Dynamique et modélisation de la Turbulence

- Cours 4

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SB Pope, «Turbulent flows» Cambridge University Press

4 axioms of O. Reynolds

The averaging operator (denoted by a bar) must fulfill th following constraints:

1. Linearity

$$\overline{a+b} = \bar{a} + \bar{b}, \quad \overline{\lambda a} = \lambda \bar{a}$$

2. Constant preservation

$$\bar{\lambda} = \lambda$$

3. Commutation with derivatives in *x*,*t*

$$\partial_x \overline{a} = \overline{\partial_x a}, \quad \partial_t \overline{a} = \overline{\partial_t a}$$

4. Be a projector

$$\overline{\bar{a}\bar{b}} = \bar{a}\bar{b}$$

Reynolds decomposition

- Turbulent variable splitting (velocity, pressure, …)

$$a(x,t) = \bar{a}(x,t) + a'(x,t)$$

« mean flow »

« turbulent » or
« fluctuating » flow

- Properties:

$$\begin{aligned} \bar{\bar{a}} &= \bar{a} \\ \overline{a'} &= 0 \\ \overline{\bar{a} b'} &= 0 \end{aligned}$$

Pdf-related parameters

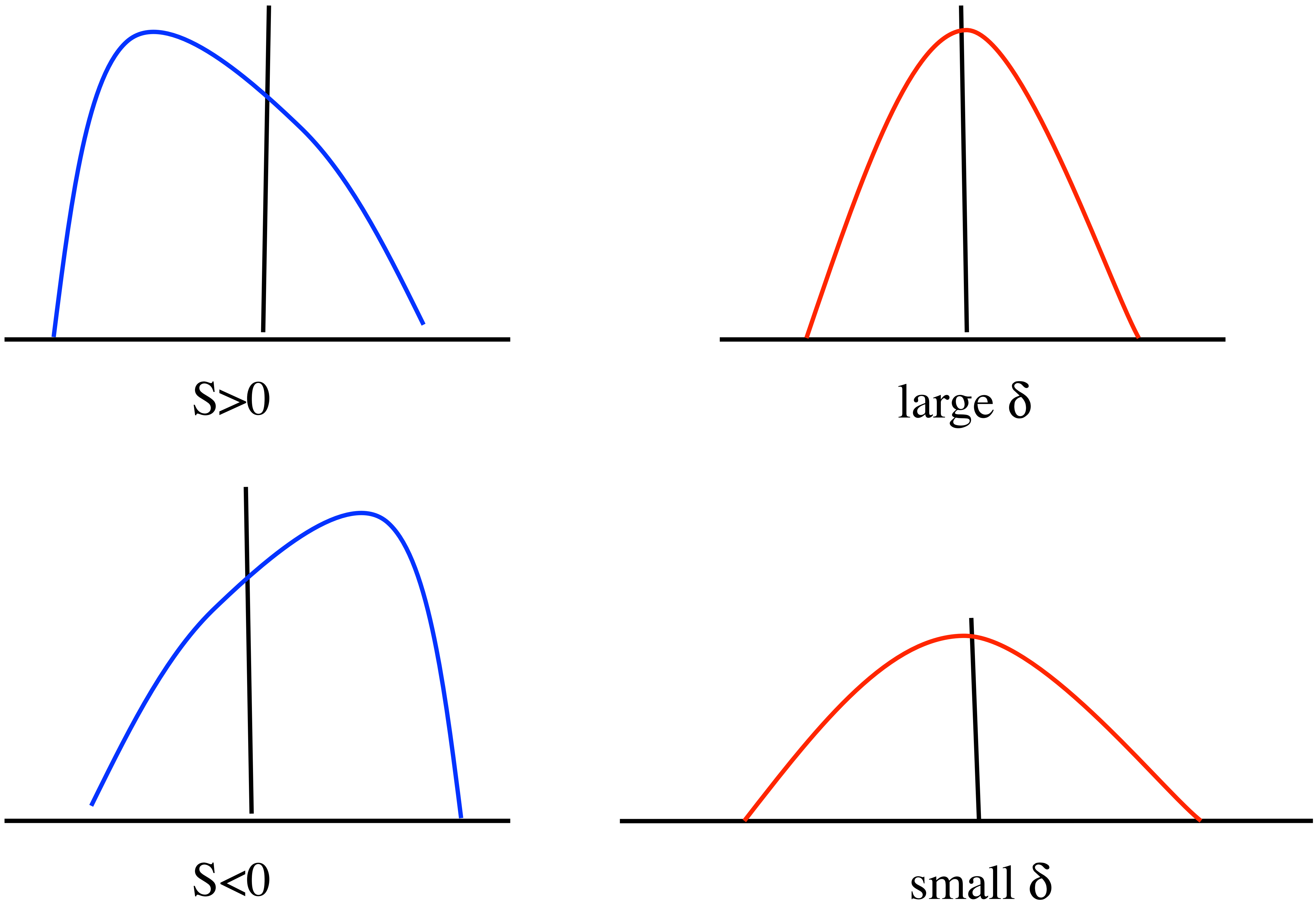
- Skewness factor

$$S = \frac{\left\langle a'^3 \right\rangle}{\left\langle a'^2 \right\rangle^{3/2}}$$

- Flatness factor

$$F = \frac{\left\langle a'^4 \right\rangle}{\left\langle a'^2 \right\rangle^{2}}$$

Rem: Gaussian process, S=0 and F=3


$S>0$
large δ
$S<0$
small δ

Governing equations in physical space

Framework:

- Incompressible flow
- Single phase flow
- Single species flow
- Newtonian flow
- --> hyp: the Navier-Stokes equations hold (rem: Clay prize)

$$\boxed{\begin{aligned} &\partial_t u_i + \partial_j (u_i u_j) = -\partial_i p + \nu \partial^2_{jj} u_i \\ &\partial_i u_i = 0 \end{aligned}}$$

Mean flow equations

Applying the filtering operator to NS eqs., one obtains

$$\partial_t \bar{u}_i + \partial_j(\overline{u_i u_j}) = -\partial_i \bar{p} + \nu \partial^2_{jj} \bar{u}_i \qquad \partial_i \bar{u}_i = 0$$

Reynolds decomposition: $\overline{u_i u_j} = \overline{(\bar{u}_i + u'_i)(\bar{u}_j + u'_j)} = \bar{u}_i \bar{u}_j + R_{ij}$

$$\boxed{\partial_t \bar{u}_i + \partial_j(\bar{u}_i \bar{u}_j + R_{ij}) = -\partial_i \overline{p} + \nu \partial^2_{jj} \bar{u}_i \qquad \partial_i \bar{u}_i = 0}$$

Coupling term between mean flow and turbulence

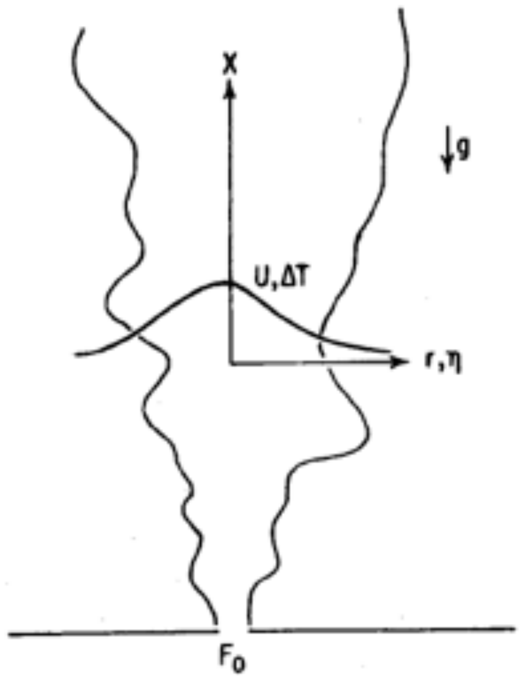
Fluctuating field equations

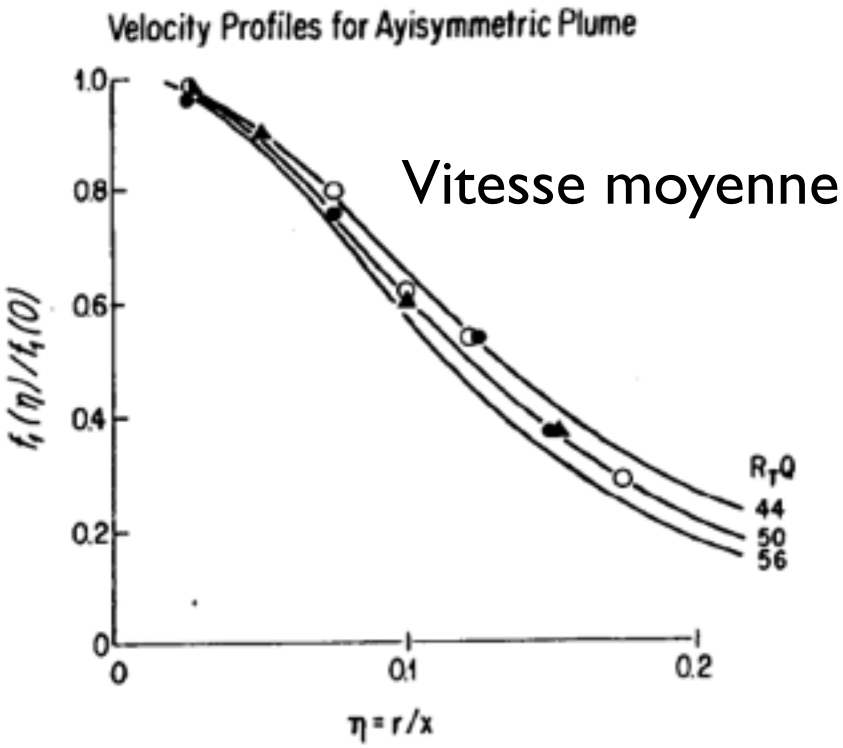
Substracting averaged momentum eq. from the original one, one has

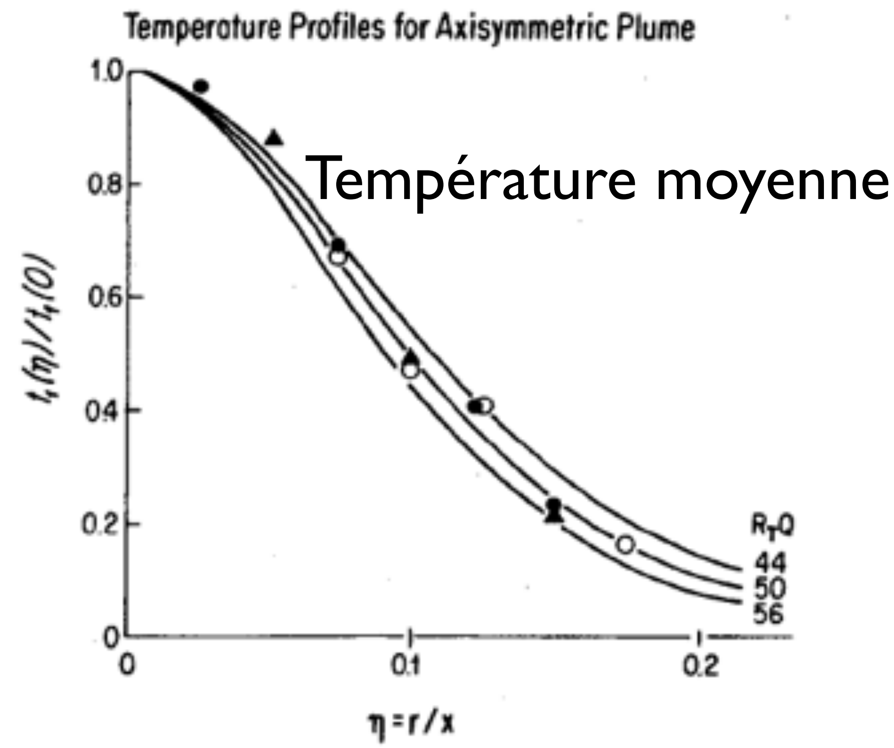
$$\boxed{\begin{aligned} &\partial_t u_i' + \partial_j(u_i u_j - \overline{u_i u_j}) = -\partial_i p' + \nu \partial^2_{jj} u_i' \\ &\partial_i u_i' = 0 \end{aligned}}$$

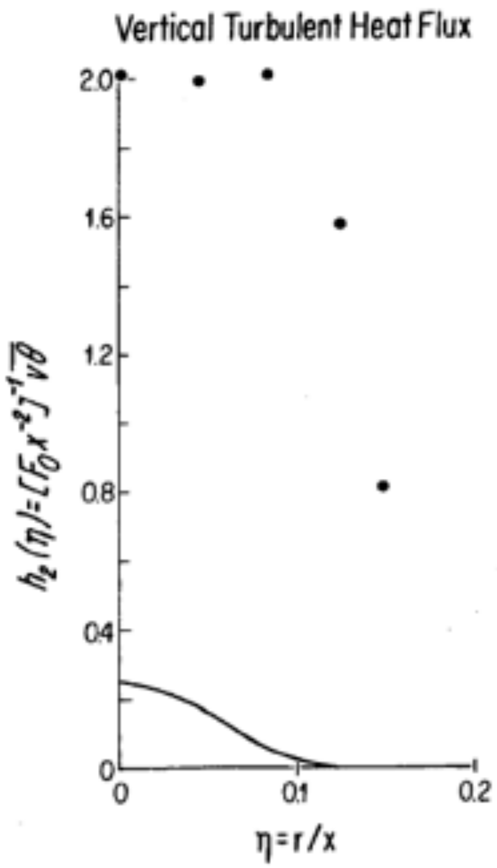
Rem: the nonlinear term can be further decomposed as follows

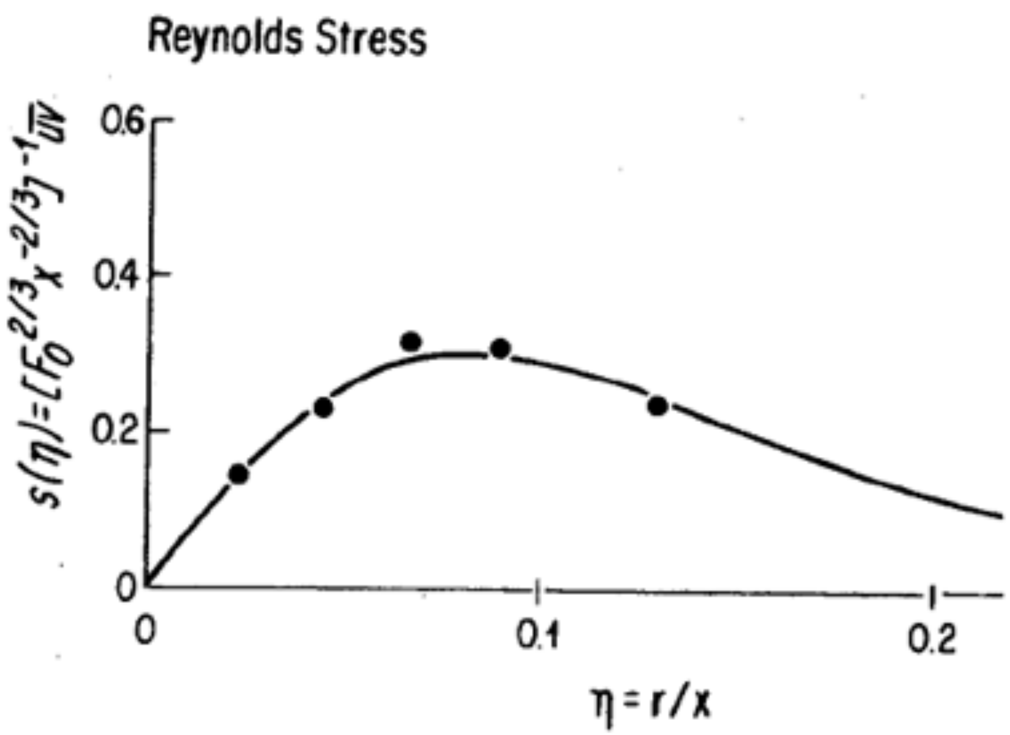
$$u_i u_j - \overline{u_i u_j} = u_i u_j - \bar{u}_i \bar{u}_j - R_{ij} = \bar{u}_i u_j' + u_i' \bar{u}_j + u_i' u_j' - R_{ij}$$

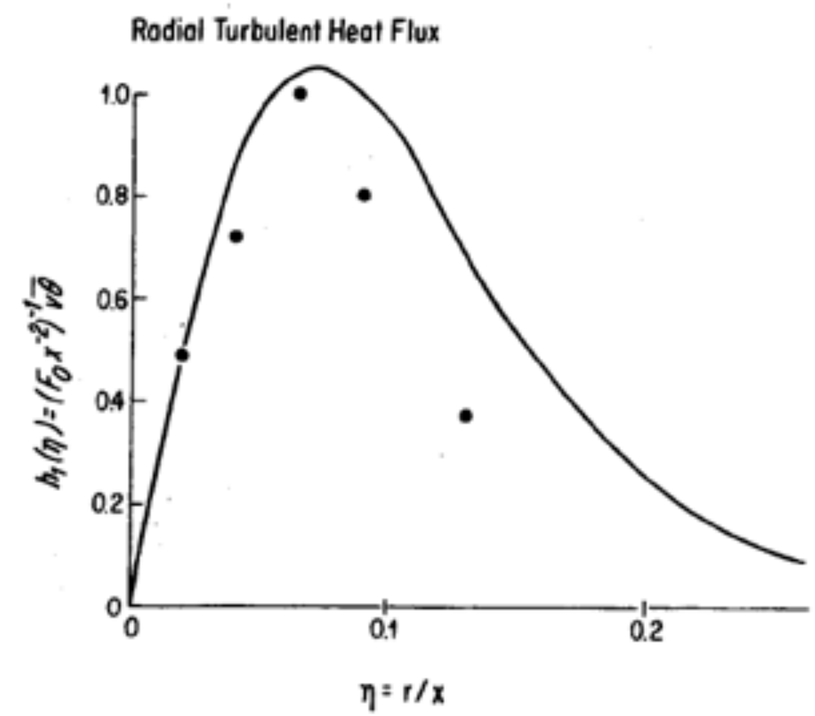

x
g
U, ΔT
r, η
F0


Velocity Profiles for Ayisymmetric Plume
Vitesse moyenne
f1(η)/f1(0)
RTQ
44
50
56
η = r/x


Temperature Profiles for Axisymmetric Plume
Température moyenne
t1(η)/t1(0)
RTQ
44
50
56
η = r/x


Vertical Turbulent Heat Flux
h2(η)=[F0x^-2]^-1 vθ
η = r/x


Reynolds Stress
s(η)=[F0^2/3 x^-2/3]^-1 uv
η = r/x


Radial Turbulent Heat Flux
h1(η)=(F0x^-2)^-1 vθ
η = r/x

Reynolds stress budget

Noticing that $\partial_t R_{ij} = \overline{u'_j \partial_t u'_i + u'_i \partial_t u'_j}$ one obtains

$$\partial_t R_{ij} + \partial_k(\bar{u}_k R_{ij}) = -(R_{ik}\partial_k \bar{u}_j + R_{jk}\partial_k \bar{u}_i)$$

Mean flow advection

Transfer term (production)

$$-\partial_k(\overline{u'_i u'_j u'_k}) - (\overline{u'_i \partial_j p'} + \overline{u'_j \partial_i p'})$$

Turbulent diffusion

Pressure power

$$+\nu \partial^2_{kk} R_{ij} - 2\nu \overline{\partial_k u'_i \partial_k u'_j}$$

Viscous diffusion

dissipation

Mean kinetic energy budget

defining $E = \frac{1}{2}\bar{u}_i\bar{u}_i$ the averaged momentum equation yields

$$\partial_t E + \partial_j(\bar{u}_j E) = \bar{u}_i f_i - \partial_i(\bar{p}\bar{u}_i)$$

Pressure diffusion

External force power

Advection along
Mean flow streamlines

$$+\partial_j(2\nu\bar{S}_{ij}\bar{u}_i) - \partial_j(\bar{u}_i R_{ij})$$

Viscous diffusion Turbulent diffusion

$$-2\nu\bar{S}_{ij}\bar{S}_{ij} + R_{ij}\partial_j\bar{u}_i$$

dissipation transfer term

$$\boxed{\bar{S}_{ij} = \frac{1}{2}(\partial_i\bar{u}_j + \partial_j\bar{u}_i)}$$

Turbulent kinetic energy budget

Denoting $K = \frac{1}{2}R_{ii} = \frac{1}{2}\overline{u_i' u_i'}$ and $S_{ij}' = \frac{1}{2}(\partial_i u_j' + \partial_j u_i')$

$$\partial_t K + \partial_k(\bar{u}_k K) = -R_{ij}\partial_j \bar{u}_i$$

Mean flow advection — Transfer term (production)

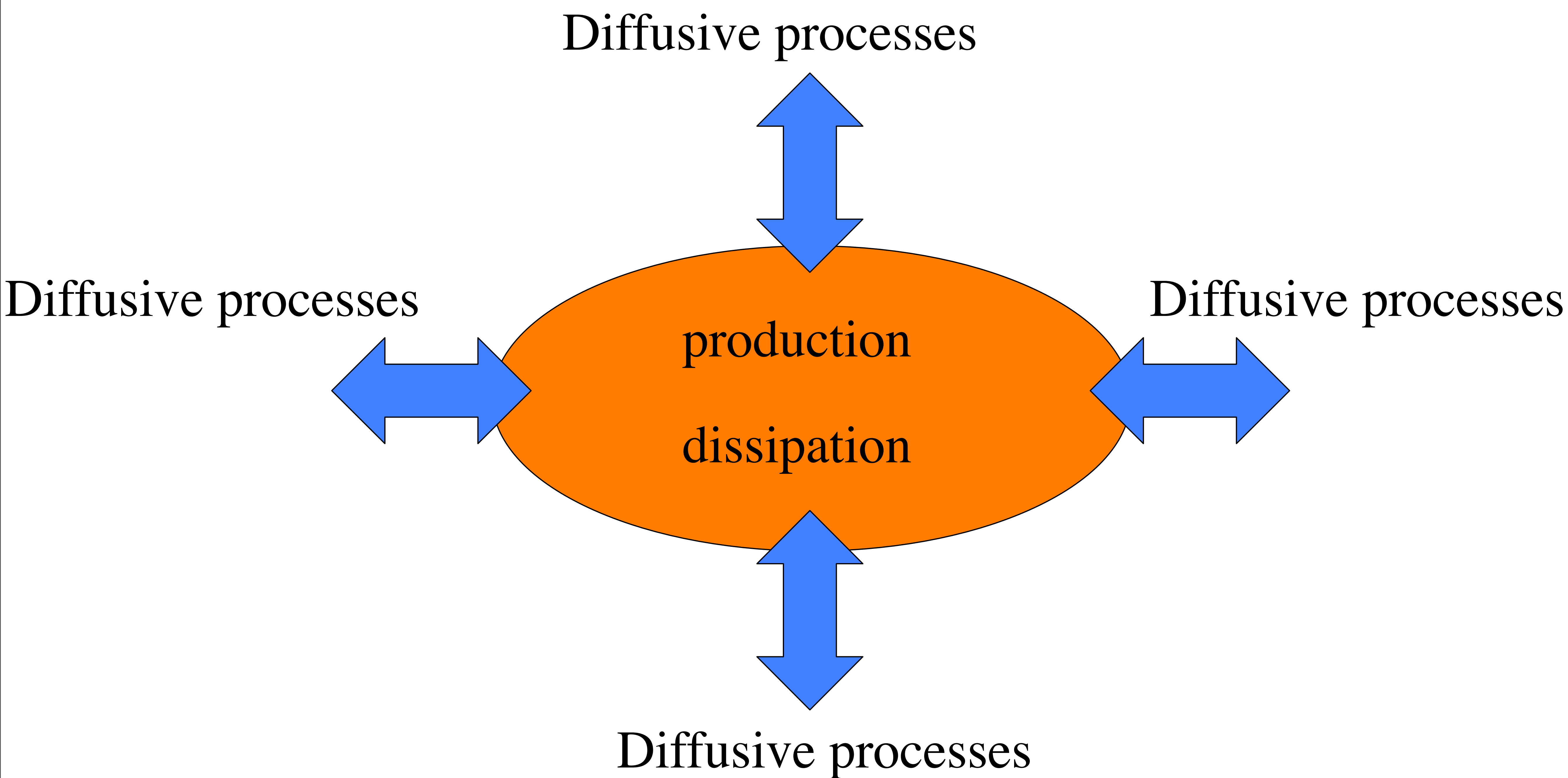
$$-\frac{1}{2}\partial_k(\overline{u_i' u_i' u_k'}) - \partial_i \overline{u_i' p'}$$

Turbulent diffusion — Pressure diffusion

$$+2\nu\partial_j \overline{S_{ij}' u_i'} - 2\nu \overline{S_{ij}' S_{ij}'}$$

Viscous diffusion — dissipation

Control volume budget



Description level hierarchy

Microscopic level:
- Boltzmann equation
- Discrete description: particule transport & collision

Local statistical average

Macroscopic level:
- Navier-Stokes equations
- Continuum mechanics: velocity, pressure, viscosity, fluxes

Reynolds statistical average

Statistical description of turbulence
- Reynolds-averaged Navier-Stokes equations
- New fluxes induced by turbulent motion