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Extreme periodic modulation of mechanical states:

enhanced parametric instabilities for functionalities in slender structures.

Nowadays, from the use of buckling for folding [1,2] (see Fig.1a) to the exploitation of fluttering piezoelectric flags for energy harvesting [3] (see fig.1b) or the benefit of parametric resonances for the reduction of parasitic signals in microelectromechanical systems [4] (see Fig.1c), elastic instabilities eventually occurring in slender elastic structures are often seen as an opportunity to seize rather than a failure to avoid. Parametric elastic instabilities, found in many engineering problems [5-6], are caused by the self-synchronized periodic modulations of the elastic state of a slender structure [7]. Although promising for elaborate functionalities, the optimal use of parametric instabilities in elastic structures is often restricted to very small scales where damping is sufficiently low for the principal and subharmonic instability regions to exist [8,9]. To overcome this drawback and fully exploit the potential of parametric instabilities for functionality at any scales, a change of paradigm is necessary. Instead of classically lowering frictions to favor parametric excitations from the small modulations of an elastic state, one could periodically impose a drastic change of elastic state to enhance dynamical instabilities at common damping, as explained by the preliminary experiment and sketches in Fig.2.

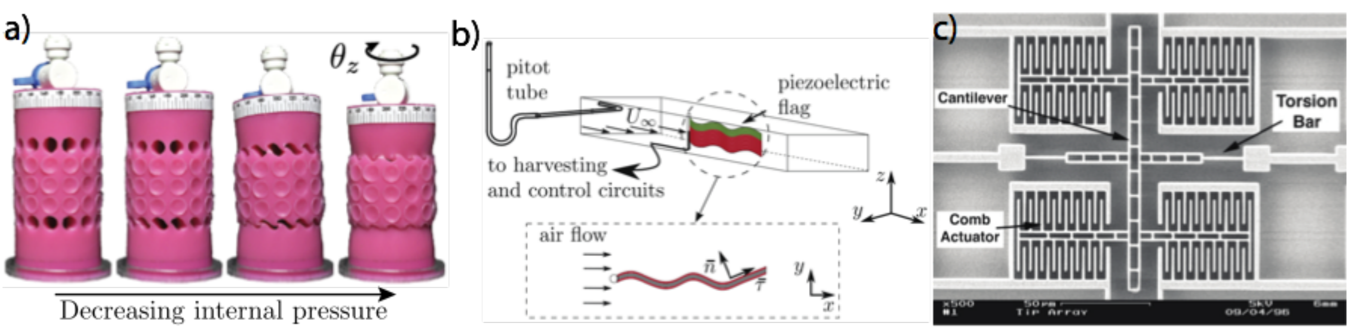

Fig. 1. Different examples of elastic instabilities for engineering functionalities. a) The collaborative local buckling triggered on the circumference of a thick elastomeric cylinder allows for its global twisting motion. This mechanism could be used as a building block for soft robotics [2]. b) It is possible to harvest the energy of an incoming flow from the fluttering instability of a piezoelectric flag [3]. c) The SEM image shows the MicroElectroMechanical System (MEMS) that exhibited a fifth subharmonic parametric resonance in 1998 [4]. In 2016, a research group exploited twenty-eight orders of parametric resonances in MEMS for multi-band vibration energy harvesting [10].

The goal of this project is to study a first archetypal example of enhanced parametric instabilities, that we denote as extreme parametric instabilities. The system to be investigated is a magnetic pendulum symmetrically placed between two attracting electromagnets, whose preliminary experiment is shown in Fig.2a). By periodically switching the electromagnets on or off, it is possible to drastically modify the mechanical state of the pendulum, as shown in Fig.2b)-c). For certain amplitude or frequency modulation parameters, this system exhibits parametric instabilities (the originally stable pendulum dynamically destabilizes) or dynamical stabilization (the naturally diverging pendulum is stabilized by the electromagnetic modulation). The rich spectral nature of parametric instabilities enhanced by the extreme modulation should offer promising opportunities for dynamical properties of structures, such as energy harvesting or discrete frequency filtering.

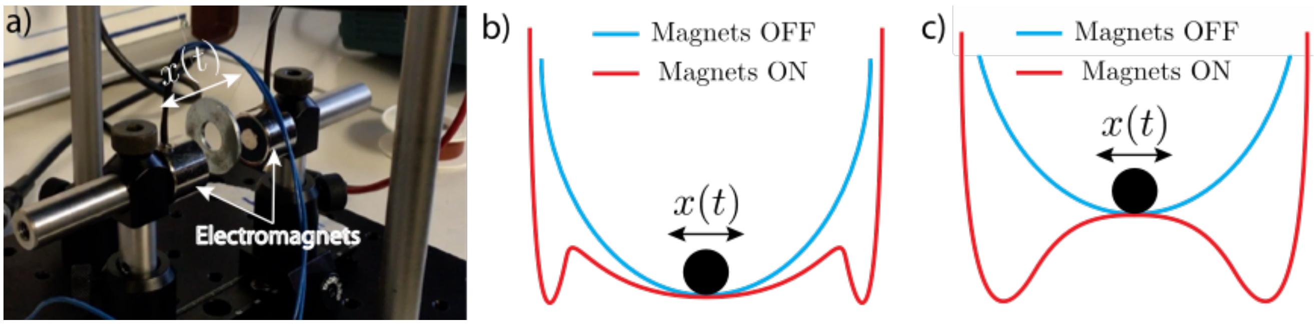

Fig. 2. Example of an extremely modulated magneto-mechanical state. a) Picture of an experiment: a pendulum whose mass is made of steel, is symmetrically placed between two electromagnets that are periodically turned on (red energy states in b-c)) and off (blue energy states in b-c)). Because the symmetric energy landscape varies drastically, parametric instabilities are enhanced. b) When working close to the locally diverging state (the local quadratic potential is almost flat), even with low and slow periodic electromagnetic modulations, highly sub-harmonic parametric instability regions are easily reached which should allow for multi-band energy vibration harvesting. c) When the pendulum periodically varies between a diverging and oscillating state, what are the optimal modulation parameters for the mass to be dynamically stable? One theoretical answer is that the stability regions could be narrow and discrete, an interesting property for amplitude or frequency filtering.

The candidate is expected to have a solid background in dynamical systems [11] as well as a deep inclination in understanding the physics behind structural mechanical problems. Starting from the work explained in [7,12] on the dynamics and stability of structures in periodic elastic states, the candidate will progressively seize the delicate concepts of parametric instabilities on the electromagnetic pendulum of Fig.2 in order to build design rules for engineering functionalities. A detailed program would be the following:

- The first step will be to understand the concepts and methodologies (Floquet forms, frequency lock-in, nonlinear limit cycles, ...) associated with the theoretical analysis of periodically-varying ordinary differential equations.
- The candidate will then need to develop mathematical and numerical models that govern the academic example of the magneto-mechanical system illustrated in Fig. 2a). Depending on the inclination of the candidate, it is possible to perform some experiments.
- The last part will be to use the developed models to identify the main mechanical and geometrical features responsible for the enhancement of parametric instabilities regarding two promising dynamical properties for functionalities:
    . The feasibility for energy harvesting by finding the configurations that allow for very high subharmonic instability regions to exist.
    . The possibility of discrete filtering by seeking the modulation functions that narrow and dissociate the regions of dynamical stabilization in the modulation parameters space.

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