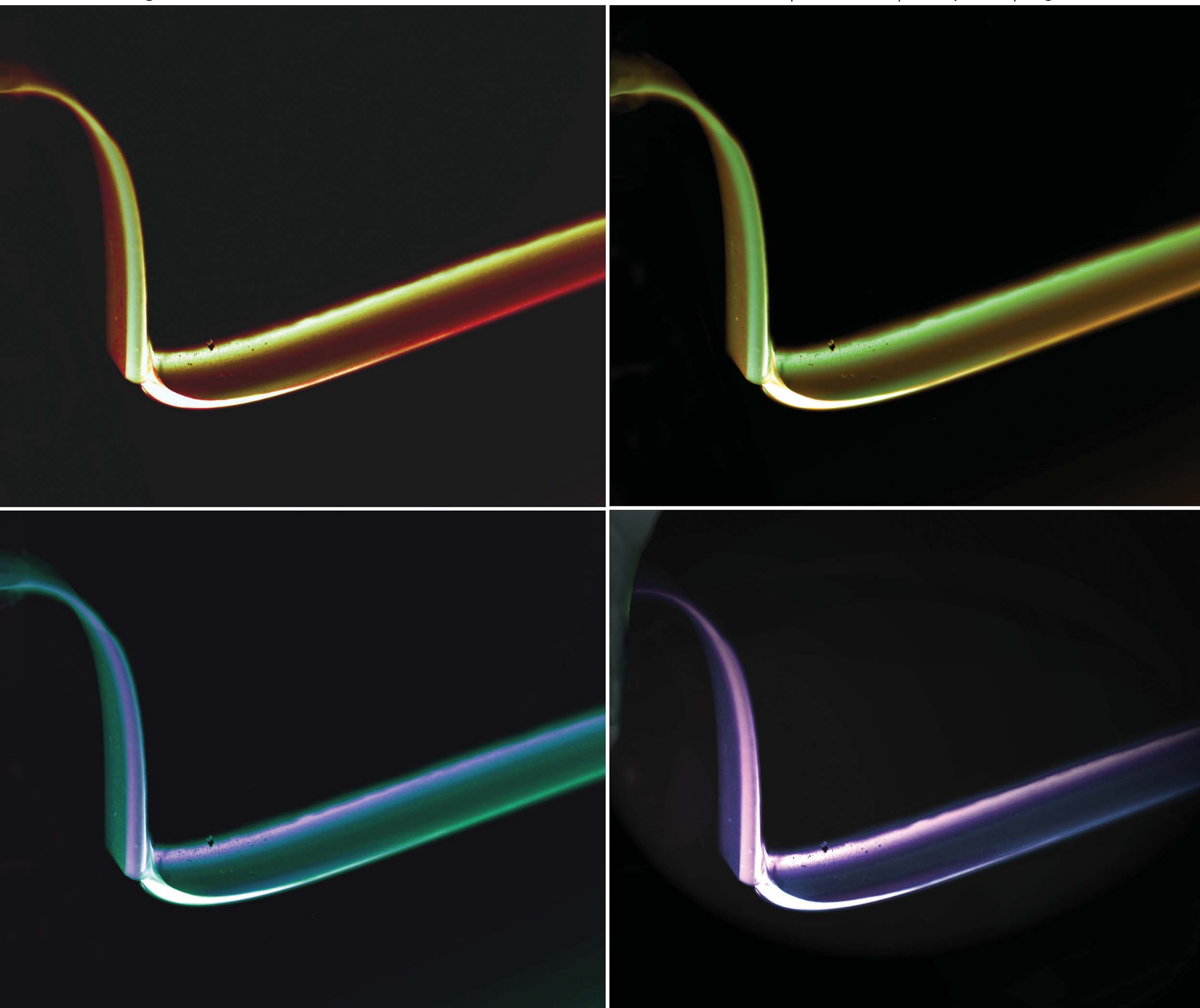


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**PAPER**

Marco Rivetti and Arnaud Antkowiak  
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# Elasto-capillary meniscus: pulling out a soft strip sticking to a liquid surface†

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Marco Rivetti‡\* and Arnaud Antkowiak\*

A liquid surface touching a solid usually deforms in a near-wall meniscus region. In this work, we replace part of the free surface with a soft polymer and examine the shape of this 'elasto-capillary meniscus', a result of the interplay between elasticity, capillarity and hydrostatic pressure. We focus particularly on the extraction threshold for the soft object. Indeed, we demonstrate both experimentally and theoretically the existence of a limit height of liquid tenable before breakdown of the compound and extraction of the object. Such an extraction force is known since Laplace and Gay-Lussac, but only in the context of rigid floating objects. We revisit this classical problem by adding the elastic ingredient and predict the extraction force in terms of the strip elastic properties. It is finally shown that the critical force can be increased with elasticity, as is commonplace in adhesion phenomena.

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## 1 Introduction

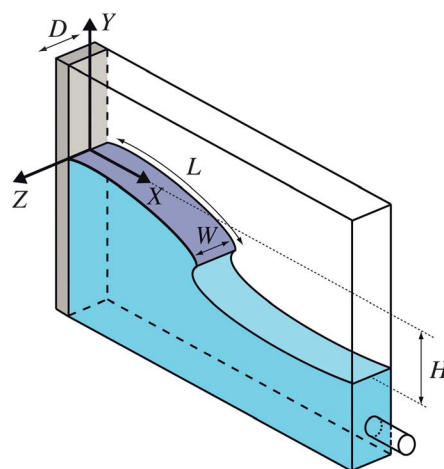
Because interfaces are sticky, it is more difficult to pull out an object resting on a liquid surface than to simply lift it. This is a common illustration of capillary adhesion. The excess force actually corresponds to the weight of the liquid column drawn behind the pulled object, but relies intimately on the surface tension as the liquid column is sculpted by capillarity into a *meniscus*. Laplace in 1805 was the first to investigate this problem in his founding monograph on capillarity.<sup>1</sup> The theory designed by Laplace predicted a maximal height for the liquid meniscus above which no more solution could be found, hence a maximal force is needed to extract an object. On Laplace's request Gay-Lussac undertook careful experiments on the traction of glass disks floating on water and obtained beautiful agreement with this first ever theory of capillarity.

Adhesion is not the only consequence of the existence of menisci. Collective behaviors such as the clustering of bubbles into rafts are also related to this capillary interface deformation. Such capillary attraction (or repulsion) has been known for long, and even used in the early 50's as a model for atomic interactions.<sup>2</sup> In the case of anisotropic particles, clustering becomes directional and allows for self-assembly or precise alignment of particles.<sup>3,4</sup>

In all these cases, there is an unavoidable elastic deformation imparted by the meniscus to the neighboring solid

substrate, which may become non-negligible for soft enough materials. Quite recently there has been a keen interest on the possibility to strongly deform an object with such elasto-capillary interactions. As a matter of fact, the variety of elasto-capillary driven shape distortion is surprisingly vast: clumping,<sup>5</sup> buckling,<sup>6</sup> wrinkling<sup>7</sup> or selectable packing,<sup>8</sup> as well as change in wetting properties<sup>9</sup> to quote a few examples, all being particularly relevant at small scales.<sup>10</sup>

Therefore, in contrast to Laplace and Gay-Lussac's experiment, elastic structures resting on liquid interfaces may experience large deformations when dipped in or withdrawn from



**Fig. 1** Elasto-capillary meniscus experiment. A thin polymer strip (violet), of length  $L$  and width  $W$ , is clamped inside a narrow rigid box partially filled with water (blue). Initially, the water level matches the embedding height. Upon withdrawal of the liquid, the soft strip and the free surface deform altogether into an elasto-capillary meniscus of height  $H$ .

Université Pierre et Marie Curie and Centre National de la Recherche Scientifique, Unité Mixte de Recherche 7190, Institut Jean Le Rond d'Alembert, 4 Place Jussieu, F-75005 Paris, France. E-mail: arnaud.antkowiak@upmc.fr

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‡ Present address: Surface du Verre et Interfaces, UMR 125, CNRS & Saint Gobain, 93303 Aubervilliers, France. E-mail: marco.rivetti@saint-gobain.com.

the liquid, because of interactions due to hydrostatic pressure and surface tension. Nature offers an elegant example of such deformations with aquatic flowers avoiding flooding of the pistil by folding their corolla as the level of water increases.<sup>11</sup> This mechanism recently inspired Reis *et al.*<sup>12</sup> to invent a smart technique to grab water from a bath using a passive 'elasto-pipette'. A different but related problem is the case of a thin polymeric film draping over a point of contact on water,<sup>13</sup> showing the appearance of wrinkles and their transition towards localized folds. Elasticity is also the key to understand some surprising phenomena involving free surfaces, for instance the difficulty in submerging hairs into a liquid by pushing them,<sup>14</sup> the water-repellent ability of striders' legs<sup>15,16</sup> or even the possibility for some insects and reptiles to walk on water.<sup>17</sup> On this topic, Burton and Bush<sup>18</sup> proposed recently a simple model to describe how elasticity modifies the buoyancy properties of a floating rod.

In the present study we propose a fully non-linear description of the shape of a soft thin object resting on (and deformed by) a liquid surface. We study particularly the quasi-static extraction of such an object from the liquid bath and we consider the deformed liquid–solid ensemble as an *elasto-capillary meniscus*, a generalization of the well-known capillary meniscus including elasticity at the interface. In the following we will detail the different regimes appearing in the problem, depending on the relative importance of elasticity, gravity and capillarity. Particular attention will be paid to the behavior of the contact line between liquid and solid, the slippage of which can seriously influence the extraction process. We finally discuss the extraction process in the light of the particular regime considered and show the possibility to increase the extraction force with a purely elastic mechanism, as is common in adhesion phenomena.

## 2 An elasto-capillary meniscus

In our experimental setup, sketched in Fig. 1, a narrow box with rigid glass walls is filled with water (density  $\rho = 10^3 \text{ kg m}^{-3}$  and surface tension  $\gamma = 72 \text{ mN m}^{-1}$ ). A thin and narrow strip, of

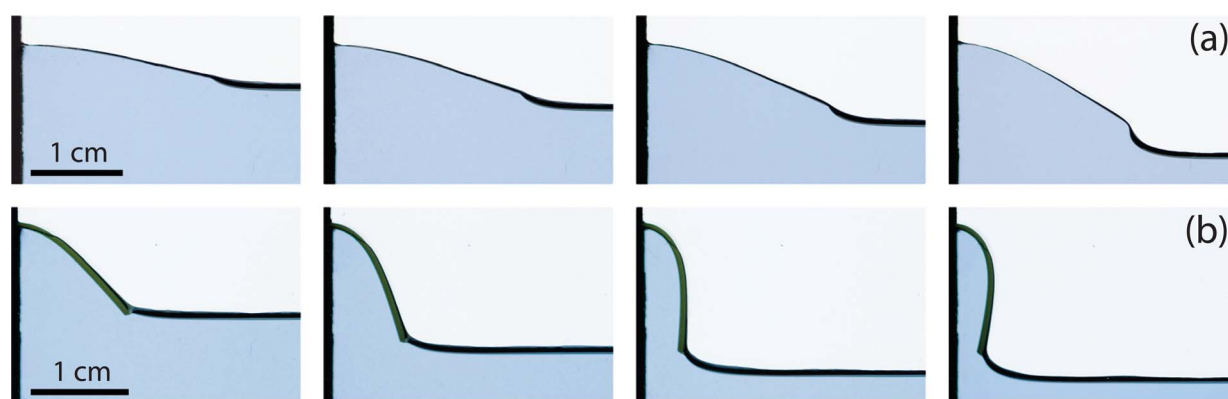
length  $L$ , width  $W$  and thickness  $e$ , is clamped at one end to the box, and is free at the other end. Strips employed in the experiments are made of polyethylene terephthalate (PET, bending stiffness per unit width  $B = 1.7 \times 10^{-4} \text{ Nm}$ ) or polyvinyl siloxane (PVS, bending stiffness per unit width  $B = 1.8 \times 10^{-5} \text{ Nm}$ ). The lateral extent of the box is  $D > W$ , such that a small gap exists between the strip and the walls in order to avoid friction.

The initial amount of water in the box is set to leave the liquid surface at the same level as that of the embedding in the wall,  $Y = 0$ . The strip floats on water and is completely flat. Then, rather than to extract the strip, water is quasi-statically withdrawn from the box with a syringe (see ESI† for a photograph of the setup), so that the liquid interface lies at distance  $H$  from the embedding. The liquid–air interface pins to the free edge of the strip, causing its deflection. Note that to ensure quasi-staticity, the level of the free surface is set to decrease at a rate lower than  $2 \text{ mm s}^{-1}$ .

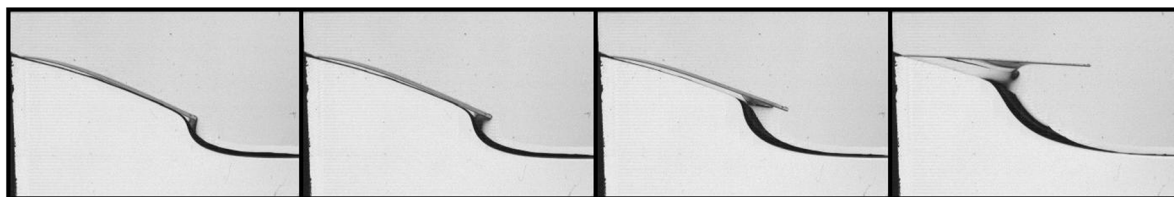
Focusing on the  $(X, Y)$  plane, we show in Fig. 2 several equilibrium configurations, obtained for one PET and one PVS strip, and for increasing values of the parameter  $H$ . Note that these pictures correspond to static configurations, as the withdrawal process has been stopped during the shot. The overall deformation of the liquid surface and the elastic strip results from a competition between capillarity, elasticity and gravity. It shall thus be referred to as an elasto-capillary meniscus in the following.

Looking at the shape of the free liquid surface, it is already quite clear that the total height of the compound can be much larger than that of a pure liquid meniscus. Note that there is no paradox however, as the height of the liquid portion still does not exceed two times the gravity-capillary length  $L_{gc} = (\gamma/\rho g)^{1/2}$ , where  $g$  denotes the gravity. The elasticity of the strip is the factor which allows such heights to be reached, and to lift a large amount of liquid.

One can observe that the main difference between the sequence in panel 2(a) and 2(b) is related to the evolution of the contact angle between the strip and the liquid. Indeed, in panel 2(a) the angle decreases with respect to  $H$ , from a value near



**Fig. 2** Typical elasto-capillary meniscus shapes. (a) Equilibrium configurations for a Mylar strip floating on water. The length of the strip here is  $L = 21 \text{ mm}$ . From left to right, the meniscus height is respectively  $H = 5.0, 7.2, 8.9$  and  $12.5 \text{ mm}$ . (b) Same snapshot sequence for a PVS strip resting on water. The length of the strip is  $L = 15.8 \text{ mm}$ . From left to right, the meniscus height is respectively  $H = 10.7, 13.6, 16.1$  and  $16.6 \text{ mm}$ .



**Fig. 3** Elastic strip extraction. Sequence illustrating the failure of the elasto-capillary meniscus, associated with air invasion from the end of the strip and violent elastic relaxation. The time intervals between two consecutive images are 168 ms, 78 ms and 27 ms.

180° in the first image to approximately 120° in the last image. Conversely in panel 2(b) the contact angle increases with respect to  $H$  up to approximately 230° in the last frame. The variety of contact angle values is a consequence of the sharp edge at the end of the strip. Here contact line pinning avoids air or water invasion in the region below or above the strip respectively, as already pointed out in Reis *et al.*,<sup>12</sup> provided that the hysteresis of the contact angle is strong enough as we will discuss later.

The height of the elasto-capillary meniscus can be increased up to a point where there is a sudden receding of the contact line associated with air invasion and a violent elastic relaxation, as depicted in Fig. 3 (see also the corresponding movie in the ESI†). This is the extraction of the strip, which is not a continuous but a violent process occurring over a typical timescale of 100 ms. It is therefore a much faster phenomenon than the withdrawal process which takes a tenth of a second. Extraction occurs for a critical height depending *a priori* on all the parameters of the problem and will be examined in detail in Section 4.

### 3 Theoretical description

In this section we present a 2D theoretical description of the shape of the elasto-capillary meniscus. On the one hand, the 2D approach for the elastic strip is justified by the fact that in all experiments  $L \gg W$  and  $L \gg e$ , which allows for the Euler-Bernoulli description of a beam. On the other hand, one can notice that the liquid interface is also deformed in the  $Z$  direction, as suggested by the thick dark shape of the interface in Fig. 2. This is the consequence of the contact between the water and the glass walls. See ESI† for a discussion on the role of this 3D effect.

#### 3.1 Liquid portion

Pressure everywhere in the liquid is given by the hydrostatic law:

$$p(Y) = p_a - \rho g(Y + H) \quad (1)$$

where  $p_a$  is the pressure in the air,  $\rho$  is the density of the liquid and  $g$  is the gravity acceleration (see Fig. 4 for notations). Whenever  $H \neq 0$ , the liquid portion such that  $Y > -H$  is in depression with respect to the atmosphere. Pressure  $p(Y)$  jumps to  $p_a$  across the curved liquid interface, due to Laplace's law  $\gamma\kappa = \Delta p$ . We introduce the angle  $\psi$  between the tangent to the meniscus and the  $X$ -direction ( $\psi$  is positive if trigonometric),

which is related to the curvature by  $\kappa = -\frac{d\psi}{dS} = -\psi'(S)$ ,  $S$  being the arc-length along the meniscus. Laplace's law can be written as:

$$\gamma\psi'(S) = \rho g(Y_m(S) + H) \quad (2)$$

where  $Y_m$  is the vertical position of the liquid interface. Using the differential relationship  $Y'_m(S) = \sin \psi(S)$ , we can finally write:

$$\psi''(S) = \frac{1}{L_{gc}^2} \sin \psi(S). \quad (3)$$

In the case of a meniscus vanishing far from the wall,  $\psi(S \rightarrow \infty) = 0$ , eqn (3) has an analytical solution,<sup>19</sup> which can be written as:

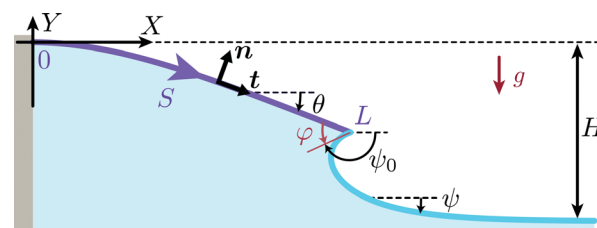
$$\psi(S) = 4 \arctan \left( \tan \frac{\psi_0}{4} \exp \frac{-S}{L_{gc}} \right). \quad (4)$$

Here  $\psi_0$  is a constant, the value of which is in general linked to the contact angle at the wall. In our system however, the contact angle changes with  $H$ , as already displayed in the previous section. We will show later the way in which  $\psi_0$  can be found.

#### 3.2 Elastic portion

The deformation of an inextensible beam can be expressed as a function of the angle  $\theta$  between the tangent to the beam and the  $X$ -direction ( $\theta$  is positive if trigonometric).

As the beam lies at the interface, it is subjected to air pressure on the upper face and water pressure on the lower face. Thus, the effective distributed force per unit width is  $\mathbf{f} = -\rho g(Y + H)\mathbf{n}$ , where  $S$  is the arc-length along the beam, spanning from 0 to  $L$ , and  $\mathbf{n} = -\sin \theta \mathbf{e}_x + \cos \theta \mathbf{e}_y$  is the unitary vector normal to the upper face. Note that we neglect the role of



**Fig. 4** Notations used in the model. This sketch is a 2D view of Fig. 1, introducing the notations used in the model. The local angle between the  $X$ -axis and the strip (violet) is denoted  $\theta$ , the one between the  $X$ -axis and the free surface (blue) is  $\psi$ , and the contact angle between the liquid and the strip is  $\varphi$ .

the weight of the polymer strip itself, as it floats on the liquid interface.

Equilibrium of internal force  $\mathbf{F}(S) = F_x(S)\mathbf{e}_x + F_y(S)\mathbf{e}_y$  and internal moment  $\mathbf{M}(S) = M(S)\mathbf{e}_z$  is given by the Kirchhoff equations, the projections of which on the three axes are:

$$F'_x(S) = -\rho g(Y(S) + H) \sin \theta(S) \quad (5a)$$

$$F'_y(S) = \rho g(Y(S) + H) \cos \theta(S) \quad (5b)$$

$$B\theta''(S) = F_x(S) \sin \theta(S) - F_y(S) \cos \theta(S) \quad (5c)$$

These equations are written per unit width. We introduced the bending constitutive relationship  $B\theta'(S) = M(S)$ , with  $B = EI/W$  the bending stiffness per unit width ( $E$  is the Young's modulus and  $I$  is the second moment of the area of the cross-section). The position of an infinitesimal element  $ds$  is related to the angle  $\theta$  by the relationships:

$$X'(S) = \cos \theta(S) \quad (6a)$$

$$Y'(S) = \sin \theta(S). \quad (6b)$$

Finally, boundary conditions are:

$$X(0) = 0 ; Y(0) = 0 \quad (7)$$

$$\theta(0) = 0 ; \theta(L) = 0$$

meaning that the strip is clamped horizontally at the origin and curvature vanishes at the end. Two more boundary conditions are related to the force applied by the liquid–air interface at the tip of the strip: here surface tension pulls the beam with a force of magnitude  $\gamma$  and an angle  $\psi_0$  with respect to the horizontal (see Fig. 4). Hence:

$$F_x(L) = \gamma \cos \psi_0 ; F_y(L) = \gamma \sin \psi_0. \quad (8)$$

It is convenient to write these last boundary conditions as a function of the contact angle  $\varphi$  between the solid and the liquid rather than  $\psi_0$ . Looking at Fig. 4 it appears that  $\psi_0 = \theta(L) + \varphi - \pi$ , therefore:

$$F_x(L) = -\gamma \cos(\theta(L) + \varphi) \quad (9a)$$

$$F_y(L) = -\gamma \sin(\theta(L) + \varphi). \quad (9b)$$

Kirchhoff eqn (5) can be written in a more compact form (see ESI† for details):

$$B\theta'''(S) + \frac{B}{2}\theta'(S)^3 + \gamma\theta'(S)\cos\varphi + \rho g(Y(S) + H) = 0 \quad (10)$$

A characteristic length scale appears naturally in this problem when a balance between elastic and hydrostatic forces applies, the elasto-hydrostatic length  $L_{eh} = (B/\rho g)^{1/4}$ . This length scale has already been introduced by Hertz<sup>20</sup> and Föppl<sup>21</sup> in their founding works on elastic plates. The elasto-hydrostatic length is also the relevant length scale when considering the buckling of an elastic beam resting on a compliant substrate,<sup>22</sup> in which the rigidity of the foundation is replaced by  $\rho g$ . We use  $L_{eh}$  to make lengths dimensionless:

$$\{s, x, y, h, \ell, \ell_{gc}\} = \frac{1}{L_{eh}} \{S, X, Y, H, L, L_{gc}\} \quad (11)$$

Dividing eqn (10) by  $\rho g L_{eh}$  leads to the system of equilibrium equations (dependence on  $s$  is now suppressed for brevity):

$$\theta''' + \frac{1}{2}\theta'^3 + \ell_{gc}^2\theta'\cos\varphi + (y + h) = 0 \quad (12a)$$

$$x' = \cos\theta \quad (12b)$$

$$y' = \sin\theta \quad (12c)$$

with boundary conditions:

$$x(0) = 0 ; y(0) = 0 ; \theta(0) = 0 \quad (13)$$

$$\theta'(\ell) = 0 ; \theta''(\ell) = \ell_{gc}^2 \sin\varphi$$

### 3.3 Matching condition

Because of the contact line pinning at the edge, the contact angle  $\varphi$  between the elastic strip and the liquid surface does not result from classical Young's construction. Rather,  $\varphi$  is set by the geometrical constraint expressing the anchoring of the meniscus at the strip edge:

$$y_{\text{strip}}(\ell) = y_{\text{meniscus}}(\ell). \quad (14)$$

This sole condition does not guarantee a constant value for  $\varphi$  and indeed, the contact angle exhibits experimentally a strong dependence with the non-dimensional depth  $h$ .

After integrating eqn (3) once, the right hand side of this condition can be rewritten as

$$y_{\text{meniscus}}(\ell) = -2\ell_{gc} \sin \frac{\psi_0}{2} - h, \quad (15)$$

leading to the following convenient end point condition for  $\varphi$ :

$$y_{\text{strip}}(\ell) = 2\ell_{gc} \cos\left(\frac{\varphi + \theta(\ell)}{2}\right) - h \quad (16)$$

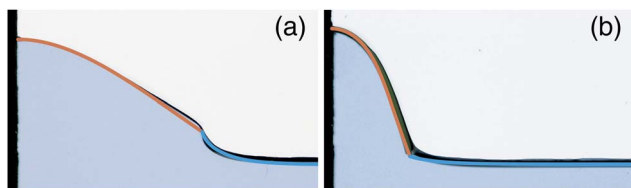
## 4 Elasto-capillary meniscus equilibrium shapes and their limit of existence

### 4.1 Predicted and observed equilibrium shapes

Typical equilibrium configurations observed in experiments for PET and PVS strips are reported in Fig. 5. These two examples correspond to different values of the three dimensionless parameters ( $h$ ,  $\ell$  and  $\ell_{gc}$ ) governing the overall shape of the structure. In order to test the assumptions made in the model, we confront these observations with theoretical predictions. Specifically, we integrate numerically the eqn (12) and (13) associated with the matching condition (16). The shape predicted with the model for the experimental governing parameters is then superimposed to the observed profiles, and a good agreement can be noted both for PVS and PET strips.

Is the observed deflection of the strip gravity-driven or capillarity-driven? To answer this question we introduce the



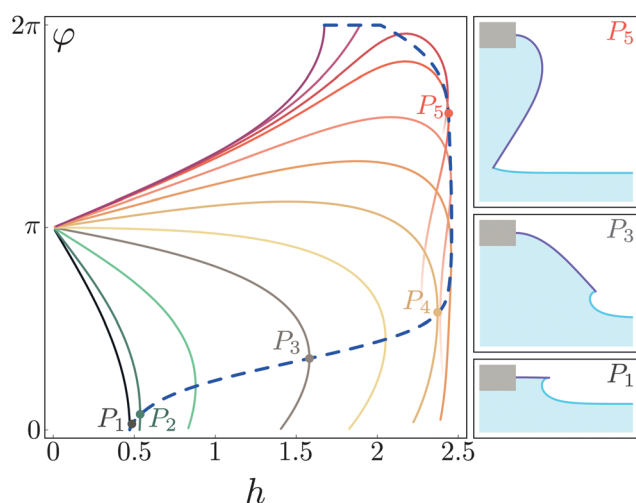


**Fig. 5** Comparison between experimental and theoretical profiles. (a) The theoretical equilibrium shape (obtained with  $l = 1.83$ ,  $l_{gc} = 0.24$  and  $h = 1.09$ ) is superimposed to its experimental counterpart (corresponding to the last frame of Fig. 2 panel a). (b) Same comparison with the second frame of Fig. 2 panel b (here  $l = 2.44$ ,  $l_{gc} = 0.48$  and  $h = 2.06$ ).

analogue of the elasto-hydrostatic length for capillarity, *i.e.* the elasto-capillary length<sup>5</sup>  $L_{ec} = (B/\gamma)^{1/2}$  which is the typical length needed for capillary bending effects to overcome the flexural rigidity of the strip. As  $L_{ec}L_{gc} = L_{eh}^2$  we see that the nondimensional parameter  $l_{gc}$  can be written as the ratio  $L_{eh}/L_{ec}$ . Hence  $l_{gc}$  is a measure of the relative importance of hydrostatic forces over capillarity in the bending process. In our experiments,  $l_{gc}$  is systematically lower than 0.5 but still of order one, meaning that the hydrostatic pressure is hardly the dominant effect in the deformation of the strip. Therefore we chose not to neglect  $l_{gc}$  in the following discussion.

## 4.2 The path towards extraction

We now investigate the nature of the extraction process *per se*, with a particular focus on the dependence of the critical height with the governing parameters. To this end, we make use of a continuation algorithm to follow the equilibrium configuration as the strip is withdrawn from the bath. These equilibria



**Fig. 6** Limit of existence for the elasto-capillary meniscus. The paths followed by equilibrium states are shown in the  $(h, \varphi)$  plane, in which  $\varphi$  is restrained to physical values between 0 and  $2\pi$ . The curves correspond to increasing values of the nondimensional strip length  $l$ , starting with a near zero value for the black curve (from bottom to top  $l = 0.01, 1, 1.5, 1.8, 2, 2.2, 2.4, 2.6, 2.8, 2.9, 3, 3.2$  respectively). It appears that for each curve a maximal value for  $h$  exists, and beyond this limit point no more solution can be found. The dashed blue line reports the path of these limit points as  $l$  is varied. Three equilibrium shapes, corresponding to the limit points  $P_1$ ,  $P_3$  and  $P_5$ , are reported on the right. Note that throughout this diagram, pinning of the contact line on the strip tip is assumed.

branches are reported in Fig. 6. Note that for comparison purposes the value of the nondimensional capillary length has been set throughout this study to  $l_{gc} = 0.24$ , corresponding to the experiment involving a strip made of PET.

We start by considering the rigid limit case  $l = 0.01$  where the elasto-hydrostatic length  $L_{eh}$  is 100 times larger than the length of the strip  $L$  (lowermost black curve in Fig. 6). When  $h = 0$ , the liquid surface is perfectly horizontal ( $\psi = 0$ ) and the apparent contact angle is  $\varphi = \pi$ . As  $h$  is increased, the contact angle drops down to zero for  $h = 2l_{gc} = 0.48$ . In this rigid case,  $h$  simply measures the true height of the liquid meniscus. This height cannot exceed the critical height  $2l_{gc}$  corresponding to a purely wetting state ( $\varphi = 0$ , see point  $P_1$ ), otherwise the matching condition (14) would not be fulfilled. This is precisely the critical height obtained by Laplace<sup>4</sup> that allowed him to predict the critical extraction force, measured experimentally by Gay-Lussac.

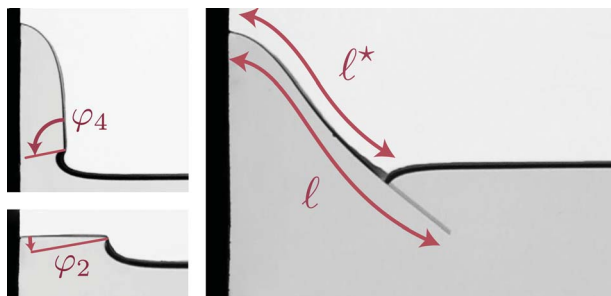
Now adding some elasticity to the problem, we consider analogous equilibrium branches for flexible strips ranging from  $l = 0.01$  up to  $l = 3.2$ . At first glance, it can be noticed that whenever  $h = 0$  the deflection of the strip is nil and so the contact angle  $\varphi$  is always equal to  $\pi$ . Starting from this flat configuration, we follow each branch of equilibrium as  $h$  varies. It is noteworthy that the addition of elasticity allows higher maximal heights in the system to be reached. This is clearly illustrated by the elasto-capillary menisci shapes at points  $P_3$  and  $P_5$  in Fig. 6.

Most of the equilibrium branches in Fig. 6 exhibit a limit (fold) point. In other words, two equilibria coexist for a given value of  $h$ , presumably one stable and the other unstable. At the critical value, these two equilibria coalesce and then disappear: *there is no more equilibrium solution past the limit point*. This behaviour is commonplace in dynamical systems (saddle-node or fold bifurcation), and was first noticed in the context of interfaces by Plateau.<sup>23</sup> More precisely, when a catenoid made of a soap film is formed between two rigid rings, there is also a limit point where two catenoids (one unstable and the other stable) merge and then cease to exist. Experimentally, beyond the limit point, the catenoid bursts into droplets, and the elasto-capillary meniscus breaks down (*i.e.* the lamella is extracted) as illustrated in Fig. 3. Note that for the two uppermost curves ( $l = 3$  and  $l = 3.2$ ) the maximum value for  $h$  no more corresponds to a limit point, but rather to a constrained maximum occurring when  $\varphi = 2\pi$ , the largest possible value for the contact angle.

The locus of these maxima, be them limit fold points or constrained maxima, is revealed in Fig. 6 with a dashed blue line. The path followed by these maxima appears non-trivial, suggesting optimum values for the elasticity to achieve largest heights and exhibiting a tumble as  $\varphi$  approaches  $2\pi$ . As these points mark the frontier where equilibria disappear, it is therefore quite natural to expect from this line to represent the *extraction limit* for the strip.

## 4.3 Pinning and slipping of the contact line

A simple way to verify the exactitude of the extraction prediction is to compare the theoretical shape of the elasto-capillary meniscus to its corresponding experimental counterpart.



**Fig. 7** Slipping of the contact line. Top left: experimental shape observed just before air invasion sets in. The strip has a nondimensional length  $\ell = 2.2$ , the limit point of which in Fig. 6 is the point  $P_4$ . The superimposed angle is the contact angle reached at the limit point  $P_4$ , and it is in good agreement with the one observed in the experiment. Bottom left: snapshot corresponding to a strip with  $\ell = 1$ . Here, air invasion is triggered for a contact angle  $\varphi$  clearly above  $\varphi_2$ , the contact angle associated with point  $P_2$ . The instability involves a slipping of the contact line as soon as the angle reaches some critical value  $\varphi_c$ . Right: equilibrium state for  $\ell = 3.68$ . The strip literally plunges in the bath, with an effective 'interfacial length'  $\ell^* < \ell$ .

In Fig. 7 (left) we show two experimental pictures taken at the moment of the extraction. In the first case ( $\ell = 2.2$  – top left image), the experimental contact angle at extraction accurately matches the angle  $\varphi_4$  expected from the theory ( $\varphi_4$  corresponds to the angle at point  $P_4$  in Fig. 6). This means that the folding point  $P_4$  accurately captures the threshold for extraction, confirming our previous analysis. However, the second case shown in Fig. 7 ( $\ell = 1.0$  – bottom left) exhibits a poor agreement with the theoretical prediction. There, the experimental critical contact angle is much larger than the value  $\varphi_2$  expected from the location of the folding point  $P_2$  in Fig. 6.

The reason for this discrepancy lies in the contact line pinning hypothesis that proves to be faulty for some values of the contact angle. Indeed, experimentally we clearly observe an advance of the contact line as soon as  $\varphi$  approaches a critical value  $\varphi_c \approx 80^\circ$ . The slippage of the contact line, associated with air invasion underneath the elastic strip, is an unstoppable process; it proceeds until the lamella is fully extracted.

Analogously, when the free surface overturns the strip tip a similar advance of the contact line can be observed. Actually it is triggered as soon as the air wedge angle is less than  $\varphi_c$ , that is, when  $\varphi > 2\pi - \varphi_c$ . But this time, the slippage is not necessarily a runaway process leading to extraction, as exemplified in Fig. 7 (right) with a *stable* configuration. There, water invasion sets in as soon as the critical angle  $2\pi - \varphi_c$  is reached. But the system now settles to a new 'plunged state' that reduces the effective 'interfacial length' of the strip  $\ell^* (< \ell)$ . In this type of configuration, a part of the strip remains stably underwater.

These experimental observations tend to depict a somewhat more complex picture of the extraction process than previously expected. Specifically, the locus of the folding points in Fig. 6 does not necessarily describe the extraction threshold. Extraction cannot be grasped without paying attention to the slipping motions of the contact line on the upper or lower side of the strip.

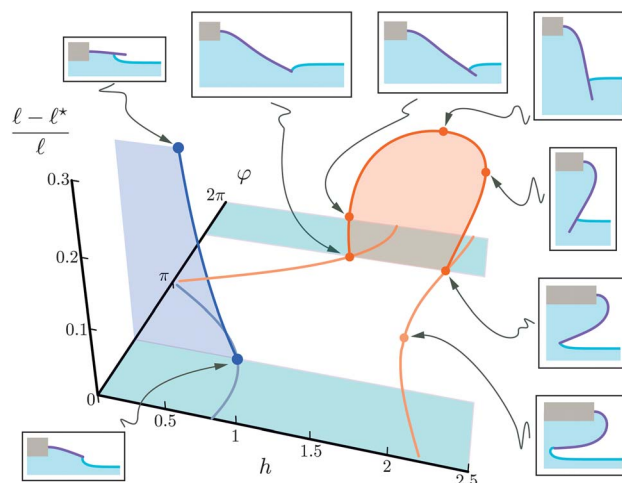
Contact line slippage can however be included in the model.<sup>24</sup> Indeed, assuming that the contact angle is constant during

slipping<sup>25</sup> (and equal to  $\varphi_c$  or  $2\pi - \varphi_c$ ), we can still search for equilibrium solutions provided that the interfacial length  $\ell^*$  is turned into a free parameter. Note that the total number of unknowns in this setting is unchanged as  $\varphi$  is known. The end condition (16) can now be used as a condition for  $\ell^*$ , and the system of leading eqn (12) and (13) can be integrated from 0 to  $\ell^*$ .

Fig. 8 displays the equilibrium branches for two values of the nondimensional length,  $\ell = 1.5$  and  $\ell = 3.4$ , when both pinning and slipping of the contact line are retained. These branches are plotted in the three-dimensional space  $(h, \varphi, (\ell - \ell^*)/\ell)$ . Here  $(\ell - \ell^*)/\ell$  represents the relative immersed/emerged portion of the strip (this portion can either be in air or water). Note that this quantity cancels out when the contact line is pinned to the strip edge.

For  $\ell = 1.5$  (blue line), the path followed by equilibrium solutions is identical to the one already shown in Fig. 6 for pure pinning. However, as the contact angle  $\varphi$  reaches the critical value  $\varphi_c$  an out-of-plane bifurcation occurs. The new branch arises from contact line slipping along the underside of the strip, as shown in the inset. This bifurcation is subcritical and the equilibrium branch monotonically tends to  $(\ell - \ell^*)/\ell = 1$ , meaning full extraction of the strip (for clarity reasons, the branch is only displayed for values such that  $(\ell - \ell^*)/\ell < 0.3$ ). A careful investigation of the nature of the bifurcations occurring at  $\varphi = \varphi_c$  suggests that those are all subcritical, *i.e.* lead to a sudden extraction of the strip.

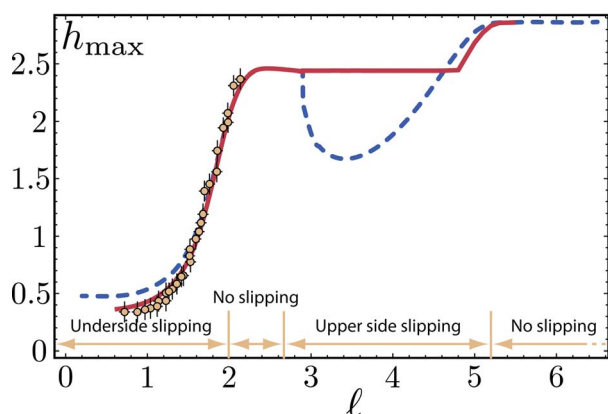
For  $\ell = 3.4$  (orange line) the contact angle now increases with  $h$ . The free surface overturns the strip edge, and a slipping bifurcation occurs when  $\varphi = 2\pi - \varphi_c$ . The nature of the bifurcation is now different, as equilibrium 'plunged states' exist for values of  $h$  larger than that at the bifurcation. Typical shapes of plunged states are shown in the insets of Fig. 8, which illustrates the slipping of the contact line above the strip. The loop



**Fig. 8** Equilibrium branches including pin-slip effects. Loci of equilibria in the  $(h, \varphi, (\ell - \ell^*)/\ell)$  space are shown for  $\ell = 1.5$  (blue line) and  $\ell = 3.4$  (orange line). As long as the contact angle  $\varphi$  lies in the range  $(\varphi_c, 2\pi - \varphi_c)$  (*i.e.* between the blue stripes), the contact line is pinned to the strip tip and equilibrium branches lie in the  $(h, \varphi)$  plane. When the contact angle reaches a critical value a bifurcation occurs. There, another branch corresponding to the slipping contact line takes off out-of-plane, as the interfacial length  $\ell^*$  no more corresponds to the actual length of the strip  $\ell$ . Reconnection to pinned states appears to be possible.

shape of the out-of-plane branch implies that the interfacial length  $\ell^*$  first decreases with  $h$  until it reaches a minimum value, and increases up to  $\ell$  again as shown in the insets. In contrast to air invasion the immersion of the strip is thus a reversible phenomenon. Surprisingly enough, the slipped states branch then exhibits a reconnection with a new in-plane (pinned states) branch. It is remarkable to point out that the two in-plane orange branches are by no way connected, even when  $\varphi$  is allowed to adopt values far beyond its physical limits (data not shown). This is the ambivalent 'pin-slip' nature of the end condition that allows for such a plentiful bifurcation diagram allowing to track equilibria along multiple branches, at variance with the catenoid case where the end condition is unique.

We now confront the experimental critical height  $h_{\max}$  reached by the elasto-capillary meniscus as a function of  $\ell$  with the model, considering both pure pinning and 'pin-slip' conditions (Fig. 9) (in the pure pinning case the dashed curve reporting the critical height is basically a translation of the limit points locus shown in Fig. 6 as a function of  $\ell$ ). Clearly the agreement between the experimental observations and the 'pin-slip' prediction is excellent. The differences between pure pinning and pin-slip are even more pronounced for larger values of  $\ell$ . Indeed, for pure pinning a strange tumbling behaviour is particularly noticeable in the region  $3 < \ell < 5$ , suggesting the odd idea that lengthening the strip shortens the critical height. This wrong prediction is actually linked to the non-monotonic shape of the limit points locus. The full model incorporating mixed 'pin-slip' end conditions allows for a correction of this prediction and smoothes out the evolution of  $h_{\max}$  with  $\ell$ . This highlights the importance of considering out-of-plane bifurcations and reconnections inherent to contact line slipping along the upper side of the strip in the extraction process. Note also that Fig. 9 exhibits the whole regime phenomenology appearing in this problem, along with their non-trivial apparition sequence. Finally, looking at Fig. 9 suggests an asymptotic behavior for  $\ell \gg 1$ , a possibility we investigate next.



**Fig. 9** Critical meniscus height versus strip length. Comparison between the critical height  $h_{\max}$  reached in experiments as a function of the nondimensional strip length  $\ell$  (orange dots) and the theoretical prediction based on the 'pin-slip' end condition (magenta curve). For reference, the results using a pure pinning end condition are shown with the dashed blue curve. The different regimes in terms of slipping of the contact line are also displayed.

#### 4.4 Universal behavior of long strips

We now present a theoretical focus on the case  $\ell \gg 1$  where the strip is very long compared to the elasto-hydrostatic length. Rescaling (12c) and (12a) with  $\tilde{s} = s/\ell$  and  $\tilde{y} = y/h$  leads to:

$$\frac{h}{\ell} \tilde{y}'(\tilde{s}) = \sin \theta(\tilde{s}) \quad (17a)$$

$$1 + \tilde{y}(\tilde{s}) + \frac{1}{\ell^3 h} \left( \theta'''(\tilde{s}) + \frac{1}{2} \theta'(\tilde{s})^3 \right) + \frac{\ell_{\text{gc}}^2}{\ell h} \theta'(\tilde{s}) \cos \varphi = 0. \quad (17b)$$

Taking  $1/\ell$  as a small parameter, we immediately obtain at leading order the external solution  $\theta = 0$  and  $\tilde{y} = -1$ . This external solution represents a strip lying flat over the liquid surface, with a contact angle  $\varphi = \pi$ . A boundary layer solution exists and reconciles the external flat solution with the boundary condition  $\tilde{y}(0) = 0$ . We introduce classically the boundary layer variable  $\tilde{S}$  as follows:  $\tilde{s} = \varepsilon \tilde{S}$ , with  $\tilde{S} = O(1)$  within the boundary layer. In terms of this variable, eqn (17a) reads:

$$\frac{h}{\ell} \frac{1}{\varepsilon} \tilde{y}'(\tilde{S}) = \sin \theta(\tilde{S}), \quad (18)$$

and dominant balance requirements immediately give the thickness of the boundary layer as  $\varepsilon \sim h/\ell$ .

Inserting this result into eqn (17b) yields:

$$\frac{1}{h^4} \left( \theta'''(\tilde{S}) + \frac{1}{2} \theta'(\tilde{S})^3 \right) + (\tilde{y}(\tilde{S}) + 1) - \frac{\ell_{\text{gc}}^2}{h^2} \theta'(\tilde{S}) = 0 \quad (19)$$

In this equation the hydrostatic pressure term is of order one, and has to be balanced by – at least – one other term. Let's first consider the case  $\ell_{\text{gc}} \ll h$ , where the hydrostatic pressure is solely balanced by elasticity. The balance can only happen if  $h \sim 1$ , consistent with the significance of the elasto-hydrostatic length  $L_{\text{eh}}$ . This constraint certainly gives clues on the asymptotic bounded behaviour of  $h_{\max}$  when  $\ell$  achieves large values, as shown in Fig. 9. In this limit (amounting to neglect capillary forces) the shape of the strip is governed by the following equation, expressed with the original variables:

$$\theta''' + \frac{1}{2} \theta'^3 + y + h = 0. \quad (20)$$

A second option in the dominant-balance analysis is to retain only hydrostatic pressure effects and surface tension ones. This corresponds typically to  $h \sim \ell_{\text{gc}}$  and  $\ell_{\text{gc}} \gg 1$ . The shape adopted by the strip in this regime is described by solutions of:

$$y + h - \ell_{\text{gc}}^2 \theta' = 0. \quad (21)$$

When derived this equation can be recast into the well known meniscus equation  $\theta'' = 1/\ell_{\text{gc}}^2 \sin \theta$ . The shape of the elastic strip is thus that of a pure capillary meniscus. Actually this is nearly the case, as solutions of this equation cannot meet the clamped boundary conditions at the wall, as eqn (21) has lost two derivation orders with respect to eqn (19). A third layer, in which elasticity is restored, can however be introduced to fulfill the correct boundary conditions of the problem.



Eventually, an appealing feature of the full eqn (19) should be pointed out: its structure bears a striking similarity with the equilibrium equation of a buckled elastic beam sitting on a liquid foundation, for which an analytical solution exists.<sup>26</sup> We have recently demonstrated that the exact solution of eqn (19) can be obtained as well and is generally not symmetric.<sup>27</sup>

## 5 Extraction force

We are now in a position to address the main question raised in this study: how does elasticity affect the extraction process? We start by introducing the (dimensionless) pulling force  $f_p$ , that is the vertical component of the force exerted by the clamp:

$$f_p = -f_y(0) = \int_0^\ell (y(s) + h) \cos \theta(s) ds - \ell_{gc}^2 \sin \psi_0 \quad (22)$$

Although the pulling force mainly balances the weight of the volume of fluid displaced by the strip, it does rely intimately on elasticity and capillarity as these effects shape the liquid volume. As the imposed height  $h$  is increased, the lifted liquid volume varies, and so does the pulling force  $f_p$ . The *extraction force*  $f_{ext}$  is therefore defined as the largest value of  $f_p$  with respect to  $h$ :  $f_{ext} = \max_h f_p(h)$ . Note that in general this maximal value is not achieved when  $h = h_{max}$ .

In order to investigate the exact role of elasticity in the extraction process, we represent in Fig. 10 the variation of the force needed to pull out a fixed-length strip with variable rigidity. More precisely the dimensional extraction force per unit width  $F_{ext} = \rho g L_{eh}^2 f_{ext}$  is represented as a function of  $L_{eh}$  for a given strip length  $L$ . For plotting purposes, we chose a strip length  $L = 27$  mm and a fixed gravito-capillary length

$L_{gc} = 2.7$  mm corresponding to our experiments, but the features displayed in Fig. 10 appear to be generic.

Different regimes in the extraction process can be readily noticed in Fig. 10. First, for very small values of  $L_{eh}$  (corresponding to  $L_{eh} < L_{gc}$ , that is  $\iota_{gc} > 1$ ), the strip is so soft that it deforms almost with no resistance under the combined action of capillarity and gravity, as would have a pure liquid interface. This capillary limit is emphasized in the leftmost thumbnail of Fig. 10 with a comparison between the elasto-capillary meniscus shape and the looping shape of a real liquid meniscus. This excellent agreement is consistent with the fact that in the limit  $\iota_{gc} > 1$ , the elasto-capillary meniscus shape is captured by eqn (21).

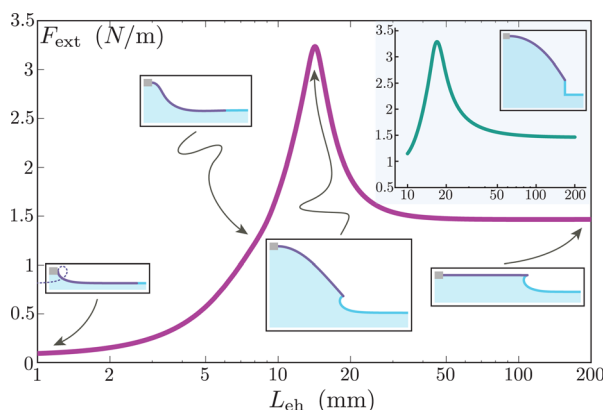
When  $L_{eh}$  reaches slightly larger values (but still such that  $L_{eh} \ll L$ , meaning  $\iota \gg 1$ ), elasticity comes into play with the elasto-hydrostatic regime. There, the strip is flat and aligned with the liquid surface outside a region of size  $\mathcal{O}(H/L_{eh})$  where the deformation is concentrated, as already discussed. The shape of the elasto-capillary meniscus in the inner region is given by solving eqn (20). Note that as long as  $\iota$  is still larger than unity, the shape of the strip exhibits a self-similar behavior as it only depends on the elasto-hydrostatic length  $L_{eh}$ , and so does the lifted liquid volume:  $F_{ext} \sim L_{eh}^2$  (i.e.  $f_{ext}$  constant). Stiffening the elastic strip therefore results in a quadratic increase of the extraction force.

For higher values of  $L_{eh}$  an adhesion peak clearly enters into the picture. This peak, centered at  $L_{eh} \simeq 0.5L$  (i.e.  $\iota \simeq 2$ ), corresponds to a cut-off of the self-similar regime imposed by end-effects, as  $L_{eh}$  becomes of the same order of magnitude as  $L$ . Here, the large deflection of the strip causes a large amount of liquid to be drawn behind, hence a maximal extraction force. For even larger values of  $L_{eh}$ , the elastic strip progressively rigidifies as the extraction force falls down to its asymptotic Laplace/Gay-Lussac limit. It should be pointed out that the extraction work, relevant in the context of imposed displacement, displays a similar adhesion peak (data not shown).

This sequence of different regimes can be captured with a rough model containing as only ingredients elasticity and gravity (inset of Fig. 10). In this model, the curvature of the strip is approximated with a linear function ramping from  $H/L_{eh}^2$  at the wall down to 0 at the tip. The liquid meniscus touching the strip tip is in turn crudely approximated with a vertical liquid wall (see the snapshot in the inset). Finding the extraction force amounts to a maximization problem: for a given  $L_{eh}$ , we look for the value of  $H$  that maximizes the liquid volume risen by the strip, taking care that the height of the liquid wall cannot exceed  $2L_{gc}$ . This simple model, disregarding both surface tension forces and nonlinear geometric displacements of the strip, recovers nicely the main features of the relationship  $F_{ext}$  vs.  $L_{eh}$ : the adhesion peak, the different limits, and the optimal shape. The quantitative agreement of this elementary model, free of adjustable parameters, certainly reveals the key physical ingredients necessary to understand the role of elasticity in capillary adhesion.

## 6 Conclusion

Elasticity is known to increase energetic costs in adhesion phenomena, and to induce hysteric cycles with different



**Fig. 10** Extraction force versus elasto-hydrostatic length. The relationship between the dimensional extraction force per unit width  $F_{ext}$  (in  $N m^{-1}$ ) and the rigidity of a fixed-length strip, here measured in terms of the elasto-hydrostatic length  $L_{eh}$  (mm). The length of the strip taken here is 27 mm and the gravito-capillary length  $L_{gc}$  has been set to 2.7 mm. The figure clearly displays an adhesion peak for a specific value of  $L_{eh}$  around  $0.5 L$ . Representative snapshots of the elasto-capillary meniscus at the moment of extraction are disposed along the curve. These snapshots correspond to different regimes; from left to right: capillary limit, elasto-hydrostatic regime, optimal adhesion, rigid limit. In the inset, a rudimentary model where the strip is approximated with a linearly varying curvature rod and the meniscus with a vertical liquid wall proves to capture the main features of the main graph, both qualitatively and quantitatively.

deformations back and forth. For example, the long polymer chains in elastomers can store all the more elastic energy when the chain is long. This energy is lost when the chains rupture and cracks initiate.<sup>28</sup> Analogously, soap bubbles gently put into contact can be deformed and remain connected even when the imposed separation exceeds the initial critical adhesion distance,<sup>29</sup> thus implying bubble conformations unexplored previously.

In this study, we have focused on a paradigm setup coupling elasticity and capillary adhesion. The elasto-capillary meniscus discussed here exhibits all the typical features of elastic adhesives: the extraction force/work possibly increased by elasticity, hysteric cycles with deformations of the strip depending on whether it touches the interface or not, the elastic energy stored released violently on extraction. This last observation, documented in Fig. 3 and in the ESI†, is also reminiscent of cavitation pockets appearing in the traction of adhesive films.<sup>30</sup>

A key point of our analysis is to highlight the existence of optimal strip stiffness corresponding to a maximal extraction force (conversely, this force can be as low as the material is soft). This could be exploited for example to improve the design of soft micro-manipulators or MEMS such as elastopipettes.<sup>12</sup> Though the maximal volume displaced by the strip does not correspond to the actual amount of liquid grabbed from an interface, the extraction force and work do have an important role in this problem as they set the limit points of extraction.

Finally, the agreement between theoretical and experimental shapes of an elasto-capillary meniscus, along with their dependence on the elasto-hydrostatic length  $L_{eh}$ , suggests new metrology tools for a precise measure of the elastic properties of soft films.

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