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A priori anisotropic goal-oriented estimates for mesh adaptation in compressible CFD

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A basic feature in finite-element method (FEM) is the initial choice of an interpolation for the unknown and the building of an integration which keeps the exactness of the discretization for a solution identical to its interpolation. A priori estimates exploid easily the central role of interpolation in FEM. We concentrate on goal-oriented *a priori* estimates. The proposed communication gathers new estimates for the two following basic models, the advection model (a) and the Laplace equation (L):

$$W^a_t + \nabla^*.(\mathbf{V}W^a) = 0 \ ; \quad -\nabla^*.\nabla W^L = f \qquad (+ \text{ initial and boundary conditions}).$$

Let $j = (g, W)$ be a functional. Adjoint state W^* is the solution of:

$$-W^{a,*}_t - \mathbf{V}.\nabla W^{a,*} = g \ ; \quad -\nabla^*.\nabla W^{L,*} = g \qquad (+ \text{ final and boundary conditions}).$$

For both cases we get an estimate expressed in terms of interpolation errors $\Pi_h W - W$ weighted by derivatives of the adjoint state W^*:

$$\begin{aligned} |(g, W^a_h - \Pi_h W^a)| &\leq \| K_1(W^{a,*}, \mathbf{V})\| \, \|\Pi_h W^a - W^a\| \\ |(g, W^L_h - \Pi_h W^L)| &\leq \|K_2(W^{L,*})\| \, \|\Pi_h W^a - W^a\| \end{aligned}$$

The extension of this analysis to the models of Fluid Mechanics for compressible gas can be done in terms of interpolation errors of algebraic functions of the unknown, for example, for Euler model (E), in terms of the 15 Euler flux components:

$$|(g, W^E_h - \Pi_h W^E)| \ \leq \| \sum_{k=1}^{k=15} K^k_1(W^{E,*})\| \, \|\Pi_h \mathcal{F}^k(W^E) - \mathcal{F}^k(W^E)\|$$

The communication presents these analyses together with demonstrative results with steady and unsteady flows.

References

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