Dynamique et modélisation de la Turbulence

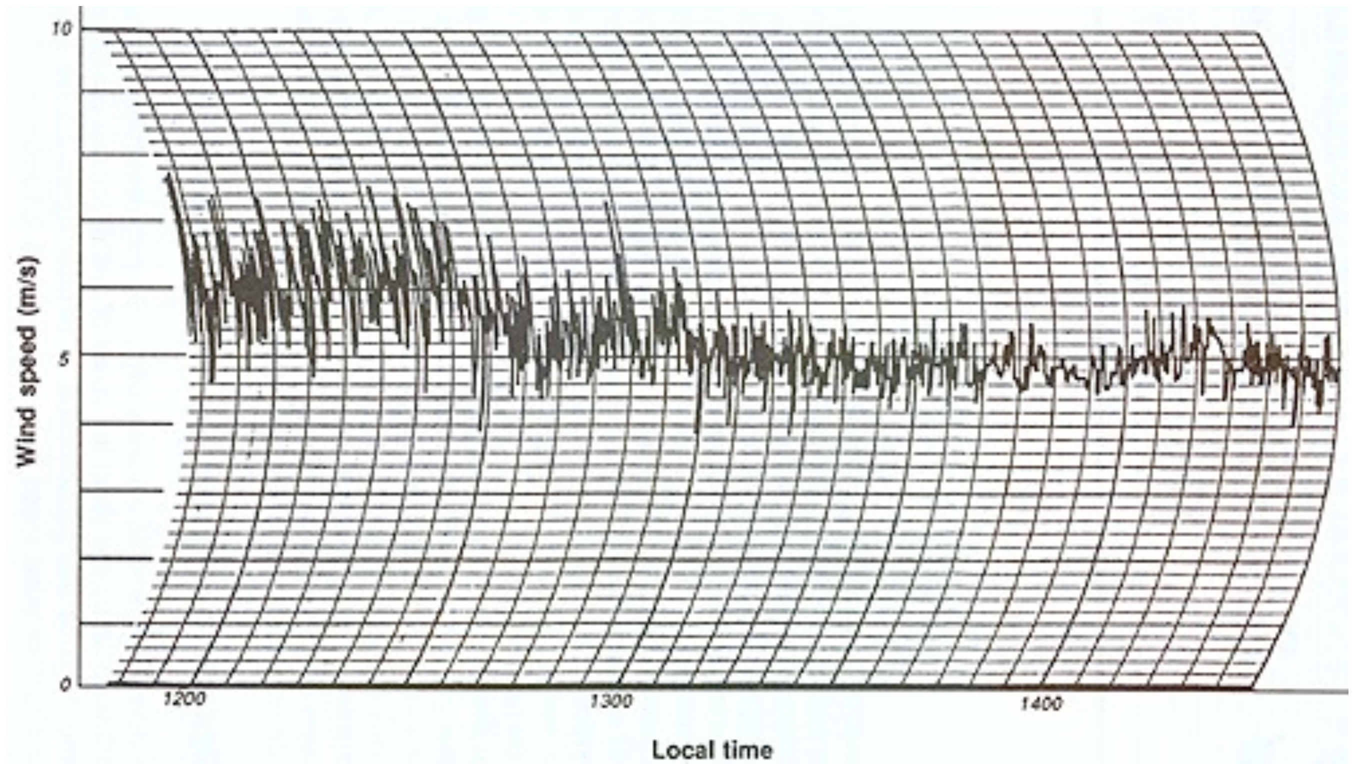
- Cours 3

sergio.chibbaro@upmc.fr

SB Pope, «Turbulent flows» Cambridge University Press

Statistical description

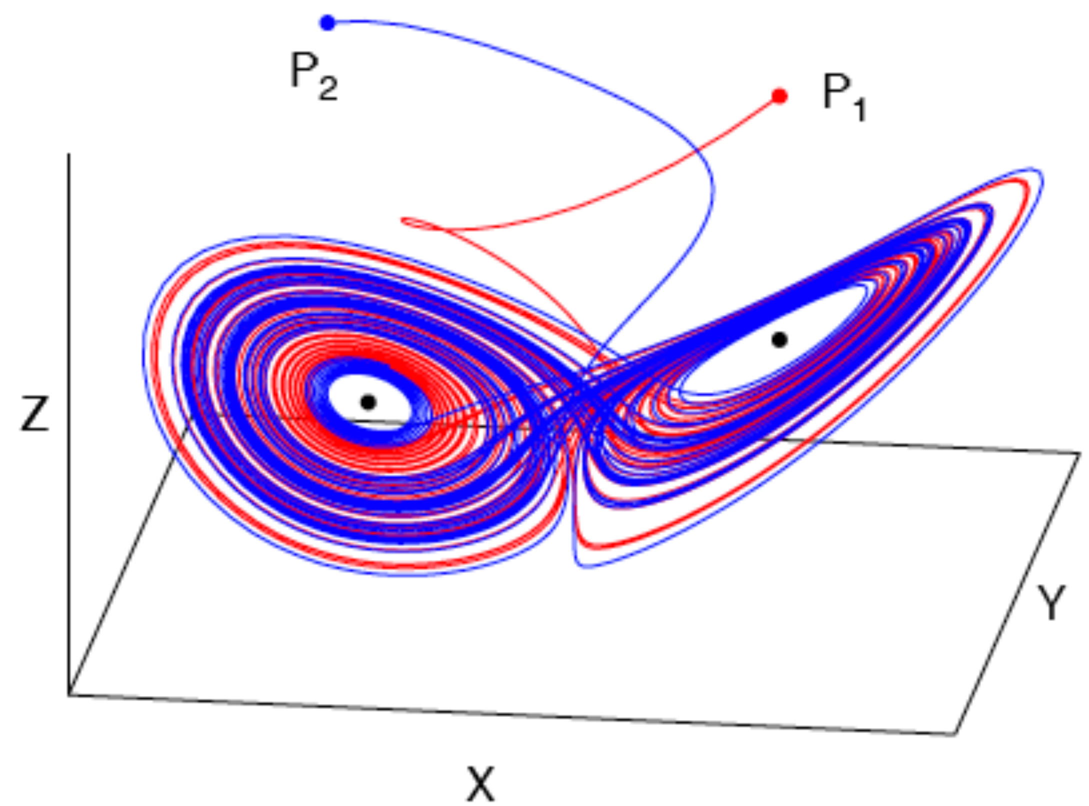
- Turbulent flows are (by definition)
  - –Highly complex
  - –Unsteady
  - –Chaotic



- --> a deterministic description/analysis is not relevant
- --> a statistical representation is required

Deterministic chaos

$$\frac{\mathrm{d}X}{\mathrm{d}t} = -\sigma X + \sigma Y$$
$$\frac{\mathrm{d}Y}{\mathrm{d}t} = -XZ + rX - Y$$
$$\frac{\mathrm{d}Z}{\mathrm{d}t} = XY - bZ\,.$$



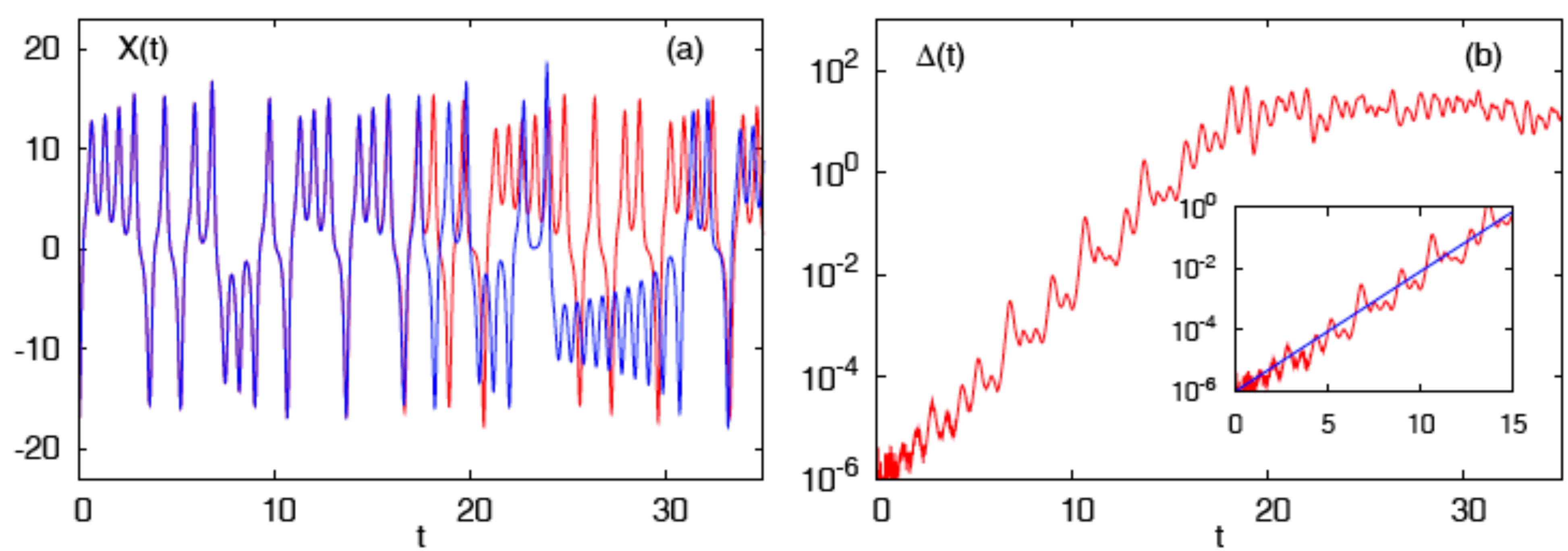


Fig. 3.7 Lorenz model: (a) evolution of reference $X(t)$ (red) and perturbed $X'(t)$ (blue) trajectories, initially at distance $\Delta(0) = 10^{-6}$. (b) Evolution of the separation between the two trajectories. Inset: zoom in the range $0 < t < 15$ in semi-log scale. See text for explanation.

Probability density function

- Prop: Let P be the pdf of a, then

$$\langle a \rangle (x,t) = \int_{-\infty}^{+\infty} u P(u) du$$

with

$$\int_{-\infty}^{+\infty} P(u) du = 1$$

Correlation function

- Def: the correlation function of $a_1(X_1)$ and $a_2(X_2)$ is defined as

$$R_{a_1a_2}(X_1,X_2) = \int_{-\infty}^{+\infty} [u - \langle a_1 \rangle (X_1)] [v - \langle a_2 \rangle (X_2)] P_{X_1X_2}(u,v)dudv$$

Ensemble average

- Def: Let ϕ be a random variable. Its ensemble averaged value is defined as

$$\bar{\phi}_i(\underline{x},t) \equiv \lim_{p \longrightarrow +\infty} \frac{1}{p} \left(\sum_{k=1,p} \phi_i^{(k)}(\underline{x},t) \right)$$

Where ϕ_i denotes independent realizations

Second order moments

Variance

$$\overline{\phi'_i\phi'_i}(\underline{x},t) = \lim_{p\longrightarrow+\infty}\frac{1}{p}\left(\sum_{k=1,p}(\phi_i^{(k)}(\underline{x},t)-\bar{\phi}_i(\underline{x},t))(\phi_i^{(k)}(\underline{x},t)-\bar{\phi}_i(\underline{x},t))\right)$$

2-points 2-times correlation

$$\begin{aligned}
\overline{\phi'_i\phi'_i}(\underline{x},\underline{y},t,t') &\equiv \overline{\phi'_i(\underline{x},t)\phi'_i(\underline{y},t')} \\
&= \overline{\phi'_i(\underline{x},t)\phi'_i(\underline{x}+\underline{r},t+\tau)} \\
&= \overline{\phi'_i\phi'_i}(\underline{x},\underline{r},t,\tau) \\
&= \lim_{p\longrightarrow+\infty}\frac{1}{p}\left(\sum_{k=1,p}(\phi_i^{(k)}(\underline{x},t)-\bar{\phi}_i(\underline{x},t))(\phi_i^{(k)}(\underline{y},t')-\bar{\phi}_i(\underline{y},t'))\right)
\end{aligned}$$

Usual 2nd-order moments

2-points velocity correlation tensor

$$R_{ij}(\underline{x}, \underline{r}, t) \equiv \overline{u'_i(\underline{x}, t) u'_j(\underline{x} + \underline{r}, t)}$$

2-points passive scalar correlation tensor

$$R_T(\underline{x}, \underline{r}, t) \equiv \overline{T'(\underline{x}, t) T'(\underline{x} + \underline{r}, t)}$$

Some important families

- Def: a random process is referred to as
  - Statistically **steady** if *moments* do not depend on time
  - Statistically **homogeneous** if *moments* are invariant by a shift in space
- Def: a random process is referred to as **isotropic** if
  - It is statistically homogeneous
  - *Moments* are invariant by rotation and mirror symmetry in space

Isotropic case

- In the isotropic case, one obtains

$$R_{ab}(M, M') = R_{ab}(|x - x'|, t, t')$$

- In the steady isotropic case, the problem is further reduced as

$$R_{ab}(M, M') = R_{ab}(|x - x'|, \tau)$$