APPLICATION OF HYBRID AND VMS-LES TURBULENT MODELS TO AERODYNAMIC SIMULATIONS

A. Belme*, A. Dervieux*, B. Koobus, S. Wornom , M-V. Salvetti**
*INRIA Sophia-Antipolis, France, **University of Pisa, Italy

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Abstract

The aim of this paper is to propose a new hybrid RANS/LES approach for the simulation of turbulent flows, wich can be interpreted as a general case of the NLDE(Non-Linear Disturbance Equation) aproach. The solution of Navier-Stokes equations is decomposed into a mean part (RANS), a perturbed/corrected part that takes into account the turbulent large-scale fluctuations and a third part made by the unresolved (SGS) fluctuations. This approach is used to simulate the flow around a circular cylinder at Reynold number 140000; results are compared with experiments and with our "classical" hybrid model (see for example [11]).

1 Introduction

The numerical simulation of turbulent flows is one of the great challenges in Computational Fluid Dynamics (CFD). It is commonly accepted that the physics of the flow of a continuous fluid is well represented by the Navier-Stokes equations. The Direct Numerical Simulation (**DNS**) (for a review, see [8]) discretizes directly the three-dimensional Navier-Stokes equations. The basic requirement for such a simulation to succeed is the use of numerical schemes of high-order accuracy and meshes fine enough to capture the smallest scales of motion, to the order of the Kolmogorov scales. However, when the ratio of inertial forces to viscous ones, quantified by the Reynolds number, increases the smallest scales become smaller, and the amount of information (handled and processed) necessary for a Navier-Stokes based prediction becomes enormous.
In order to deal with the complex flows associated with higher Reynolds numbers and complex geometries, as those encountered in practical engineering applications, turbulence modeling was introduced.
One of the most widely used turbulence approach is RANS (Reynolds Averaged Navier-Stokes), in which only the time or ensemble averaged flow is solved. Due to nonlinearities, the pure mathematical averaging of the Navier-Stokes system introduces new terms. The closure of the new system needs to be obtained from phenomenological information provided by the study of simplified flows. Ensemble averaging and the need of extra information for closure are indeed two limitations of RANS. However, the RANS models made the prediction of high Reynolds number flows possible.

In the Large Eddy Simulation (LES) approach, the reduction of the simulation unknowns is obtained through the application of a spatial filter to the Navier-Stokes equations. In most cases, the filter size is strictly related to the typical size of the computational grid (grid scale). Only the set of scales larger than the grid-scale, which we also call globally “grid-scale”, is computed explicitly, while the small scales (subgrid-scale, **SGS**) are modeled.
For solving complex unsteady flows as the flow around bluff-bodies, the Large-Eddy Simulation (LES) approach gives generally more accurate predictions than the computationally cheaper RANS models, and can also deliver an increased

level of details. While RANS methods provide averaged results, LES is able to predict some instantaneous flow characteristics and to resolve important turbulent flow structures. Initiated by a few pioneering papers like [14], a new class of models has recently been proposed in the literature which combines RANS and LES approaches. The purpose is to obtain simulations as accurate as in the LES case in some part of the flow but at reasonable computational costs.

In the perspective of the simulation of massively separated unsteady flows in complex geometry, as occur in many cases of engineering or industrial interest, we are primarily interested in the so-called universal hybrid models, which should be able to automatically switch from RANS to LES throughout the computational domain. Among the universal hybrid models described in the literature, the Detached Eddy Simulation (DES) has received the largest attention. The DES approach [13] is generally based on the Spalart-Allmaras RANS model modified in such a way that far from solid walls and with refined grids, the simulation switches to the LES mode with a one-equation SGS closure. Another hybrid approach has been proposed, the *Limited Numerical Scales* (LNS) [1], in which the blending is obtained by taking the minimum of the RANS and LES eddy-viscosities. An example of validation of LNS for the simulation of bluff-body flows is given in [2].

A major difficulty in combining a RANS model with a LES one is due to the fact that the unknown of the RANS model is a mean flow, which can be in many cases a steady one. In particular, RANS does not naturally allow for time-fluctuations, due to its tendency to damp them and to "perpetuate itself", as explained in [13]. On the other hand, LES needs a significant level of fluctuations in order to accurately model the turbulent flow. The abrupt passage from a RANS region to a LES one may produce the so-called "modeled stress depletion" [13]. We examine here a more general strategy for blending RANS and LES approaches in a hybrid model [11, 10]. To this purpose, as in [7], the flow variables are decomposed in a RANS part (i.e. the averaged flow field), a correction part that takes into account the turbulent large-scale fluctuations, and a third part made of the unresolved or SGS fluctuations. The basic idea is to solve the RANS equations and to correct the obtained averaged flow field by adding the resolved fluctuations in a hybrid mode. The hybrid model involves a blending parameter which allows a smooth passing from RANS to LES.

In the case where the RANS is computed separately, the spurious influence of the LES fluctuations on the RANS mean flow is avoided. With only one field to calculate, the computational time is lower. It is then interesting to analyse the merits of the two options from the standpoint of predictivity.

This paper is organised as follows : We first give some features and the description of our hybrid model followed by a few explanations on the near wall treatment. Then we give a description of the new hybrid model as a general case of a NLDE approach, and lastly some numerical results for the flow around a circular cylinder at Reynolds number 140K will be presented.

2 Modelling

2.1 Non-Linear Disturbance Equation formulation

Following Labourasse and Sagaut [7], the following decomposition of the flow variables is adopted:

$$W = \underbrace{<W>}_{RANS} + \underbrace{W^c}_{correction} + W^{SGS}$$

where $<W>$ are the RANS flow variables, obtained by applying an averaging operator to the Navier-Stokes equations, W^c are the remaining resolved fluctuations (i.e. $<W>+W^c$ are the flow variables in LES) and W^{SGS} are the unresolved or SGS fluctuations.

Writing the Navier-Stokes equations for the averaged flow $\langle W \rangle$ and applying a filtering operator, the LES equations are obtained and we get first a closure term given by a RANS turbulence model and then a SGS term. An equation for the resolved fluctuations W^c can thus be derived (see also [7]).

The basic idea of the proposed hybrid model is to solve the equation for the averaged flow in the whole domain and to correct the obtained averaged flow by adding the remaining resolved fluctuations (computed through the equation of the resolved fluctuations), wherever the grid resolution is adequate for a LES.

To identify the regions where the additional fluctuations must be computed, we introduce a *blending function*, θ, smoothly varying between 0 and 1. When $\theta = 1$, the RANS approach is recovered, wherever $\theta < 1$, additional resolved fluctuations are computed.

Thus, the equations for the averaged flow and for the correction term in the proposed hybrid model become respectively:

$$\begin{aligned} &\frac{\partial \langle W \rangle}{\partial t} + \nabla \cdot F_c(\langle W \rangle) + \\ &\nabla \cdot F_v(\langle W \rangle) = -\tau^{RANS}(\langle W \rangle) \end{aligned} \tag{1}$$

$$\begin{aligned} &\frac{\partial W^c}{\partial t} + \nabla \cdot F_c(\langle W \rangle + W^c) - \\ &\nabla \cdot F_c(\langle W \rangle) + \nabla \cdot F_v(W^c) = \\ &(1-\theta)\left[\, \tau^{RANS}(\langle W \rangle) - \tau^{LES}(\langle W \rangle + W^c)\right] \end{aligned} \tag{2}$$

where $\tau^{RANS}(\langle W \rangle)$ is the closure term given by a RANS turbulence model, and $\tau^{LES}(W)$ is given by one of the SGS closures described in [12] [15] [9].

To avoid the solution of two different systems of PDEs and the consequent increase of required computational resources, Eqs. (1) and (2) can be recast together in a more classical way as:

$$\begin{aligned} &\frac{\partial W}{\partial t} + \nabla \cdot F_c(W) + \nabla \cdot F_v(W) = \\ &-\theta\, \tau^{RANS}(\langle W \rangle) - (1-\theta)\tau^{LES}(W) \end{aligned} \tag{3}$$

where W stands now for $\langle W \rangle + W^c$.

Clearly, if only Eq. (3) is solved, $\langle W \rangle$ is not available at each time step. Two different options are possible: either to use an approximation of $\langle W \rangle$ obtained by averaging and smoothing of W, in the spirit of VMS, or to simply use in Eq. (3) $\tau^{RANS}(W)$. This second option has been firstly tested by our team.
The novelty we bring up today is to simply use two different systems connected with each other. The RANS system (1) is firstly solved, followed by the hybrid (2) one. This involves the resolution of two different systems at each time step.

2.2 Hybridisation

The most popular option in hybridisation consists in identifying a region of the computational domain, generally a close neighborhood of the wall, where the RANS model is applied, the rest of the domain being treated with the LES model (see the DES litterature). In the other main option, the decision will depend on the comparison of characteristic scales of both models. For example, in the LNS method, the minimum of the two turbulent viscosities is chosen.

As a possible choice for θ, the following function is used in the present study:

$$\theta = F(\xi) = tanh(\xi^2) \tag{4}$$

where ξ is the *blending parameter*, which should indicate whether the grid resolution is fine enough to resolve a significant part of the turbulence fluctuations, i.e. to obtain a LES-like simulation. The choice of the *blending parameter* is clearly a key point for the definition of the present hybrid model. In the present study, the RANS model is of $k-\varepsilon$ type [3]. Different options are proposed and investigated, namely: the ratio between the eddy viscosities given by the LES and the RANS closures, $\xi_{VR} = \mu_s/\mu_t$, which is also used as a blending parameter in LNS [1], and $\xi_{LR} = \Delta/l_{RANS}$, l_{RANS} being a typical length in the RANS approach, i.e. $l_{RANS} = k^{3/2}\varepsilon^{-1}$ and, Δ measures the local mesh size.

A few words about the $k-\varepsilon$ we use. For flows with complex geometry, we found it preferable to build a formulation that does not need the distance to the wall. The low Reynolds $k-\varepsilon$ formulation of [4, 3] enjoys this property. Furthermore, this low Reynolds k-eps model was designed to improve the prediction of the standard k-eps one for adverse pressure gradient flows, including separated flows. In order to get a robust formulation applicable to high Reynolds number, we combine it with Reichardt's wall law which takes into account the whole boundary layer (see for example [5]).

2.3 Variational Multiscale LES modelling

For the LES mode, we consider the Variational Multi-Scale approach, in which the flow variables are decomposed as $W = \overline{W} + W'$, where $\overline{W}$ are the large resolved scales (LRS) and W' are the small resolved scales (SRS). We follow here the VMS approach proposed in [6] for the simulation of compressible turbulent flows through a finite volume/finite element discretization on unstructured tetrahedral grids. In order to obtain the VMS flow decomposition, basis and test functions can be expressed as: $\chi_l = \overline{\chi}_l + \chi'_l$ and $\phi_l = \overline{\phi}_l + \phi'_l$, in which the overbar denotes the basis functions spanning the finite dimensional spaces of the large resolved scales and the prime those spanning the SRS spaces. As in [6], the basis functions of the LRS space are defined through a projector operator in the LRS space, based on spatial average on macro cells, which are obtained by an agglomeration process. Finally, a key feature of the VMS approach is that the SGS model is added only to the smallest resolved scales. Eddy-viscosity models are used here, and, hence, the SGS terms are discretized analogously to the viscous fluxes. In this context, the Galerkin projection for the computation of the LES approximation W^{VMS} is:

$$\begin{array}{l}\left(\dfrac{\partial W^{VMS}}{\partial t},\chi_l\right) + \left(\nabla\cdot F_c(W^{VMS}),\chi_l\right) + \\ \left(\nabla\cdot F_v(W^{VMS}),\phi_l\right) = -\left(\tau^L(W'),\phi'_l\right) \\ \quad l = 1,N\end{array} \tag{5}$$

For defining more precisely the expression of τ^L in Eq.(7), three different eddy-viscosity models have been considered, namely those proposed by Smagorinsky [12] and Vreman [15] and the so-called WALE model [9]. The eddy-viscosity introduced by these models, within the VMS approach, is computed as a function of the SRS flow variables, and the filter width Δ has been selected as the cubic root of the volume of each tetrahedron. Finally, the model constant has been set equal to 0.1 for the Smagorinsky and WALE models and to 0.158 for the Vreman one.

2.4 Global formulation

Thus, the Galerkin projection of the equations for the averaged flow and for the correction term in the proposed hybrid model become respectively:

$$\begin{array}{l}\left(\dfrac{\partial\langle W\rangle}{\partial t},\psi_l\right) + (\nabla\cdot F_c(\langle W\rangle),\psi_l) \\ +(\nabla\cdot F_v(\langle W\rangle),\phi_l) = -\left(\tau^{RANS}(\langle W\rangle),\phi_l\right) \\ \quad l = 1,N\end{array} \tag{6}$$

$$\begin{array}{l}\left(\dfrac{\partial W^c}{\partial t},\psi_l\right) + (\nabla\cdot F_c(\langle W\rangle + W^c),\psi_l) - \\ (\nabla\cdot F_c(\langle W\rangle),\psi_l) + (\nabla\cdot F_v(W^c),\phi_l) = \\ (1-\theta)\left[\left(\tau^{RANS}(\langle W\rangle),\phi_l\right) - \left(\tau^{LES}(W'),\phi'_l\right)\right] \\ \quad l = 1,N\end{array} \tag{7}$$

where $\tau^{RANS}(\langle W\rangle$ is the closure term given by a RANS turbulence model, W' and ϕ'_l denote the small resolved component of $\langle W\rangle + W^c$ and ϕ_l, and $\tau^{LES}(W')$ is given by one of the SGS closures.

3 Numerical Results

In this numerical part, we evaluate the performance of our new hybrid model for the simulation on unstructured grids of the flow around a circular cylinder. The obtained numerical results are contrasted with those predicted by RANS and various hybrid models, and compared with experimental data.

Flow around a circular cylinder (Hybrid RANS/LES) The new proposed hybrid model (*Fluctuation Correction Model*, FCM) has been implemented in our CFD software AERO, on a MPI (message passing interface) parallel platform, in Fortran 95 language, and applied to the simulation of the flow around a circular cylinder at $Re = 140000$ (based on the far-field velocity and the cylinder diameter). The considered mesh is unstructured, tetrahedral, rather coarse with 458K vertices.

Spacial discretization of three-dimensional unsteady Navier-Stokes equations is based on a mixed finite volume/element formulation and high-order accuracy is obtained with the MUSCL scheme. A first-order implicit scheme is used for time advancing with a CFL condition number going up to 100.

The V6 scheme has been used and the numerical parameter γ_s, which controls the amount of numerical viscosity introduced in the simulation, has been set equal to 0.3.

Figure 3 shows the pressure distribution on the cylinder surface averaged in time on homogeneous z direction for the new proposed hybrid model, compared with the RANS and hybrid RANS-LES [11] and the experiments of Jones.

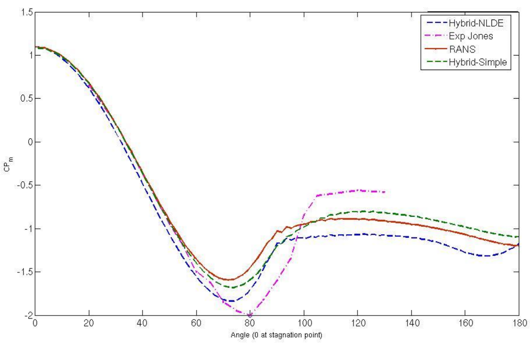

Fig. 1 Time-averaged and z-averaged pressure distribution on the surface of the cylinder, experiment: Jones

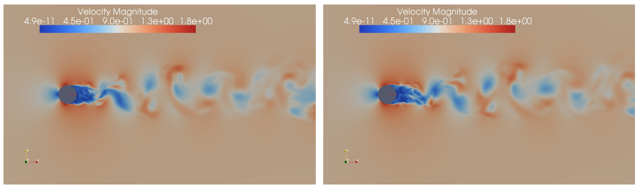

Fig. 2 Two-Field model: velocity fields for hybrid double field approach (left) and for the classical hybrid model based on one equation (right)

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