Influence of an imposed flow on the stability of a gravity current in a near horizontal duct

S. M. Taghavi,[1] T. Séon,[2,a)] D. M. Martinez,[1] and I. A. Frigaard[2,3]
[1]Department of Chemical and Biological Engineering, University of British Columbia, 2360 East Mall, Vancouver, British Columbia V6T 1Z3, Canada
[2]Department of Mathematics, University of British Columbia, 1984 Mathematics Road, Vancouver, British Columbia V6T 1Z2, Canada
[3]Department of Mechanical Engineering, University of British Columbia, 6250 Applied Science Lane, Vancouver, British Columbia V6T 1Z4, Canada

(Received 16 October 2009; accepted 23 December 2009; published online 24 March 2010)

We study experimentally the effect of a mean flow imposed on a buoyant exchange flow of two miscible fluids of equal viscosity in a long tube oriented close to horizontal. We measure the evolution of the front velocity V_f as a function of the imposed velocity V_0. At low V_0, an exchange-flow dominated regime is found, as expected, and is characterized here by Kelvin–Helmholtz-like instabilities. With increasing V_0 we observed that the flow becomes stable. Here also V_f increases linearly with V_0 with slope of >1. At large V_0 we find $V_f \sim V_0$. © *2010 American Institute of Physics*. [doi:10.1063/1.3326074]

Gravity currents of a heavy fluid displacing a lighter fluid over a nearly horizontal surface are widespread both in natural (oceanography, hydrology, and atmospheric sciences) and industrial systems (chemical and petroleum engineering).[1–3] Such flows are driven by buoyancy, but the physical mechanisms that limit the flow may be inertia or viscosity depending on the geometric configuration, the mean flow, and the type of fluids.[4] These processes, often studied in unconfined geometries,[5,6] are strongly influenced by flow confinement. The lock-exchange problem in both confined[4,7–10] or unconfined geometries[11–13] has been previously investigated in sloping channels. The present letter is an extension of this work in which we superimpose a mean flow on the lock-exchange problem in a confined geometry.[14] Specifically, we measure and analyze the evolution of the front velocity V_f of the displacing fluid as a function of the mean flow velocity V_0 for fluids with different densities but of equal viscosities. We report the influence of the pressure-driven flow on the flow stability for the case of displacement of a lighter fluid by a heavier fluid in an unstable configuration, i.e., the heavier fluid is displacing the lighter fluid, in a long nearly horizontal tube.

In recent work in this area, Séon *et al.*[4,8] studied the lock-exchange problem and reported on the different flow regimes as a function of tube angle θ, measured relative to vertical, for a number of fluids of different densities and for different values of their common viscosity. No pressure driven flow was imposed in this case. For tubes far from horizontal, they observed mixing at the interface between the two fluids[15] and characterized the results as a function of the ratio of the buoyancy to inertial forces. By equating these terms, these authors advanced the argument that the characteristic velocity V_t for the exchange flow in this configuration is related to the square root of the density differences between the fluids ($V_t = \sqrt{\mathrm{At}gd}$) and defined the Reynolds number $\mathrm{Re}_t = \sqrt{\mathrm{At}gd}d/\nu$.

Here At is the Atwood number, defined as $\mathrm{At} = (\rho_2 - \rho_1)/(\rho_2 + \rho_1)$, ρ_i are the densities of the displaced ($i=1$) and displacing ($i=2$) fluids, respectively, g is acceleration due to gravity, d is the diameter of the tube, and ν is the common kinematic viscosity of the fluids. In contrast to this, for the case of tubes close to horizontal they observed an exchange flow without mixing. Here, a quasiparallel flow was observed in each fluid layer, indicating that the inertial forces are negligible. In this limit, they concluded that the buoyancy force must be balanced by the viscous force and defined this as the viscous flow regime. The transition between the inertial and viscous flow regimes was found to occur for $\mathrm{Re}_t \cos\theta \simeq 50$.

In the present letter we demonstrate that the imposition of a pressure-driven flow on an exchange flow strongly influences the front velocity and the physical mechanisms that dissipate energy. Below, after discussing the experimental setup, the front velocity is presented as a function of the mean flow velocity V_0 from which we identify three different flow regimes. Following this, we report an important finding of this work: the transition of the flow from inertia-dominated behavior to viscous-dominated behavior with increasing energy introduced into the system by increasing V_0.

The study was performed in a 4 m long, 19 mm diameter transparent tube with a gate valve located 80 cm from one end (Fig. 1). The tube was mounted on a frame which could be tilted to a given angle. Initially, the lower part of the tube was filled with water colored with a small amount of ink and the upper part of the tube by a denser salt-water solution. The tube was fed by gravity from an elevated tank. The imposed flow rate was controlled by a valve and measured by a rotameter and a magnetic flowmeter located downstream of the tube. Experiments were conducted using water as the

[a)]Author to whom correspondence should be addressed. Electronic mail: seon@math.ubc.ca.

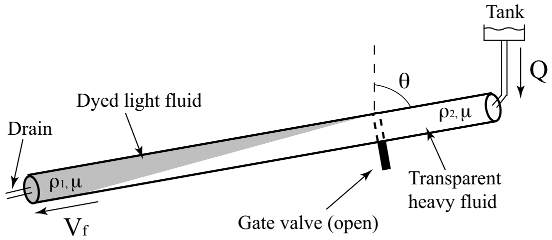

FIG. 1. Schematic view of experimental setup. The shape of the interface is illustrative only. More realistic shapes are given in Fig. 3 where the interface shape was found to evolve both spatially and temporally. Here V_f represents the front velocity of the displacing fluid (transparent).

common fluid with salt (NaCl) as a weighting agent to densify one of the fluids. A large number of experiments were conducted over the ranges of $[V_0, \mathrm{At}] \in [0-350\ \mathrm{mm\ s^{-1}}, 10^{-3}-4\times10^{-2}]$. Our imaging system consisted of two digital cameras with images recorded at a frame rate of 2 Hz. The tube was back lit, and after opening the valve, images were obtained at regular time intervals, which enable us to create spatiotemporal diagrams of the mean concentration profiles along the tube. The displacement of the front with time was marked on these diagrams by a sharp boundary between domains of the different relative concentrations of the fluids. The front velocity V_f is equal to the slope of these boundaries. It was found that the slope of this boundary was essentially linear after a couple of diameters within experimental uncertainty.

We discuss now the experimental relation between V_f and V_0. A representative case is illustrated in Fig. 2 for $\theta=83°$, $\mathrm{At}=10^{-2}$, and $\mu=10^{-3}$ Pa s. Phenomenologically, we observe three distinct behaviors as V_0 is increased from zero. (i) As $V_0 \to 0$, we observe an exchange-flow dominated regime: the imposed flow has only a slight influence on the dynamics of the exchange flow. For the case depicted in this figure, we are in the inertial regime,[8] since $\mathrm{Re}_t \cos\theta = 101.3 > 50$, and the flow develops some shear instabilities at the interface. (ii) In the second regime, the balance between pressure gradient and dissipative forces still exists but the mean flow becomes stronger than the buoyancy driven flow. This controls its dynamic. The main feature here is a linear relationship between V_f and V_0 ($V_f/V_0 \approx 1.3$ in this representative example). We have conducted experiments at various At in this regime, as shown in the inset of Fig. 2, and we observe that the slope V_f/V_0 does not vary significantly with At. This is interesting because the fact that V_f is different from V_0 indicates that buoyancy has a role to play, but this effect is almost independent of the Atwood number. Moreover we emphasize that this linear relationship is found for cases for which the first regime may be either inertial or viscous, as this is the case, respectively, for $\mathrm{At}=10^{-2}$ ($\mathrm{Re}_t \cos\theta > 50$) and $\mathrm{At}=10^{-3}$ ($\mathrm{Re}_t \cos\theta < 50$). These observations need more understanding and will be discussed in more depth in a companion paper. (iii) For $V_0 > 150-200\ \mathrm{mm\ s^{-1}}$, we observe a second linear regime with $V_f \approx V_0$. This third regime is displayed partially on the main curve and more completely in the inset of Fig. 2. It is defined by the buoyancy forces becoming negligible compared with the imposed pressure gradient. In this case it occurs when the imposed flow is turbulent ($V_0 \gtrsim 150$, see inset, which corresponds to $\mathrm{Re} \gtrsim 3000$). As a result, the two fluids mix (see inset) and are completely displaced ($V_f \approx V_0$). We also observe a transitional zone between the second and third regimes in Fig. 2.

We now focus on the influence of the imposed flow on the stability of the system. To illustrate this we show in Fig. 3 images from the flows of Fig. 2 for three different representative imposed velocities V_0. Figure 3 displays images of the 70 cm long section of the tube, tilted at $\theta=83°$, taken 30 cm below the gate, out of view on the right hand side, for the same density contrast. The heavier transparent fluid is moving downward, i.e., from right to left. In Fig. 3(a) we observe an inertial gravity current where, behind the front, pseudo-interfacial shear instabilities (Kelvin–Helmholtz-like) develop and induce mixing between the two fluids transversally across the section. This low mean flow case ($V_0 = 8.6\ \mathrm{mm\ s^{-1}}$) is in the first regime (see Fig. 2) where the flow is driven by a balance between buoyancy and inertia (since here $\mathrm{Re}_t \cos\theta > 50$).[8] In Fig. 3(b) with an increased imposed flow we observe a stable flow in which there are no Kelvin–Helmholtz instabilities at the interface. Consequently there is no mixing between the two fluids. Moreover, the front height is small and the slope of the interface with respect to the pipe axis is constant and weak. We infer that the velocity field is quasiparallel and is therefore under conditions where the lubrication approximation becomes valid; the flow dissipates its energy through viscosity. In comparison to Fig. 3(a), this behavior appears quite counterintuitive since more energy is being injected into the system as V_0 is greater than in the previous case. As the mean flow approaches a Poiseuille flow, the flow is inherently stable in this range of Reynolds number. This demonstrates a key observation of

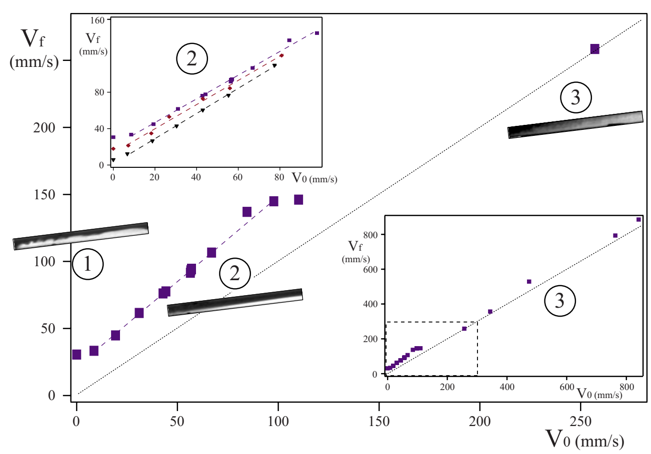

FIG. 2. (Color online) Variation in the front velocity V_f as a function of mean flow velocity V_0 for $\theta=83°$, $\mathrm{At}=10^{-2}$, and $\mu=10^{-3}$ Pa s. The dashed line is a linear fit of data points in the mean flow dominated regime (slope 1.3), whereas the dotted line shows the slope 1 of the final mean flow regime. The top left inset displays the second regime for different Atwood numbers: $\mathrm{At}=10^{-2}$, $\mathrm{At}=4\times10^{-3}$, and $\mathrm{At}=10^{-3}$, and the dashed lines have respective slopes: 1.3, 1.33, and 1.38. The bottom right inset displays the same data as the main curve for a higher range of mean flow values; the dashed square represents the range of the main plot. The insets are pictures of a 20 cm long section of tube, 80 cm below the gate valve in the corresponding flow domains.

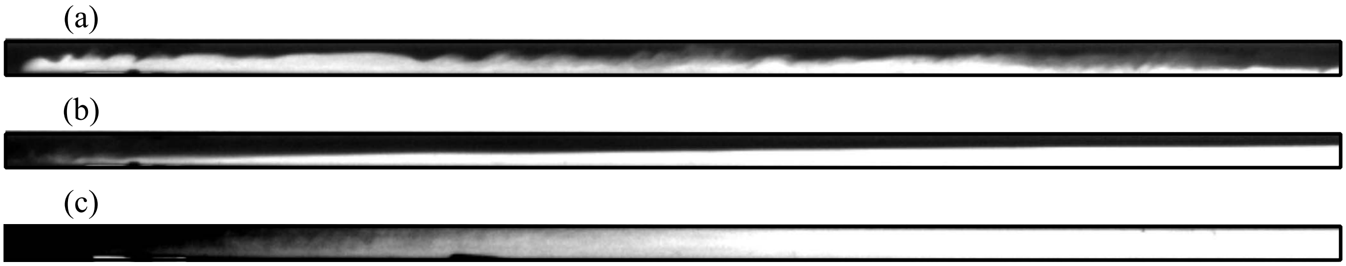

FIG. 3. Three snapshots of video images taken for different mean flows and showing the flow stability induced by the Poiseuille flow. These images are obtained for $\theta=83°$, $\mathrm{At}=10^{-2}$, and $\mu=10^{-3}$ Pa s and mean flow velocities (a) $V_0=8.6$ mm s^{-1}, (b) $V_0=71$ mm s^{-1}, and (c) $V_0=343$ mm s^{-1} (the corresponding buoyant velocity is $V_f^{V_0=0}=30.55$ mm s^{-1}). The field of view is 700×19.6 mm^2 and taken 30 cm below the gate valve. The images are taken at (a) 33 s, (b) 12 s, and (c) 5 s after opening the valve.

this letter: even though the Reynolds number is increased, the imposed flow stabilizes the initial inertial exchange flow by making the streamlines quasiparallel. Further, as stability results from a quasiparallel approximation, a small perturbation can break this fragile geometry and induce the propagation of a local burst along the interface. When such a burst appears, it induces transverse mixing. Finally, if the mean flow velocity [see Fig. 3(c)] is further increased, i.e., much higher than the buoyant velocity, the flow reaches the third regime where buoyancy forces are negligible. In this case, the stretched interface combined with the transverse mixing induced by the turbulent mean flow results in a complete displacement. The two pure fluids are separated by a mixing zone.

If we consider the pure exchange flow in this configuration, Séon *et al.*[4] showed that this exchange flow can become viscous by using a lubrication approximation argument. This regime appears when the transverse gravity component is strong enough to block the development of the instabilities at the interface, which occurs when the tube is close to horizontal. In this regime, except in the particular case of a horizontal tube, the front always reaches a constant velocity defined by the balance of buoyancy and viscous forces. In this case, Séon *et al.* showed that the quasiparallel approximation is not valid everywhere in the domain. The front usually appears in the form of an inertial "bump" with a velocity equal to $\sqrt{\mathrm{At}gh}$, where h (height of the front) adapts itself to maintain a front velocity equal to the viscous bulk velocity. Thus, we note that the steadiness of the velocity in this regime implies that this height remains constant with time.

Such a viscous exchange flow with an inertial bump is displayed on the top image of Fig. 4. This sequence displays a 45 cm section of the tube, a few centimeters below the gate valve (out of view on the right hand side). The images are plotted every $\Delta t=0.5$ s, and this sequence corresponds to an experiment conducted at $\theta=87°$, where the mean flow ($V_0=77.4$ mm s^{-1}) was imposed after the first image. We observe in this sequence that the inertial bump disappears under the effect of the mean flow. Indeed, the top of the bump seems to move faster than its base, or in other words, the Poiseuille velocity gradient spreads the initial shape of the bump out. This demonstrates that the lubrication approximation, which could not be valid close to the inertial bump for the exchange flow configuration, is now valid everywhere due to the mean flow (except perhaps very close to the front). Indeed, the only way for the inertial bump to disappear is to be subjected to a laminar flow in this region and this can only be achieved when the streamlines in this region are parallel. Therefore, the flow is now dominated by the Poiseuille flow and the buoyancy driven flow becomes a correction.

To conclude, these experiments have allowed us to quantify the influence of a pressure-driven flow on the well studied configuration of a buoyant flow of two miscible fluids of different densities in the confined geometry of a long tube close to horizontal. We showed the existence of three regimes as a function of V_0. In the first regime, i.e., $V_0\rightarrow0$, the influence of mean flow is negligible and therefore, the dynamics is governed by the balance between the buoyant and resistive forces (which depends on the fluid properties and can be either viscous or inertial). In the second regime, defined for higher values of the mean flow, the front velocity varies linearly with the imposed flow velocity. This result is in a good agreement with previous theoretical work[16] for the case of a laminar flow between parallel plates. Moreover in this regime the imposed flow stabilizes the unstable buoyant flow by making the streamlines more parallel. In other words, it tends to decrease the inertial term in the governing Navier–Stokes equations. We have seen that this inertial term, which was not negligible at the front for the laminar exchange flow (presence of the inertial bump), is removed by

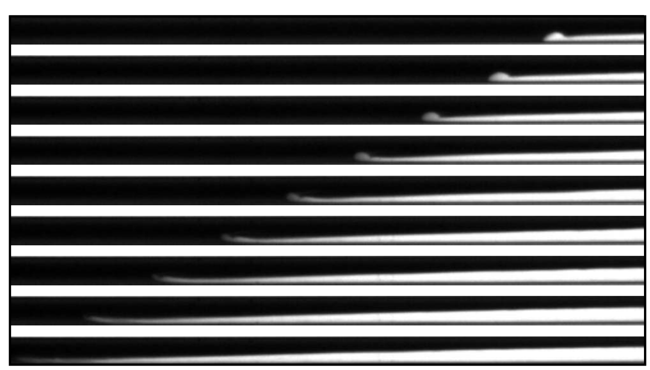

FIG. 4. Sequence of images showing the initial bump shape spread out by the Poiseuille velocity gradient. This sequence is obtained for $\theta=87°$, $\mathrm{At}=10^{-2}$, and $\mu=10^{-3}$ Pa s, and the mean flow ($V_0=77.4$ mm s^{-1}) is imposed after the first image (top one). The field of view is 452×20 mm^2 and taken a few centimeters below the gate valve. The sequence starts 7 s after opening the gate valve and the time interval between images is $\Delta t=0.5$ s.

a sufficiently strong imposed flow. A different way of viewing this is to note that when $V_0=0$, the instabilities at the surface of the current are due to the shear created by the exchange flow (due to buoyancy). If a mean flow is imposed, the relative influence of buoyancy decreases compared with that of the pressure gradient: the velocity gradient at the surface will decrease whereas the stratification remains unchanged. Thus, the local gradient Richardson number increases and the flow becomes more stable. Obviously, both explanations require quantifying. On the other hand, it is expected that higher buoyancy forces would not stabilize the flow. Indeed in this case, the mean flow required to stabilize the buoyant flow may itself be unstable, and so the flow would transit from an unstable buoyancy dominated regime to a turbulent pressure-driven regime. Finally, in the third regime, defined when the buoyancy forces are negligible, the mean flow is turbulent. The two fluids are displaced at the mean flow velocity and a mixing zone separates the two pure fluids. In this turbulent regime, we can expect that for a suitably strong mean flow and over long enough time scale, the mixing zone will spread diffusively governed by turbulent Taylor[17] dispersion. Thus, $V_f \sim V_0$ may not be strictly valid in this regime for longer times.

This research has been carried out at the University of British Columbia, supported financially by NSERC and Schlumberger through CRD Project No. 354717-07. T.S. also acknowledges the additional financial support from PIMS through a postdoctoral fellowship. The authors also thank K. Thielmann for assisting in running the experiment and the reviewers for helpful comments.

[1] T. B. Benjamin, "Gravity currents and related phenomena," J. Fluid Mech. **31**, 209 (1968).
[2] T. H. Ellison and J. S. Turner, "Turbulent entrainment in stratified flows," J. Fluid Mech. **6**, 423 (1959).
[3] J. E. Simpson, *Gravity Currents in the Environment and the Laboratory*, 2nd ed. (Cambridge University Press, Cambridge, England, 1997).
[4] T. Séon, J. Znaien, D. Salin, J. P. Hulin, E. J. Hinch, and B. Perrin, "Transient buoyancy-driven front dynamics in nearly horizontal tubes," Phys. Fluids **19**, 123603 (2007).
[5] N. Didden and T. Maxworthy, "The viscous spreading of plane and axisymmetric gravity currents," J. Fluid Mech. **121**, 27 (1982).
[6] D. Hoult, "Oil spreading in the sea," Annu. Rev. Fluid Mech. **4**, 341 (1972).
[7] T. Séon, J. P. Hulin, D. Salin, B. Perrin, and E. J. Hinch, "Buoyant mixing of miscible fluids in tilted tubes," Phys. Fluids **16**, L103 (2004).
[8] T. Séon, J. P. Hulin, D. Salin, B. Perrin, and E. J. Hinch, "Buoyant driven miscible front dynamics in tilted tubes," Phys. Fluids **17**, 031702 (2005).
[9] T. Séon, J. P. Hulin, D. Salin, B. Perrin, and E. J. Hinch, "Laser-induced fluorescence measurements of buoyancy driven mixing in tilted tubes," Phys. Fluids **18**, 041701 (2006).
[10] T. Séon, J. Znaien, E. J. Hinch, B. Perrin, D. Salin, and J. P. Hulin, "Front dynamics and macroscopic diffusion in buoyant mixing in a tilted tube," Phys. Fluids **19**, 125105 (2007).
[11] V. K. Birman, J. E. Martin, and E. Meiburg, "The non-Boussinesq lock-exchange problem. Part 2. High-resolution simulations," J. Fluid Mech. **537**, 125 (2005).
[12] V. K. Birman, B. A. Battandier, E. Meiburg, and P. F. Linden, "Lock-exchange flows in slopping channels," J. Fluid Mech. **577**, 53 (2007).
[13] J. O. Shin, S. B. Dalziel, and P. F. Linden, "Gravity currents produced by lock exchange," J. Fluid Mech. **521**, 1 (2004).
[14] K. C. Sahu, H. Ding, P. Valluri, and O. K. Matar, "Pressure-driven miscible two-fluid channel flow with density gradients," Phys. Fluids **21**, 043603 (2009).
[15] S. A. Thorpe, "Experiments on the instability of stratified shear flows: Miscible fluids," J. Fluid Mech. **46**, 299 (1971).
[16] S. M. Taghavi, T. Seon, D. M. Martinez, and I. A. Frigaard, "Buoyancy-dominated displacement flows in near-horizontal channels: The viscous limit," J. Fluid Mech. **639**, 1 (2009).
[17] G. I. Taylor, "Dispersion of matter in turbulent flow through a pipe," Proc. R. Soc. London, Ser. A **223**, 446 (1954).