



27/04/2006

milieux granulaires

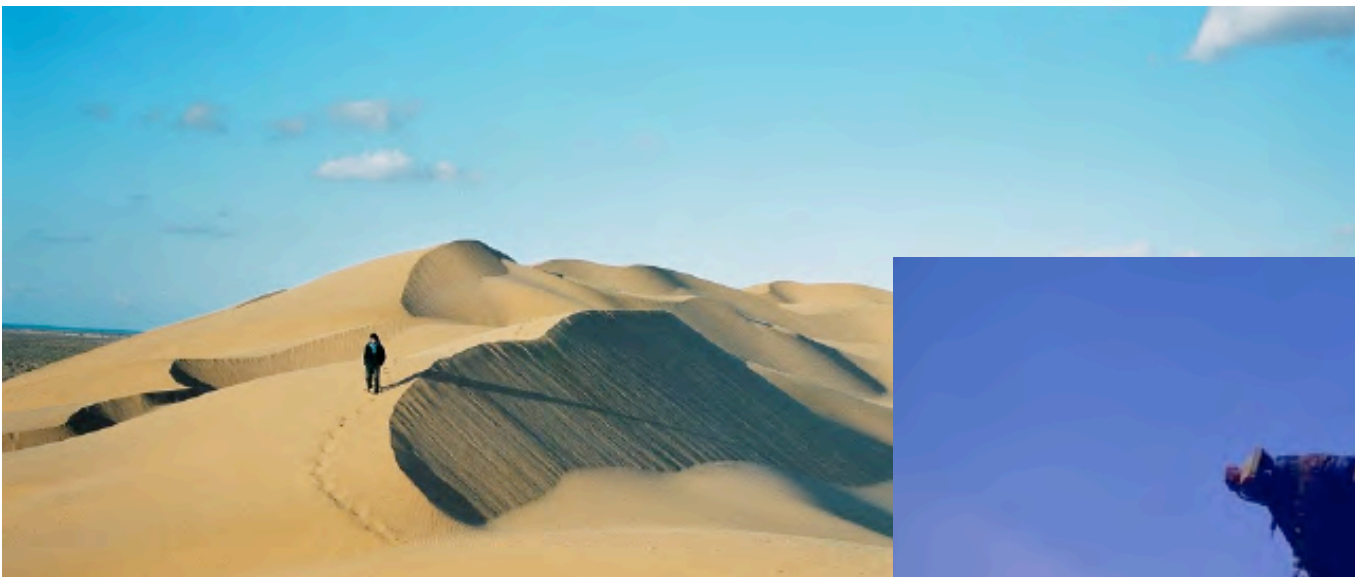
Lagrée P.-Y.

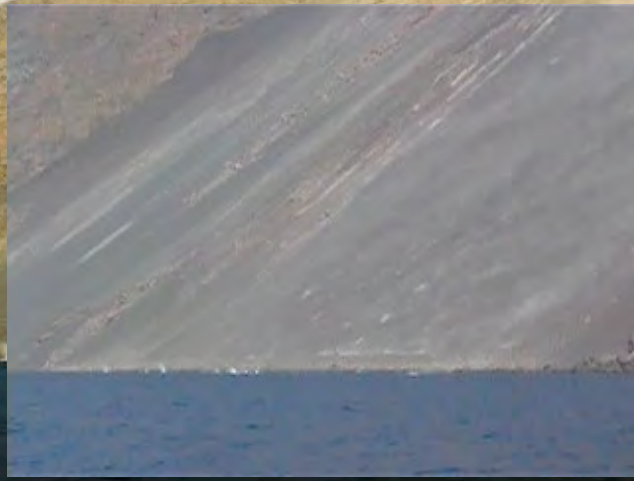
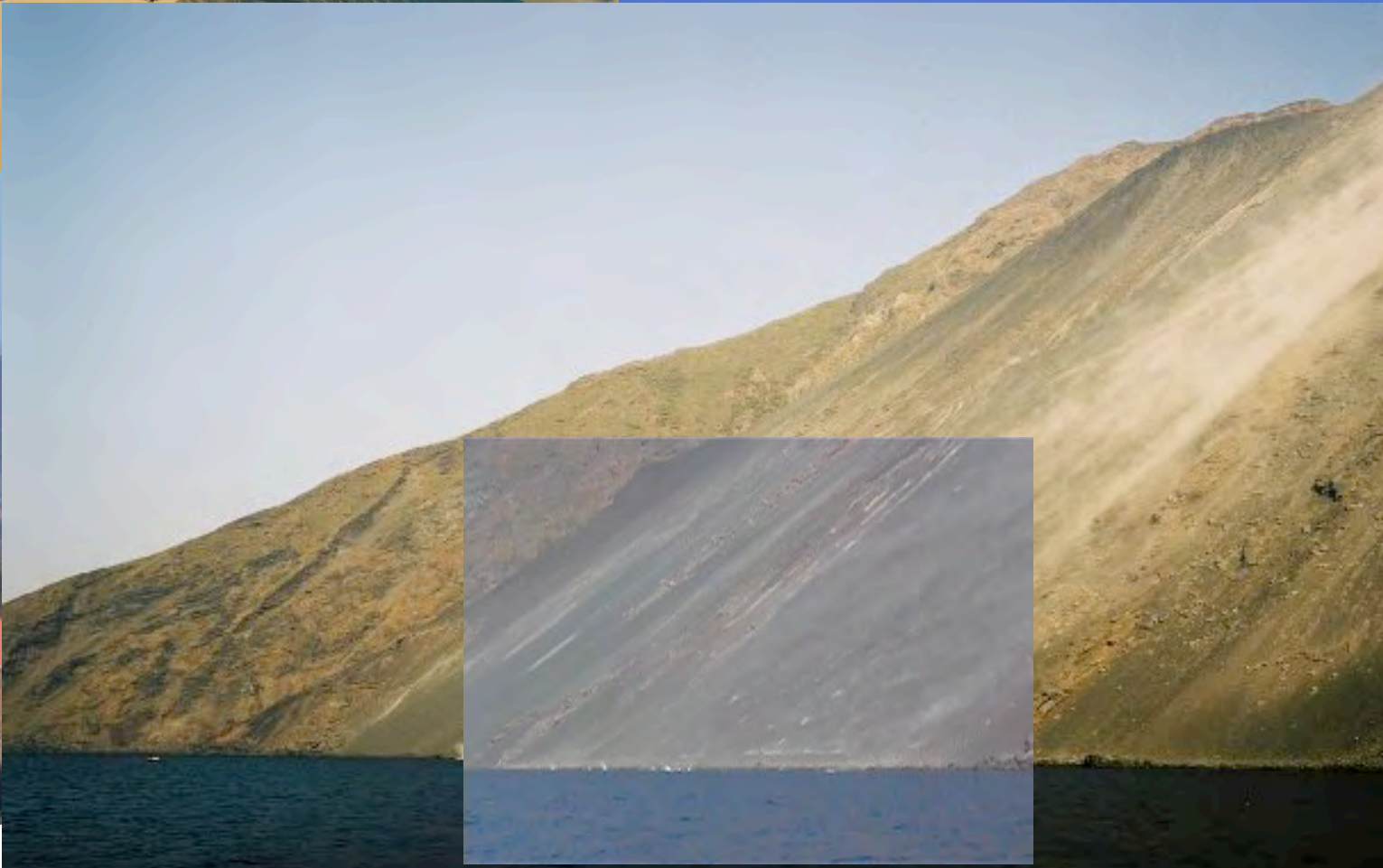
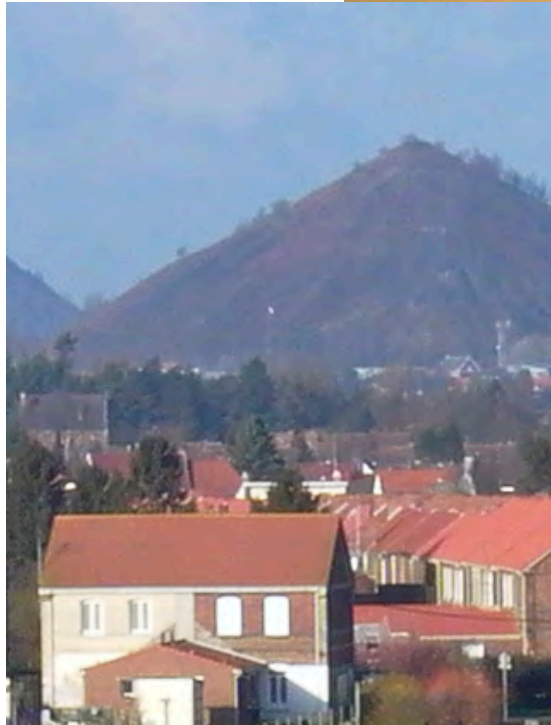
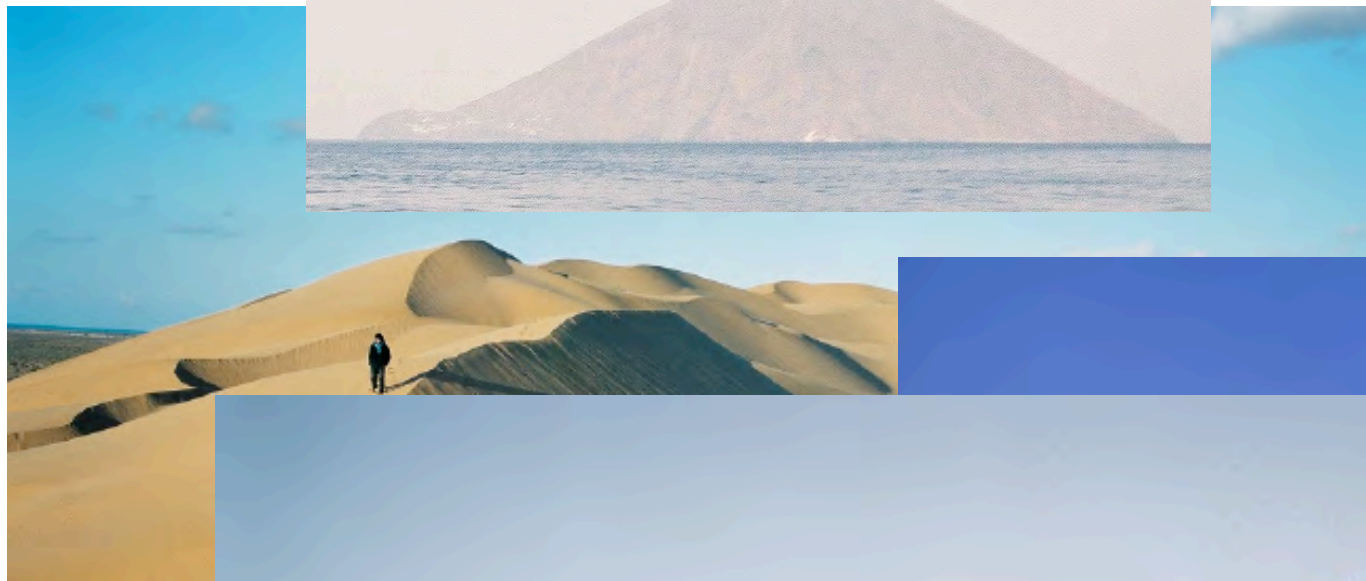
Lhuillier D., Josserand C., Kouakou K.K.J.

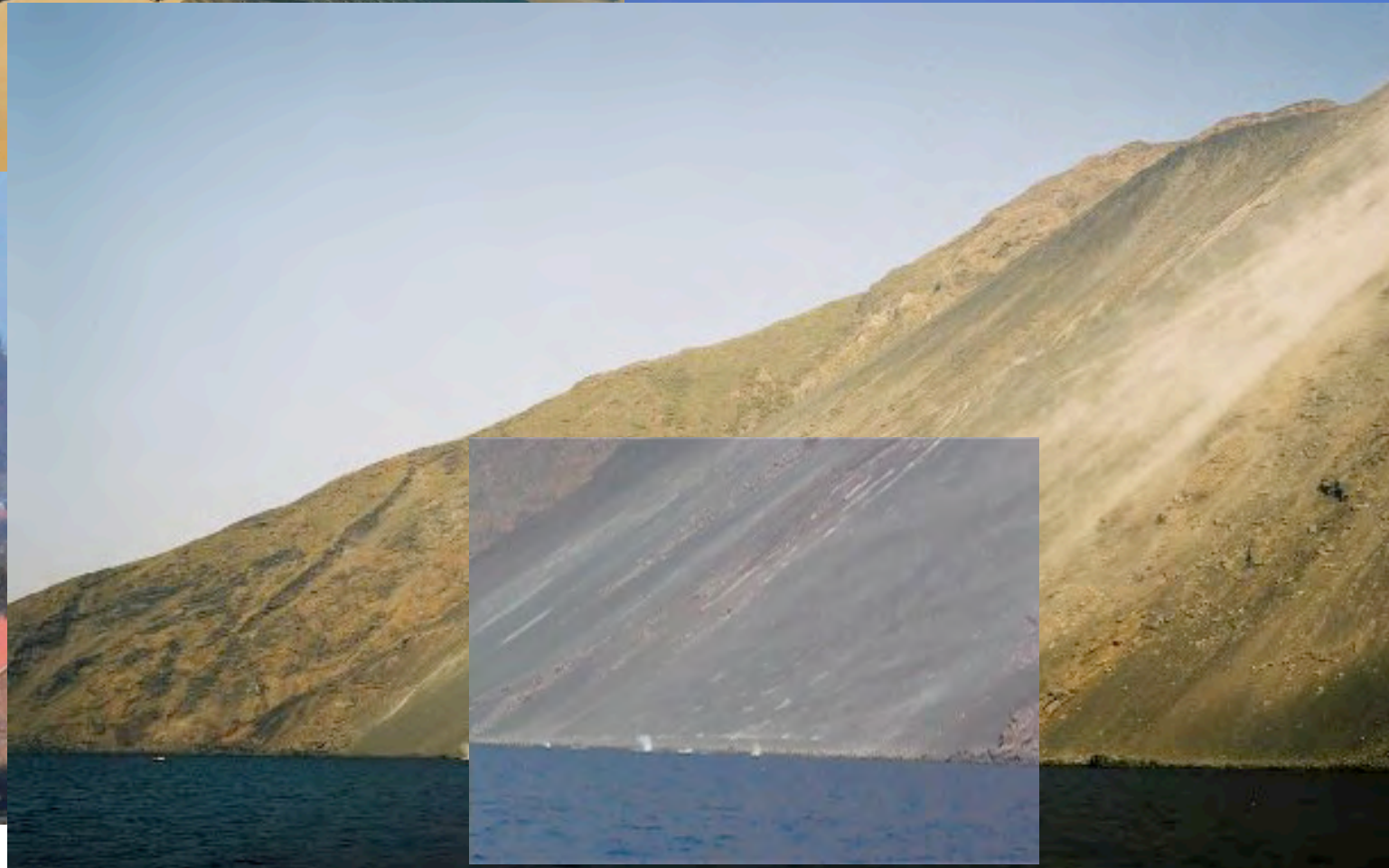
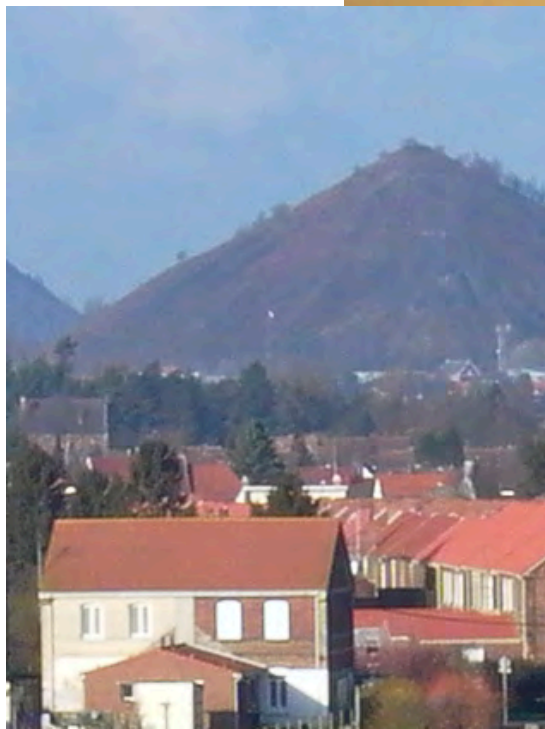


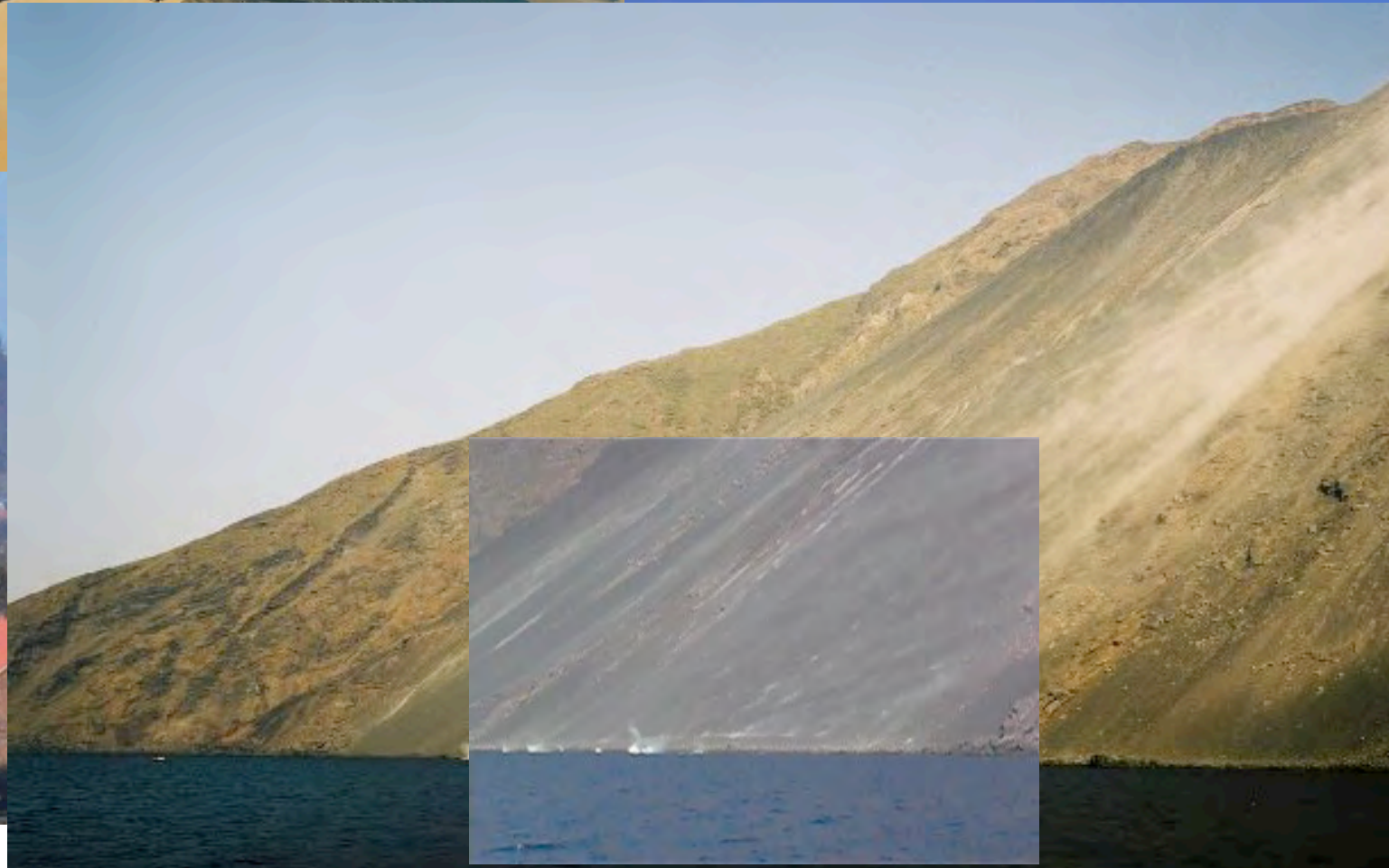
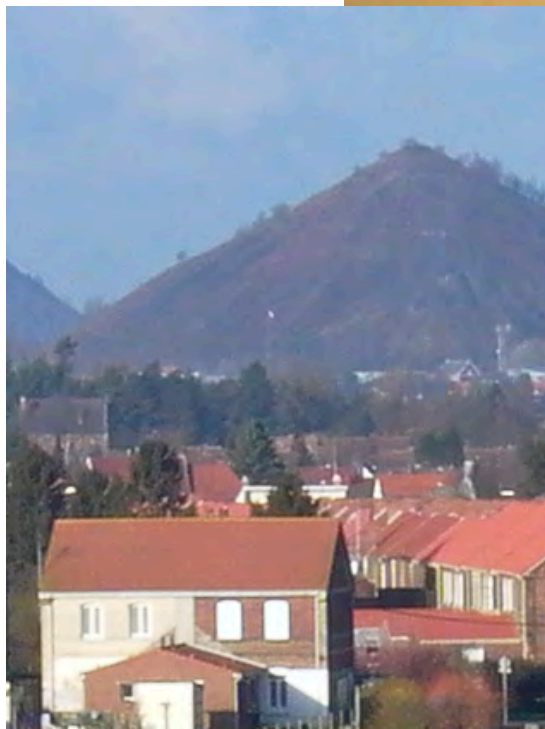
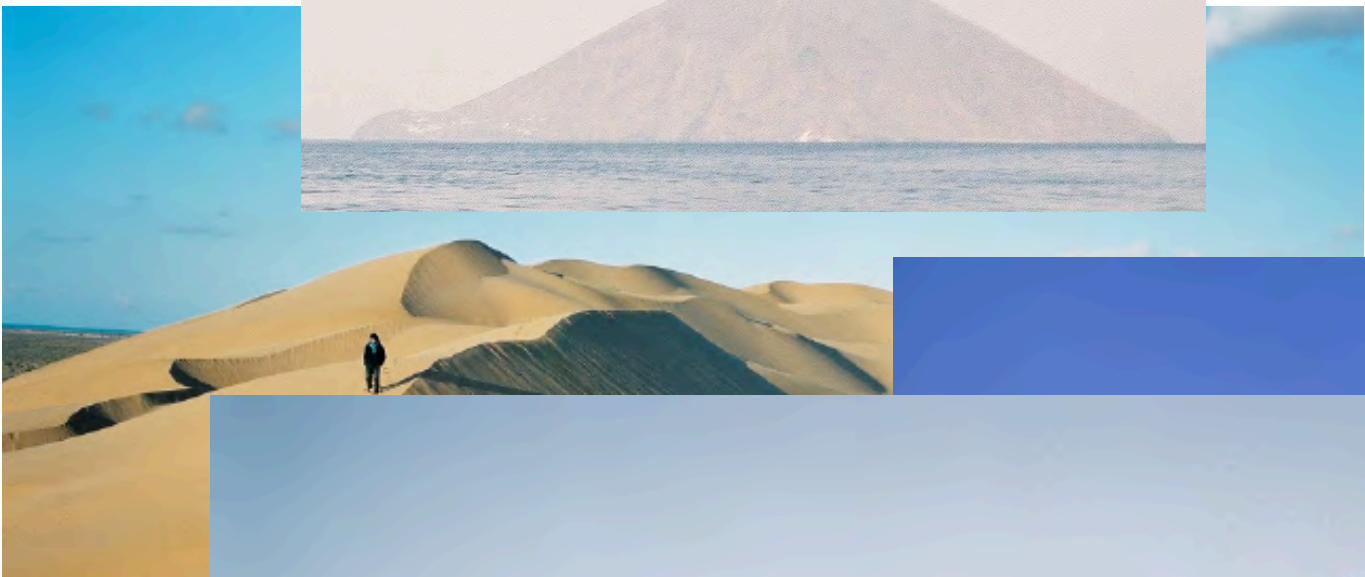
- une loi de comportement pour les milieux granulaires secs
- stabilité d'un fond érodable, rides

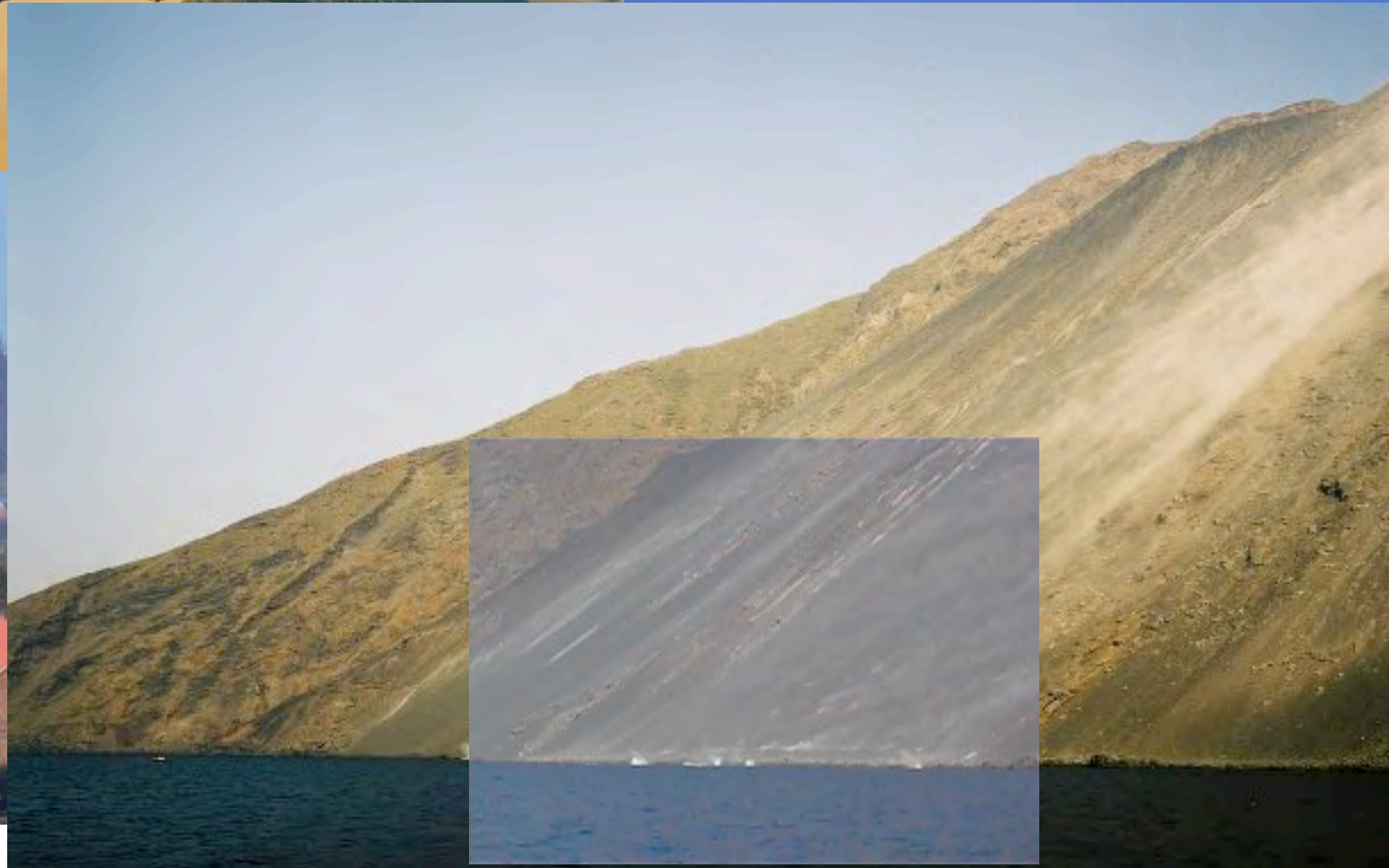
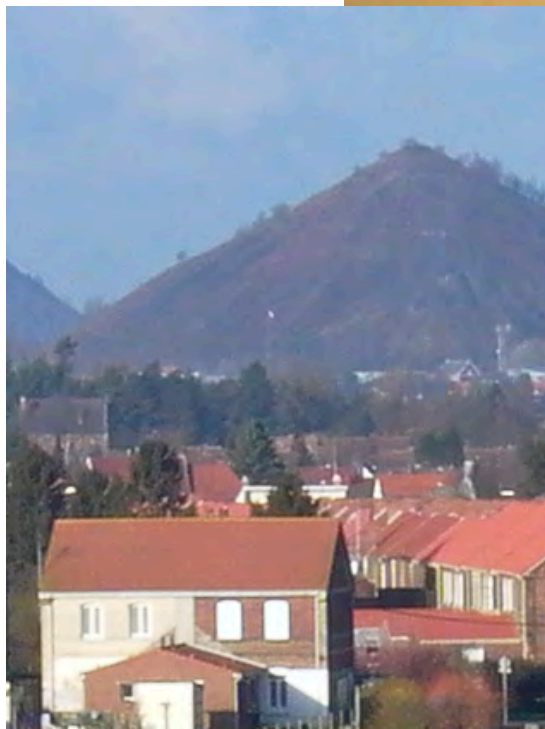


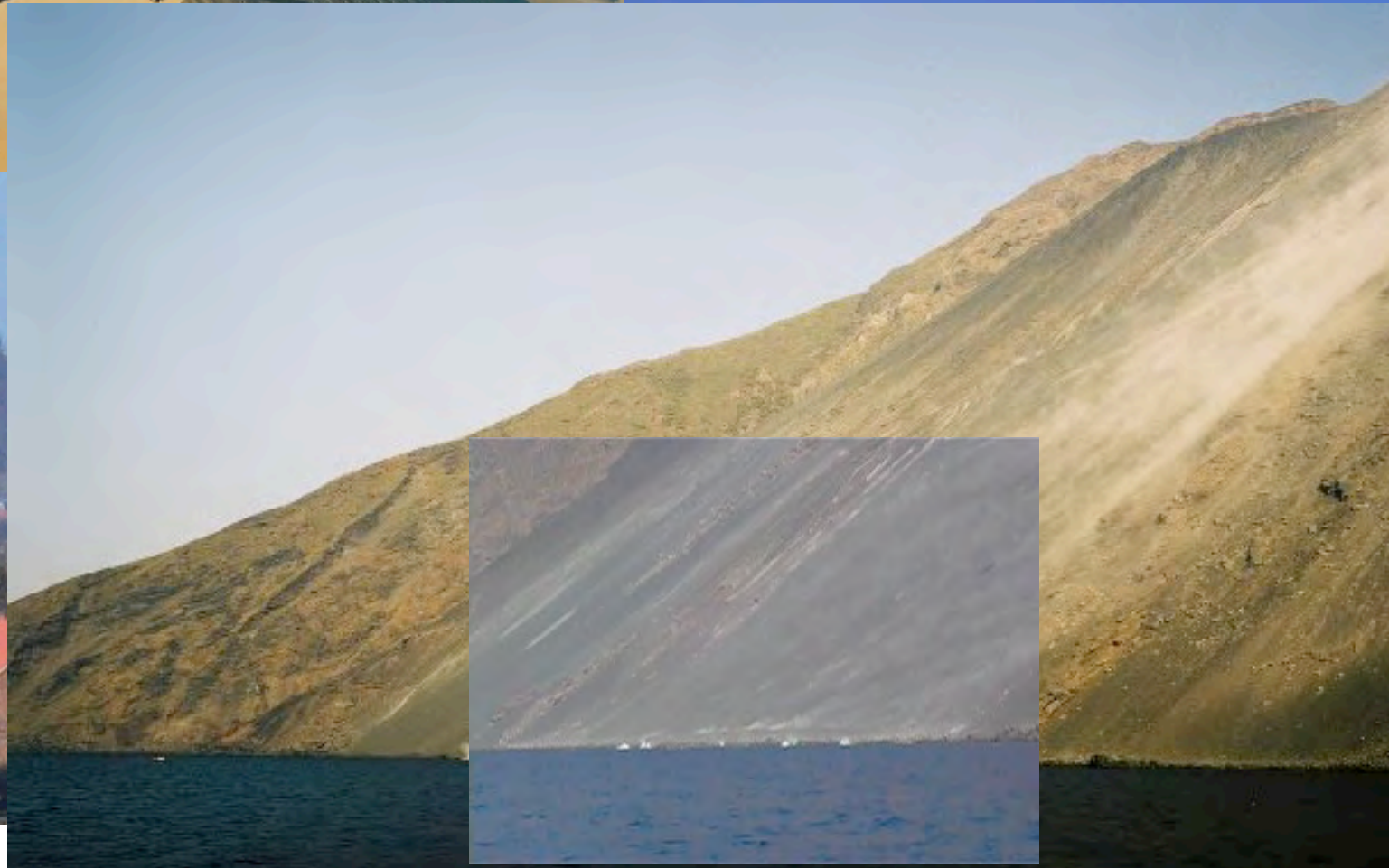
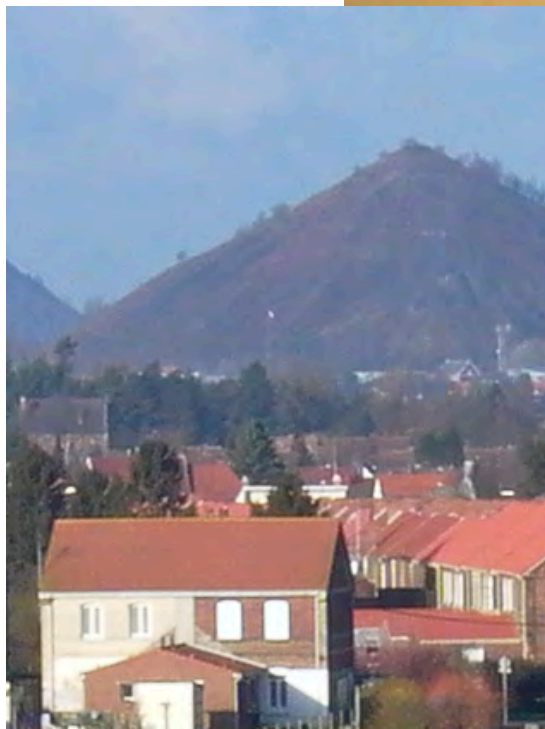
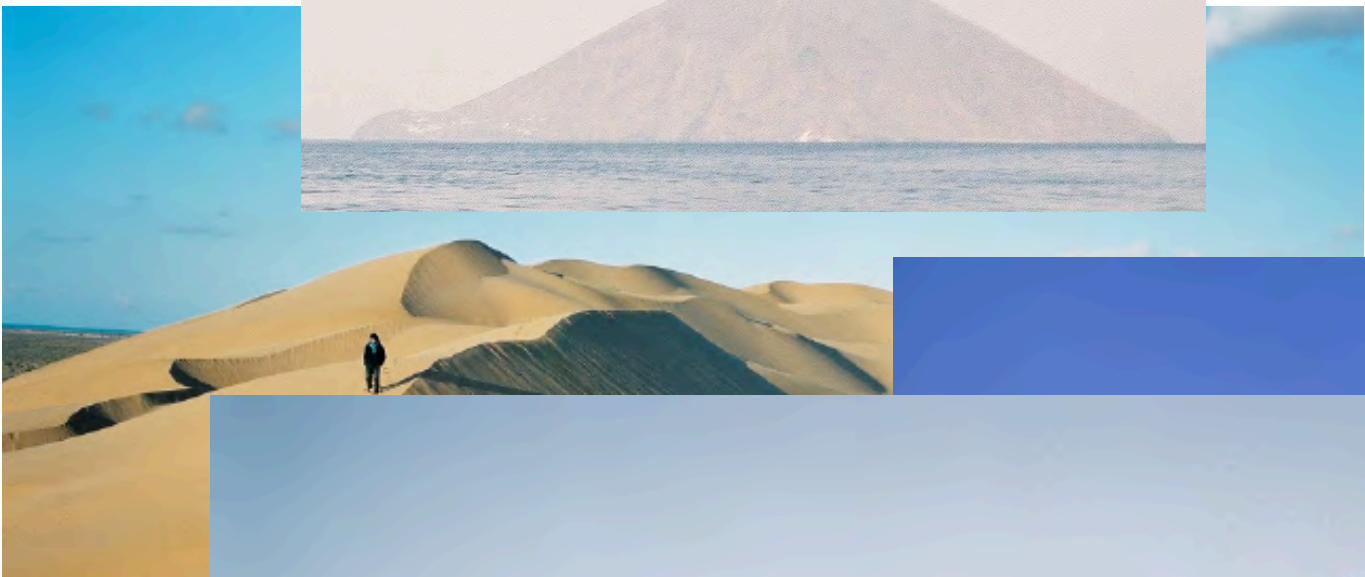


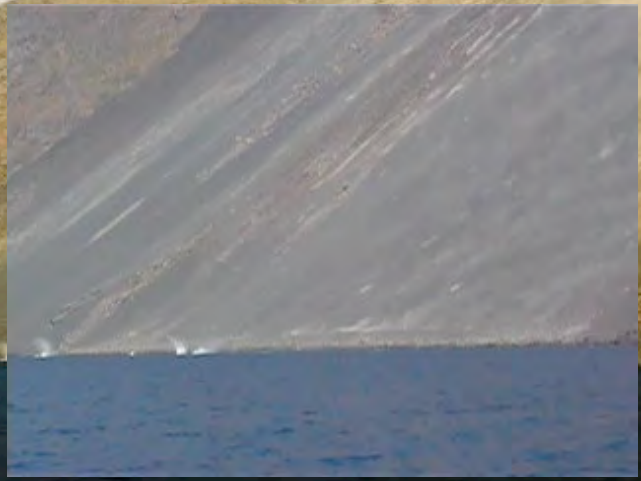
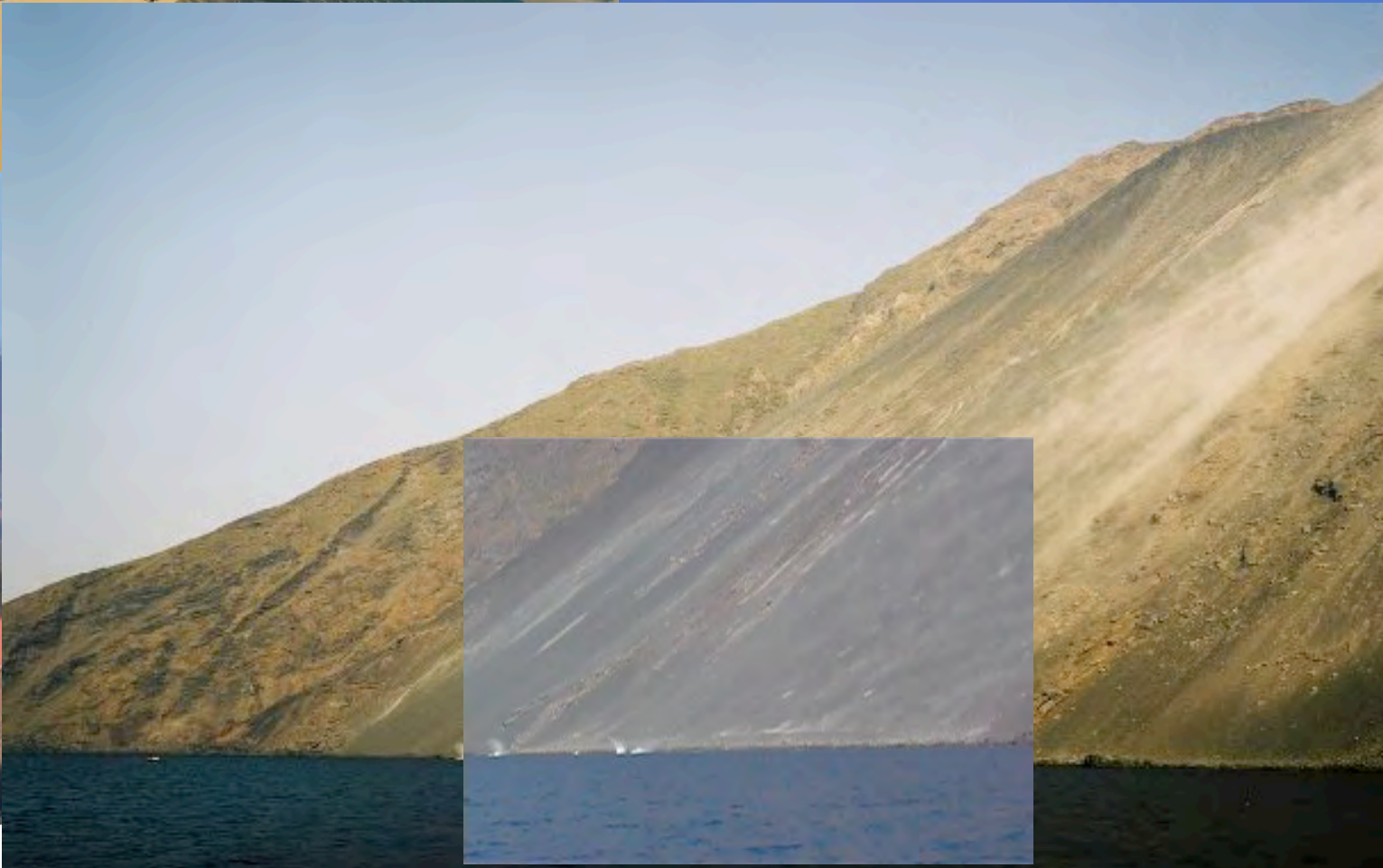
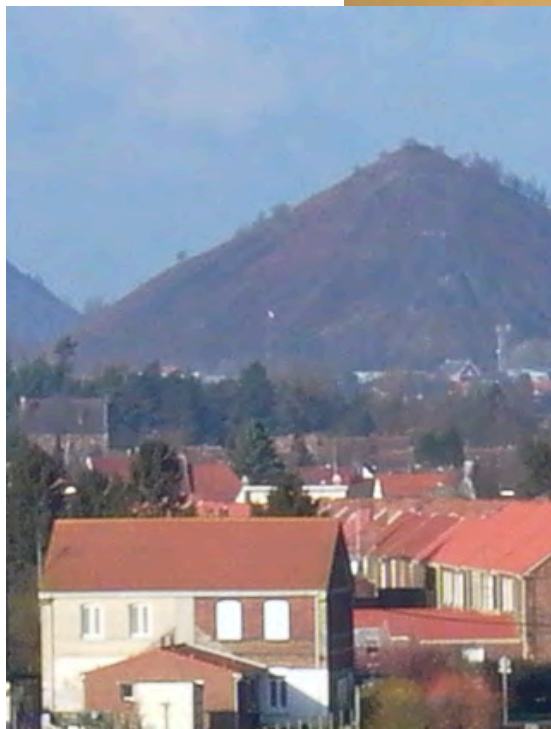
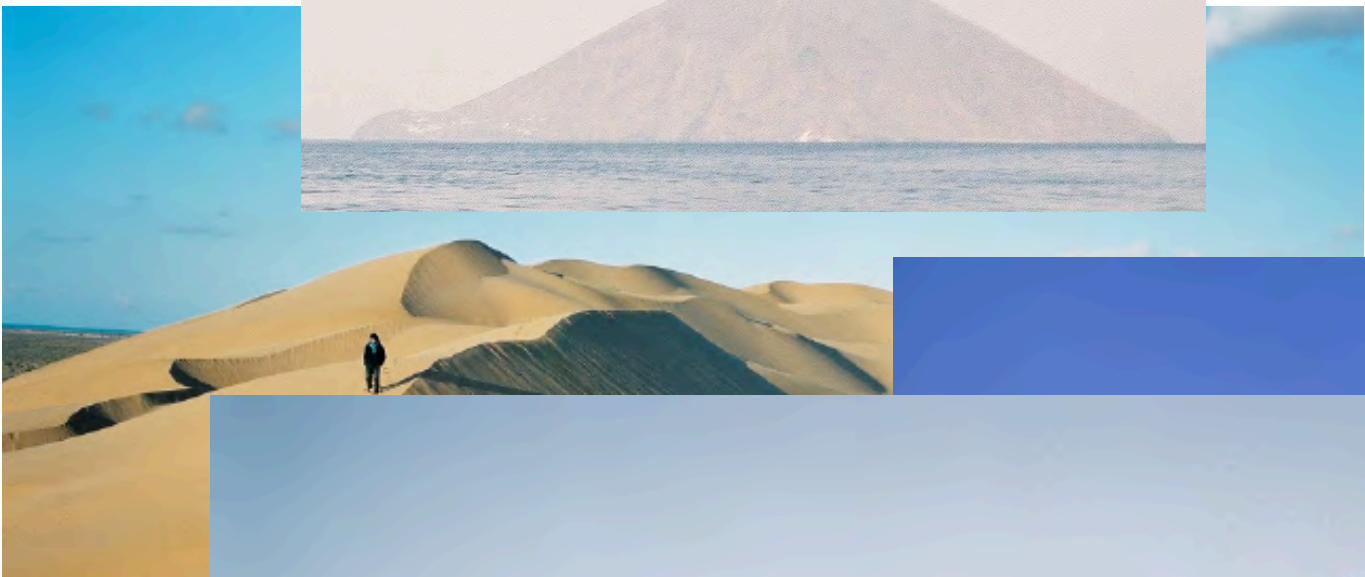


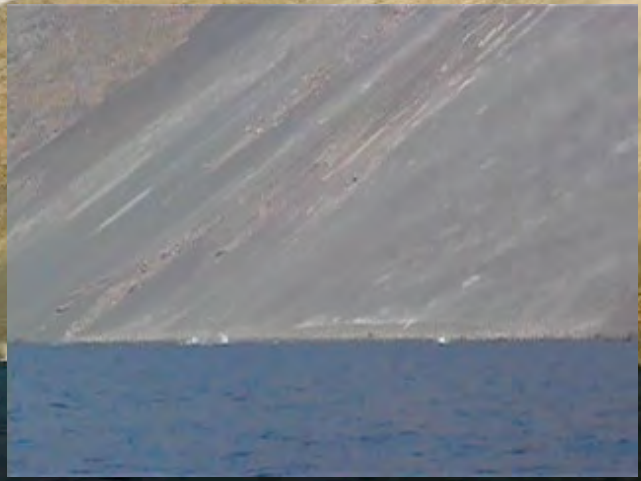
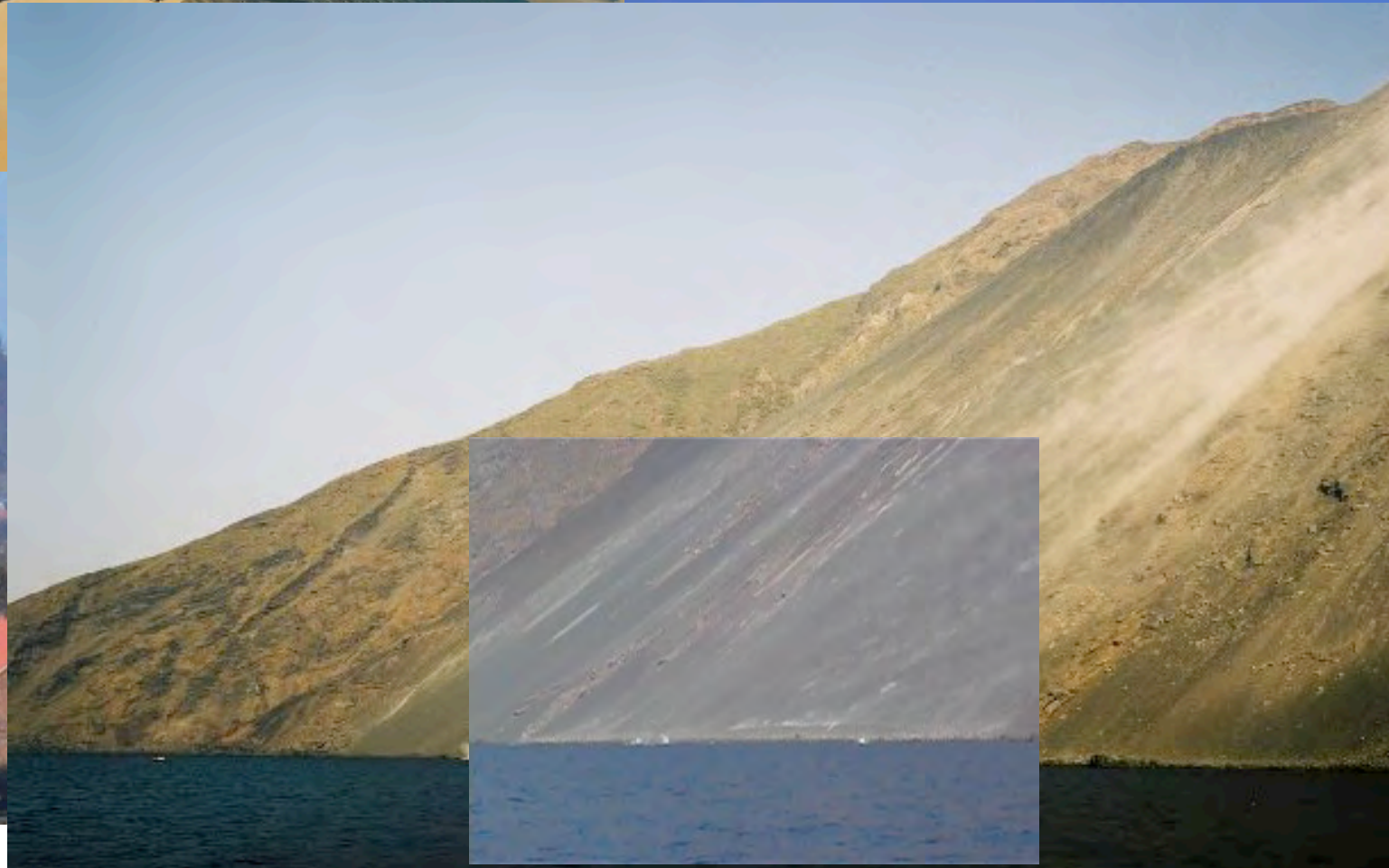
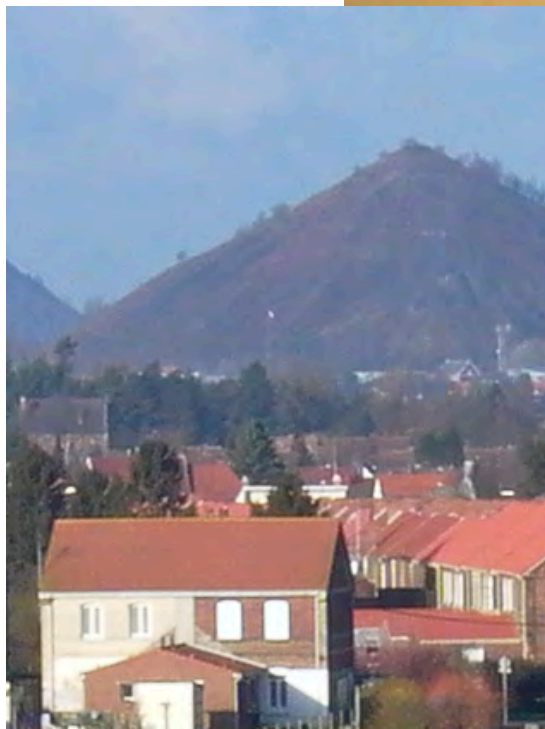
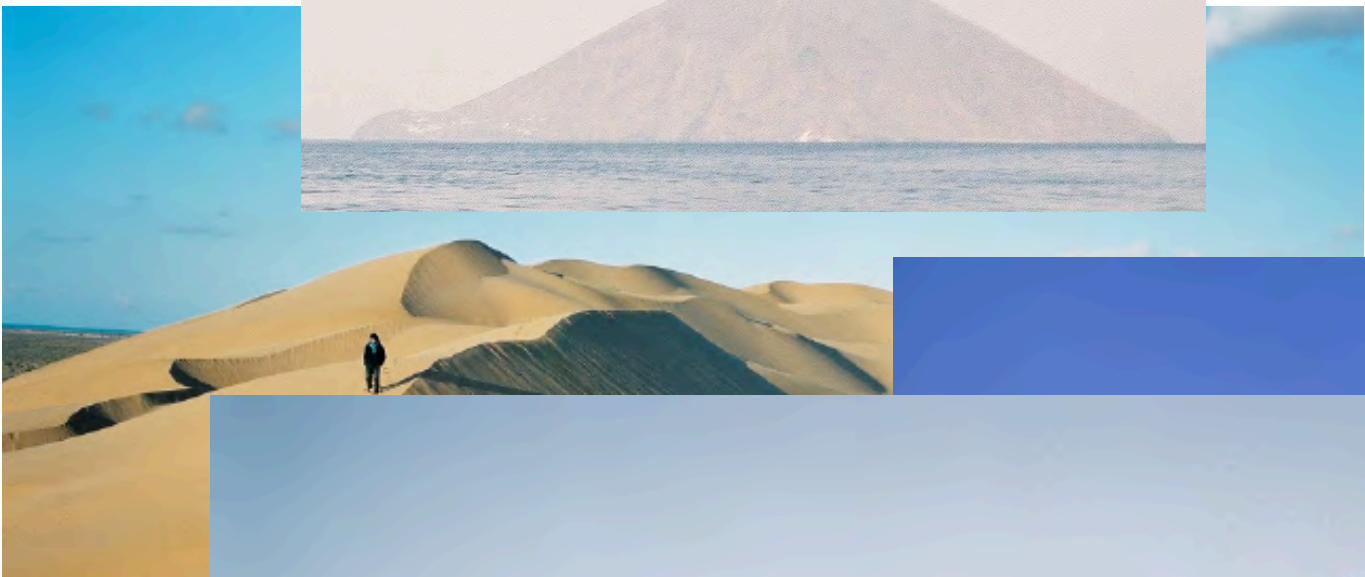


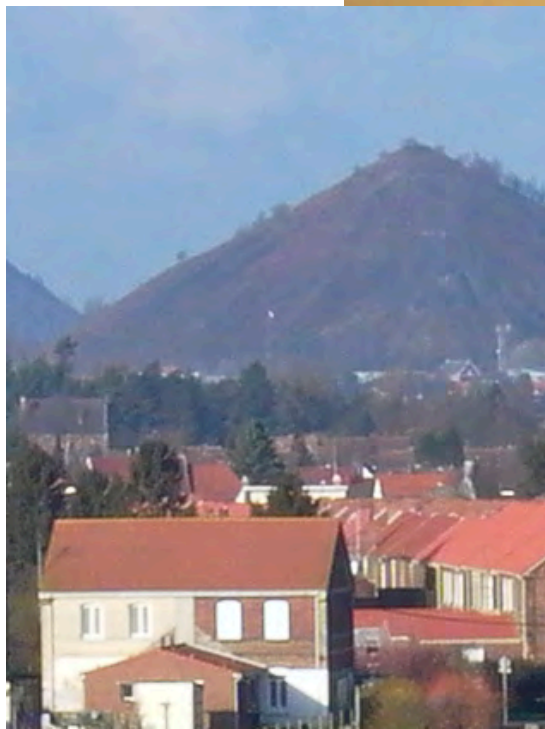


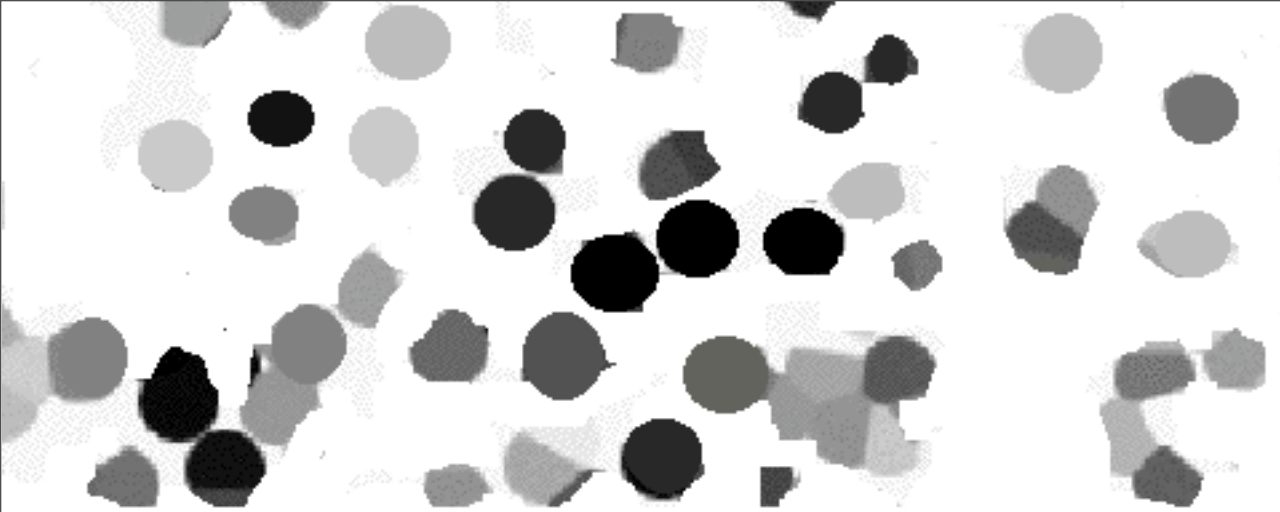






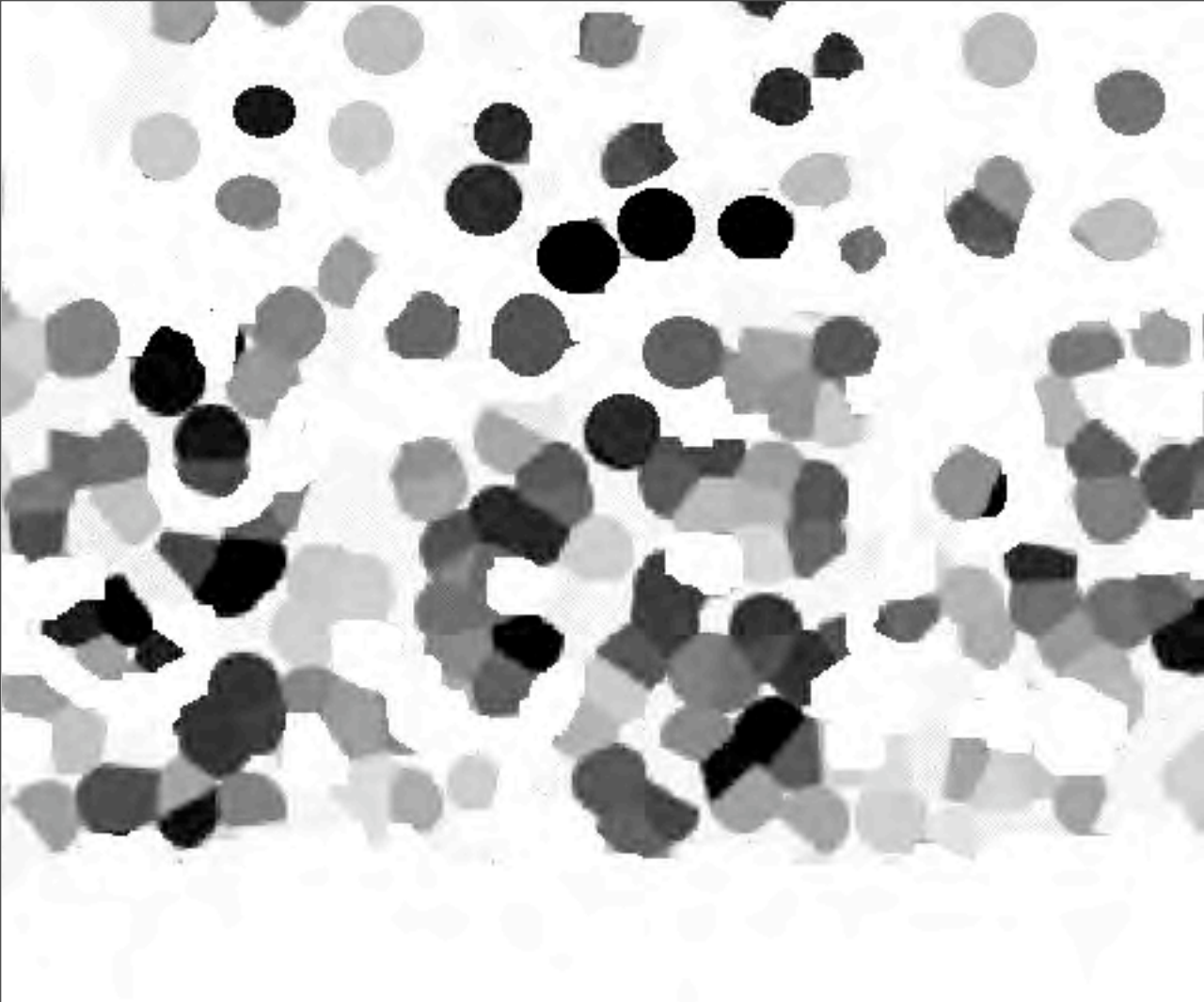






grains

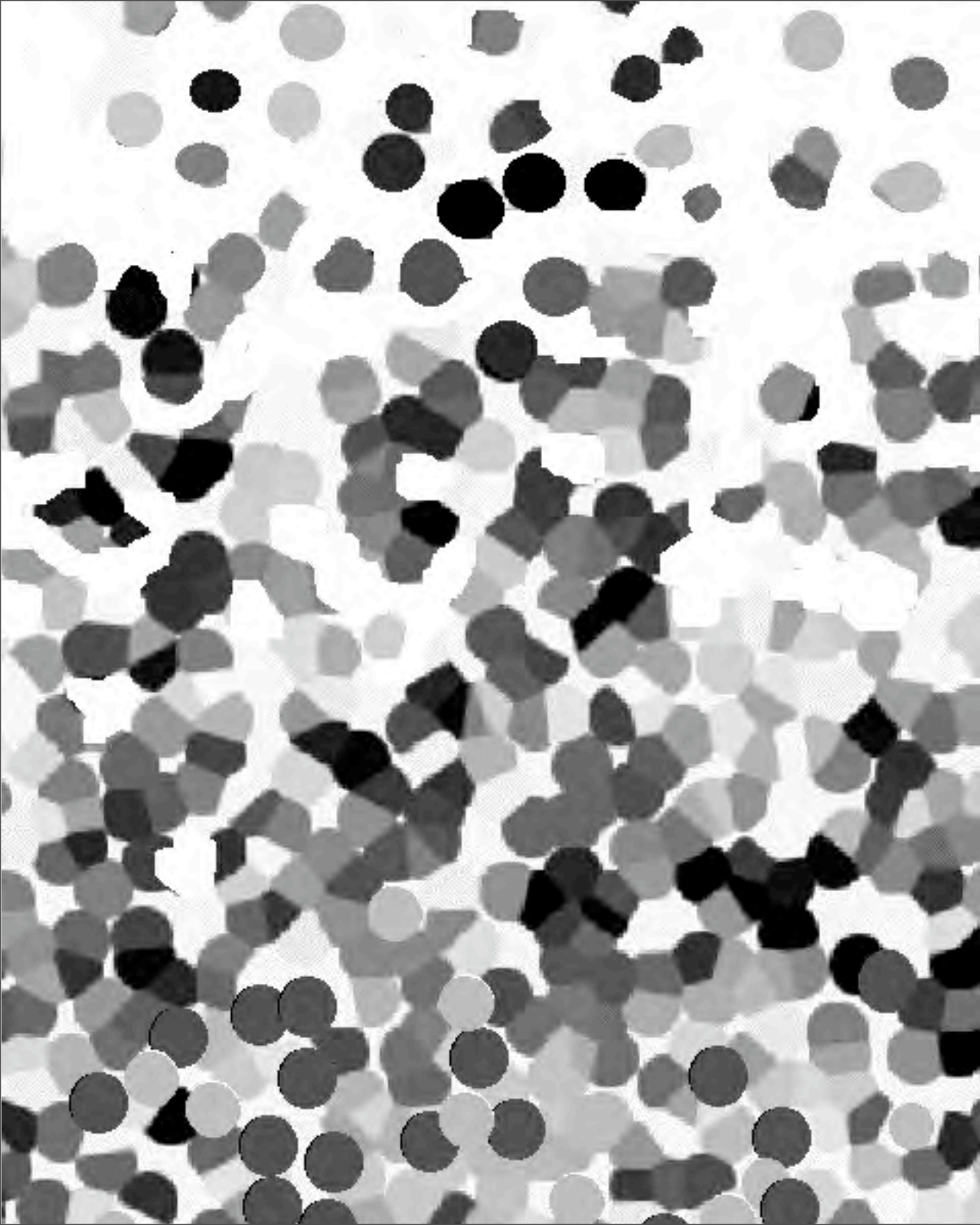
impacts:
suspension



grains

impacts:
suspension

matériau granulaire:
contacts

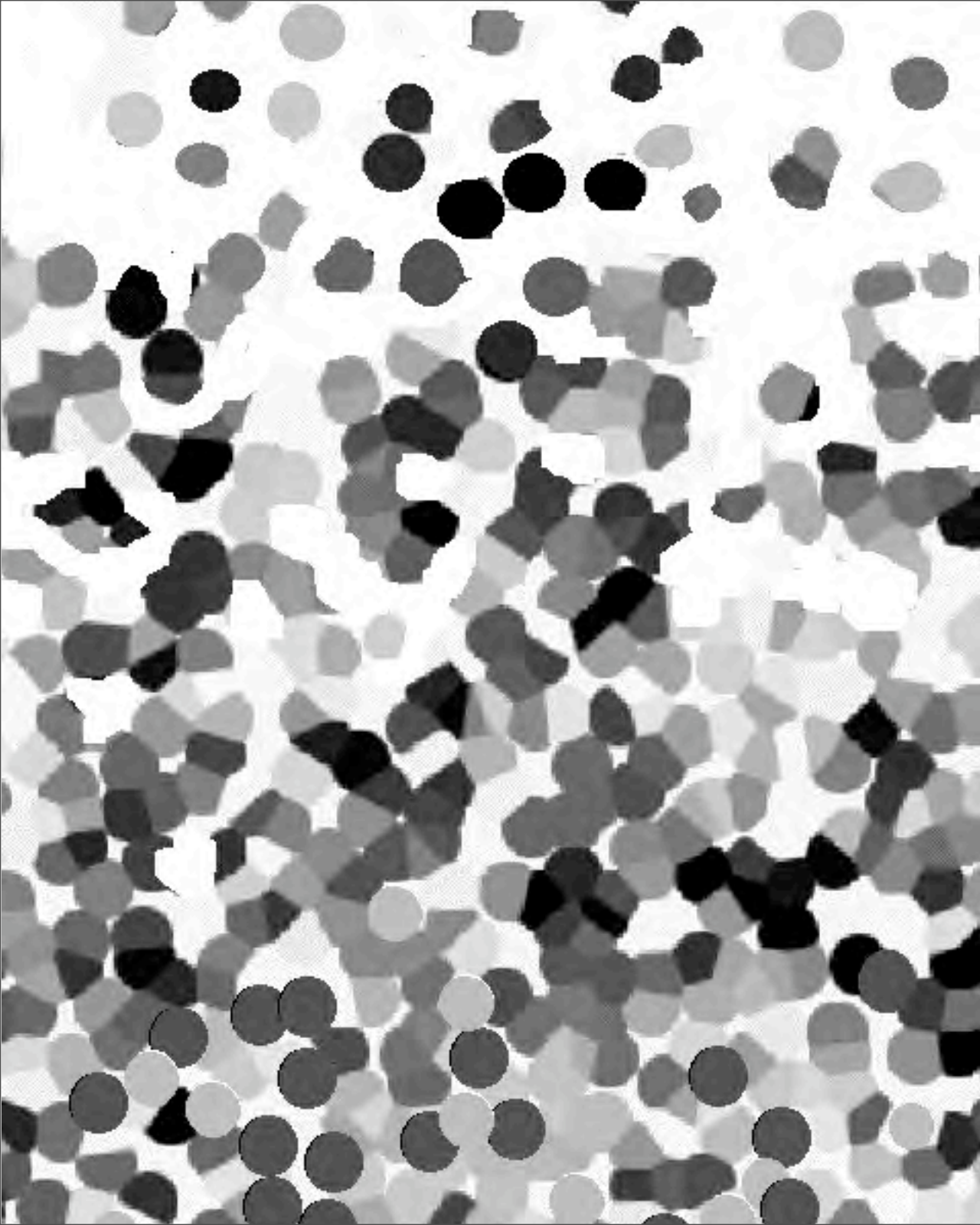


grains

impacts:
suspension

matériau granulaire:
contacts

Matériau comme
un solide:

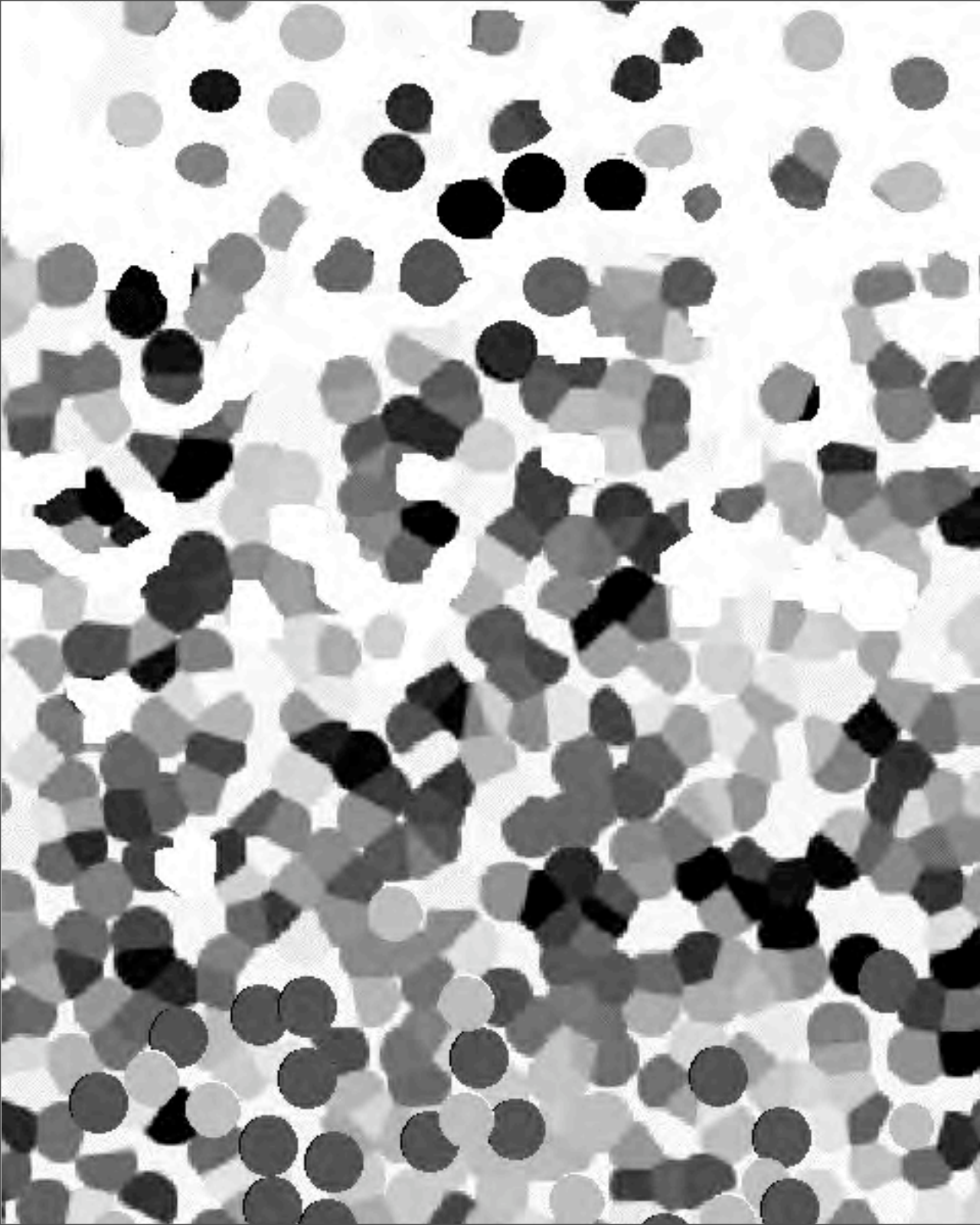


grains

Fraction Volumique

ϕ

“Compaction”

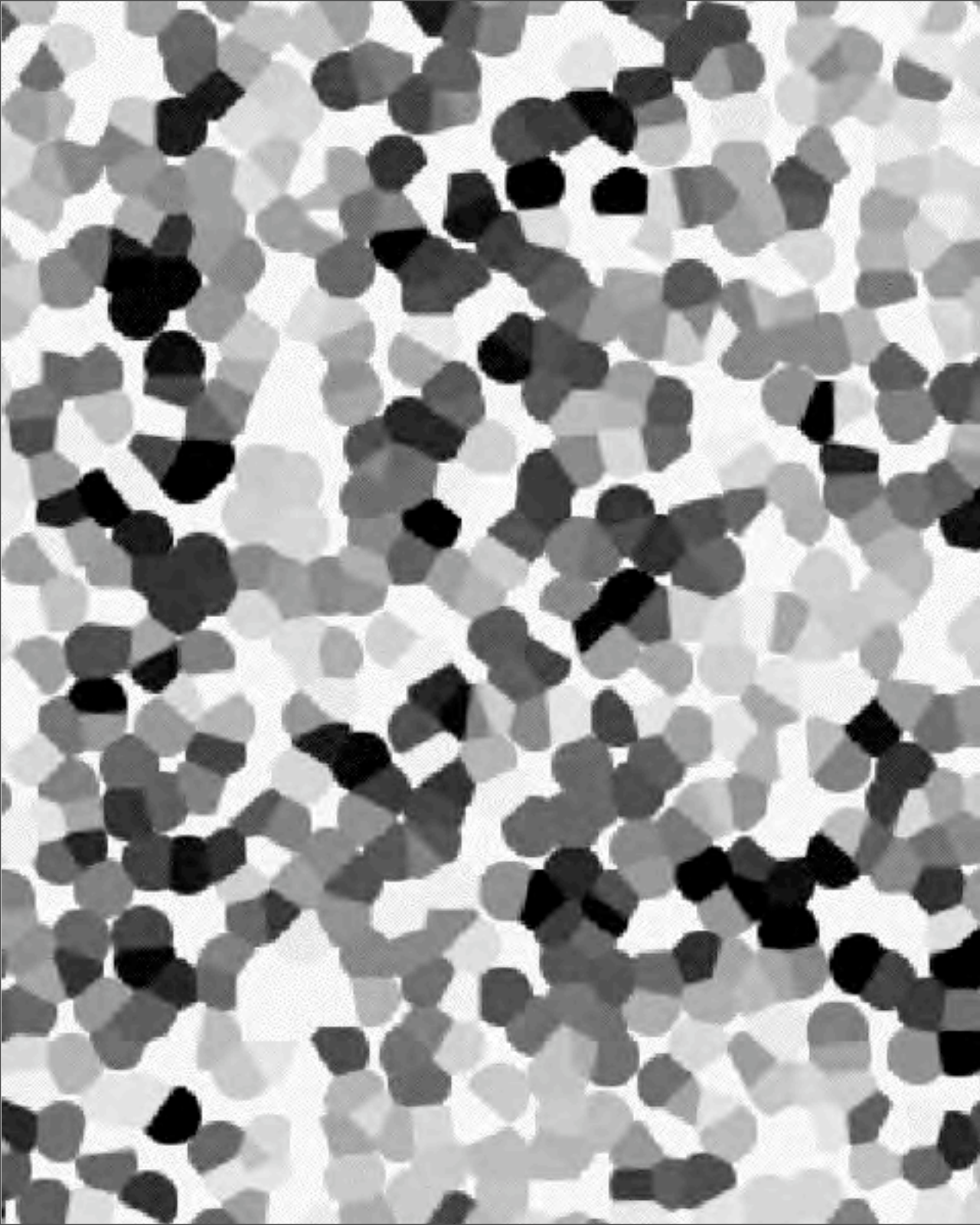


grains

ϕ_{min} 0.5 (2D) 0.55 (3D)

$\phi_{min} < \phi < \phi_{Max}$

ϕ_{max} 0.8 (2D) 0.65 (3D)



~~grains~~

hypothèse de milieu
continu



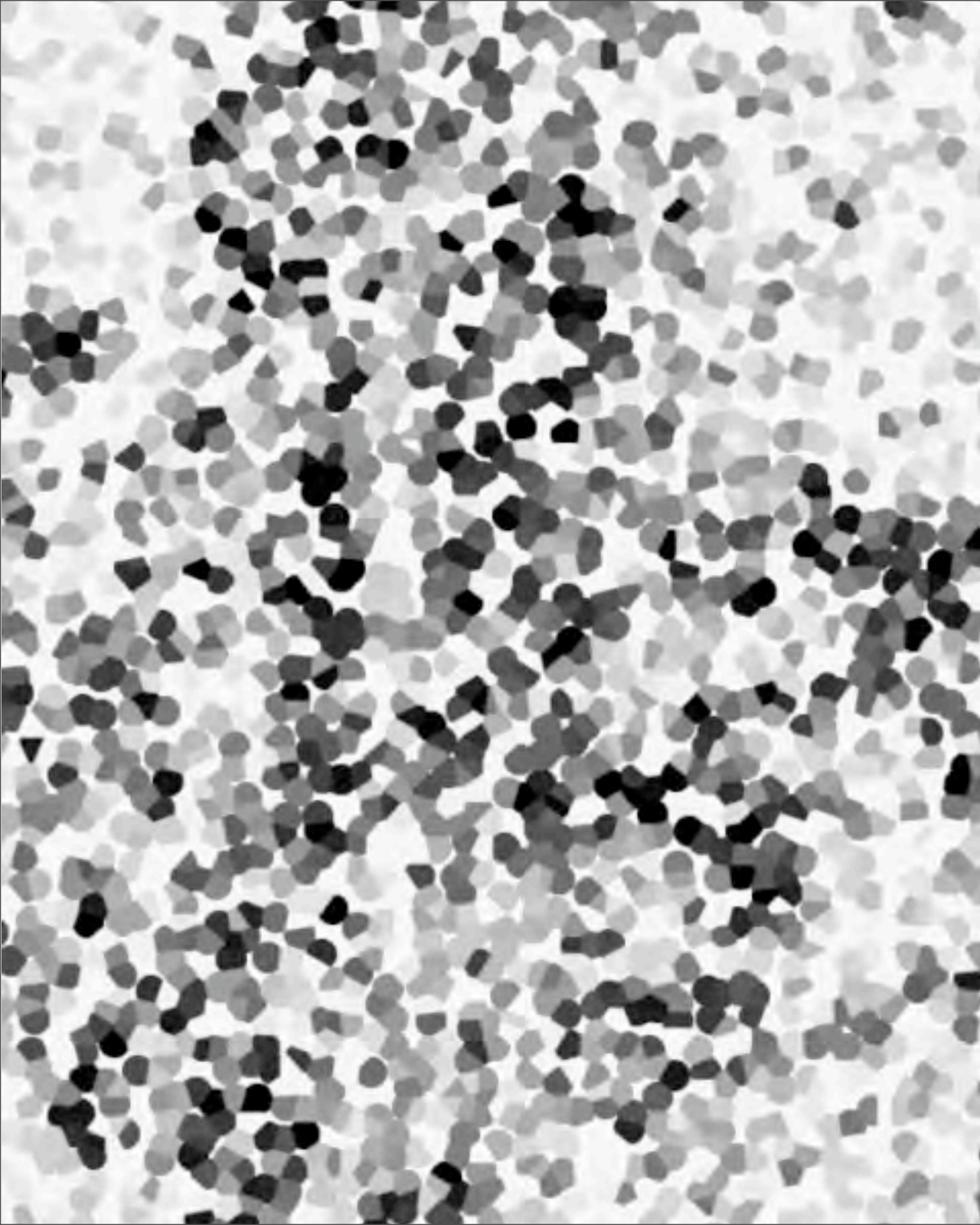
~~grains~~

hypothèse de milieu
continu



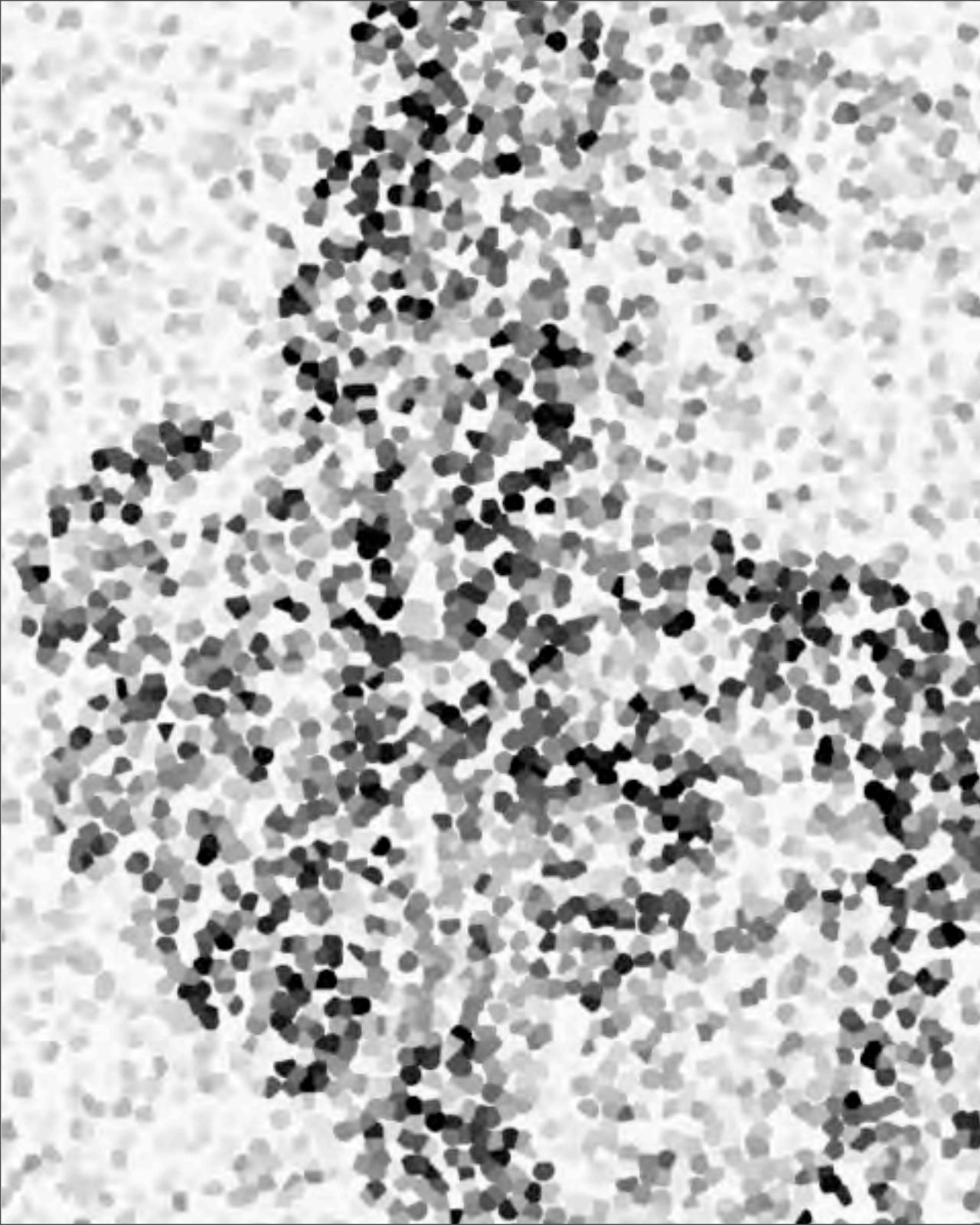
~~grains~~

hypothèse de milieu
continu



~~grains~~

hypothèse de milieu
continu



~~grains~~

hypothèse de milieu
continu



~~grains~~

hypothèse de milieu
continu

~~grains~~

hypothèse de milieu
continu





~~grains~~

hypothèse de milieu
continu

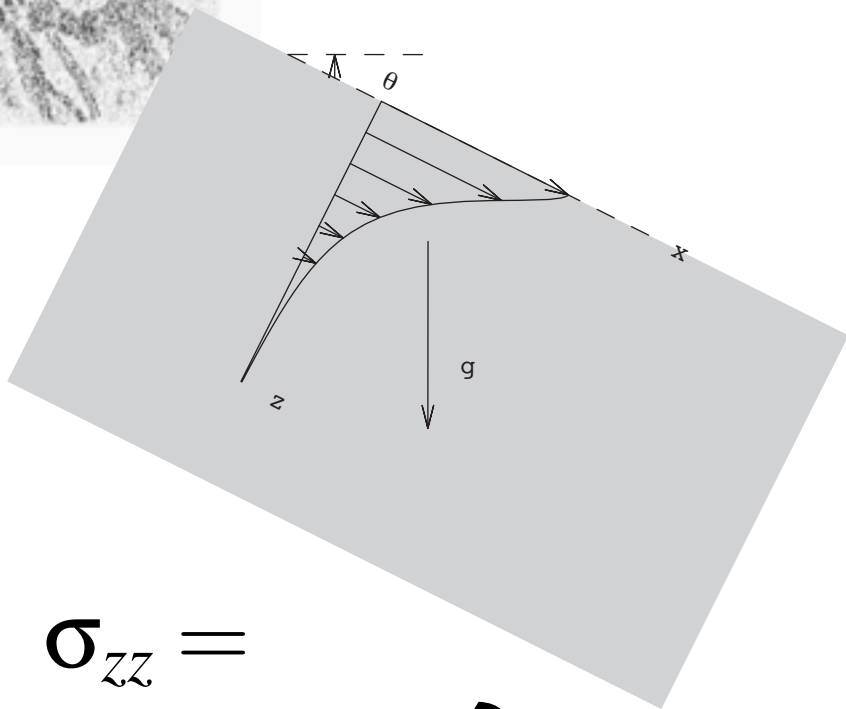


~~grains~~

hypothèse de milieu
continu



hypothèse de milieu continu



$$-\frac{\partial \sigma_{xz}}{\partial z} + \phi \rho g \sin(\theta) = 0$$
$$-\frac{\partial \sigma_{zz}}{\partial z} + \phi \rho g \cos(\theta) = 0$$

$$\sigma_{zz} =$$

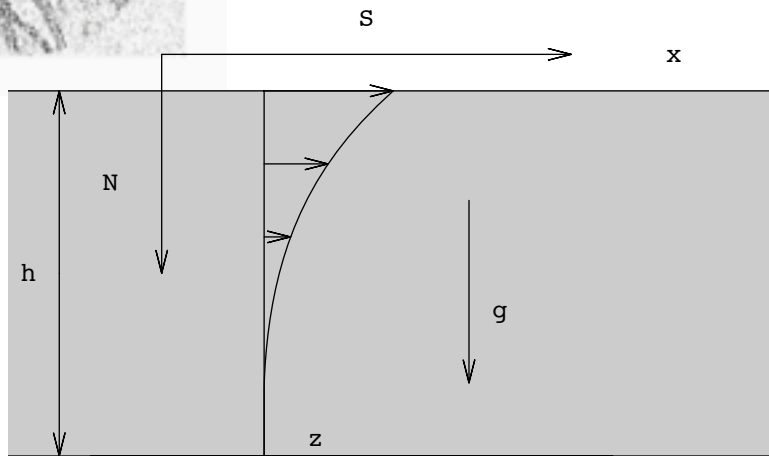
?

$$\sigma_{xz} =$$

équations constitutives simplifiées



hypothèse de milieu continu



$$\sigma_{zz} =$$

?

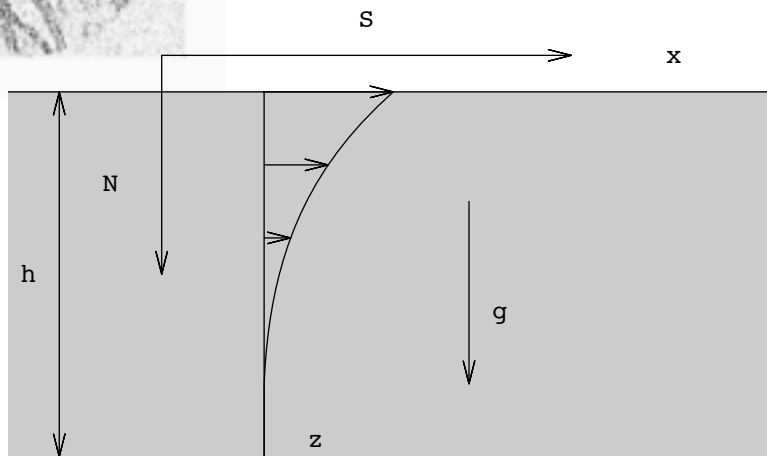
$$\sigma_{xz} =$$

équations constitutives simplifiées



hypothèse de milieu continu

Savage 82, Johnson Jackson 87



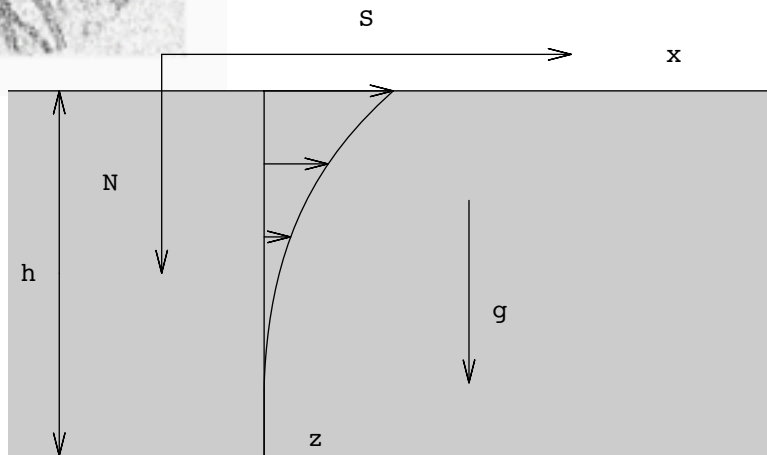
$$\sigma_{zz} = \text{pression de contact} + \text{contrainte}(\phi, (\nabla V)^S, \omega - \frac{1}{2} \nabla \times V)$$

$$\sigma_{xz} =$$

pas de fluide interstitiel, pas de force cohésive, matériau sec



hypothèse de milieu continu



$$\sigma_{zz} = \text{pression de contact} + \text{contrainte}(\phi, (\nabla V)^S, \omega - \frac{1}{2} \nabla \times V)$$
$$\sigma_{xz} =$$

relation tensorielle 3D, trop compliqué!

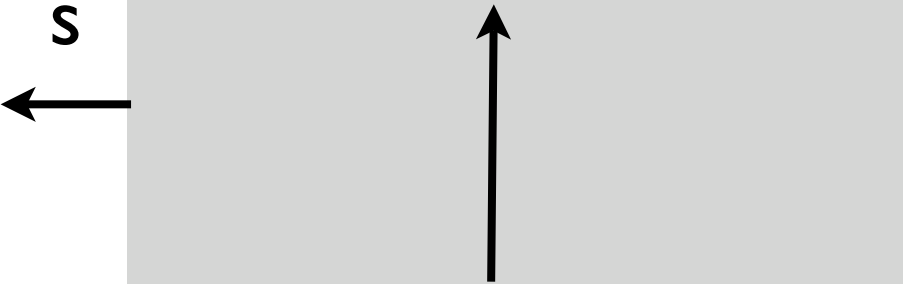
écoulement 2D simple



friction solide

n

s



$$\sigma_{zz} =$$

$$\sigma_{xz} = \mu \sigma_{zz} \text{ "Coulomb"}$$

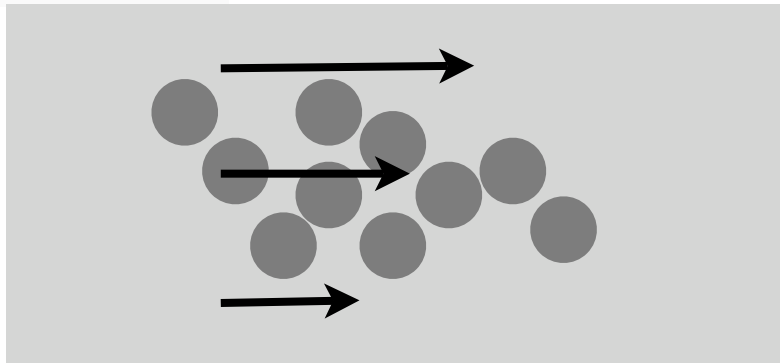


$V(z)$

viscosité $\frac{\partial V}{\partial z}$

$$\sigma_{zz} =$$

$$\sigma_{xz} = \mu \sigma_{zz} \text{ "Coulomb" } + \text{ "cisaillement de Bagnold" }$$

 $V(z)$ 

$$\frac{\text{longueur}^2 \text{ vitesse}}{\text{longueur}} \frac{\partial V}{\partial z}$$

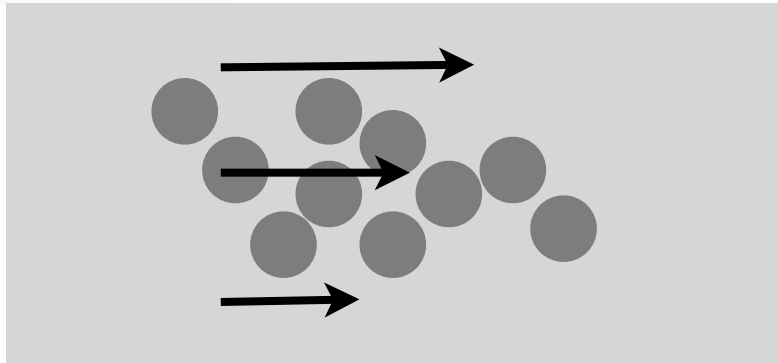
$$\sigma_{zz} =$$

$$\sigma_{xz} = \mu \sigma_{zz} \text{ "Coulomb" } + \text{ "cisaillement de Bagnold" }$$



$V(z)$

D taille du grain



$$\rho D^2 \mu_T(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$

$$\sigma_{zz} =$$

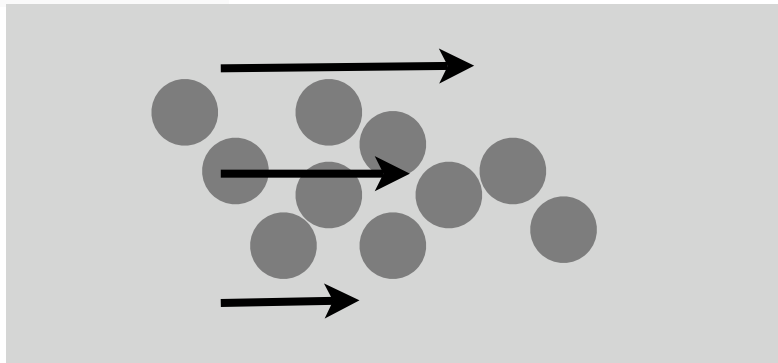
$$\sigma_{xz} = \mu \sigma_{zz} + \rho D^2 \mu_T(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$



$V(z)$

D taille du grain

$$\rho D^2 \mu_T(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$



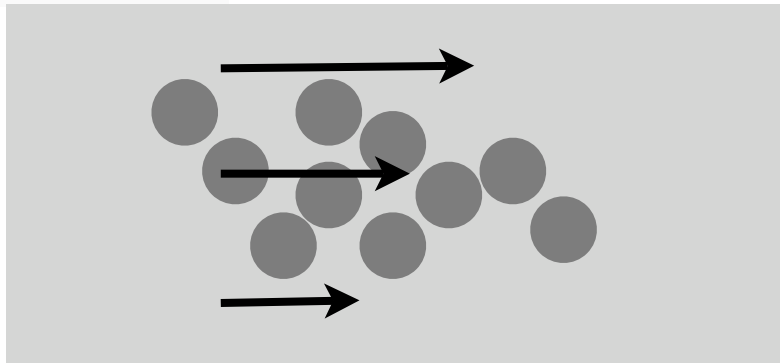
$$\sigma_{zz} =$$

“cisaillement de Bagnold”

$$\sigma_{xz} = \mu \sigma_{zz}$$

+

$$\rho D^2 \mu_T(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$

 $V(z)$ 

$$\sigma_{zz} =$$

“dilatance de Reynolds”

$$\sigma_{xz} = \mu \sigma_{zz}$$

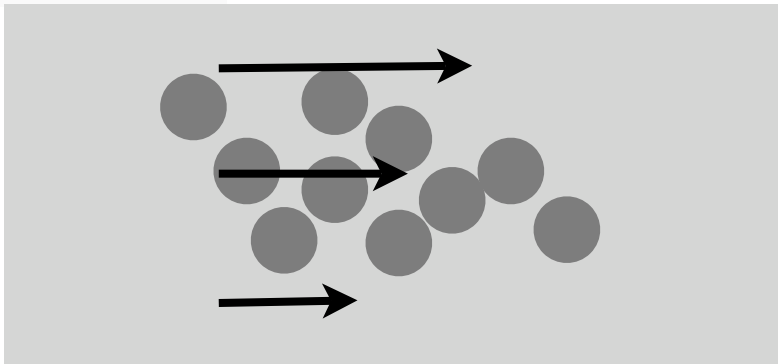
+

$$\rho D^2 \mu_T(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$

cisailler le milieu diminue la compaction



$V(z)$



$$\sigma_{zz} =$$

“dilatance de Reynolds”

$$\sigma_{xz} = \mu \sigma_{zz}$$

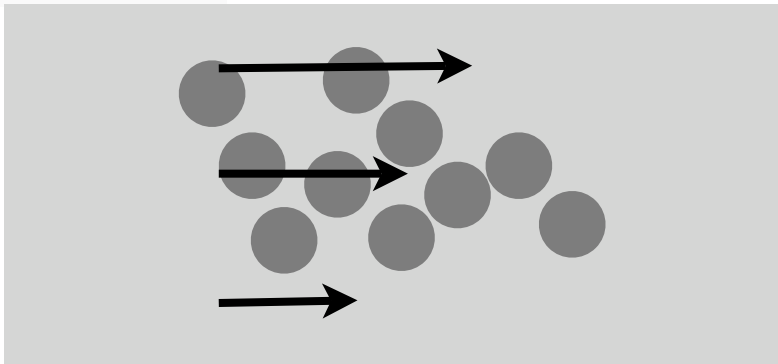
+

$$\rho D^2 \mu_T(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$

cisailler le milieu diminue la compaction



$V(z)$



$$\sigma_{zz} =$$

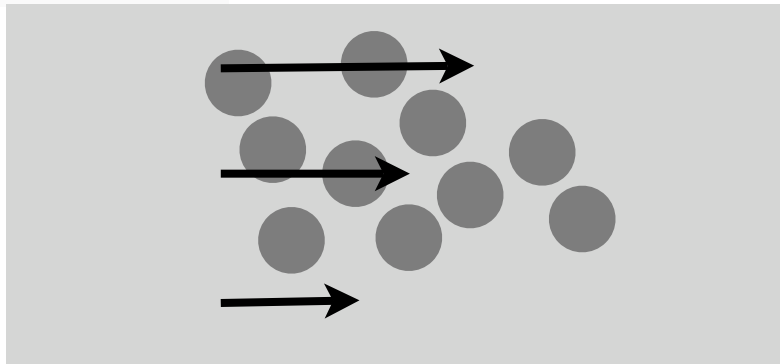
“dilataance de Reynolds”

$$\sigma_{xz} = \mu \sigma_{zz}$$

+

$$\rho D^2 \mu_T(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$

cisailler le milieu diminue la compaction

 $V(z)$ 

$$\sigma_{zz} =$$

“dilatance de Reynolds”

$$\sigma_{xz} = \mu \sigma_{zz}$$

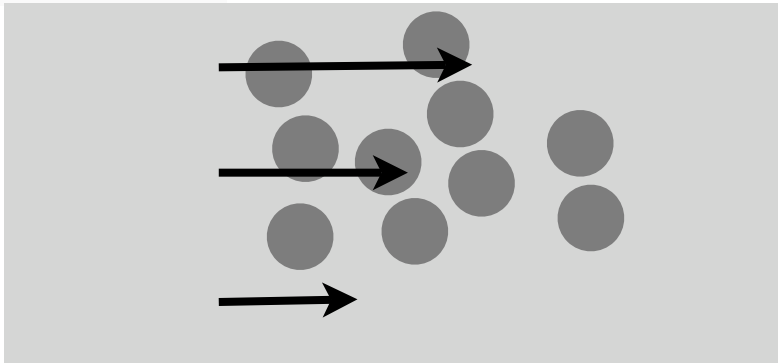
+

$$\rho D^2 \mu_T(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$

cisailler le milieu diminue la compaction



$V(z)$



$$\sigma_{zz} =$$

“dilataance de Reynolds”

$$\sigma_{xz} = \mu \sigma_{zz}$$

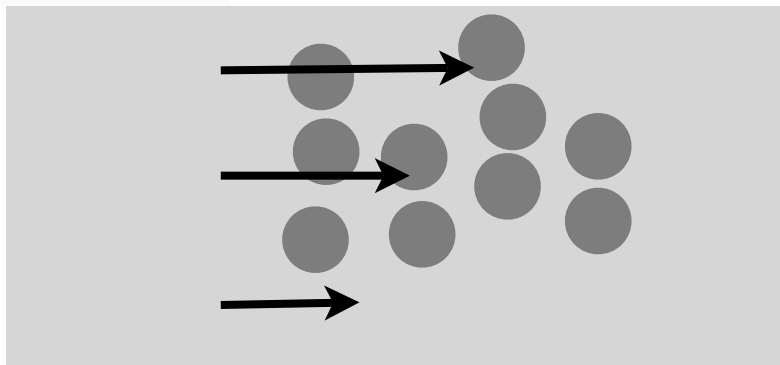
+

$$\rho D^2 \mu_T(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$

cisailler le milieu diminue la compaction



$V(z)$



$$\sigma_{zz} =$$

“dilataance de Reynolds”

$$\sigma_{xz} = \mu \sigma_{zz}$$

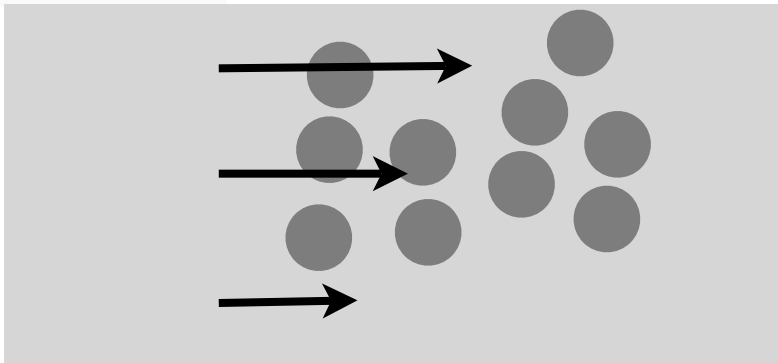
+

$$\rho D^2 \mu_T(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$

cisailler le milieu diminue la compaction



$V(z)$



$$\sigma_{zz} =$$

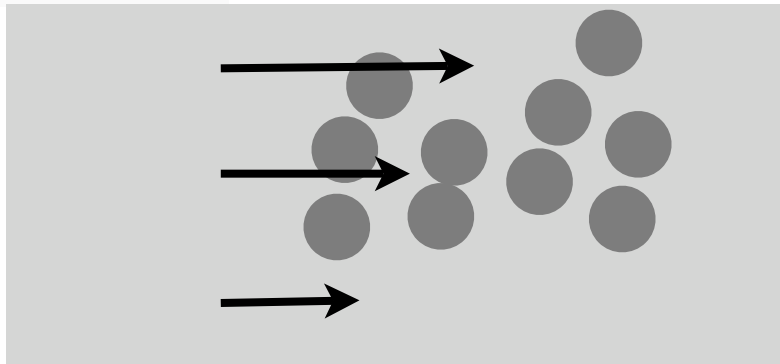
“dilataance de Reynolds”

$$\sigma_{xz} = \mu \sigma_{zz}$$

+

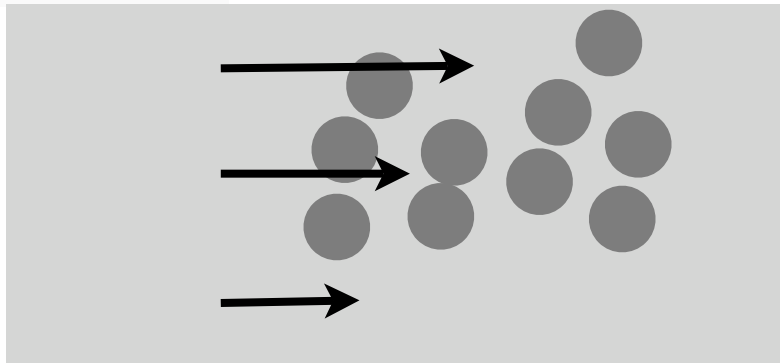
$$\rho D^2 \mu_T(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$

cisailler le milieu diminue la compaction

 $V(z)$ 

$$\begin{aligned}\sigma_{zz} &= & + & \rho D^2 \mu_N(\phi) \left(\frac{\partial V}{\partial z}\right)^2 \\ \sigma_{xz} &= \mu \sigma_{zz} & + & \rho D^2 \mu_T(\phi) \left(\frac{\partial V}{\partial z}\right)^2\end{aligned}$$

cisailler le milieu diminue la compaction

 $V(z)$ 

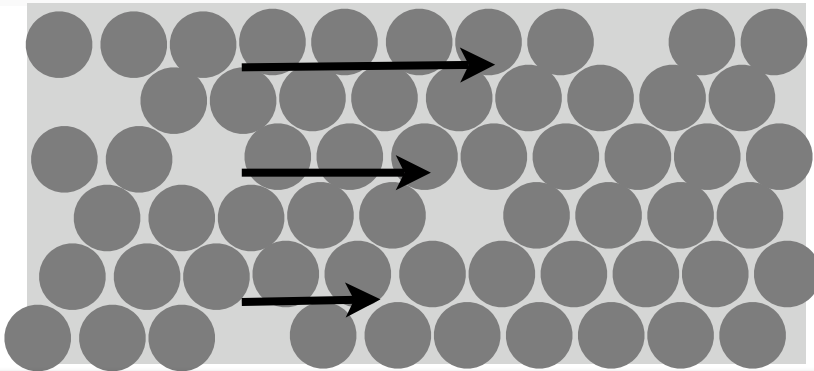
$$\begin{aligned}\sigma_{zz} &= + \rho D^2 \mu_N(\phi) \left(\frac{\partial V}{\partial z}\right)^2 \\ \sigma_{xz} &= \mu \sigma_{zz} + \rho D^2 \mu_T(\phi) \left(\frac{\partial V}{\partial z}\right)^2\end{aligned}$$

ou, à compaction constante, cisailier augmente la “pression”



pression granulaire

volume libre



$$\sigma_{zz} = \text{“pression granulaire”}$$

+

$$\rho D^2 \mu_N(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$

$$\sigma_{xz} = \mu \sigma_{zz}$$

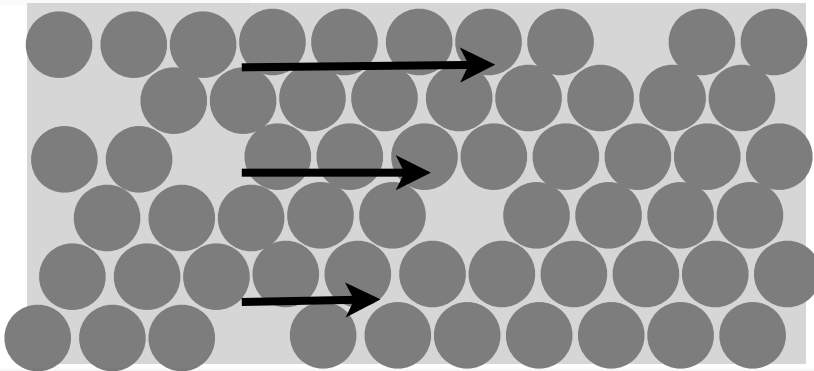
+

$$\rho D^2 \mu_T(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$



pression granulaire

micro-configurations ayant
la même fraction volumique



$$\sigma_{zz} = \text{“pression granulaire”}$$

+

$$\rho D^2 \mu_N(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$

$$\sigma_{xz} = \mu \sigma_{zz}$$

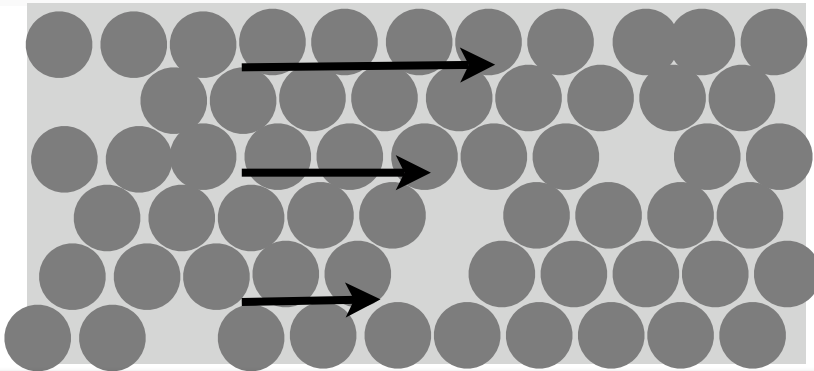
+

$$\rho D^2 \mu_T(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$



pression granulaire

micro-configurations ayant
la même fraction volumique



$$\sigma_{zz} = \text{“pression granulaire”}$$

+

$$\rho D^2 \mu_N(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$

$$\sigma_{xz} = \mu \sigma_{zz}$$

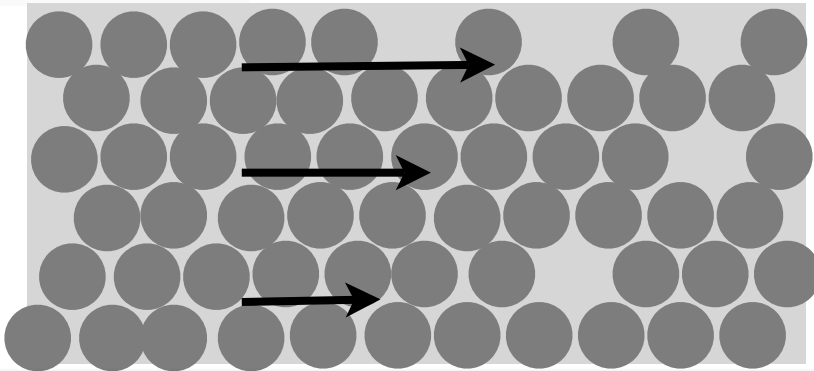
+

$$\rho D^2 \mu_T(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$



pression granulaire

micro-configurations ayant
la même fraction volumique



$$\sigma_{zz} = \text{“pression granulaire”}$$

+

$$\rho D^2 \mu_N(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$

$$\sigma_{xz} = \mu \sigma_{zz}$$

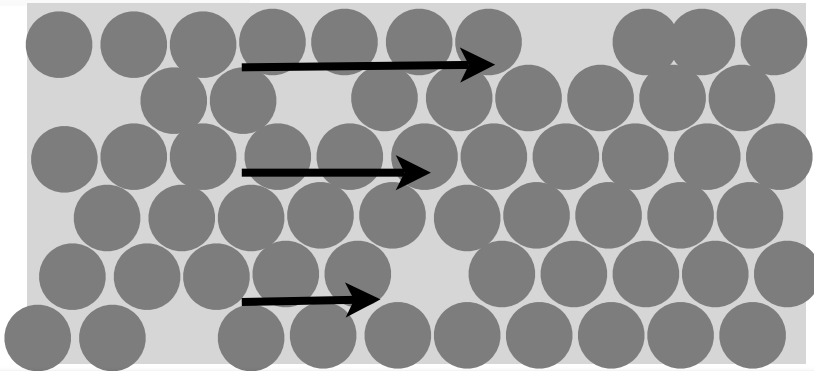
+

$$\rho D^2 \mu_T(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$



pression granulaire

micro-configurations ayant
la même fraction volumique



$$\sigma_{zz} = \text{“pression granulaire”}$$

+

$$\rho D^2 \mu_N(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$

$$\sigma_{xz} = \mu \sigma_{zz}$$

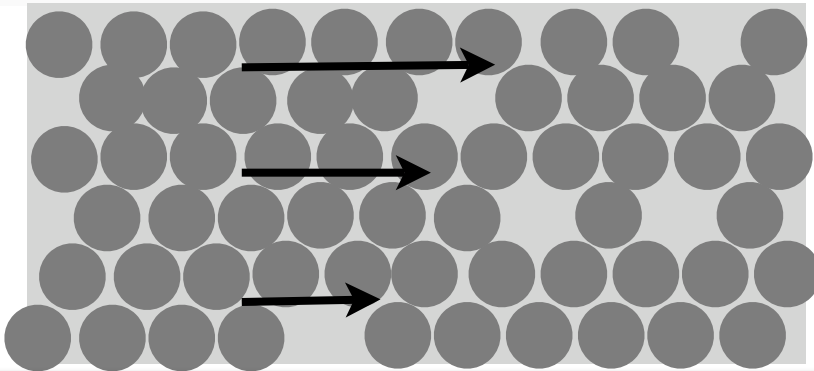
+

$$\rho D^2 \mu_T(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$



pression granulaire

micro-configurations ayant
la même fraction volumique



$$\sigma_{zz} = \text{“pression granulaire”}$$

+

$$\rho D^2 \mu_N(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$

$$\sigma_{xz} = \mu \sigma_{zz}$$

+

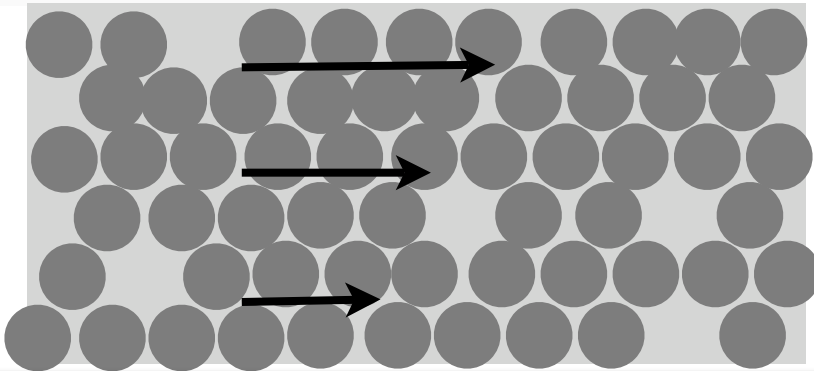
$$\rho D^2 \mu_T(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$



pression granulaire

nombre de micro-configurations

$$\Omega(\phi)$$



$$\sigma_{zz} = \text{“pression granulaire”}$$

+

$$\rho D^2 \mu_N(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$

$$\sigma_{xz} = \mu \sigma_{zz}$$

+

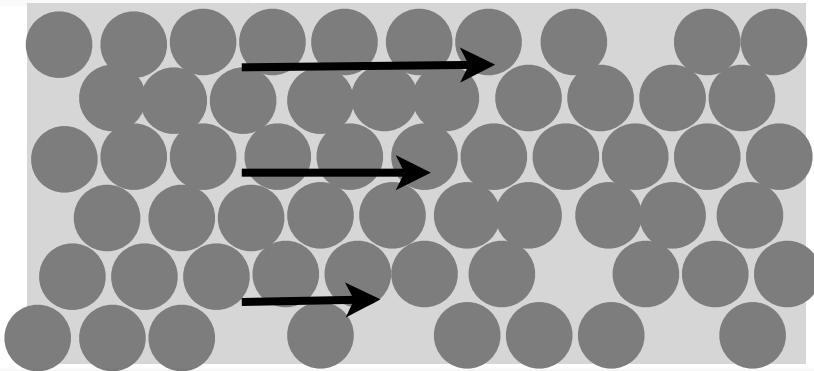
$$\rho D^2 \mu_T(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$



pression granulaire

entropie

$$S(\phi)$$



$$\sigma_{zz} = \text{“pression granulaire”}$$

+

$$\rho D^2 \mu_N(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$

$$\sigma_{xz} = \mu \sigma_{zz}$$

+

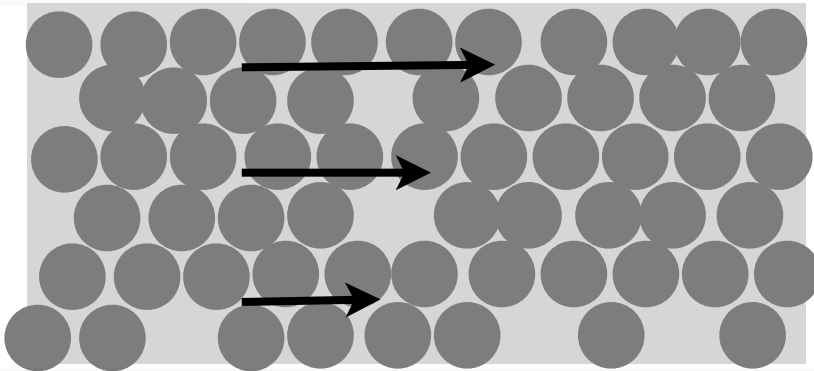
$$\rho D^2 \mu_T(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$



pression granulaire

pression

$$p(\phi) = -\frac{\partial S(\phi)}{\partial \phi}$$



$$\sigma_{zz} = p(\phi)$$

+

$$\rho D^2 \mu_N(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$

$$\sigma_{xz} = \mu \sigma_{zz}$$

+

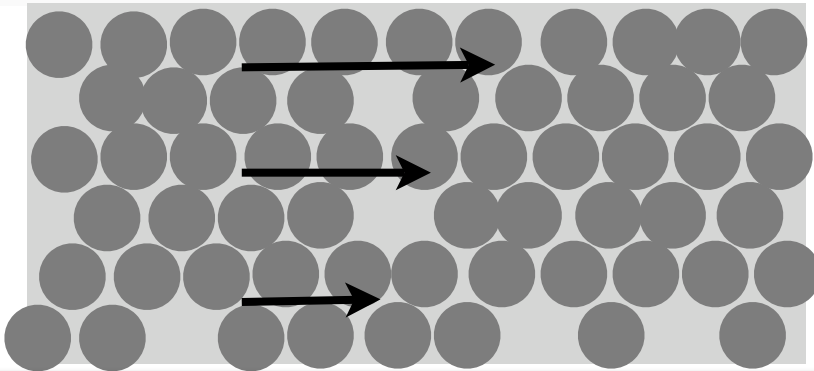
$$\rho D^2 \mu_T(\phi) \left(\frac{\partial V}{\partial z} \right)^2$$



pression granulaire

pression

$$p(\phi) = -P_0 \ln\left(\frac{\phi_M - \phi_m}{\phi_M - \phi}\right)$$



$$\sigma_{zz} = p(\phi)$$

+

$$\rho D^2 \mu_N(\phi) \left(\frac{\partial V}{\partial z}\right)^2$$

$$\sigma_{xz} = \mu \sigma_{zz}$$

+

$$\rho D^2 \mu_T(\phi) \left(\frac{\partial V}{\partial z}\right)^2$$

Savage 98, Kanatani 80



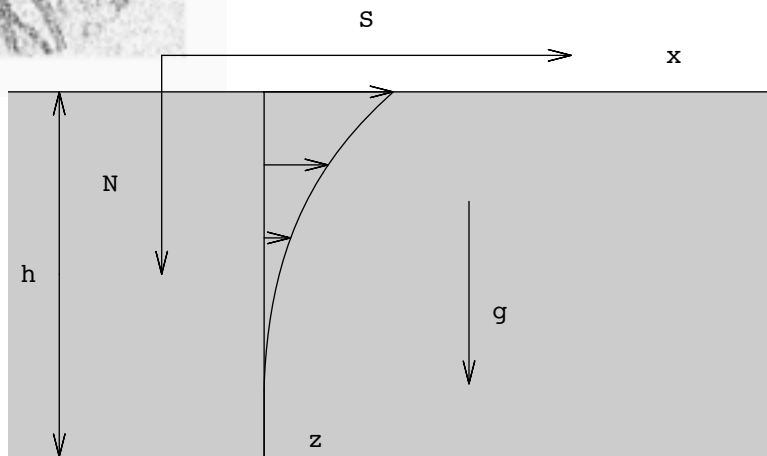
choix particulier des dépendances des coefficients

μ_T / μ_N indépendant de la compaction

$$p(\phi) = P_0 \text{Log} \left(\frac{\phi_M - \phi_m}{\phi_M - \phi} \right) \text{ et } \mu_T = \mu_{T0} \left(\frac{\phi_M - \phi_m}{\phi_M - \phi} \right)^2$$



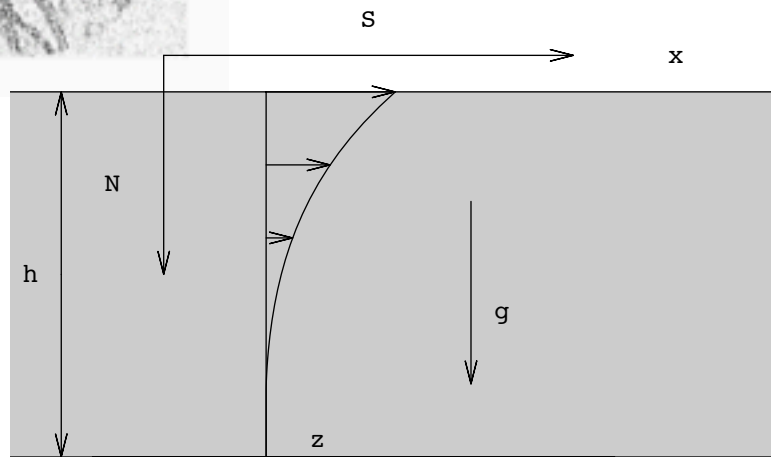
en résumé



$$\sigma_{zz} = \text{“pression granulaire”} + \text{“dilatance de Reynolds”}$$

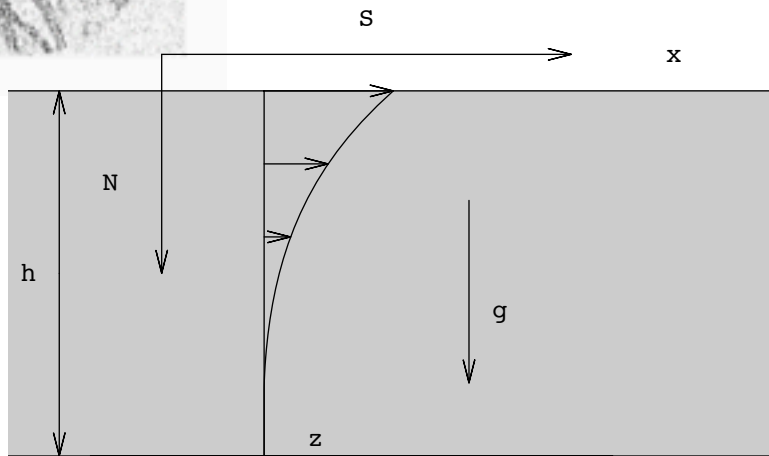
$$\sigma_{xz} = \mu\sigma_{zz} \text{“Coulomb”} + \text{“cisaillement de Bagnold”}$$

équations constitutives simplifiées



$$\begin{aligned}\sigma_{zz} &= p(\phi) & + & \rho D^2 \mu_N(\phi) \left(\frac{\partial V}{\partial z}\right)^2 \\ \sigma_{xz} &= \mu \sigma_{zz} & + & \rho D^2 \mu_T(\phi) \left(\frac{\partial V}{\partial z}\right)^2\end{aligned}$$

équations constitutives simplifiées

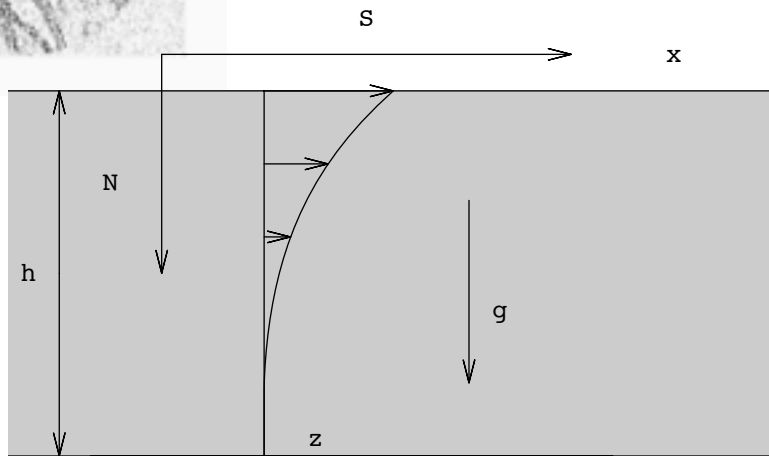


$$1 = p(\phi) / \sigma_{zz} + \rho D^2 \mu_N(\phi) \left(\frac{\partial V}{\partial z} \right)^2 / \sigma_{zz}$$

$$\sigma_{xz} / \sigma_{zz} = \mu + \rho D^2 \mu_T(\phi) \left(\frac{\partial V}{\partial z} \right)^2 / \sigma_{zz}$$

variable d'inertie sans dimension

$$I = \frac{\left(D \frac{\partial V}{\partial z} \right)}{\sqrt{\sigma_{zz} / \rho}}$$

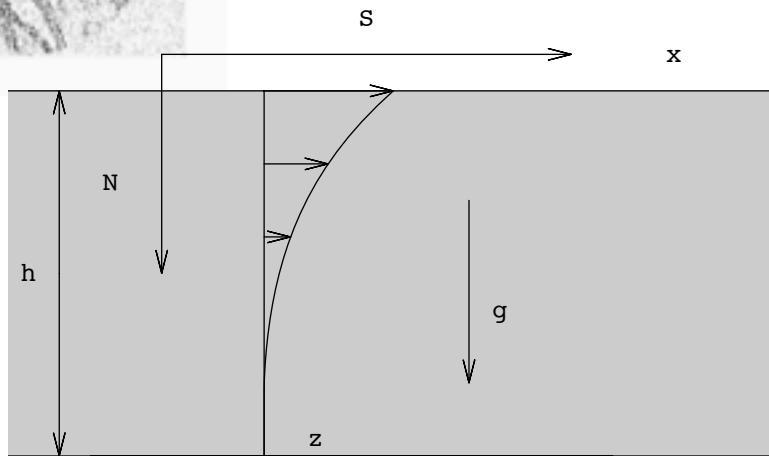


relation entre ϕ , I et σ_{zz}/P_0

$$\sigma_{xz}/\sigma_{zz} = \mu + \rho D^2 \mu_T(\phi) \left(\frac{\partial V}{\partial z} \right)^2 / \sigma_{zz}$$

variable d'inertie sans dimension

$$I = \frac{\left(D \frac{\partial V}{\partial z} \right)}{\sqrt{\sigma_{zz}/\rho}}$$

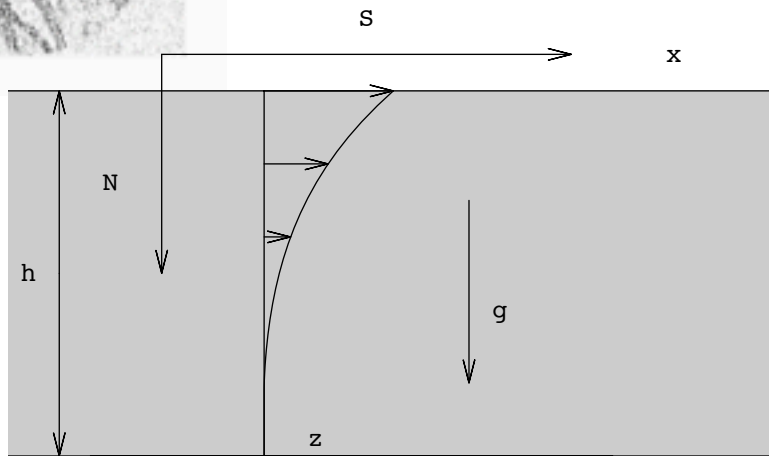


relation entre ϕ , I et σ_{zz}/P_0

$$\sigma_{xz}/\sigma_{zz} = \mu(\sigma_{zz}/P_0, I)$$

variable d'inertie sans dimension

$$I = \frac{(D \frac{\partial V}{\partial z})}{\sqrt{\sigma_{zz}/\rho}}$$



$$\sigma_{xz} / \sigma_{zz} = \mu(I)$$

avec

$$I = \frac{\left(D \frac{\partial V}{\partial z}\right)}{\sqrt{\sigma_{zz} / \rho}}$$

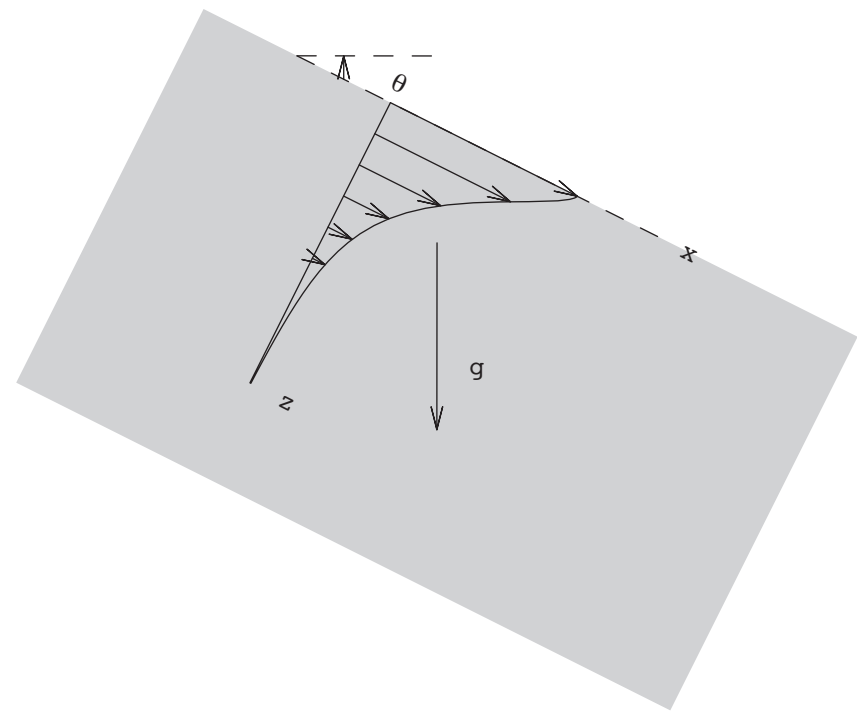
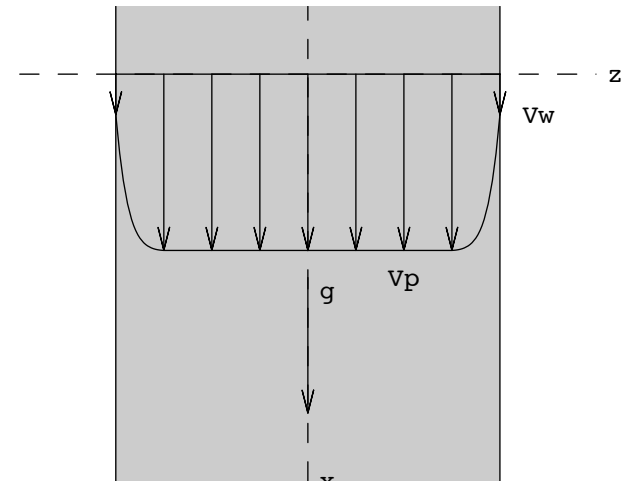
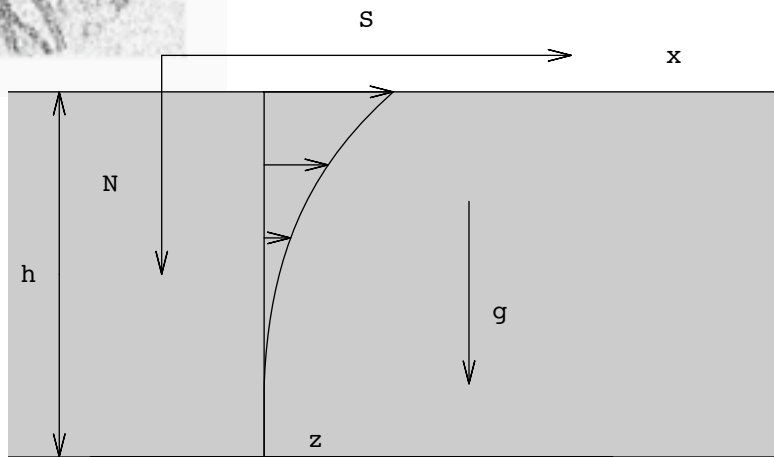
Relation "MIDI"

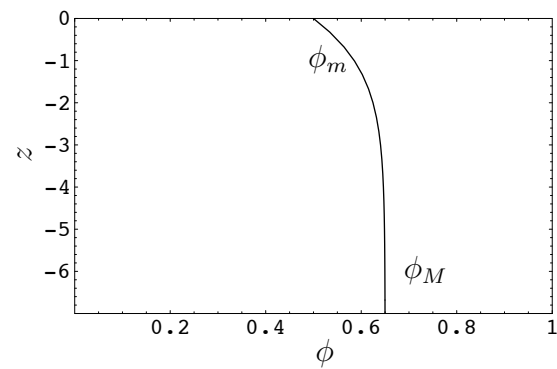
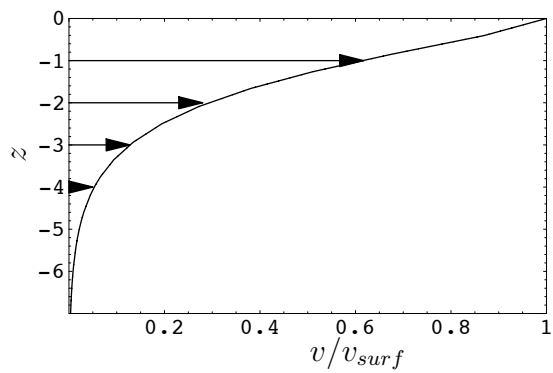
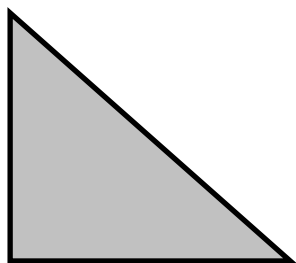
Pouliquen

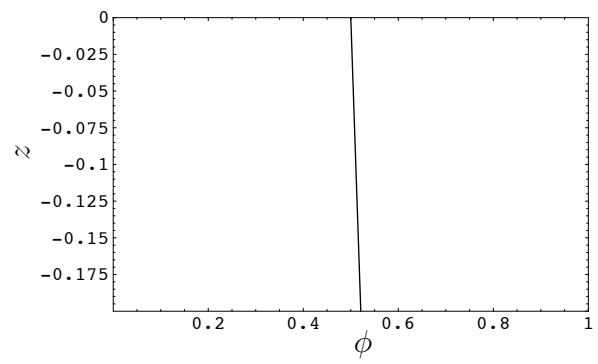
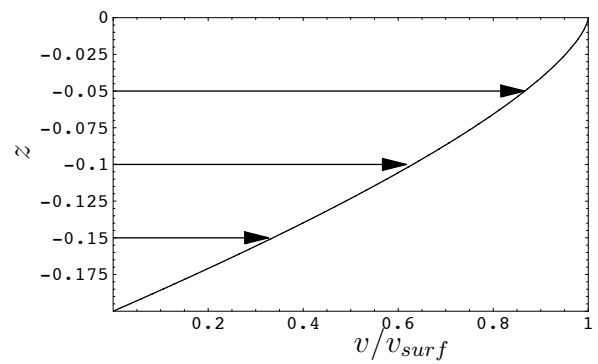
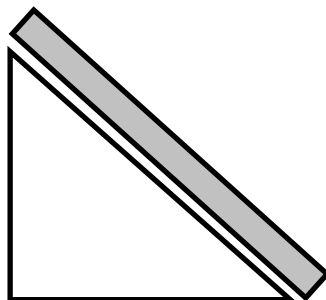
$$I = \frac{\left(D \frac{\partial V}{\partial z}\right)}{\sqrt{P / \rho}}$$

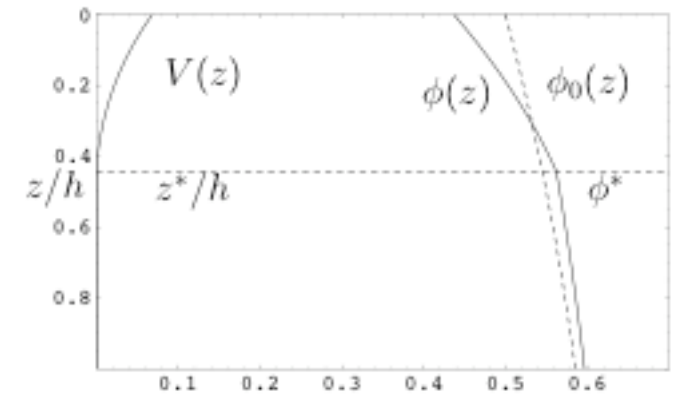
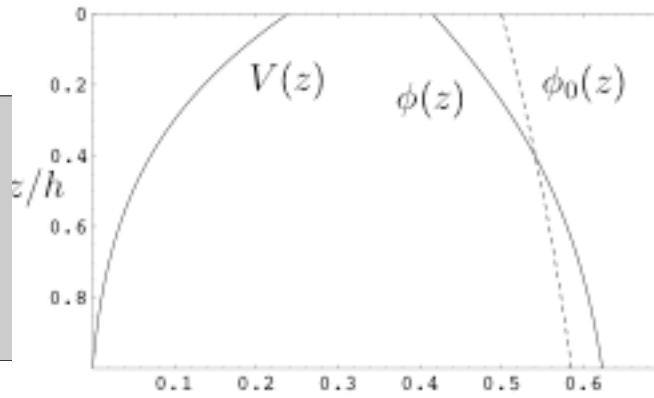
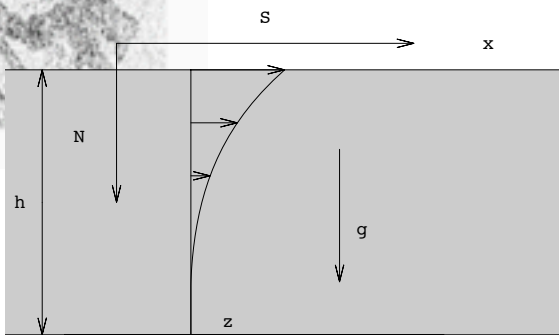


Écoulements cisailés

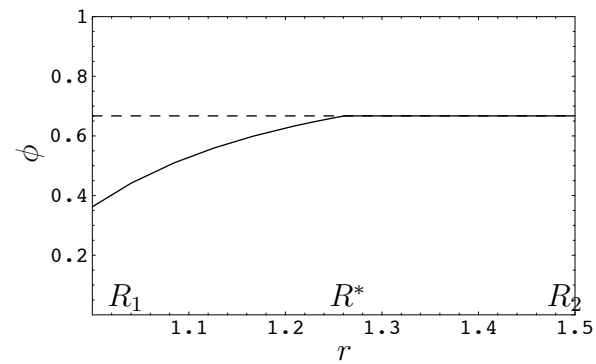
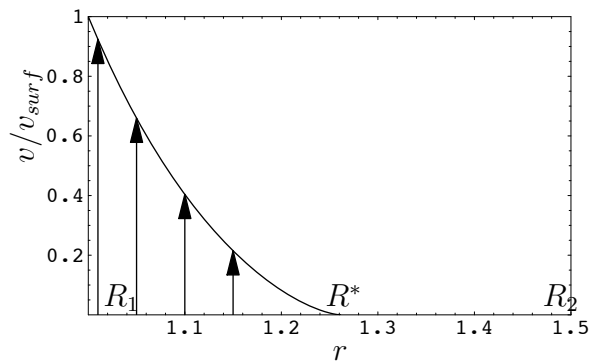
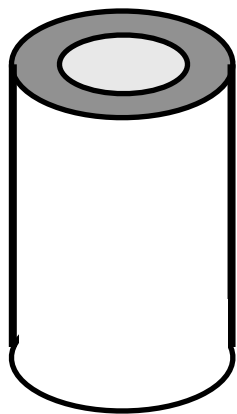




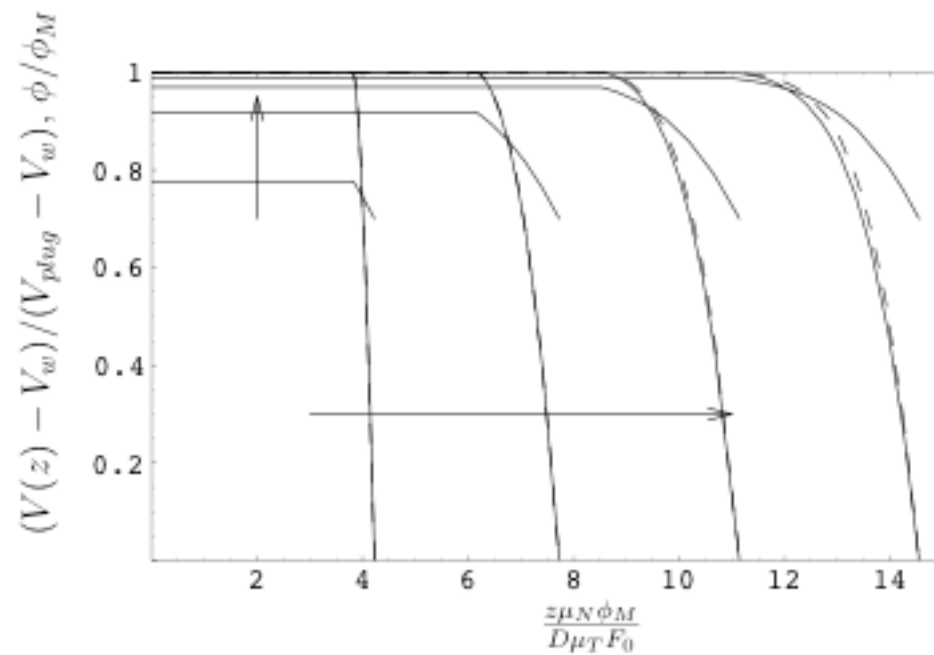
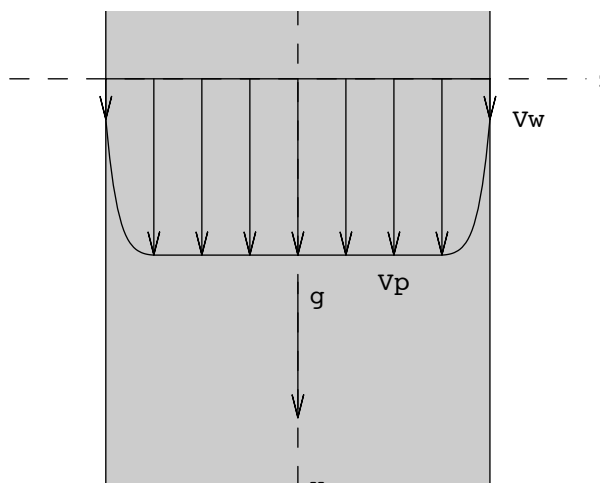




localisation



localisation



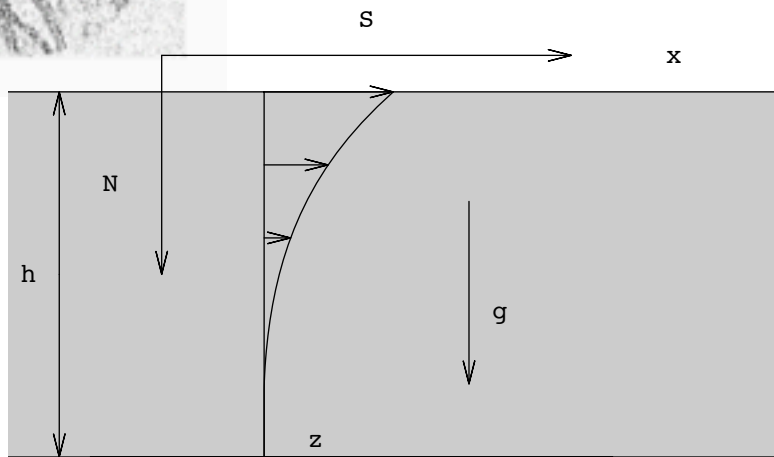
localisation



conclusion de la première partie

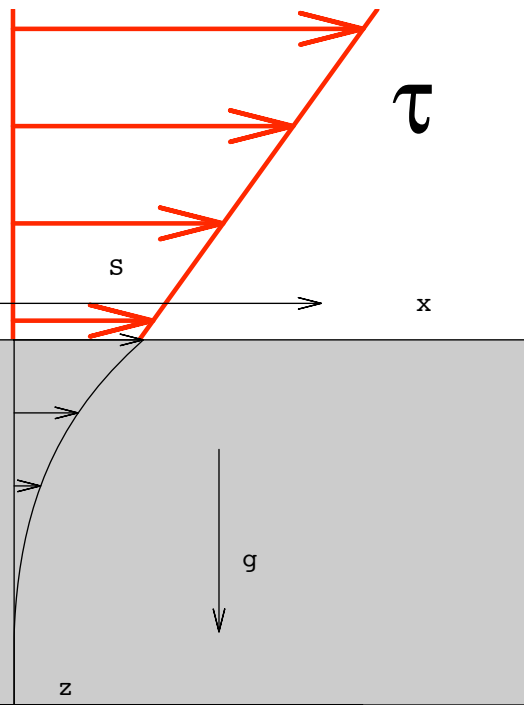
- description continue
- Mélange de “vieilles” idées:
Savage, Bagnold, Kanatani...
- Pressure Granulaire
- Quelques exemples simples
- perspectives?
 - changer les relations (F , μ ...)
 - introduire le temps?
 - ω près des parois?
- Fermetures pour Saint-Venant...





Résultat

$$V, \phi \rightarrow q$$



Résultat

$$V, \phi \rightarrow q$$

$$q(\tau)$$



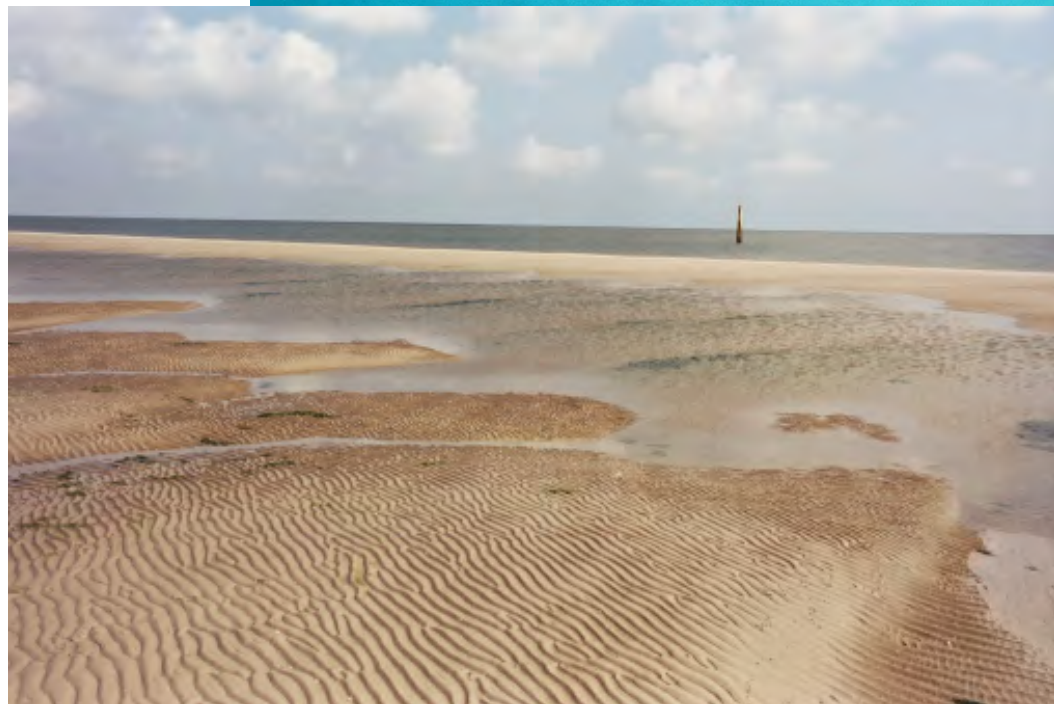


stabilité des fonds érodables



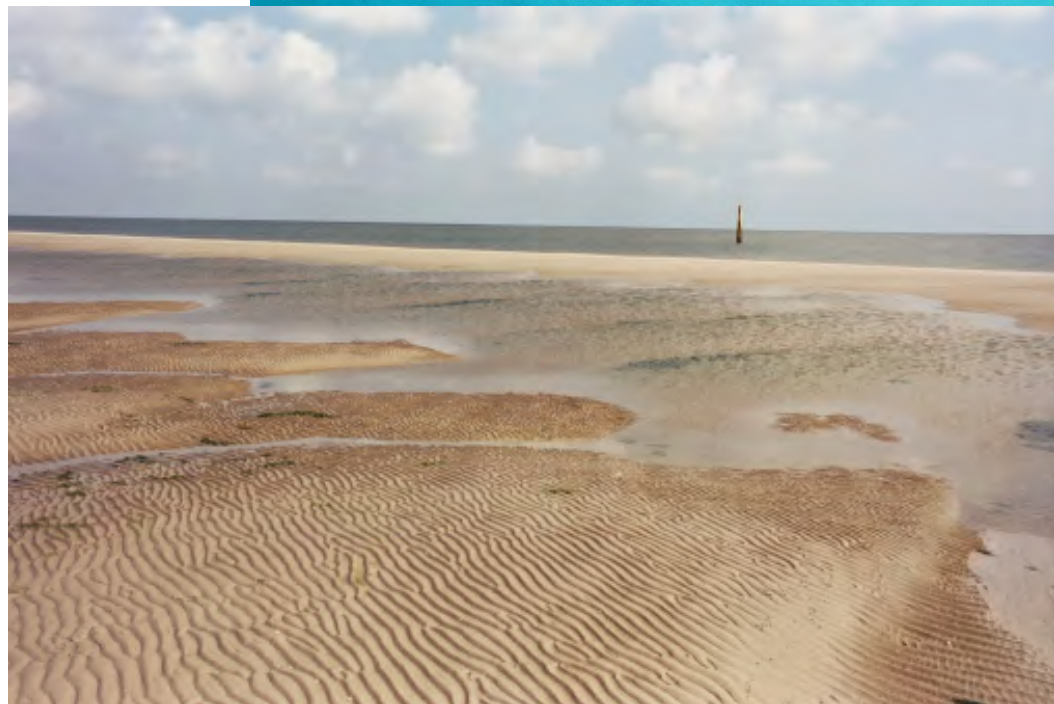


stabilité des fonds érodables





stabilité des fonds érosibles



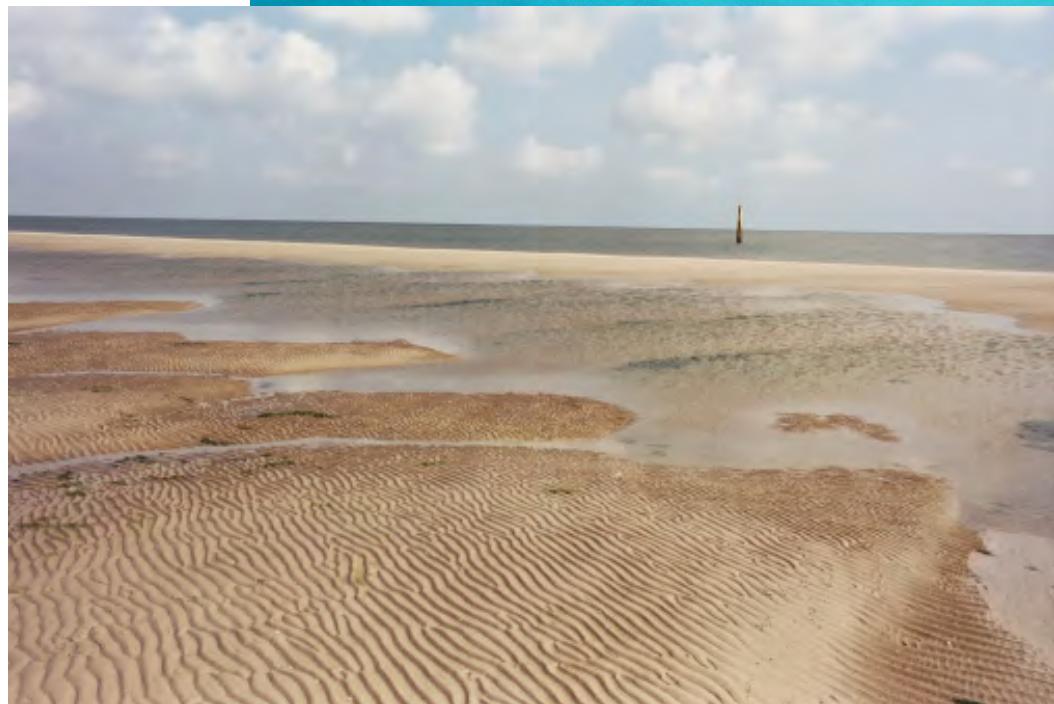


stabilité des fonds érosionnables



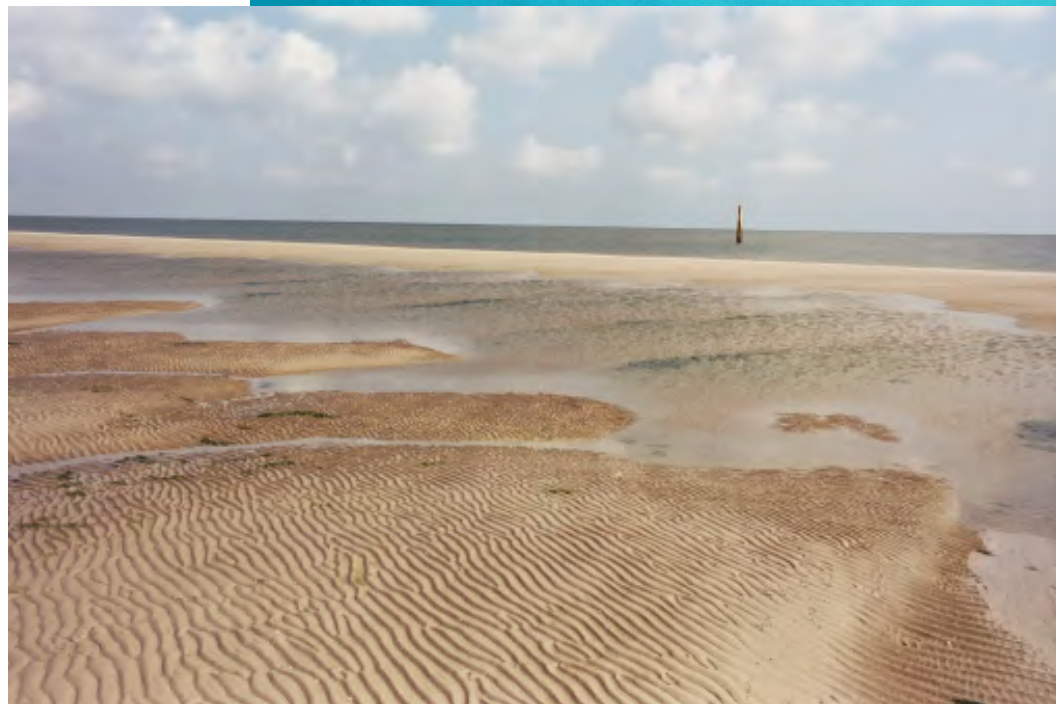


stabilité des fonds érosibles



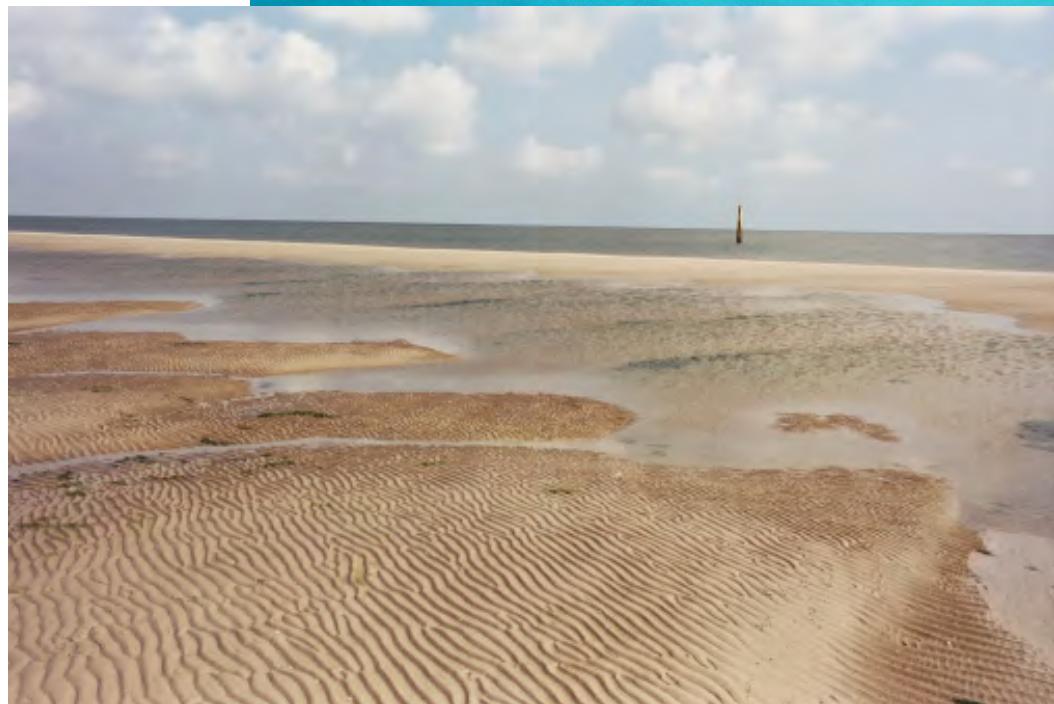


stabilité des fonds érodables





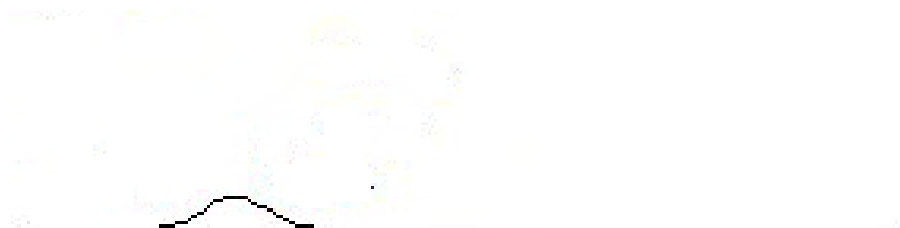
stabilité des fonds érosibles





- pour un sol donné $f(x, t)$

- ...





- pour un sol donné $f(x, t)$
- nous devons calculer l'écoulement ($u(x, y, t)$).





- pour un sol donné $f(x, t)$
- nous devons calculer l'écoulement $(u(x, y, t))$.



- l'écoulement érode le sol.



- pour un sol donné $f(x, t)$
- nous devons calculer l'écoulement ($u(x, y, t)$).



- l'écoulement érode le sol.
- ce qui change le sol.



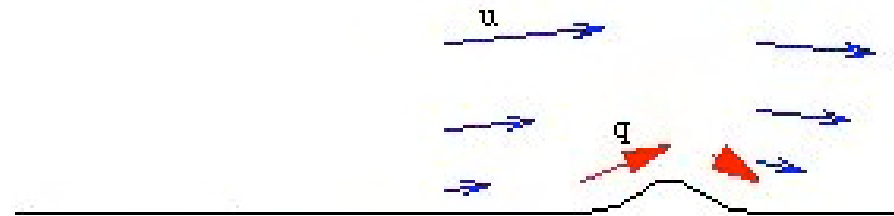
- pour un sol donné $f(x, t)$
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- l'écoulement érode le sol.
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- *etc*



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- pour un sol donné $f(x, t)$
- nous devons calculer l'écoulement ($u(x, y, t)$).



- l'écoulement érode le sol.
- ce qui change le sol.
- *etc*

nous aller présenter une description simplifiée pour l'écoulement et
et pour l'interaction avec le sol.



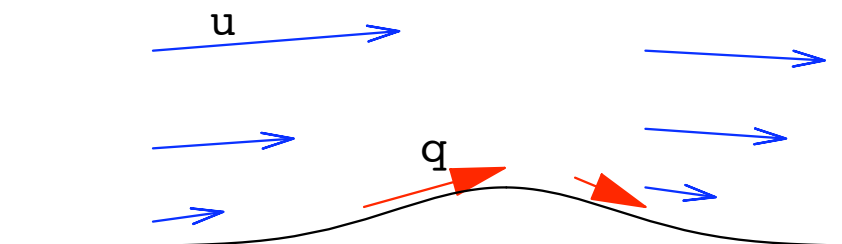
Conservation de la masse pour les sédiments :

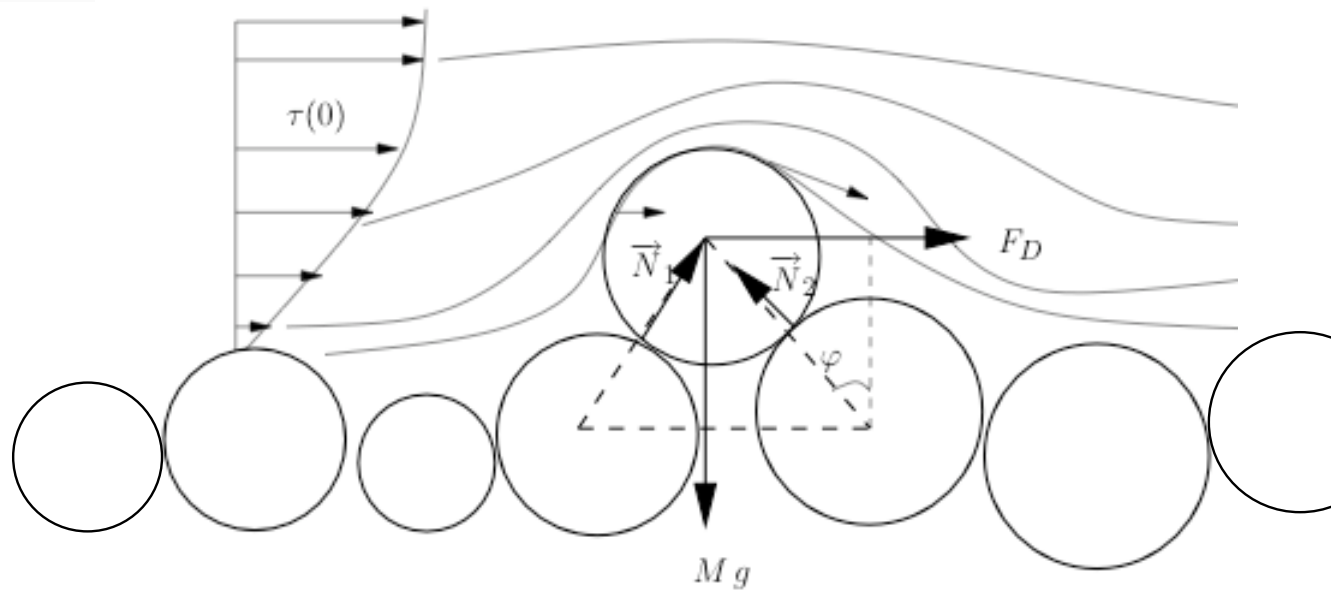
$$\frac{\partial f}{\partial t} = -\frac{\partial q}{\partial x}$$

Problème :

Quelle est la relation entre q et l'écoulement ?

indication : plus u est grand, plus l'érosion est importante et plus grand est q
 q semble être proportionnel au cisaillement pariétal

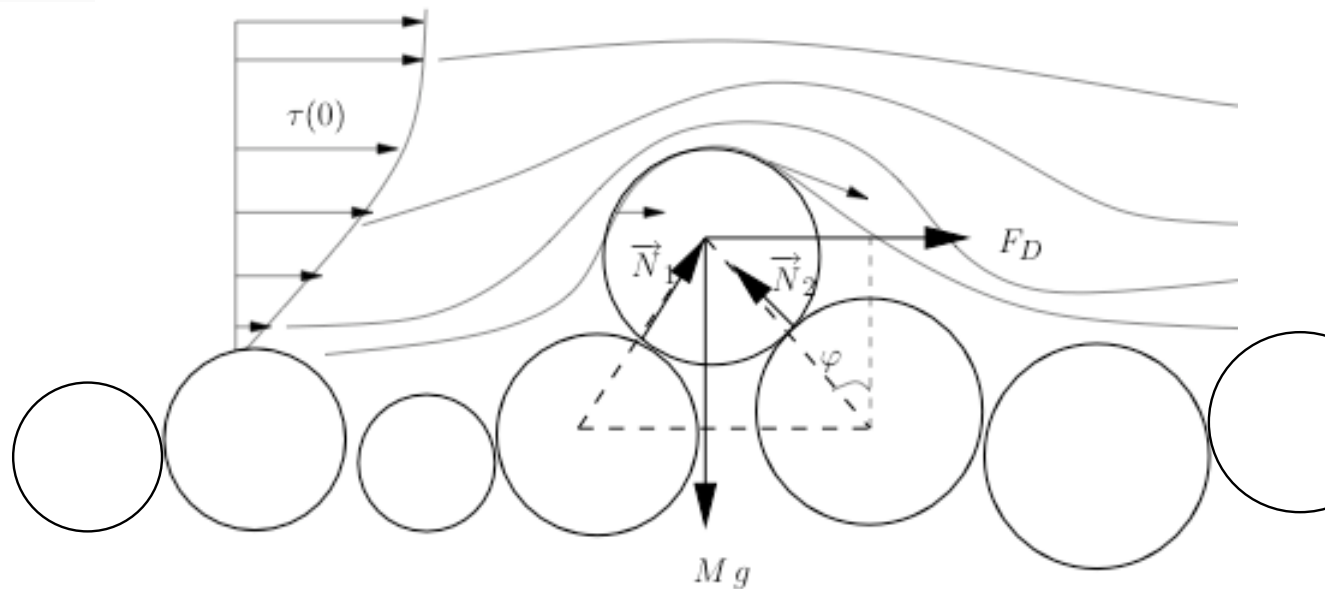




Contrainte supérieure à un seuil $\tau > \tau_s$

Nombre de Shield

$$\frac{\tau}{(\rho_p - \rho)gD}$$



Les lois d'entraînement de M. Scipion Gras
sur les torrents des Alpes (Annales des ponts et Chaussées, 1857, 2^e semestre) résumées par du Boys 1879 :

“un caillou posé au fond d'un courant liquide, peut être déplacé par l'impulsion des filets qui le rencontrent : le mouvement aura lieu si la vitesse est supérieure à une certaine limite qu'il (S. Gras) nomme vitesse d'entraînement. Cette vitesse limite dépend de la densité, du volume et de la forme du caillou ; elle dépend aussi de la densité du liquide et de la profondeur du courant.”



dans la littérature :

$$q_s = E\varpi(\tau^a(\tau - \tau_s)^b)$$

si $(\tau - \tau_s) > 0$ alors $\varpi(\tau - \tau_s) = (\tau - \tau_s)$ sinon $\varpi((\tau - \tau_s)) = 0$.

avec une correction de pente pour le seuil :

$$\tau_s + \Lambda \frac{\partial f}{\partial x},$$

a, E coefficients, $a = 0, b = 3$ ou $a = b = 1$ ou $a = 1/2, b = 1$ ou ...



écrire l'équation de conservation de la masse

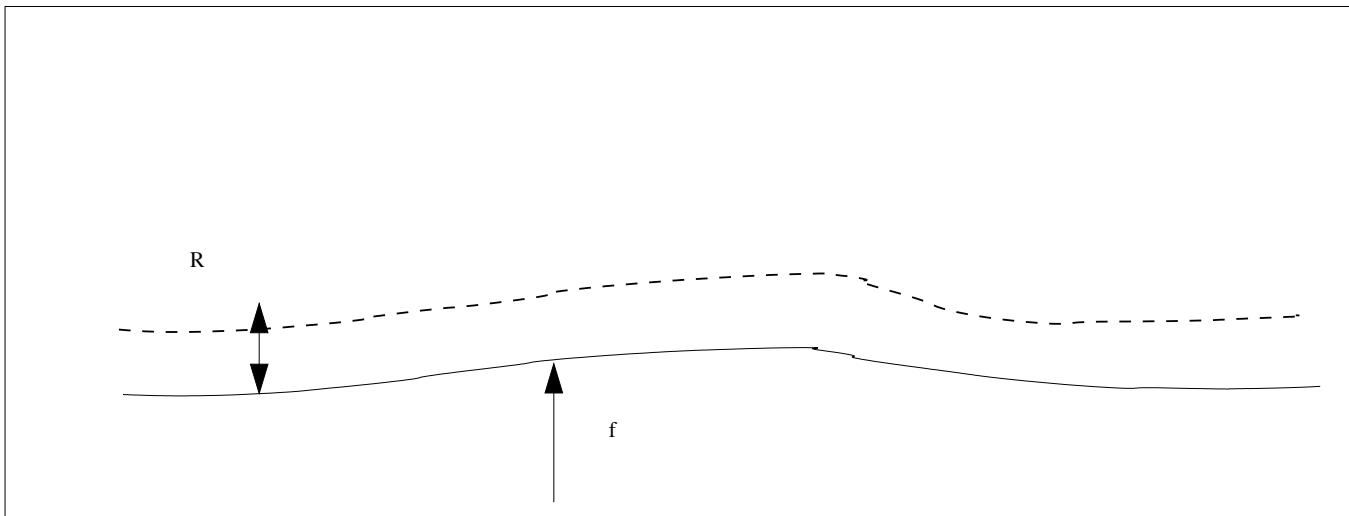
Kroy/ Hermann/ Sauermann 02, Lagrée 03, Valance Langlois 05, Charru Hinch 06



écrire l'équation de conservation de la masse

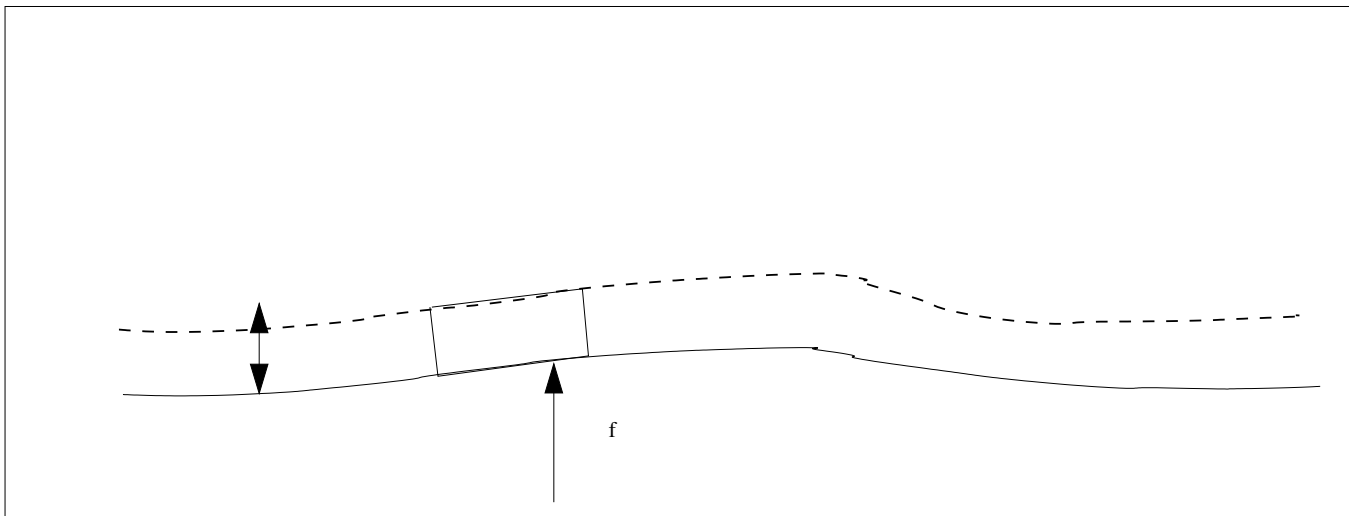
ce qui rentre - ce qui sort

Kroy/ Hermann/ Sauermann 02, Lagrée 03, Valance Langlois 05, Charru Hinch 06



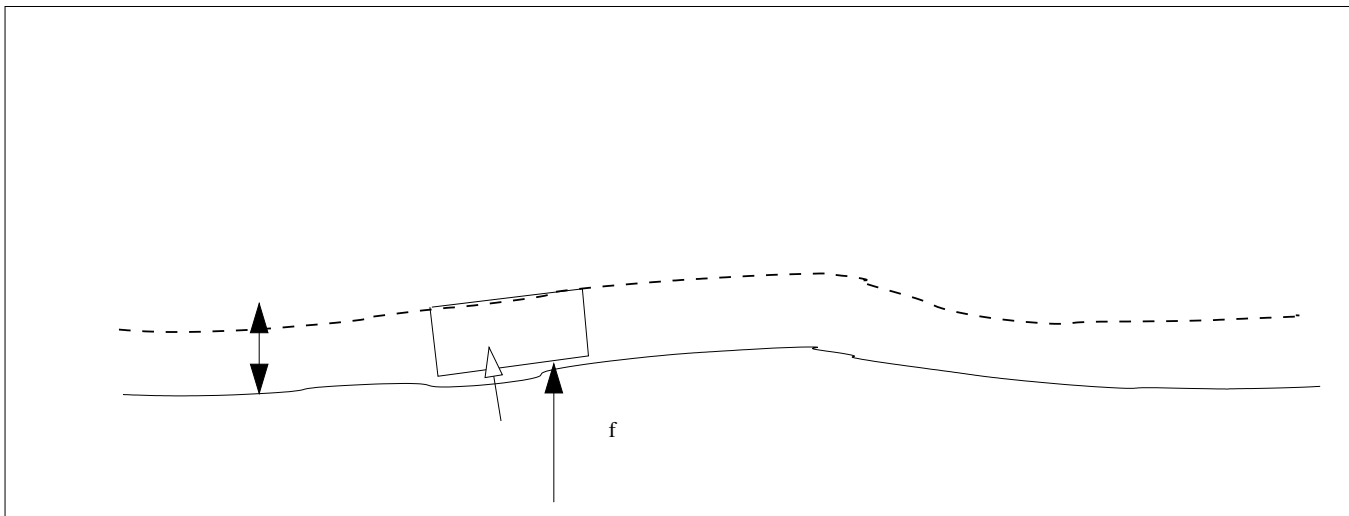
$$\frac{\partial R}{\partial t} = \dots$$

$$\frac{\partial f}{\partial t} = \dots$$



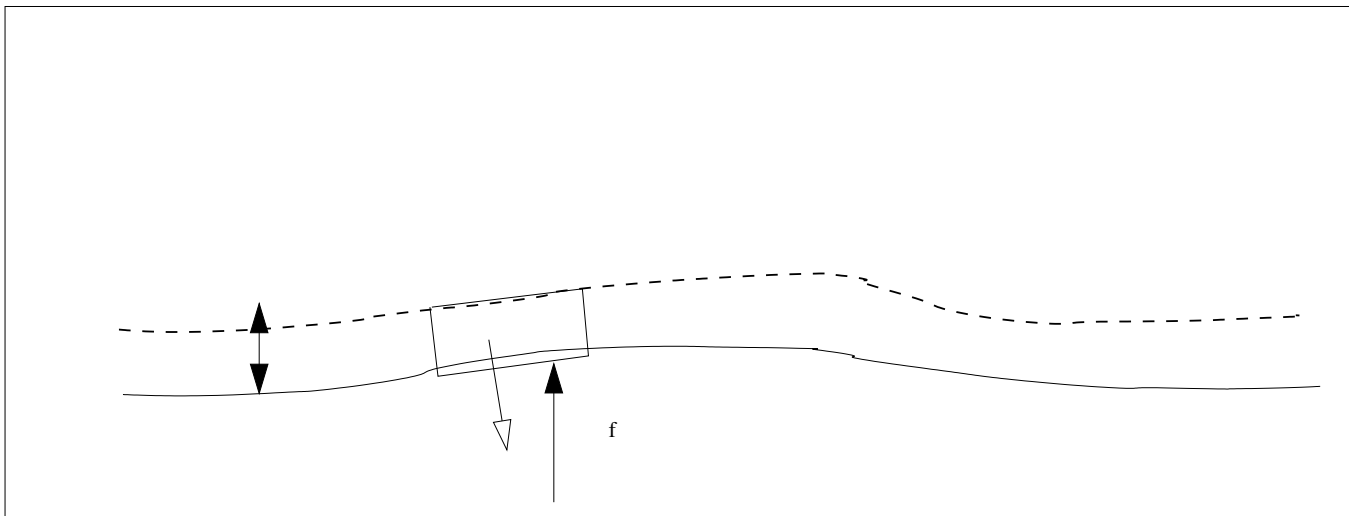
$$\frac{\partial R}{\partial t} = \dots$$

$$\frac{\partial f}{\partial t} = \dots$$



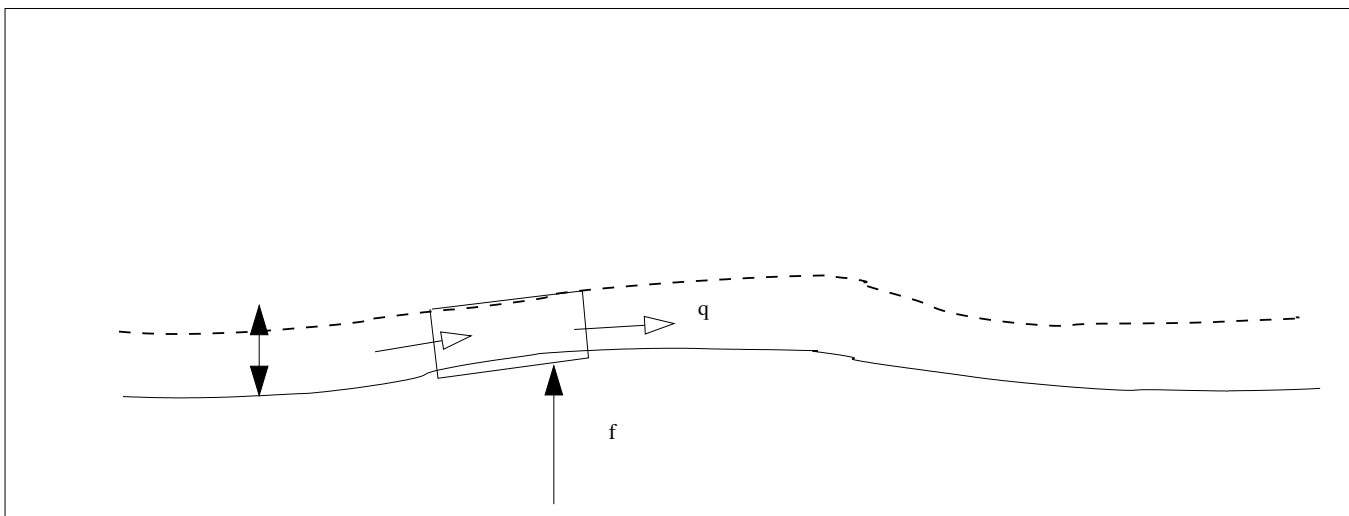
$$\frac{\partial R}{\partial t} = \dots + \Gamma$$

$$\frac{\partial f}{\partial t} = -\Gamma$$



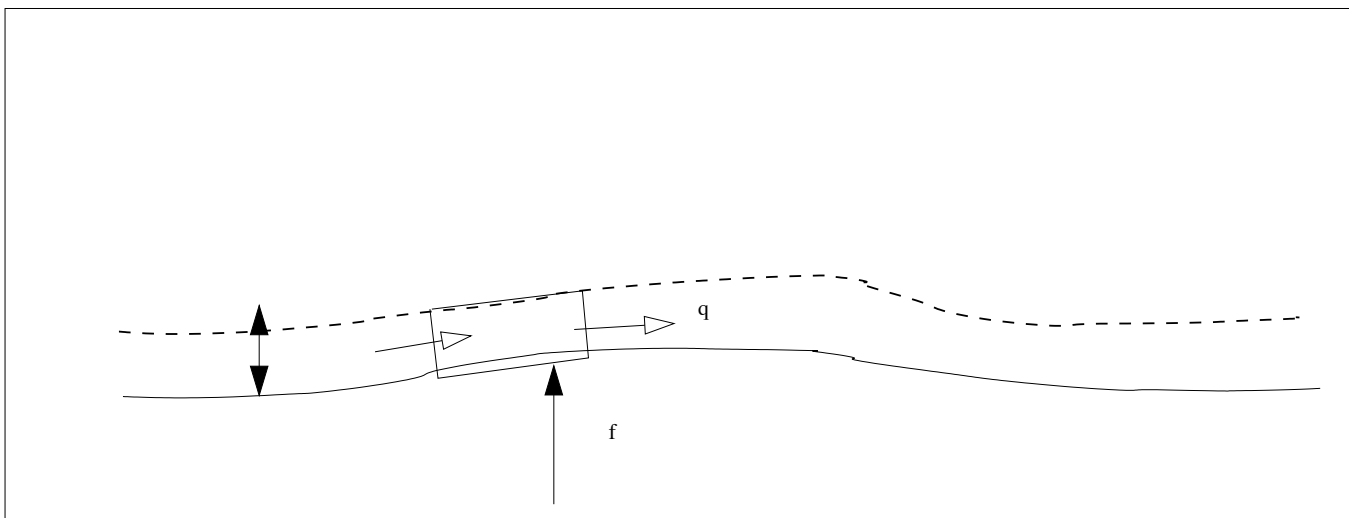
$$\frac{\partial R}{\partial t} = \dots + \Gamma$$

$$\frac{\partial f}{\partial t} = -\Gamma$$



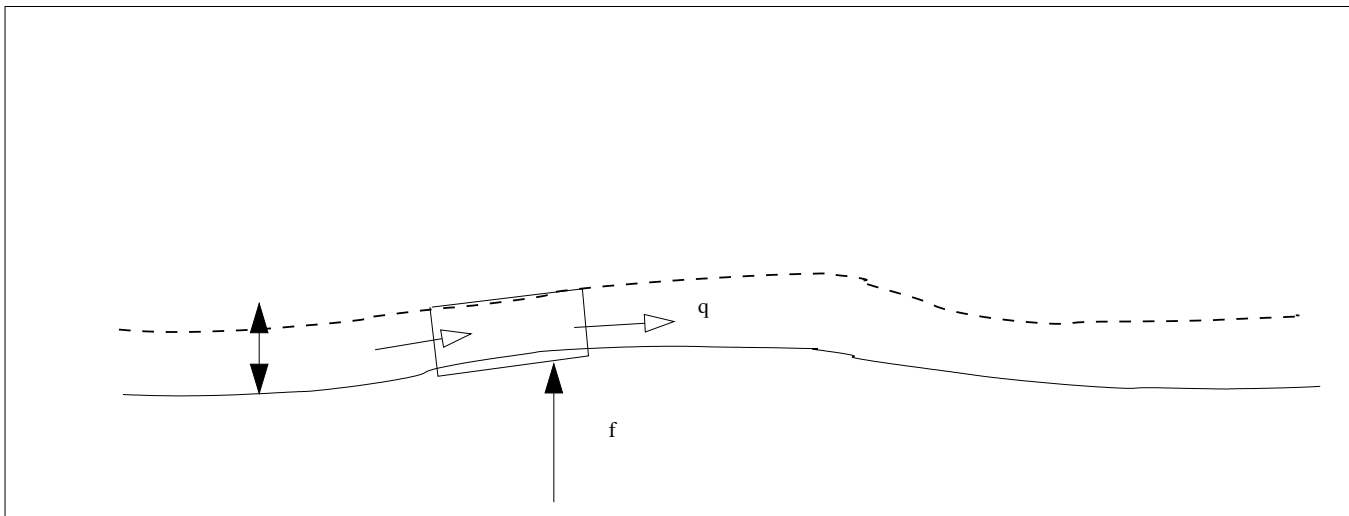
$$\frac{\partial R}{\partial t} = -\frac{\partial q}{\partial x} + \Gamma$$

$$\frac{\partial f}{\partial t} = -\Gamma$$



$$\frac{\partial R}{\partial t} = -\frac{\partial q}{\partial x} + \Gamma$$

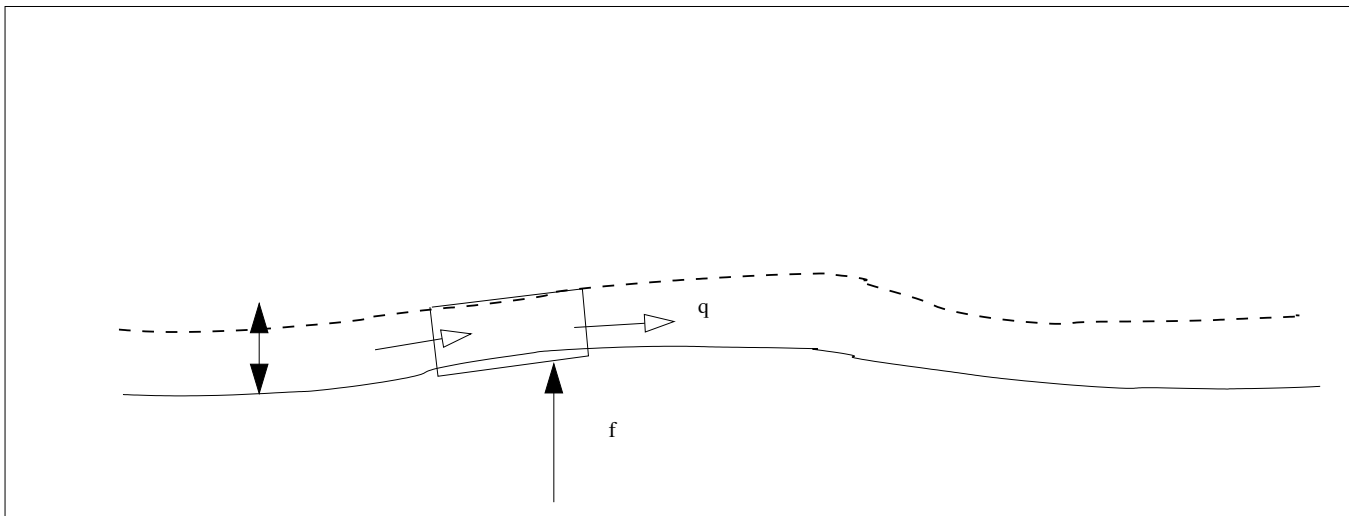
$$\frac{\partial f}{\partial t} = -\Gamma$$



$$\frac{\partial R}{\partial t} = -\frac{\partial q}{\partial x} + \Gamma$$

$$\frac{\partial f}{\partial t} = -\Gamma$$

$\Gamma = (\text{érosion}) - (\text{déposition})$



$$\frac{\partial R}{\partial t} = -\frac{\partial q}{\partial x} + \Gamma$$

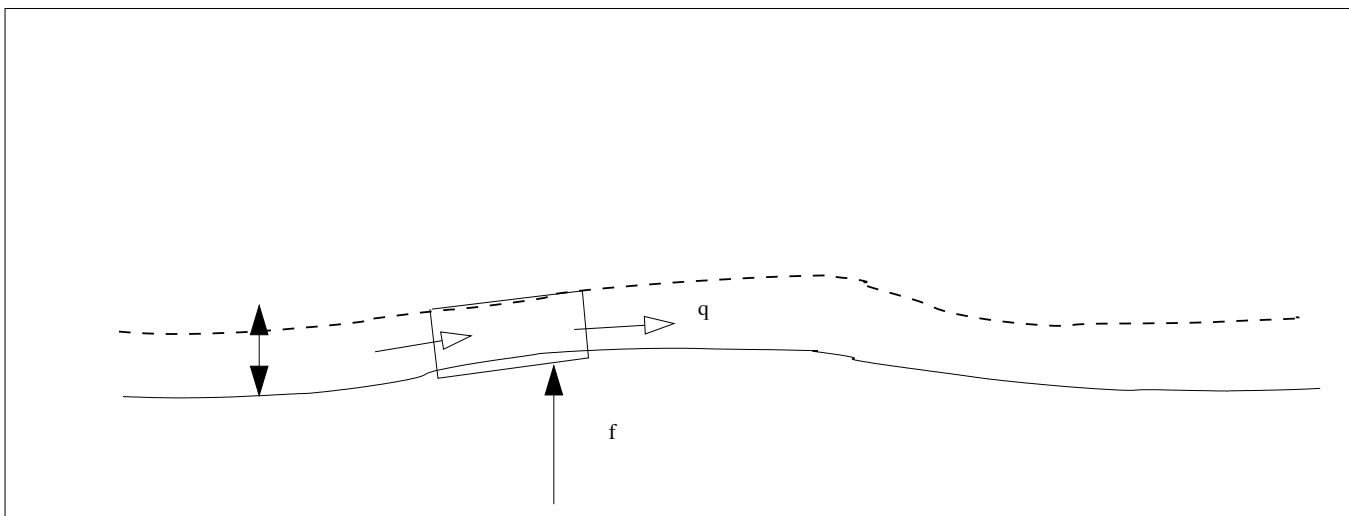
$$\frac{\partial f}{\partial t} = -\Gamma$$

$$\Gamma = (\text{érosion}) - (\text{déposition})$$

$$-(\text{déposition}) \propto -R$$

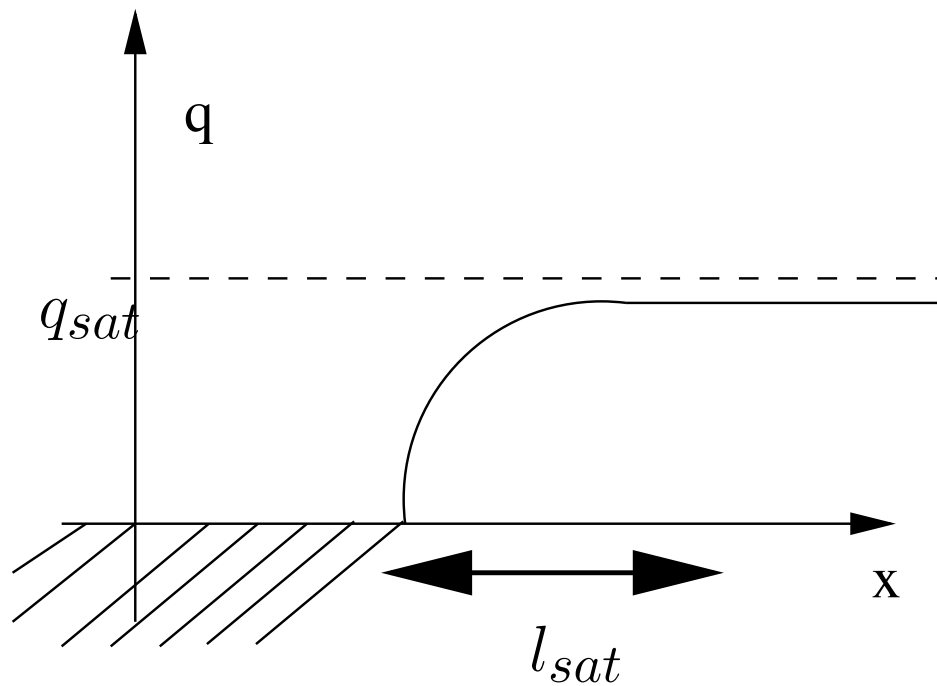
$$\text{érosion} \propto (\tau - \tau_s)$$

$$\text{et } q \propto R$$



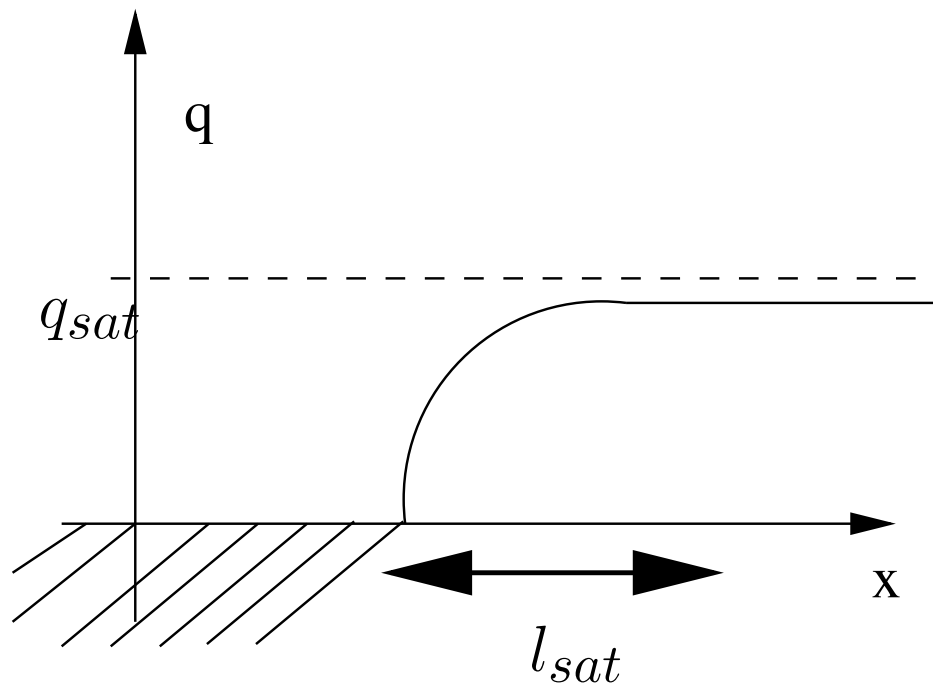
$$l_s \frac{\partial q}{\partial x} + q = q_s \qquad \frac{\partial f}{\partial t} = - \frac{\partial q}{\partial x}$$

$$q_s = E \mathfrak{W} (\tau - \tau_s)$$



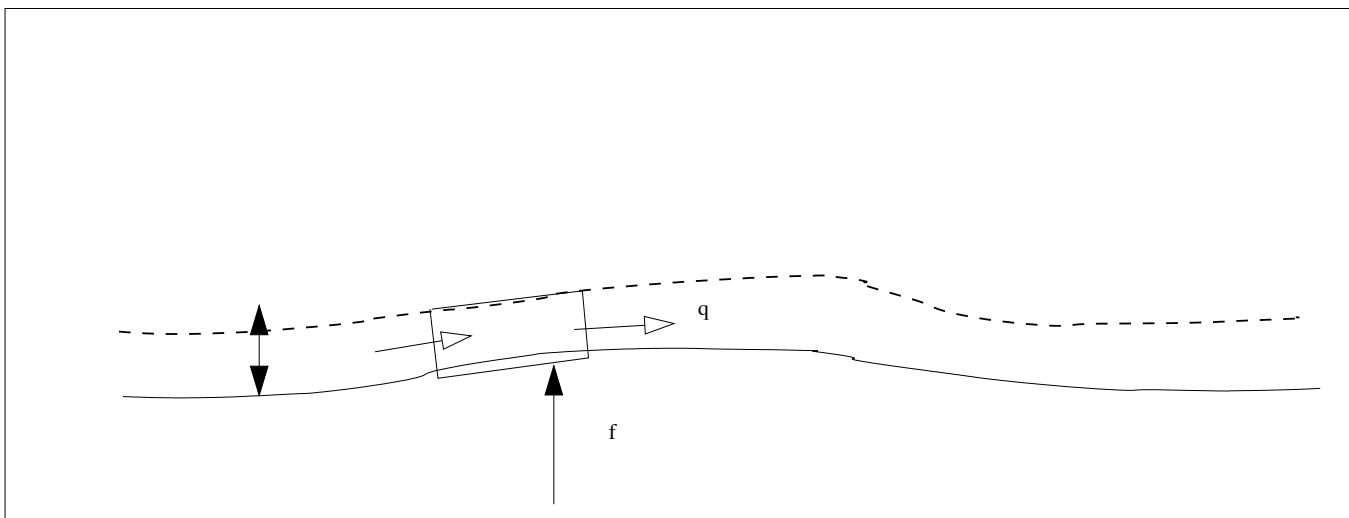
Sauerman, Kroy, Hermann 01, Andreotti Claudin Douady 02,

$$l_{sat} \frac{\partial q}{\partial x} + q = q_{sat}$$



Du Boy (1879) :

“une fois une certaine quantité de matières en mouvement sur le fond du lit, la vitesse des filets liquides devient trop faible pour entraîner davantage : le cours d'eau est alors saturé. Un cours d'eau non saturé tend à le devenir en entraînant une partie des matériaux qui composent son lit, et en choisissant de préférence les plus petits.”



$$l_s \frac{\partial q}{\partial x} + q = q_s \qquad \frac{\partial f}{\partial t} = - \frac{\partial q}{\partial x}$$

$$q_s = E\omega \left(\tau - \tau_s - \Lambda \frac{\partial f}{\partial x} \right)$$



Le fluide

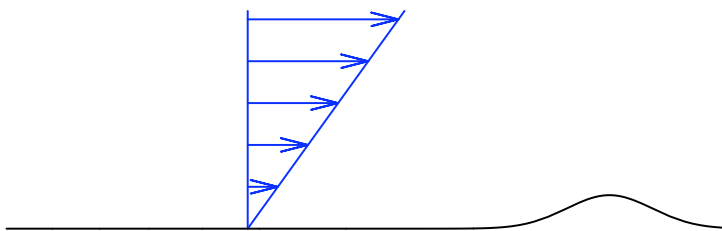
Résolution numérique des équations de Navier Stokes.
En "vrai" : la viscosité change... turbulence...

Ici on présente des simplifications sévères :

- Ecoulement quasistationnaire
- Solution Asymptotique de N.S. : théorie laminaire visqueuse à $Re = 0$
- Solution Asymptotique de N.S. : théorie laminaire visqueuse à $Re = \infty$
Triple Deck Stewartson 69/ Neiland 69 (en fait *Double Deck* Smith 80)
En fait Fowler 01
- Solutions linéarisées



Cas très visqueux $Re = 0$

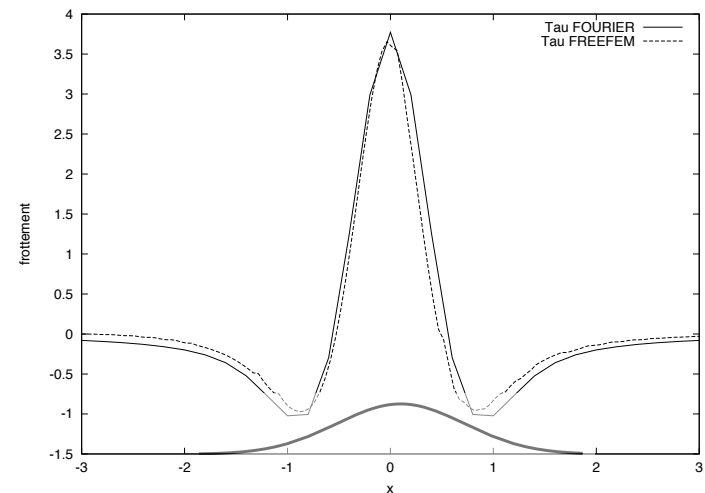
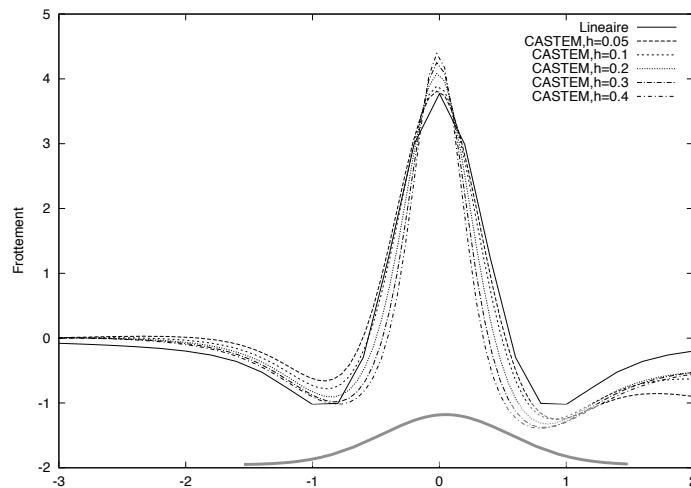


Écoulement cisailé sur la bosse $f(x, t)$ à faible nombre de Reynolds

$$f(x, t) \text{ donne } \tau = 1 + \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{f'}{x-\xi} d\xi$$



perturbation d'un écoulement cisailé $Re = 0$



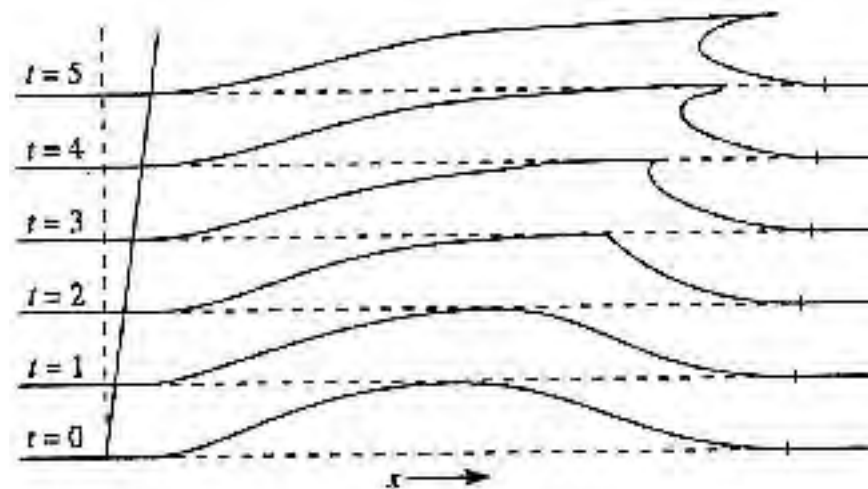
perturbation du frottement pariétal calculé par CASTEM $\frac{1}{h_0} \frac{\partial \bar{u}}{\partial y}$ pour $0.05 < h_0 < 0.4$
(hauteur) et $Re = 1$
perturbation calculée avec FreeFem.



si $q = \tau - \tau_s$

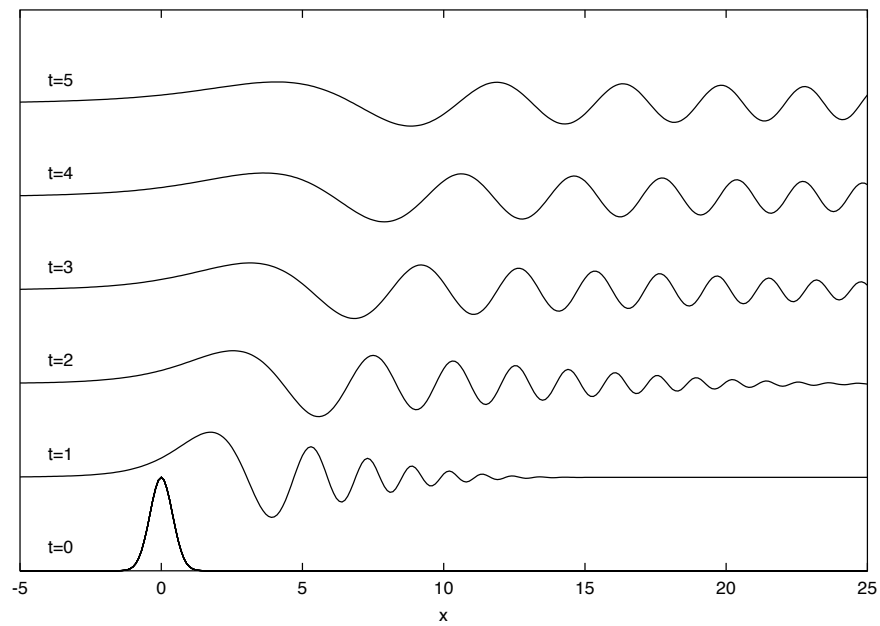
$$\frac{\partial f}{\partial t} = -\frac{1}{\pi} \frac{\partial}{\partial x} \int \frac{f'}{x - \xi} d\xi.$$

on obtient l'équation de Benjamin -Ono linéaire.



dessin Original d'Exner 1925

même équation, mais cas $Re = \infty$



stable dispersif

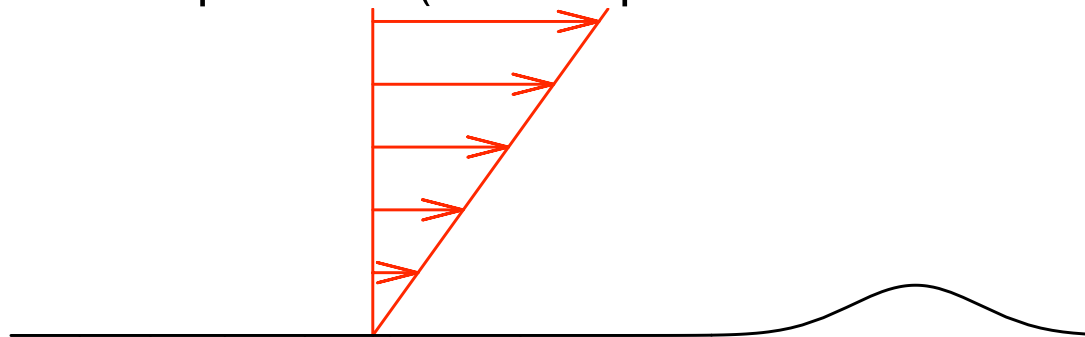


Solution Asymptotique "double deck theory"

Effets visqueux toujours prédominants au fond

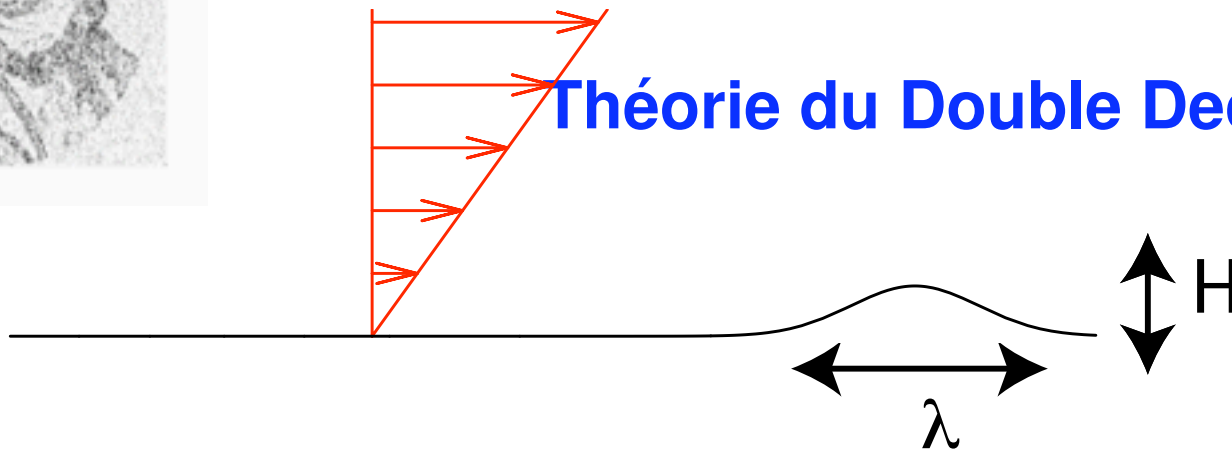
Perturbation d'un écoulement cisailé

Résolution Non Linéaire possible (avec séparation : courant de retour)





Théorie du Double Deck



Pour une bosse de longueur d'ordre λ et de hauteur d'ordre $H \ll \delta$:

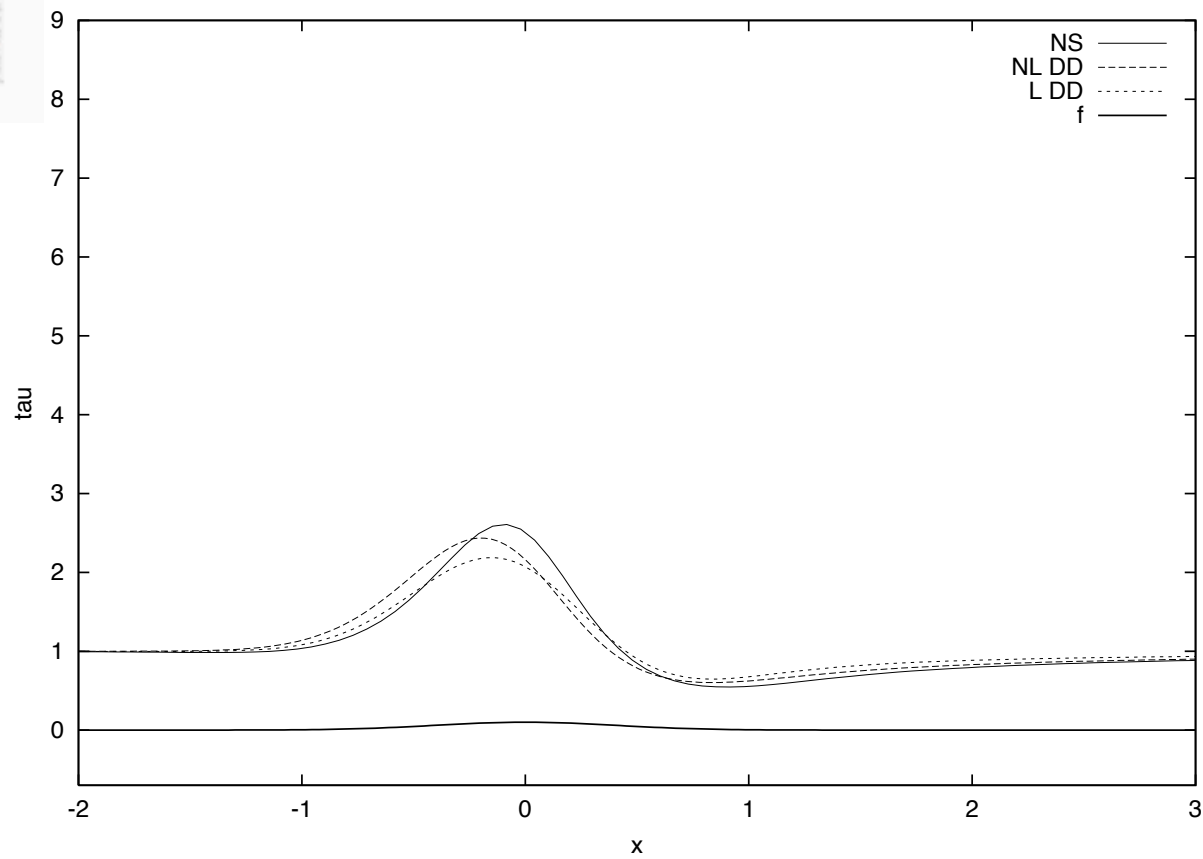
$$\tau = \mu U'_0 (\bar{U}'_S (1 + (\frac{U'_0}{\nu \lambda})^{1/3} H \tilde{c})), \text{ avec } \tilde{c} = FT^{-1} [FT[\tilde{f}] 3Ai(0) (-i2\pi \tilde{k}) \bar{U}'_S]^{1/3}$$

la fonction du temps \bar{U}'_S est un nombre d'ordre 1.

$$(\frac{U'_0}{\nu \lambda})^{1/3} H \leq 1$$



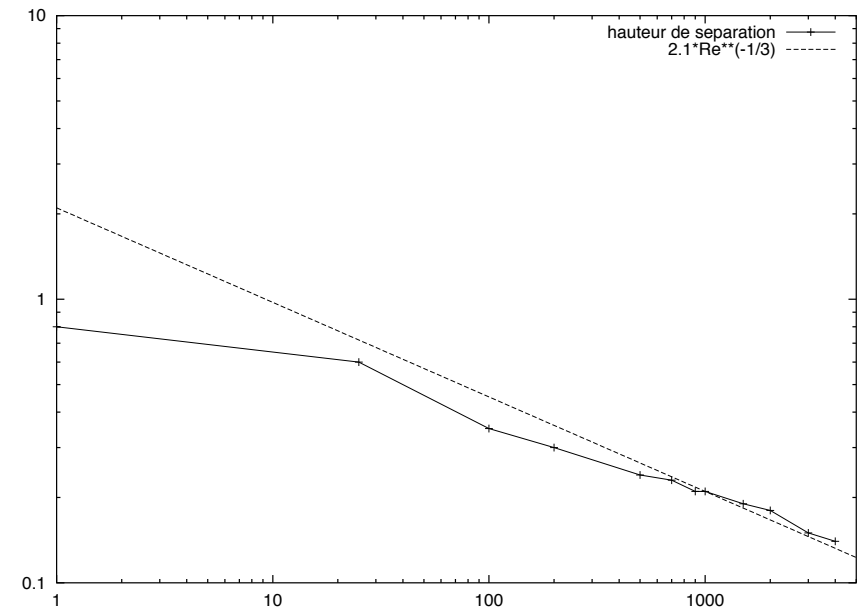
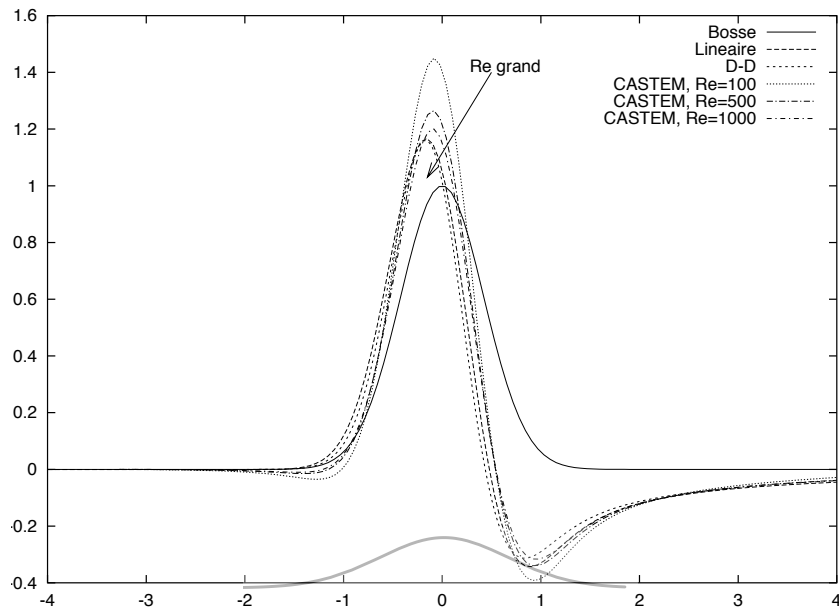
NS/ Double Deck Linéaire et Non-Linéaire



$$h = 0.1, Re = 1000$$



Comparaison avec Navier Stokes



pas mal !

conclusion : la perturbation du cisaillement est en avance par rapport au sommet de la bosse.

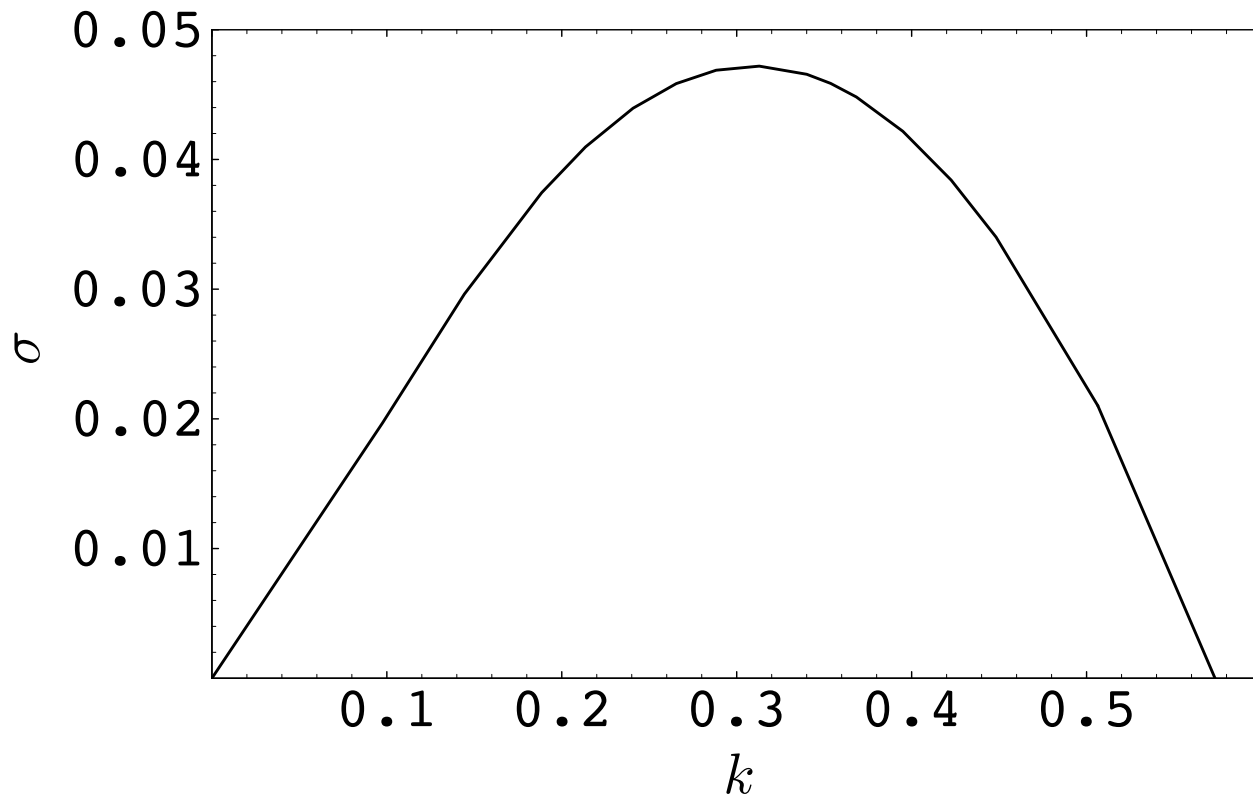
Sol complètement érodable



$$\tau = TF^{-1}[(3Ai(0))(-ik)^{1/3}TF[f]]$$

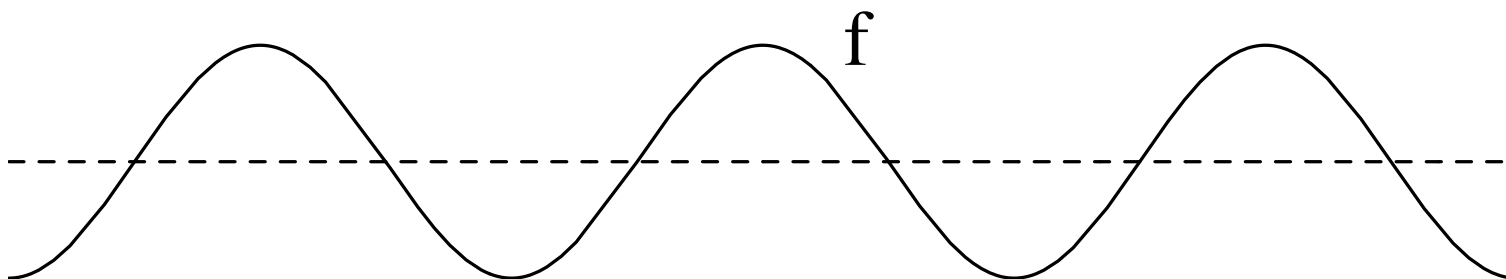
$$l_s \frac{\partial q}{\partial x} + q = \varpi(\tau - \tau_s)$$

$$\frac{\partial f}{\partial t} = -\frac{\partial q}{\partial x}$$





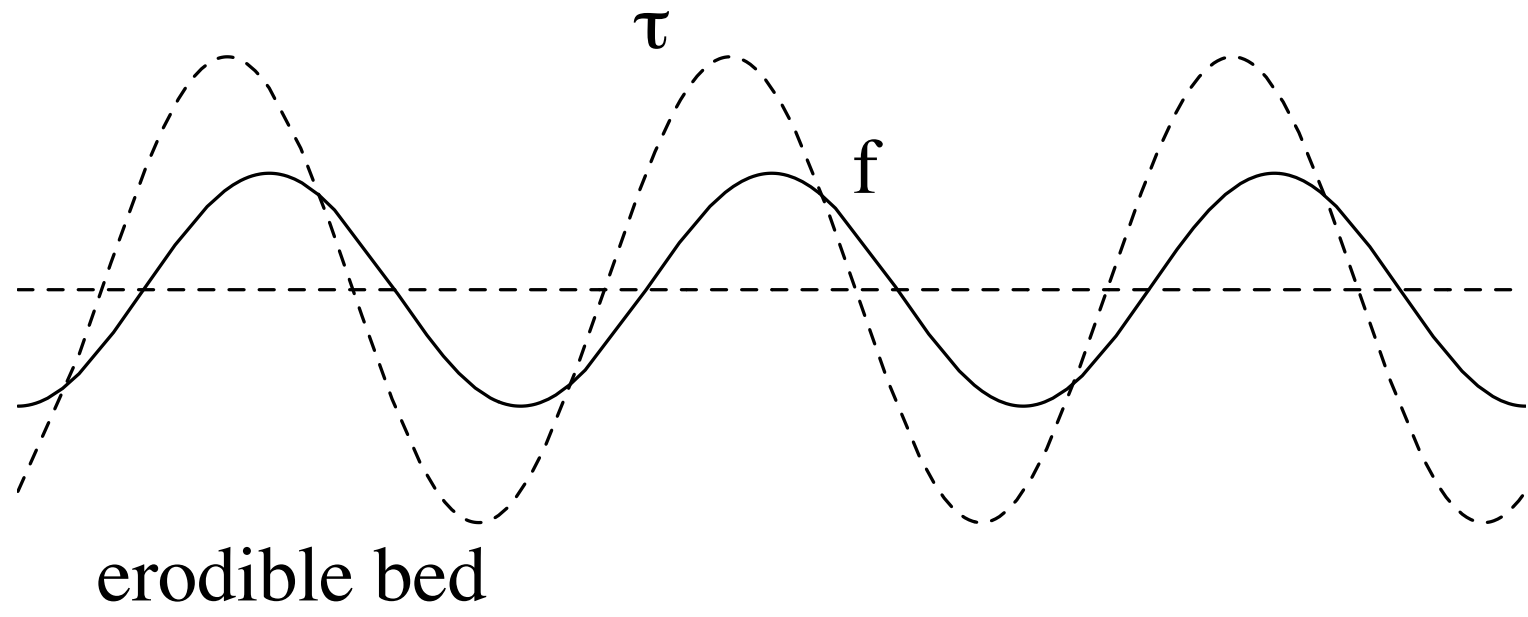
fluid



erodible bed

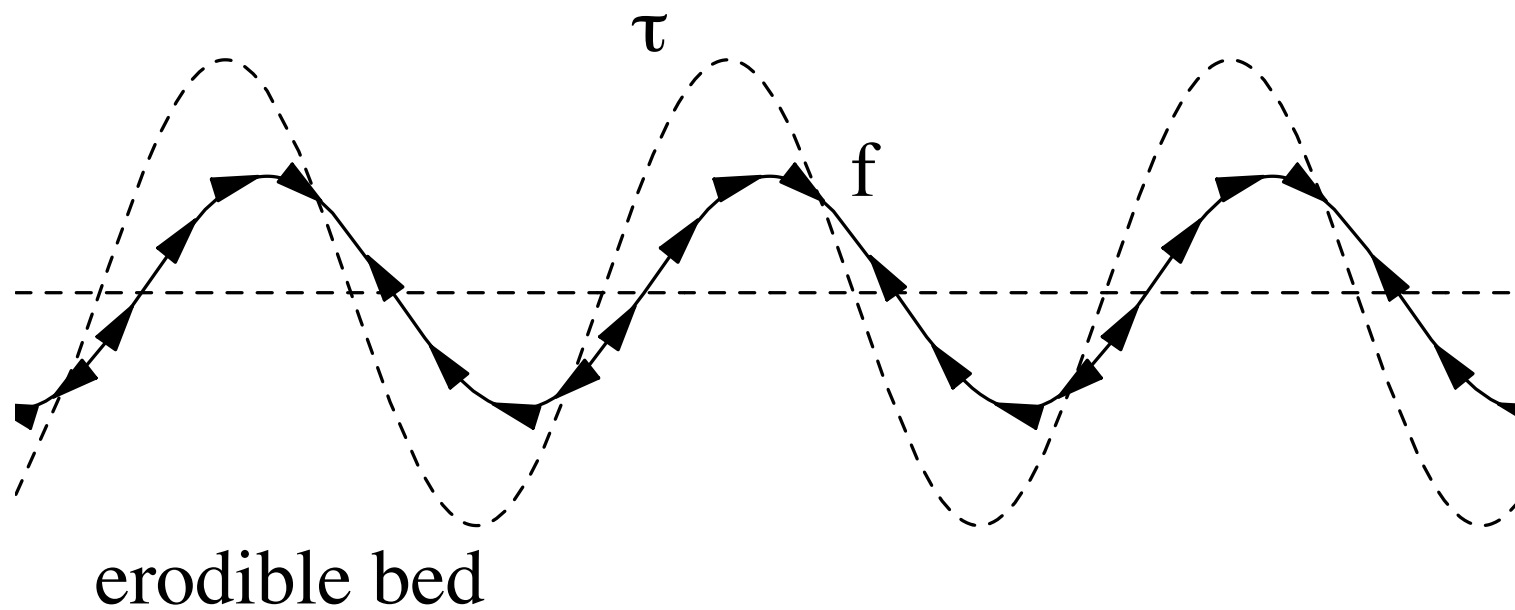


fluid \longrightarrow



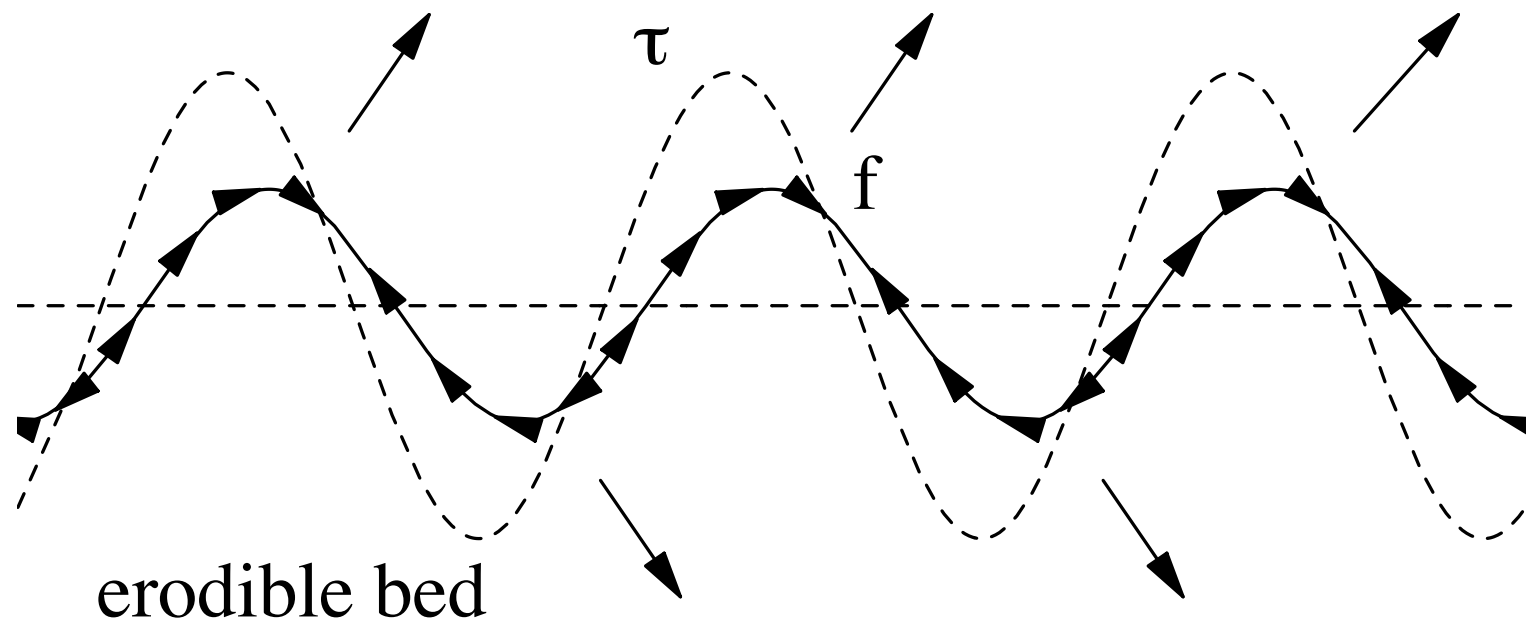


fluid \longrightarrow



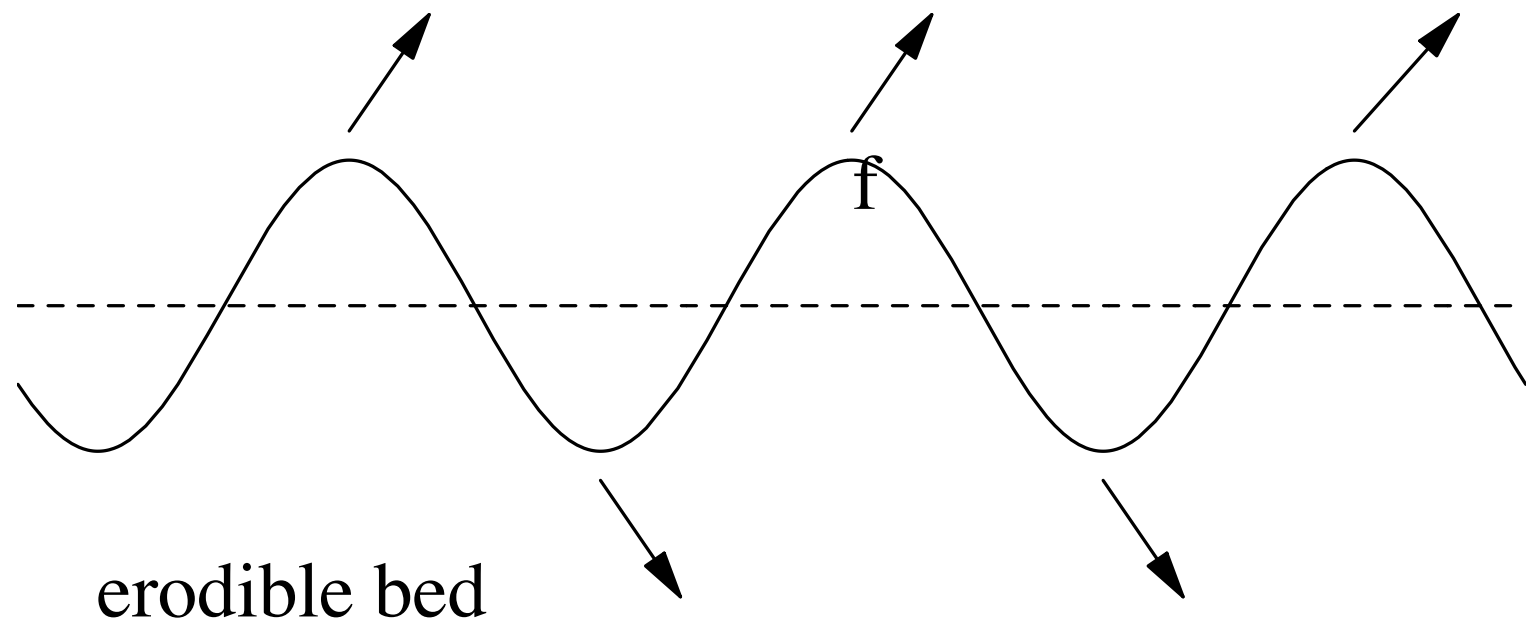


fluid \longrightarrow



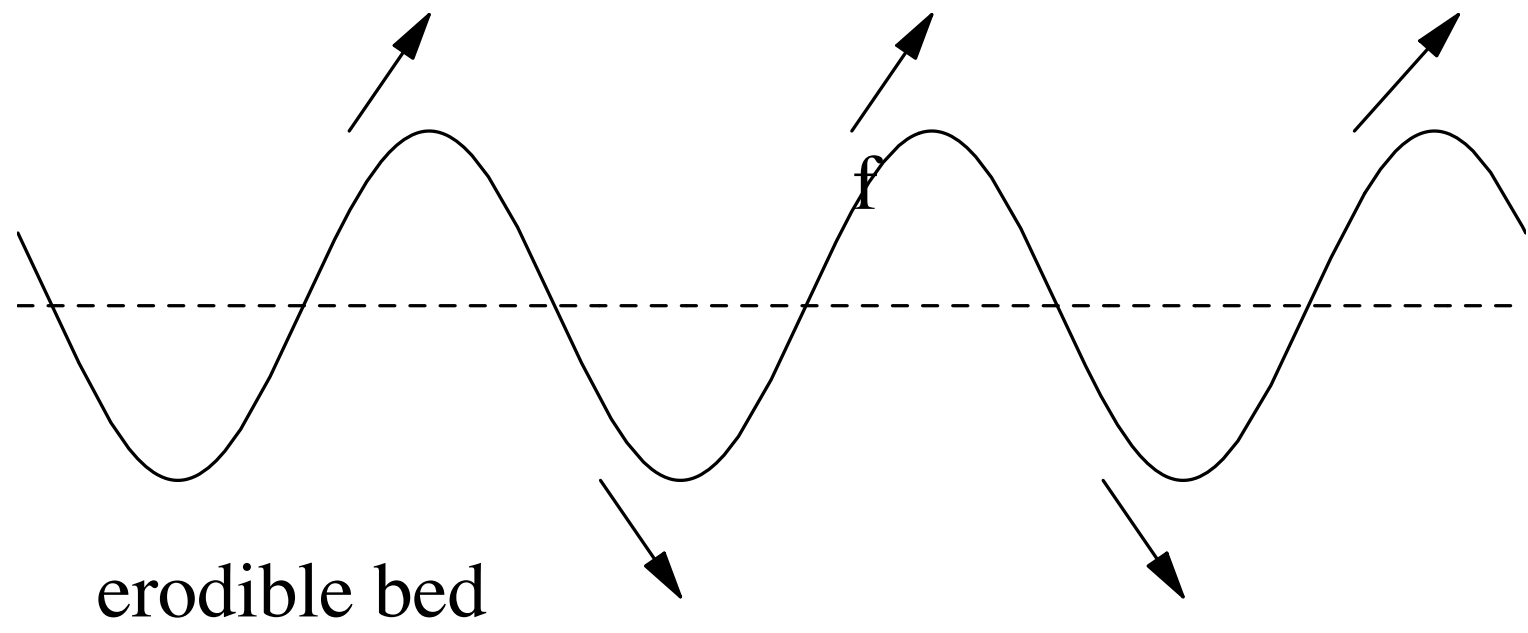


fluid 



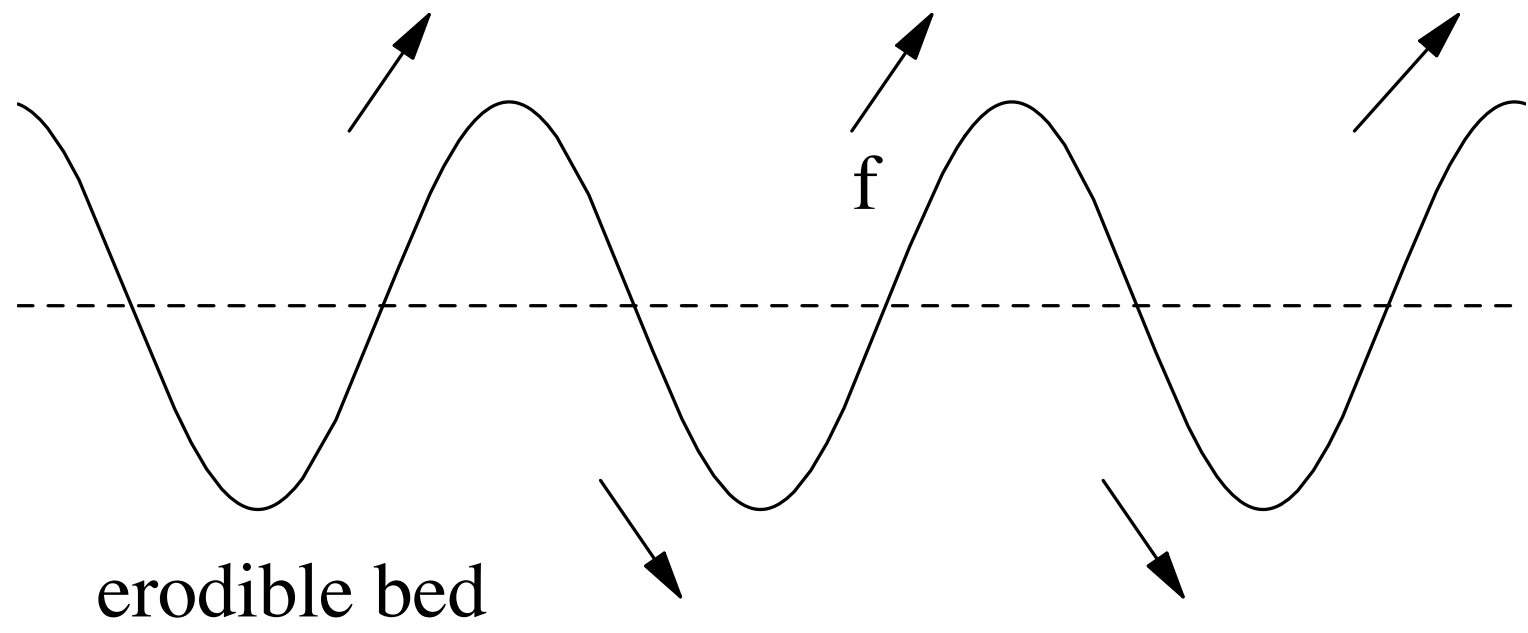


fluid \longrightarrow



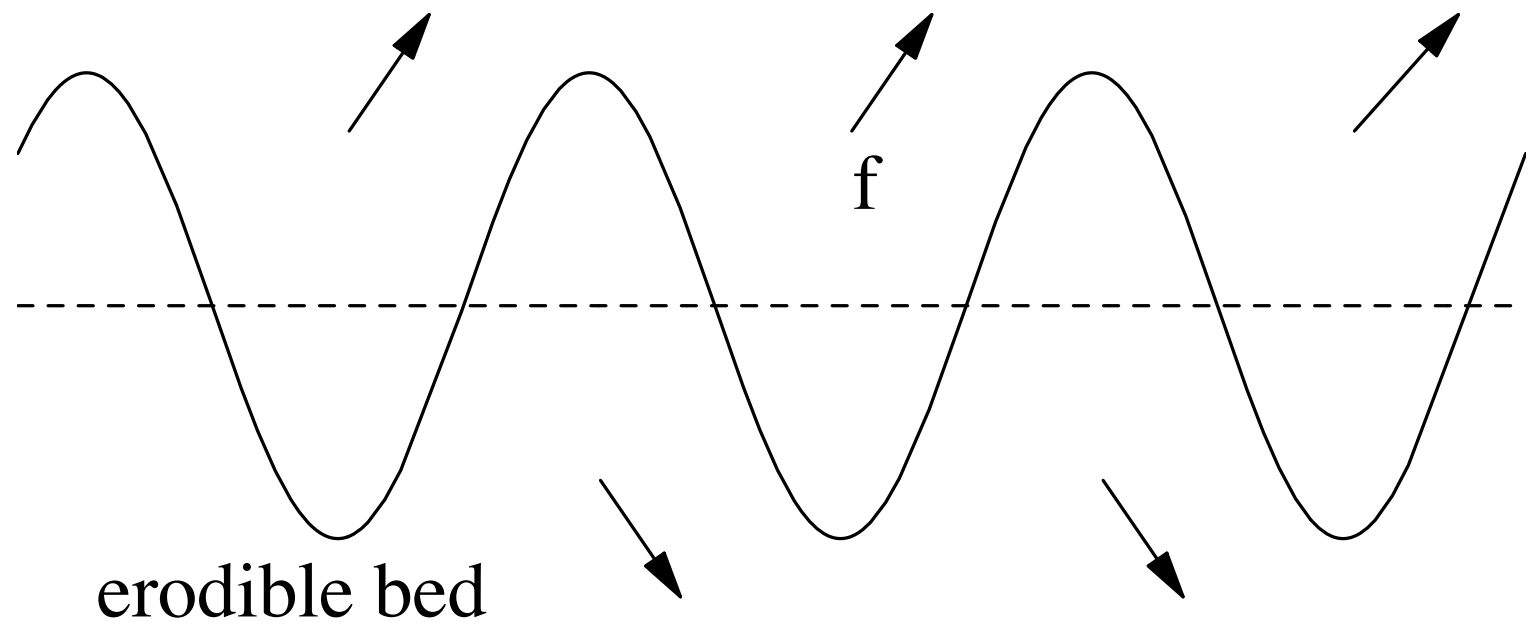


fluid 



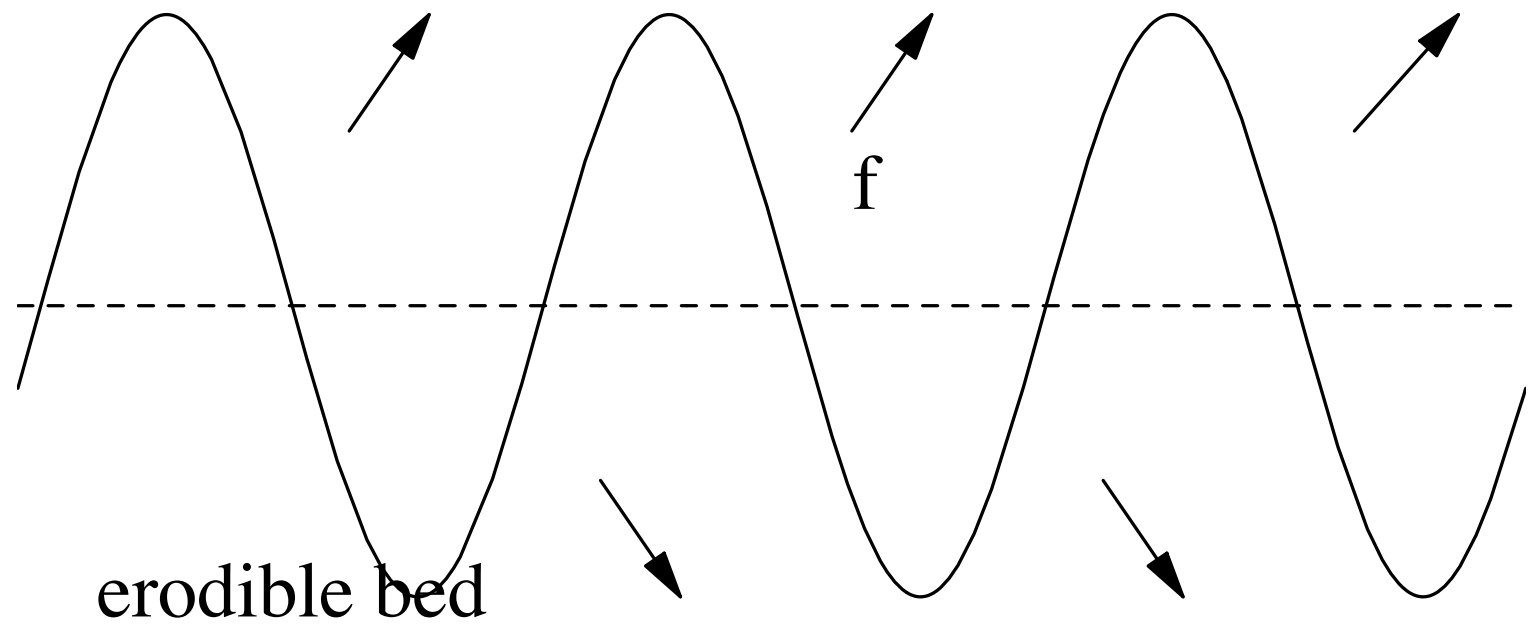


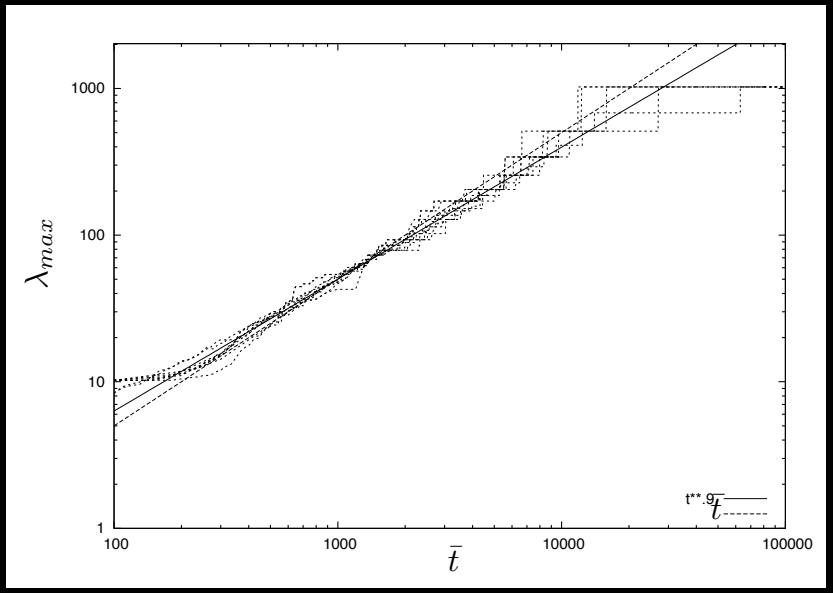
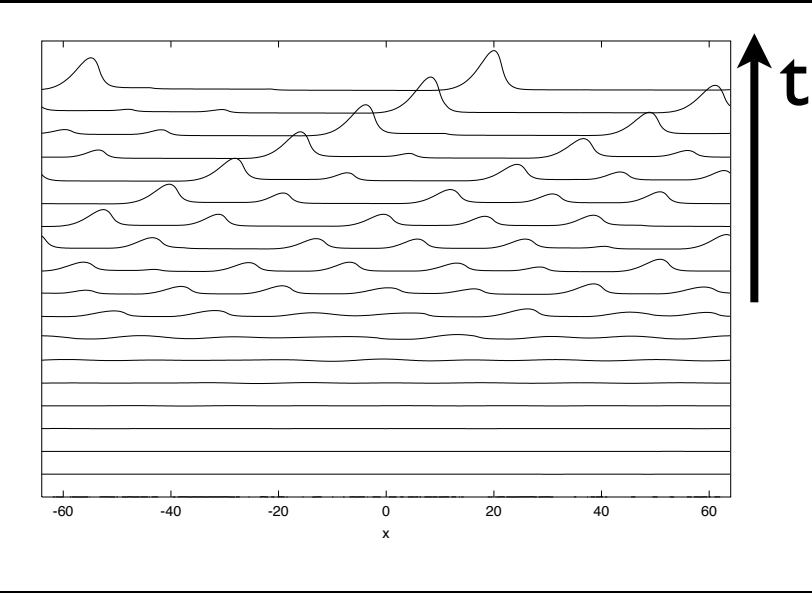
fluid 





fluid 

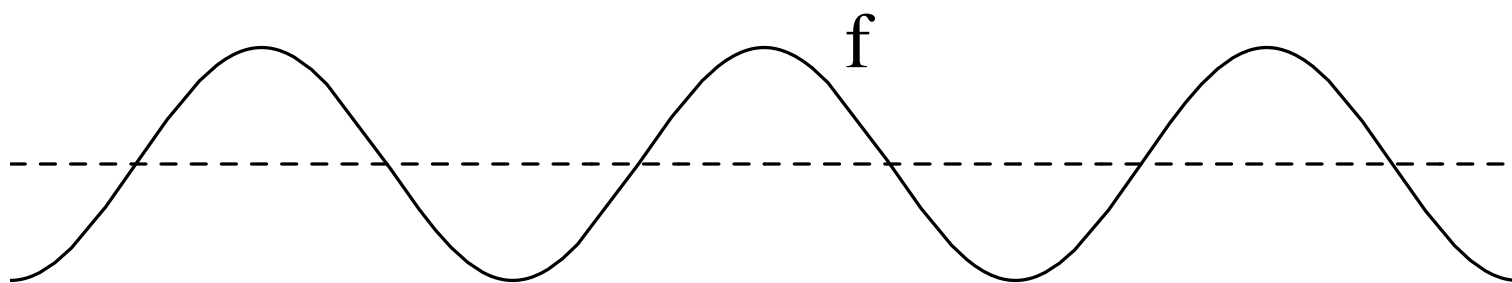






cas oscillant

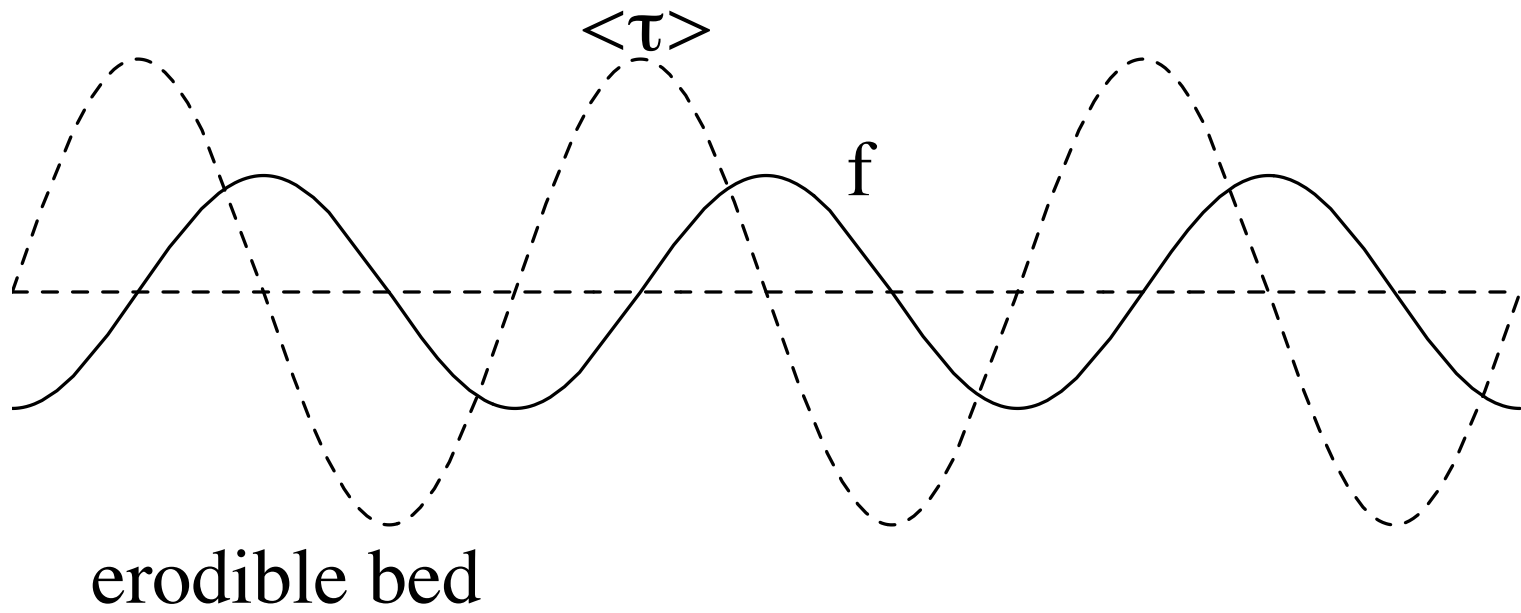
fluid



erodible bed

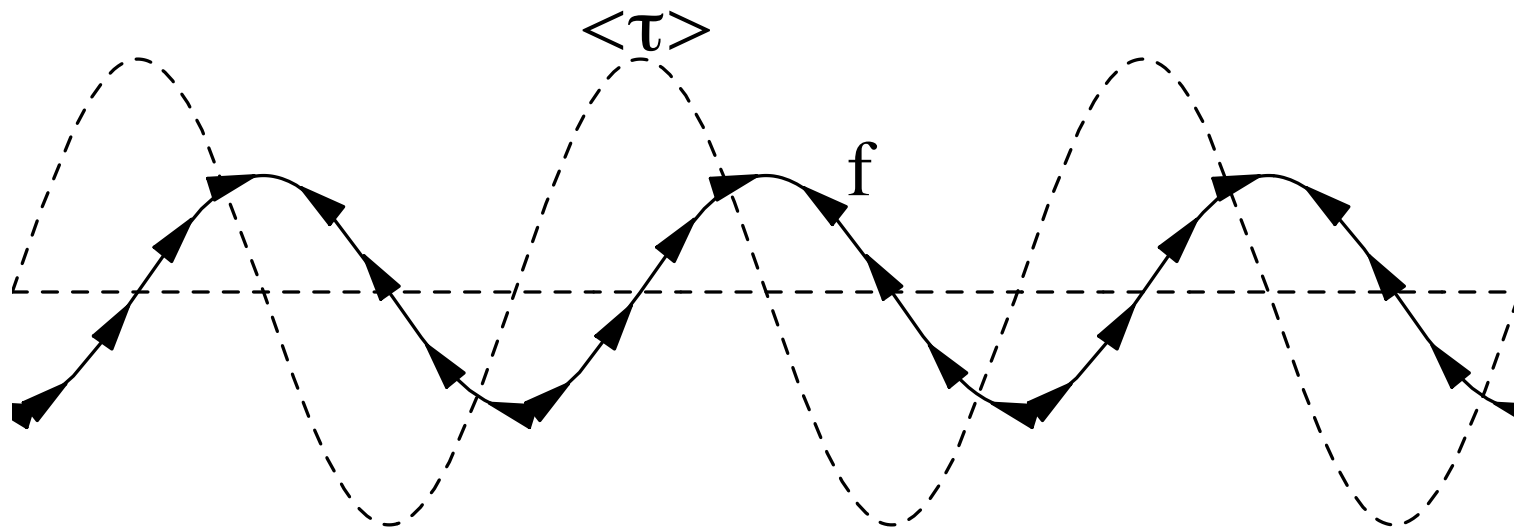


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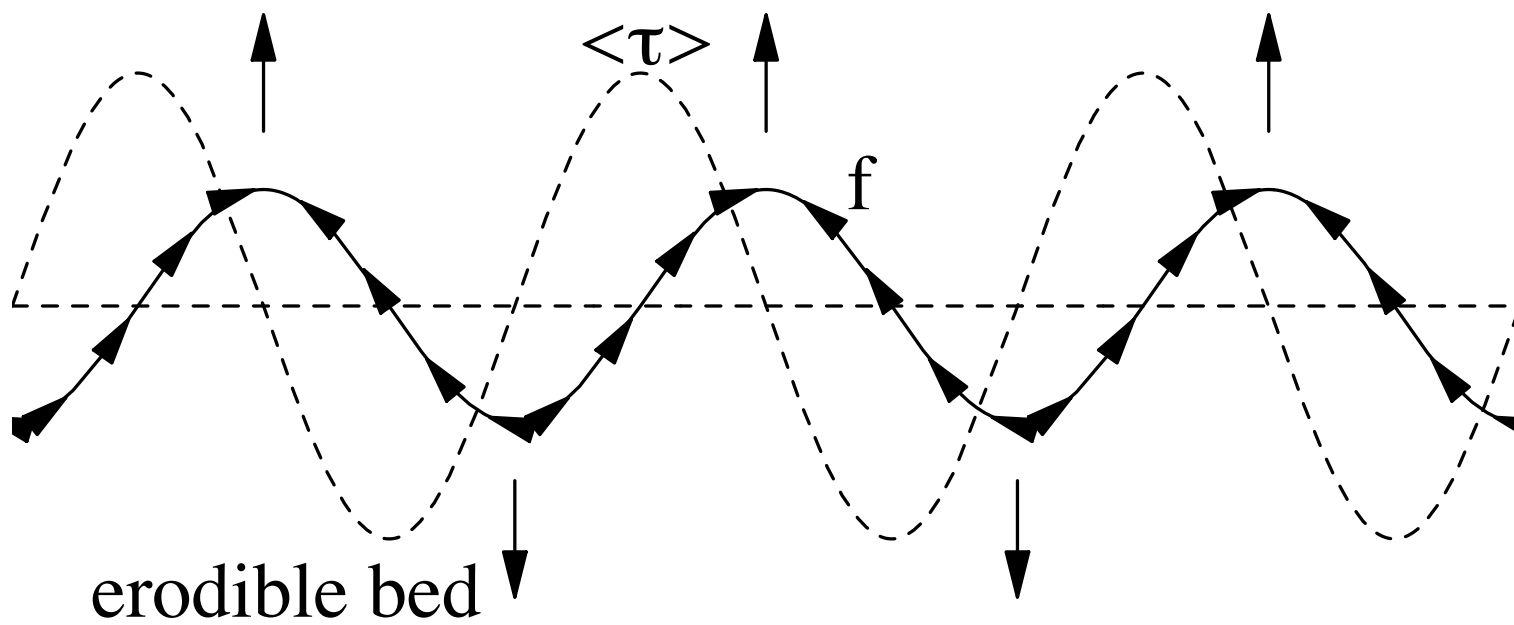
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erodible bed

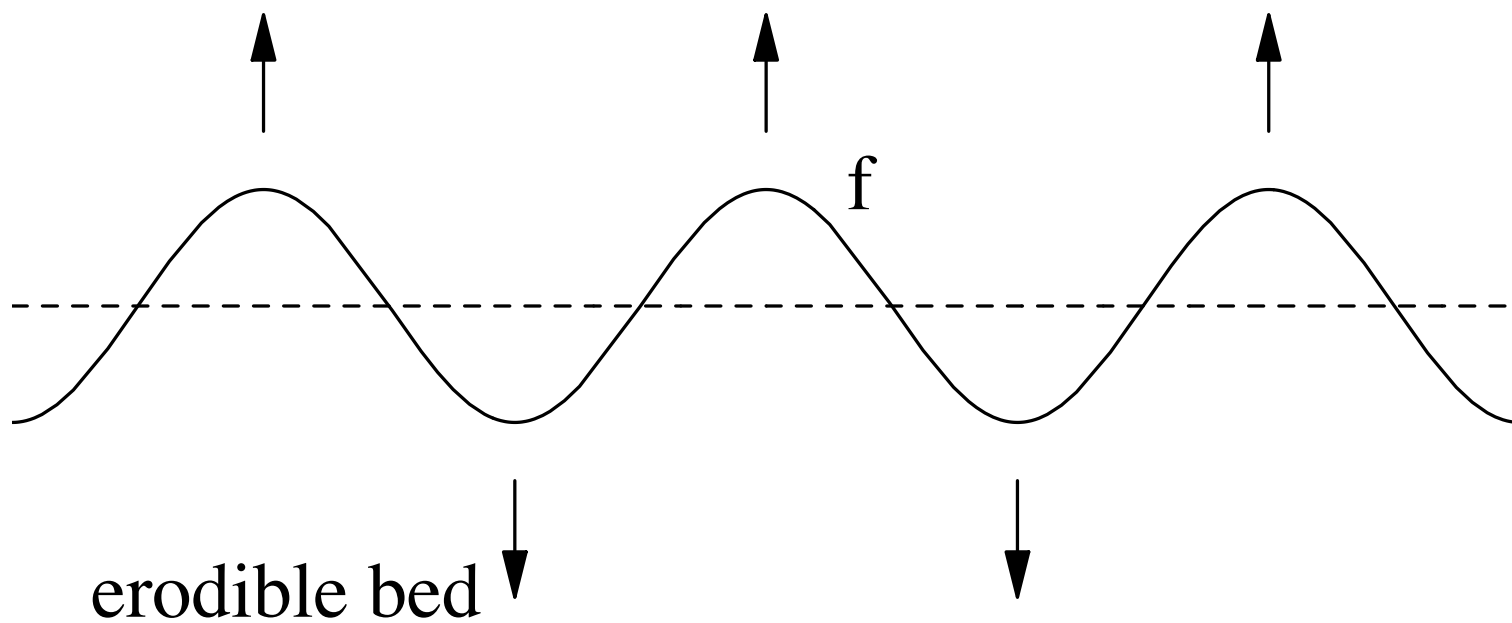


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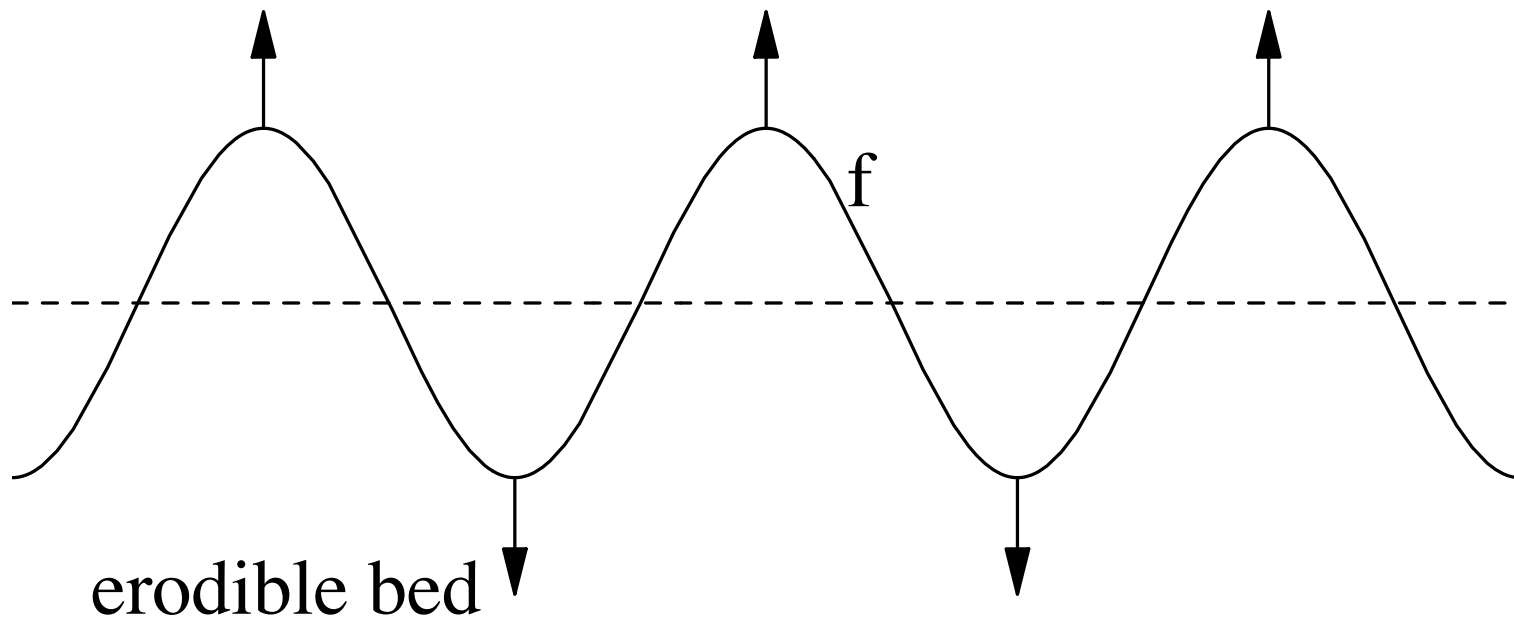


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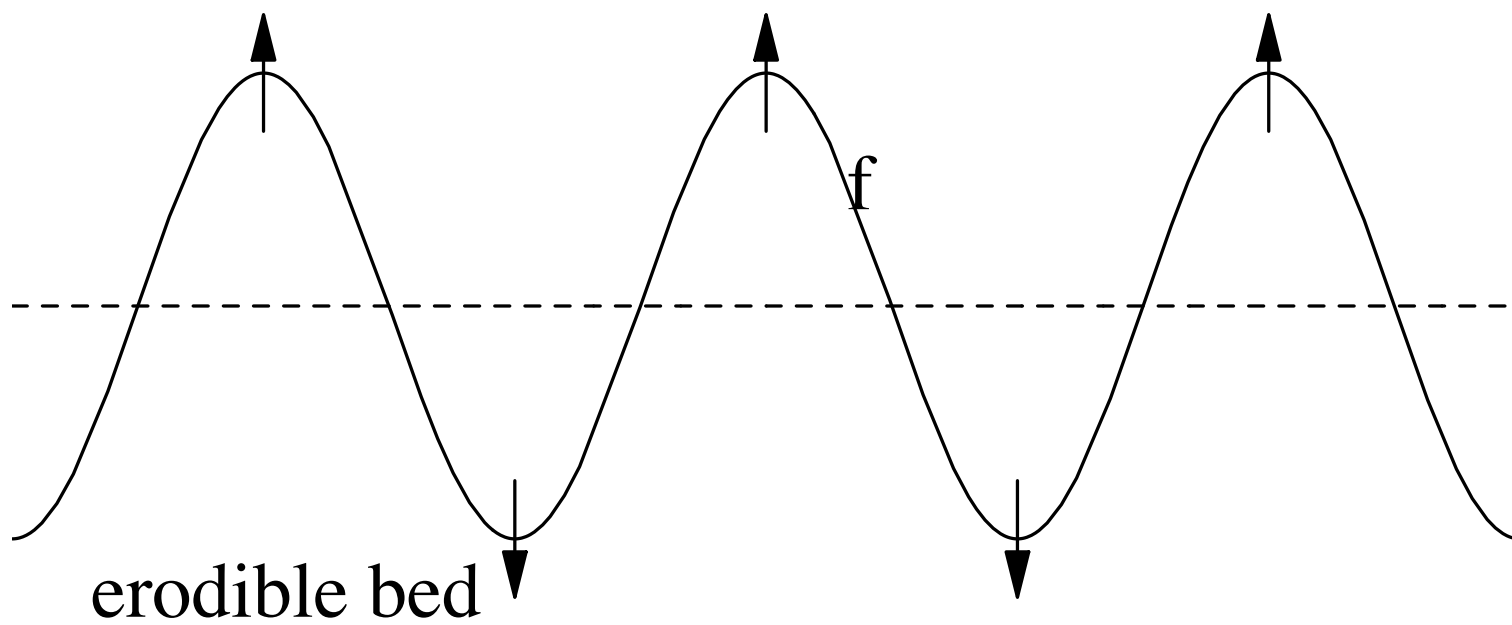


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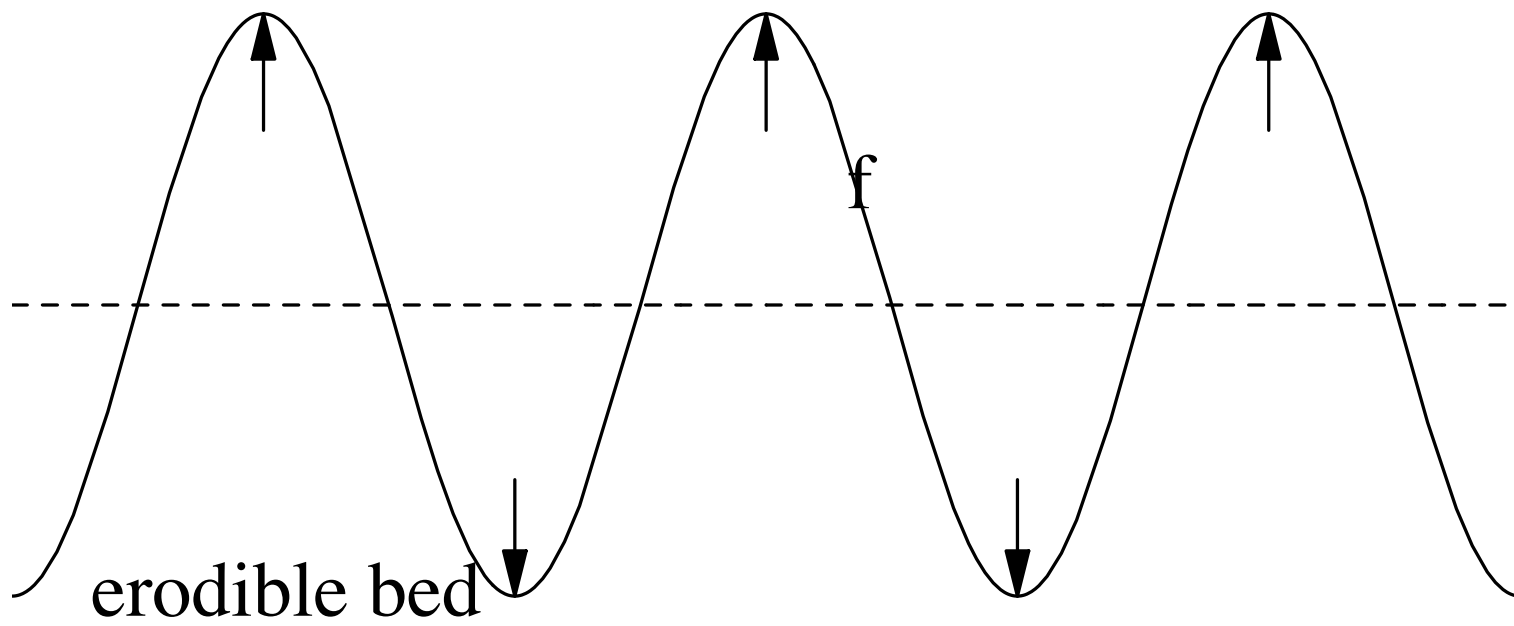


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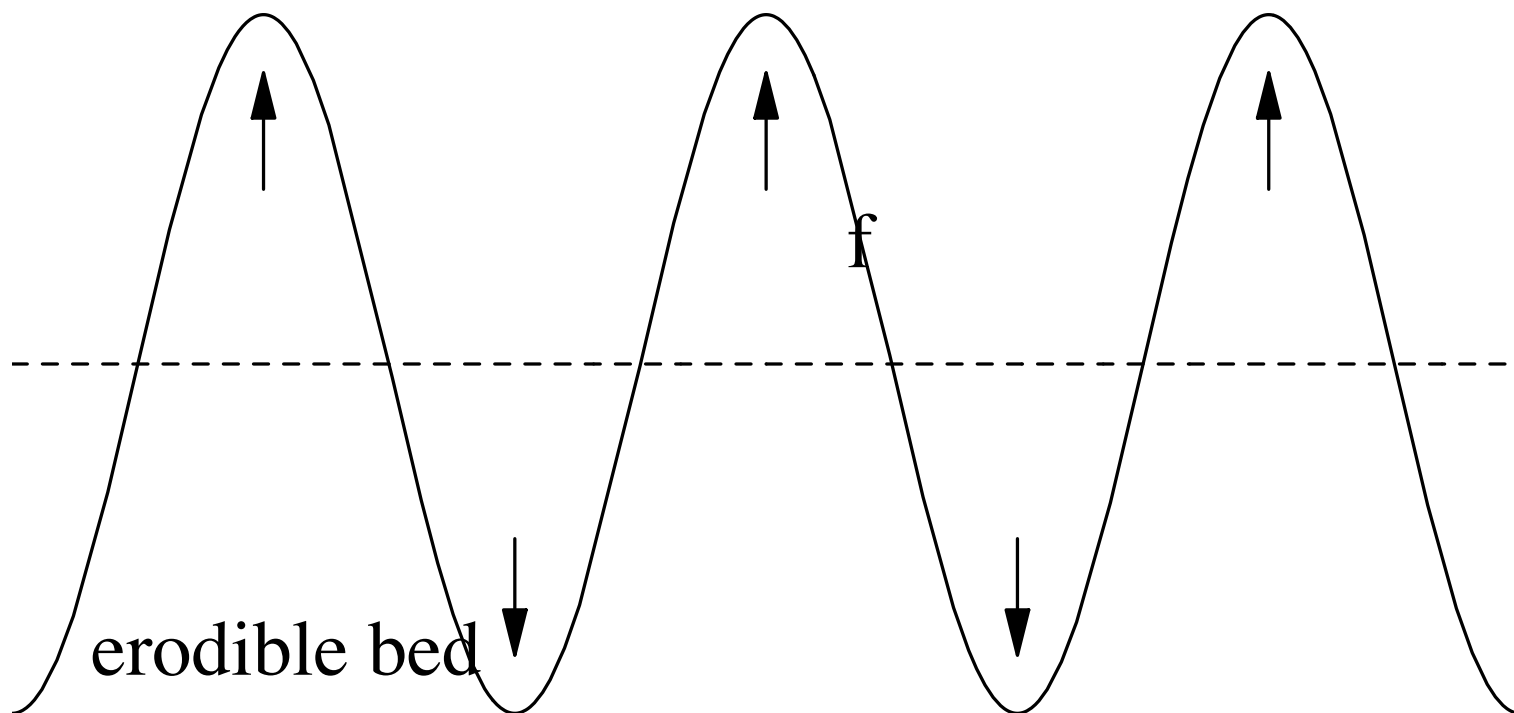


fluid



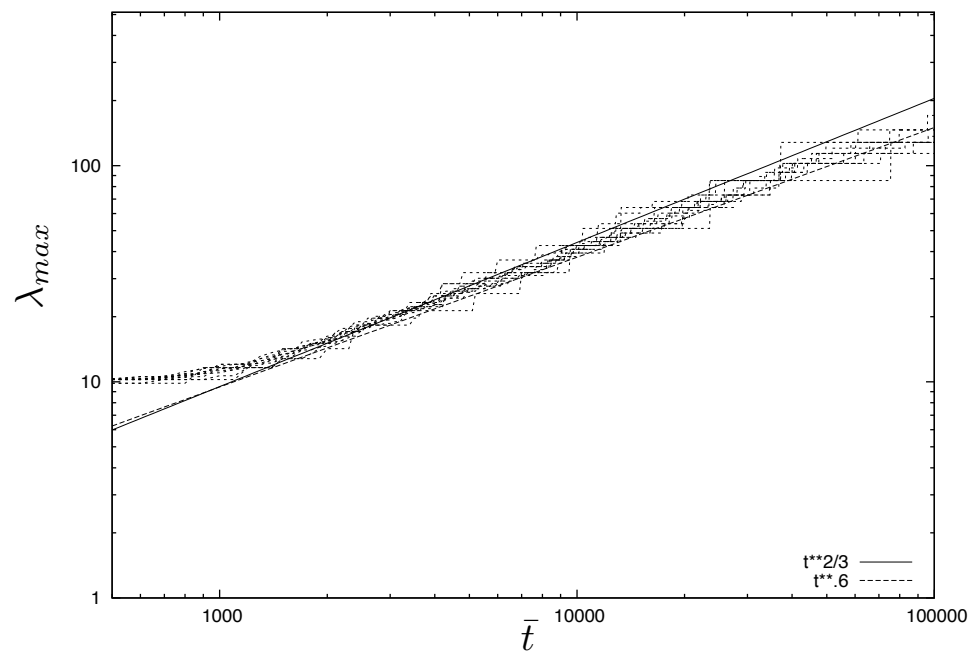
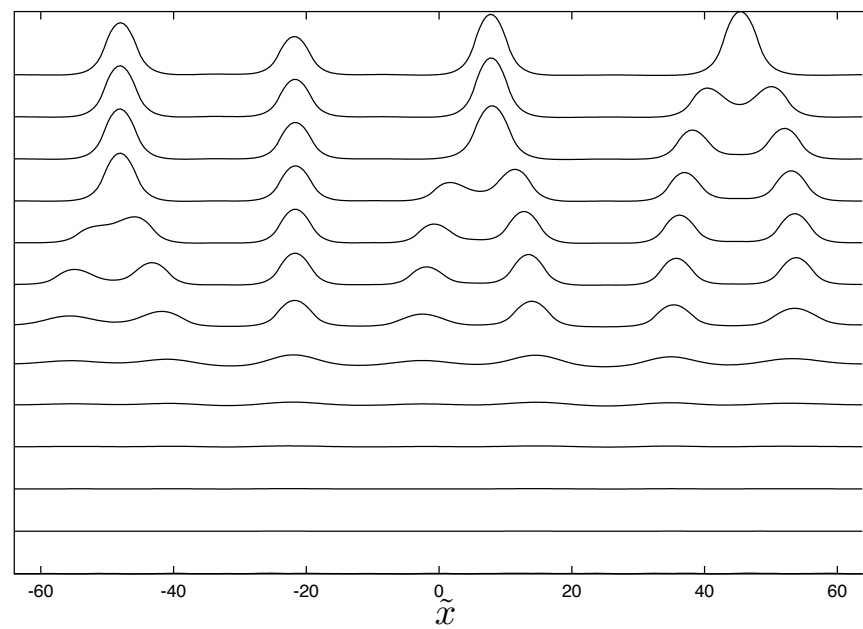


fluid



erodible bed

f





Quelques vérifications expérimentales grossières

Betat, Kruelle, Frette, and Rehberg (2002) :

La longueur d'onde la plus instable est environ $9cm$, $\sigma^* = 3 \cdot 10^{-3} s^{-1}$

$$\lambda^* \simeq 15cm.$$

$$\sigma^* = 1.2 \cdot 10^{-3} s^{-1},$$

C'est l'ordre de grandeur correct.

σ^* augmente avec le cisaillement, observé

λ^* augmente avec $U_0'^{-1}$, pas observé



Quelques vérifications expérimentales grossières

Cas Oscillant Rousseaux Stegner Wesfreid (2004)

$\lambda_{initial} \simeq 0.5cm.$

$$\lambda^* \simeq 0.26cm.$$

C'est l'ordre de grandeur correct.

La formule prédit $\lambda^* \propto d^{3/2}$, $\lambda^* \propto \delta^2$, et $\lambda^* \propto \nu^{-1}$.

c'est dans le bon sens

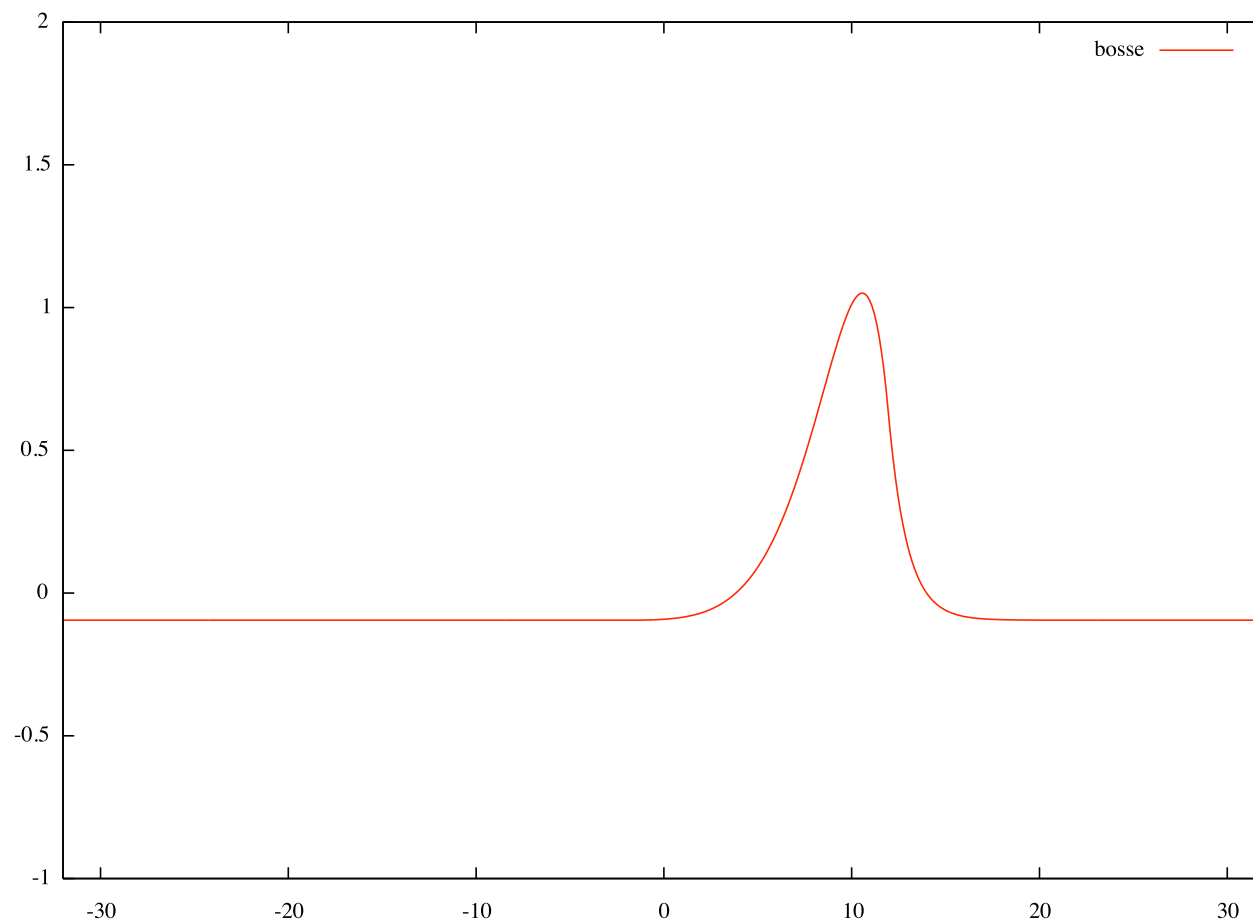
Rousseau et coll. ont fitté : $\lambda_{max} \propto \text{Log}(t)$ (Cahn-Hillard)

$$\lambda_{max} \propto t^{2/3}$$



Bosse finale : une pseudo "dune"

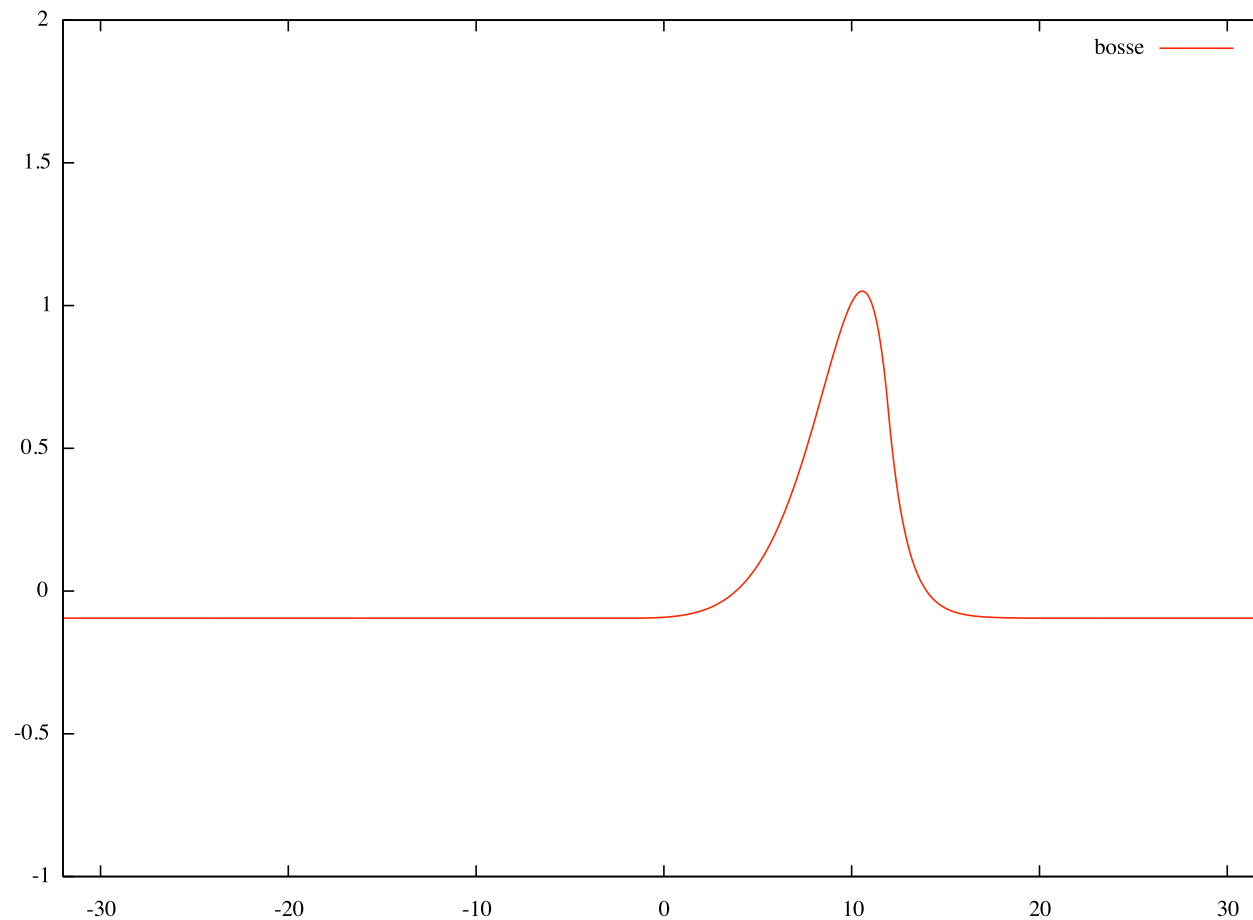
En fin de murissement, il ne reste qu'une bosse qui occupe toute la boîte.





Bosse finale : une pseudo "dune"

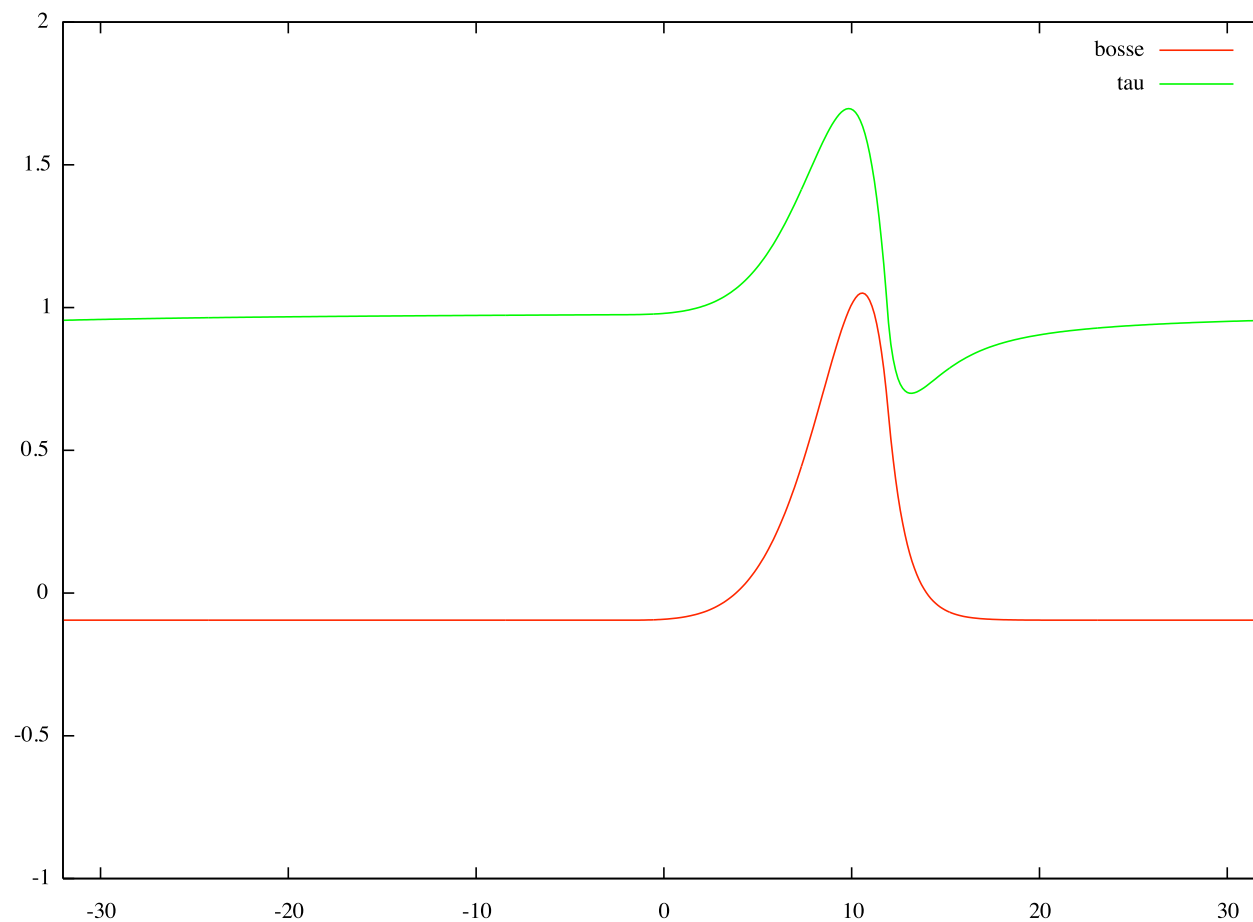
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Bosse finale : une pseudo "dune"

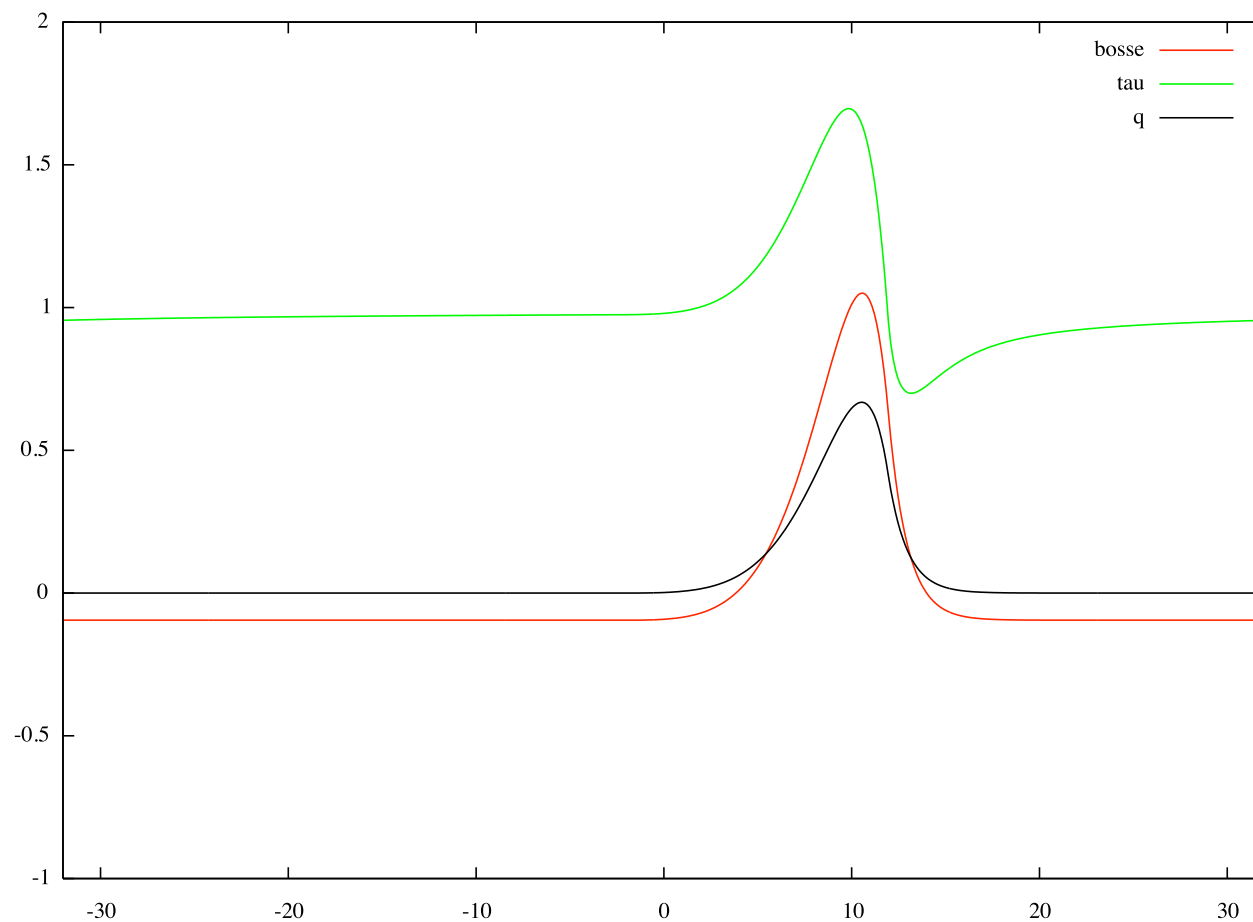
En fin de murissement, il ne reste qu'une bosse qui occupe toute la boîte.





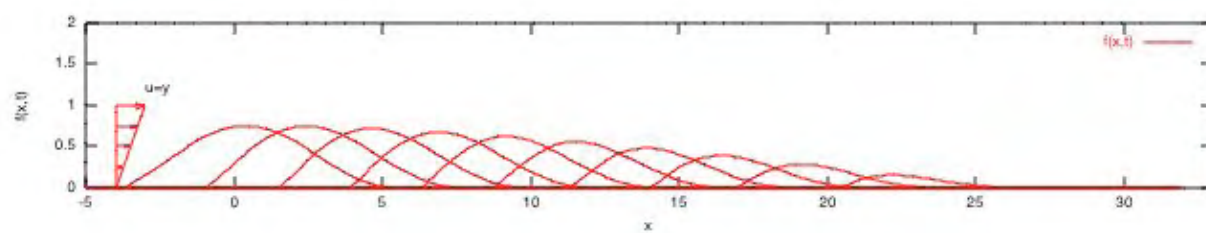
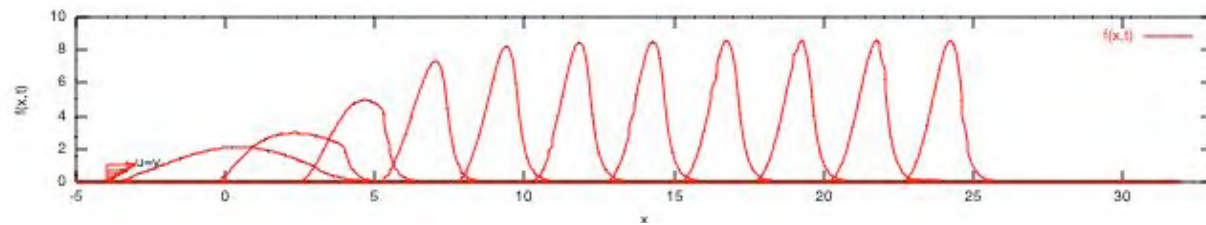
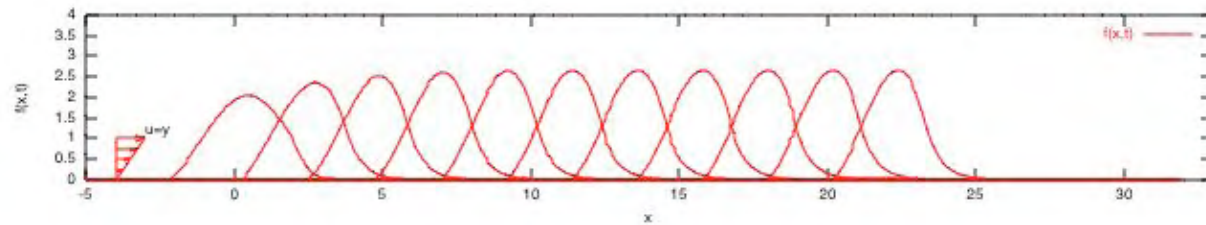
Bosse finale : une pseudo "dune"

En fin de murissement, il ne reste qu'une bosse qui occupe toute la boîte.





Déplacement d'une "dune" dans un écoulement cisailé :





Problème Auto Semblable

rescalons $x = Lx^*$, on doit avoir $f = L^{1/3}f^*$ pour que τ soit invariant

$$\tau = L^{-1/3}L^{1/3}TF^{-1}[(3Ai(0))(-ik^*)^{1/3}TF[f^*]] = \tau^*$$

$$q = q^*$$

$$\int f dx = m \text{ d'où } L^{4/3} = m \text{ avec } \int f^* dx^* = 1$$

$$\left(\frac{l_s}{L}\right) \frac{\partial q^*}{\partial x^*} + q^* = \varpi(\tau^* - \tau_s)$$

$$\frac{\partial f^*}{\partial t^*} = -\frac{\partial q^*}{\partial x^*}$$

$$t = L^{4/3}t^* \text{ and } c = L^{-1/3}c^* \text{ d'où } c = m^{-1/4}c^*$$



Problème Auto Semblable

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$$t = L^{4/3}t^* \text{ and } c = L^{-1/3}c^* \text{ d'où } c = m^{-1/4}c^*$$

$1/c$ proportionnel à $m^{1/4}$ et fonction de $l_s^{-1}m^{3/4}$



Self Similarity

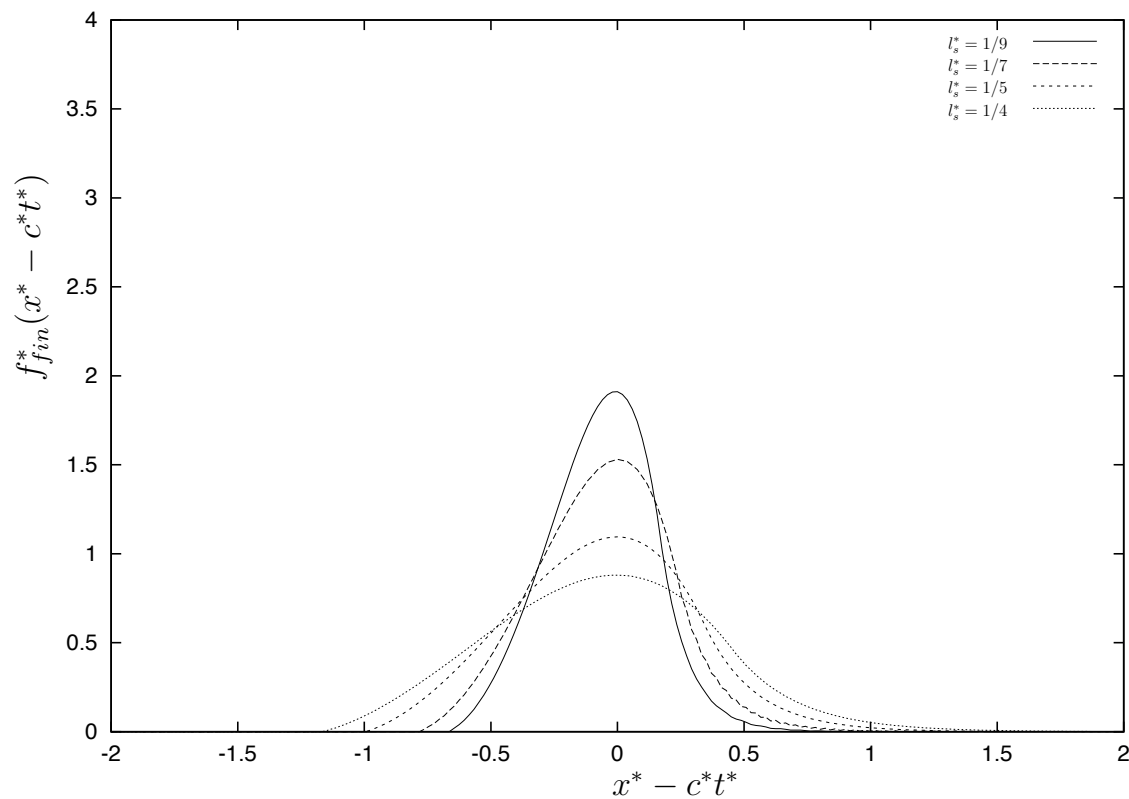


Fig. 8. "Dunes" of unit mass with $l_s^* = 1/4, 1/5, 1/7, 1/9$ ($\tau_s = 0.9$). The smaller l_s^* is, the thinner and higher the "dune" is.

selfsimilarity, unit mass $m = 1$, different $l_s^{-1} m^{3/4}$.



formes finales lin/ non lin

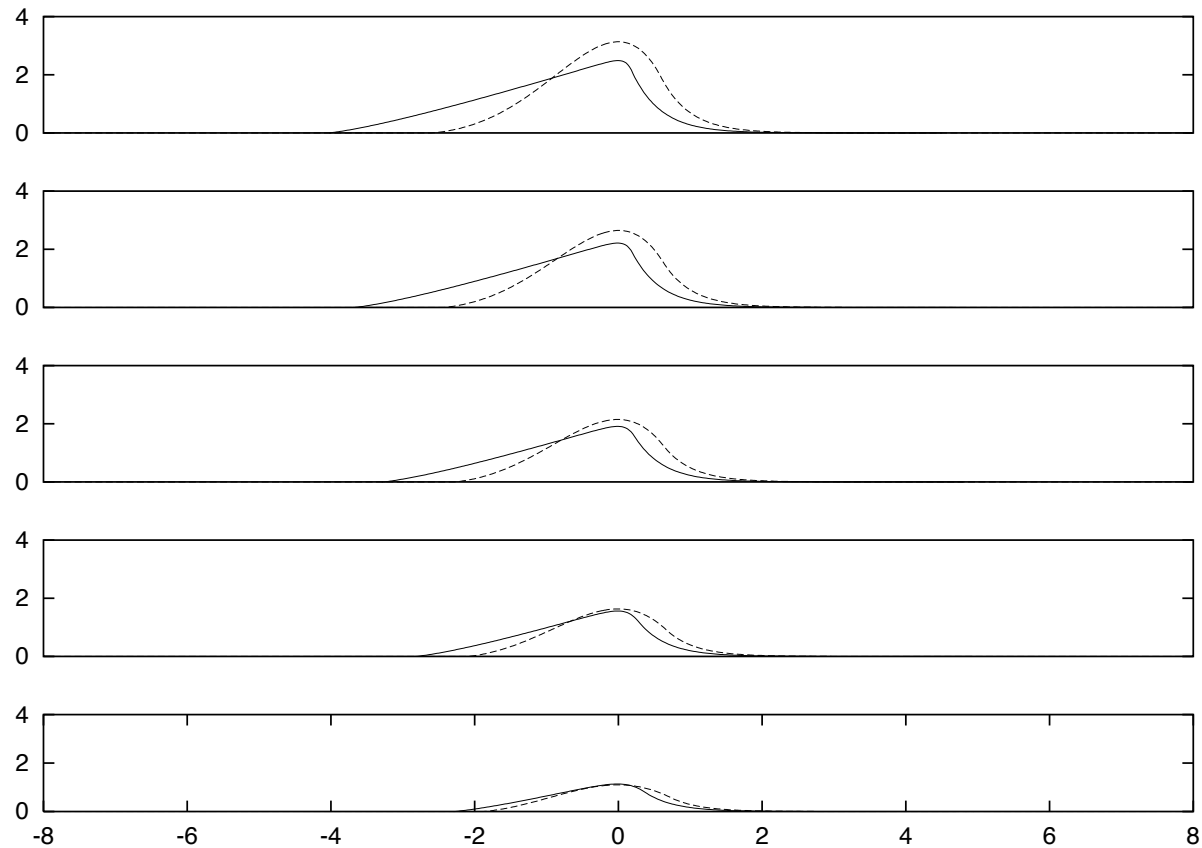
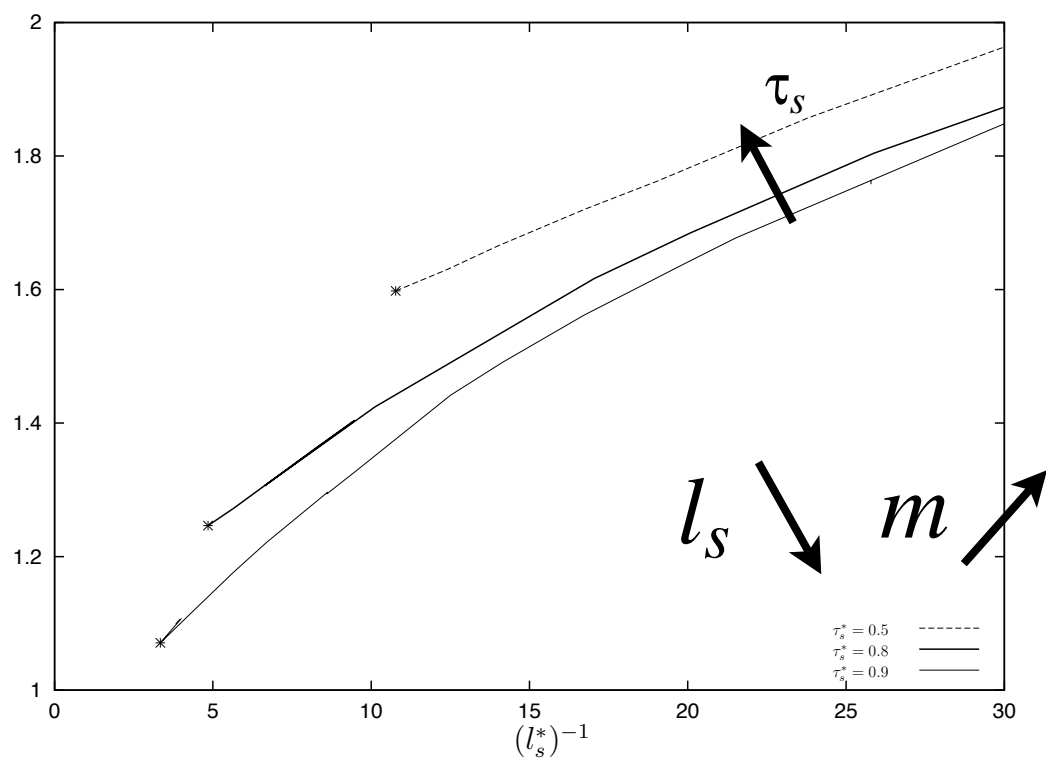


Fig. 5. The non-linear final moving "dune" solution $f_{fin}(x - ct)$ is represented with solid lines, the linear solution is represented with dashed lines, and $\tau_s = 0.9$, $1/l_s = 2.5$, $m = 2, 3, 4, 5$ (bottom curve to top curve).



$$cm^{1/4} c^*$$

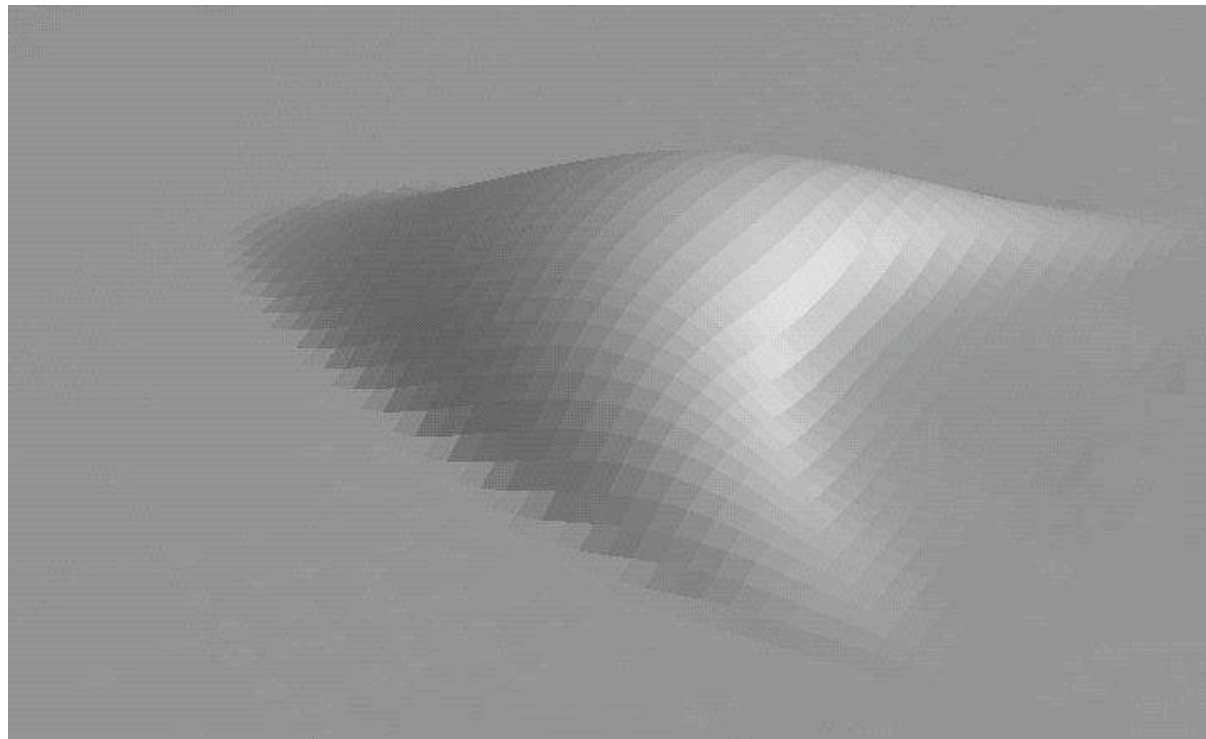


$$l_s^{-1} m^{3/4}$$

la vitesse est en $m^{-1/4}$

il y a une taille minimale





A "dune" in a shear flow



Conclusion

- pas trop réaliste
- mais justification du calcul du cisaillement (comparé à NS), qui autorise la séparation
- ordres de grandeur corrects
- temps de calcul assez court
- "coarsening"
- prédiction de la dépendance spéciale de la vitesse de la pseudo dune en $m^{-1/4}$



Perspectives

- Application à un cas spécial : Hele Shaw
- Cas turbulent "Interacting Boundary Layer"
- mieux comprendre le 3D
- application aux méandres



