Topological imaging of buried defects using experimental measurements on silione specimens

Master internship, 2021
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Backgound and objectives

Non-destructive evaluation (NDE) of a solid (for instance a building, a mechanical part, or even a part of the human body) aims at detecting and identifying defects that could be buried (cavities, fractures, tissue damage) wihtout deteriorating this solid. If one restrains the broad area of NDE to mechnical tests and measurements, the displacements or strains are typically recorded at the free surfaces of the solid when it is submitted to static or dyamical loads. Comparing these measurements to the expected reponse of a reference defect-free solid then enables to assert the presence or absence of defects. Sometimes these defects can even be characterized, i.e. their position, nature, size etc. can be determined. Finding a good identification methodology is also called an *inverse problem*, while the direct problem would be predicting the response of the solid, knowing its full state including defects.

The goal of this internship is to characterize parallelepipedal silicone specimens in which known defects such as marbles or coins have been buried. Strains and/or displacement measurements will be peformed during mechanical tests e.g. compression tests in several directions (Figure 1). The simple geometry and the choice of silicone, that reaches large strains in the linear elasticity regime, are expected to facilitate the specimens elaboration and the measurements. The focus of the internship will then be the implementation of a relevant identification method, and its critical evaluation.

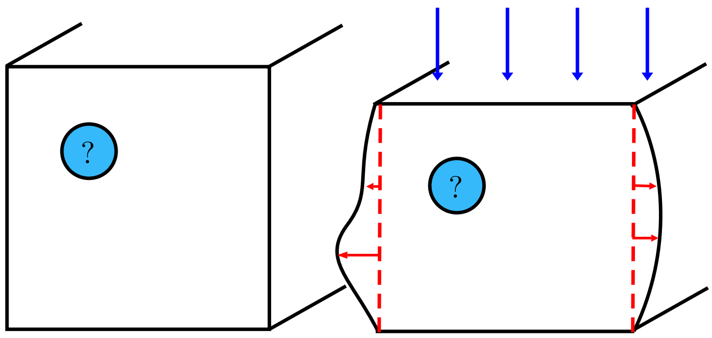

Figure 1: Schematics of the considered problem : a defect is buried in a cubical specimen, and must be found from strain measurements when submitted to static loads.

Numerous methods have been proposed to tackle similar inverse problems [1]. Among them, the *topological imaging* methods and especially the computation of *topological derivative* maps [5, 3, 2] were assessed by theoretical arguments, numerous numerical experimentations, and some laboratory experiments implying dynamical loads (see Figure 2). Charaterizing a solid from measurements coming from static loads is *a priori* more difficult, since these measurements carry less information than dynamical data. To address this issue, *high-order* topological derivatives [2, 4] might be of use: they drastically increase the quality of the reconstruction in some cases, see Figure 3.

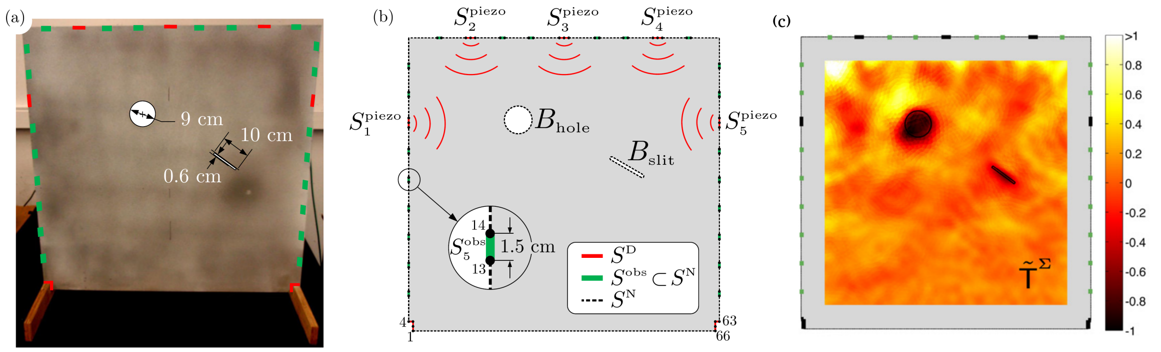

Figure 2: Imaging a perforated plate using dynamical excitation and boundary displacement measurements. (a) Experimental setting, (b) location of sources and measurement points and (c) topological derivative map computed by combining the data from loads at several frequencies. The most negative values (red) indicate the location of holes. Figures from [5].

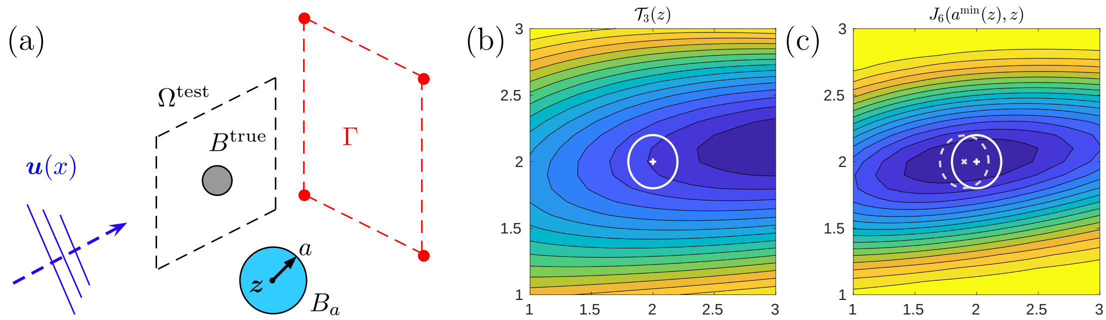

Figure 3: Numerical experiment: identification of a spherical penetrable obstacle $B^{\rm true}$ from measurements of scattered waves at four coptors on a surface Γ, and by comparison with "test" obstacle B_a. (a) Experiment setting and (b-c) topological derivative maps in the plane $\Omega^{\rm test}$. In this configuration, the sole topological derivative (b) does not provide a correct localisation (the "blue spot" indicating the most negative values is outside the obstacle $B^{\rm true}$) while *higher-order* toplogical derivatives (c) provide a precise location of the obstacle and an estimate of its size (the "best guess" B_a is plotted in dashed lines). Figures from [3].

Organisation

The internship will necessarily include the following steps:

1. bibliographical study on topological imaging methods and their validity and efficiency domain (geometries of solids and defects, static vs. dynamical measurements ...),
2. based on this study, choice of an exprimental setting: specimens and relevant mechanical loads to test the efficiency of topological derivative-based methods,
3. implementation of the method, and numerical experiments (generating artificial measurements and try to adress the inverse problem) to validate the implementation,
4. experiments: defective specimens production, mechanical tests, data treatment,
5. conclusions on the efficiency (or inefficiency) of the method and/or on the relevance of the chosen experimental setting,
6. writing an internship report and preparing a presentation.

Depending on the obtained results, some of these steps may be developed: more elaborate experimental setting, comparisons with other methods such as the *reciprocity gap* [1] (the same numerical tools can be used for both methods); and a scientific paper might be written.

If experiments are impossible to perform due to the sanitary crisis, the internship will focus on theoretical and numerical aspects of the proposed method.

Practical informations

Time: 5 to 6 months, start expected about March 2020.
Location: Institut d'Alembert, Sorbonne Université, campus Pierre et Maris Curie, Paris, France.
Salary: Sorbonne Université internship stipend ($\approx$ 570€/ month).
Expected profile: The candidate must have a good knowledge of the mathematical formulation of linear elasticity, especially on variational form. Prior knowledge of NDE methods / inverse problems would be appreciated but are not mandatory. He or she should have a tase for programmation, as the project will need both a finite element software (for instance Abaqus, CASTEM, Fenics, FreeFem++ ...) and a script language (for instance Python or Matlab). The choice of these tools will depend on the candidate proficiency. A taste for experimentation is also necessary.
Candidacy: The candidates must send CV (including recommendations if possible), grade records and cover letter to both internship supervisors (adresses below). Interview may be programmed. *All questions are welcome.*
Supervisors :

- Arnaud Lazarus : arnaud.lazarus@sorbonne-universite.fr, http://www.ida.upmc.fr/ alazarus/
- Rémi Cornaggia : remi.cornaggia@sorbonne-universite.fr

References

[1] Marc Bonnet and Andrei Constantinescu. Inverse problems in elasticity. *Inverse problems*, 2005.

[2] Marc Bonnet and Rémi Cornaggia. Higher order topological derivatives for three-dimensional anisotropic elasticity. *ESAIM: Mathematical Modelling and Numerical Analysis*, 51(6):2069–2092, 2017.

[3] Rémi Cornaggia. *Development and use of higher-order asymptotics to solve inverse scattering problems*. Ph. D. thesis, ENSTA, Université Paris Saclay and CEGE, University of Minnesota, September 2016.

[4] Antonio André Novotny, Jan Sokołowski, and Antoni Żochowski. Topological derivatives of shape functionals. Part III: Second-order method and applications. *Journal of Optimization Theory and Applications*, 181(1):1–22, Apr 2019.

[5] Roman Tokmashev, Antonin Tixier, and Bojan B Guzina. Experimental validation of the topological sensitivity approach to elastic-wave imaging. *Inverse Problems*, 29(12):125005, 2013.