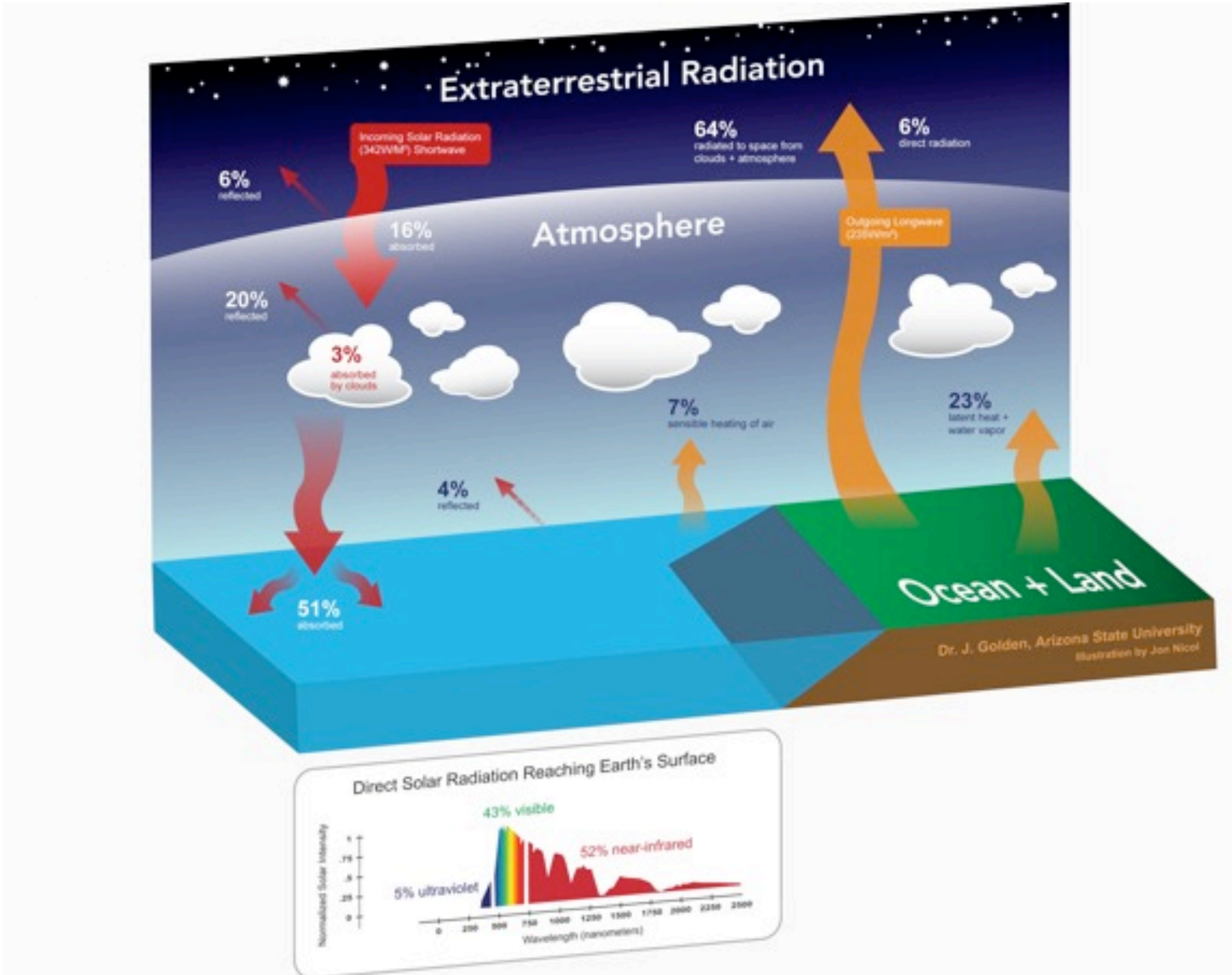
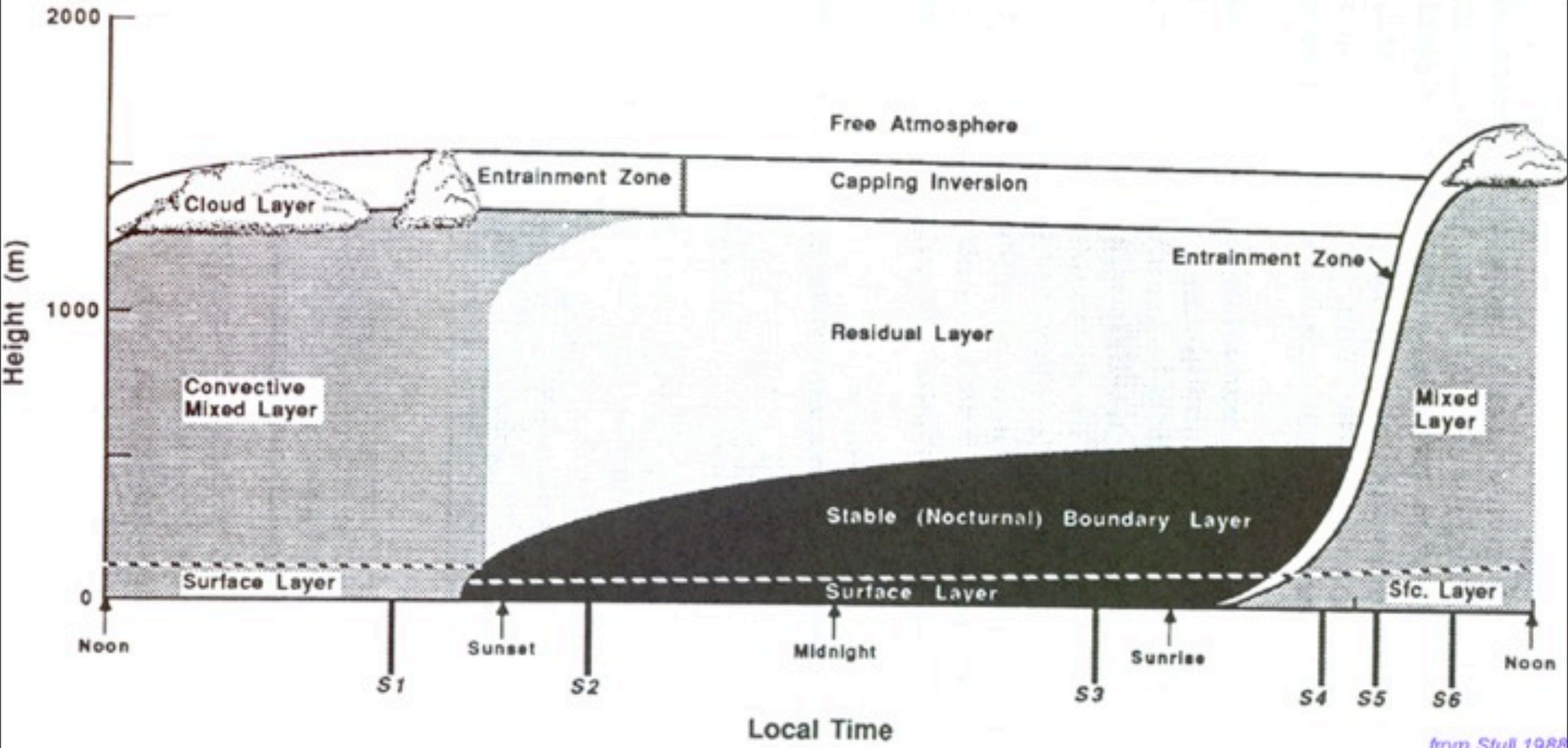
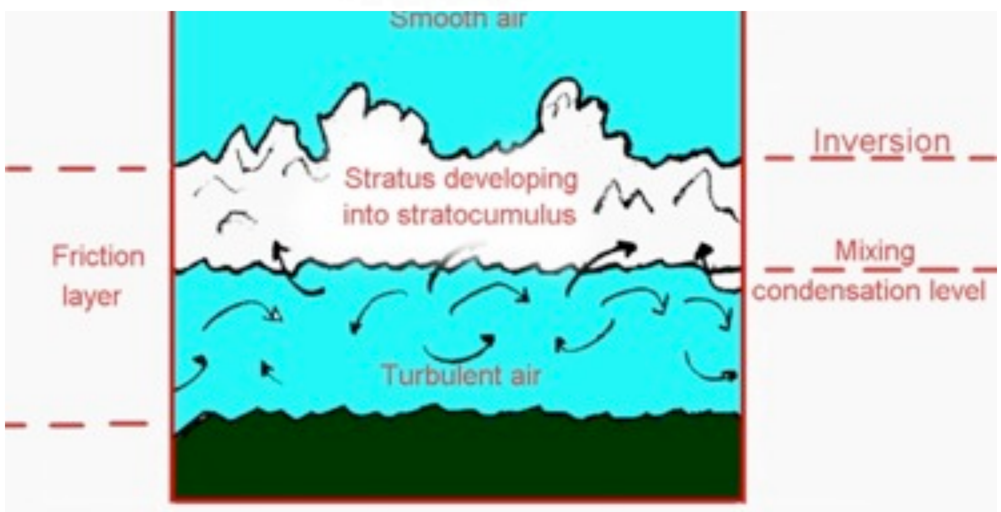
Turbulent boundary layer

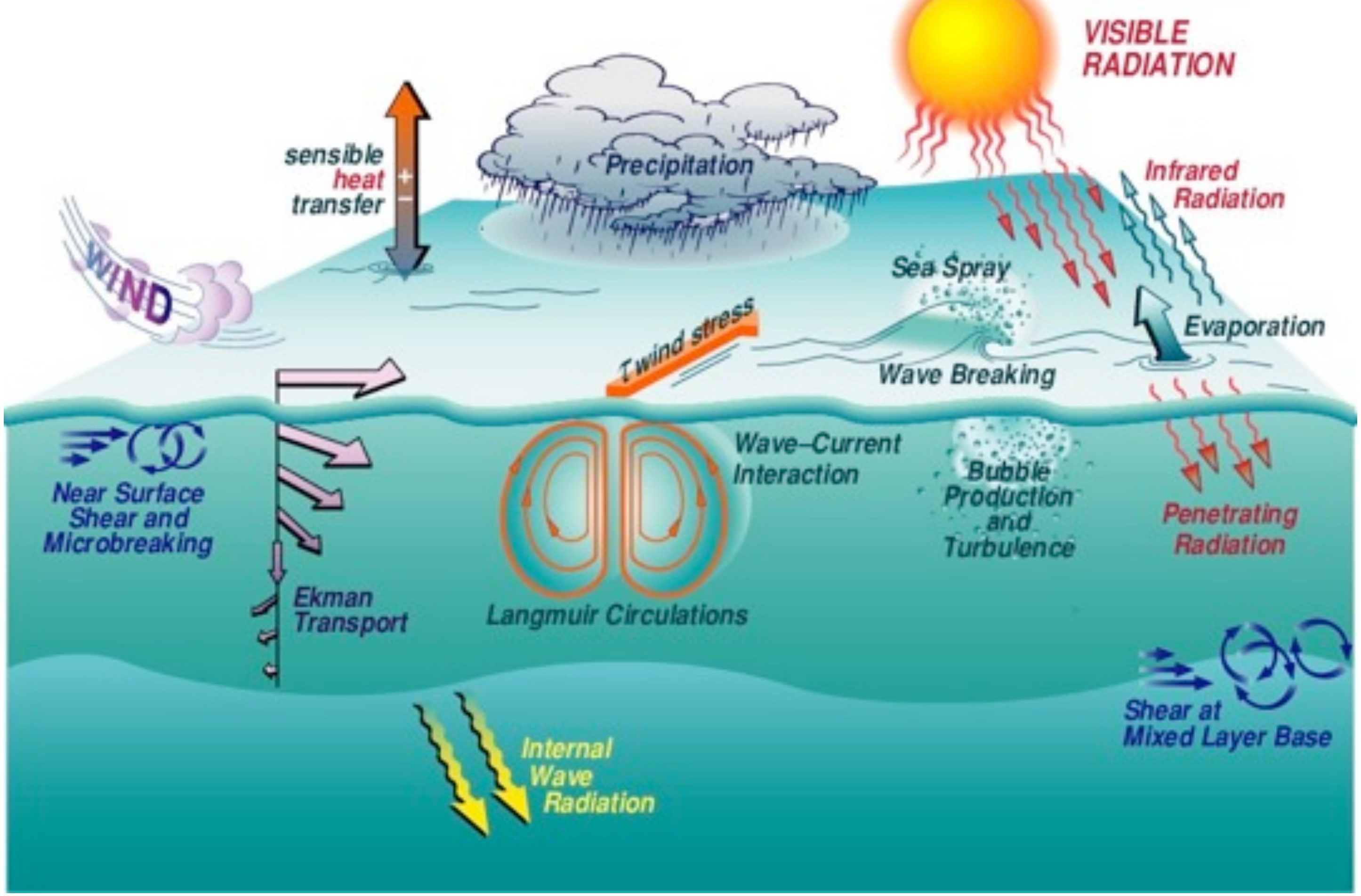
0. Are they so different from laminar flows ?
1. Three main effects of a solid wall
2. Statistical description: equations & results
3. Mean velocity field: classical asymptotic theory
4. Rugosity
5. Coherent structures & turbulence dynamics
6. Turbulent drag: generation & control

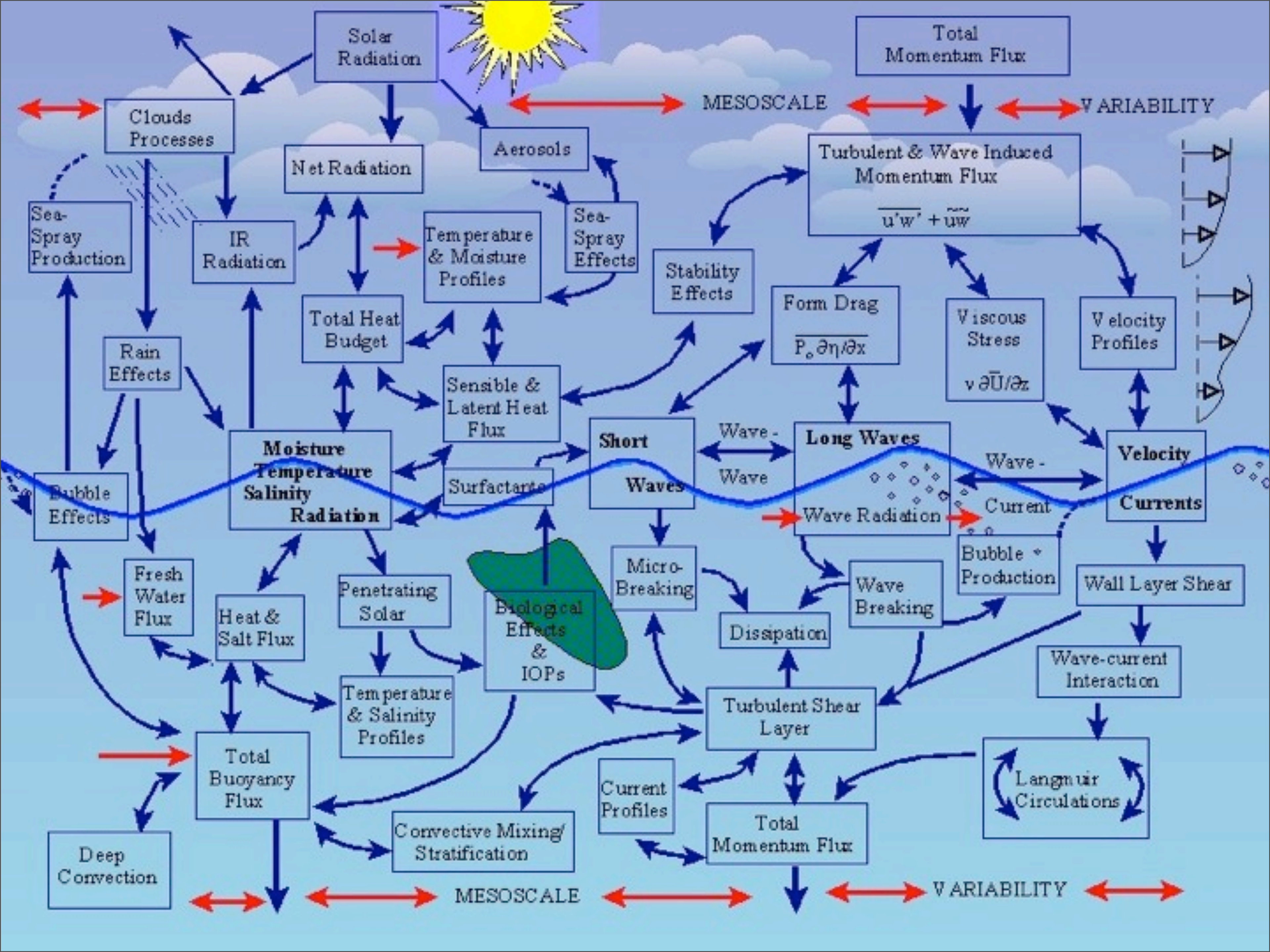
Example: Planetary Boundary Layer




2000
1000
0
Height (m)
Free Atmosphere
Entrainment Zone
Capping Inversion
Cloud Layer
Entrainment Zone
Residual Layer
Convective Mixed Layer
Mixed Layer
Stable (Nocturnal) Boundary Layer
Surface Layer
Surface Layer
Sfc. Layer
Noon
Sunset
Midnight
Sunrise
Noon
S1
S2
S3
S4
S5
S6
Local Time
from Stull 1988

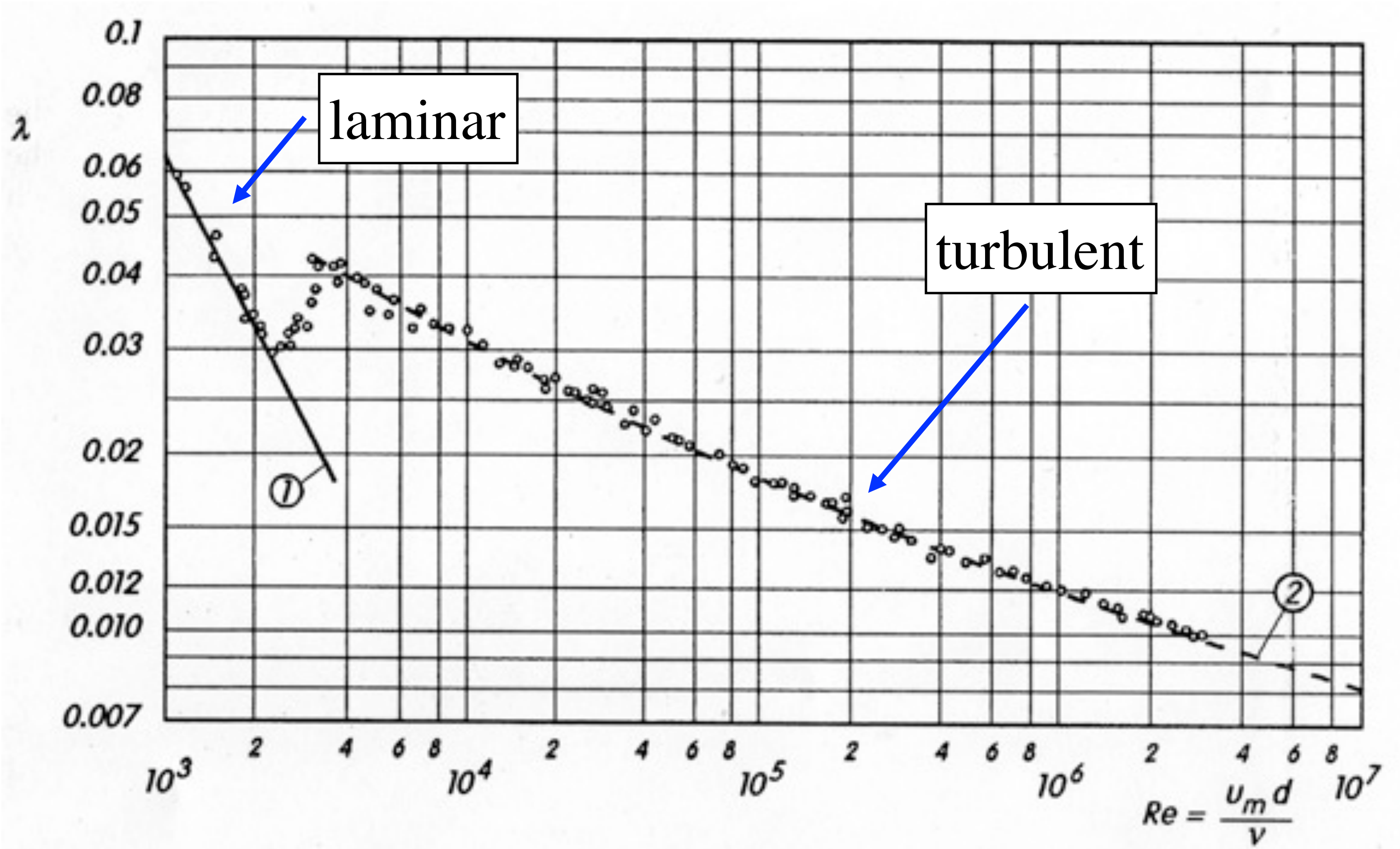

Smooth air
Inversion
Stratus developing into stratocumulus
Friction layer
Mixing condensation level
Turbulent air


VISIBLE RADIATION
sensible heat transfer
Precipitation
Infrared Radiation
WIND
Sea Spray
Evaporation
τ wind stress
Wave Breaking
Wave–Current Interaction
Near Surface Shear and Microbreaking
Bubble Production and Turbulence
Penetrating Radiation
Ekman Transport
Langmuir Circulations
Shear at Mixed Layer Base
Internal Wave Radiation



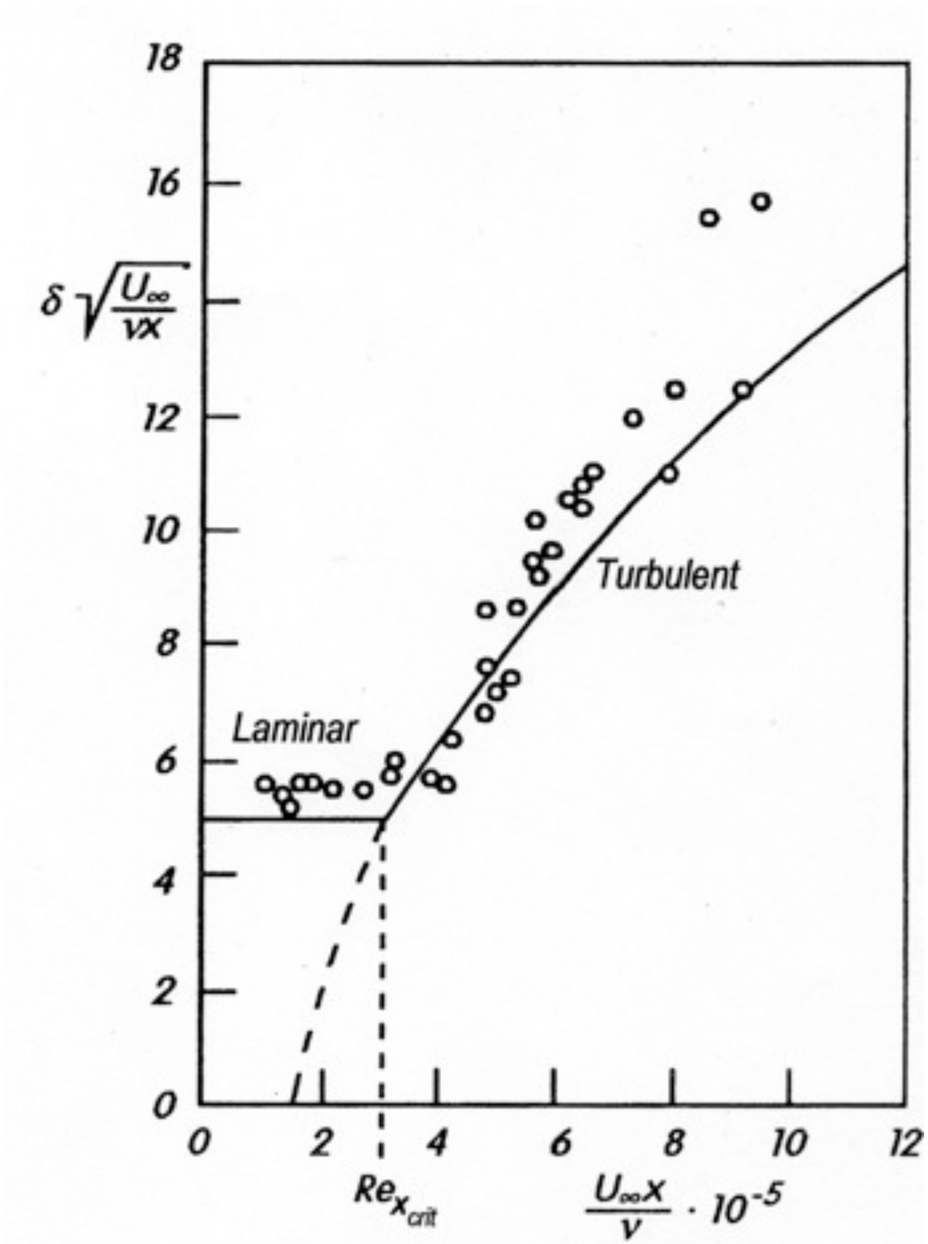
Solar Radiation
Total Momentum Flux
MESOSCALE
VARIABILITY
Clouds Processes
Net Radiation
Aerosols
Turbulent & Wave Induced Momentum Flux
$\overline{u'w'} + \overline{\tilde{u}\tilde{w}}$
Sea-Spray Production
IR Radiation
Temperature & Moisture Profiles
Sea-Spray Effects
Stability Effects
Form Drag
$\overline{P_o \partial\eta/\partial x}$
Viscous Stress
$\nu \partial\overline{U}/\partial z$
Velocity Profiles
Total Heat Budget
Rain Effects
Sensible & Latent Heat Flux
Moisture Temperature Salinity Radiation
Short Waves
Wave - Wave
Long Waves
Wave - Current
Velocity Currents
Bubble Effects
Surfactants
Wave Radiation
Bubble Production
Micro-Breaking
Wave Breaking
Wall Layer Shear
Fresh Water Flux
Heat & Salt Flux
Penetrating Solar
Biological Effects & IOPs
Dissipation
Wave-current Interaction
Temperature & Salinity Profiles
Turbulent Shear Layer
Total Buoyancy Flux
Current Profiles
Langmuir Circulations
Total Momentum Flux
Convective Mixing/ Stratification
Deep Convection
MESOSCALE
VARIABILITY

Flat-plate BL skin friction factor



(Schlichting, 8th edn)

Boundary layer thickness evolution



(Schlichting, 8th edn)

Remarks

- Turbulent flows are very different from their laminar counterpart:
  - Increased skin friction/pressure loss
    Increased thickness
  - Increased heat/mass transfer properties
  - --> technological importance !
- What are the associated physical mechanisms ?
- Understanding is required to design/optimize systems and control devices

The three effects of a solid wall

- Hypotheses about the solid surface:
  - Impermeable
  - Infinitely rigid
  - Plane
  - Non-reactive, cold
  - No-slip boundary condition holds (beware of micro/nano-channel dynamics !)

Cont'd

1- ***The shear effect***: the no-slip boundary condition involves the existence of a mean shear (matching with outer flow condition)

- Anisotropic TKE production term $-R_{12}\partial_y \bar{u}_1(y)$
- Anisotropy forcing

Cont'd

2- ***Viscous effects***: the mean velocity decreases when approaching the wall

- --> the local Reynolds number diminishes
- --> viscous effects are more important near the wall

- Effect 1: viscous diffusion $\nu \partial_{kk} R_{ij}$
- Effect 2: dissipation $2\nu \overline{\partial_k u'_i \partial_k u'_j}$

Cont'd

3- ***Effects due to the impermeability assumption***

- Kinematic « splash » effect: structures impinging the wall induce a redistribution of TKE from wall-normal toward tangential velocity components
  - --> damping of the wall-normal Reynolds stress
  - --> increase of the 2 other diagonal stresses
  - --> increase of anisotropy

Note: also present in shear-free boundary layer (e.g. boundary layer developing above a moving belt)

Cont'd

– Dynamic « echo » effect: a non-local modification of the pressure field is induced.

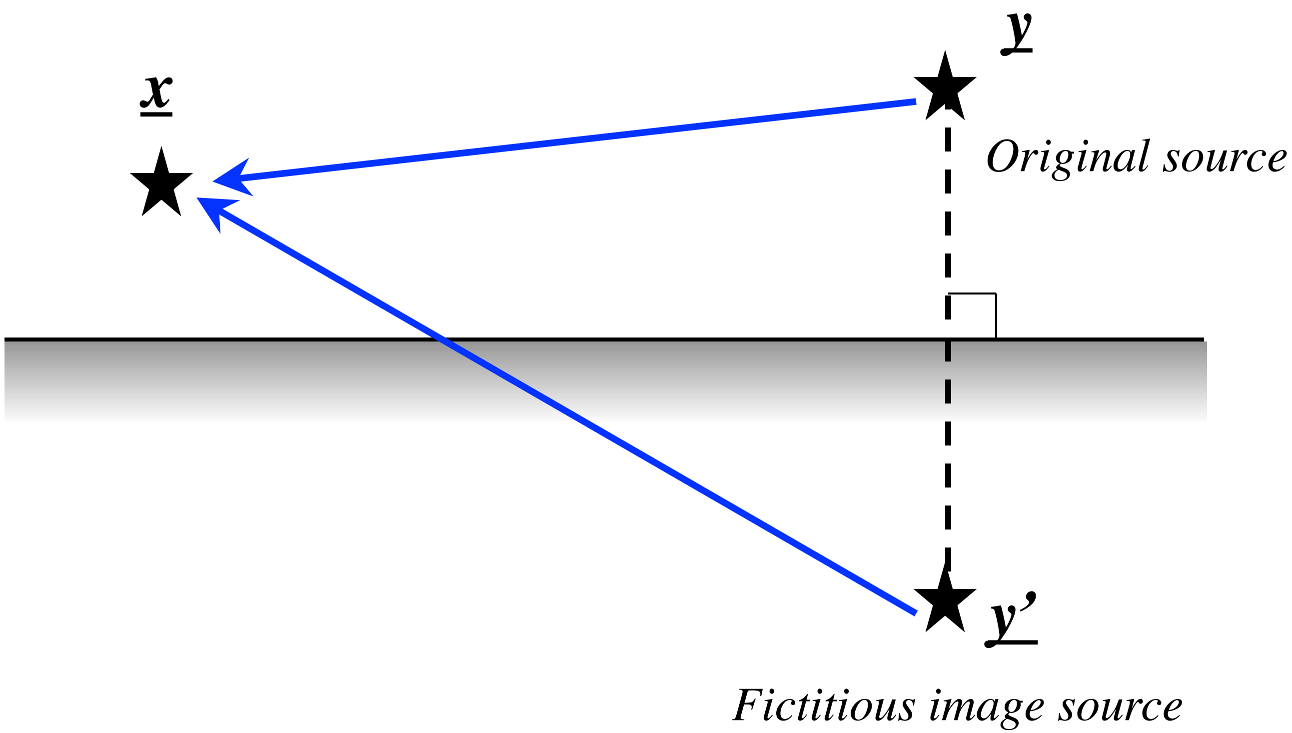
Let us consider the Poisson equation for pressure:

$$\nabla^2 p = f(\underline{x},t) \Longrightarrow p(\underline{x},t) = -\frac{1}{4\pi}\int \frac{f(\underline{y},t)}{|\underline{x}-\underline{y}|}d\underline{y}$$

One can see that the Green-function-based solution (defined for an unbounded domain) must be modified to account for the solid surface

Cont'd

Idea: image source model



$$\Longrightarrow p(\underline{x},t) = -\frac{1}{4\pi}\int_{\text{half-plane}} \frac{f(\underline{y},t)}{|\underline{x}-\underline{y}| + |\underline{x}-\underline{y}'|} d\underline{y}$$

Present framework

- Additional assumption: quasi-parallel flow
  - Almost true for ‘canonical’ zero-pressure gradient flat plate boundary layer
  - Exact condition for internal flows in straight pipes and plane 2D channels

Boundary layer: a multiple scale problem

- External region (far from the wall)
  - High local *Re*
  - Characteristic velocity scale = external velocity
  - Characteristic length-scale = geometry fixed (BL thickness, pipe/channel radius)
- Internal region (near the wall)
  - Viscous effects & impermeability effects important
  - Characteristic velocity scale = friction velocity
  - Characteristic lengthscale = viscous length

$$u_* \equiv \sqrt{\frac{\tau_*}{\rho}} = \sqrt{\nu \left.\frac{d\bar{u}}{dy}\right|_{\text{paroi}}} \qquad l_* \equiv \frac{\nu}{u_*} = \sqrt{\frac{\nu}{\left.\frac{d\bar{u}}{dy}\right|_{\text{paroi}}}}$$

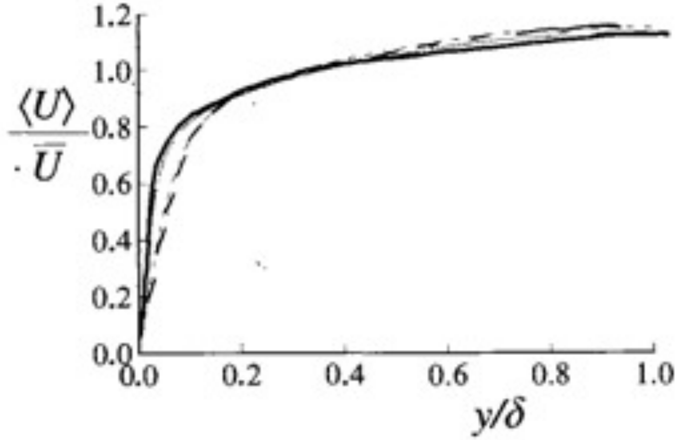


Fig. 7.2. Mean velocity profiles in fully developed turbulent channel flow from the DNS of Kim *et al.* (1987): dashed line, Re = 5,600; solid line, Re = 13,750.

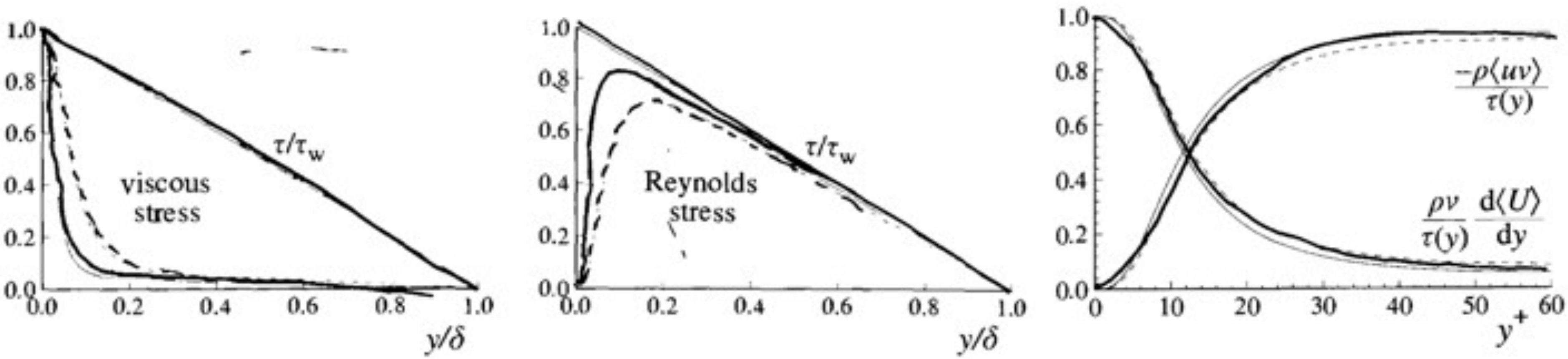


Fig. 7.3. Profiles of the viscous shear stress, and the Reynolds shear stress in turbulent channel flow: DNS data of Kim *et al.* (1987): dashed line, Re = 5,600; solid line, Re = 13,750.

Mean flow: equations & results

Mean momentum equations simplify as

$$
\begin{aligned}
0 &= -\frac{\partial \bar{p}}{\partial x} + \nu \frac{d^2 \bar{u}}{dy^2} - \frac{dR_{12}}{dy} = -\frac{\partial \bar{p}}{\partial x} + \frac{d}{dy}\left[\nu \frac{d\bar{u}}{dy} - R_{12}\right] \\
0 &= -\frac{\partial \bar{p}}{\partial y} - \frac{dR_{22}}{dy} = -\frac{\partial}{\partial y}\left[\bar{p} + R_{22}\right] \\
0 &= -\frac{dR_{32}}{dy}
\end{aligned}
$$

$$\bar{p}(x,y) + R_{22}(y) = \bar{p}(x,0) = \bar{p}_0(x)$$

$$0 = -\frac{d}{dx}\bar{p}_0(x) + \frac{d}{dy}\left[\nu \frac{d\bar{u}}{dy} - R_{12}\right]$$

Mean flow classical theory

- Mean velocity profile can be predicted (at least partially):
  - Using phenomenological analysis (von Karman & Prandtl, early 1930s)
  - Using Asymptotic Matched Expansions (Isakson & Millikan, late 1930s + later works)

Phenomenological analysis

Momentum equation with zero-pressure-gradient hypothesis

$$0 = \frac{d}{dy}\left[\nu \frac{d\bar{u}}{dy} - R_{12}\right]$$

Integrating once in the vertical direction between *0* and *y*

$$\nu \frac{d}{dy}\bar{u}(y) - R_{12}(y) = \nu \frac{d}{dy}\bar{u}(0) \equiv \frac{\tau_*}{\rho} \equiv u_*^2$$

A priori unknown turbulent term

A priori known

- Phenomenological hypotheses
  - R_{12} is constant and negative
  - friction velocity is relevant to describe fluctuations
  - It is possible to define a **turbulent viscosity**

$$-R_{12} = \nu_t \frac{d}{dy}\bar{u}(y)$$

$$\nu_t = [L^2][T^{-1}] \longrightarrow \nu_t(y) = \kappa_{\mathrm{VK}} u_* y$$

Von Karman constant: 0.38-0.41

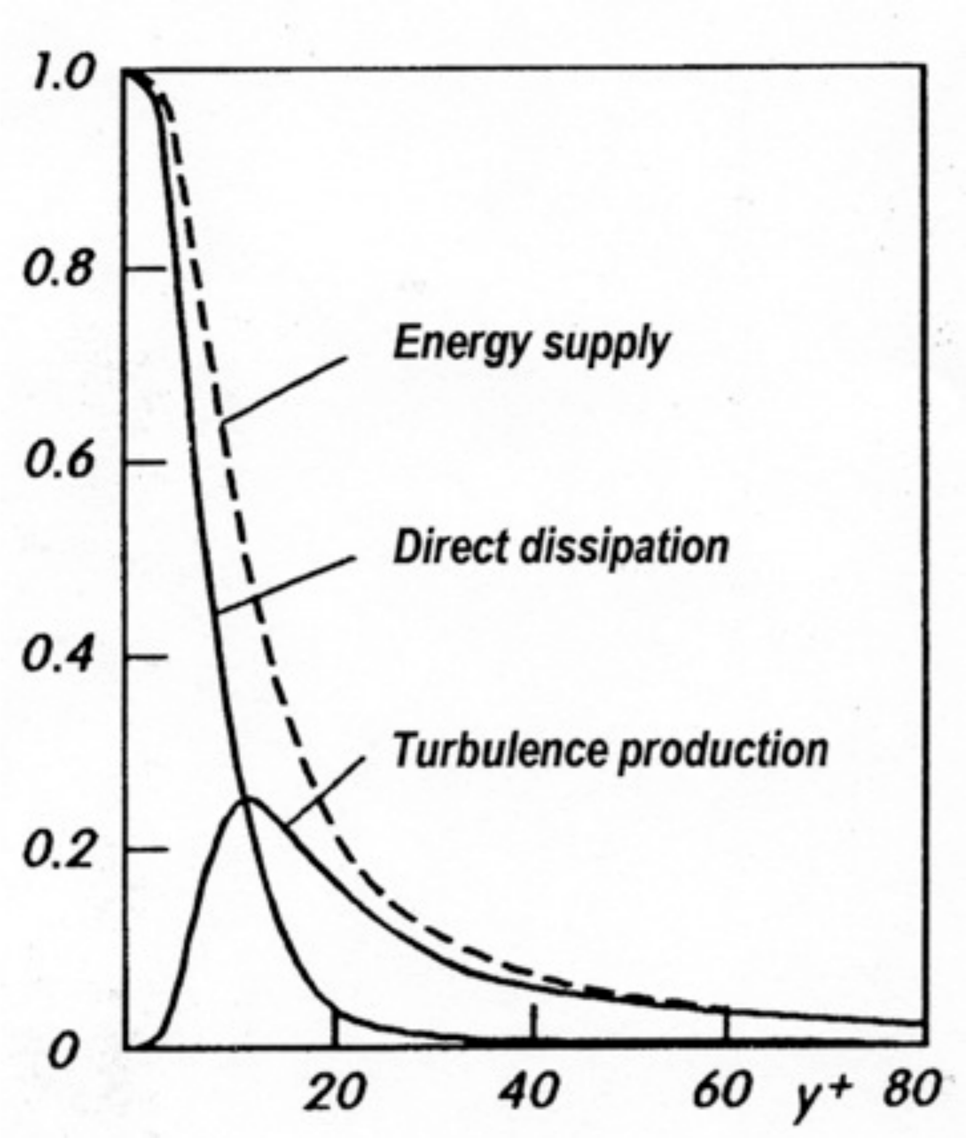
Negligible molecular viscosity assumption

$$u_*^2 = \kappa_{\text{VK}} u_* y \frac{d}{dy}\bar{u}(y)$$

$$\Rightarrow \quad \frac{\bar{u}}{u_*} = \frac{1}{\kappa_{\text{VK}}} \ln\left(y u_* / \nu\right) + B$$

- Logarithmic solution
- Logarithmic layer, inertial layer, constant shear layer
- not consistent with no-slip boundary condition !

Mean flow kinetic energy balance



(Schlichting, 8th edn)

Matched Asymptotic Expansions

Dimensional analysis:

$$\bar{u} = \bar{u}\left(y, \nu, \frac{d\bar{p}_0}{dx}, h\right), \quad R_{12} = R_{12}\left(y, \nu, \frac{d\bar{p}_0}{dx}, h\right)$$

Symmetry condition at channel centerline:

$$\frac{d\bar{u}}{dy}(y = h) = R_{12}(y = h) = 0, \quad \bar{u}(y = h) = u_c$$

No-slip condition at solid walls:

$$\bar{u}(y = 0) = \bar{u}(y = 2h) = 0, \quad R_{12}(y = 0) = R_{12}(y = 2h) = 0$$

Integrating momentum in the vertical direction, taking *y=2h*

$$-h\frac{d}{dx}\bar{p}_0 = u_*^2$$

Fundamental equation for MAE analysis:

$$-R_{12}(y) + \nu\frac{d\bar{u}}{dy}(y) = u_*^2\left(1 - \frac{y}{h}\right)$$

Dimensionless formulation

$$\langle U\rangle = u_\tau F_0\left(\frac{y}{\delta}, \mathrm{Re}_\tau\right) \qquad \frac{\mathrm{d}\langle U\rangle}{\mathrm{d}y} = \frac{u_\tau}{y}\Phi\left(\frac{y}{\delta_\nu}, \frac{y}{\delta}\right) \qquad \left(\frac{y}{\delta_\nu}\right)/\left(\frac{y}{\delta}\right) = \mathrm{Re}_\tau$$

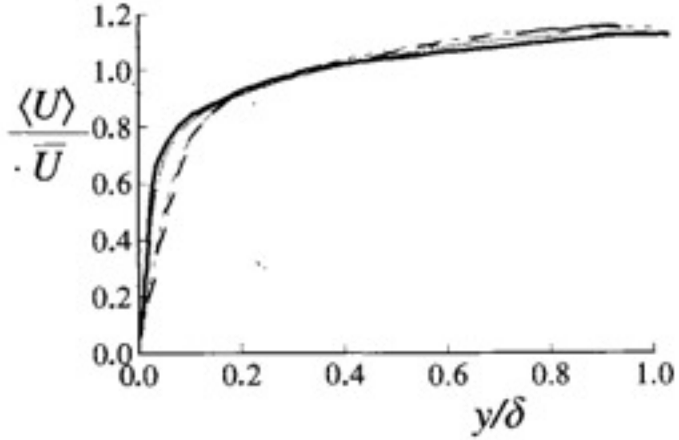


Fig. 7.2. Mean velocity profiles in fully developed turbulent channel flow from the DNS of Kim *et al.* (1987): dashed line, Re = 5,600; solid line, Re = 13,750.

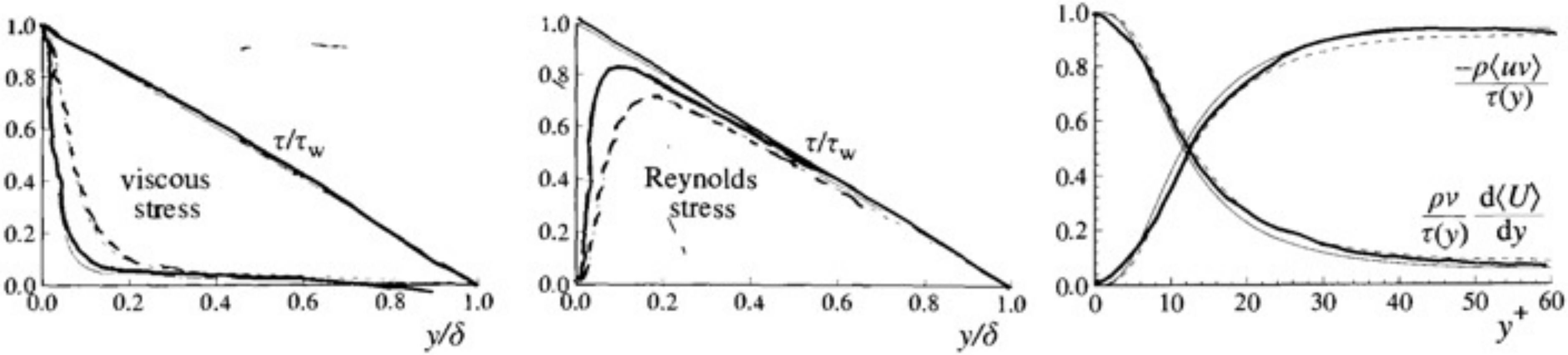


Fig. 7.3. Profiles of the viscous shear stress, and the Reynolds shear stress in turbulent channel flow: DNS data of Kim *et al.* (1987): dashed line, Re = 5,600; solid line, Re = 13,750.

Inner layer

$$\Phi_1\left(\frac{y}{\delta_v}\right) = \lim_{y/\delta \to 0} \Phi\left(\frac{y}{\delta_v}, \frac{y}{\delta}\right)$$

$$u^+ \equiv \frac{\langle U \rangle}{u_\tau}$$

$$\frac{\mathrm{d}u^+}{\mathrm{d}y^+} = \frac{1}{y^+}\Phi_1(y^+)$$

$$u^+ = f_w(y^+)$$

$$f_w(y^+) = \int_0^{y^+} \frac{1}{y'}\Phi_1(y')\,\mathrm{d}y'$$

Viscous sublayer

$$f'_w(0) = 1$$

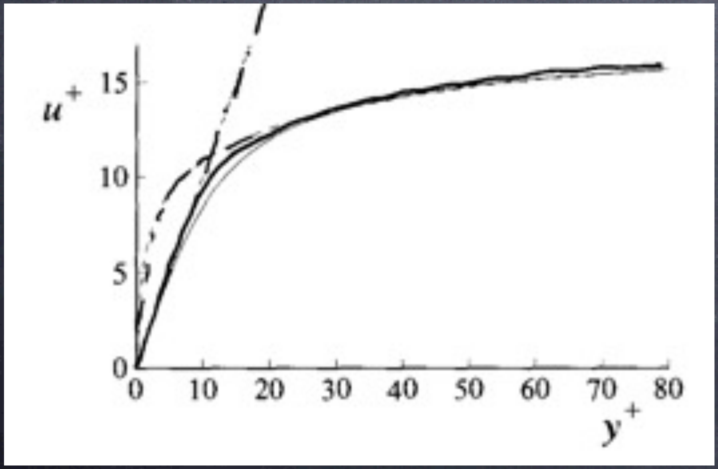
$$f_w(y^+) = y^+ + \mathcal{O}(y^{+2})$$

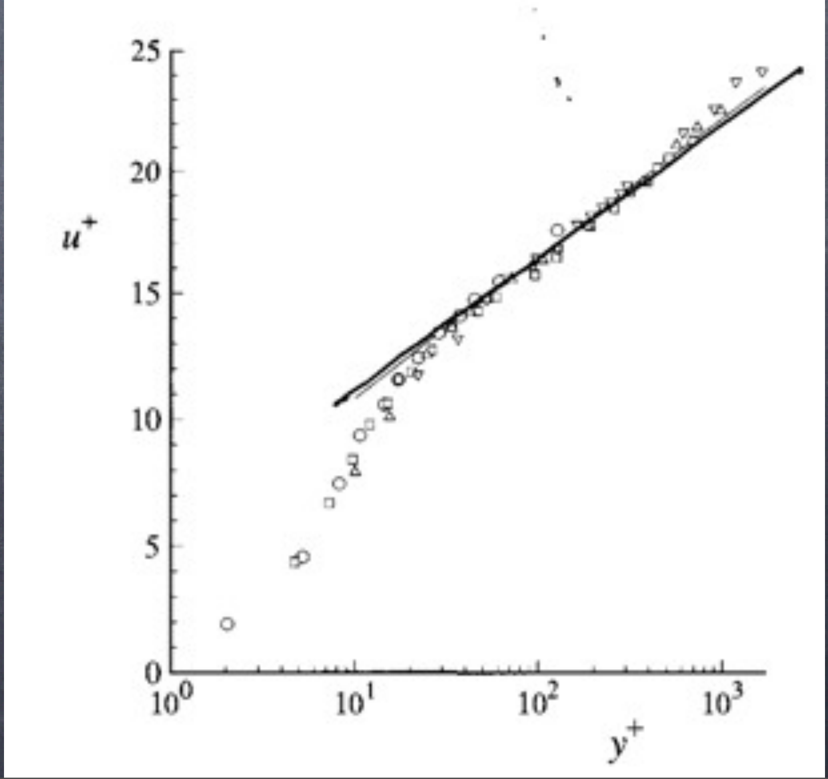
Log Law

$$\Phi_I(y^+) = \frac{1}{\kappa}, \qquad \text{for } \frac{y}{\delta} \ll 1 \text{ and } y^+ \gg 1$$

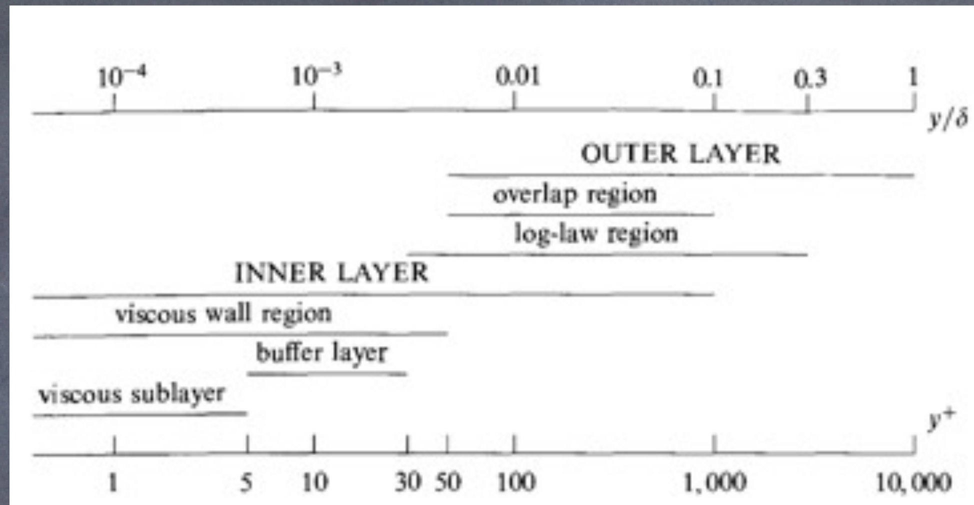
$$u^+ = \frac{1}{\kappa} \ln y^+ + B$$

$$\kappa = 0.41, \qquad B = 5.2$$





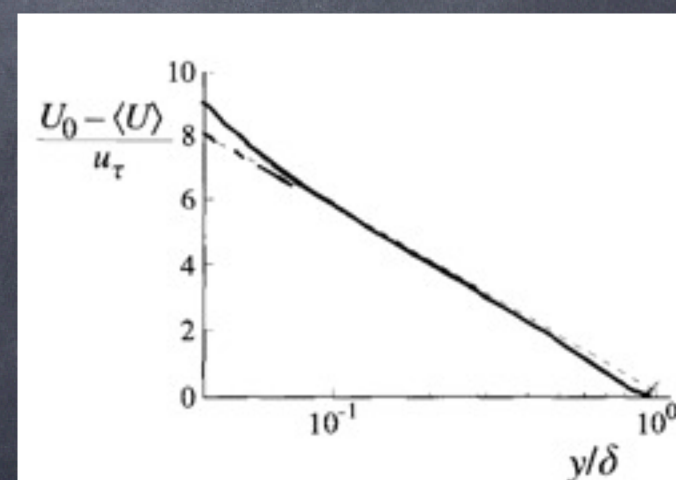
Velocity defect law



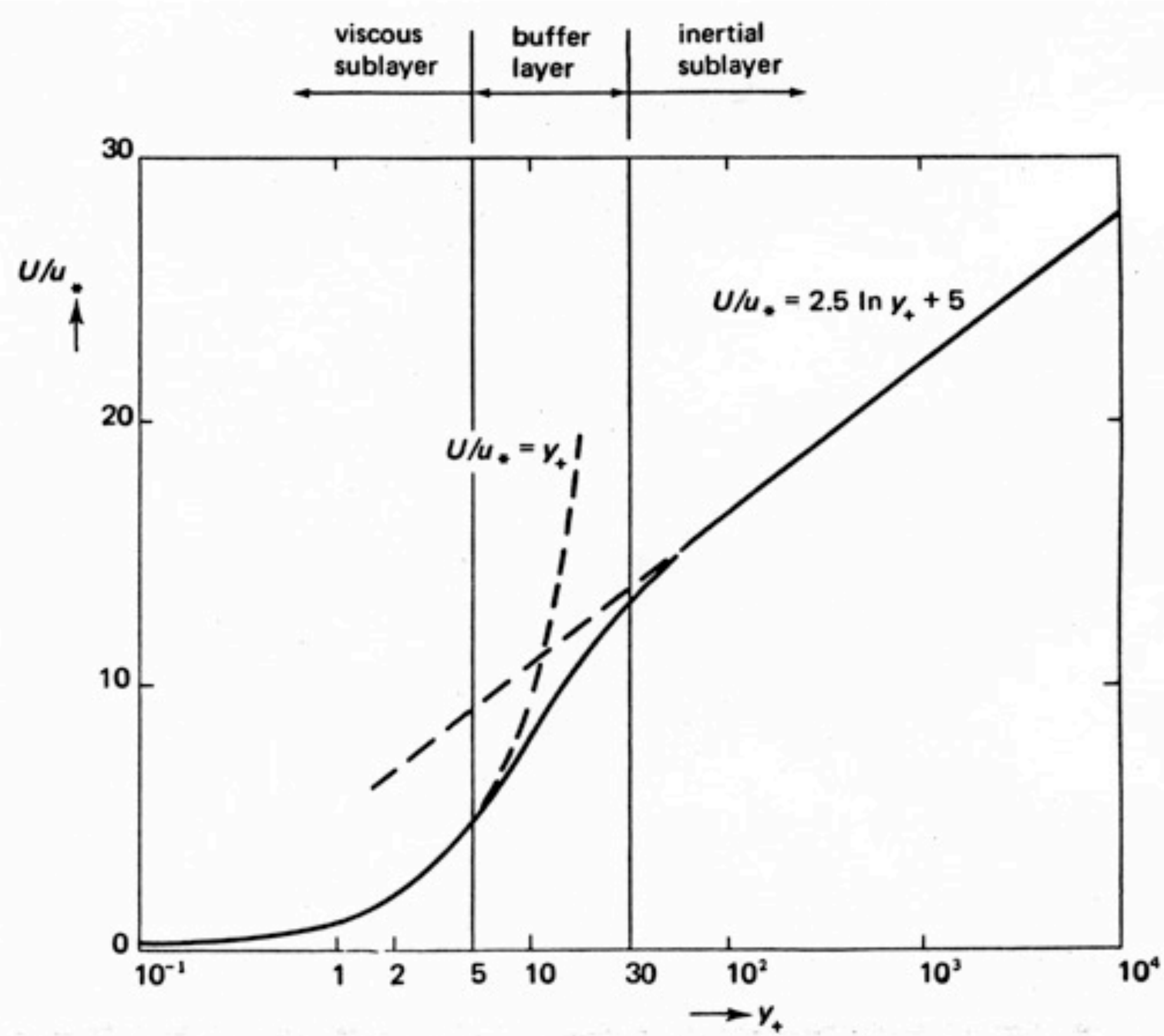
$$\lim_{y/\delta_\nu \to \infty} \Phi\left(\frac{y}{\delta_\nu}, \frac{y}{\delta}\right) = \Phi_0\left(\frac{y}{\delta}\right)$$

$$\frac{U_0 - \langle U \rangle}{u_\tau} = F_D\left(\frac{y}{\delta}\right)$$

$$F_D\left(\frac{y}{\delta}\right) = \int_{y/\delta}^{1} \frac{1}{y'} \Phi_0(y')\, dy'$$



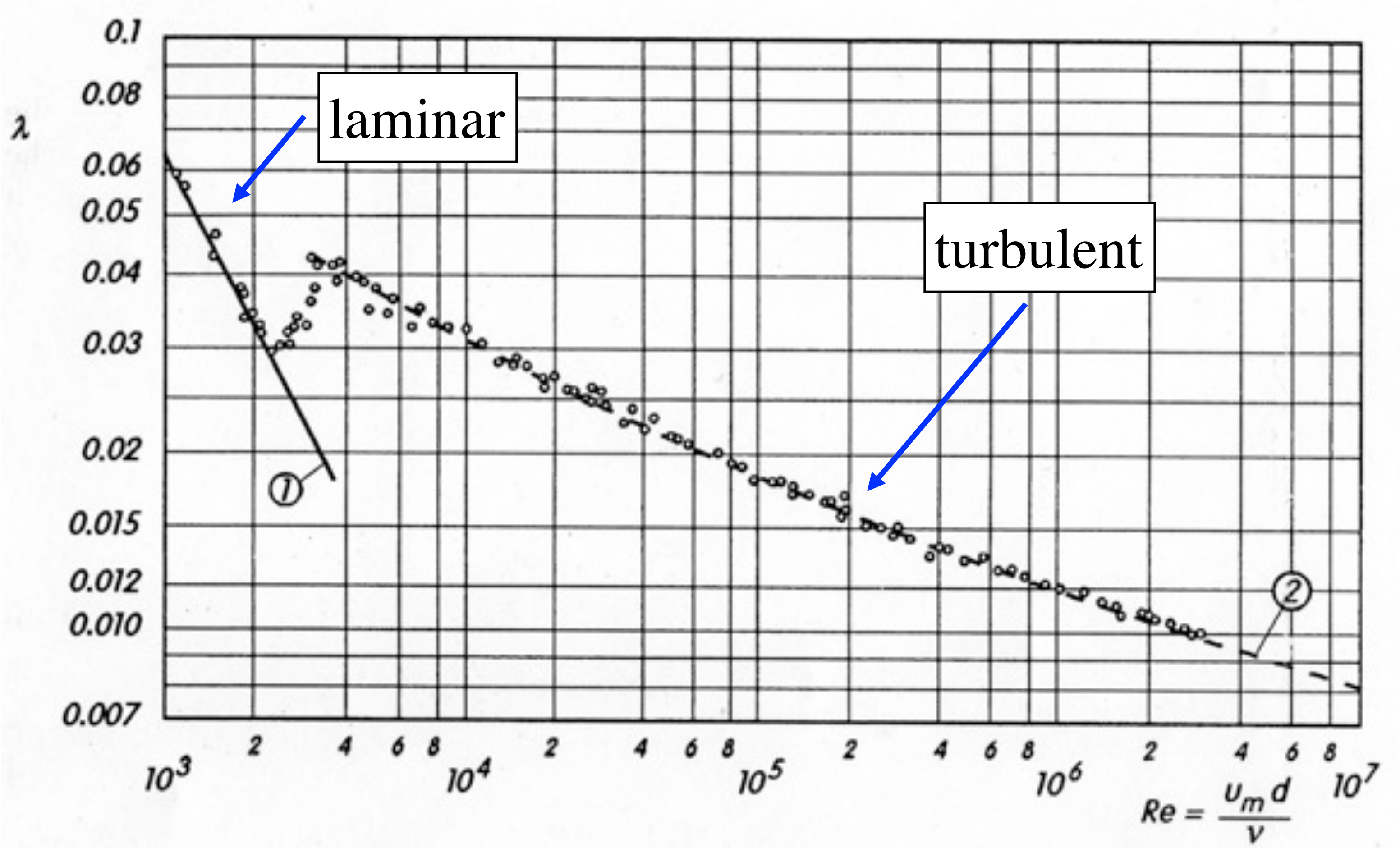
The channel flow case



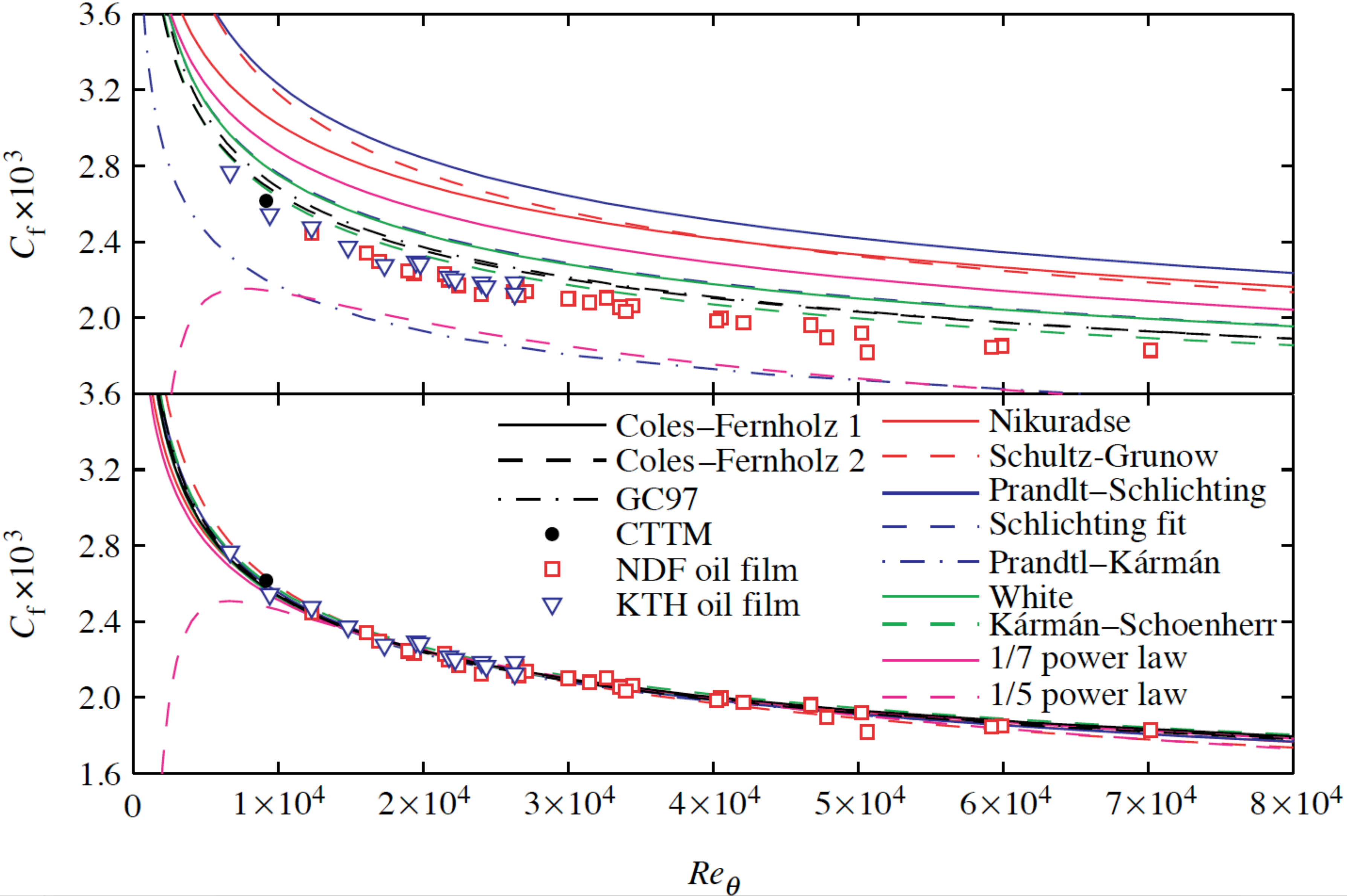
Typical « universal » mean velocity profile

(Tennekes & Lumley)

Turbulent Drag: generation & control



Many available formulas



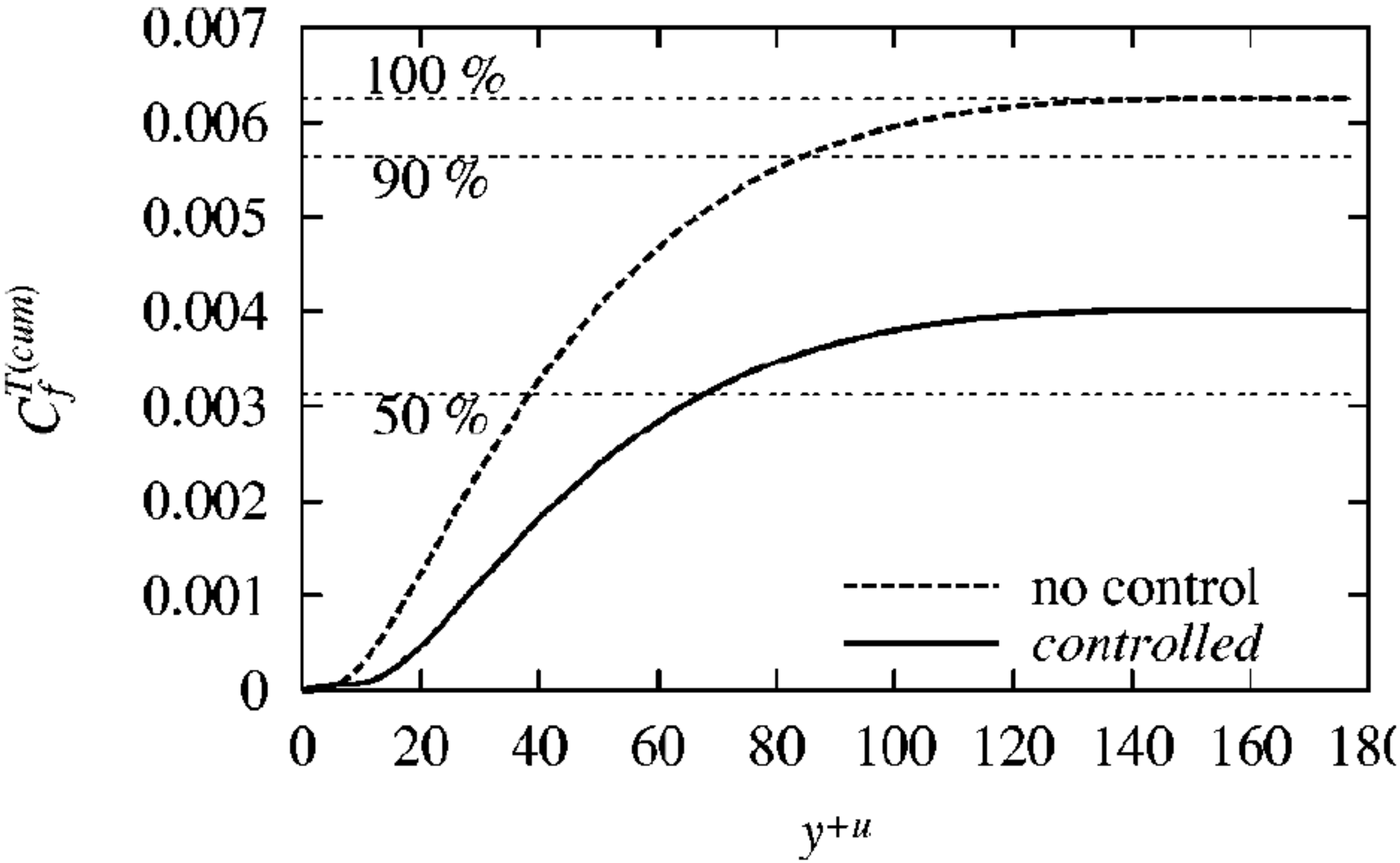
Non-local FIK formula (2002)

- Skin friction is a local quantity
  - Difficult to measure in wind tunnel
  - Highly sensitive to errors
  - Poor understanding of generation by turbulent events
- Non-local formula much more helpful
- Triple integration of momentum equation:

$$C_f = 12\left[\frac{1}{Re_b} - \int_0^1 2(1-y)R_{12}(y)dy + \frac{1}{2}\int_0^1 (1-y^2)\left(I_x'' + \frac{\partial p''}{\partial x} + \frac{\partial}{\partial t}\bar{u}\right)dy\right]$$

$$\phi''(x,y,t) \equiv \bar{\phi}(x,y,t) - \tilde{\bar{\phi}}(x,t), \quad \tilde{\bar{\phi}}(x,t) \equiv \int_0^1 \bar{\phi}(x,y,t)dy$$

Drag reduction: example



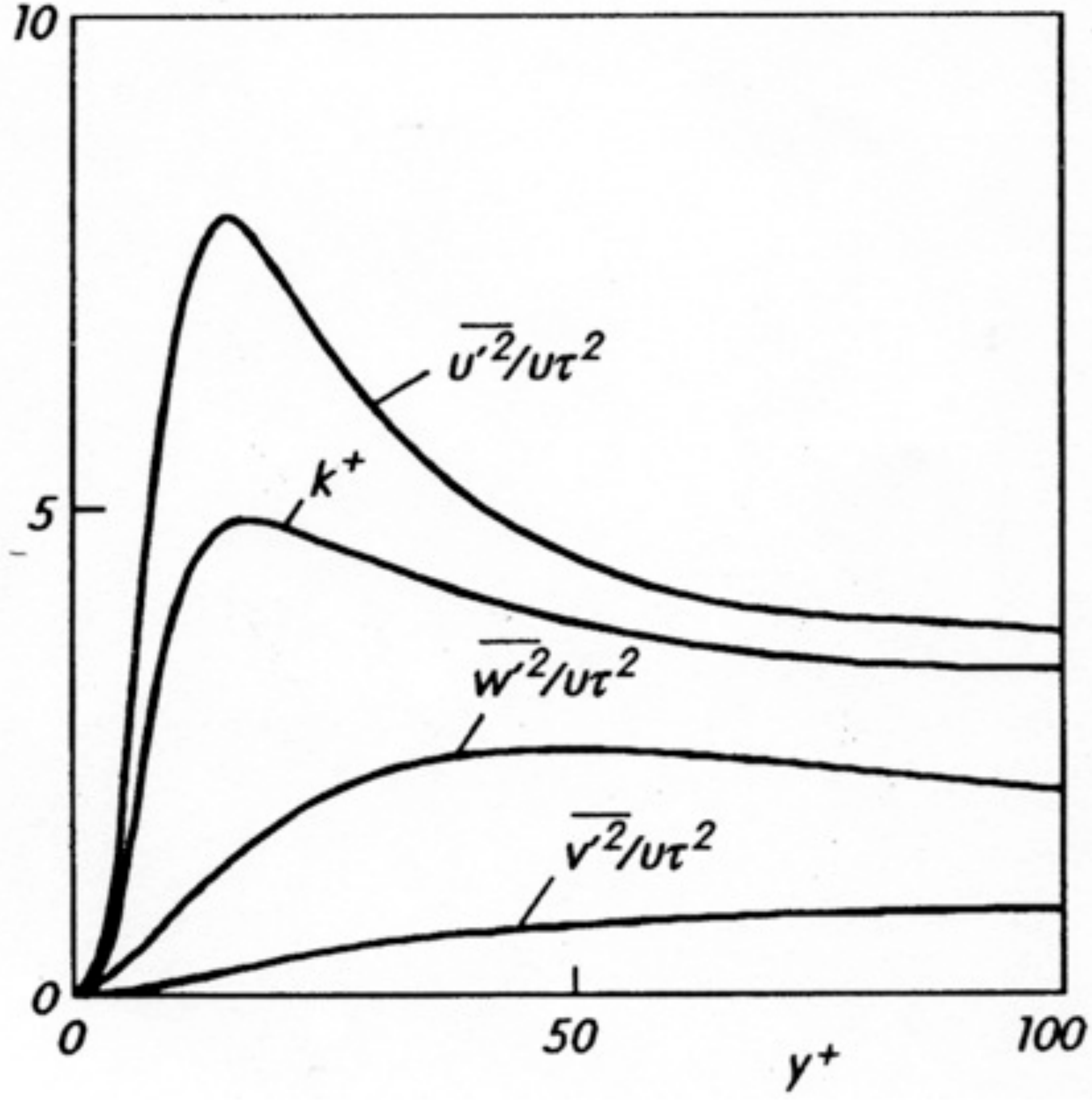
Reynolds stresses & TKE balance

Governing equations for Reynolds stresses and TKE read

$$
\begin{aligned}
0 &= -2R_{12}\frac{d\bar{u}}{dy} + \frac{d}{dy}\left(-\overline{u'u'v'} + \nu\frac{d}{dy}R_{11}\right) + \Pi_{11} - \varepsilon_{11}\\
0 &= \frac{d}{dy}\left(-\overline{v'(v'v' + 2p')} + \nu\frac{d}{dy}R_{22}\right) + \Pi_{22} - \varepsilon_{22}\\
0 &= \frac{d}{dy}\left(-\overline{v'w'w'} + \nu\frac{d}{dy}R_{33}\right) + \Pi_{33} - \varepsilon_{33}\\
0 &= -R_{22}\frac{d\bar{u}}{dy} + \frac{d}{dy}\left(-\overline{u'(v'v' + p')} + \nu\frac{d}{dy}R_{12}\right) + \Pi_{12} - \varepsilon_{12}\\
0 &= -R_{12}\frac{d\bar{u}}{dy} + \frac{d}{dy}\left(-\frac{1}{2}\overline{v'(u'u' + v'v' + w'w')} - \overline{p'v'} + \nu\frac{d}{dy}\mathcal{K}\right) - \varepsilon
\end{aligned}
$$

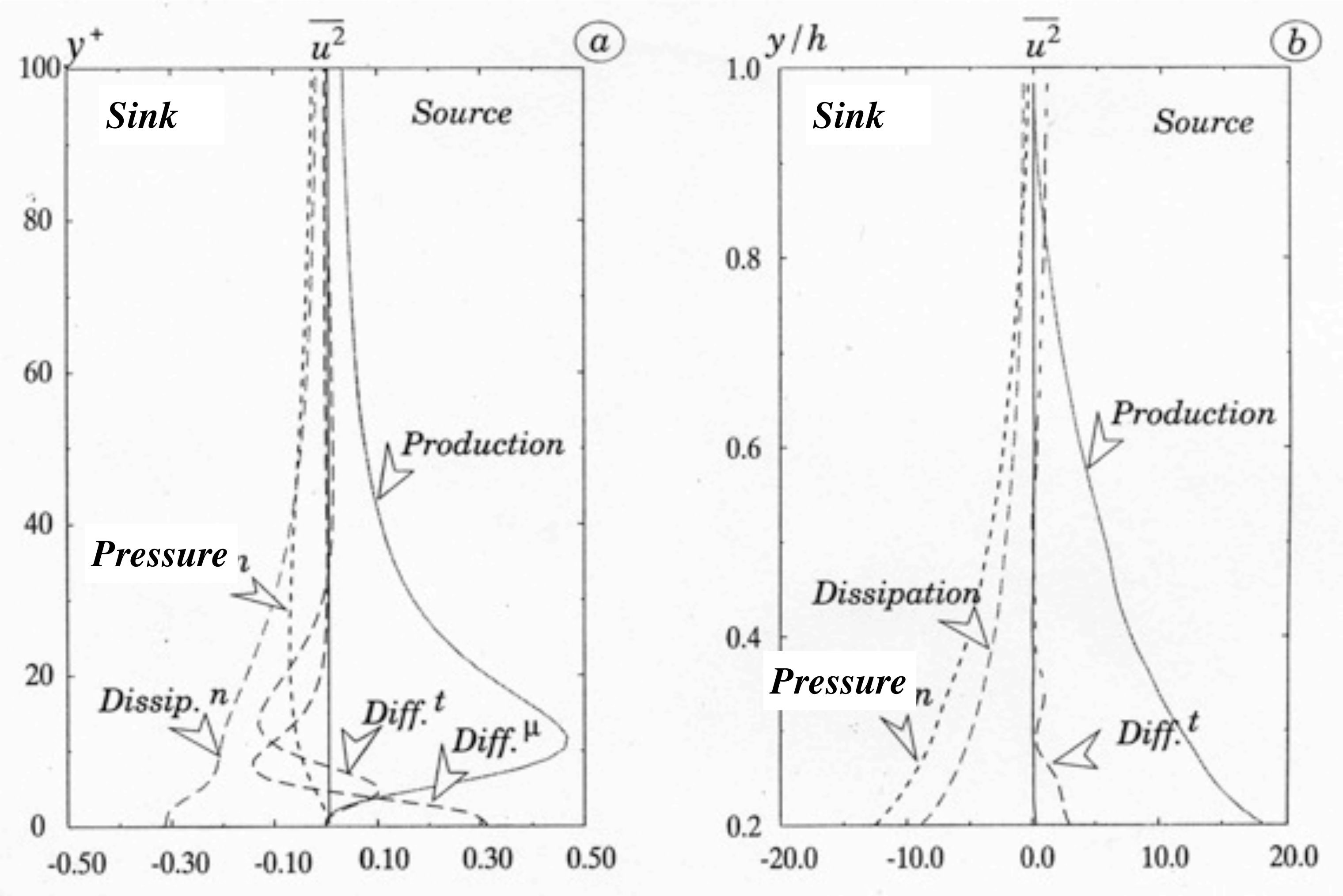
Main difference with homogeneous shear: wall-normal diffusion term (turbulent+pressure+viscous contributions)

Typical profiles (channel flow, inner layer)



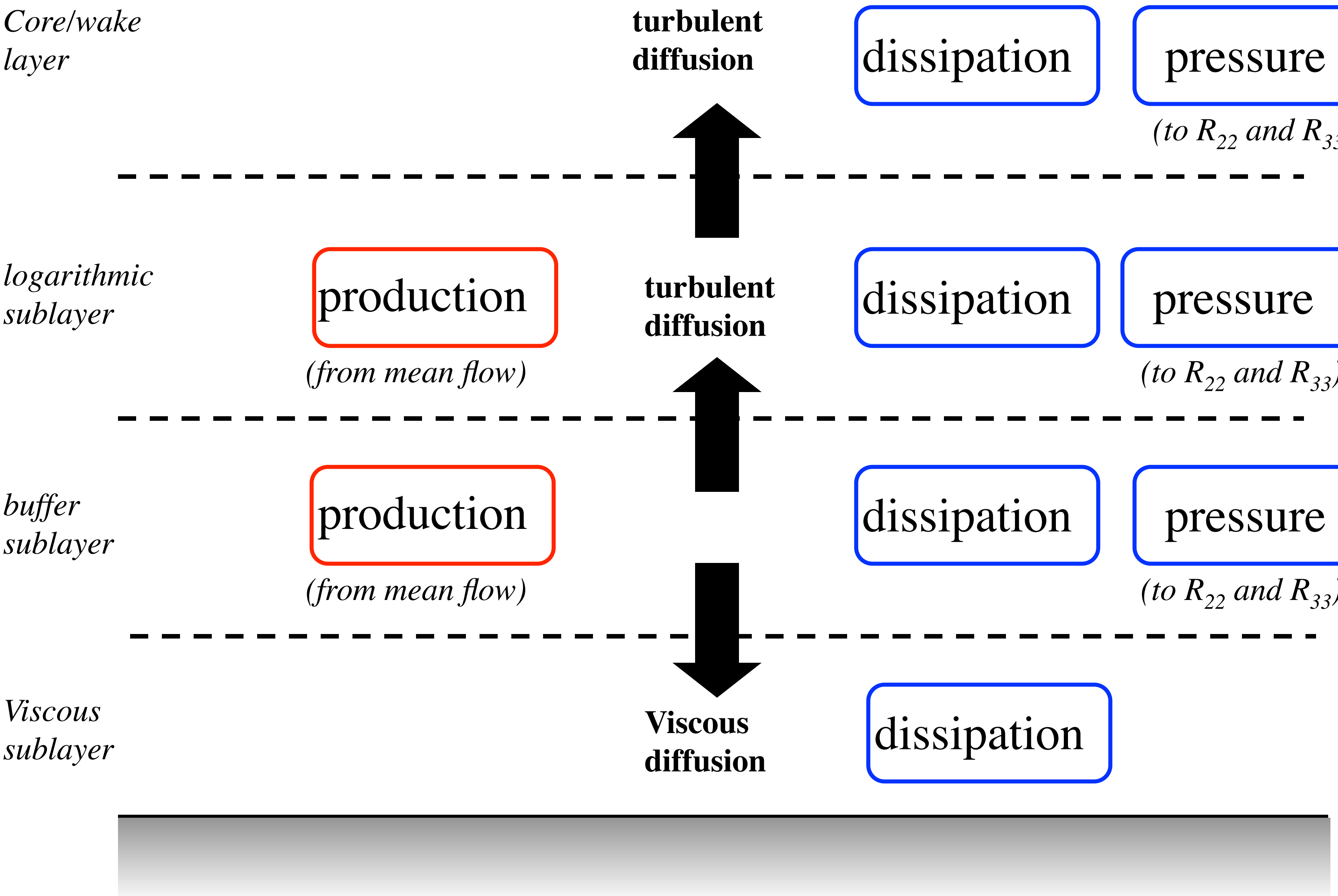
(Schlichting, 8th edn)

Streamwise RST balance - R_{11}

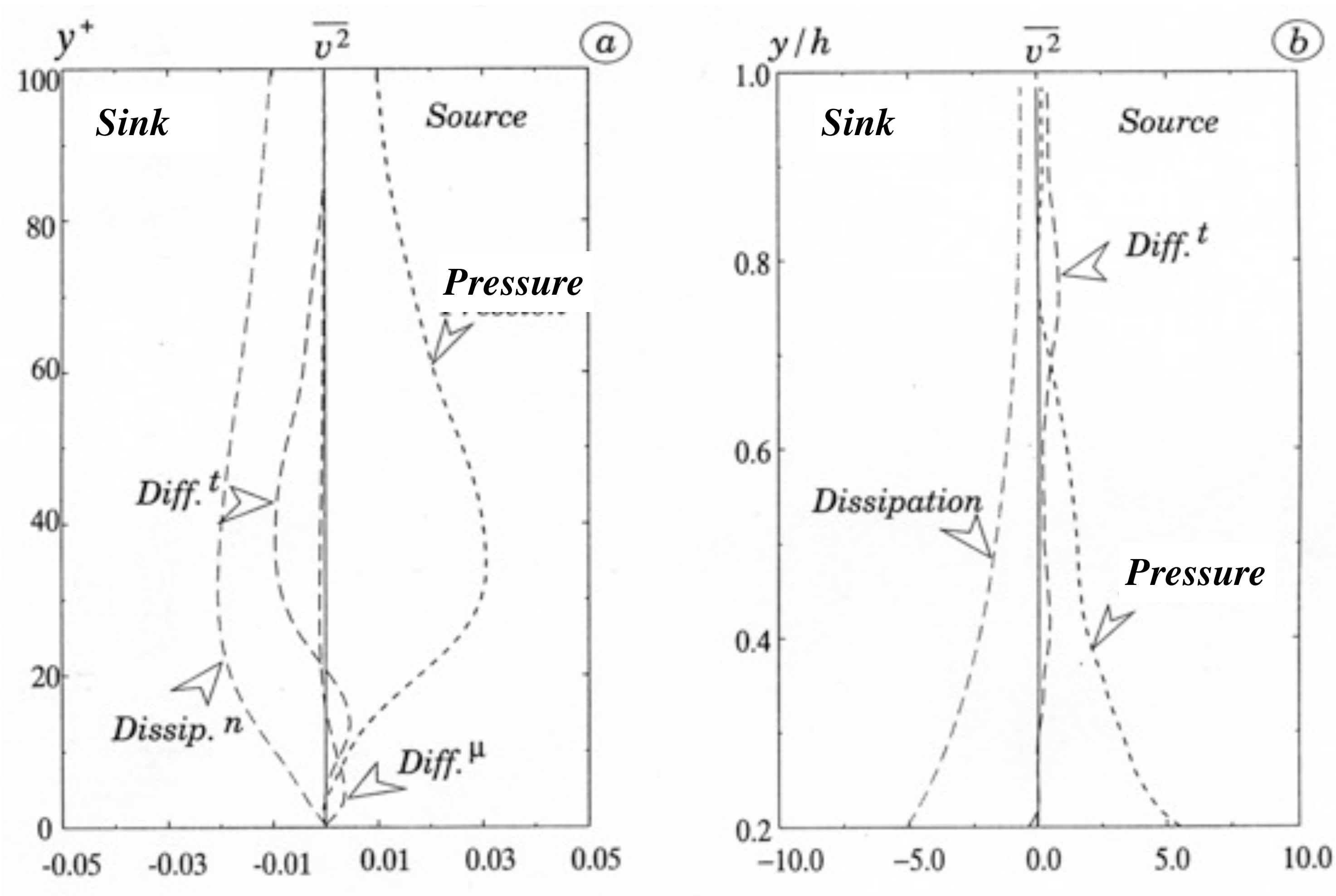


(Chassaing)

STREAMWISE REYNOLDS STRESS BUDGET

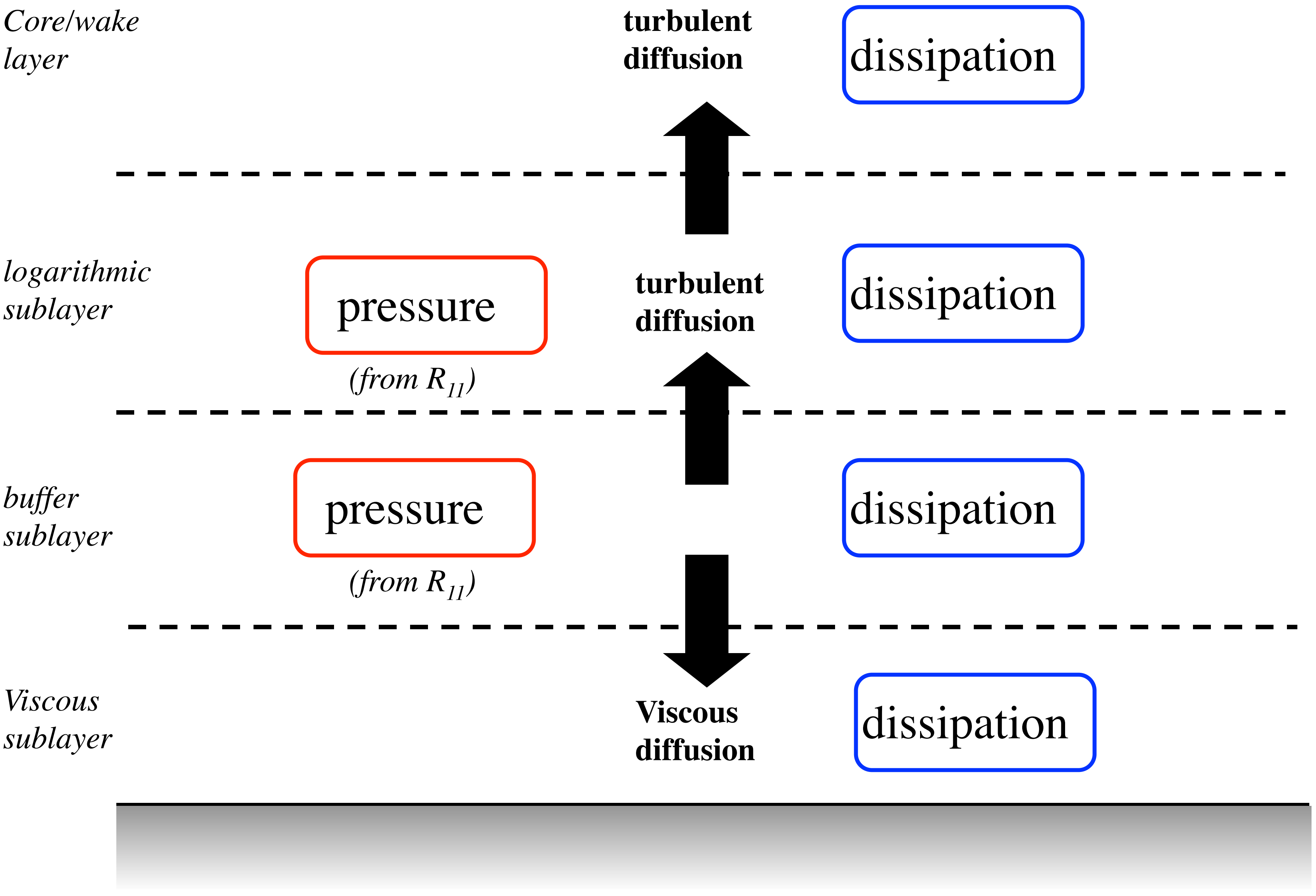


Wall-normal RST balance - R_{22}

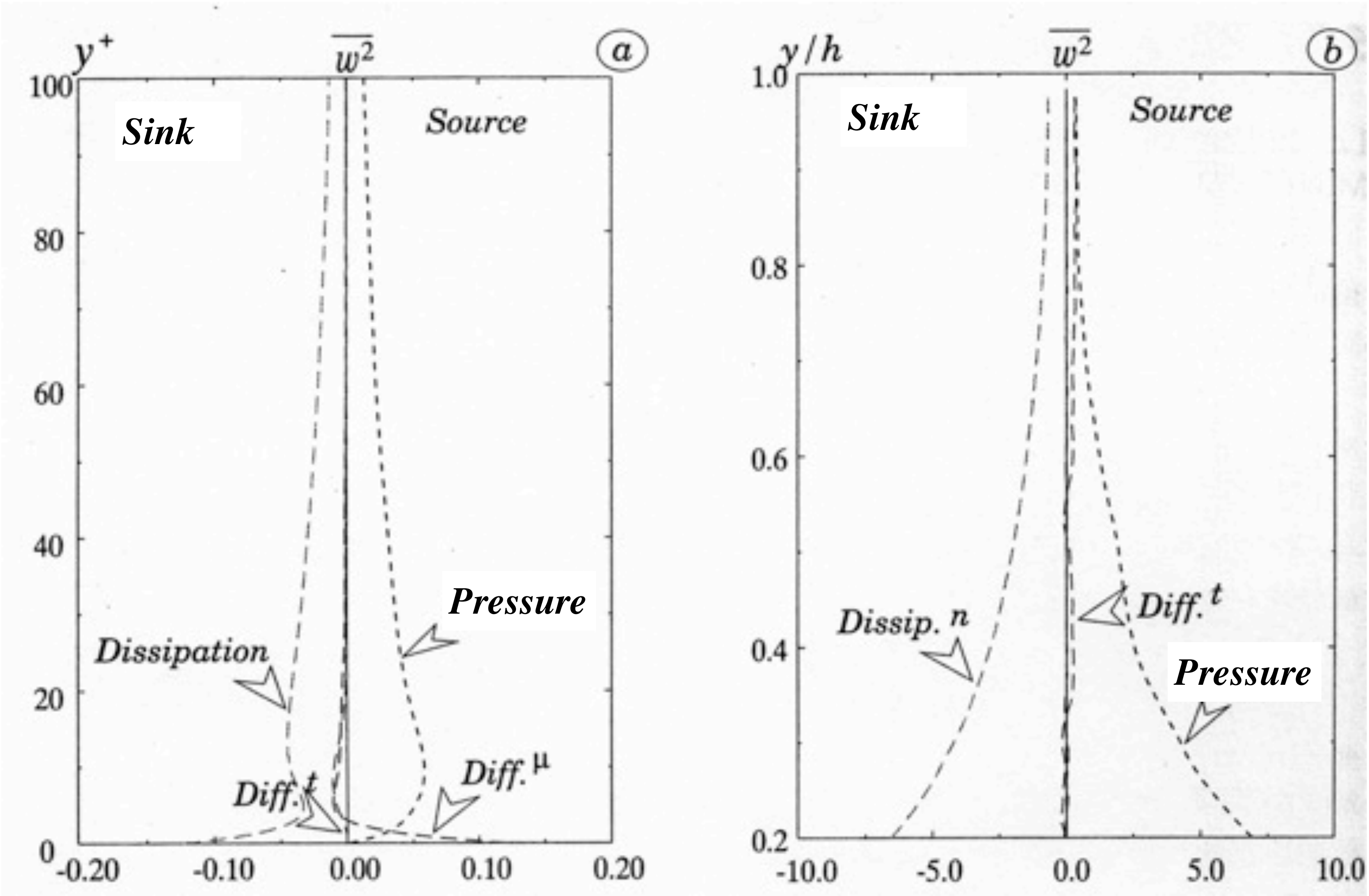


(Chassaing)

WALL-NORMAL REYNOLDS STRESS BUDGET

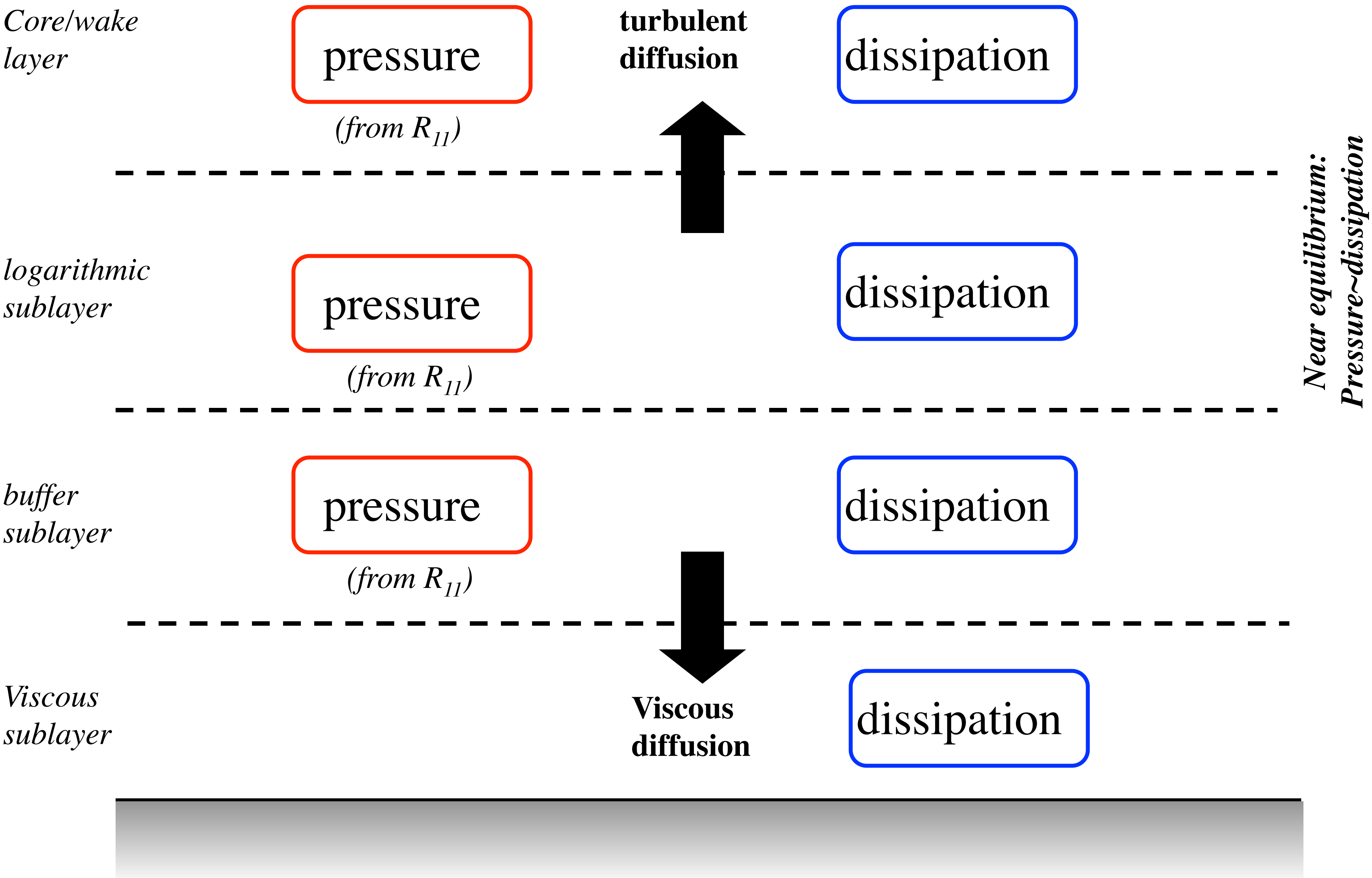


Spanwise RST balance - R_{33}

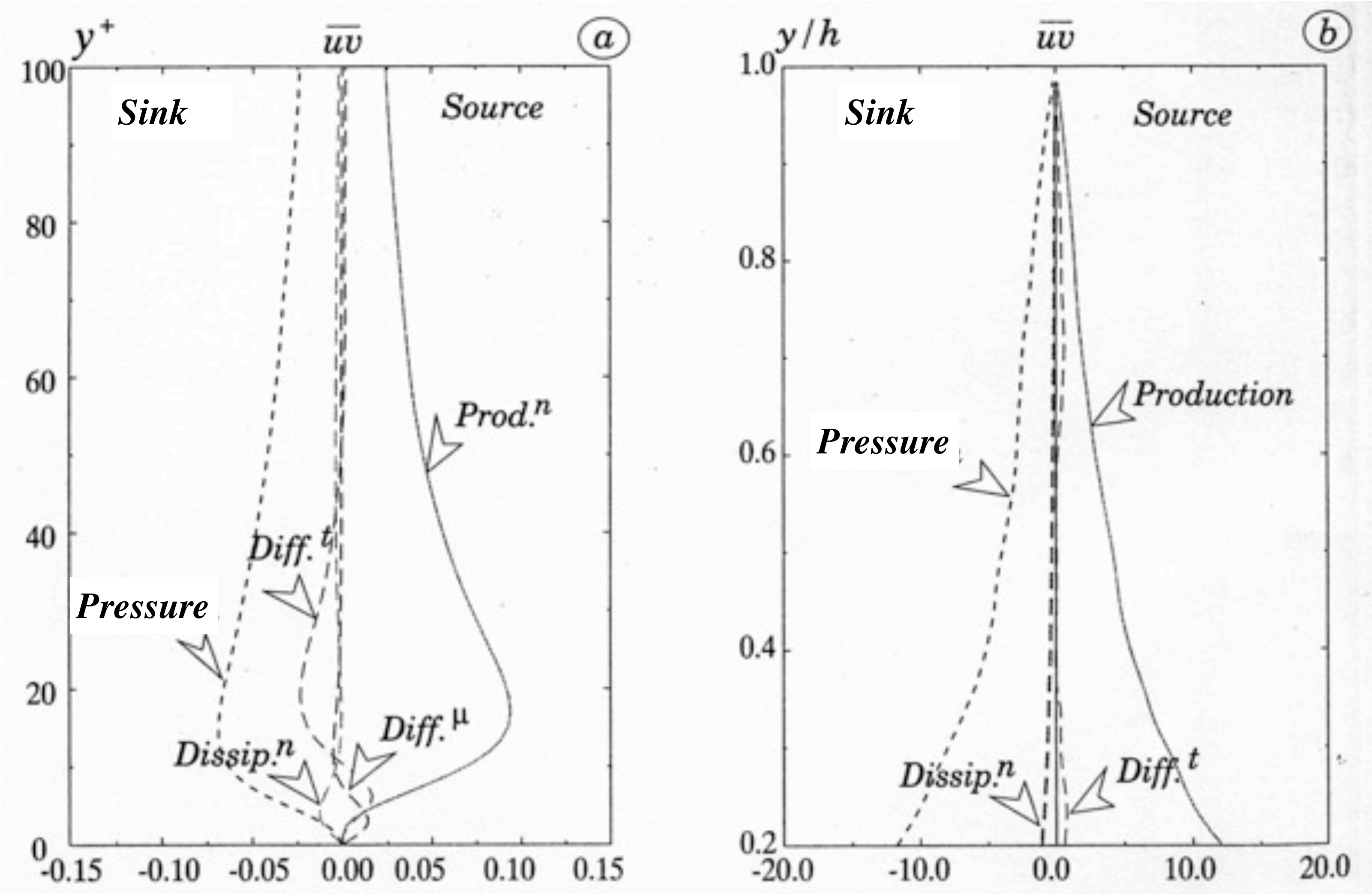


(Chassaing)

SPANWISE REYNOLDS STRESS BUDGET

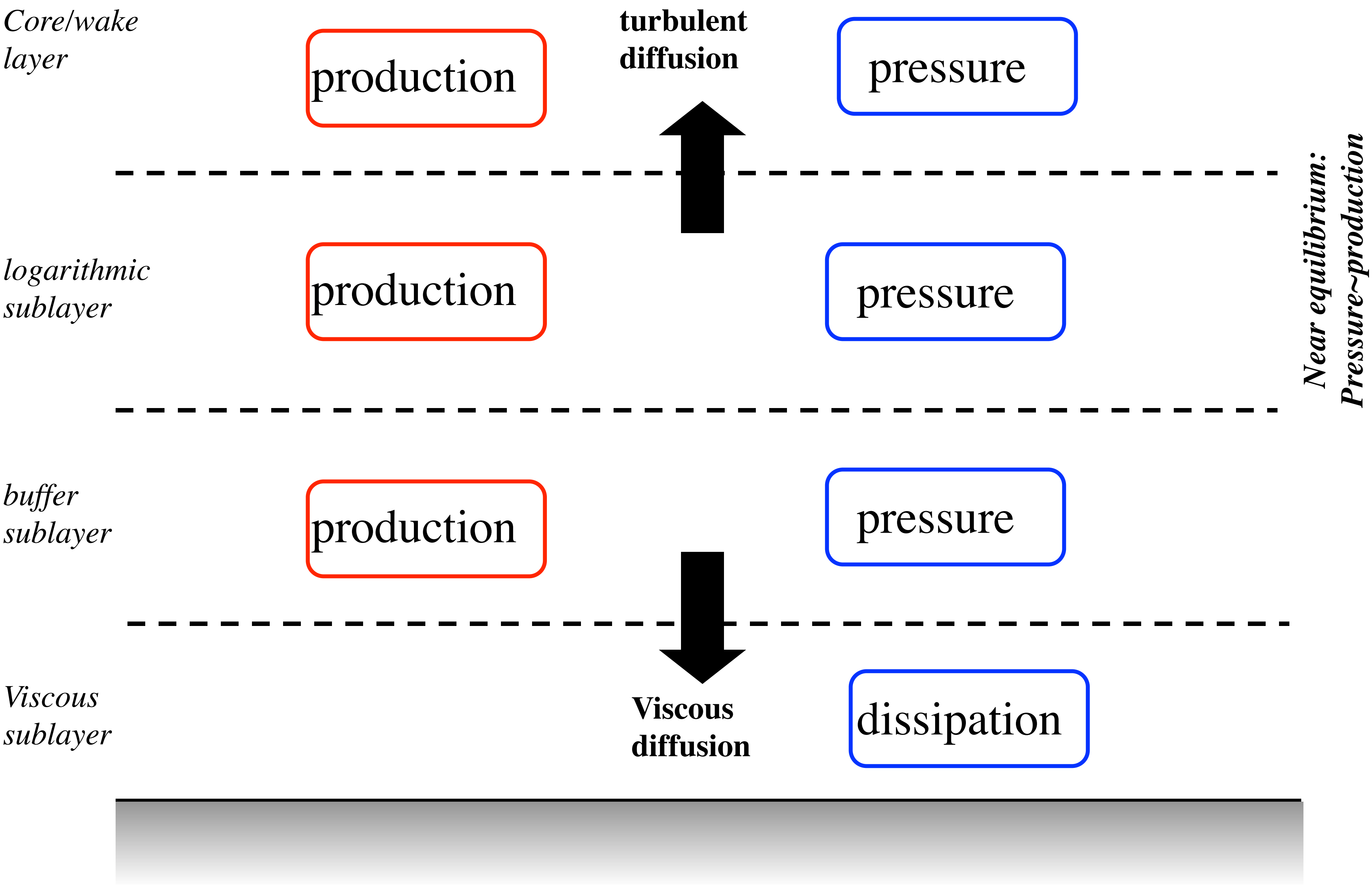


Shear stress balance - R_{12}

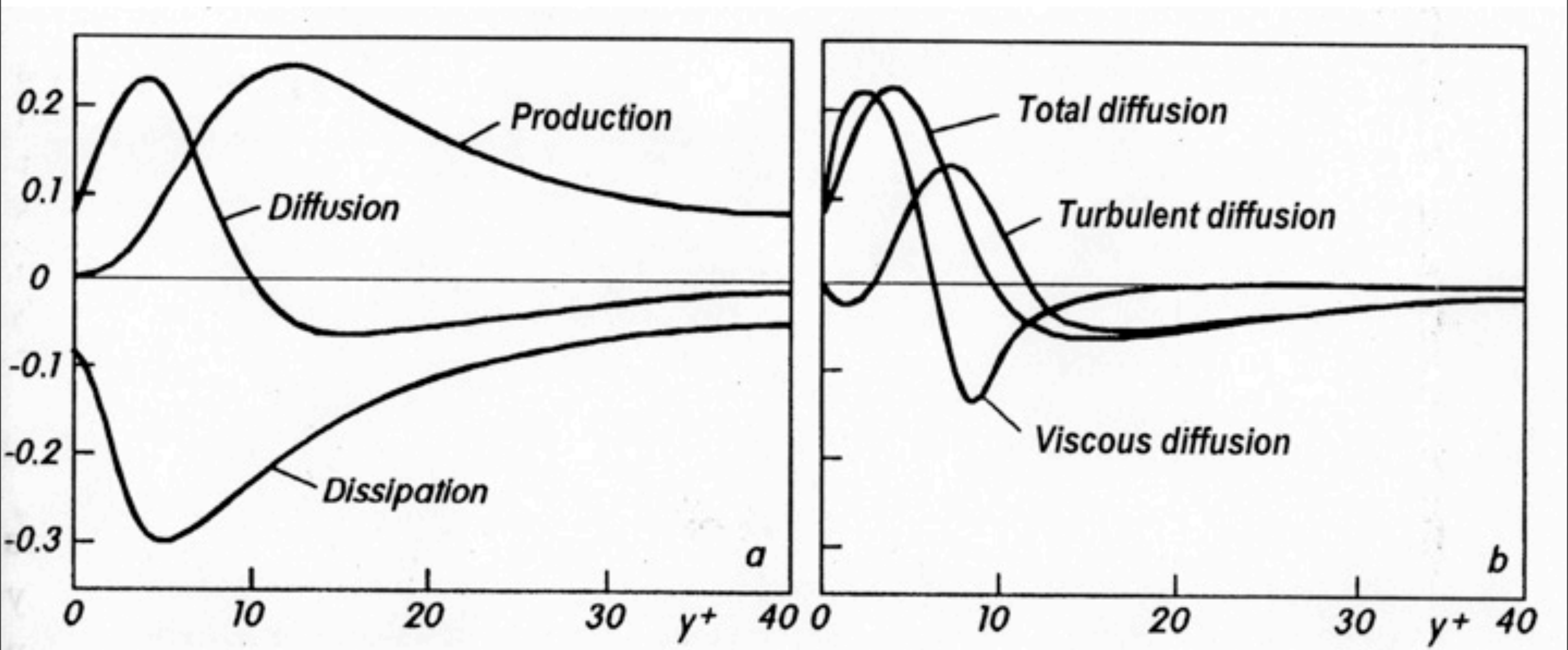


(Chassaing)

SHEAR STRESS BUDGET

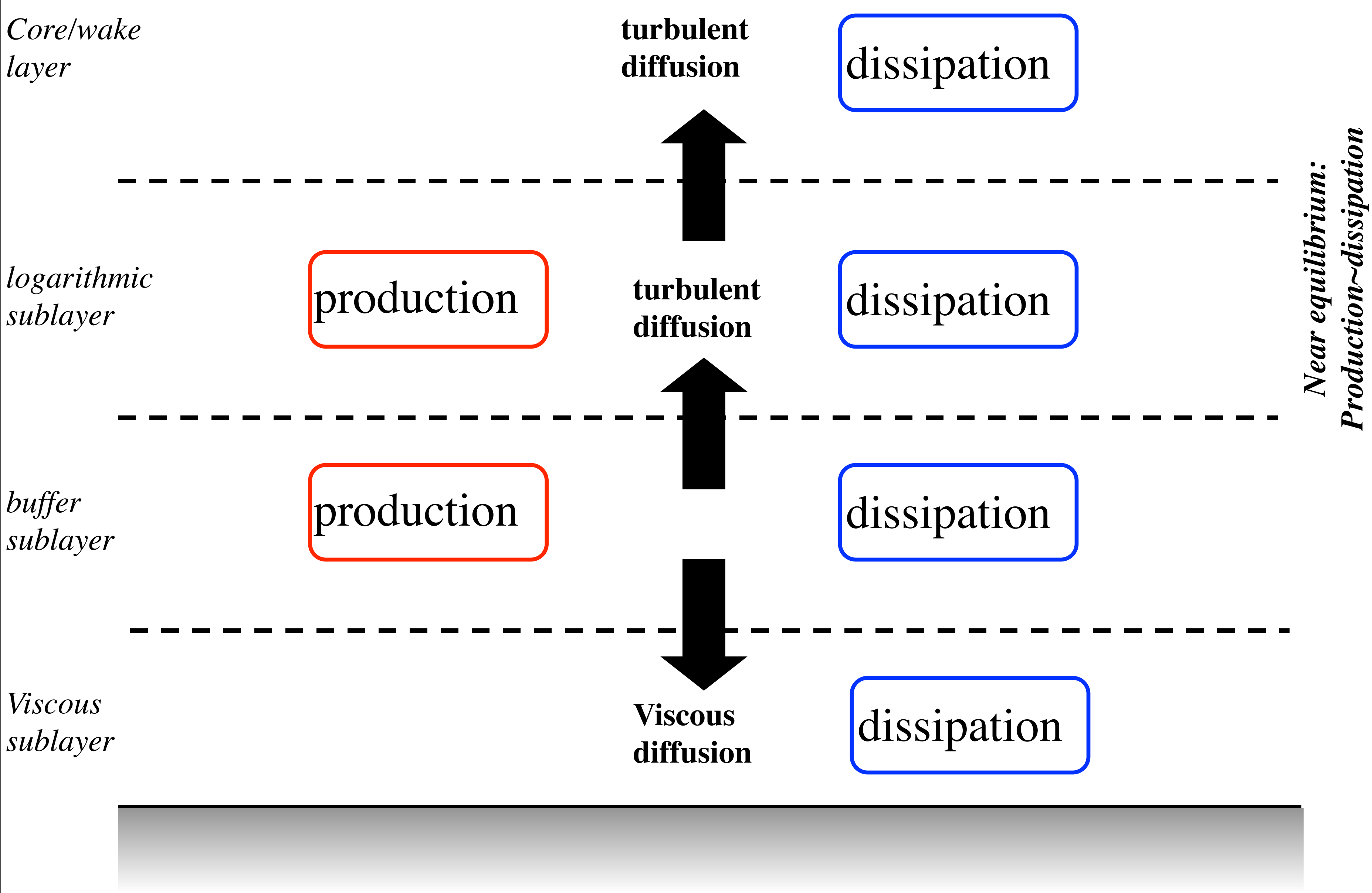


TKE balance

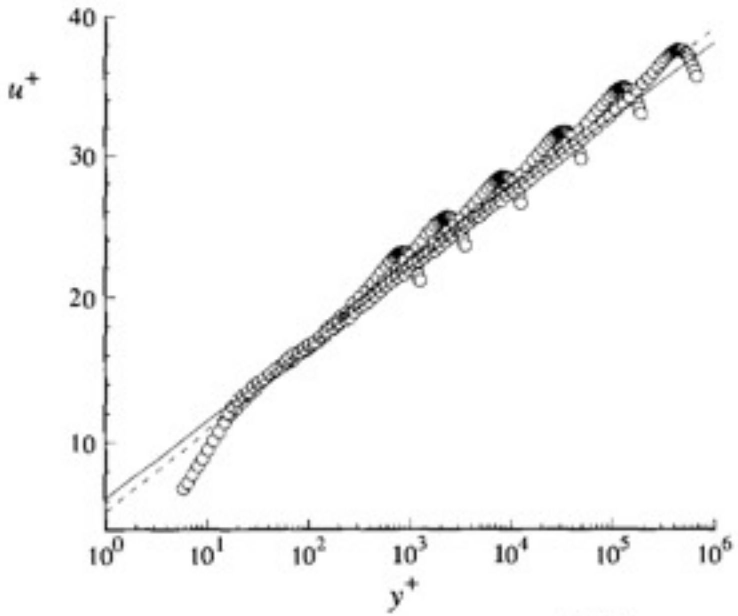


(Schlichting, 8th edn)

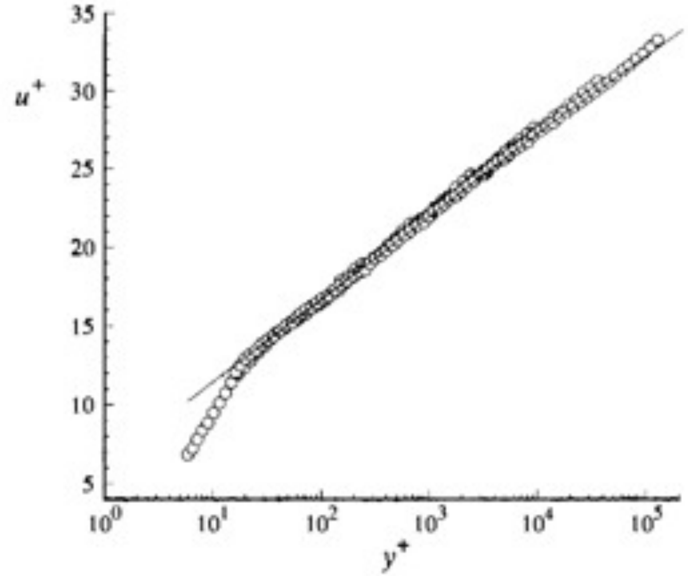
GLOBAL TKE BUDGET



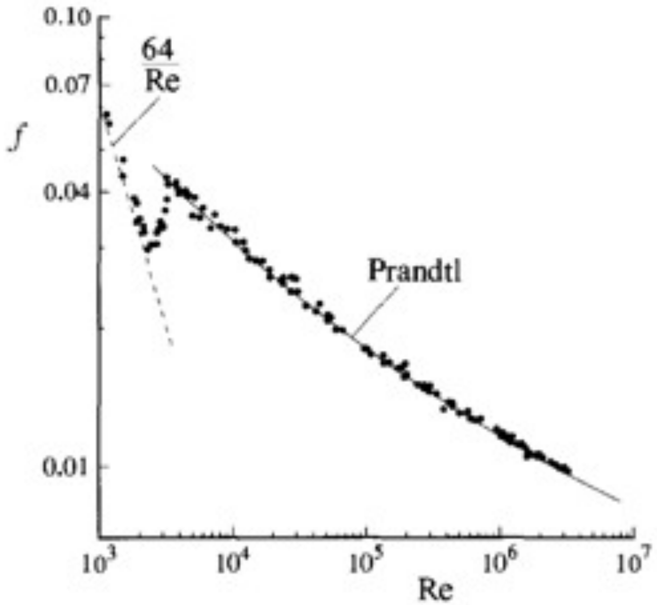
Pipe flows



40
30
20
10
u^+
10^0 10^1 10^2 10^3 10^4 10^5 10^6
y^+

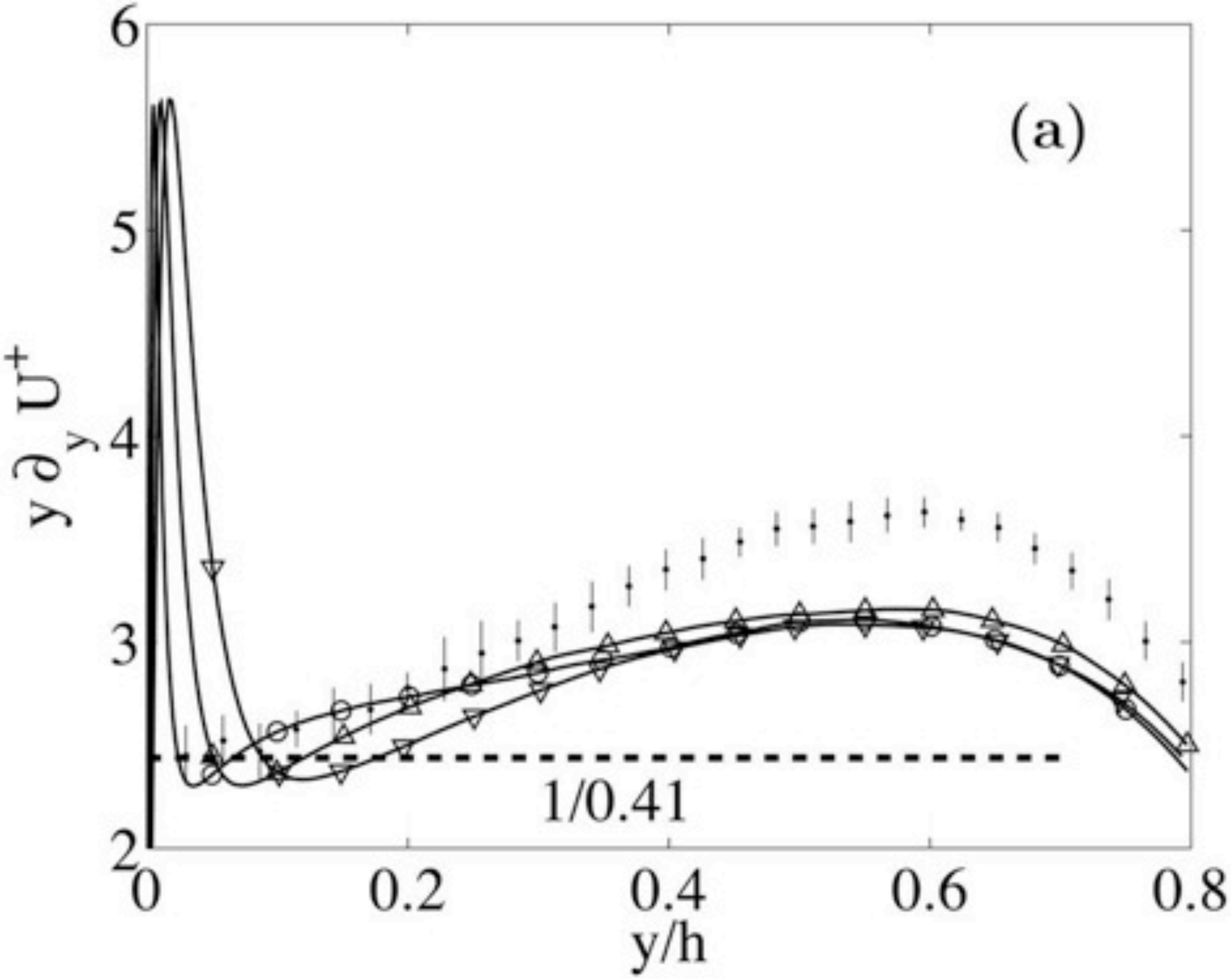


35
30
25
20
15
10
5
u^+
10^0 10^1 10^2 10^3 10^4 10^5
y^+



0.10
0.07
0.04
0.01
f
$\frac{64}{Re}$
Prandtl
10^3 10^4 10^5 10^6 10^7
Re

Is asymptotic theory valid ?



(Hoyas & al., 2006)

Strong deviations in internal flows observed --> open issue

Is the Log Law observable?

(with available experimental setups & computers)

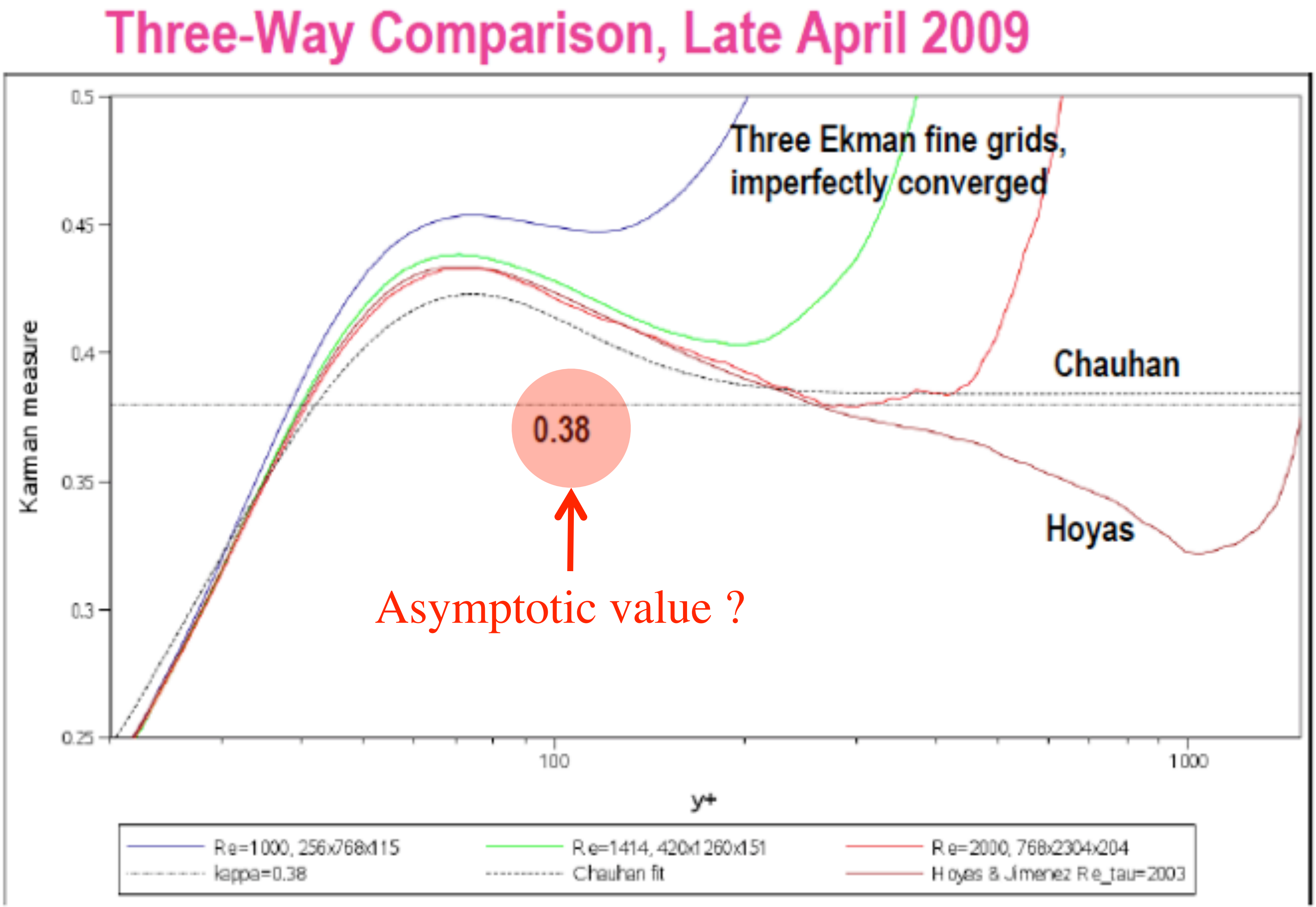
Logarithmic layer extent: $y^+ > 30, \quad y^+ < 0.1\delta^+$

→ Necessary condition: $\delta^+ \geq 300$

1-decade Log Layer: $\delta^+ \geq 3000$

- Larger by a factor about 10 than existing DNS
- Almost equal to maximum reached in wind tunnels (Lille, Melbourne)
- Lower by a factor about 10-100 than real applications !

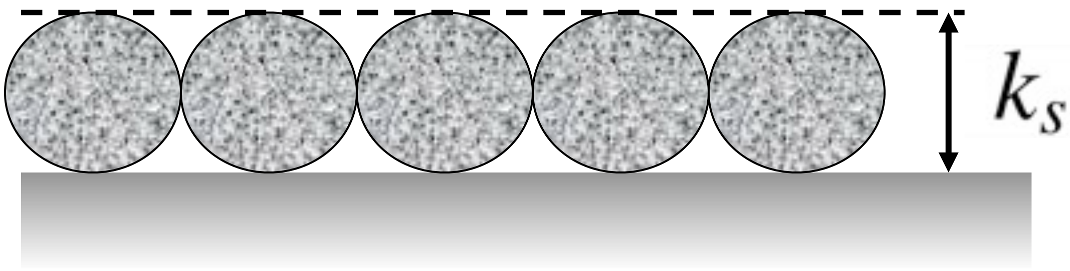
What about « universal constants »?



Roughness effects

- Previous results hold for « ideally smooth » surfaces
- Real materials are not perfect
  - --> a new lengthscale is involved to describe rugous walls:
  - --> how are previous results modified ?

Sand roughness k_s



- Def: sand roughness = height of ideal spherical sand grains
- Hyp: in the inertial layer, one can write

$$\bar{u}^+(y^+) = \frac{1}{\kappa}\ln(y^+) + C^+(k_s^+)$$

Smooth/fully rough surfaces

- Smooth surface: $\lim_{k_s^+ \longrightarrow 0} C^+(k_s^+) = 5.0$
- Rough surface:
  - logarithmic law can be rewritten as

$$\bar{u}^+(y^+) = \frac{1}{\kappa}\ln\left(\frac{y}{k_s}\right) + \underbrace{\frac{1}{\kappa}\ln(k_s^+) + C^+(k_s^+)}_{C_r^+(k_s^+)}$$

  - **fully rough regime** (1<< $k^+{}_s$): viscosity independent solution

$$\lim_{k_s^+ \longrightarrow +\infty} C_r^+(k_s^+) = 8.0 \qquad \textit{(experimental value)}$$

Roughness length y_k

- Logarithmic law can be rewritten as

$$\lim_{k_s^+ \longrightarrow +\infty} \bar{u}^+(y^+, k_s^+) = \frac{1}{\kappa} \ln\left(\frac{y}{y_k}\right), \quad y_k = \frac{\nu}{u_*} e^{-\kappa C^+(k_s^+)}$$

- Fully rough regime $y_k = k_s e^{-8\kappa} = 0.04 k_s$
- y=0 chosen so that the logarithmic law holds

Equivalent sand roughness

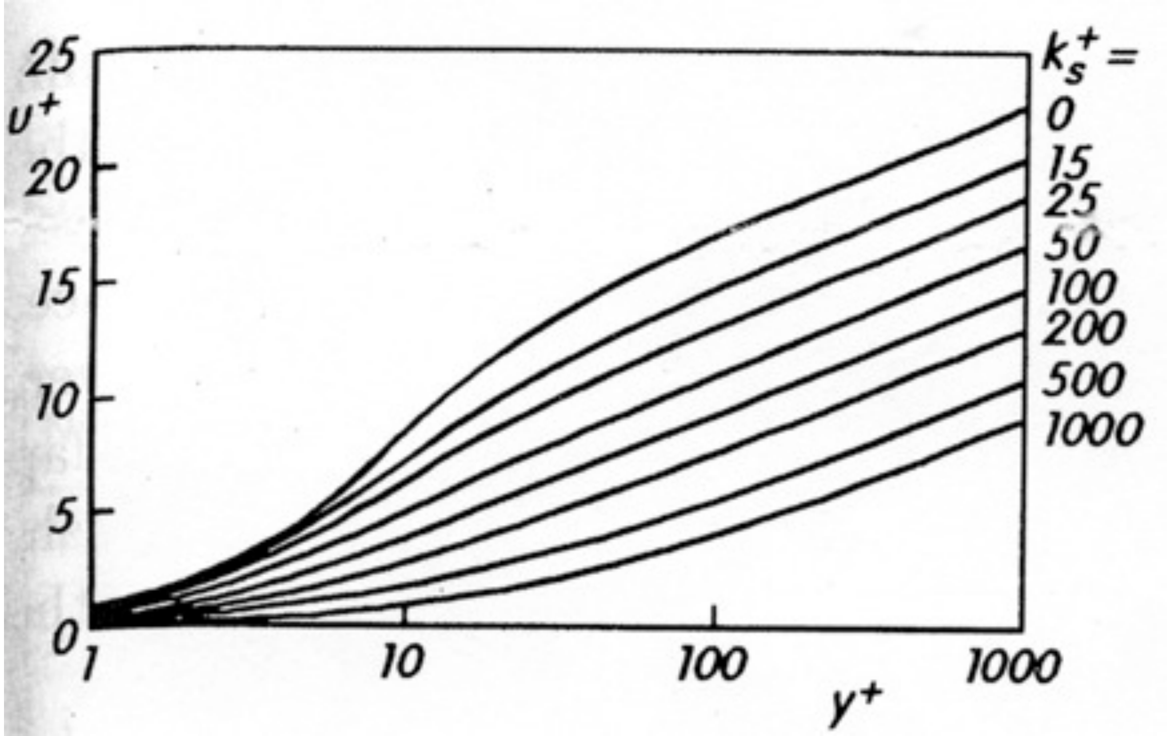
- An equivalent sand roughness can be determined for each technical roughness:

$$k_{s,eq} = \exp\left(\kappa \lim_{y \longrightarrow 0}\left[8.0 + \frac{1}{\kappa}\ln y - u^{+}(y)\right]\right)$$

Measured in laboratory experiment

Roughness regimes

(Schlichting, 8th edn)



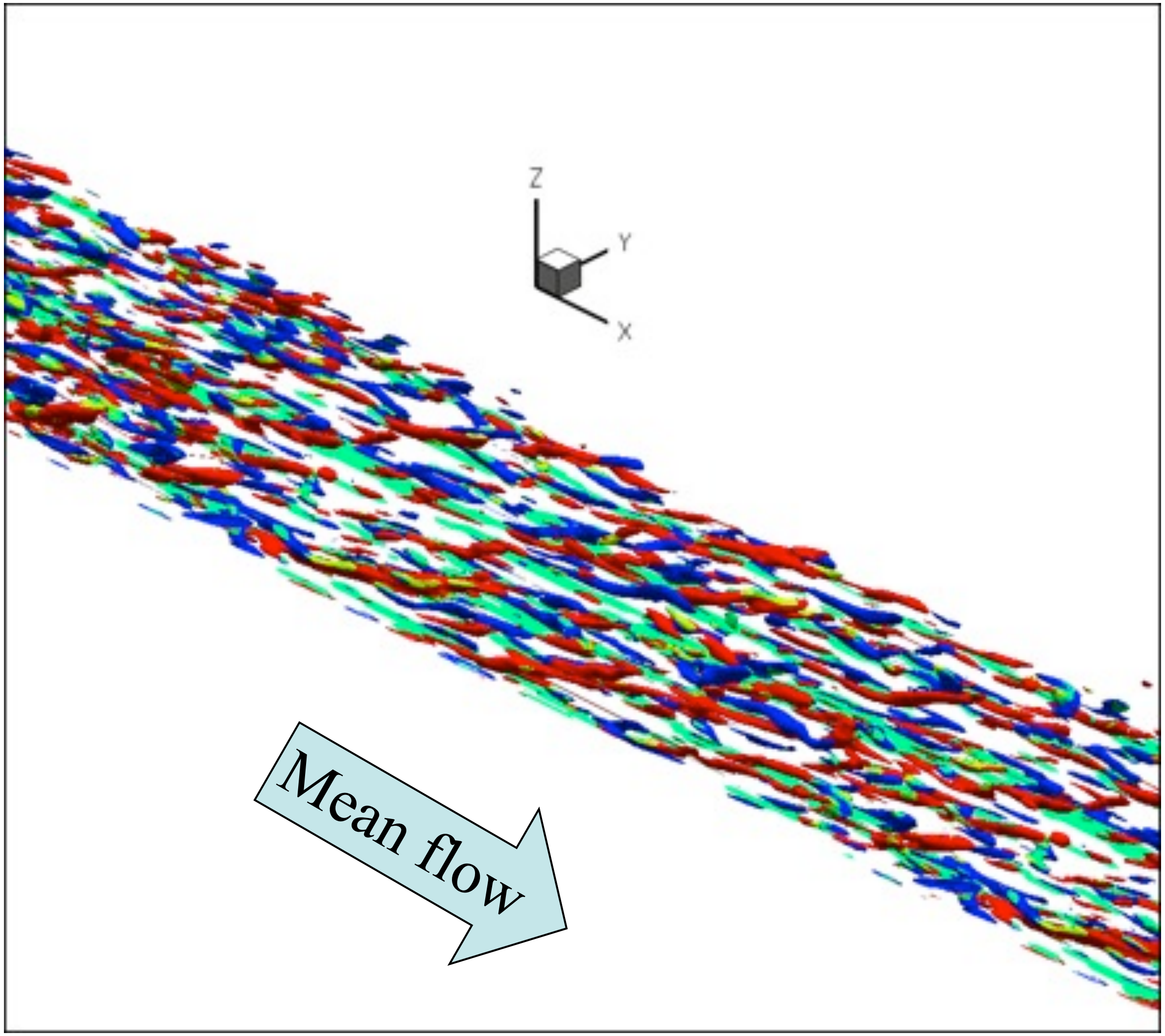
regime	height	constant
Hydraulically smooth	$0 \leq k^+_s \leq 5$	$C^+ \sim 5.0$
Transition regime	$5 \leq k^+_s \leq 70$	$C^+(k^+_s)$
Fully rough	$70 \leq k^+_s$	$C^+_r \sim 5.0$

Coherent structures & turbulence dynamics

- The dynamics is associated with a very complex instantaneous flow organization
- Several types of flow structures are observed
- Each layer exhibits different coherent events
- Identification of the exact role of each structure is still an open controversial issue

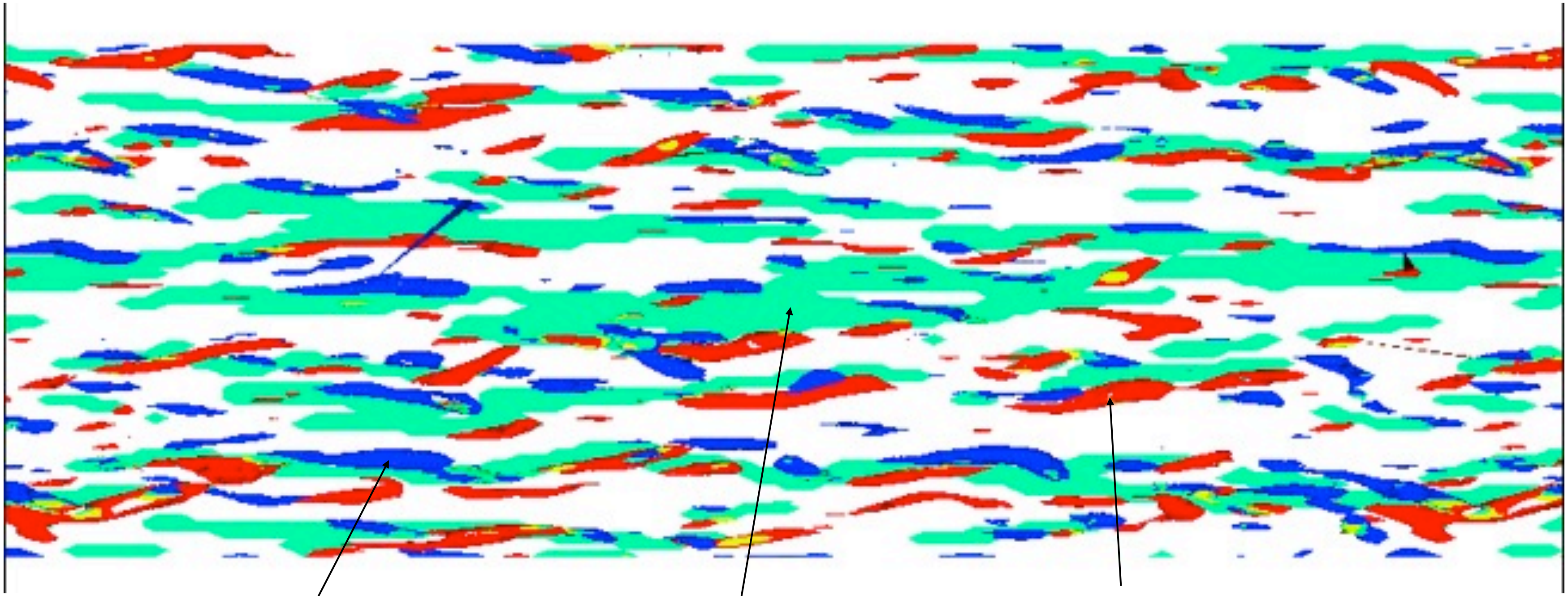
Near-wall region structures

Q-criterion colored by streamwise vorticity



(Pamies & Garnier, ONERA)

Cont'd



Vortex with $\omega_x < 0$

Vortex with $\omega_x > 0$

Region with low instantaneous streamwise velocity

(Pamies & Garnier, ONERA)

What is observed in viscous/buffer layers:

- Low/high-speed Streamwise velocity streaks : sinuous arrays of alternating streamwise jets superimposed on the mean shear *(Kim & al., 1971)*
  - Average spanwise wavelength z^+=50-100 *(Smith & al., 1983)*
  - Average streamwise length x^+=1000
  - Wall shear is higher than the average at locations where the jets point forward (resp. backward) for high speed (resp. low speed) streaks

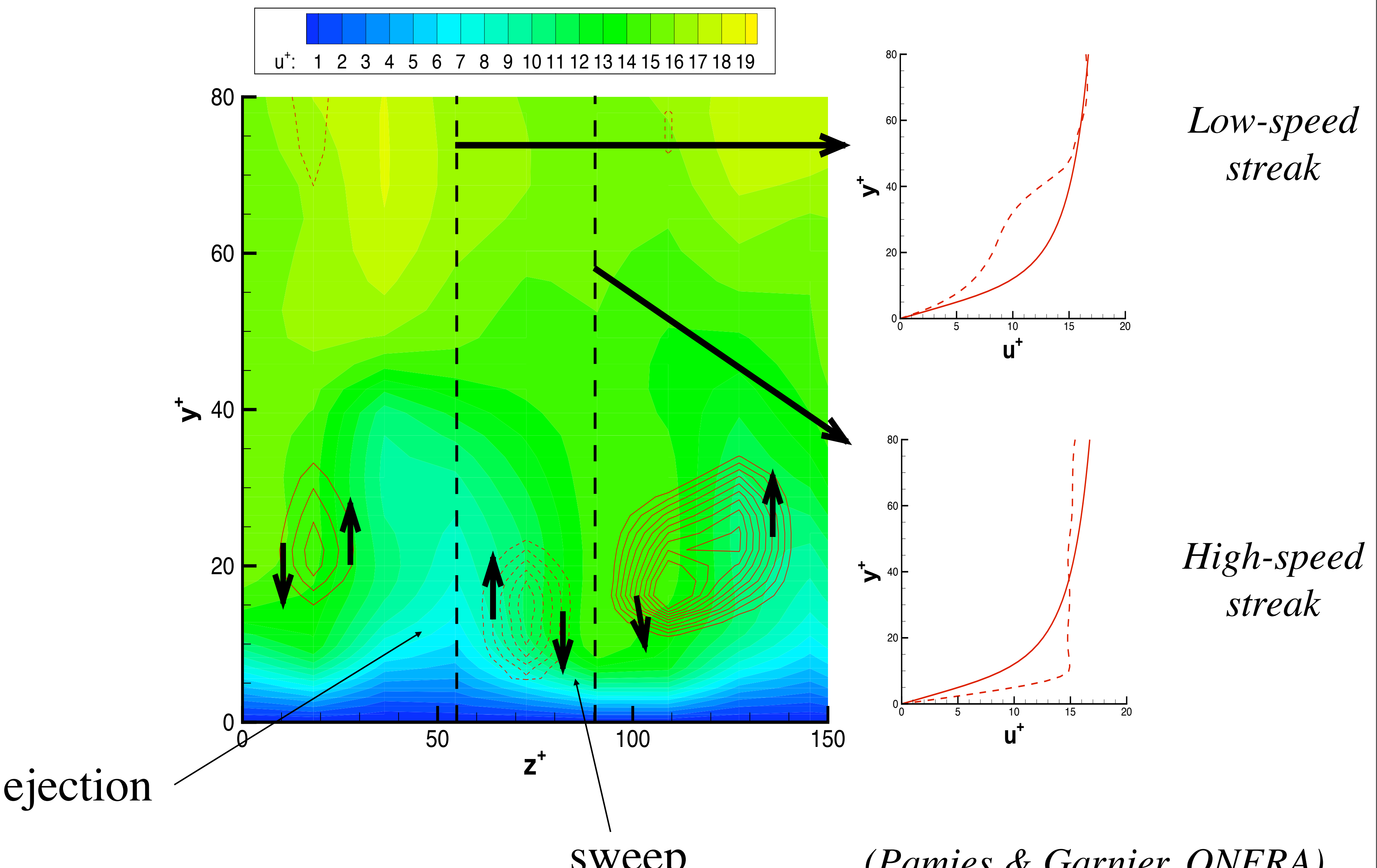
Cont'd

- Quasi-streamwise vortices
  - Slightly tilted from the wall
  - Stay in the near-wall region only for $x^+=200$ *(Jeong & al., 1997)*
  - Several vortices are associated with each streak, with longitudinal spacing $x^+=400$
  - Some of them are connected to legs of hairpin vortices in the log layer, but most merge in uncoherent vorticity away from the wall
  - Are advected at speed $c^+=10$

Vortices, streaks & turbulent drag

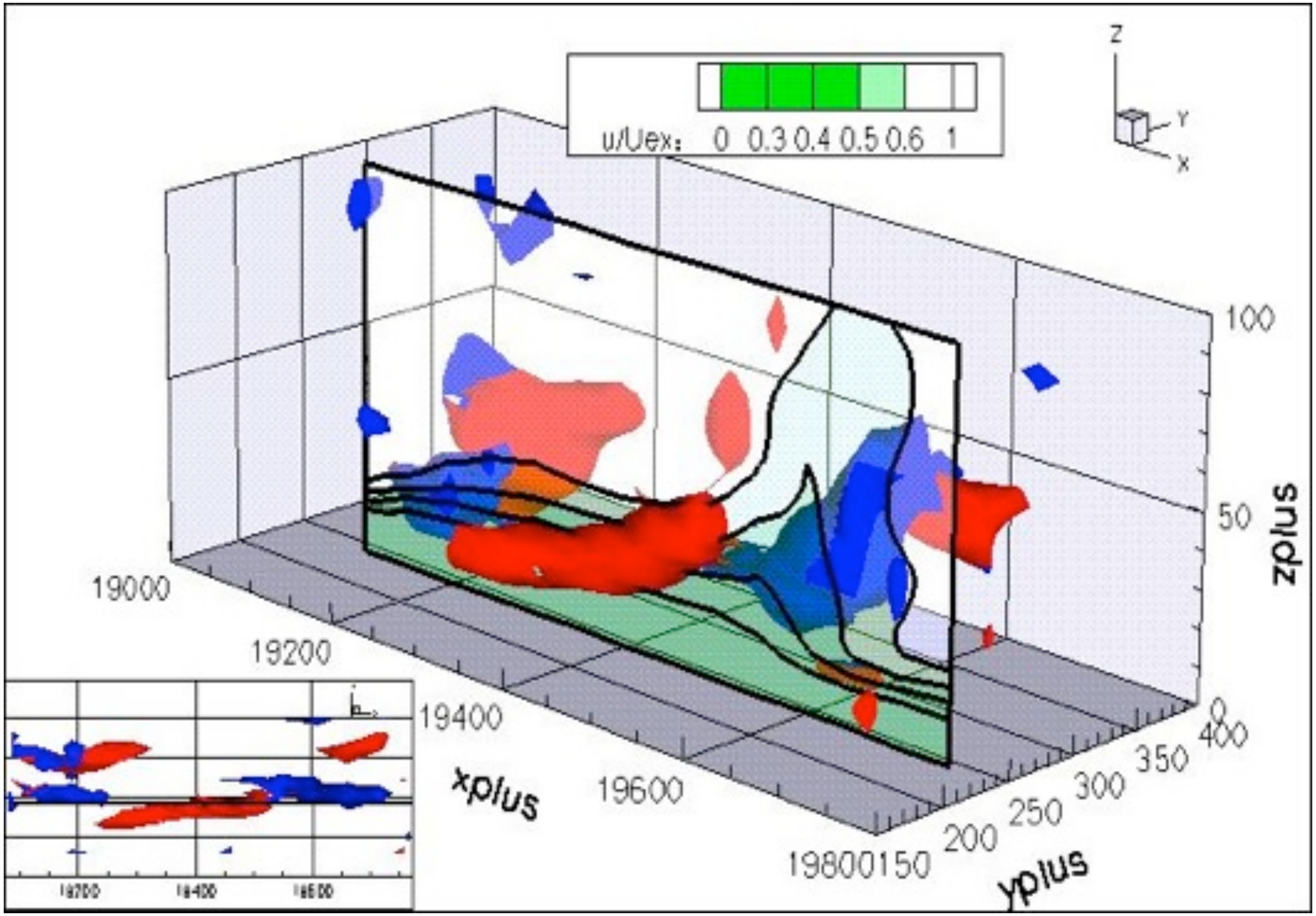
- Quasi-streamwise vortices :
  - cause the streaks by advecting the mean shear *(Blackwelder & Eckelman, 1979)*
  - Are independent of the presence of the wall *(Rashidi & Banerjee, 1990)*
  - Are responsible for the turbulent drag *(Orlandi & Jimenez, 1994)*

Cont'd



(Pamies & Garnier, ONERA)

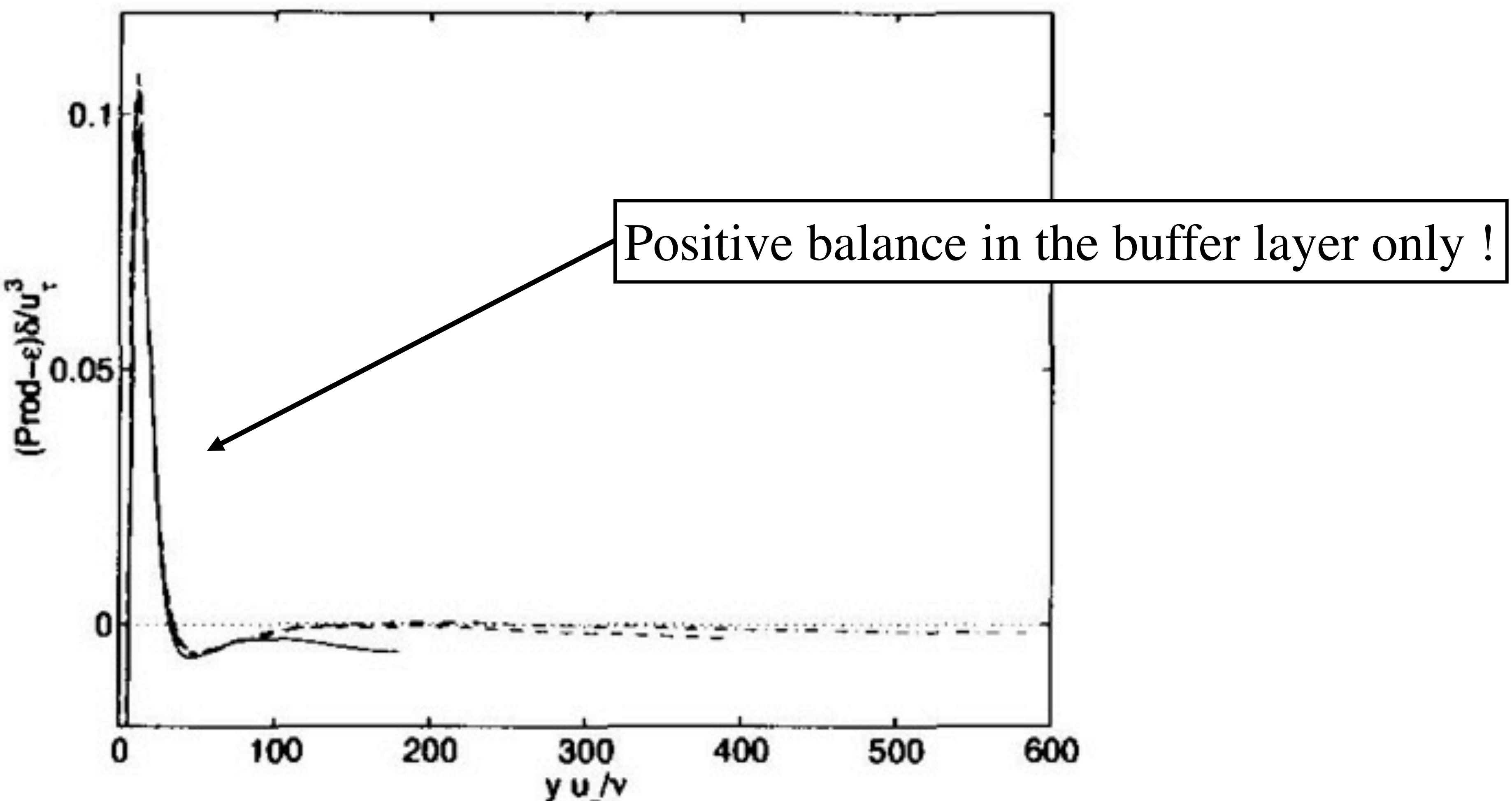
Cont'd



(Pamies & Garnier, ONERA)

Autonomous cycle and SSP

TKE production/dissipation balance



(Jimenez, 1999)

Cont'd

- The fact that TKE balance is positive in a single small part of the full channel raises several question:
  - Existence of a « turbulent engine » located in the buffer layer, which feds the rest of the flow ?
  - Is this mechanism (if any) autonomous, i.e. independent of the flow in the outer layer
  - If any, may it be understood/modelled ?

Autonomous cycle in the buffer layer

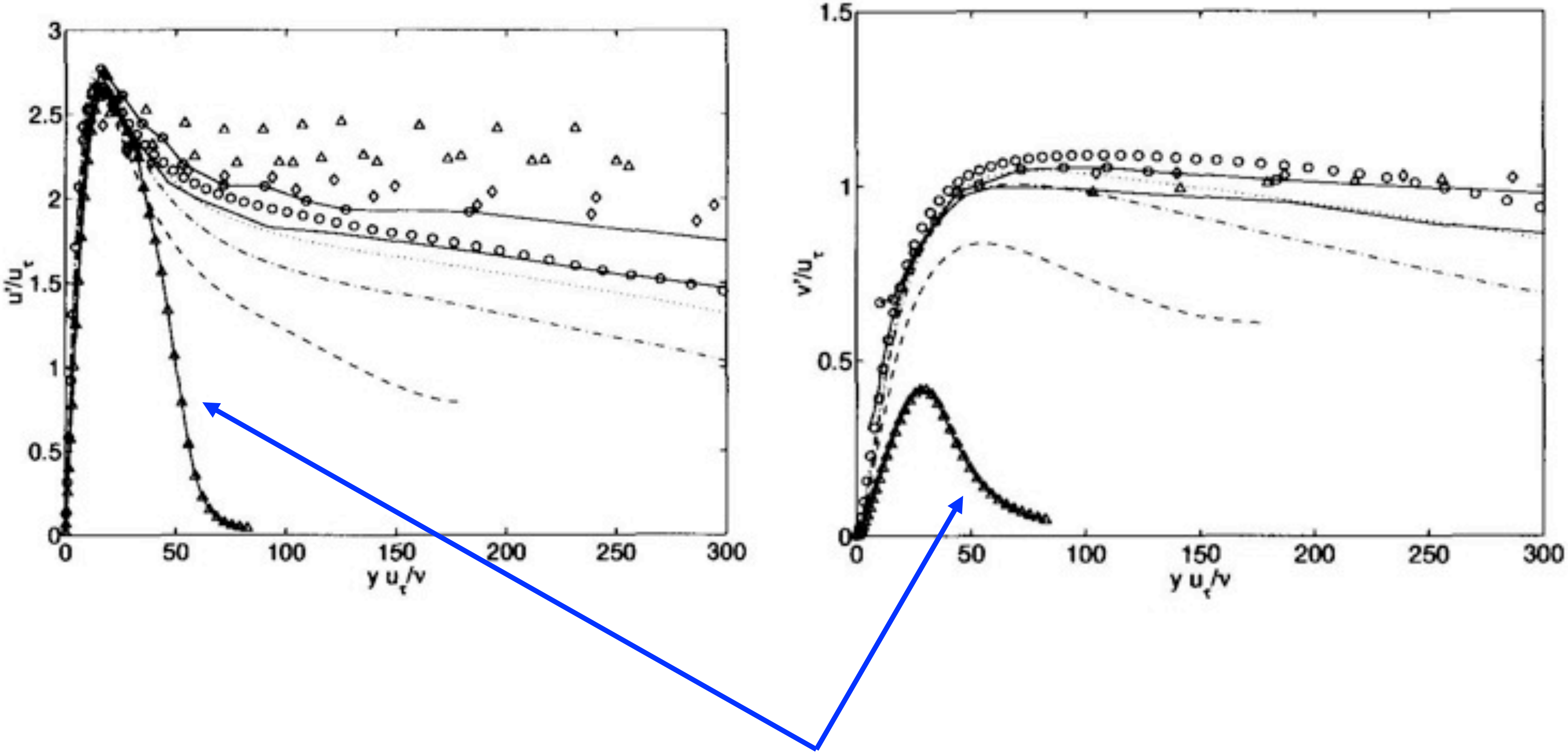
Numerical simulations make it possible to prove the existence of an autonomous cycle in the buffer layer:

1. Streamwise vortices extract energy from the mean flow to create alternating streaks of streamwise velocity
2. Streaks experience inflectional instabilities
3. Perturbations regenerate the vortices

Cont'd

- Features of the autonomous cycle:
  - Located in region $10 \leq y^+ \leq 60$
  - Independent from the outer flow
  - The main role of the solid wall is to sustain the main shear
  - Global turbulence level decays if the cycle is killed

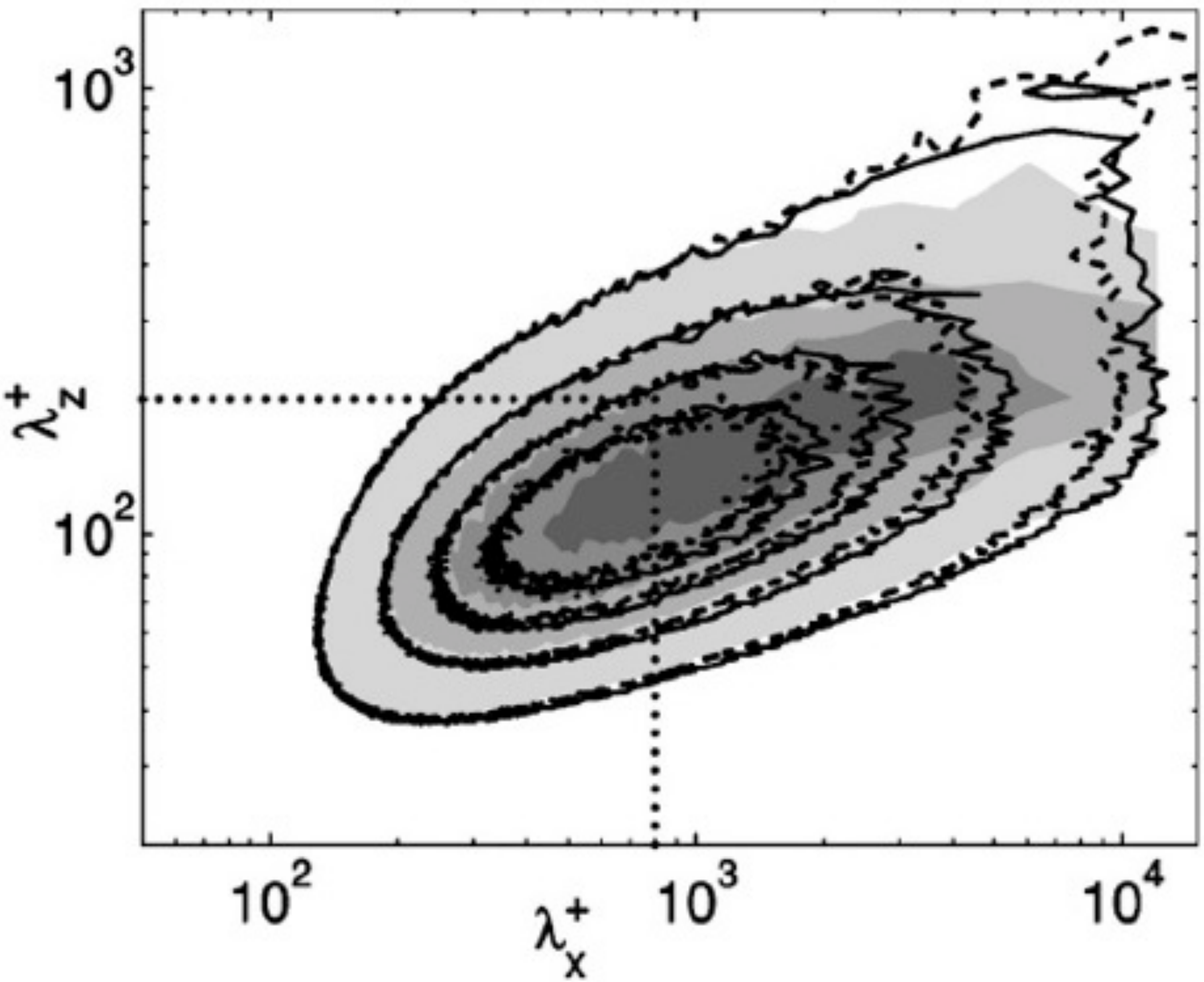
Cont’d



Low-dimensional autonomous flow

(Jimenez, 1999)

$$\phi_{uu}(k_x, k_z, y) \equiv k_x k_z \overline{\hat{u}\hat{u}^*}(k_x, k_z, y)$$

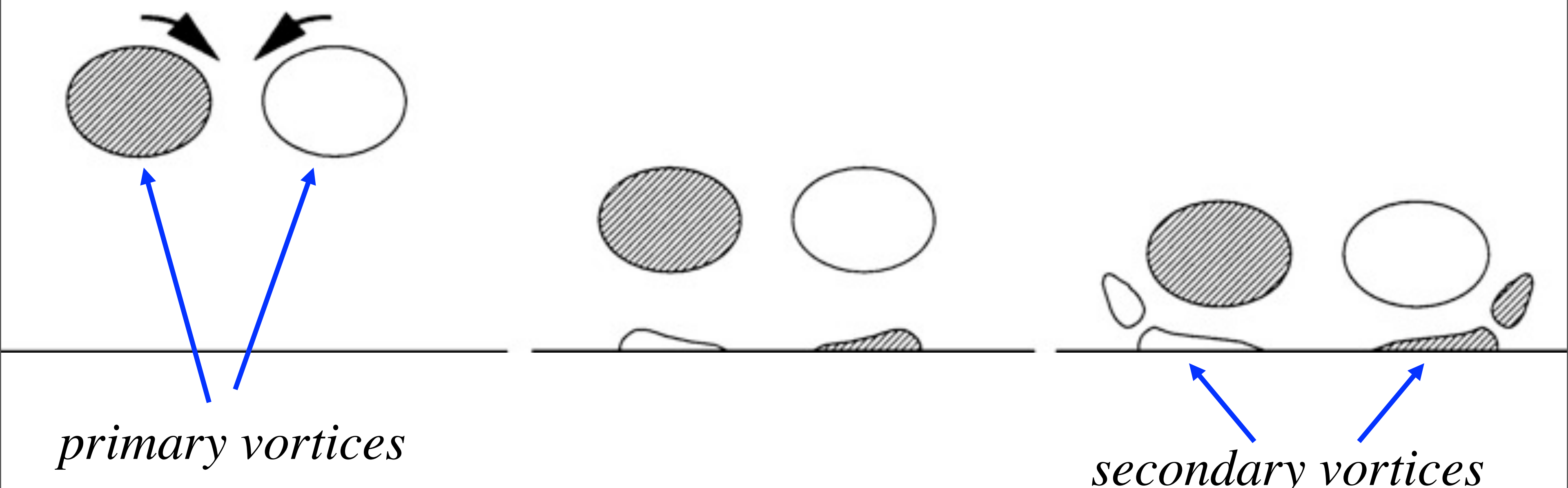


Shaded: autonomous cycle

Lines: full channel computations

(Jimenez & al., 2001)

Example of another vorticity generation mechanism at the wall



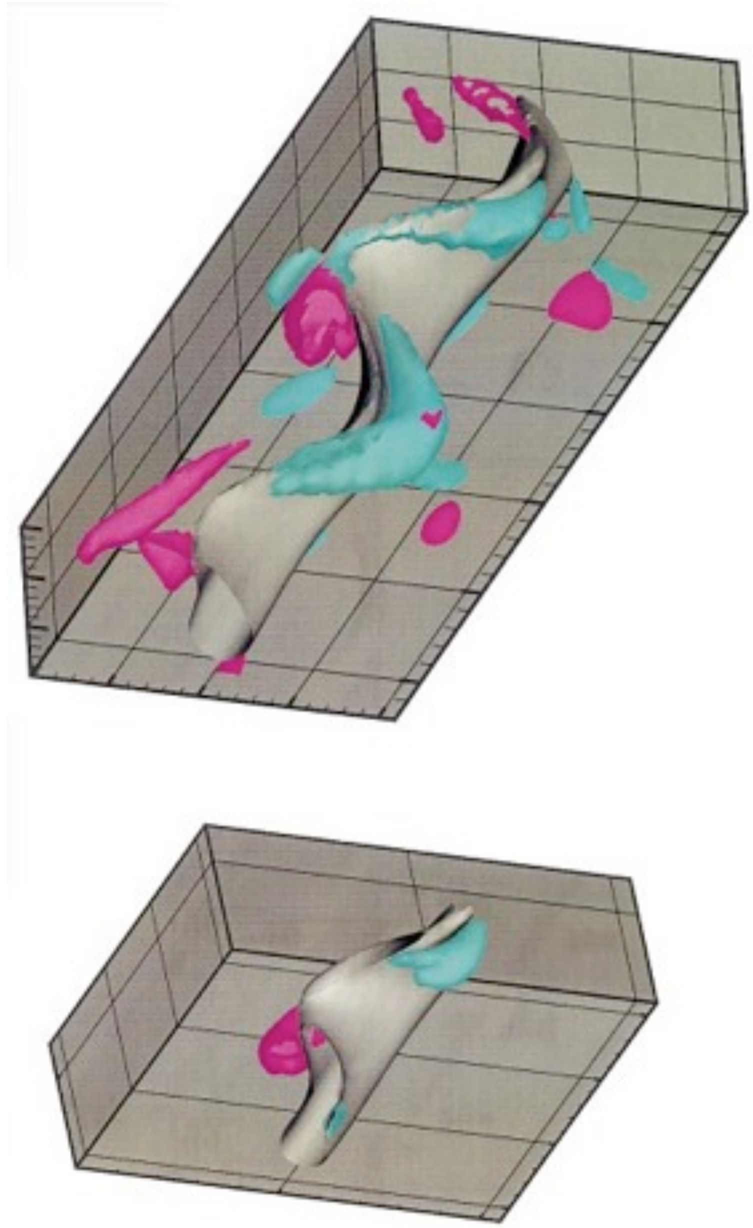
--> occurs certainly, but is not dominant

(Jimenez & al., 1999)

Minimal wall flow *(Jimenez & Moin, 1991)*

- Concept: what is the size of the smallest box in which the cycle is sustained ?
- Numerical experiments lead to $\lambda_x^+ \approx \lambda_z^+ \approx 150$
- Typical pattern: one wavy low-speed streak flanked by two quasi-streamwise vortices

Cont’d

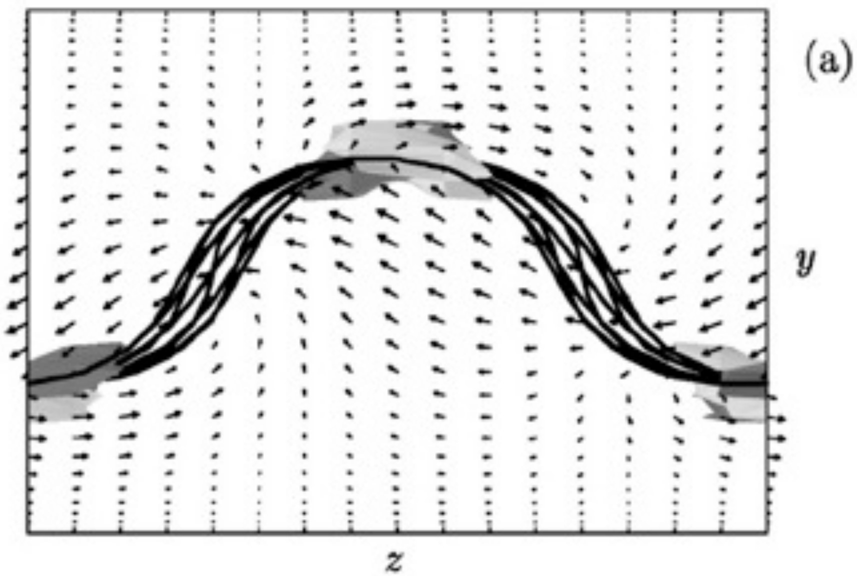


(Jimenez & al., 2001)

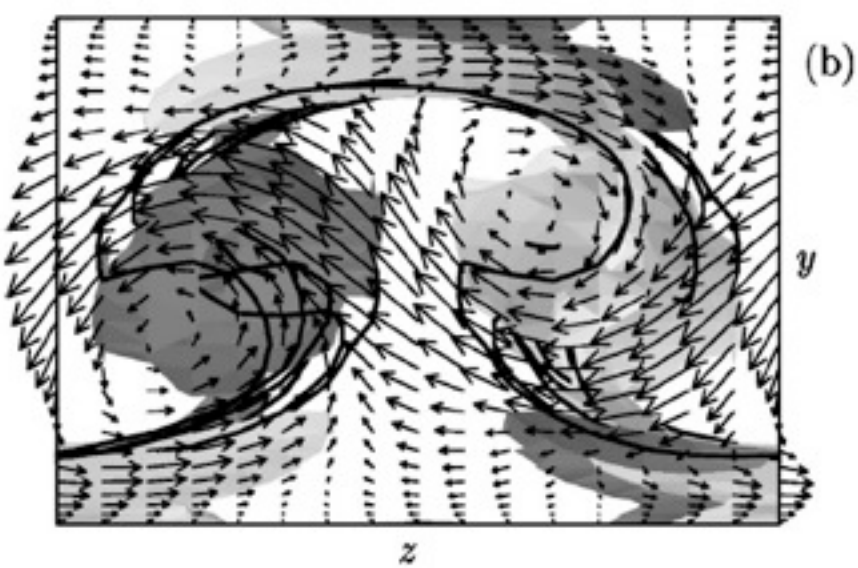
Bridging with theory

- May the autonomous minimal cycle be related to a theoretical model ?
- --> there are several attempts to find exact analytical nonlinear solutions of the Navier-Stokes equations with similar features
  - Steady solutions: the 'minimal flow' is interpreted as a deviation of the flow from a fixed point in phase space
  - Unsteady periodic solutions

E.g. Nagata's steady waves (1990)
(periodic solutions of Couette flow)



Streak dominated mode



Vortex dominated mode