$\sigma_k = 1 \qquad \nu_t \gg \nu$

①

$\bar{u} = 0 \qquad \frac{\partial}{\partial x_j}\langle\,\rangle = 0$

(a) $$\begin{cases} \frac{\partial k}{\partial t} = -\varepsilon \\ \frac{\partial \varepsilon}{\partial t} = -\frac{\varepsilon^2}{k} C_{\varepsilon_2} \end{cases}$$

(b) $$\frac{dk}{d\varepsilon} = \frac{\varepsilon}{\varepsilon^2/k \, C_{\varepsilon_2}} = \frac{1}{C_{\varepsilon_2}} \frac{k}{\varepsilon} \Rightarrow C_{\varepsilon_2} \frac{dk}{k} = \frac{d\varepsilon}{\varepsilon}$$

$$C_\varepsilon \ln k \sim \ln \varepsilon \Rightarrow k^{C_{\varepsilon_2}} \sim \varepsilon$$

$$\Rightarrow \frac{\partial k}{\partial t} \sim k^{C_{\varepsilon_2}} \Rightarrow \boxed{\beta_1 = C_{\varepsilon_2}}$$

(c) $$k(t) = k(0)\left(1 + \frac{t}{t_0}\right)^n \Rightarrow$$

$$\frac{\partial k}{\partial t} = k(0)\frac{n}{t_0}\left(1 + \frac{t}{t_0}\right)^{n-1} \sim k^\beta = k(0)^\beta \left(1 + \frac{T}{T_0}\right)^{\beta n}$$

$$\Rightarrow n - 1 = \beta n \qquad \beta = \frac{n-1}{n} = 1 - \frac{1}{n} = C_{\varepsilon_2}$$

=> dubbio

[redacted] $$\varepsilon = -\frac{\partial k}{\partial t} = -\frac{n k}{T_0\left(1 + \frac{t}{t_0}\right)^{\cancel{n}+1}} \quad (-n+1) \sim k^c$$

$$-n + 1 = \frac{C_{\varepsilon_2}}{C_{\varepsilon_2 - 1}}$$

d) n n'est pas universel, peut varier d'un cas à l'autre selon le spectre

$$k = k(t) \sim t^n = t^{\frac{1}{1-c_{\varepsilon_2}}}$$

(2) $|\underline{s}| = S$ $\quad\quad || \; P = \nu_t S^2$ et $\nu_t = C_\mu \dfrac{k^2}{\varepsilon} \; ||$

(a)
$$\begin{cases} \dfrac{\partial k}{\partial t} = C_\mu \dfrac{k^2}{\varepsilon} S^2 - \varepsilon \\[2ex] \dfrac{\partial \varepsilon}{\partial t} = \dfrac{\varepsilon}{k}\left(C_{\varepsilon_1} C_\mu \dfrac{k^2}{\varepsilon} S^2 - C_{\varepsilon_2} \varepsilon \right). \end{cases}$$

(b) $k(t) = k(0) e^{\lambda S t}$ $\quad\quad \varepsilon(t) = \varepsilon(0) e^{\lambda S t}$

$$\cancel{k(0)} \lambda S = C_\mu \frac{k^{\cancel{2}}(0)}{\varepsilon(0)} S^2 - \varepsilon(0) \Rightarrow \lambda S = C_\mu \frac{k(0)}{\varepsilon(0)} S^2 - \frac{\varepsilon(0)}{k(0)}$$

$$\lambda S = C_\mu C_{\varepsilon_1} \frac{k(0)}{\varepsilon(0)} S^2 - C_{\varepsilon_2} \frac{\varepsilon(0)}{k(0)}$$

$$\Rightarrow \begin{cases} \lambda = C_{\varepsilon_1} C_\mu \dfrac{k(0)}{\varepsilon(0)} S - C_{\varepsilon_2} \dfrac{\varepsilon(0)}{k(0) S} \\[2ex] \lambda = C_\mu \dfrac{k(0) S}{\varepsilon(0)} - \dfrac{\varepsilon(0)}{k(0) S} \end{cases} \quad\quad \frac{k(0) S}{\varepsilon(0)} = \xi$$

$$\lambda = C_{\varepsilon_1} C_\mu \xi - \frac{C_{\varepsilon_2}}{\xi} = C_\mu \xi - \frac{1}{\xi}$$

$$\Rightarrow \xi \lambda = C_{\varepsilon_1} C_\mu \xi^2 - C_{\varepsilon_2} = C_\mu \xi^2 - 1$$

$$(C_\mu C_{\varepsilon_1} - C_\mu)\xi^2 - (C_{\varepsilon_2} - 1) = 0$$

$$\xi^2 = \frac{C_{\varepsilon_2} - 1}{C_\mu (C_{\varepsilon_1} - 1)}$$ ok

$$\lambda = C_\mu \xi - \frac{1}{\xi} \Rightarrow \lambda^2 = C_\mu^2 \xi^2 + \frac{1}{\xi^2} - 2 C_\mu$$

Insérant ξ^2 $\Rightarrow$

$$\lambda^2 = C_\mu^{\cancel{2}} \frac{(C_{\varepsilon_2} - 1)}{\cancel{C_\mu} (C_{\varepsilon_1} - 1)} + \frac{C_\mu (C_{\varepsilon_1} - 1)}{C_{\varepsilon_2} - 1} - 2 C_\mu$$

$$\lambda^2 = \frac{C_\mu \left[(C_{\varepsilon_2} - 1)^2 + (C_{\varepsilon_1} - 1)^2 - 2 (C_{\varepsilon_2} - 1)(C_{\varepsilon_1} - 1) \right]}{(C_{\varepsilon_2} - 1)(C_{\varepsilon_1} - 1)}$$

$$= C_\mu \frac{(C_{\varepsilon_1} - C_{\varepsilon_2})^2}{(C_{\varepsilon_1} - 1)(C_{\varepsilon_2} - 1)} = f(C_{\varepsilon_1}, C_{\varepsilon_2}, C_\mu)$$

(c) $P = \nu_t S^2 \qquad \nu_t = C_\mu \frac{k^2}{\varepsilon}$

$$\frac{P}{\varepsilon} = \frac{\nu_t S^2}{\varepsilon} = C_\mu \frac{k^2}{\varepsilon} \frac{S^2}{\varepsilon} = C_\mu \left(\frac{k S}{\varepsilon} \right)^2 = C_\mu \xi^2$$

$$= C_\mu \frac{C_{\varepsilon_2} - 1}{C_\mu (C_{\varepsilon_1} - 1)} = f(C_{\varepsilon_1}, C_{\varepsilon_2})$$

③

$$\bar{u} = (\bar{u}(y), 0, 0)$$

$$P = -\overline{u'v'}\,\frac{d\bar{u}}{dy}$$

$$u^+(y^+) = \frac{1}{\kappa}\log(y^+) + C \qquad \kappa = 0.41 \quad C = 5.1$$

(a) $u_\tau = \sqrt{\nu \frac{d\bar{u}}{dy}(0)}$

(b) $\frac{du^+}{dy^+} = \frac{1}{\kappa}\frac{1}{y^+} \qquad \frac{d\bar{u}}{dy} = \frac{u_\tau}{\kappa y}$

(c) $-\overline{u'v'}(y) = \text{cost.} = u_\tau^2$ Zone Logarithmique $(P = \varepsilon)$

$$P = -\overline{u'v'}\,\frac{d\bar{u}}{dy} = \frac{u_\tau^3}{\kappa y}$$

(d) $\nu_t = C_\mu \frac{k^2}{\varepsilon} \qquad P = \nu_t |S|^2 = \nu_t \left|\frac{d\bar{u}}{dy}\right|^2 = \frac{u_\tau^3}{\kappa y}$

$$\nu_t = \frac{u_\tau^3}{\kappa y}\frac{1}{u_\tau^2}\kappa^2 y^2 = u_\tau \kappa y$$

(e) $\nu_t = u_\tau \kappa y = C_\mu \frac{k^2}{\varepsilon} \qquad P = \varepsilon \Rightarrow$

$$u_\tau \kappa y = C_\mu k^2 / u_\tau^3 * \kappa y \quad \Rightarrow \quad \boxed{C_\mu = \frac{u_\tau^4}{k^2}} \sim \text{cf. } 0.09$$

(f) $$\begin{cases} P = \varepsilon \\ 0 = \frac{\varepsilon}{k}\left(C_{\varepsilon 1} - C_{\varepsilon 2}\right)\frac{u_\tau^3}{\kappa y} + \frac{\partial}{\partial y}\left(\frac{u_\tau \kappa y}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial y}\right) \end{cases}$$

$$\varepsilon = \frac{u_\tau^3}{\varkappa y} \Rightarrow$$

$$\nu_t = u_\tau \varkappa y = C_\mu \frac{k^2}{\varepsilon} = C_\mu \frac{k^2}{\frac{u_\tau^3}{\varkappa y}} \Rightarrow \frac{u_\tau^4}{C_\mu} = k^2 \quad \text{(vedi domanda (c))}$$

$$\Rightarrow \boxed{k = \frac{u_\tau^2}{\sqrt{C_\mu}}} \qquad P\frac{\varepsilon}{k} = \frac{\varepsilon}{k}\frac{u_\tau^3}{\varkappa y} = \left(\frac{u_\tau^3}{\varkappa y}\right)^2 \frac{\sqrt{C_\mu}}{u_\tau^2}$$

$$\Rightarrow 0 = \left(\frac{u_\tau^3}{\varkappa y}\right)^2 \frac{\sqrt{C_\mu}}{u_\tau^2}\left(C_{\varepsilon_1} - C_{\varepsilon_2}\right) + \frac{u_\tau \varkappa}{\sigma_\varepsilon}\frac{d\varepsilon}{dy} + \frac{u_\tau \varkappa y}{\sigma_\varepsilon}\frac{d^2\varepsilon}{dy^2}$$

$$0 = \left(\frac{u_\tau^3}{\varkappa y}\right)^2 \frac{\sqrt{C_\mu}}{u_\tau^2}\left(C_{\varepsilon_1} - C_{\varepsilon_2}\right) \underbrace{- \frac{u_\tau \cancel{\varkappa}}{\sigma_\varepsilon}\frac{u_\tau^3}{\cancel{\varkappa} y^2} + \frac{u_\tau \cancel{\varkappa} y}{\sigma_\varepsilon}\frac{2 u_\tau^3}{\cancel{\varkappa} y^{\cancel{3}2}}}_{= \frac{u_\tau^4}{\sigma_\varepsilon}\frac{1}{y^2}}$$

$$\sigma_\varepsilon = -\frac{\cancel{u_\tau^4} \varkappa^2 y^2 \cancel{u_\tau^2}}{(C_1 - C_2) y^2 \cancel{u_\tau^6}\sqrt{C_\mu}} = \frac{-\varkappa^2}{\sqrt{C_\mu}\left(C_{\varepsilon_1} - C_{\varepsilon_2}\right)} \quad //$$

4

(a)

$$u(0) = 0 \Rightarrow R_{ij}(0) = 0$$

$$k(0) = \frac{1}{2}\sum_i \overline{u_i'^2}(0) = 0 \qquad R_{ij}(0) = 0$$

$$\Rightarrow k(0) = \overline{u'u'} = 0$$

$$\frac{dk}{dy}(0) = \overline{u'\frac{du'}{dy}}\Big|_0 = 0$$

(b) éq. k

$P = \overline{u_i'u_j'} \frac{\partial \bar{u}_i}{\partial y_j} \underset{y \to 0}{\longrightarrow} 0$ $\qquad \nu_t = C_\mu \frac{k^2}{\varepsilon} \underset{y \to 0}{\longrightarrow} 0$

$P \approx \nu_t |\bar{S}|^2$ et $\bar{S}(0) \neq 0$

pour $y \to 0$

$\Rightarrow \quad 0 = -\varepsilon(0) + \nu \frac{d^2 k(0)}{dy^2}$

$\frac{d^2 k}{dy^2} = + \frac{\varepsilon(0)}{\nu}$ $\qquad k(y) = A + By + \frac{\varepsilon(0) y^2}{2\nu}$ $\qquad \frac{dk}{dy} = B + \frac{\varepsilon(0) y}{\nu}$

$k(0) = 0 \quad \frac{dk}{dy}(0) = 0 \quad \Rightarrow \quad A = B = 0$

$k(y) = \frac{\varepsilon(0)}{2\nu} y^2$ pour $y \approx 0$ $\qquad \Rightarrow \quad \varepsilon(0) = \lim_{y \to 0} \frac{2 k(y) \nu}{y^2}$

ok

(c) éq. (2) $\quad \frac{\varepsilon}{k}(C_1 P - C_2 \varepsilon) + \nu \frac{d^2 \varepsilon}{dy^2}$

$\lim_{y \to 0} \Rightarrow$ car $\nu_t \to 0$ $\quad P \to 0$ mais $k \sim y^2$ pour $y \to 0$

et $\varepsilon \to \varepsilon(0) \sim$ cst. $\quad \Rightarrow \quad \boxed{\frac{\varepsilon}{k} \to \infty}$

$\frac{dk}{dt} = -\varepsilon \quad \Rightarrow \quad \frac{\varepsilon}{k} \sim t^{-1}$ $\qquad$ échelle de temps divergeant

effets visqueux négligés $\qquad$ (effets linéaires)

la plus petite échelle de temps $\boxed{\tau_\eta = \left(\frac{\nu}{\varepsilon}\right)^{1/2}}$ $\qquad$ temps Kolmogorov