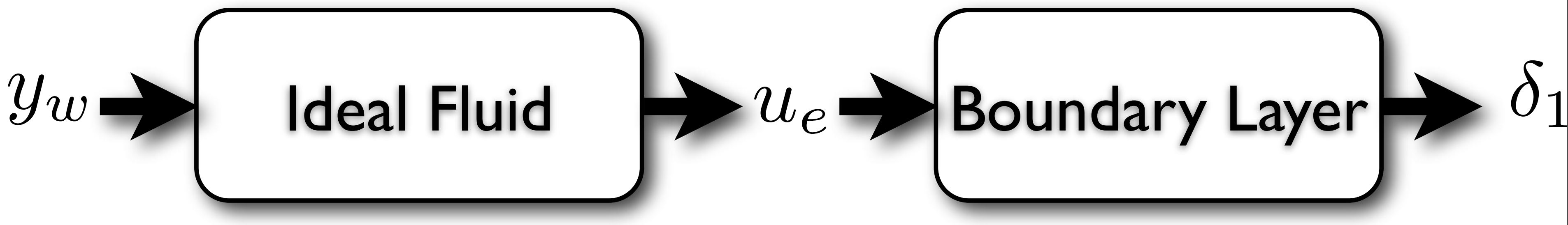
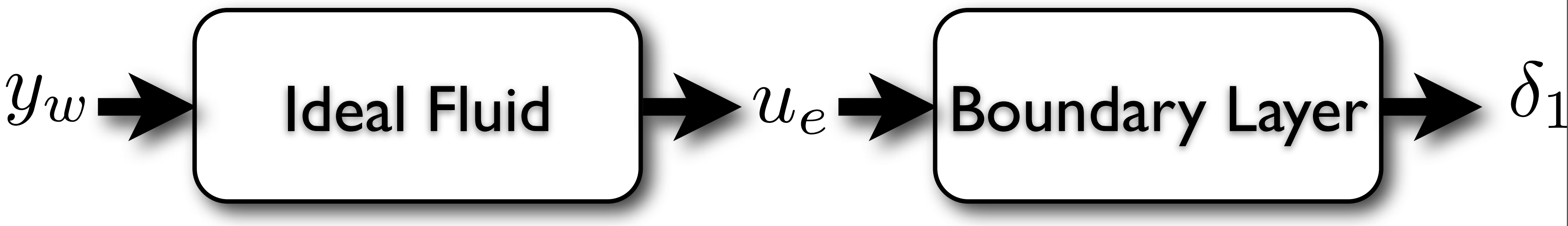
ideal fluid
boundary layer
L
inviscid viscous interaction

pressure- blowing velocity
panel methods/
finite diff...

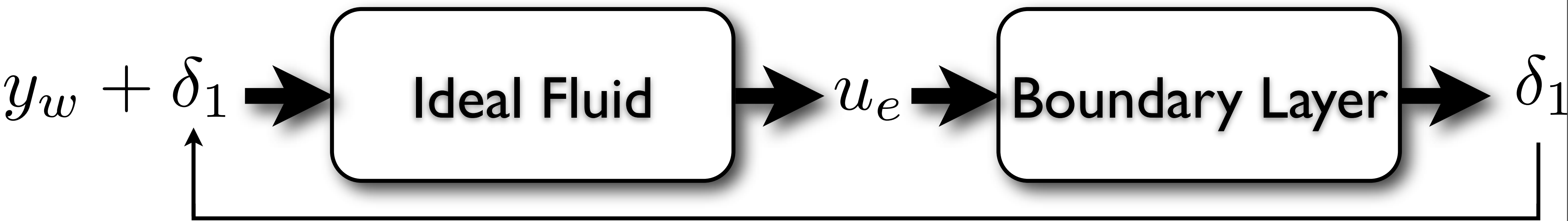


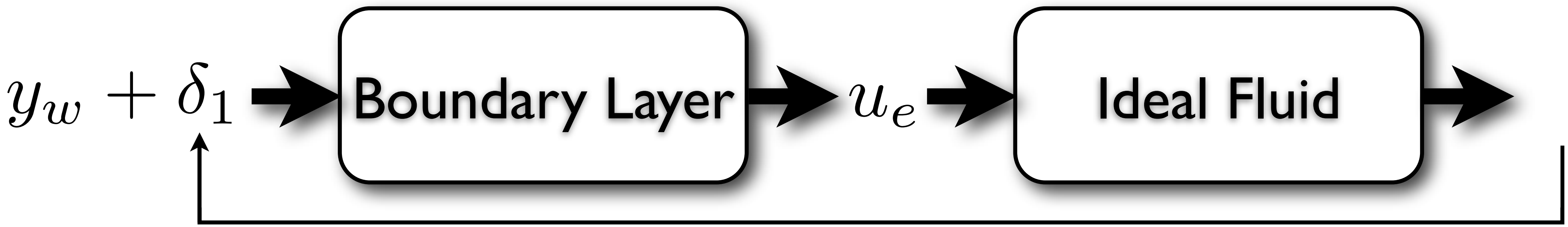
Keller Box,
Finite differences....

Finite elements

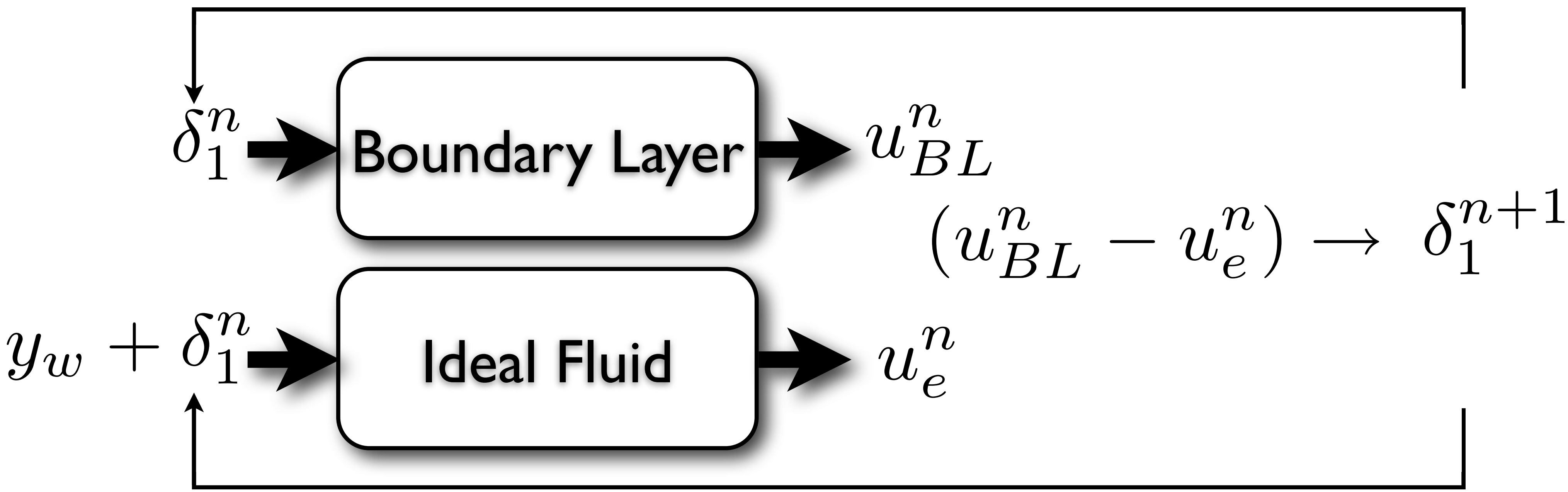


y_w
Ideal Fluid
u_e
Boundary Layer
δ_1

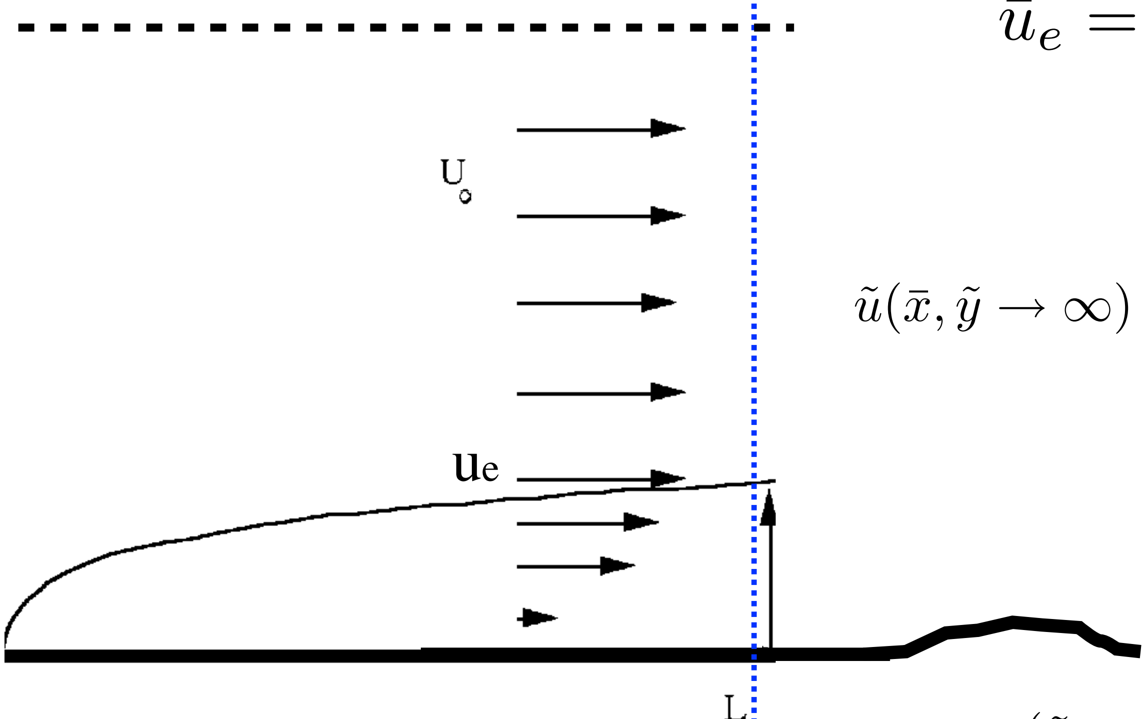

$y_w + \delta_1$
Ideal Fluid
u_e
Boundary Layer
δ_1



$y_w + \delta_1$
Boundary Layer
u_e
Ideal Fluid



$$\delta^{n+1} = \delta^n + \lambda(u_{BL}^n - u_e^n)$$



inviscid viscous interaction

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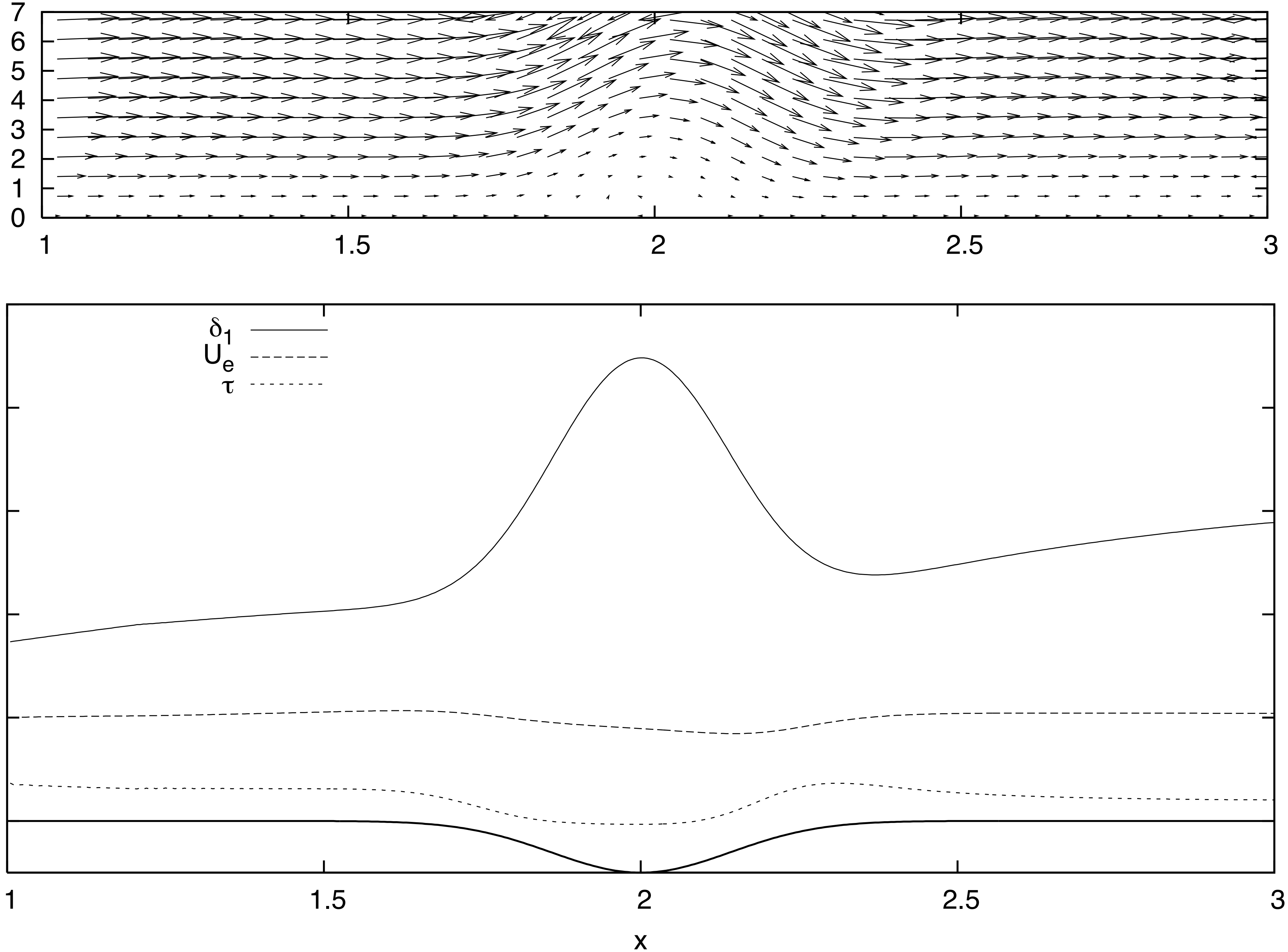
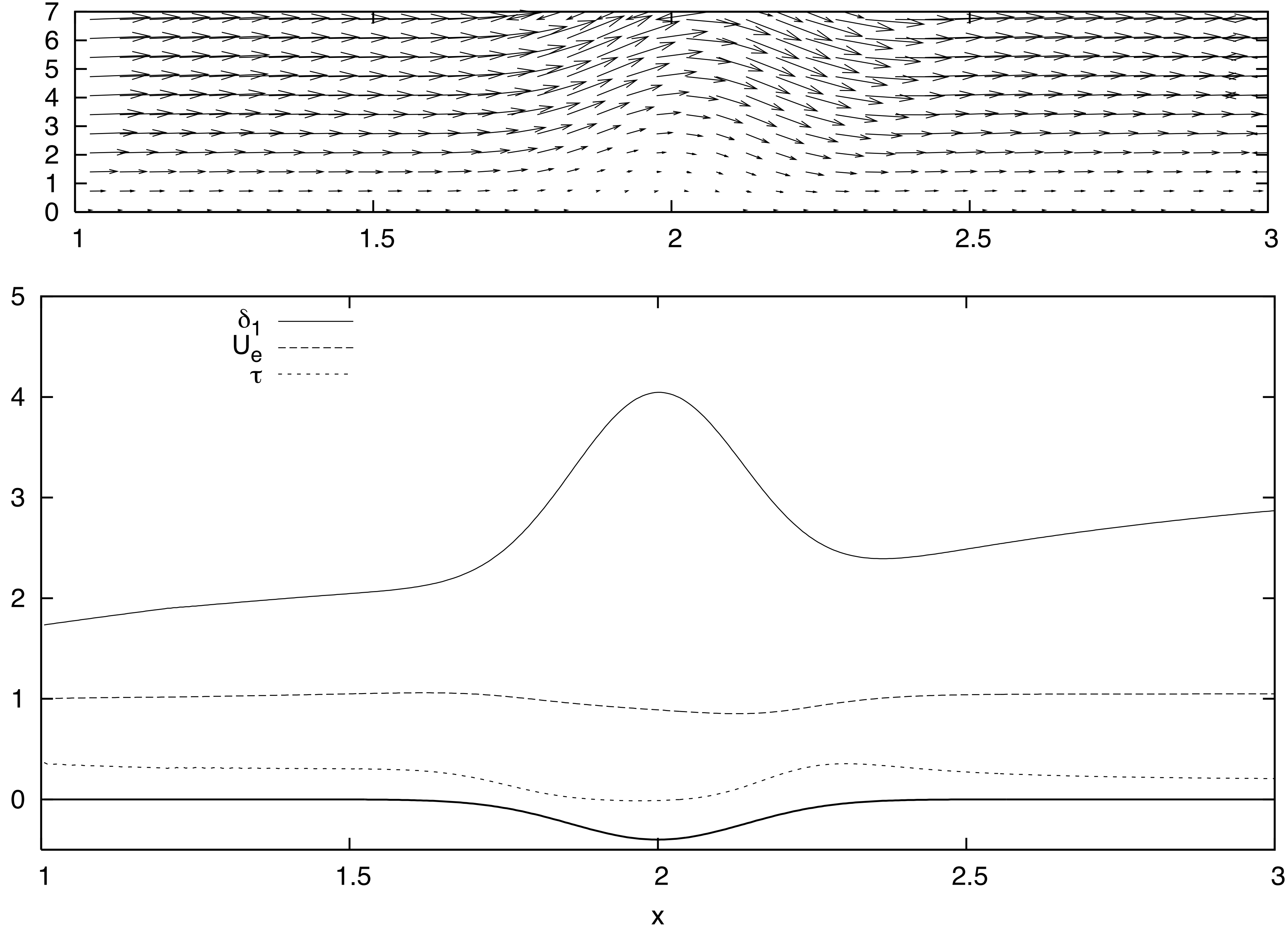
subsonic





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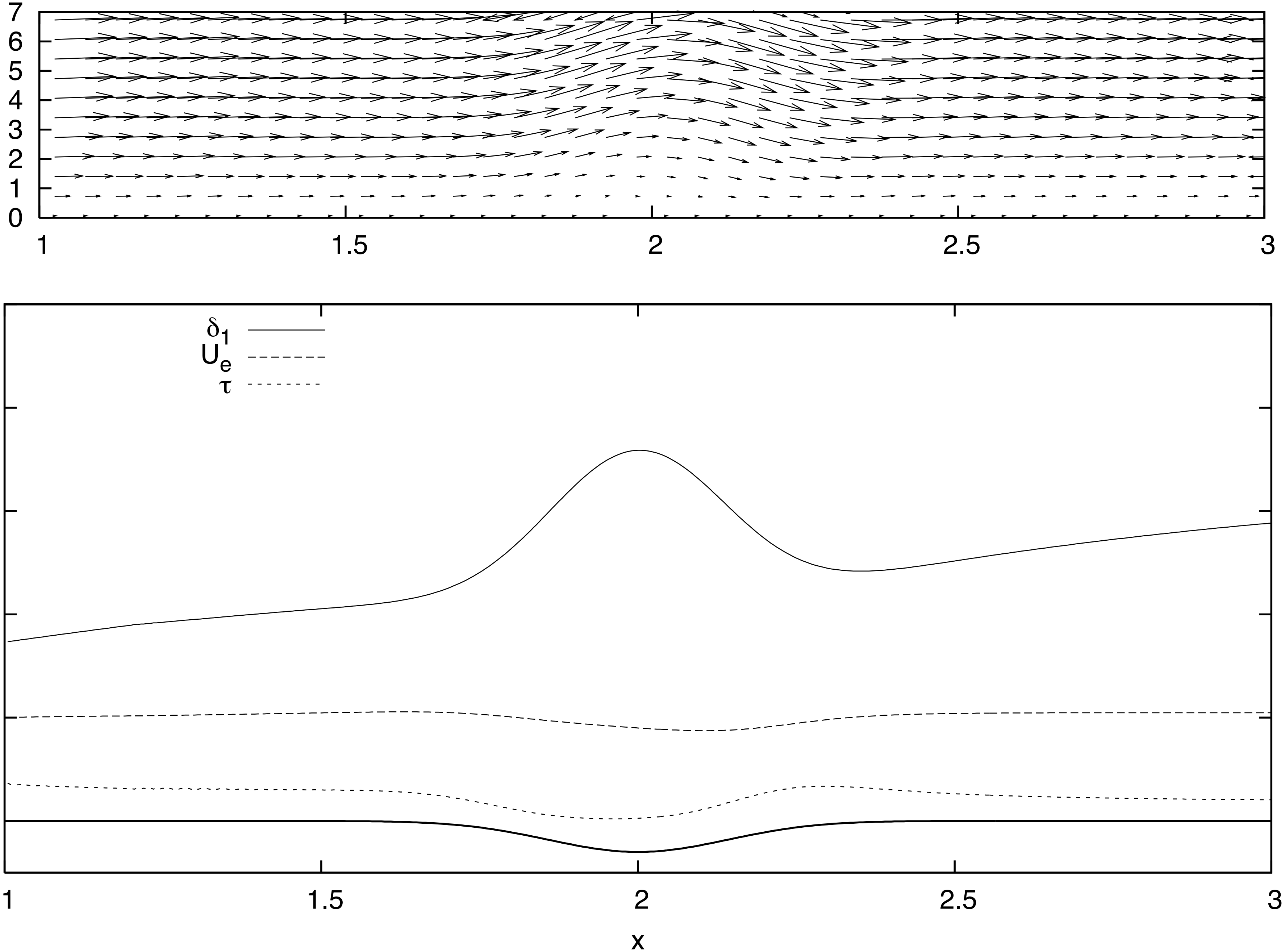
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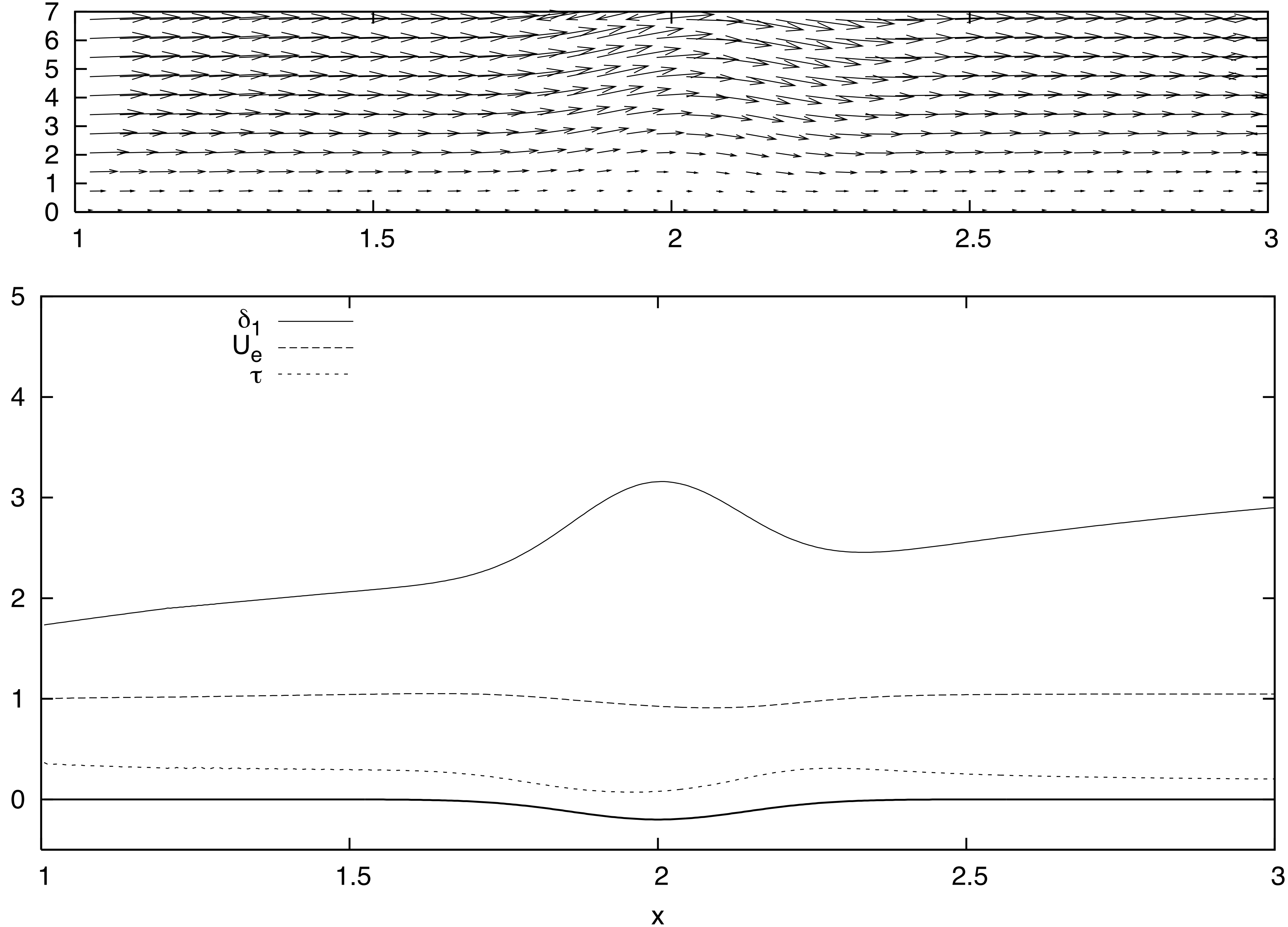

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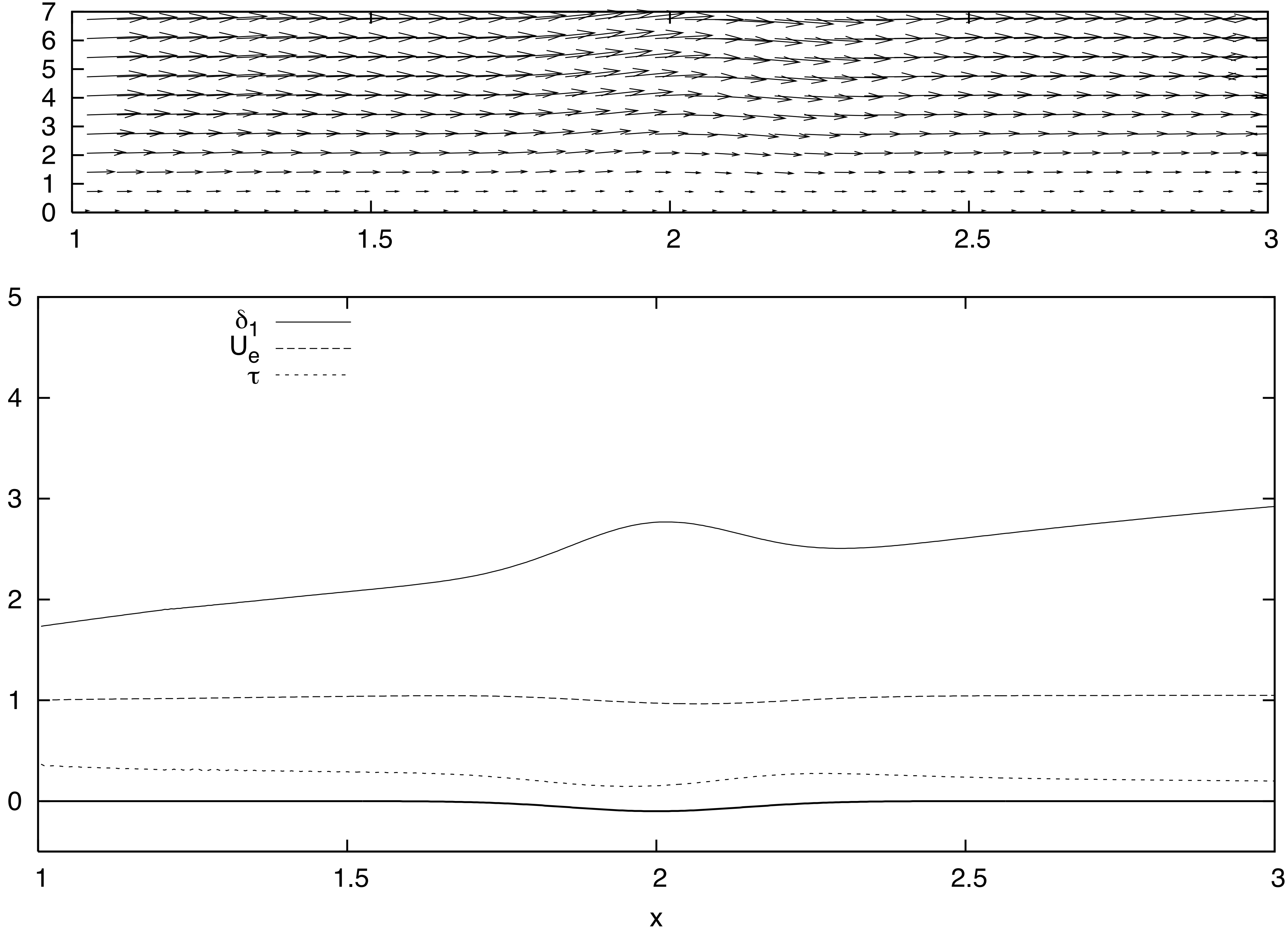

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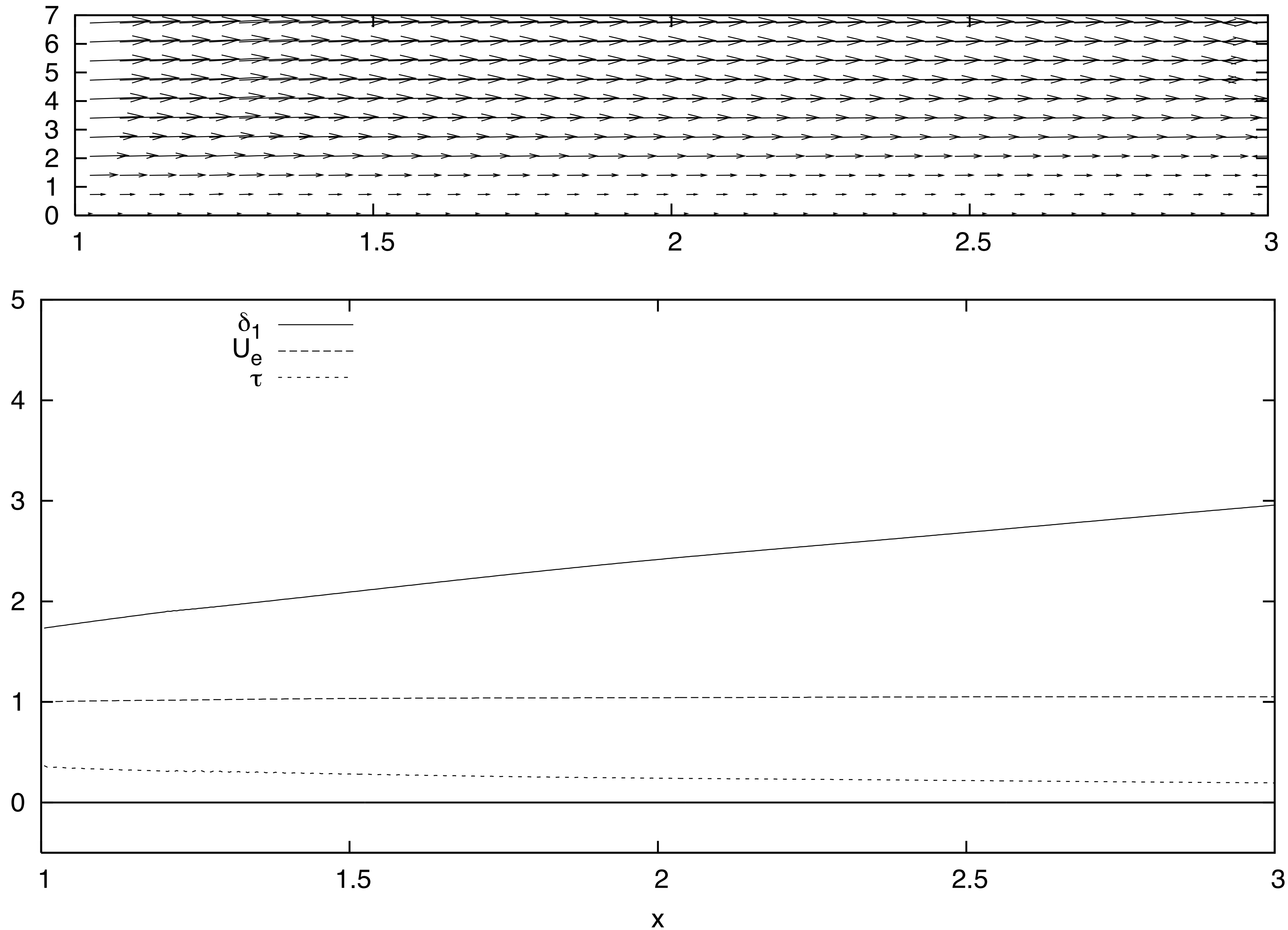

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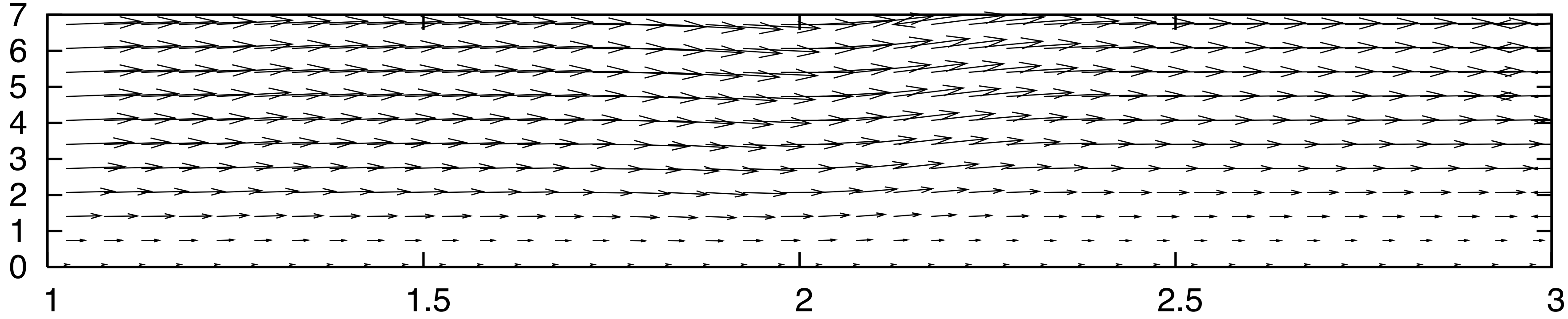

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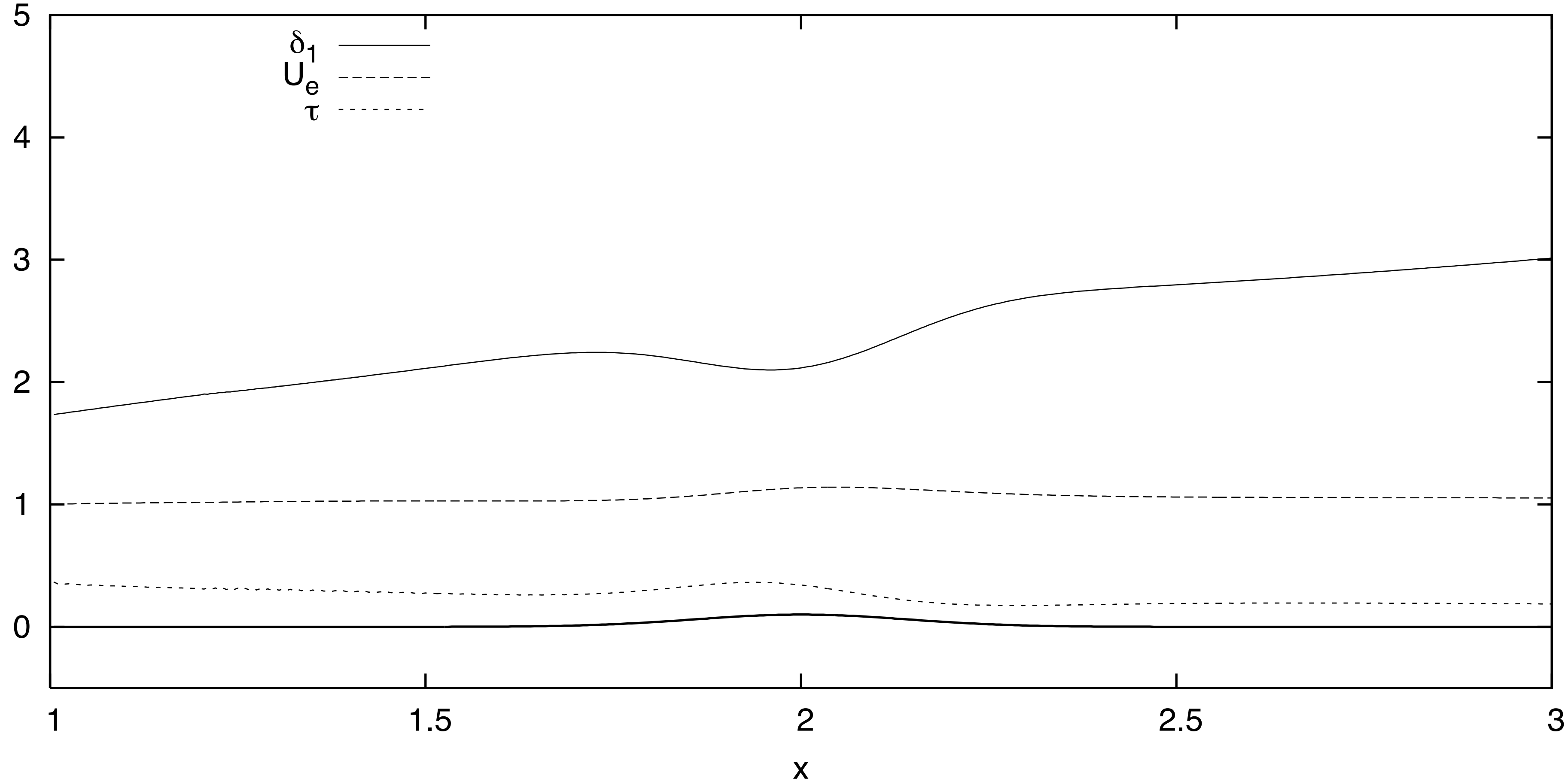

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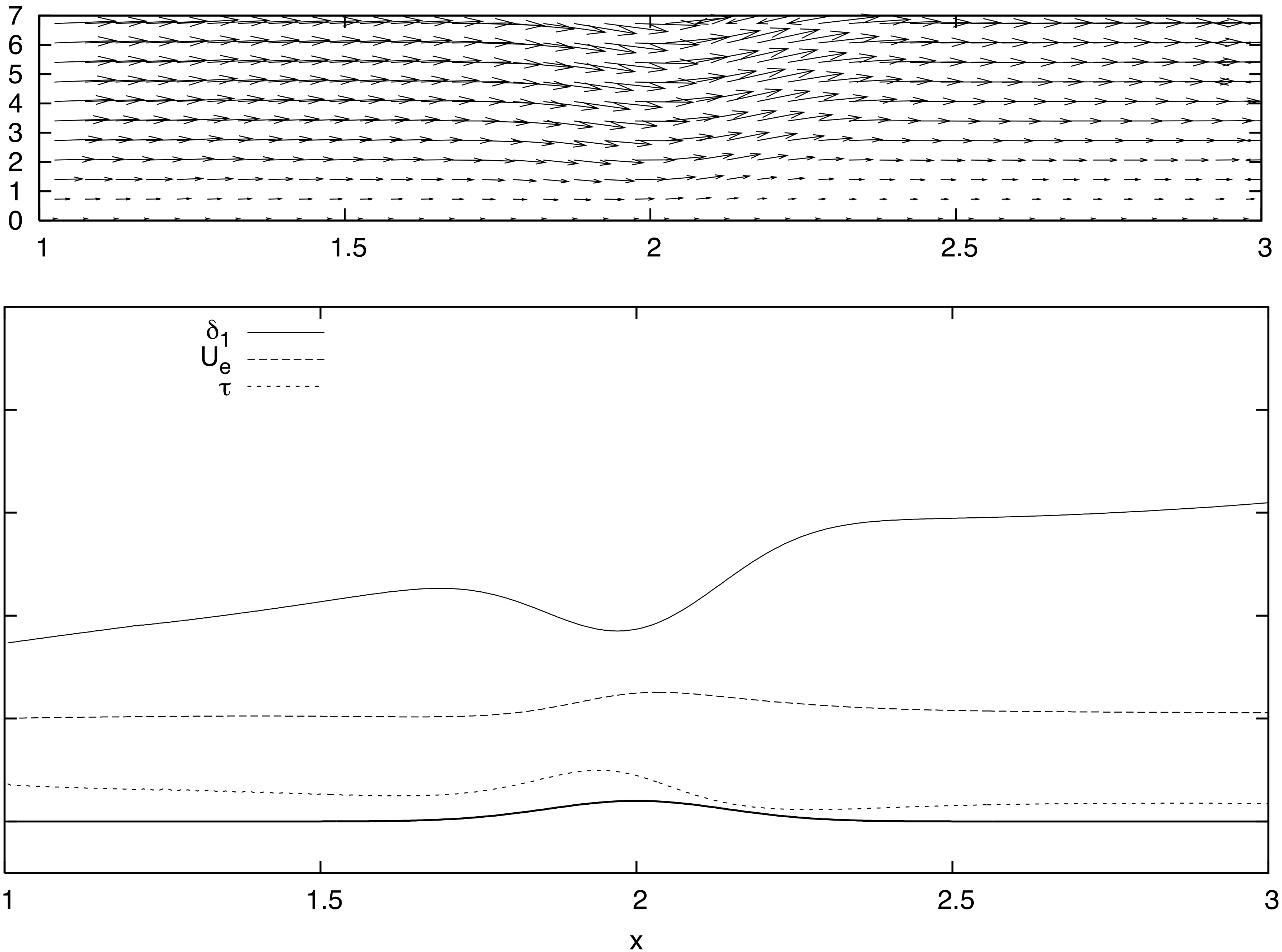

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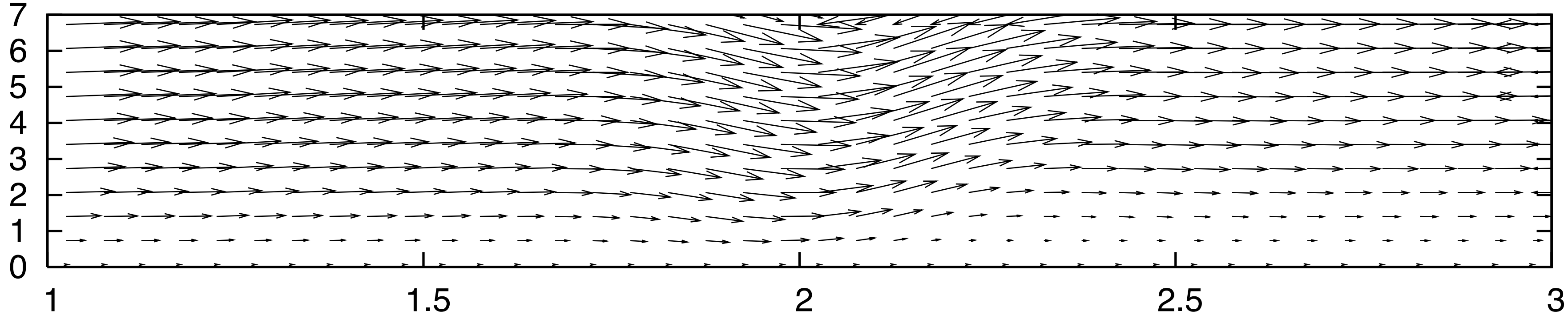

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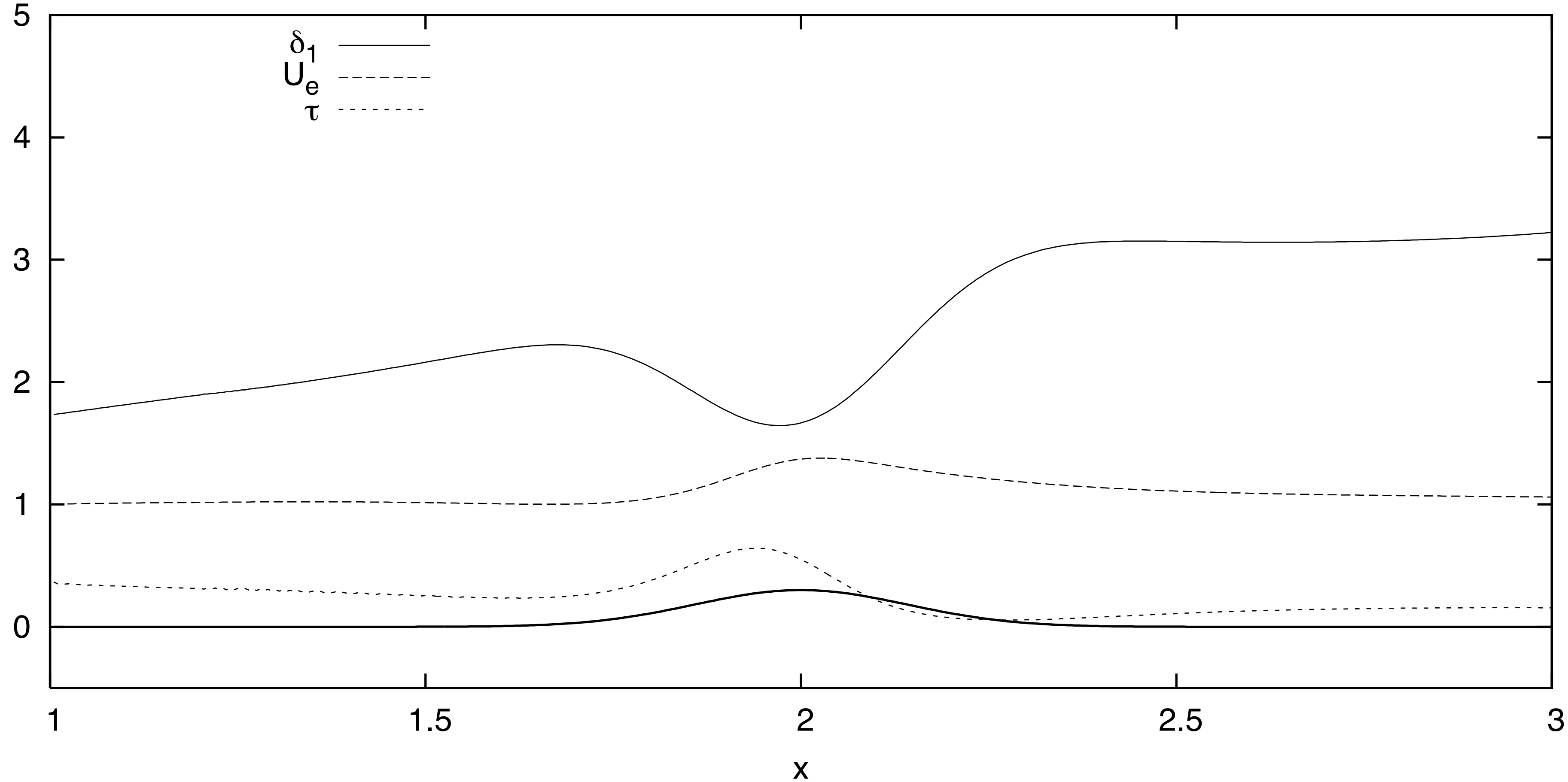

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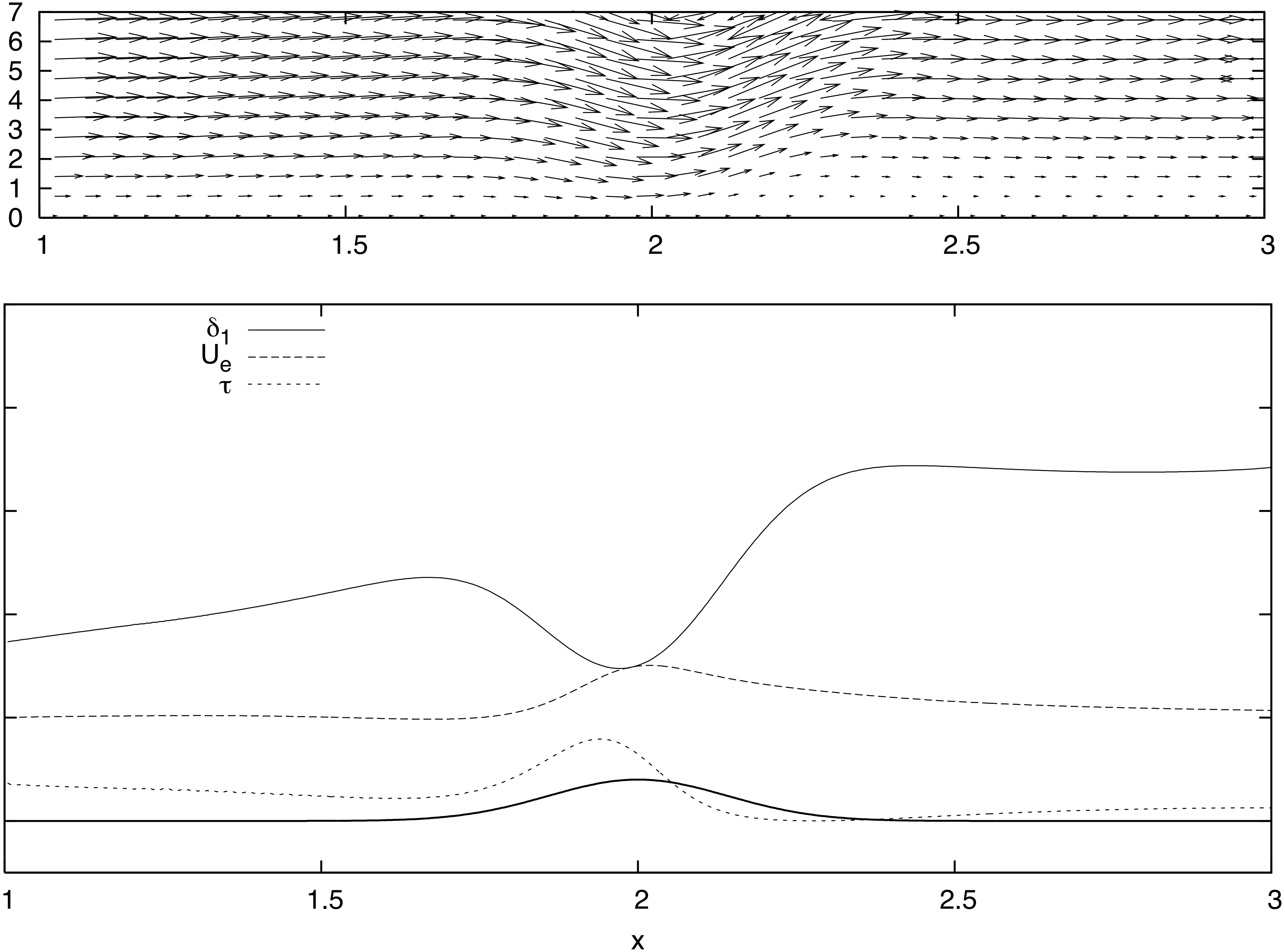

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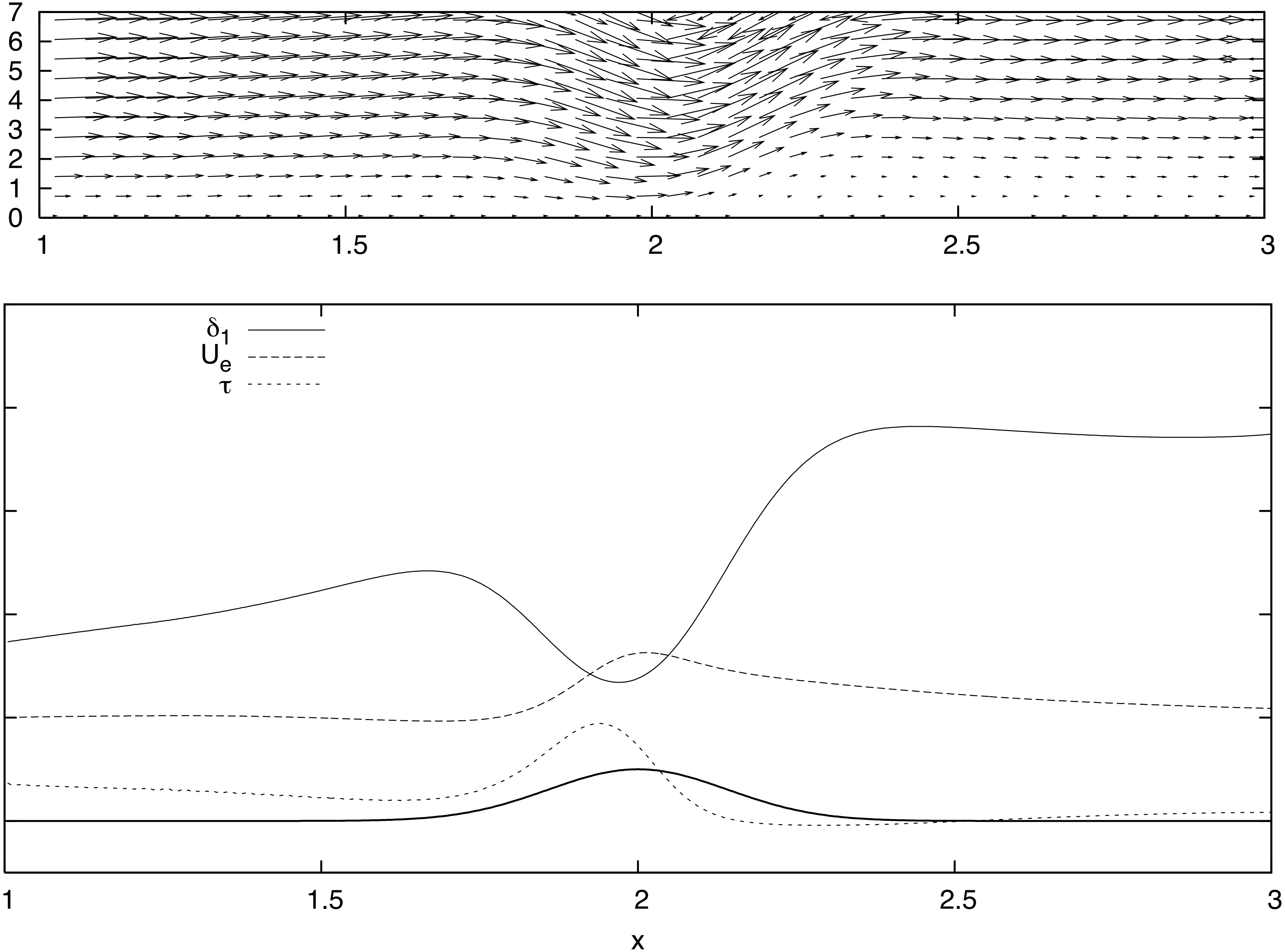

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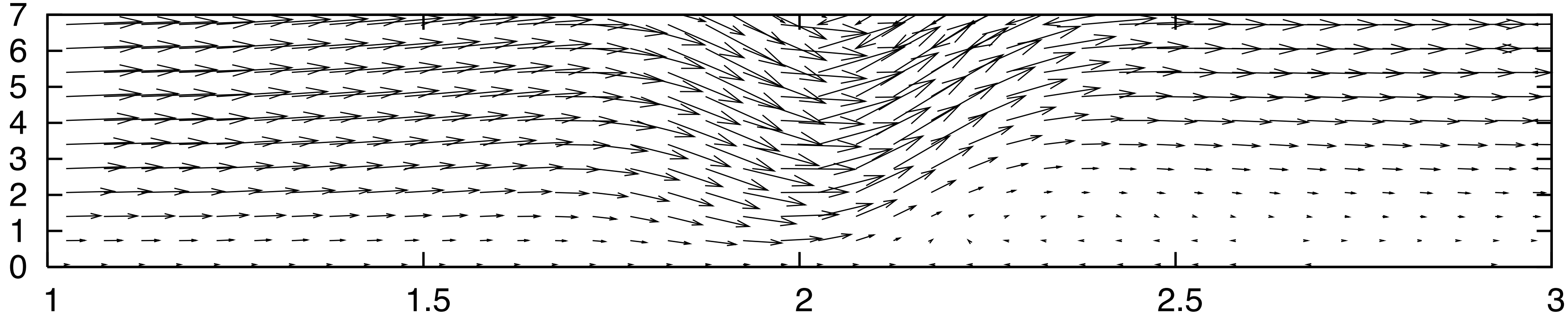

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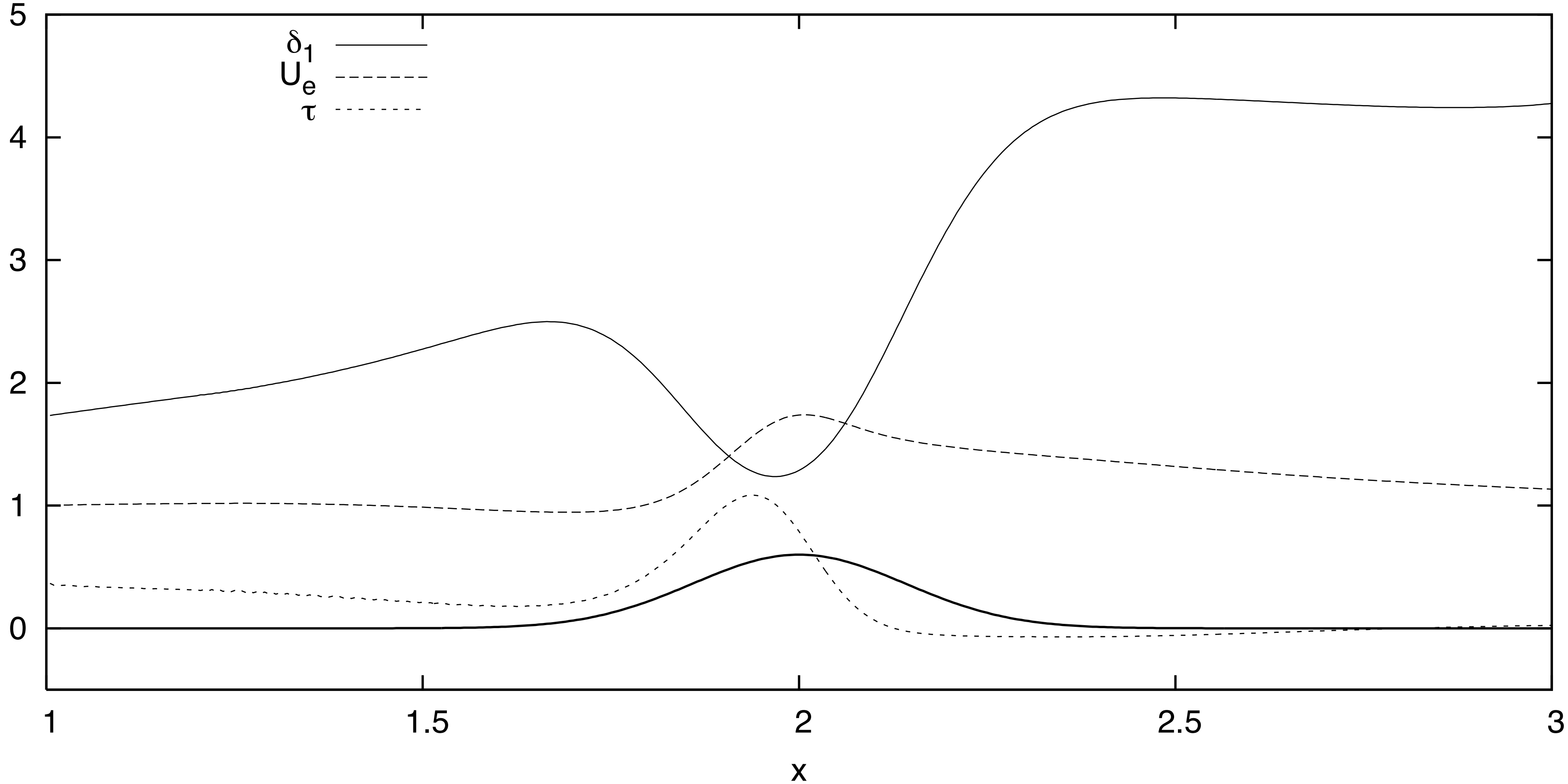

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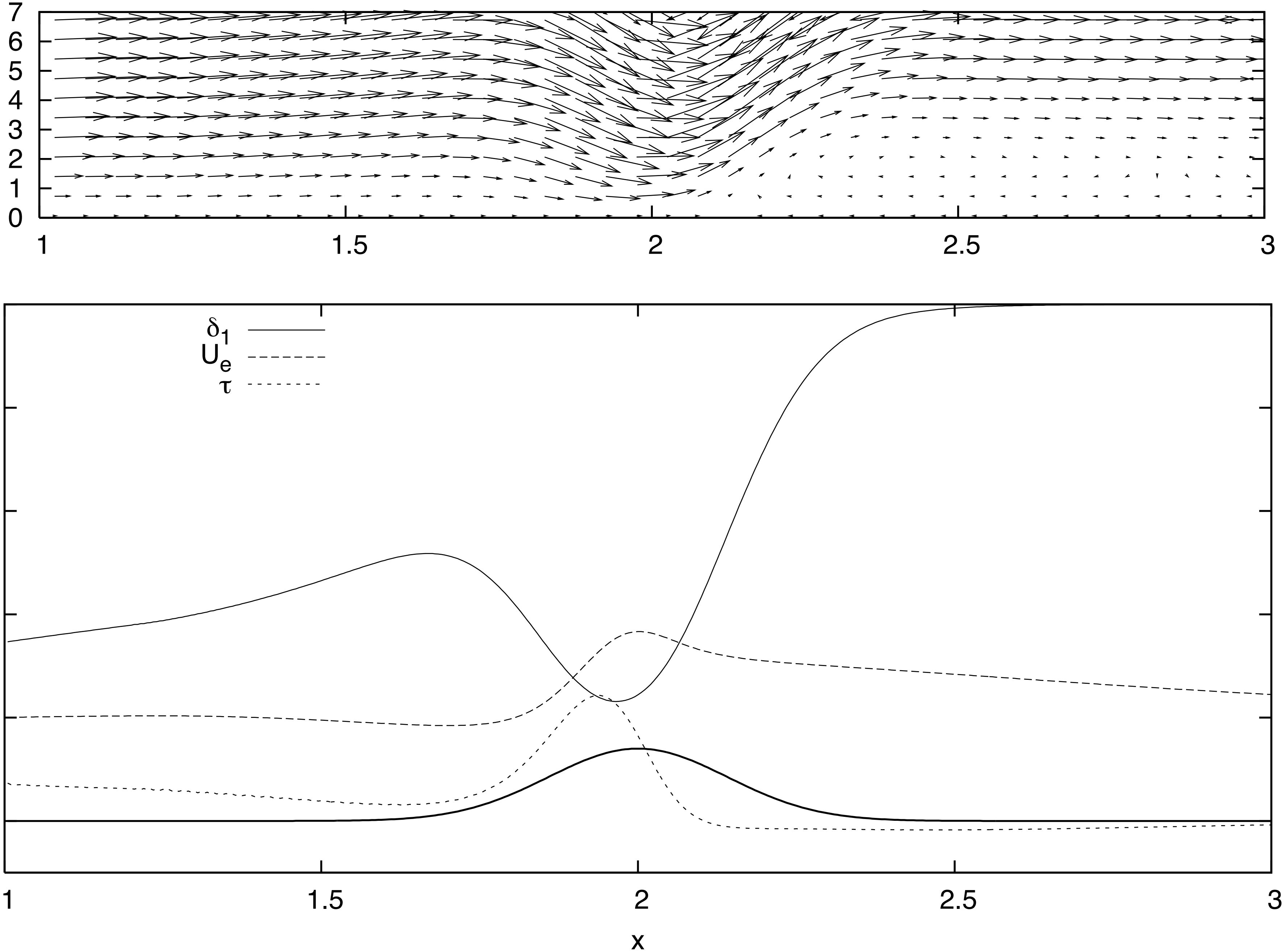

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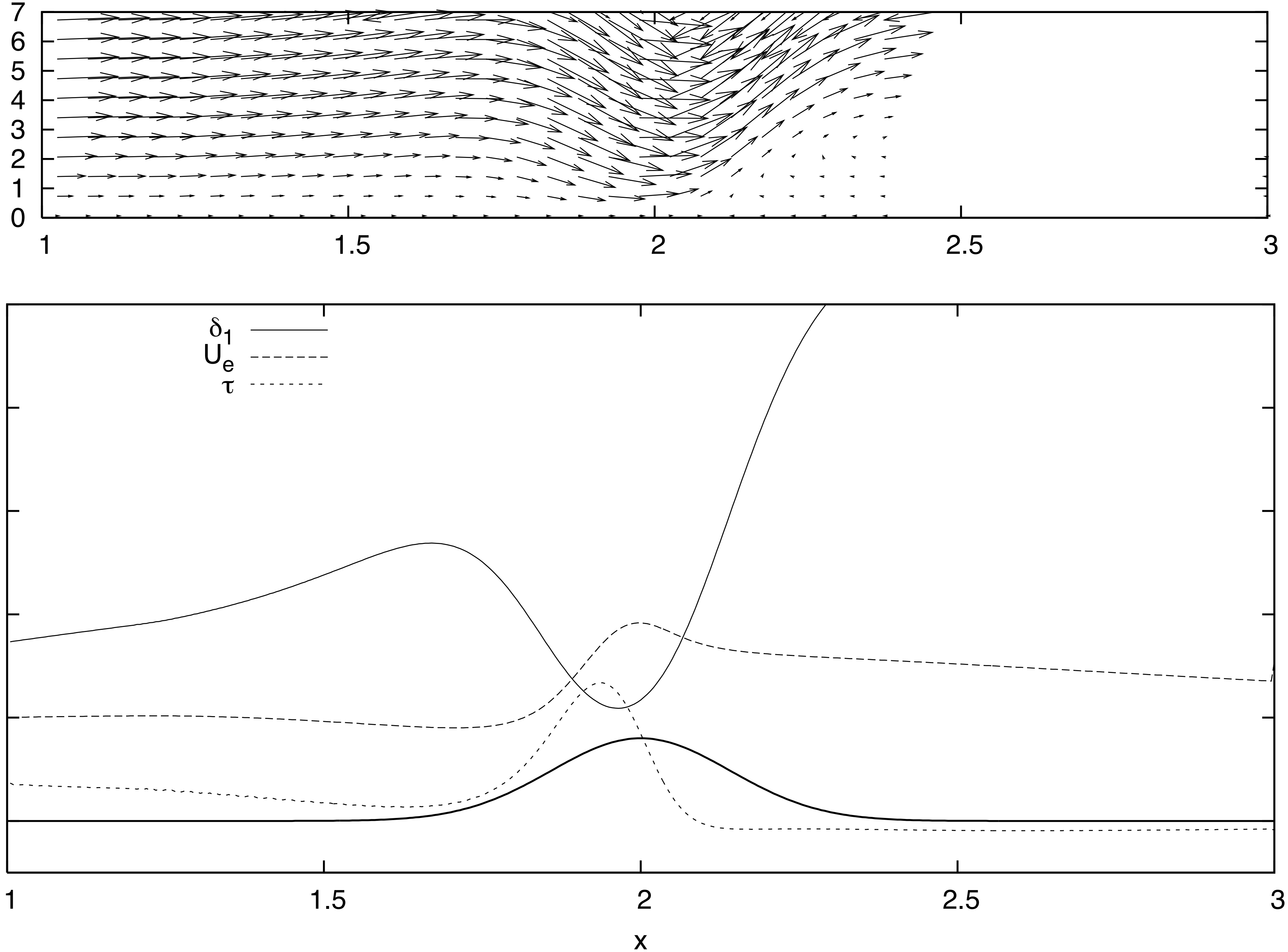

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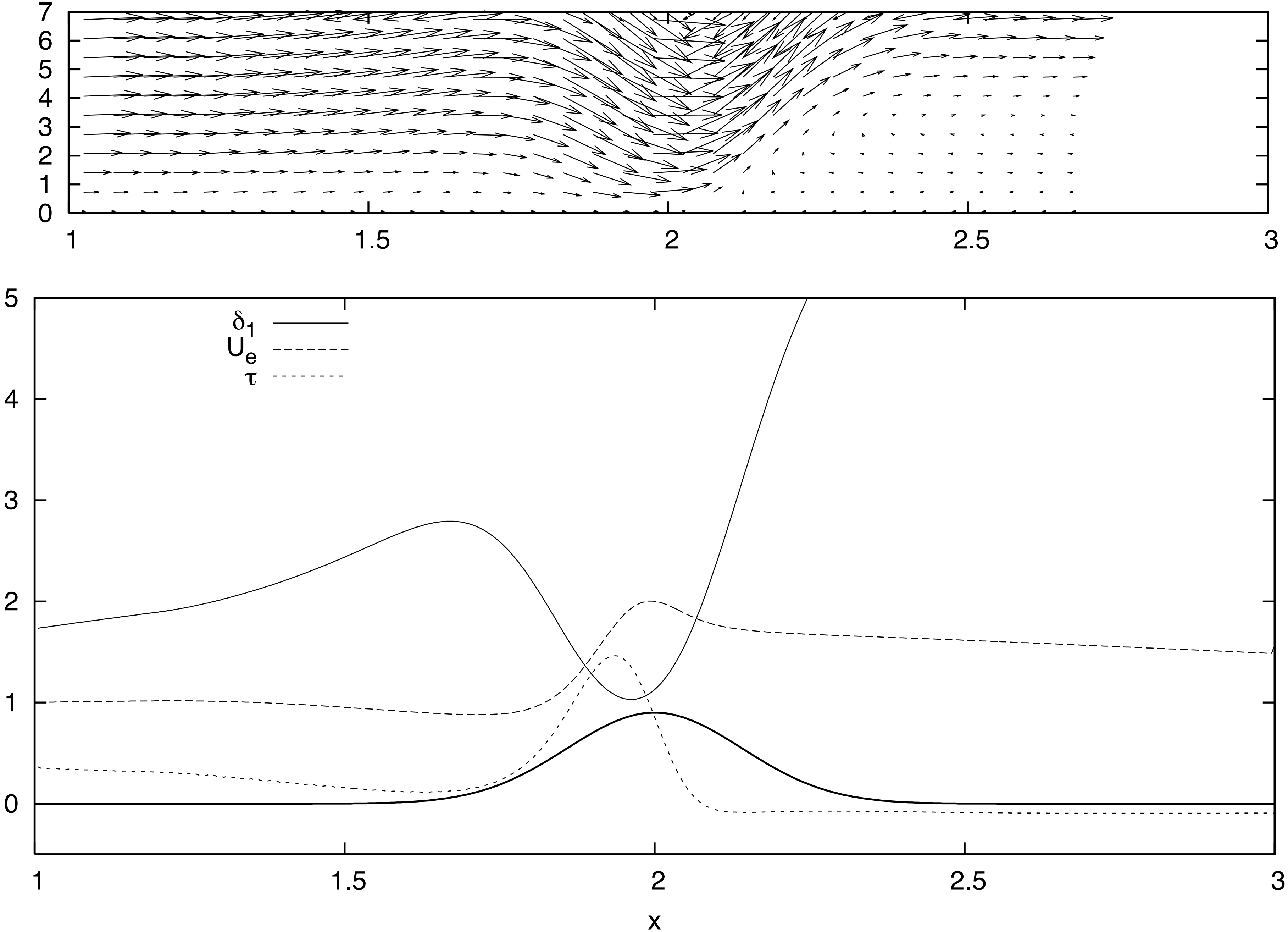
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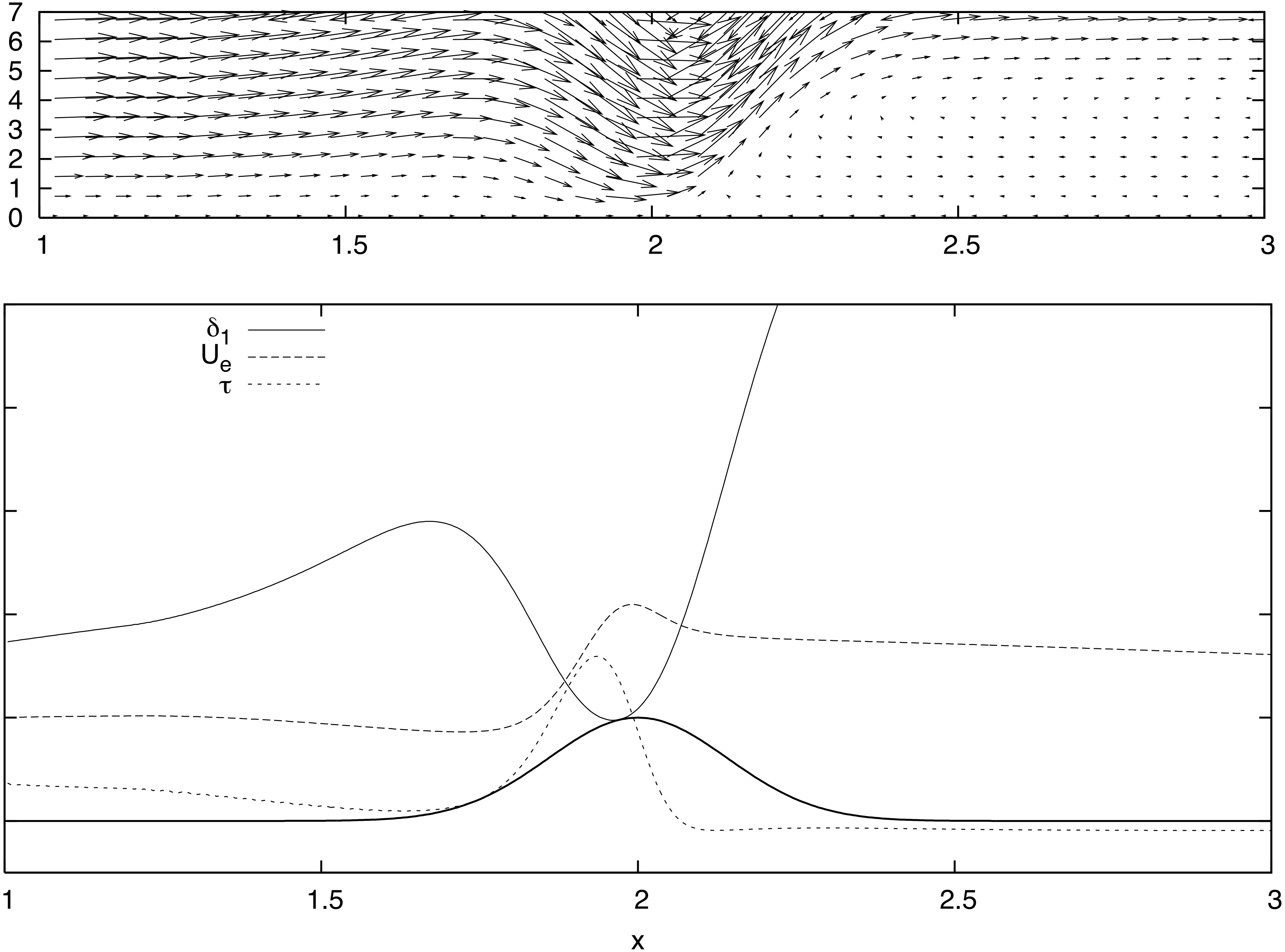

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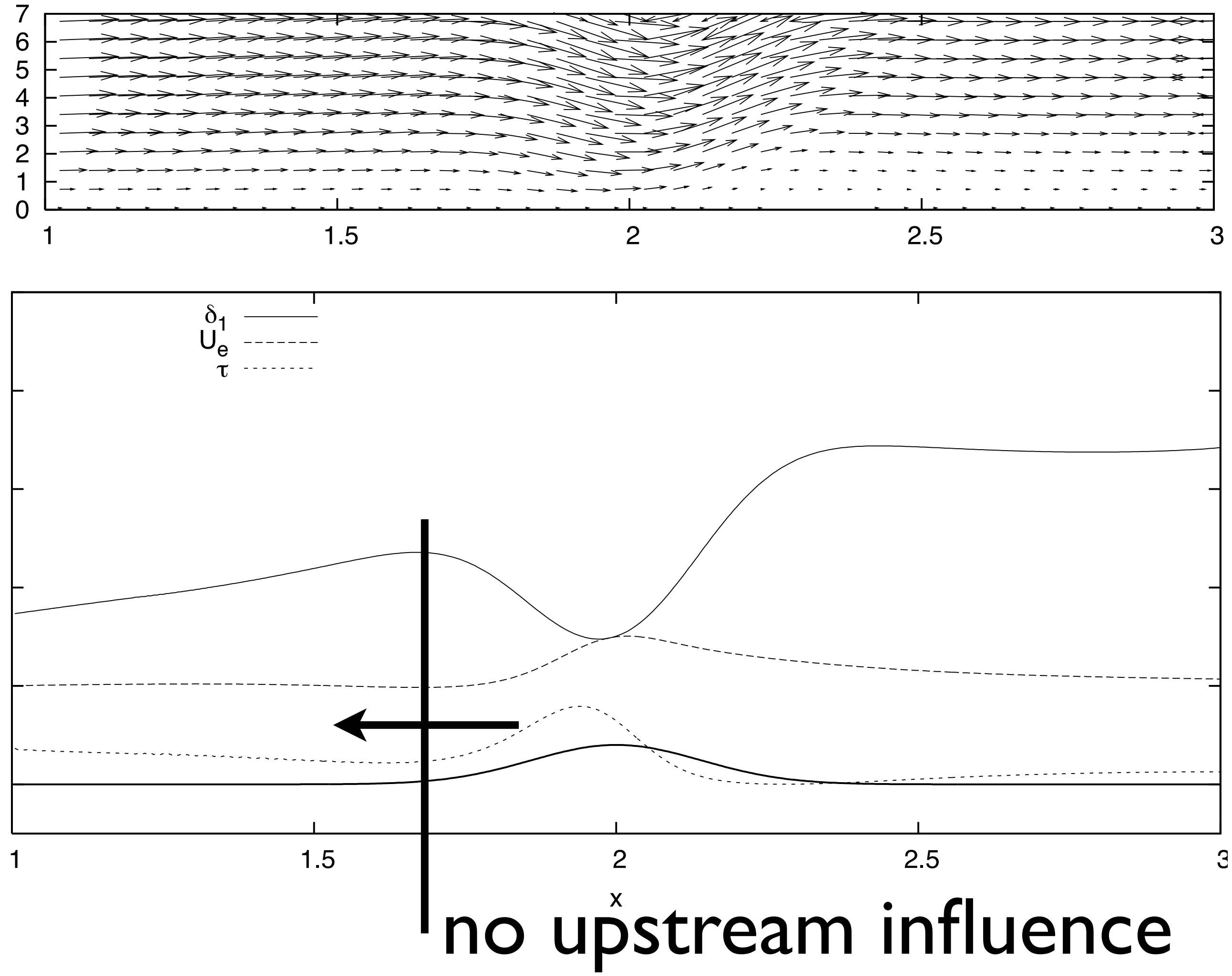


Figure 16: Incompressible flow [click to launch the movie, Adobe Reader required]. Top the velocity field $\tilde{u}, \tilde{v}$ (Prandtl transform), bottom the wall, here a bump, the displacement thickness $\tilde{\delta}_1$ (starting from Blasius value 1.7 in $\bar{x} = 1$), the skin friction (starting from Blasius value 0.3 in $\bar{x} = 1$) and the outer velocity starting from Ideal Fluid value 1 in $\bar{x} = 1$. A positive disturbance of the wall increases the velocity and decreases the displacement. Separation may occur after the bump, or before the tough.

- convective diffusive balance
- Prandtl equation with different boundary conditions
- separation of the flow near the wall
- key role of the displacement thickness
- interaction between two layers

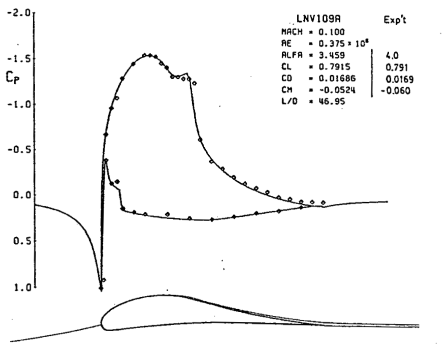
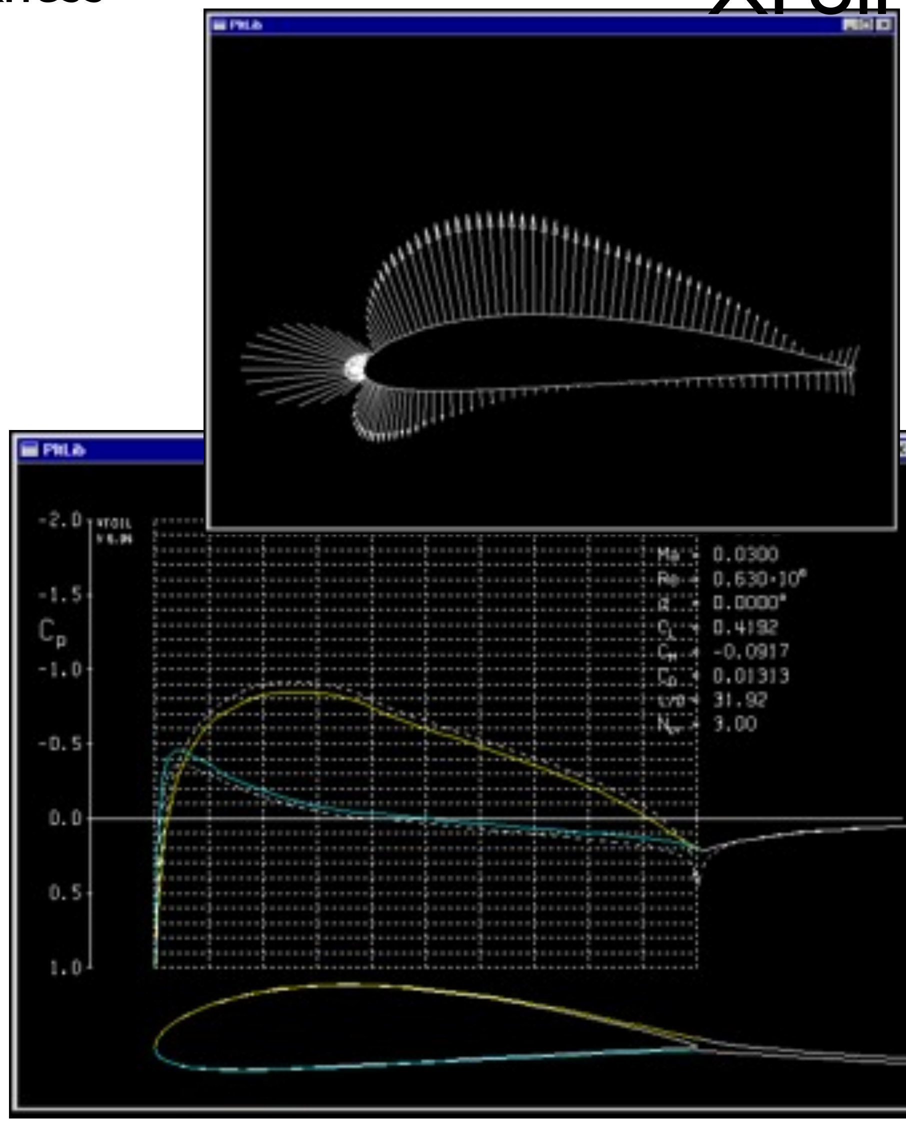


Fig. 9 LNV109A calculated and experimental pressure distributions.

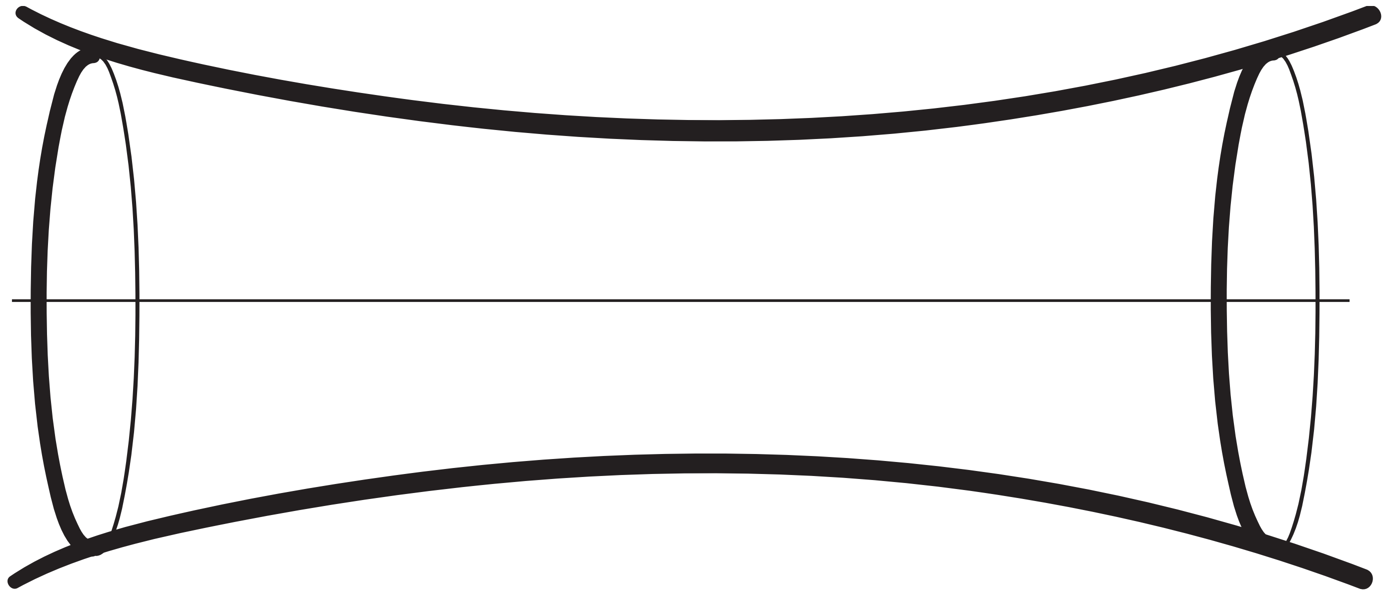
xample of comparison of IBL computation, Drela & Giles [7]

XFoil

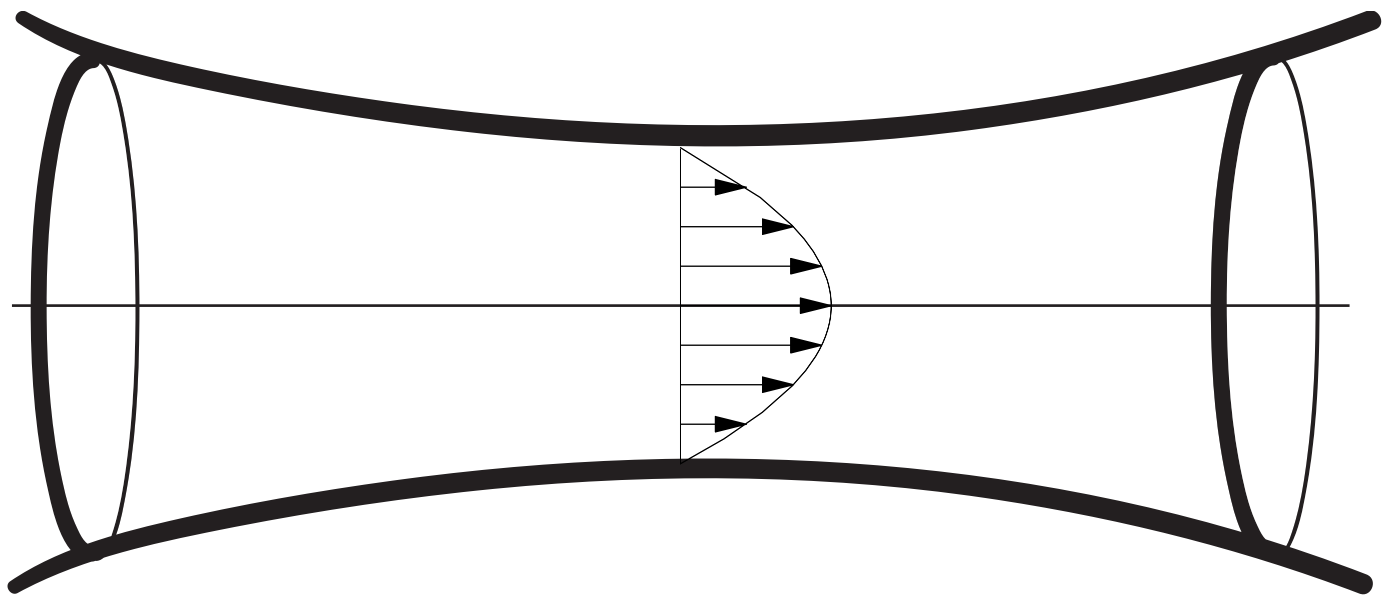


Le Balleur, 1977
Veldman 81 NLR in Amsterdam
Carter 79, Jameson at Stanford.
Cebeci applied IVI at Boeing
Lock & Williams 87 RAE
Neiland and Sychev at the TsAGI in USSR

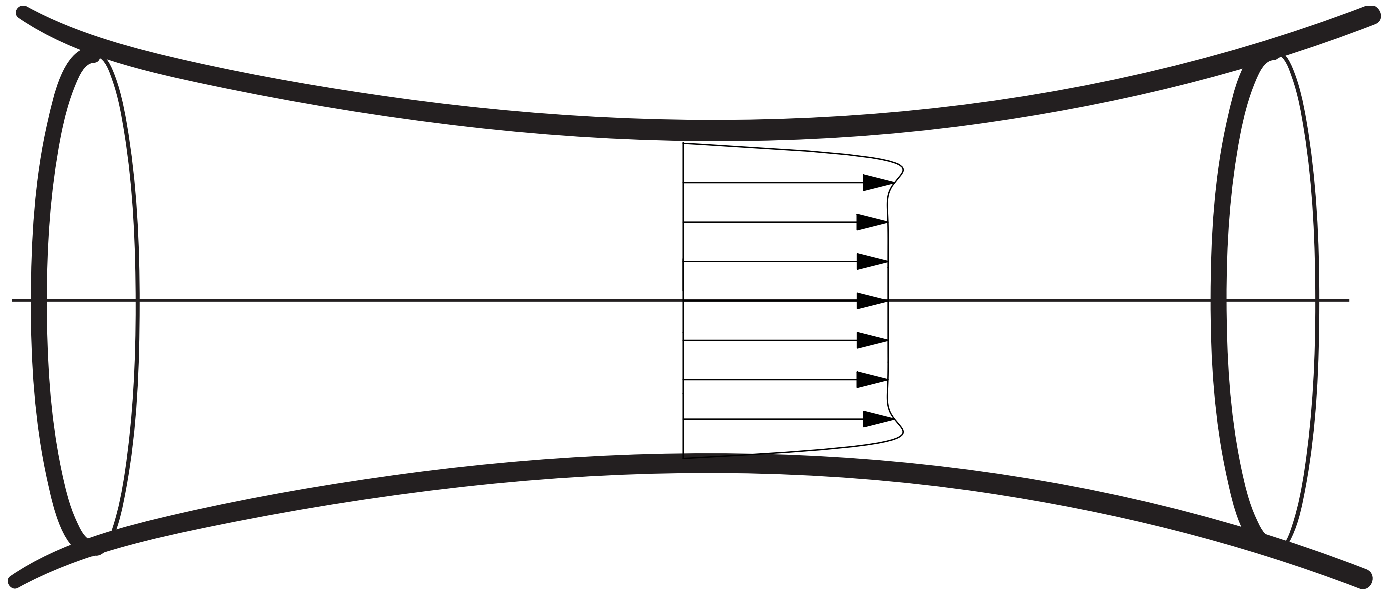
- now, we present a set of equation which includes IBL in tubes: RNSP



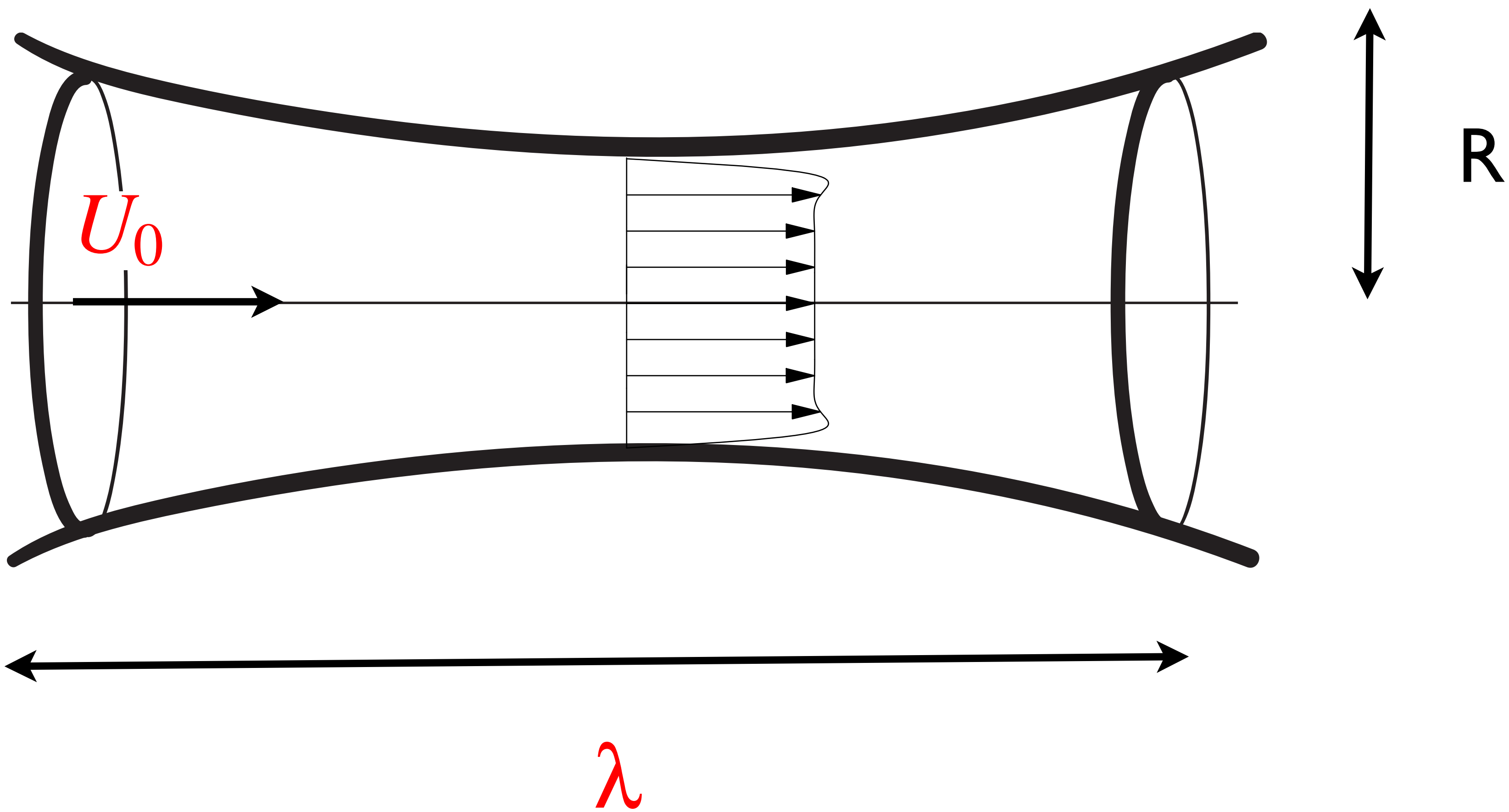
straight pipe, smooth walls, symmetry



RNSP Equations

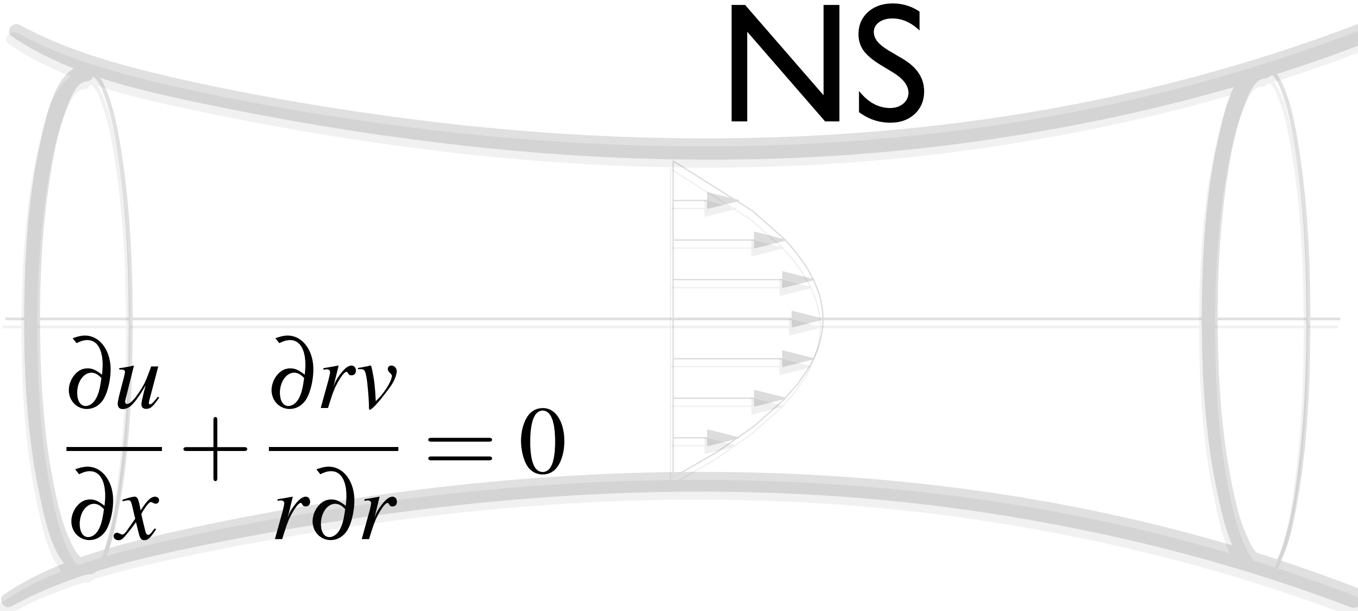


- simplified set
- deduced from orders of magnitude



$$R << \lambda$$

NS

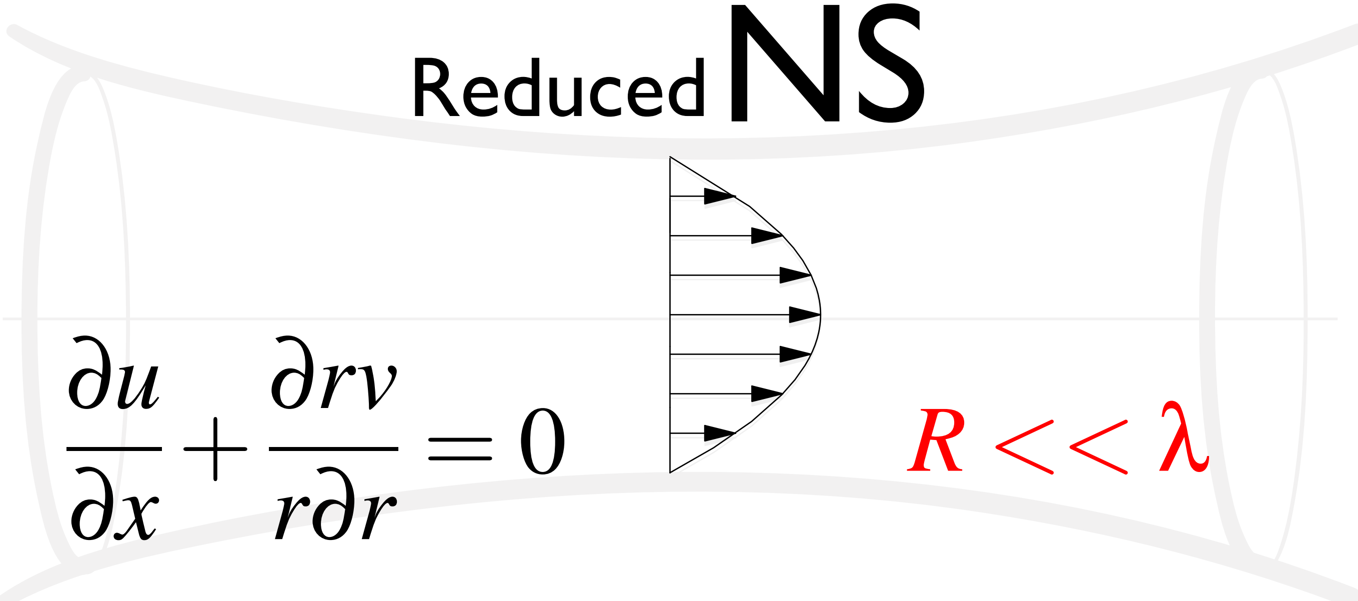


$$\frac{\partial u}{\partial x} + \frac{\partial r v}{r \partial r} = 0$$

$$\frac{\partial u}{\partial t} + u \frac{\partial}{\partial x} u + v \frac{\partial}{\partial r} u = -\frac{\partial p}{\rho \partial x} + \nu \frac{\partial^2}{\partial x^2} u + \nu \frac{\partial}{r \partial r} r \frac{\partial u}{\partial r}$$

$$\frac{\partial v}{\partial t} + u \frac{\partial}{\partial x} v + v \frac{\partial}{\partial r} v = -\frac{\partial p}{\rho \partial r} + \nu \frac{\partial^2}{\partial x^2} v + \nu \frac{\partial}{r \partial r} r \frac{\partial v}{\partial r}$$

Reduced NS



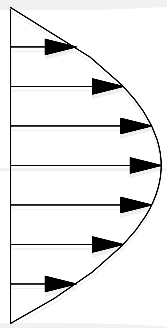
$$\frac{\partial u}{\partial x}+\frac{\partial rv}{r\partial r}=0$$

$R << \lambda$

$$\frac{\partial u}{\partial t}+u\frac{\partial}{\partial x}u+v\frac{\partial}{\partial r}u=-\frac{\partial p}{\rho\partial x}+\nu\cancel{\frac{\partial^2}{\partial x^2}}u+\nu\frac{\partial}{r\partial r}r\frac{\partial u}{\partial r}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial}{\partial x}v+v\frac{\partial}{\partial r}v=-\frac{\partial p}{\rho\partial r}+\nu\cancel{\frac{\partial^2}{\partial x^2}}v+\nu\frac{\partial}{r\partial r}r\frac{\partial v}{\partial r}$$

RNS/P



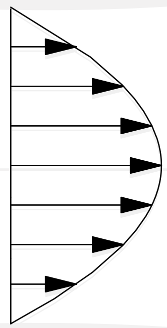
$$\frac{\partial u}{\partial x} + \frac{\partial r v}{r \partial r} = 0$$

$R << \lambda$ $\qquad V \sim U_0 \frac{R}{\lambda}$

$$\frac{\partial u}{\partial t} + u\frac{\partial}{\partial x}u + v\frac{\partial}{\partial r}u = -\frac{\partial p}{\rho \partial x} + \nu \cancel{\frac{\partial^2}{\partial x^2}} u + \nu \frac{\partial}{r \partial r} r \frac{\partial u}{\partial r}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial}{\partial x}v + v\frac{\partial}{\partial r}v = -\frac{\partial p}{\rho \partial r} + \nu \cancel{\frac{\partial^2}{\partial x^2}} v + \nu \frac{\partial}{r \partial r} r \frac{\partial v}{\partial r}$$

RNS/P



$$\frac{\partial u}{\partial x} + \frac{\partial rv}{r\partial r} = 0$$

$$p \sim \rho U_0^2$$

$$R << \lambda$$

$$V \sim U_0 \frac{R}{\lambda}$$

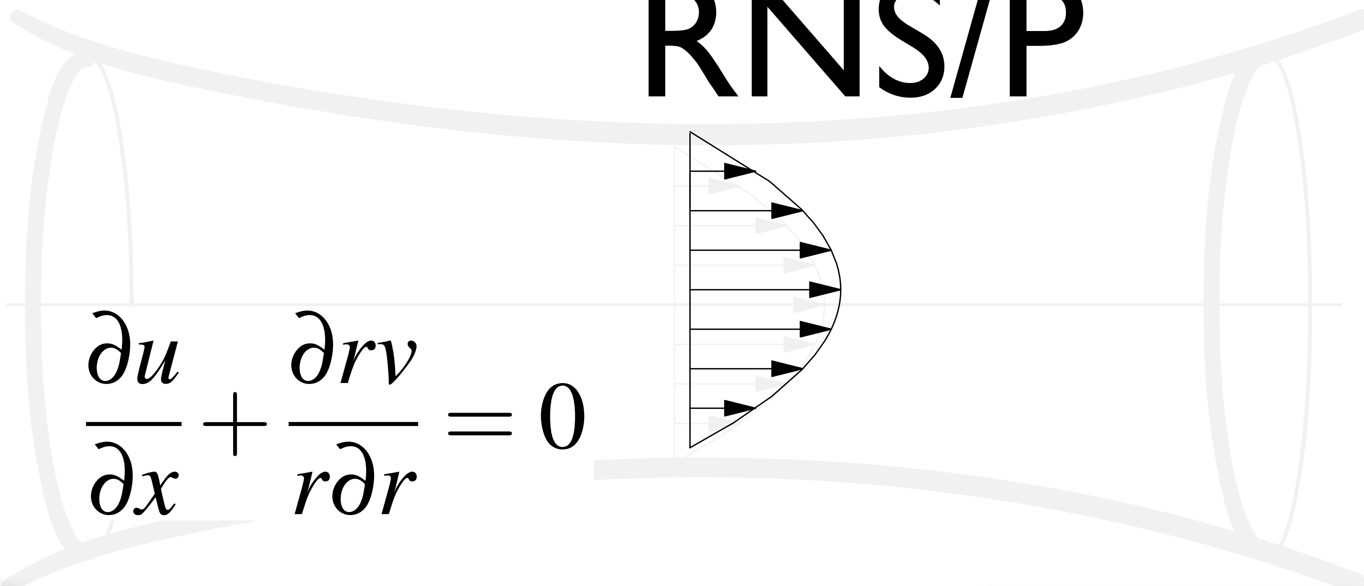
$$\frac{\partial u}{\partial t} + u\frac{\partial}{\partial x}u + v\frac{\partial}{\partial r}u = -\frac{\partial p}{\rho \partial x} + \nu \cancel{\frac{\partial^2}{\partial x^2}} u + \nu \frac{\partial}{r\partial r} r \frac{\partial u}{\partial r}$$

$$\cancel{\frac{\partial v}{\partial t}} + u\cancel{\frac{\partial}{\partial x}}v + v\cancel{\frac{\partial}{\partial r}}v = -\frac{\partial p}{\rho \partial r} + \nu \cancel{\frac{\partial^2}{\partial x^2}} v + \nu \frac{\partial}{r\partial r} \cancel{r} \frac{\partial v}{\partial r}$$

$$\frac{U_0^2}{\lambda} \sim \nu \frac{U_0}{R^2}$$

$$\lambda = R Re$$

RNS/P



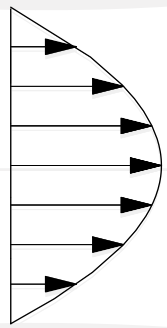
$$\frac{\partial u}{\partial x}+\frac{\partial rv}{r\partial r}=0$$

$$\boxed{\frac{\partial u}{\partial t}}+u\frac{\partial}{\partial x}u+v\frac{\partial}{\partial r}u=-\frac{\partial p}{\rho\partial x}+\boxed{\nu\frac{\partial}{r\partial r}r\frac{\partial u}{\partial r}}$$

$$0=-\frac{\partial p}{\rho\partial r}$$

RNS/P

Prandtl



$$\lambda = R Re$$

$$p \sim \rho U_0^2$$

$$V \sim U_0 \frac{R}{\lambda}$$

$$\frac{\partial u}{\partial x} + \frac{\partial r v}{r \partial r} = 0$$

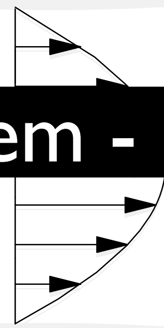
$$u \frac{\partial}{\partial x} u + v \frac{\partial}{\partial r} u = -\frac{\partial p}{\rho \partial x} + \nu \frac{\partial}{r \partial r} r \frac{\partial u}{\partial r}$$

$$0 = -\frac{\partial p}{\rho \partial r}$$

for the pipe itself

RNS/P

Prandtl



Parabolic Problem - Marching Problem

$$\frac{\partial u}{\partial x} + \frac{\partial rv}{r\partial r} = 0$$

$$u\frac{\partial}{\partial x}u + v\frac{\partial}{\partial r}u = -\frac{\partial p}{\rho\partial x} + \nu\frac{\partial}{r\partial r}r\frac{\partial u}{\partial r}$$

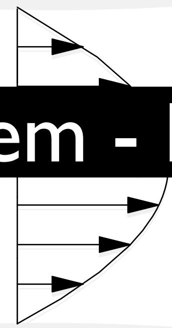
$$0 = -\frac{\partial p}{\rho\partial r}$$

first velocity given, no slip at the wall.
Pressure drop is a result

RNS/P

Prandtl

Parabolic Problem - Marching Problem



$$\frac{\partial u}{\partial x} + \frac{\partial rv}{r\partial r} = 0$$

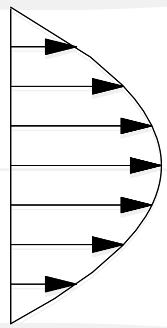
$$u\frac{\partial}{\partial x}u + v\frac{\partial}{\partial r}u = -\frac{\partial p}{\rho\partial x} + \nu\frac{\partial}{r\partial r}r\frac{\partial u}{\partial r}$$

$$0 = -\frac{\partial p}{\rho\partial r}$$

This system of equations is a good candidate to compute a large variety of flow in pipes (or in 2D) as long as there is no upstream influence

RNS/P

Prandtl



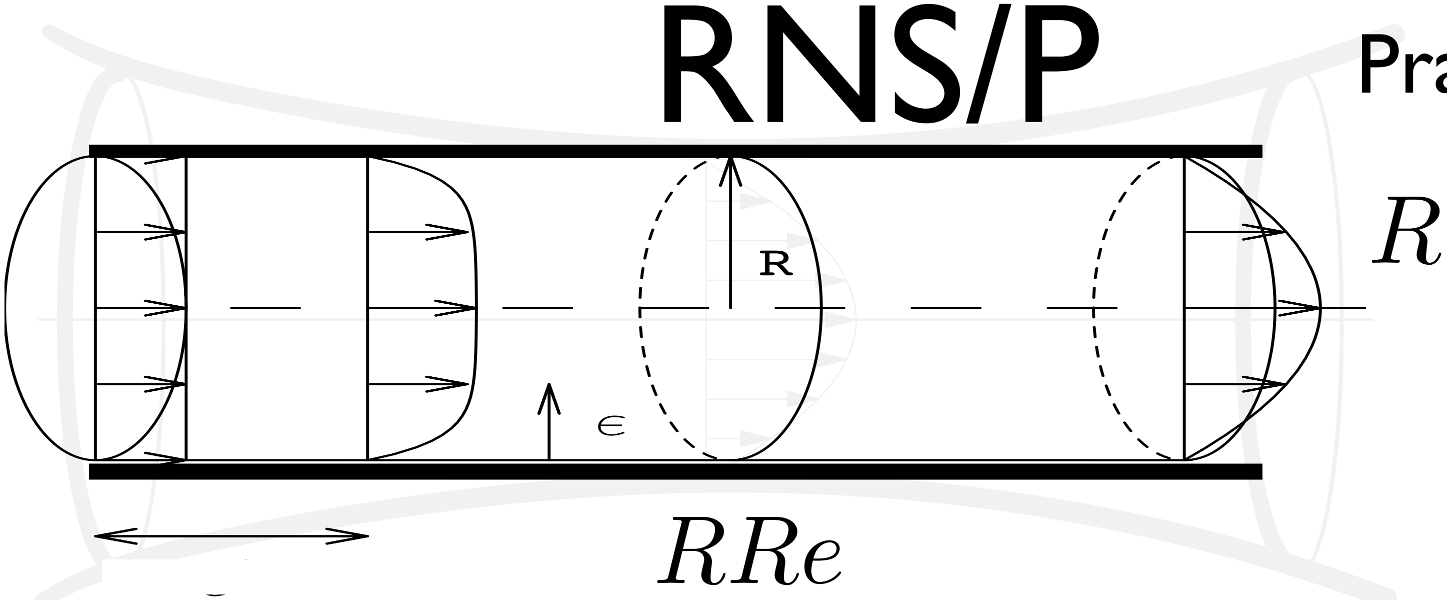
$$\frac{\partial u}{\partial x} + \frac{\partial r v}{r \partial r} = 0$$

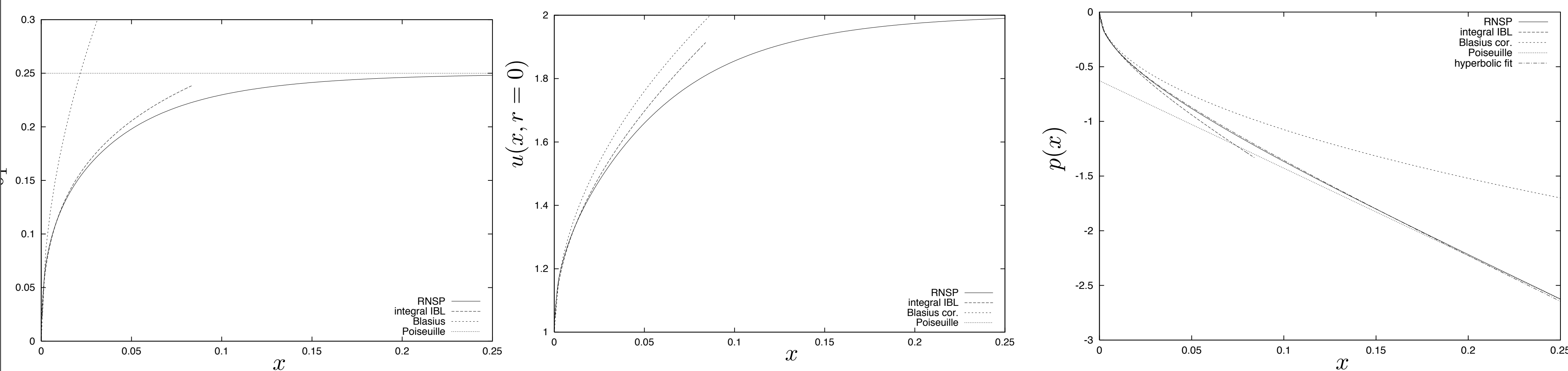
$$u\frac{\partial}{\partial x}u + v\frac{\partial}{\partial r}u = -\frac{\partial p}{\rho \partial x} + \nu\frac{\partial}{r\partial r}r\frac{\partial u}{\partial r}$$

$$0 = -\frac{\partial p}{\rho \partial r}$$

this system is a kind of Graetz formulation...

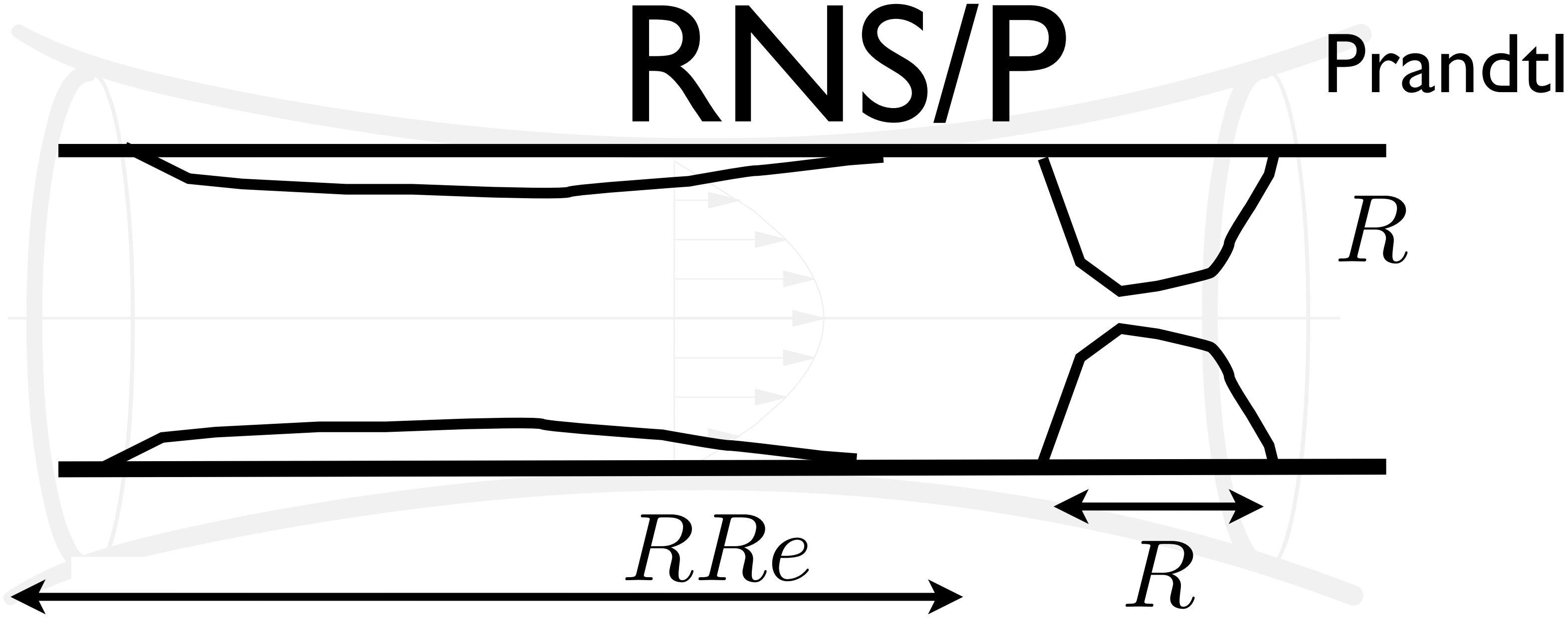
RNS/P





this system is a kind of Graetz formulation...

RNS/P

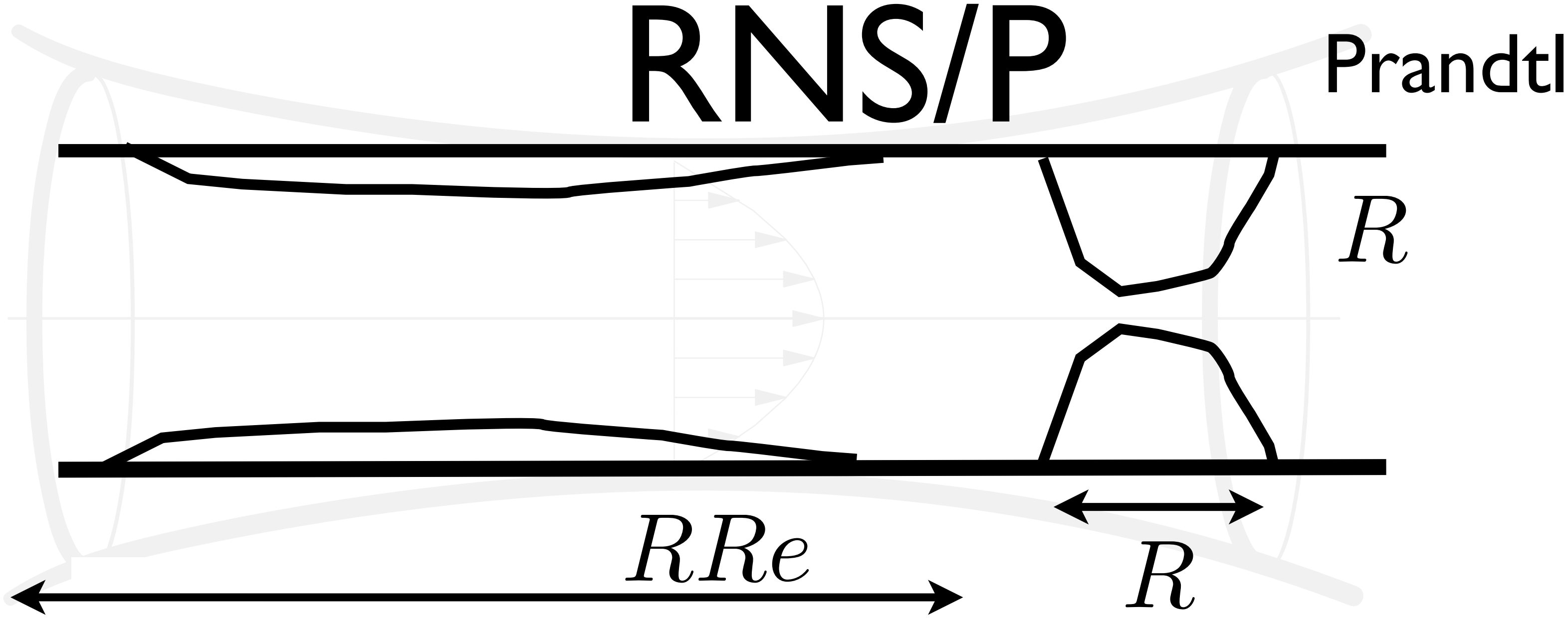


this system valid ONLY for long bumps RRe

and valid for large Reynolds

but we will test it after for short ones R

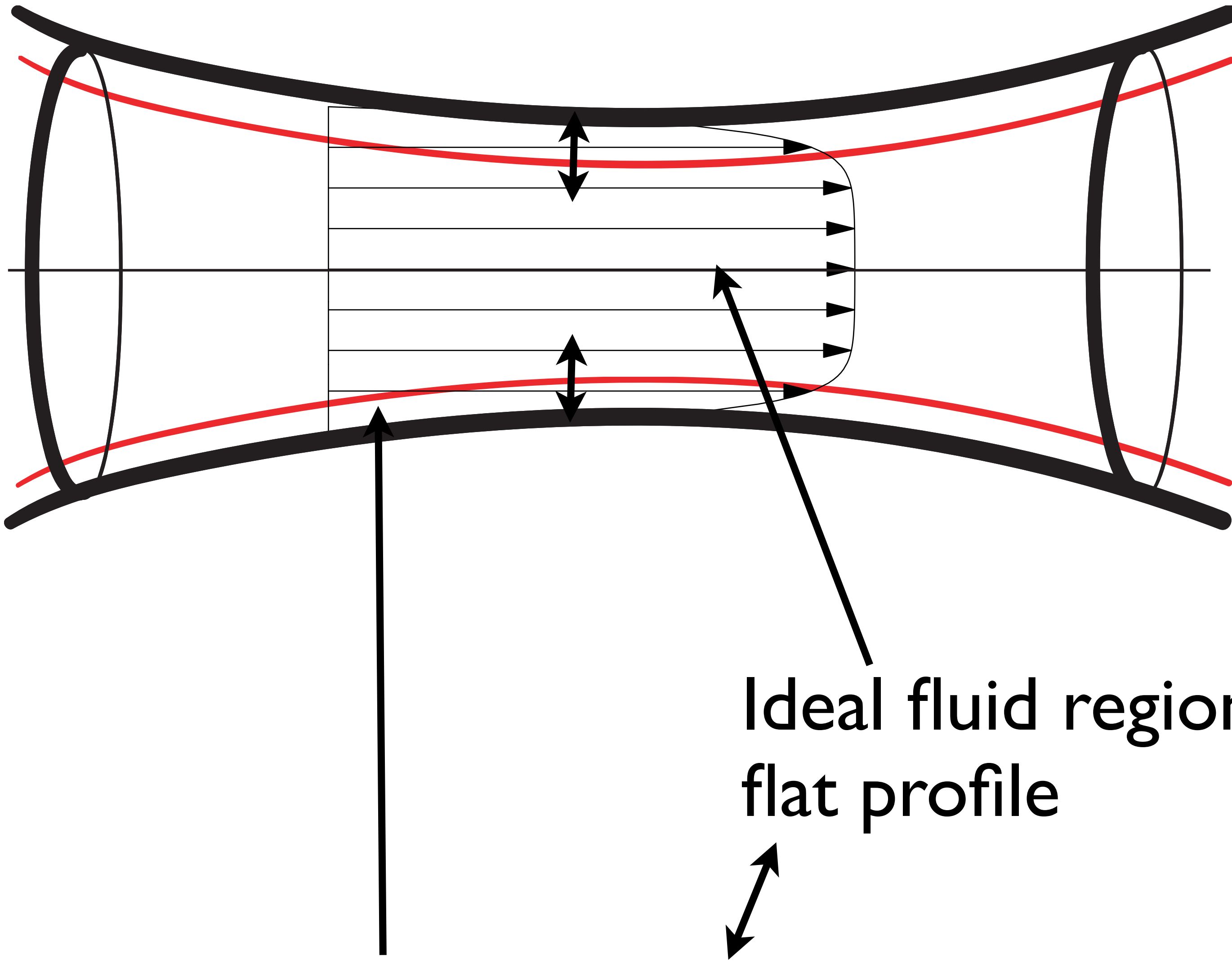
RNS/P



this system valid ONLY for long bumps RRe
and valid for large Reynolds
but we will test it after for short ones R

and it works!

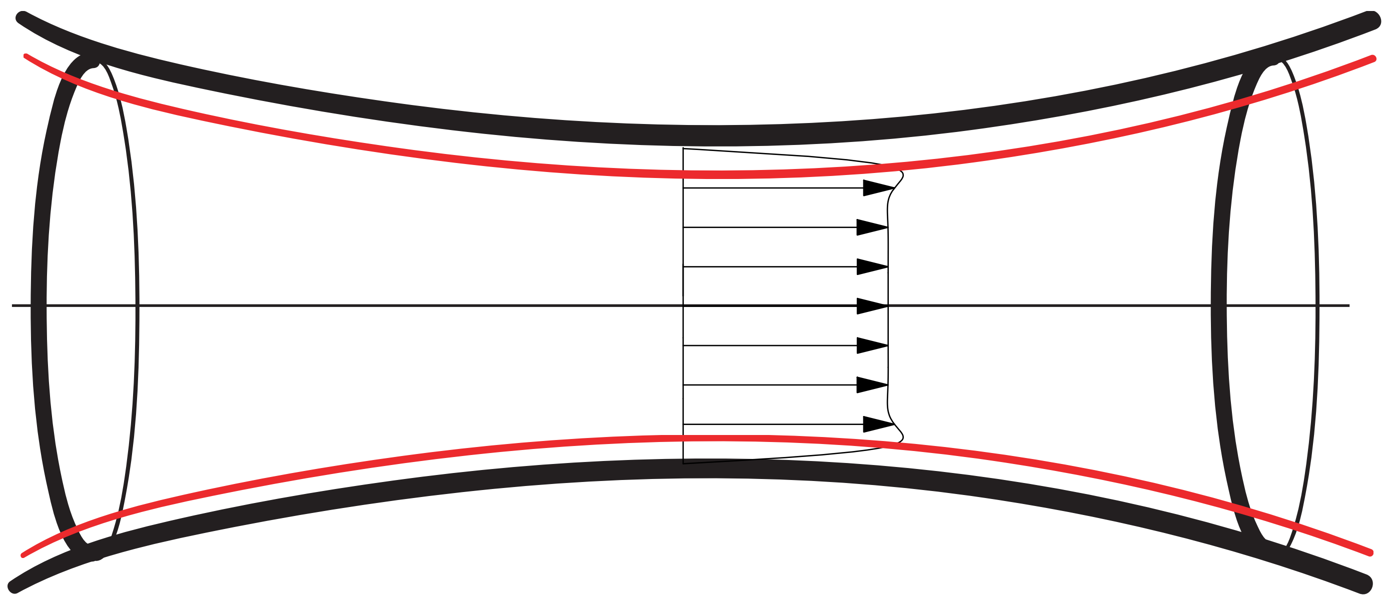
Interactive Boundary Layer



Ideal fluid region
flat profile

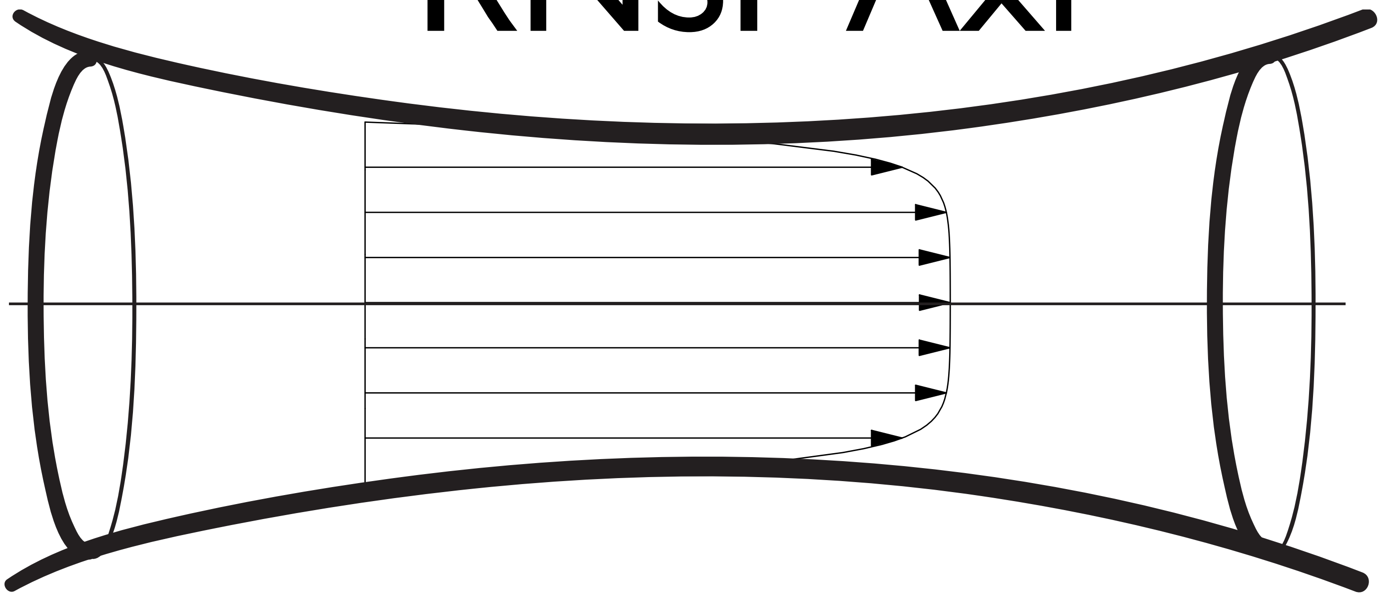
Viscous region: boundary layer

Interactive Boundary Layer



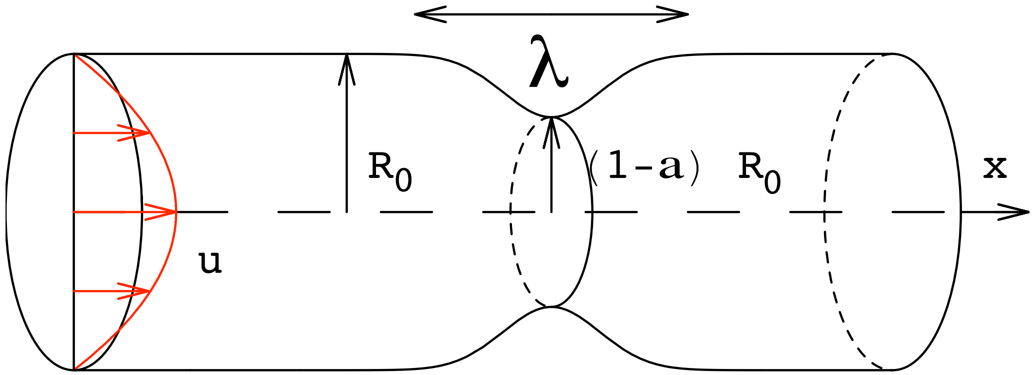
IBL is included in a larger system: RNSP

RNSP Axi



- Flow in a stenozed vessel
- steady, rigid wall

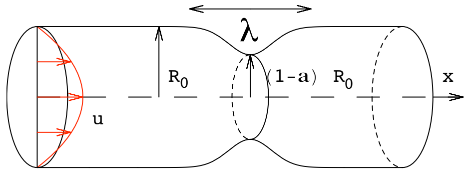
RNSP Scales



Using:

$x^* = xR_0Re$, $r^* = rR_0$, $u^* = U_0u$, $v^* = \frac{U_0}{Re}v$,
$p^* = p_0^* + \rho_0 U_0^2 p$ and $\tau^* = \frac{\rho U_0^2}{Re}\tau$

the following partial differential system is obtained from Navier Stokes as $Re \to \infty$:



RNSP: Reduced Navier Stokes/ Prandtl System

$$
\begin{aligned}
\frac{\partial}{\partial x}u + \frac{\partial}{r\partial r}rv &= 0,\\
(u\frac{\partial}{\partial x}u + v\frac{\partial}{\partial r}u) &= -\frac{\partial p}{\partial x} + \frac{\partial}{r\partial r}(r\frac{\partial}{\partial r}u),\\
0 &= -\frac{\partial p}{\partial r}.
\end{aligned}
$$

Parabolic Problem - Marching Problem

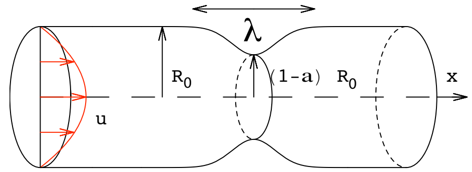
- axial symmetry ($\partial_r u = 0$ and $v = 0$ at $r = 0$),

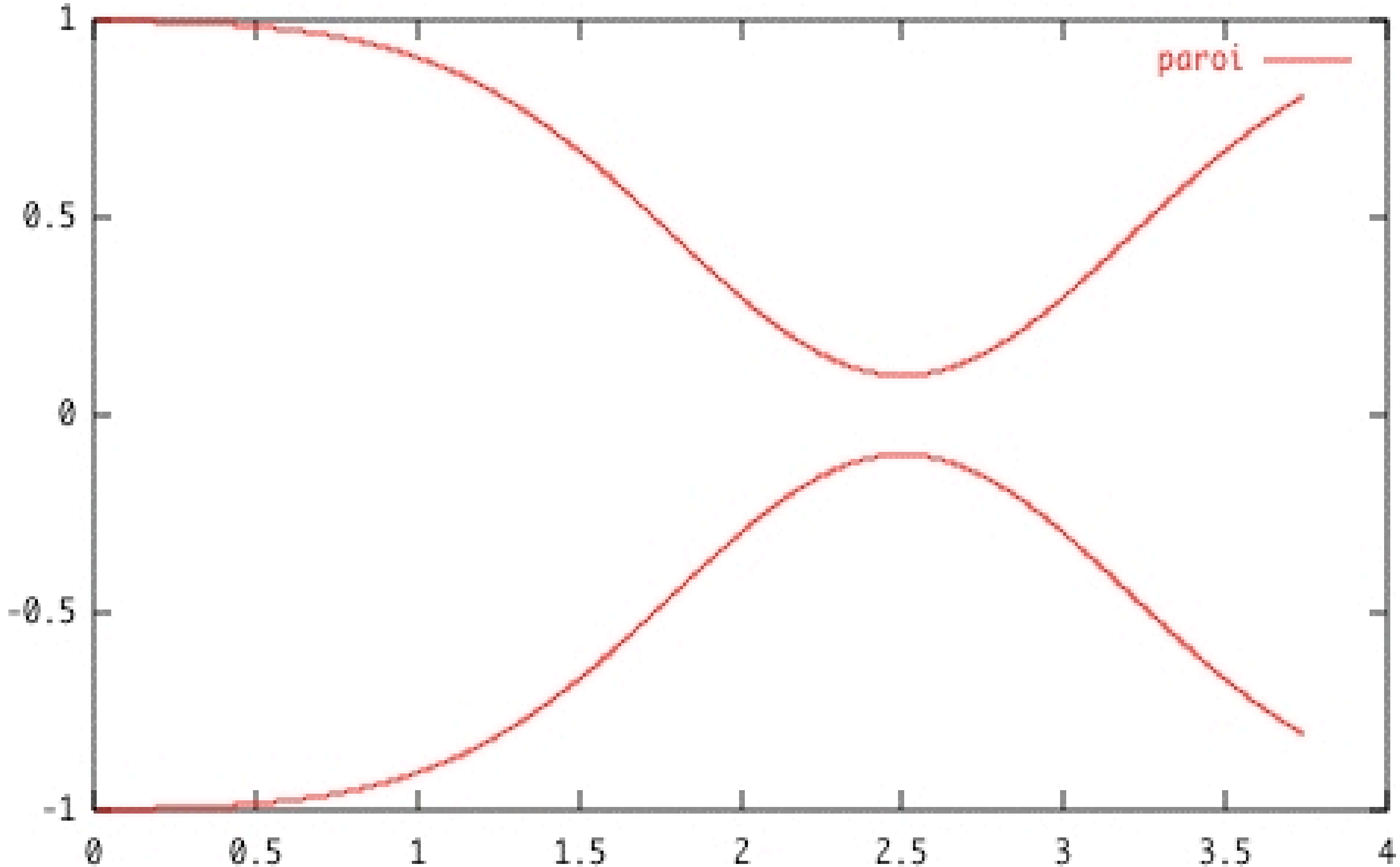
- no slip condition at the wall ($u = v = 0$ at $r = 1 - f(x)$),

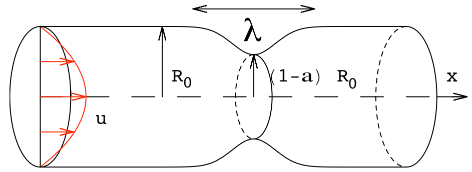
- the entry velocity profiles ($u(0, r)$ and $v(0, r)$) are given

- *no* output condition in $x_{out} = \frac{x^*_{out}}{R_0 Re}$

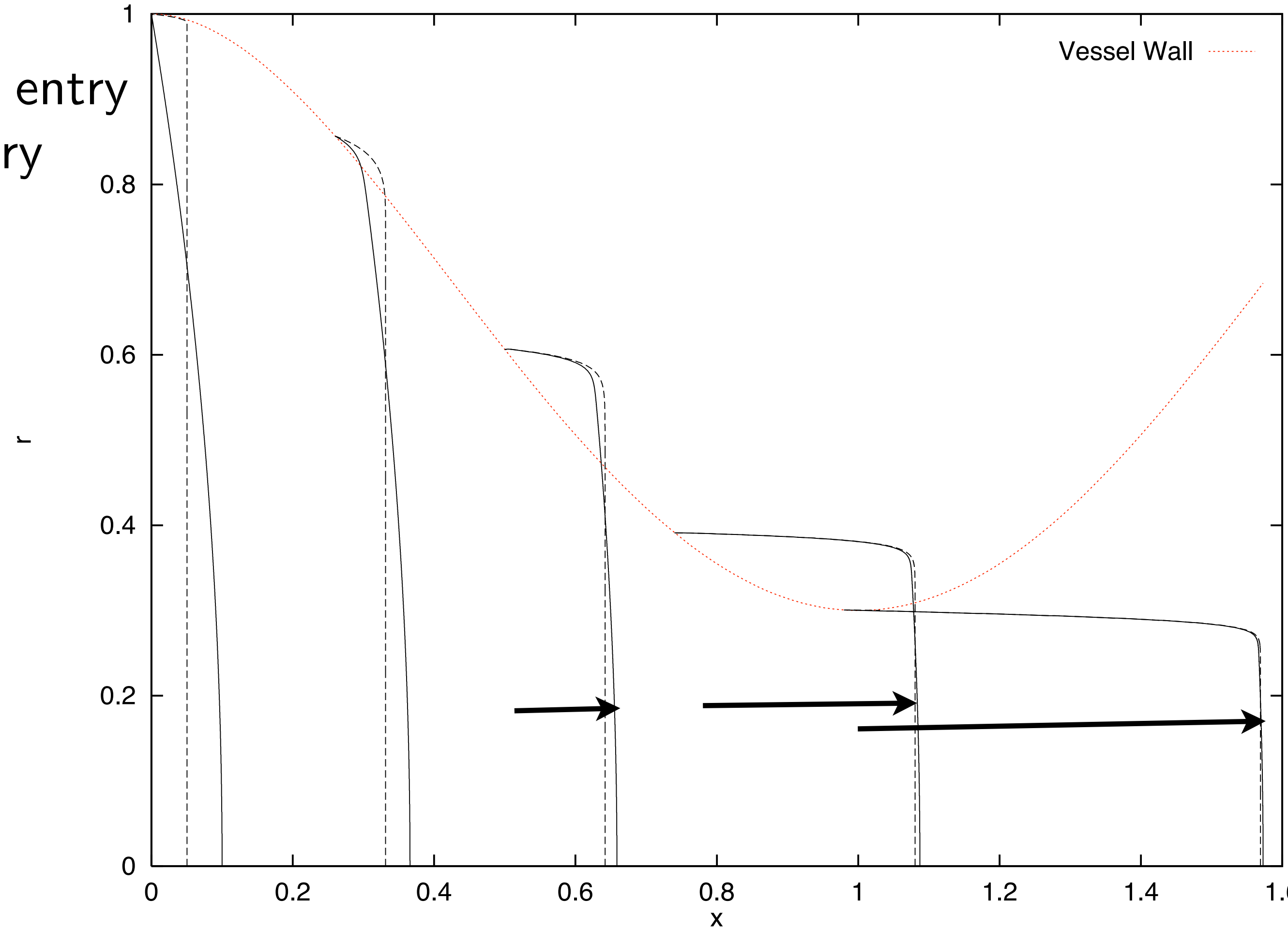
- streamwise marching, even when flow separation.

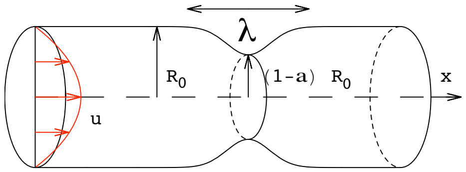

λ
R_0
(1-a) R_0
x
u


paroi
1
0.5
0
-0.5
-1
0
0.5
1
1.5
2
2.5
3
3.5
4

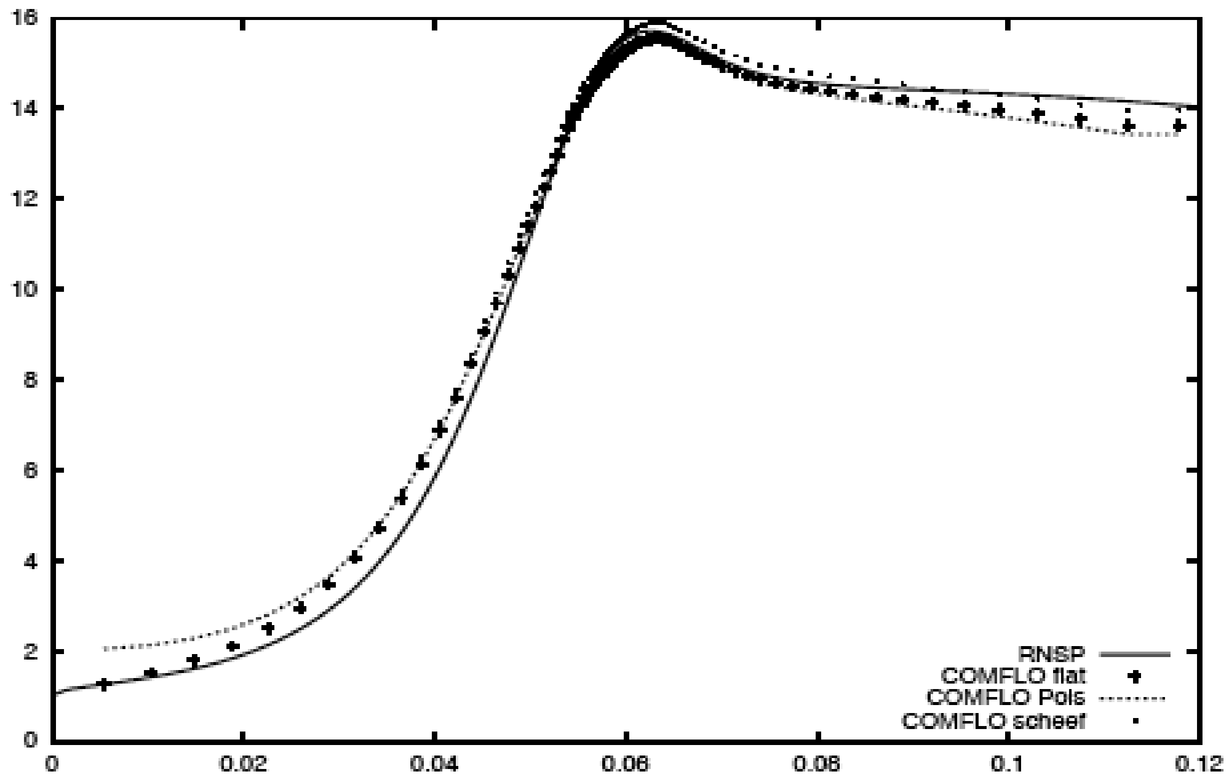


Evolution of the velocity profile along the convergent part of a 70% stenosis ($Re = 500$) ;
solid line: Poiseuille entry
broken line: flat entry

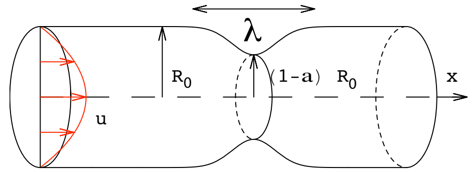




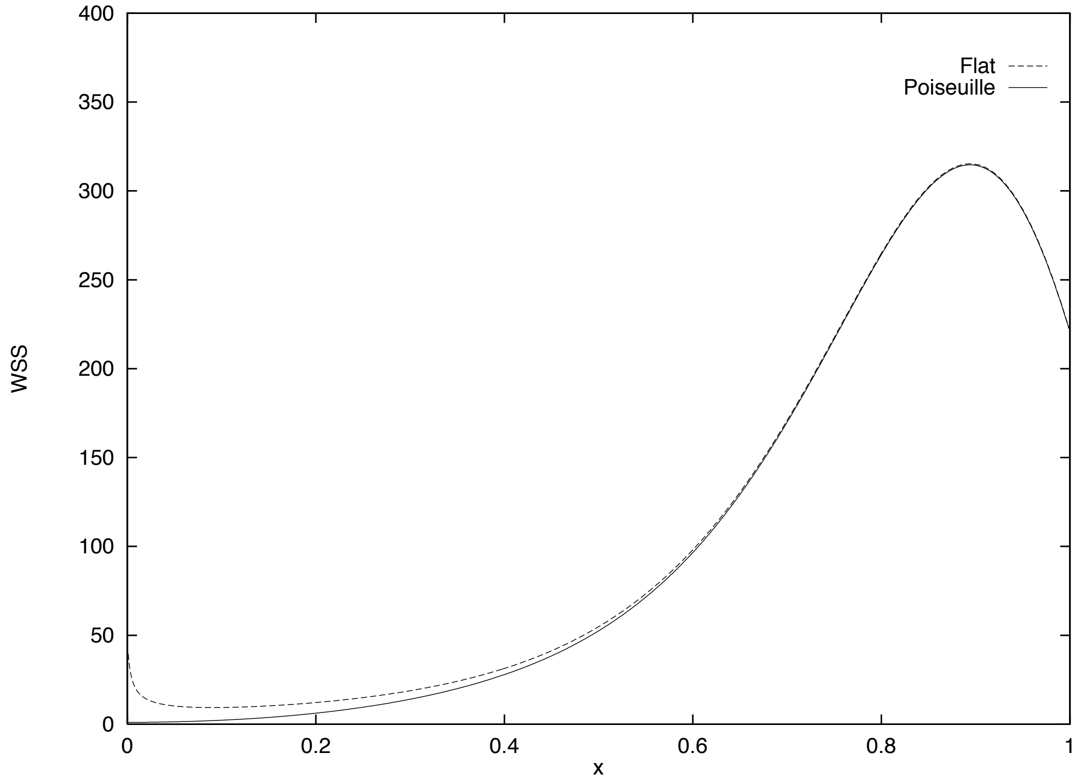
Testing asymmetry in the entry profile



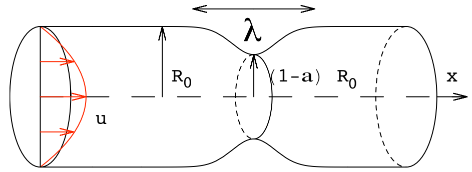
The velocities in the middle for Comflo and RNS.
Comflo uses here 50X50X100 points. Dimensionless scales!

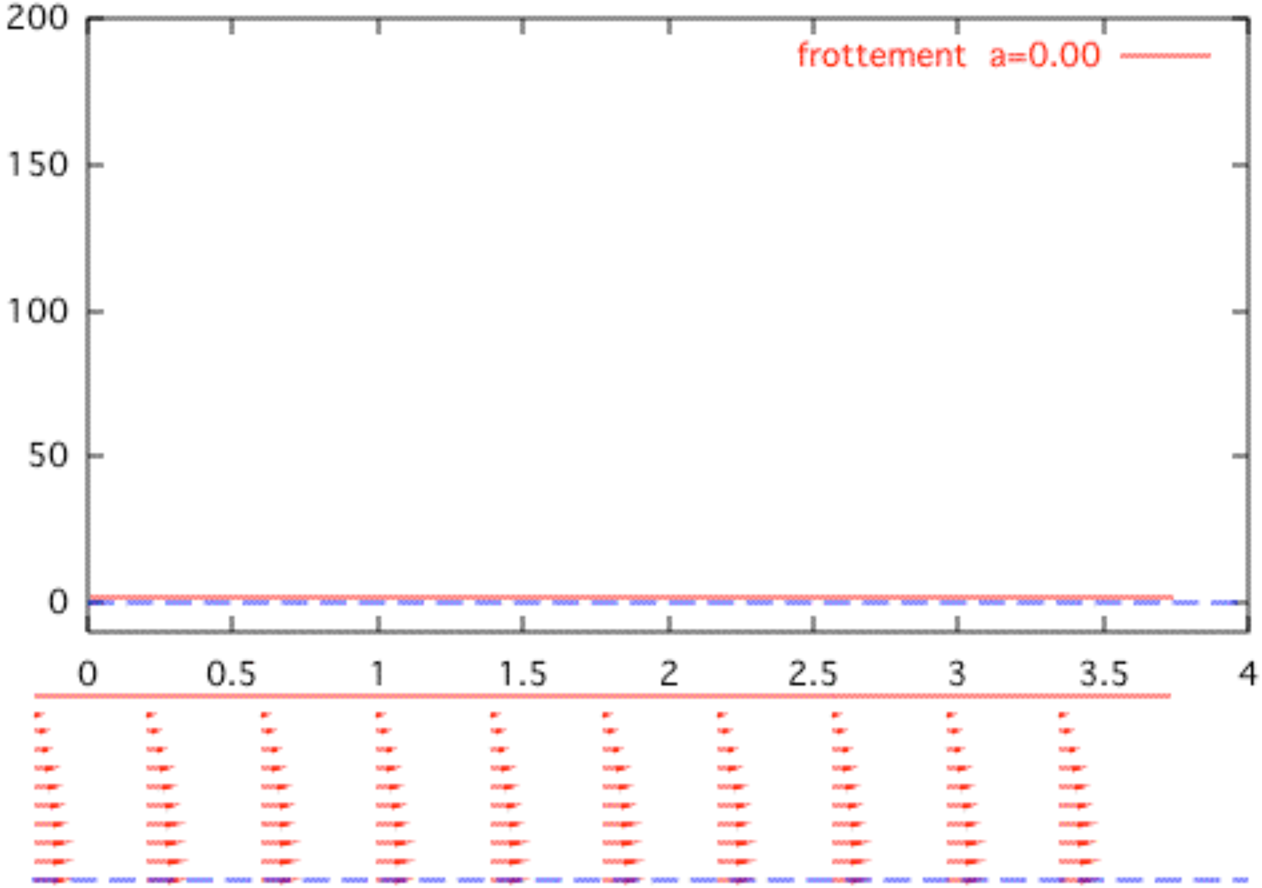


Wall Shear Stress

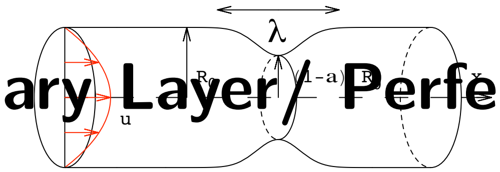


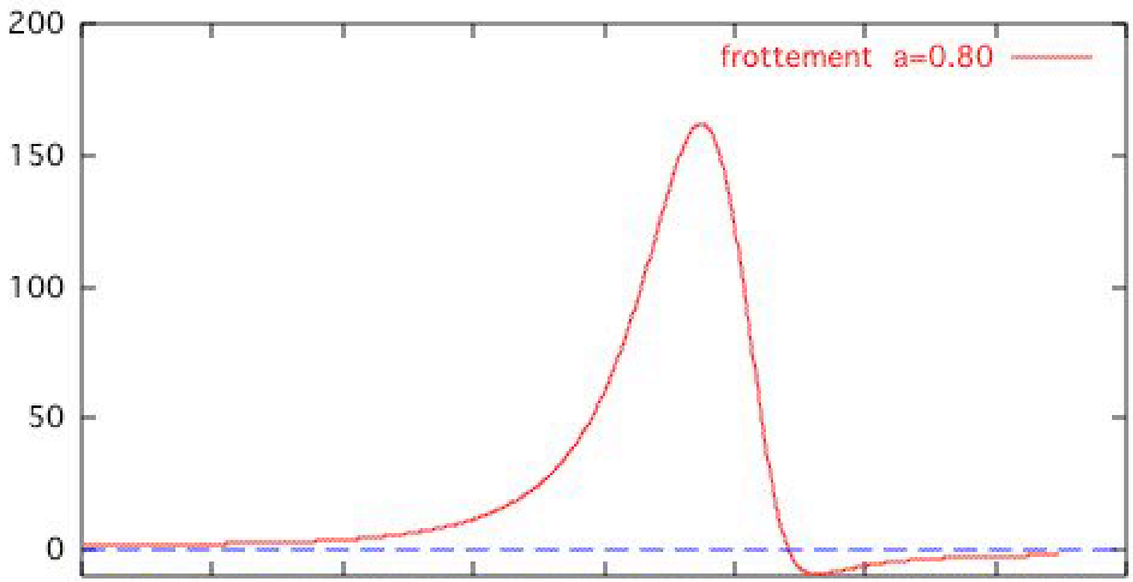
Evolution of the WSS distribution along the convergent part of a 70% stenosis ($Re = 500$) ; solid line: Poiseuille entry profile ; broken line: flat entry profile.


λ
R0
(1-a) R0
x
u


frottement a=0.00
200
150
100
50
0
0
0.5
1
1.5
2
2.5
3
3.5
4

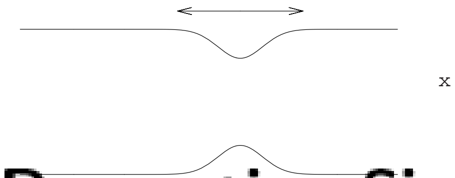
Boundary Layer / Perfect Fluid





thin boundary layer

Inviscid core



IBL integral: 1D equation Simplified Shear Stress

- variation of velocity (flux conservation) $\quad U_0 \to U_0/(1-\alpha-\delta_1)^2$

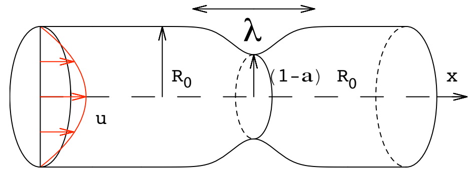
- acceleration: boundary layer $\delta_1 \simeq \frac{\lambda}{\sqrt{Re_\lambda}}$, with $Re_\lambda = \frac{\lambda U_0}{(1-\alpha)^2\nu} = \frac{Re\lambda}{(1-\alpha)^2}$

- WSS = (variation of velocity)/(boundary layer thickness) $=\frac{(Re/\lambda)^{1/2}}{(1-\alpha)^3}$

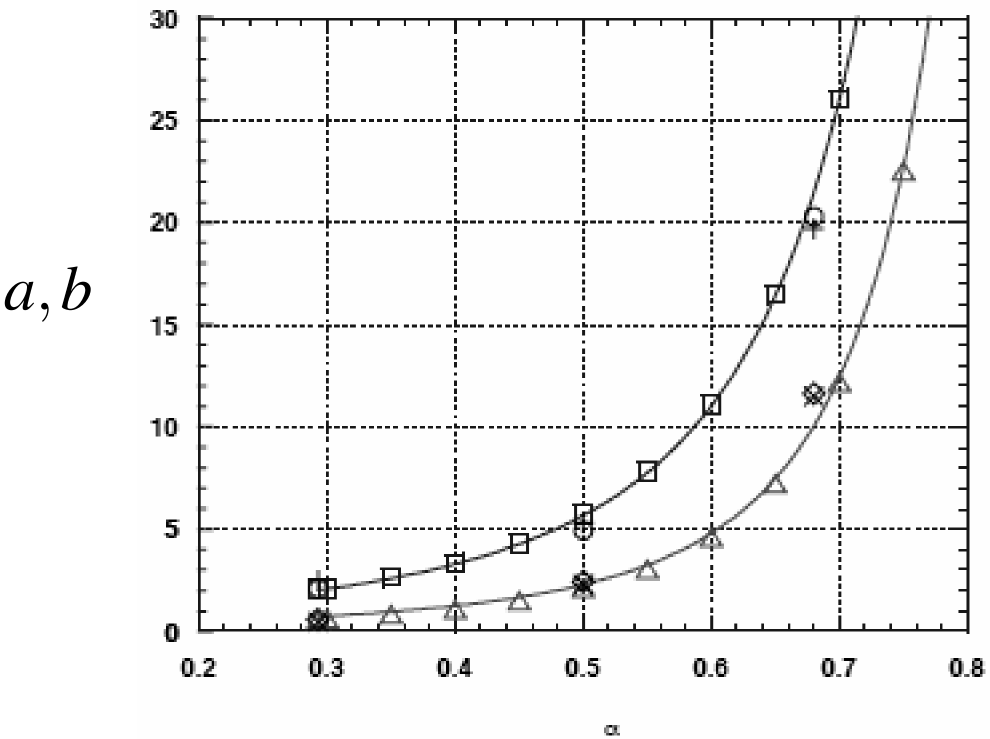
A simple formula as been settled:

$$WSS = (\mu\frac{\partial u^*}{\partial y^*})/((\mu\frac{4U_0}{R})) \sim .22\frac{((Re/\lambda)^{1/2}+3)}{(1-\alpha)^3}$$

Reynolds number is no longer Re but $Re\lambda$ and $(Re/\lambda)^{1/2}$ is the inverse of the relative boundary layer thickness.



IBL integral: Comparison with Navier Stokes (Siegel et al. 1994)



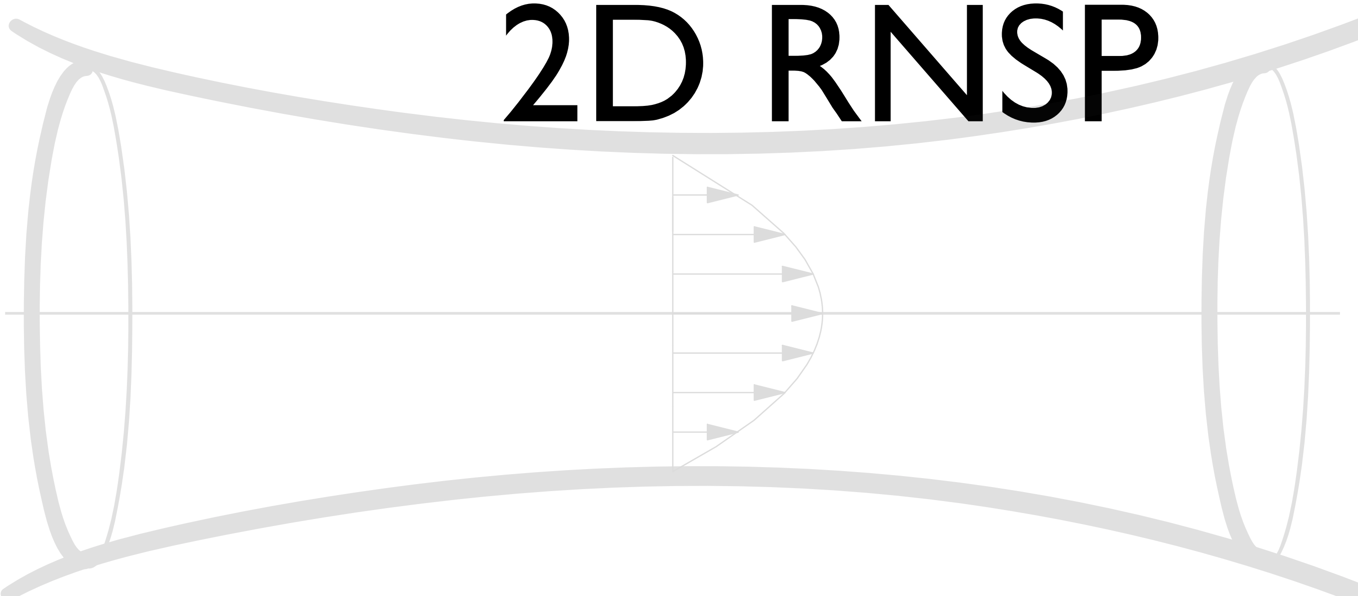
$$WSS = aRe^{1/2} + b$$

Coefficient a and b for the maximum WSS.
solid lines with $\triangle$ and "square" : coefficient a and b obtained using the IBL integral method ;

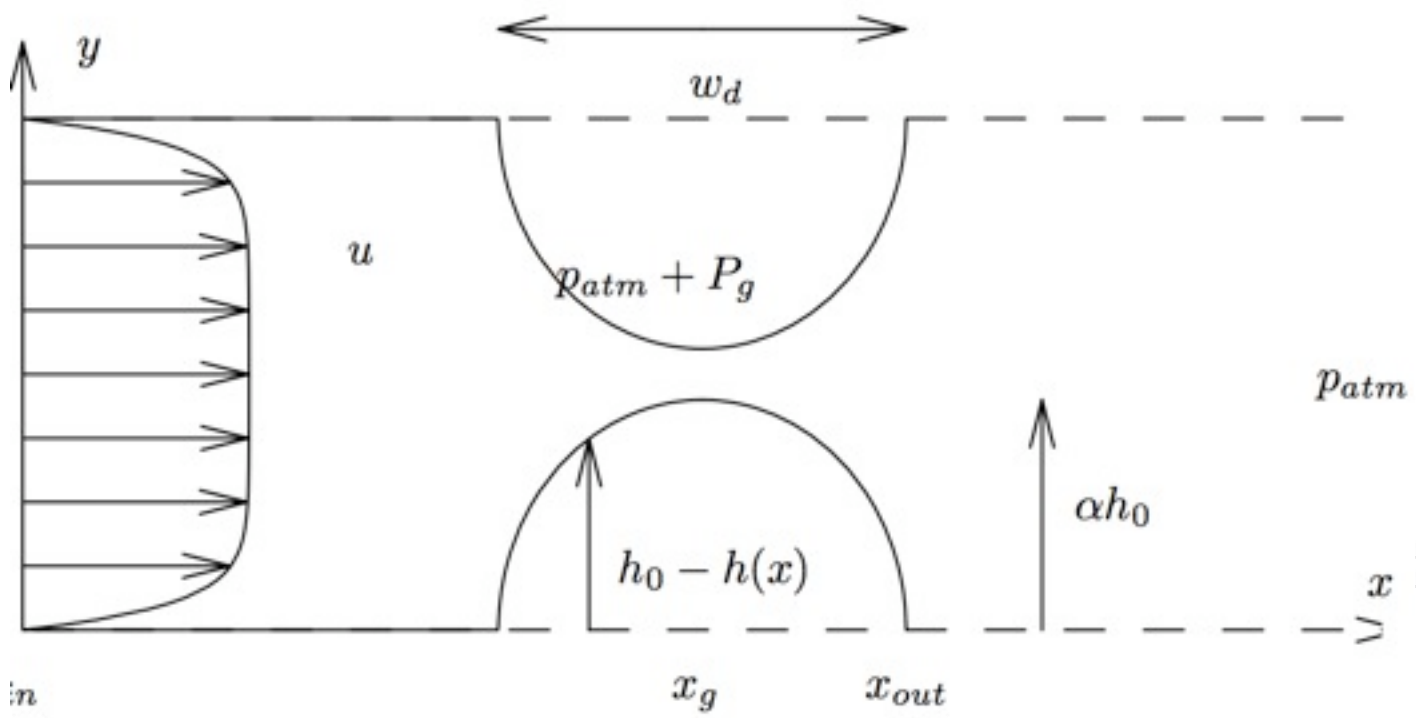
$\diamond$: coefficient a derived from Siegel for $\lambda = 3$;
$\times$: coefficient a derived from Siegel for $\lambda = 6$;
$\bigcirc$: coefficient b derived from Siegel for $\lambda = 3$;
$+$: coefficient b derived from Siegel for $\lambda = 6$.

$$WSS = (\mu \frac{\partial u}{\partial y})/(\mu \frac{4U_0}{R}) \simeq 0.22 \frac{(Re/\lambda)^{1/2} + 3}{(1-\alpha)^3}$$

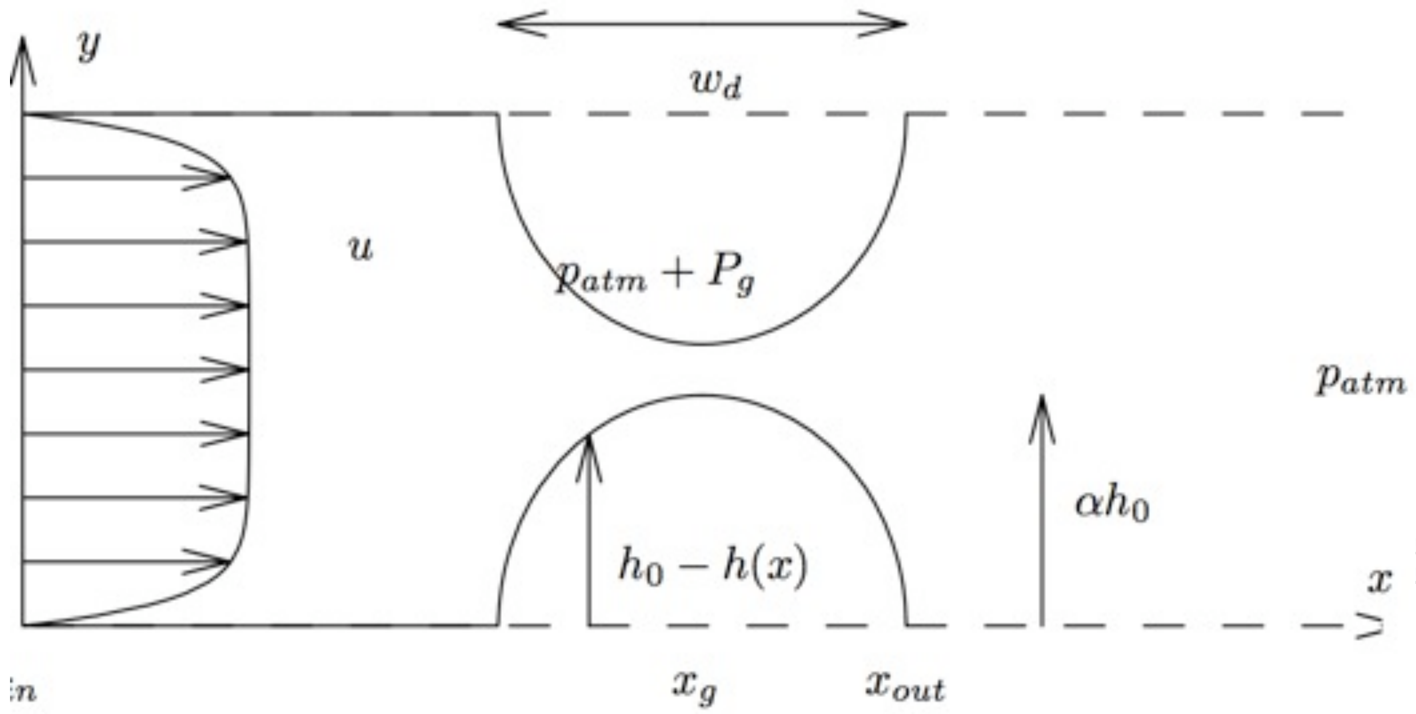
2D RNSP



- Flow in a 2D stenozed vessel
- steady, rigid wall



- Flow in a stenozed vessel
- steady, rigid wall

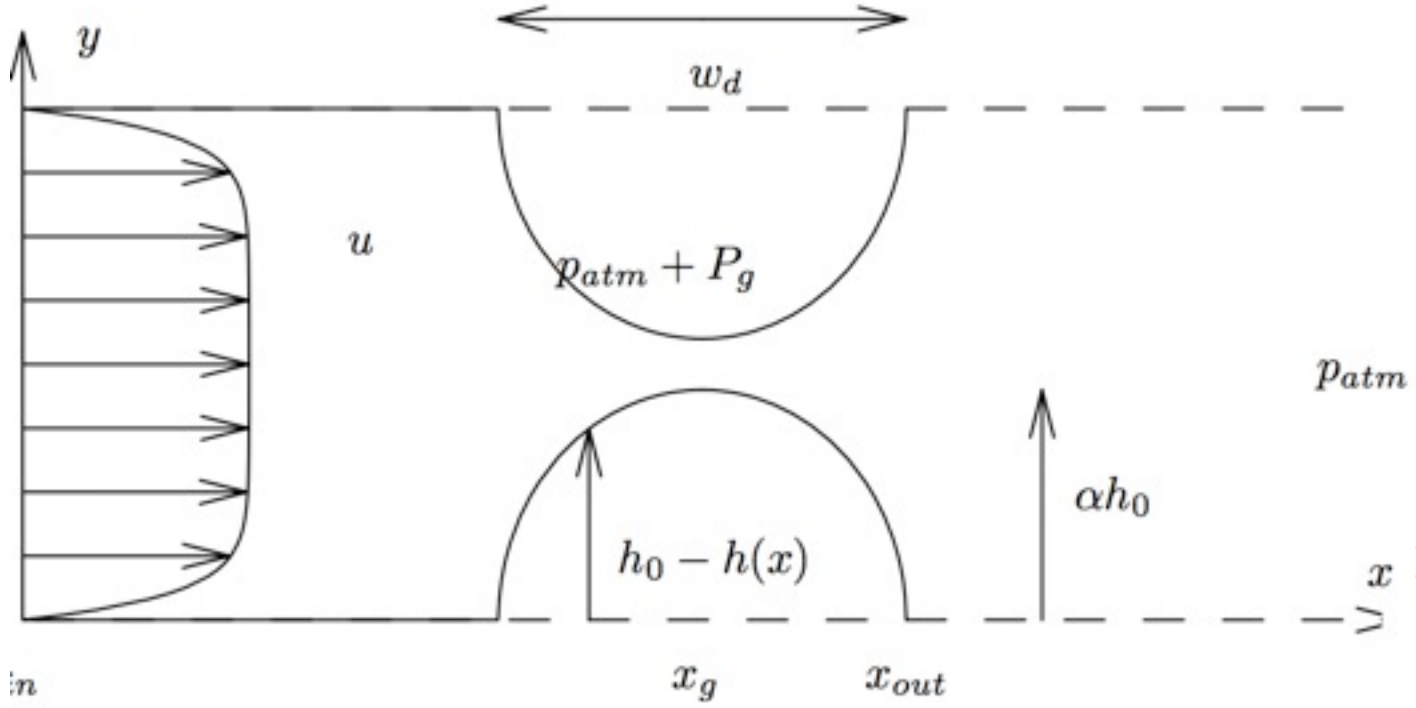


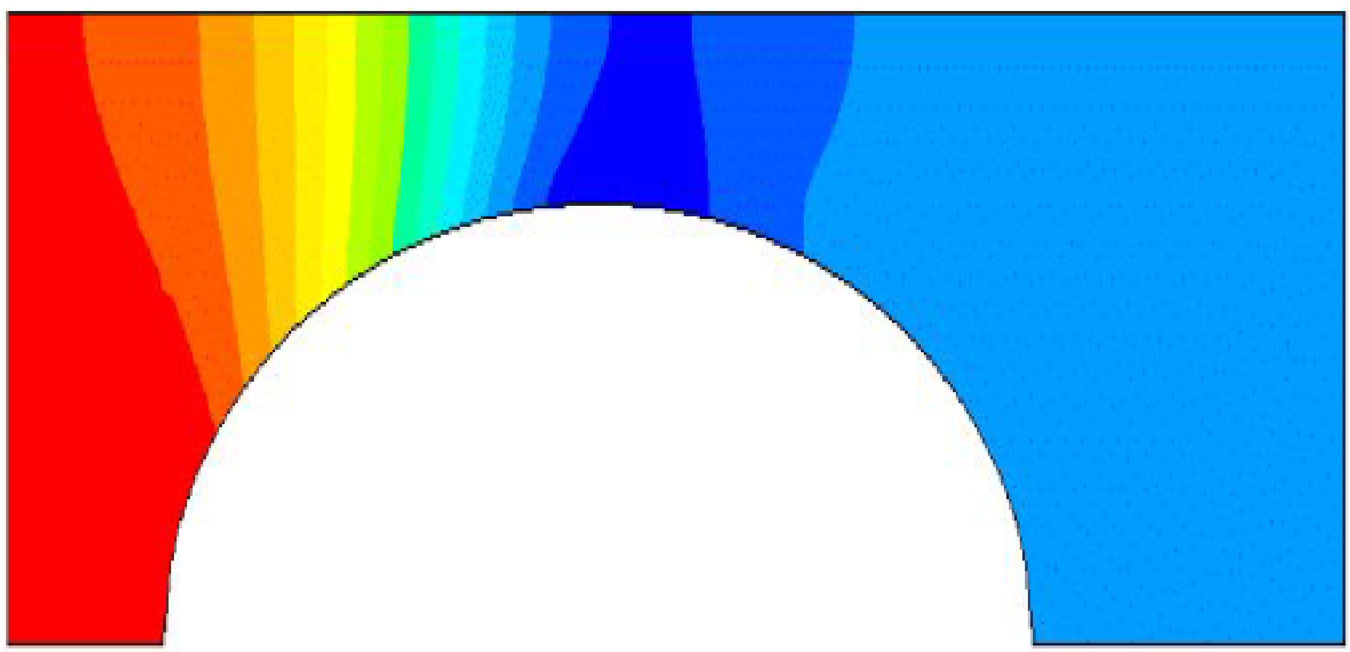
$$\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0$$

$$u\frac{\partial}{\partial x}u + v\frac{\partial}{\partial y}u = -\frac{\partial}{\partial x}p + \frac{\partial^2}{\partial y^2}u$$

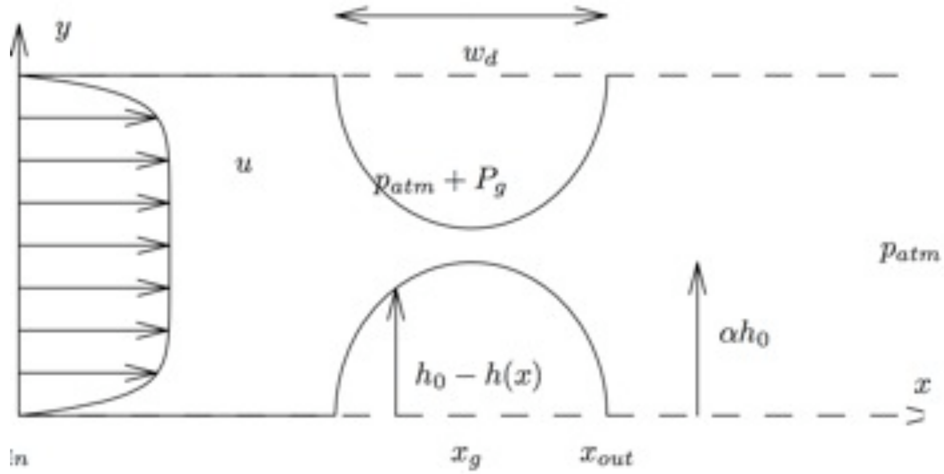
$$0 = -\frac{\partial}{\partial y}p$$

RNSP non dimensional





CAST3M



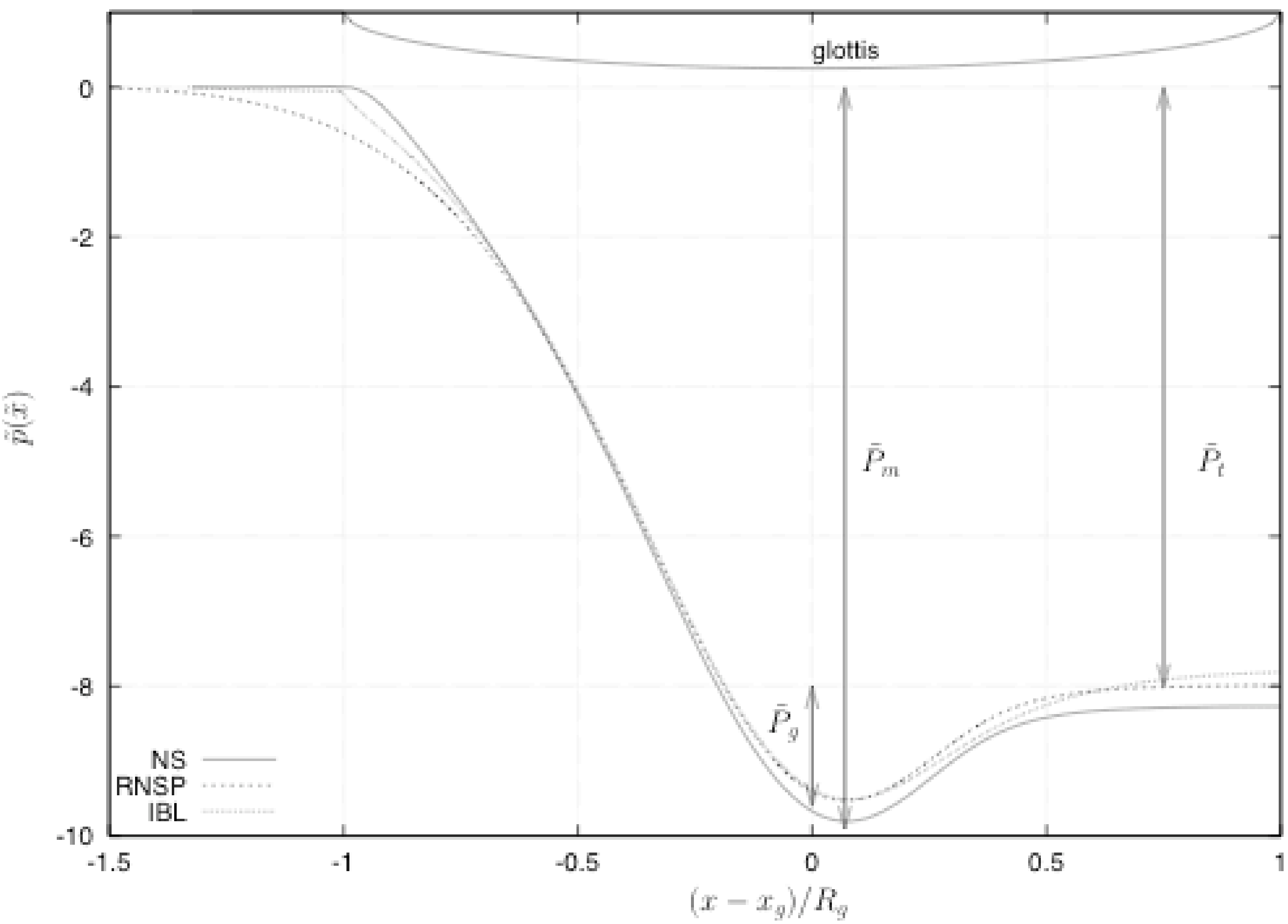
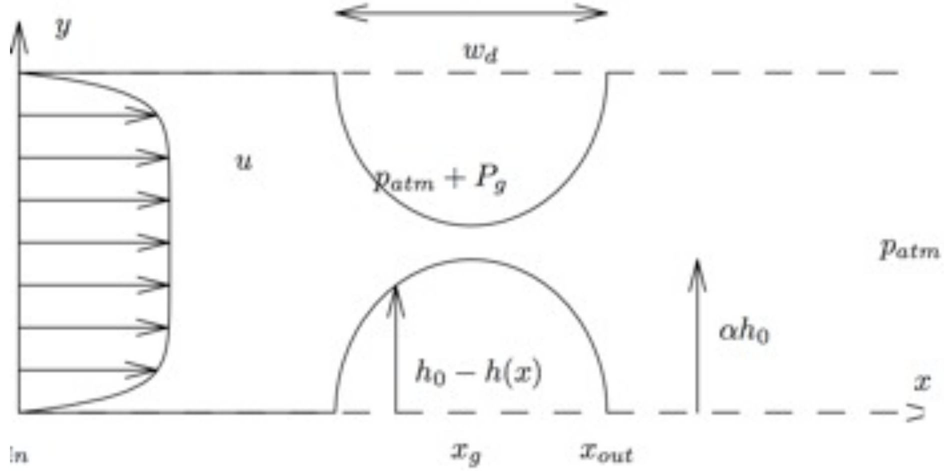
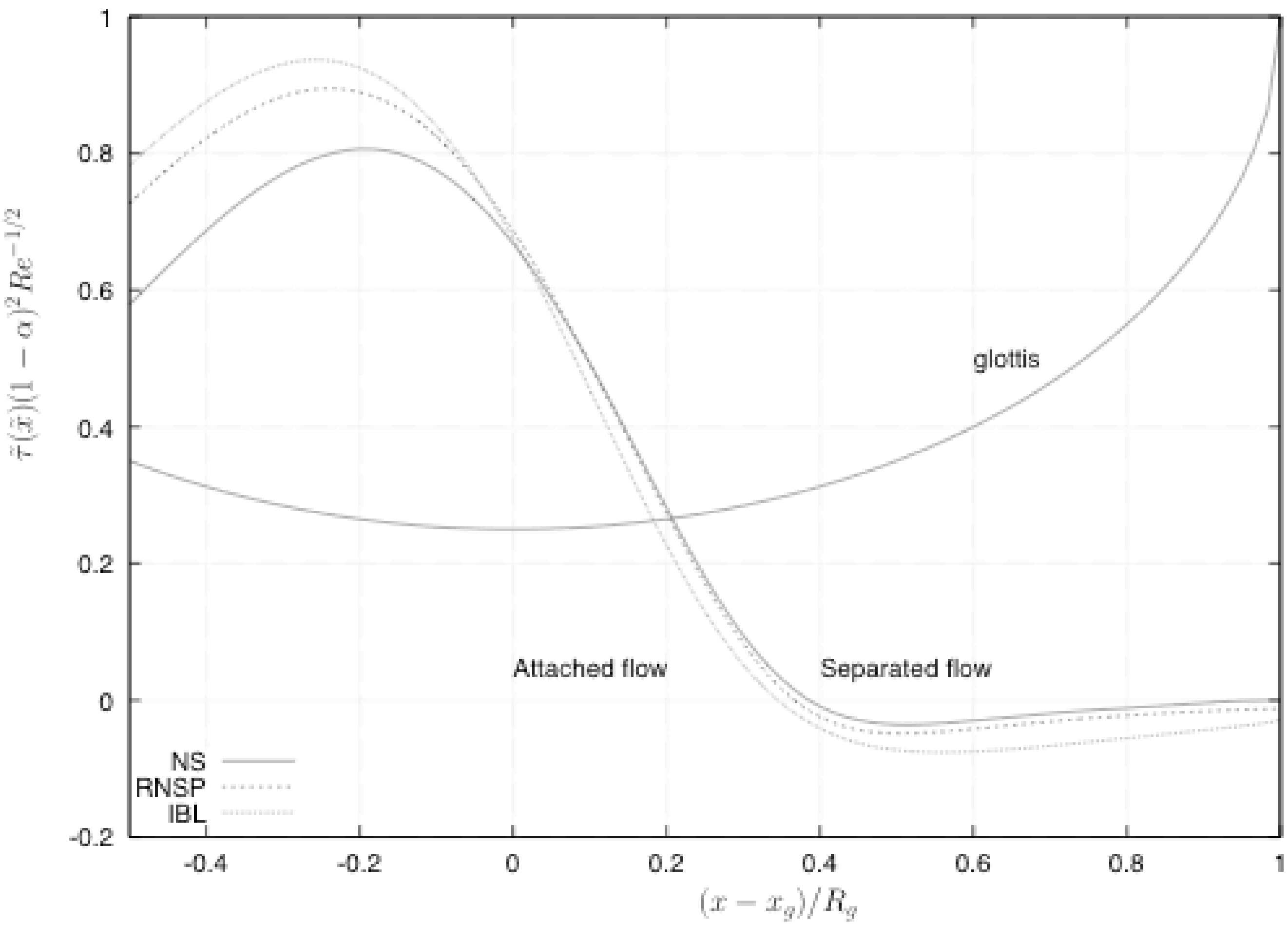


Fig. 2 A comparison between computed non-dimensional pressure for the three models (NS, IBL and RNSP, in this last case the wall has

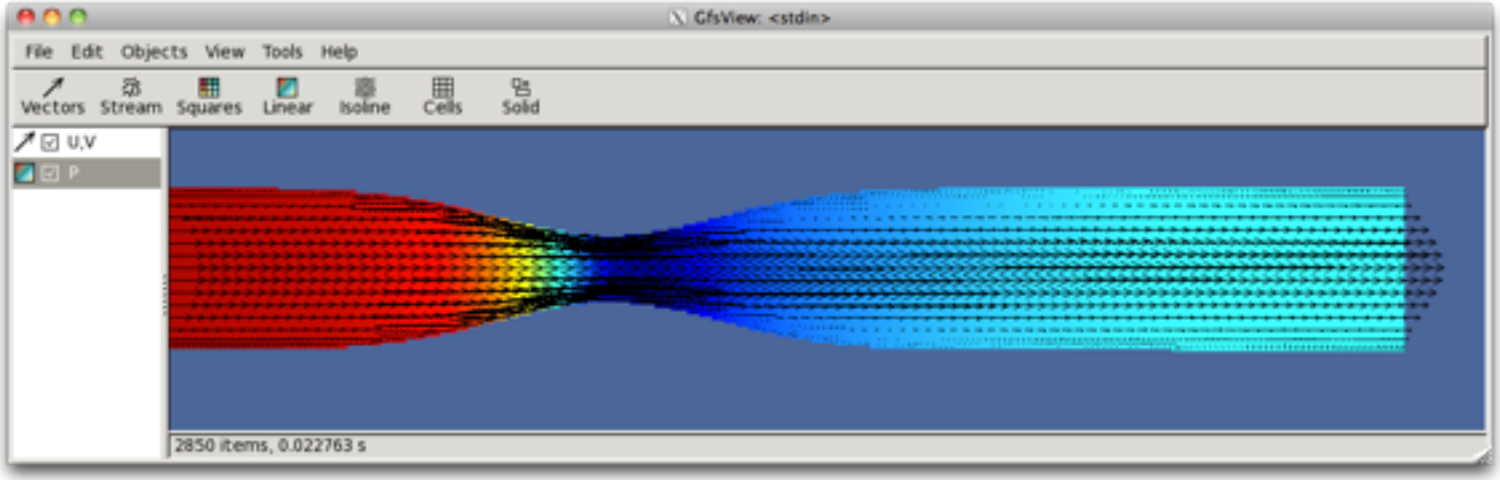


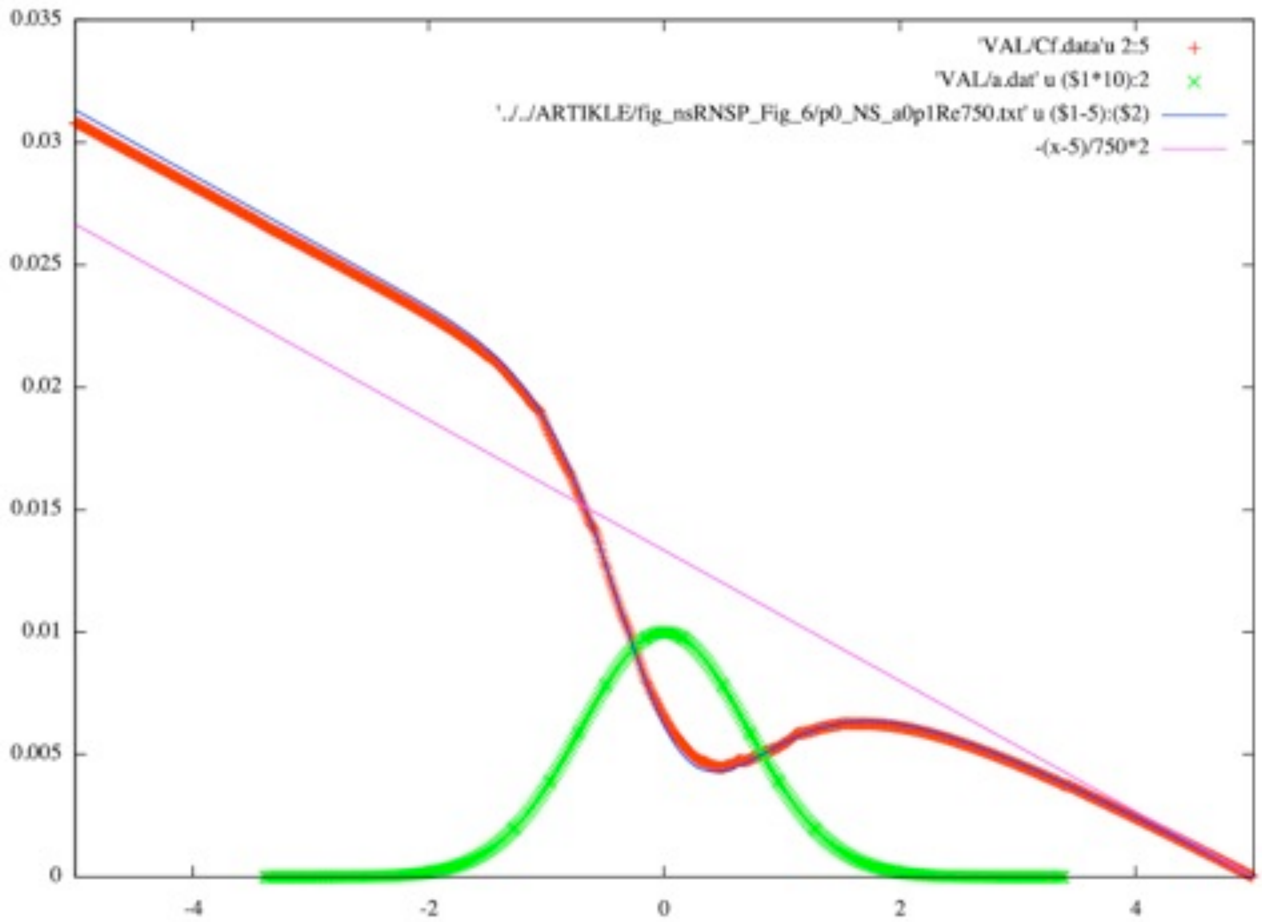
y
w_d
u
$p_{atm} + P_g$
p_{atm}
αh_0
$h_0 - h(x)$
x
x_g
x_{out}



$\bar{\tau}(\bar{x})(1-\alpha)^2 Re^{-1/2}$
$(x - x_g)/R_g$
glottis
Attached flow
Separated flow
NS
RNSP
IBL

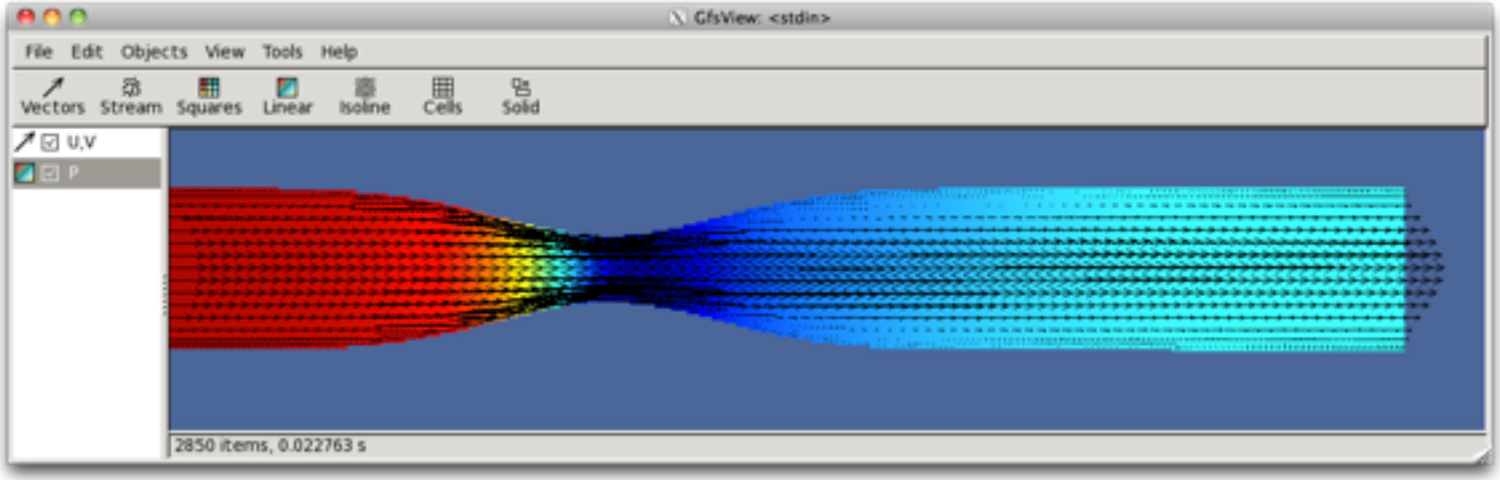
other comparisons:
Gerris Flow Solver
RNSP with FreeFem++

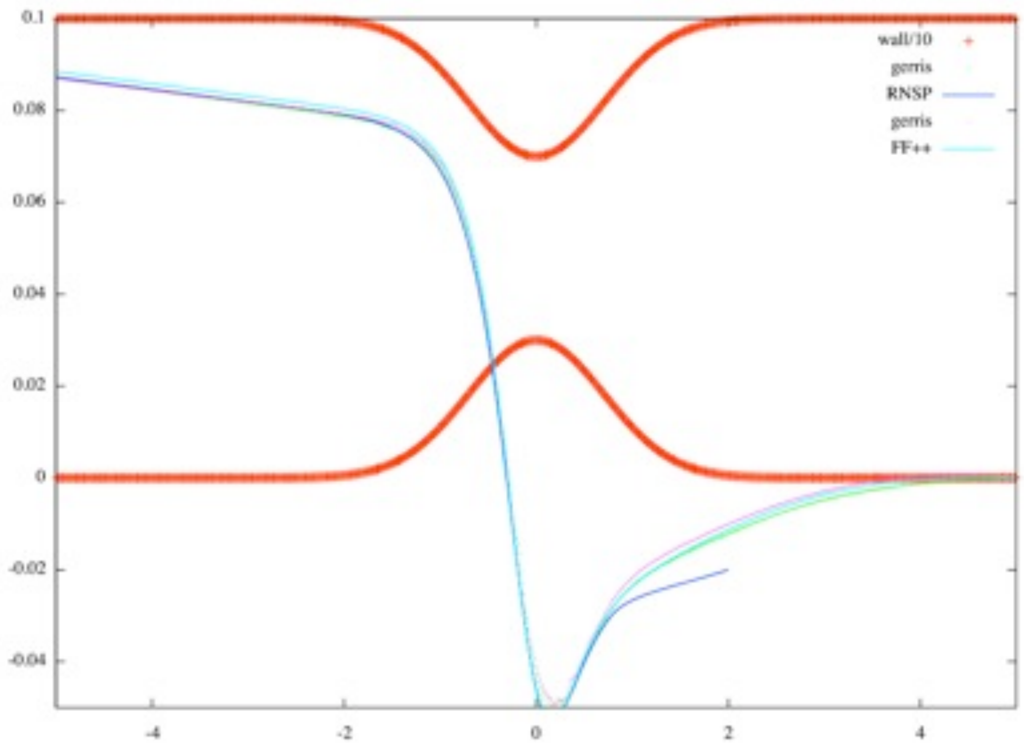


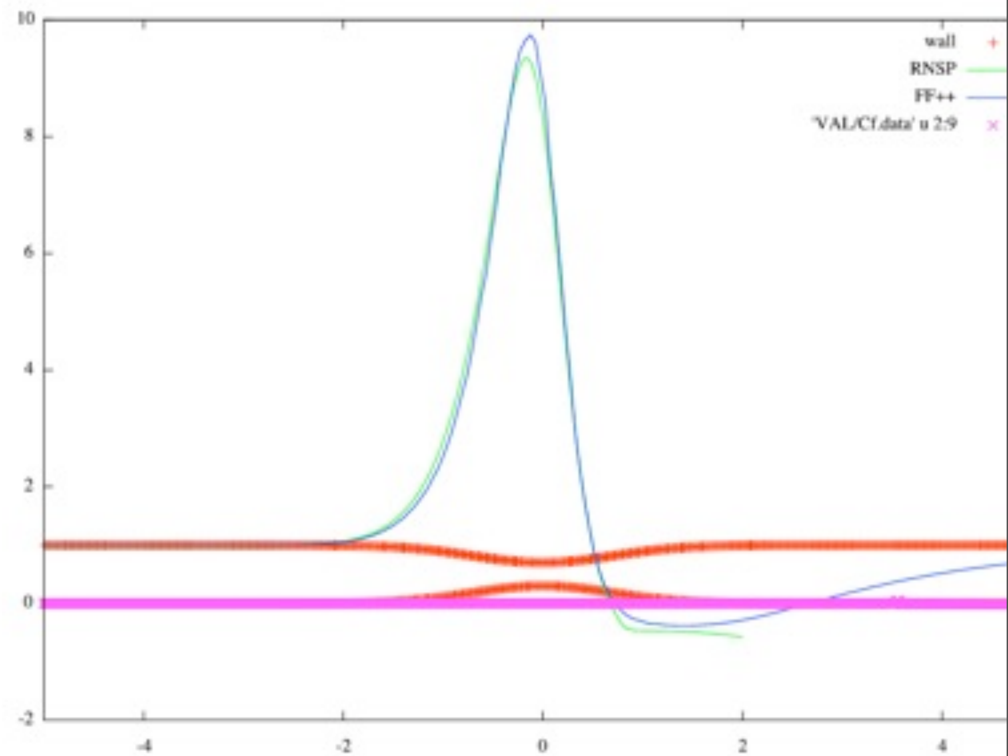


- RNSP Faster, allows separated flow

other comparisons:
Gerris Flow Solver
RNSP with FreeFem++
RNSP FiniteDifferences





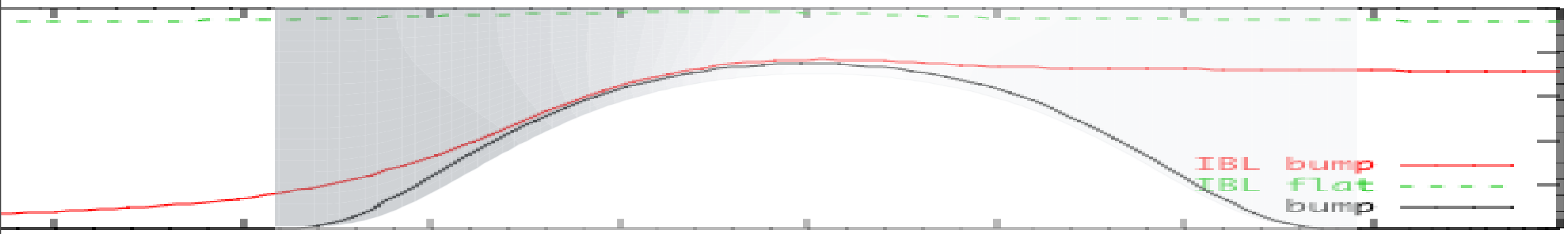


- RNSP Faster, allows separated flow

non symmetrical case

- RNSP
- Flow in a stenozed vessel
- steady, rigid wall
- modified integral method to take into account the transverse variation of pressure
- NS

non symmetrical case



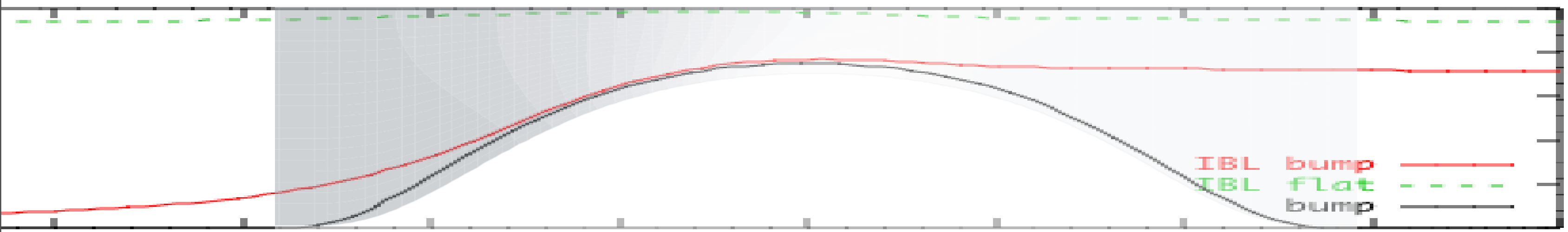
$$u = U_0(\xi) + \varepsilon u_1(\xi, y) + \varepsilon^2 u_2(\xi, y) + \cdots,$$
$$v = \varepsilon v_1(\xi, y) + \varepsilon^3 v_3(\xi, y) + \cdots,$$
$$p = p_0(\xi) + \varepsilon p_1(\xi, y) + \varepsilon^2 p_2(\xi, y) + \cdots,$$

$$U_0 \frac{\partial U_0}{\partial \xi} = -\frac{\partial p_0}{\partial \xi},$$

$$\varepsilon \frac{\partial U_0}{\partial \xi} + \varepsilon \frac{\partial v_1}{\partial y} = 0.$$

$$v_1(\xi, y_b = f_h) = U_0 \frac{\partial f_b}{\partial \xi}, \qquad v_1(\xi, y_h = 1 - f_h) = -U_0 \frac{\partial f_h}{\partial \xi},$$

non symmetrical case



$$U_0(\xi) = \frac{1}{1 - f_b(\xi) - f_h(\xi)}, \qquad P_0(x) = \frac{1}{2} - \frac{1}{2}\left(\frac{1}{1 - f_b(\xi) - f_h(\xi)}\right)^2.$$

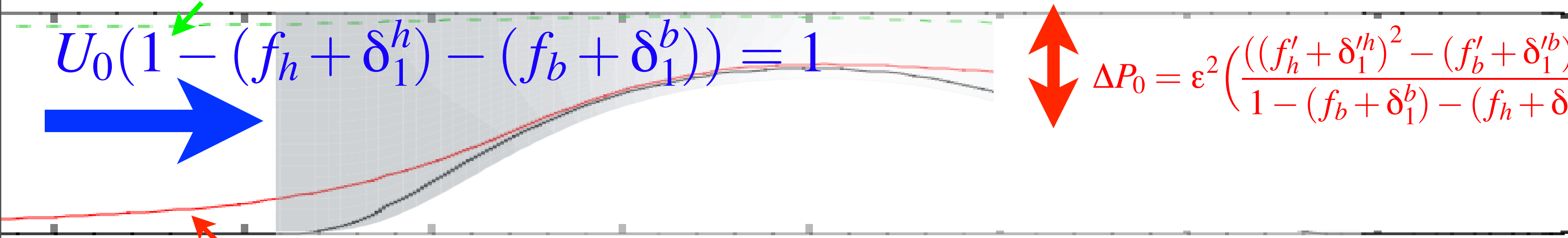
$$v_1(\xi, y) = U_0 \frac{\partial f_b}{\partial \xi} + \frac{y - f_b}{1 - f_h - f_b}\left(-U_0 \frac{\partial f_b}{\partial \xi} - U_0 \frac{\partial f_h}{\partial \xi}\right).$$

next order

$$\varepsilon^2 U_0 \frac{\partial v_1}{\partial \xi} = -\varepsilon^2 \frac{\partial p_2}{\partial y},$$
$$\varepsilon^3 \frac{\partial U_0 u_2}{\partial \xi} = -\varepsilon^3 \frac{\partial p_2}{\partial \xi},$$
$$\varepsilon^3 \frac{\partial u_2}{\partial \xi} + \varepsilon^3 \frac{\partial v_3}{\partial y} = 0.$$

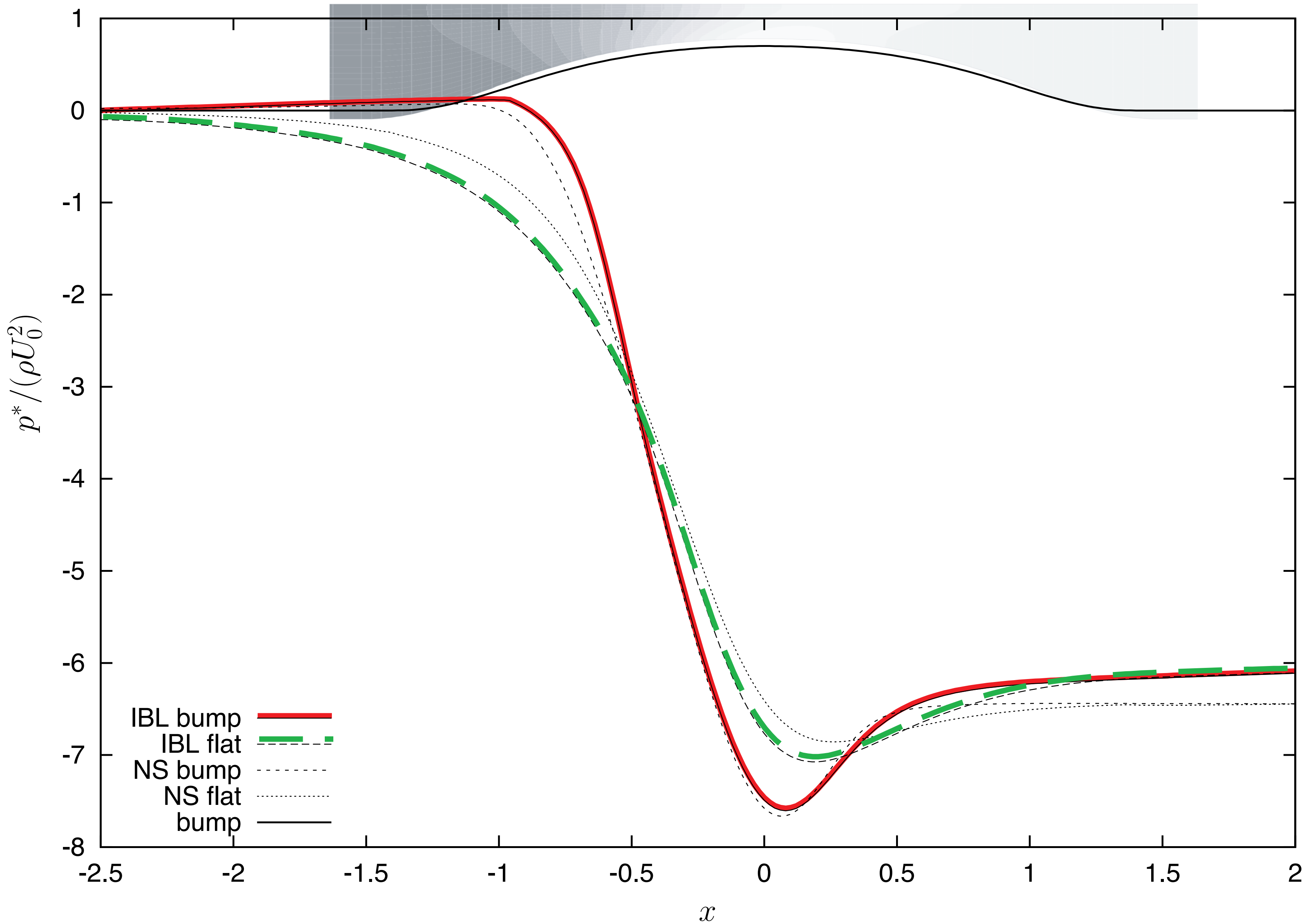
$$\varepsilon^2 \big(p_2(\xi, y_h) - p_2(\xi, y_b)\big) = \varepsilon^2 \left(\frac{(f_h'(\xi)^2 - f_b'(\xi)^2)}{1 - f_b(\xi) - f_h(\xi)} + \frac{(f_h''(\xi) - f_b''(\xi))}{2} \right)$$

$$\frac{d}{dx}\left(\frac{\delta_1^h}{H}\right)+\frac{\delta_1^h}{u_e^h}\left(1+\frac{2}{H}\right)\frac{du_e^h}{dx}=\frac{f_2 H}{\delta_1^h u_e^h}, \qquad \delta_1^h = F(p_e^h)$$



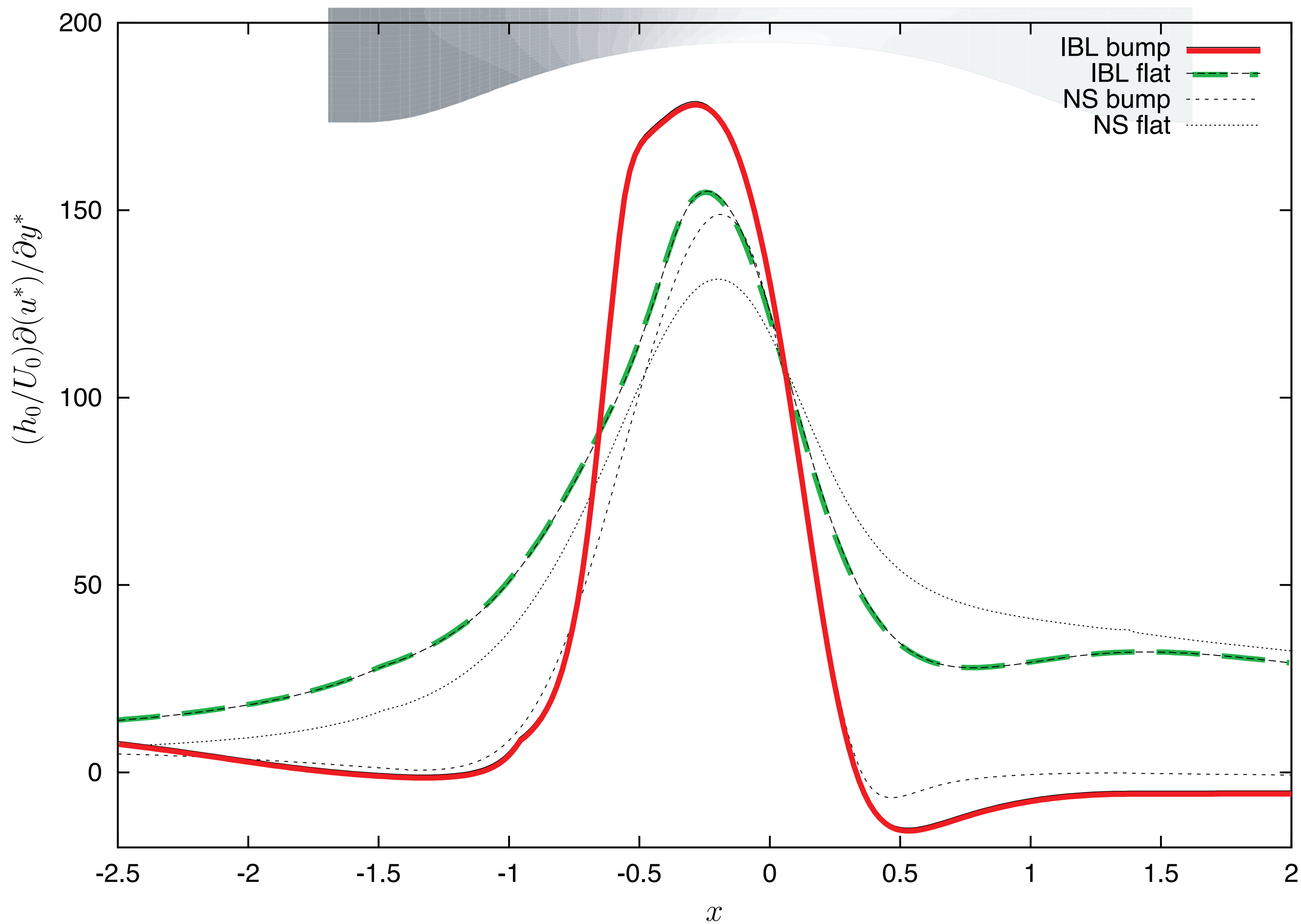
$$\Delta P_0=\varepsilon^2\left(\frac{((f_h'+\delta_1'^h)^2-(f_b'+\delta_1'^b)^2)}{1-(f_b+\delta_1^b)-(f_h+\delta_1^h)}+\frac{(f_h''+\delta_1''^h-f_b''-\delta_1''^b)}{2}\right).$$

$$\frac{d}{dx}\left(\frac{\delta_1^b}{H}\right)+\frac{\delta_1^b}{u_e^b}\left(1+\frac{2}{H}\right)\frac{du_e^b}{dx}=\frac{f_2 H}{\delta_1^b u_e^b}, \qquad \delta_1^b = F(p_e^b)$$



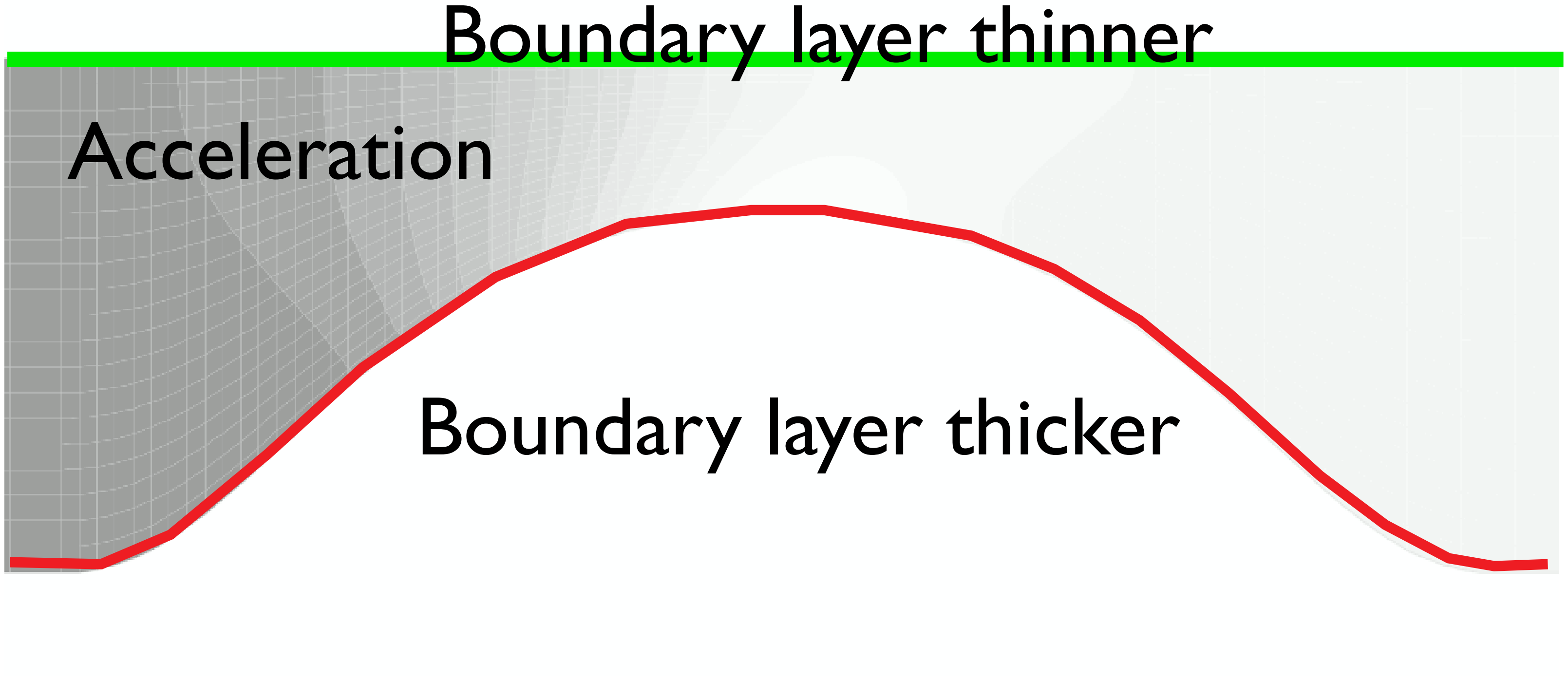
1
0
-1
-2
-3
-4
-5
-6
-7
-8
$p^*/(\rho U_0^2)$
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
x
IBL bump
IBL flat
NS bump
NS flat
bump

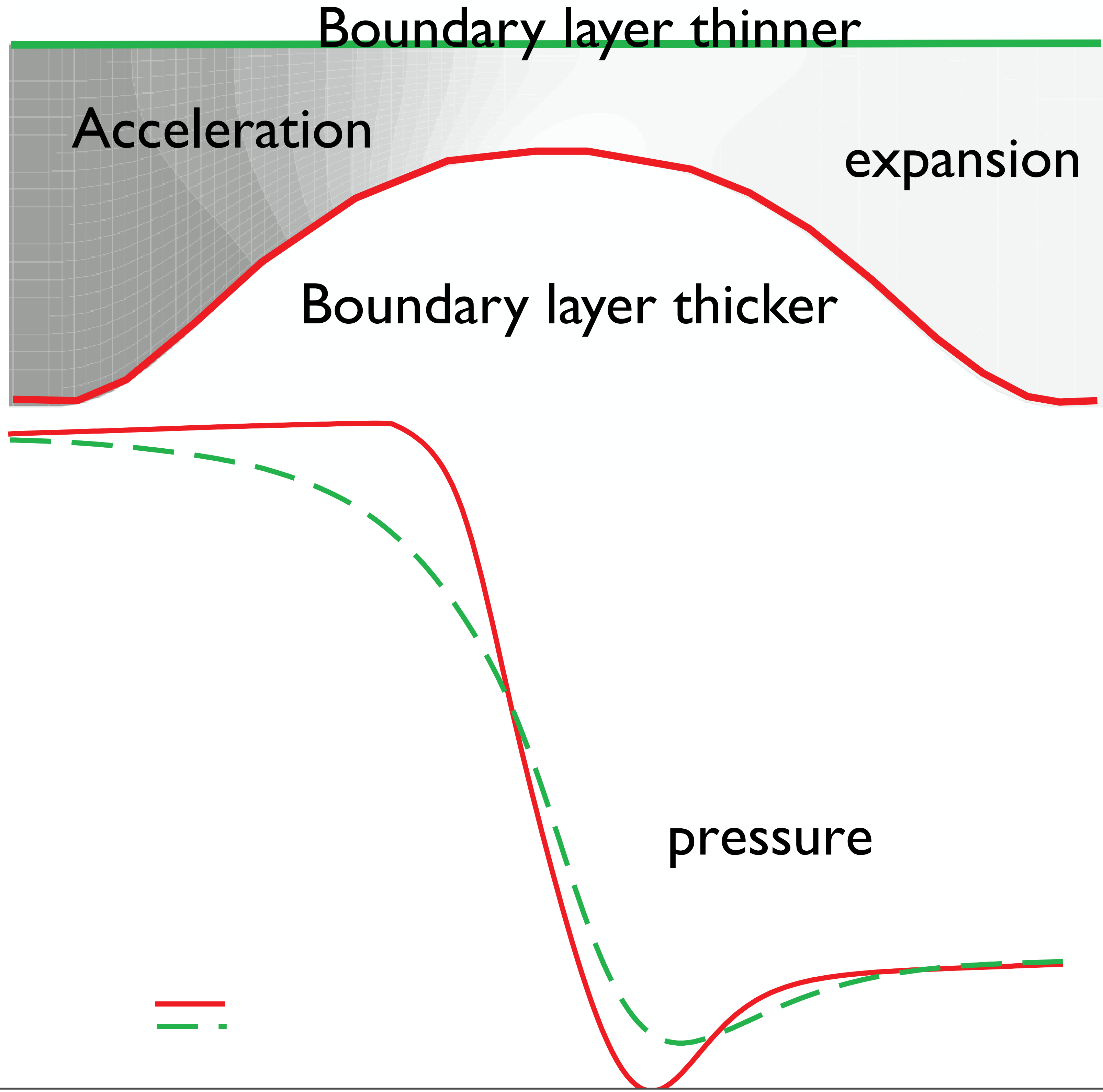
NS: FreeFem++

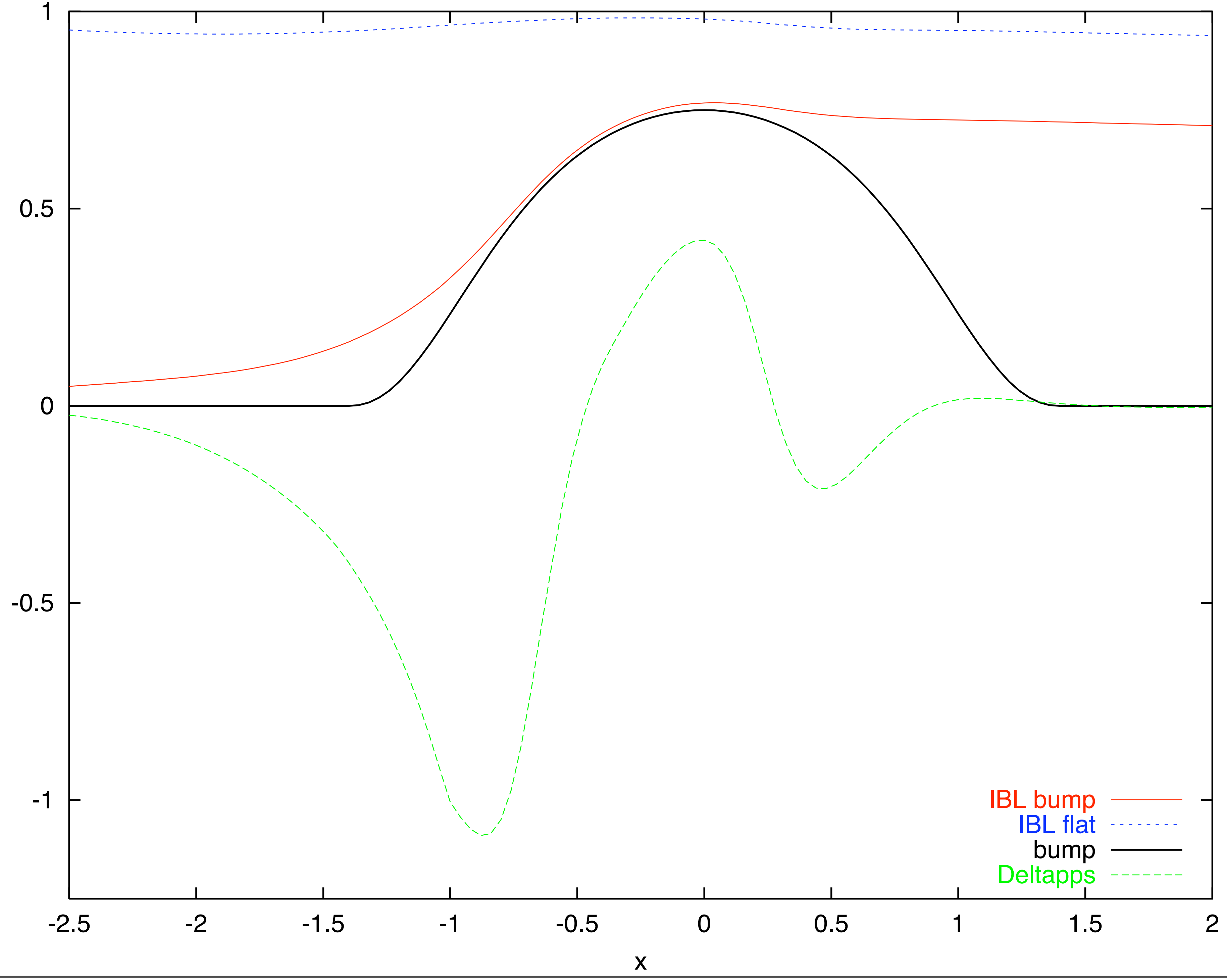


IBL bump
IBL flat
NS bump
NS flat
$(h_0/U_0)\partial(u^*)/\partial y^*$
200
150
100
50
0
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
x

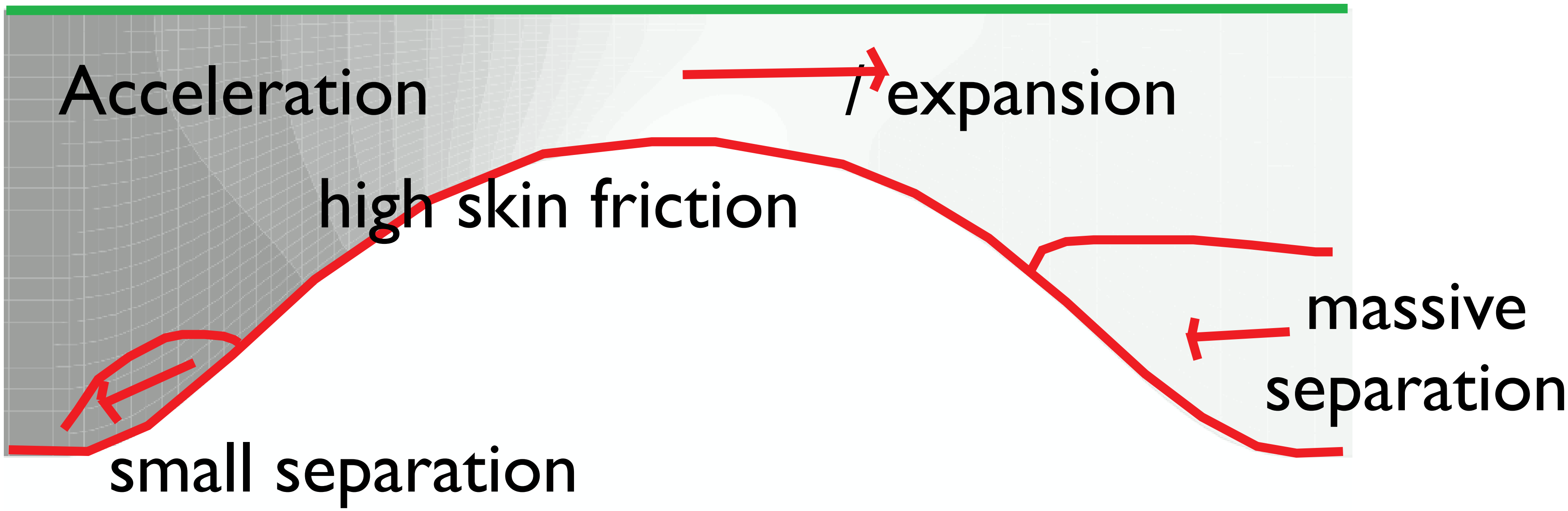
NS: FreeFem++


Boundary layer thinner
Acceleration
Boundary layer thicker


Boundary layer thinner
Acceleration
expansion
Boundary layer thicker
pressure



jeudi 8 avril 2010


Acceleration
/ expansion
high skin friction
massive
separation
small separation

Conclusion

- starting from Navier Stokes
- set of simple equations RNSP: Prandtl in the pipe
- set of more simple equations Integral
- valid for long bump
- BUT Good agreement with full Navier Stokes for O(1) bumps
- BUT Good agreement with full NS at moderate Re
- “explains” the features of the flow
- upstream influence in non symetrical case
- used to compute fluid structure interaction in Sleep Apnea