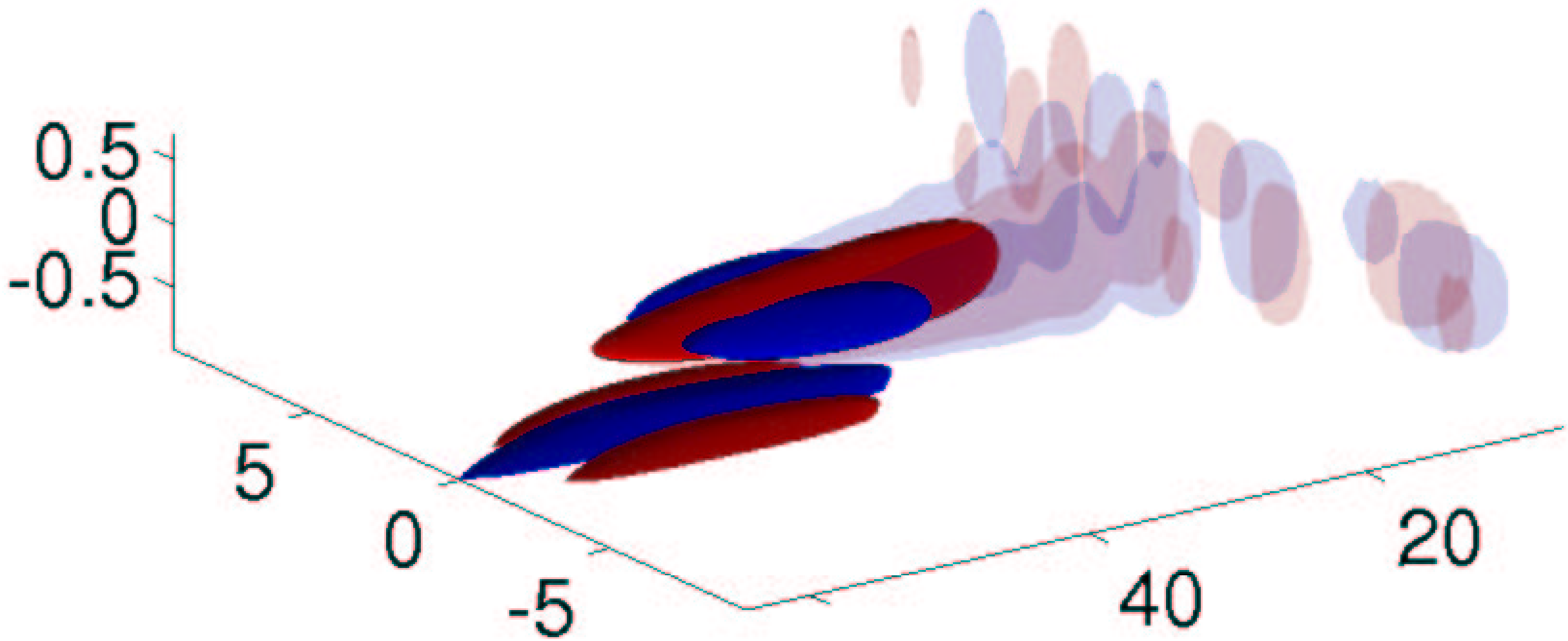
KTH Mechanics

Coupling sensors and actuators for flow control



Jérôme Hœpffner,

Supervisor Dan Henningson

In this talk

Describe how the actuators uses the sensor information in optimal control of Channel flow.

Framework

linear feedback control theory

transition to turbulence

Keywords

Reactive control (feedback)

Transfer function

Why using reactive control?

Act on the mean flow

And affect the stability of small perturbations

→ control effort of the order of magnitude of the mean flow

Act on the fluctuations

And prevent them from growing and disrupting the mean flow

→ control effort on the order of magnitude of the fluctuations

In a transitionnal case, the fluctuations are of much smaller amplitude than the mean flow.

Information and action

Sensors

Streamwise skin friction fluctuations

Spanwise skin friction fluctuations

Wall pressure fluctuations

Actuators

Blowing and suction at the walls

Only flow quantities at the wall are available.

The control problem

Stochastic disturbances f, g, q_0

(External sources, sensor noise, unknown initial condition)

Actuation and sensing u, y

$$\begin{cases} \dot{q} = Aq + B_1 f + B_2 u, \quad q(0) = q_0, \\ y = Cq + g, \end{cases}$$

Feedback control

$$u = \mathcal{G}(y)$$

Which is the optimal mapping $\mathcal{G}$?

Solution of the control problem

$$\text{Plant} \begin{cases} \dot{q} = Aq + B_1 f + B_2 u \\ y = Cq + g. \end{cases}$$

$$\text{Estimator} \begin{cases} \dot{\hat{q}} = A\hat{q} + B_2 u - v \\ \hat{y} = C\hat{q}. \end{cases}$$

$$\text{Feedback} \quad v = L\tilde{y} = L(y - \hat{y}), \quad u = K\hat{q}.$$

Decouple into an estimation problem and a full information problem. Solve two Riccati equations to get the optimal L and K.

Transfer function:

$$u(t) = \int_0^\infty \underbrace{-K\mathrm{e}^{(A+BK+LC)\tau} L}_{G(\tau)} \, y(t-\tau) d\tau.$$

Selected literature

- Hu H. H. & Bau, H.H. 1994 *Feedback control to delay or advance linear loss of stability in planar Poiseuille flow*
  Use of proportional controller : $u(t) = Ky(t)$
- Joshi, S. S., Speyer, J. L. & Kim, J. 1995 *Modeling and control of two dimensional Poiseuille flow*
  Introduction of the optimal feedback control method (LQG, or H_2).
- Högberg, M. & Bewley, T. 2002 *Spatially localised convolution kernels for decentralised control and estimation of plane channel flow*
  Decomposition of the control into state estimation and full information control.
  Spatial localisation of the feedback law.

LQG (or H_2) feedback control

LQG for

Linear

Use of a linear model for the dynamics

$\longrightarrow$ Use of linearised Navier–Stokes equations

Quadratic

A quadratic objective function

$\longrightarrow$ minimise the energy of flow fluctuations

Gaussian

Gaussian disturbances to the flow

$\longrightarrow$ Use a covariance model for the disturbances

Fondamental achievement of control theory

Physical assumptions

- Dense array of sensors and actuators

  We know all the wall information

- Periodic domain in the two homogeneous direction

  For Fourier transform and temporal study

- Low amplitude for the fluctuations

  to use the linearised Navier–Stokes equations

Why a numerical study?

- Sensing and actuation

  Dense arrays of actuators and sensors are difficult to implement in experiment.

- Computational time

  With the actual formulation we need to run an on-line simulation of the flow.

- Understanding

  There is still many issues to be addressed on disturbance modeling, choice of control objective, and feedback formulation for flow applications.

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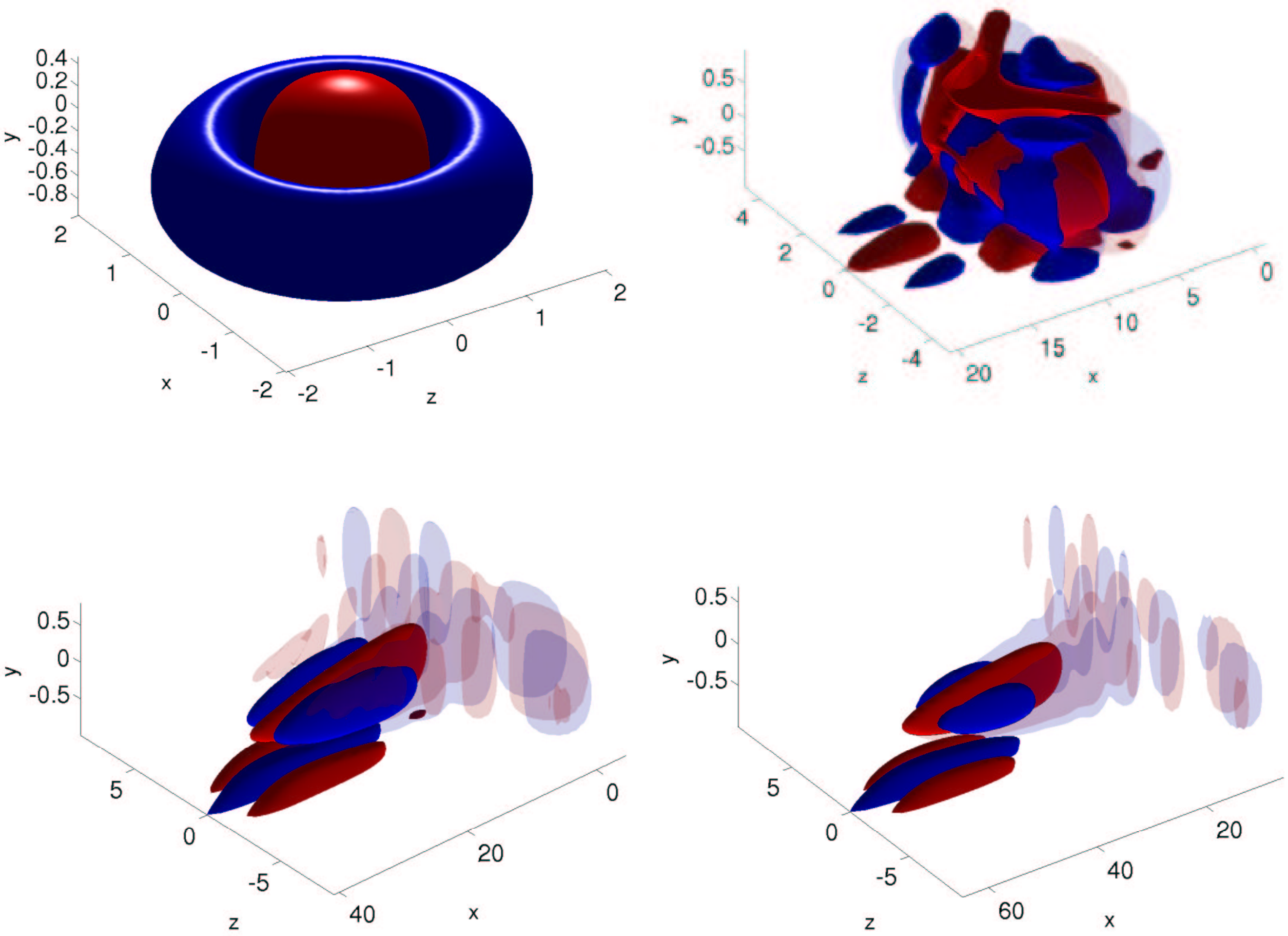
Test case

Axisymetric localised initial condition

Wall normal velocity for original flow and controlled flow, Time 0, 10, 70, 90.



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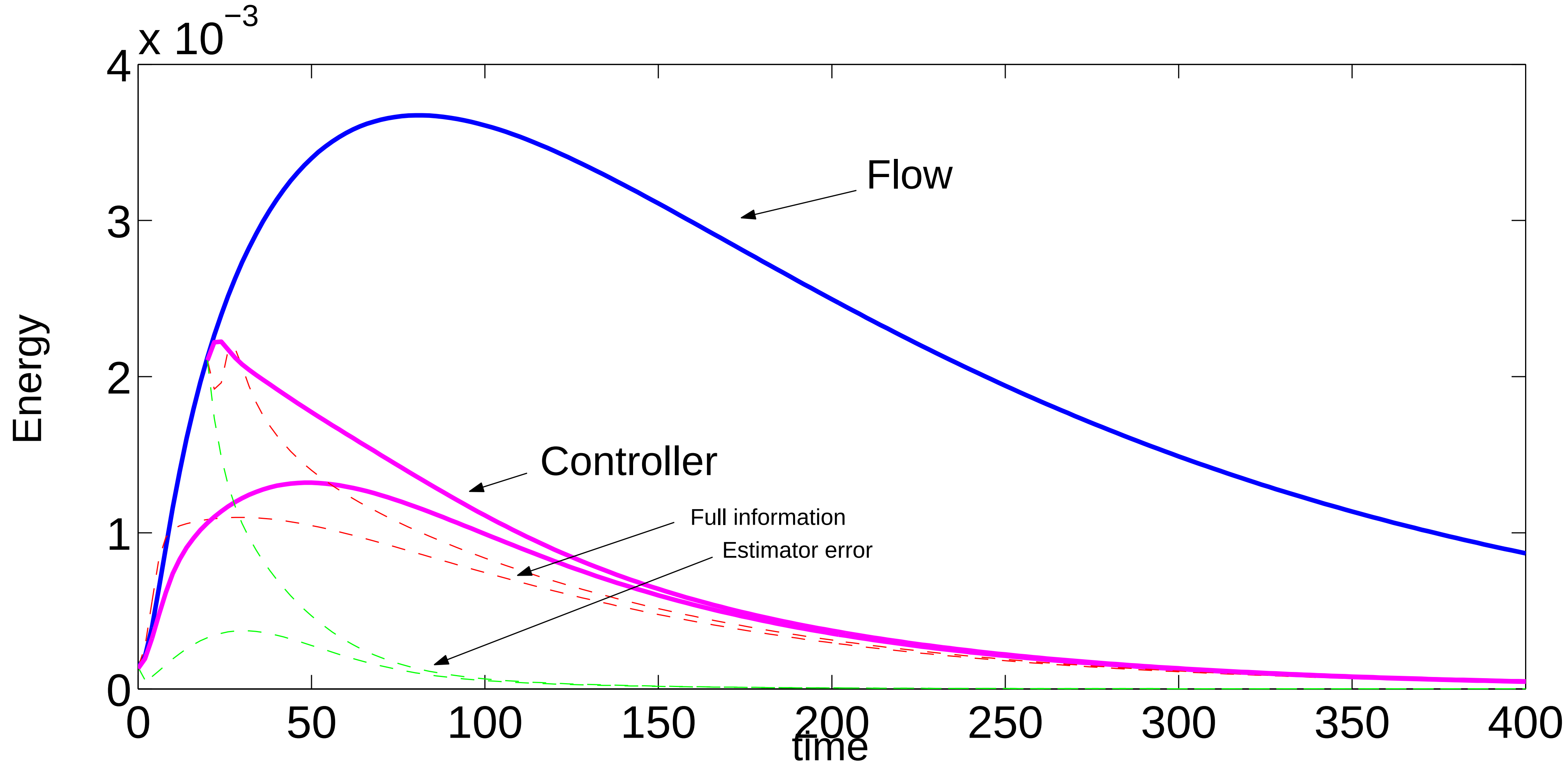
Performance of the control

Turn on the controller at time 0 and time 20



Transfer function formulation

The linear mapping can be written in the transfer function formalism:

$$u(0,0,t) = \mathcal{G}(y) = \int_x \int_z \int_0^\infty G(x,z,\tau) y(x,z,t-\tau) d\tau$$

Convolution of the measurement history over the wall.

τ is the time lag

Potential instability of the TF

The closed loop is stable by construction

$$\begin{cases} \dot{q} = Aq + B_1 f + B_2 u, \quad q(0) = q_0, \\ y = Cq + g, \end{cases}$$

$$u = \mathcal{G}(y)$$

But $\mathcal{G}$ is not guaranteed to be stable.

The interconnection of unstable systems can be stable.

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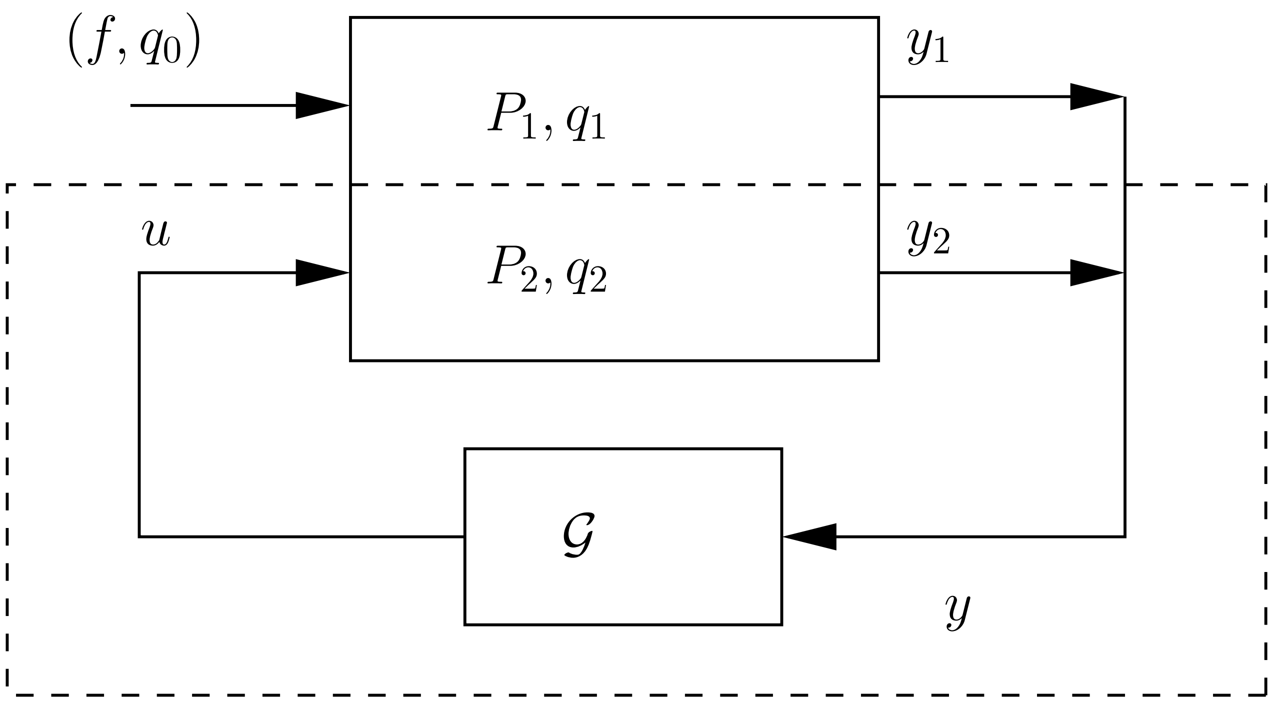
How to continue?
Redefine of the input



Because the input y should be dependent on the output u

The control affects the measurement.

Split y into y_1 and y_2

$$y = y_1 + y_2$$

$$q = q_1 + q_2$$

$u = \mathcal{G}^*(y_1)$ is the optimal control.

Now we continue with $\mathcal{G}^*$

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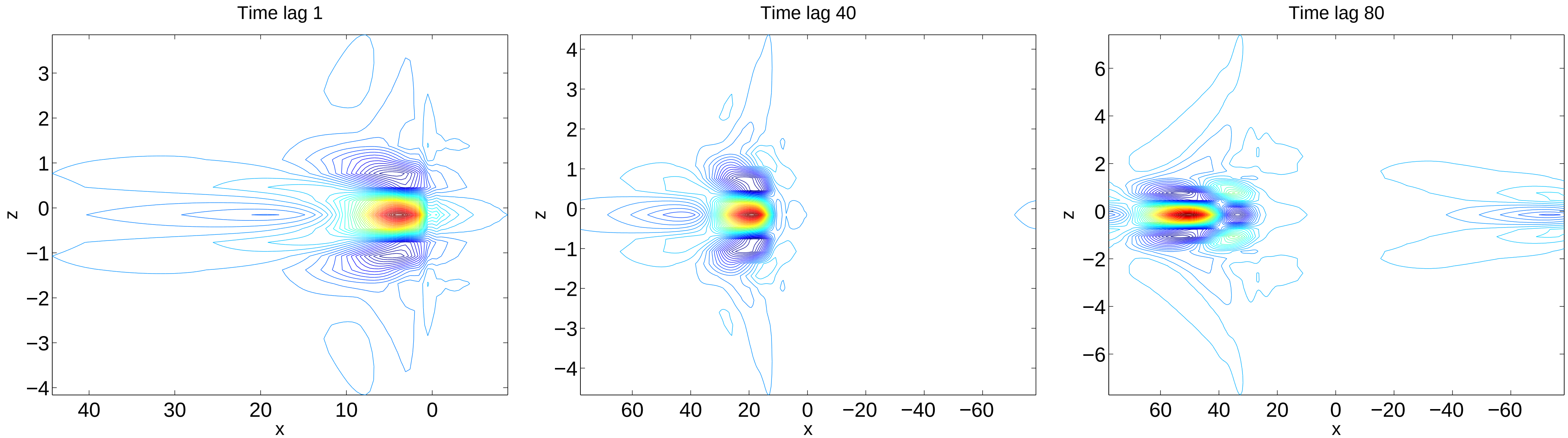
The TF in the channel

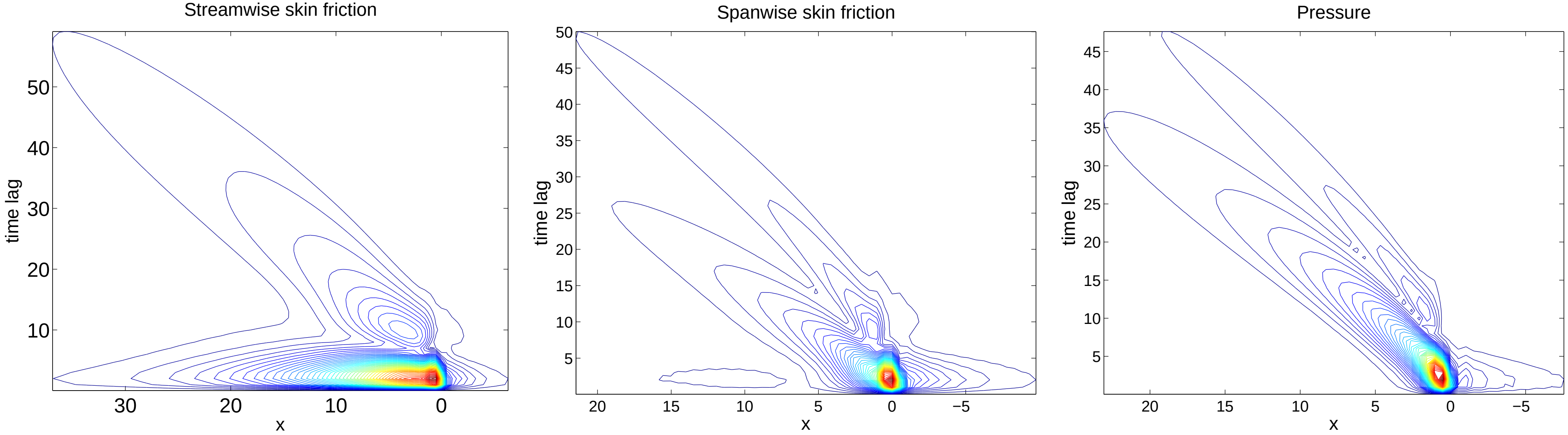
For selected time lags τ=1, 40, 80.

Streamwise skin friction measurement.

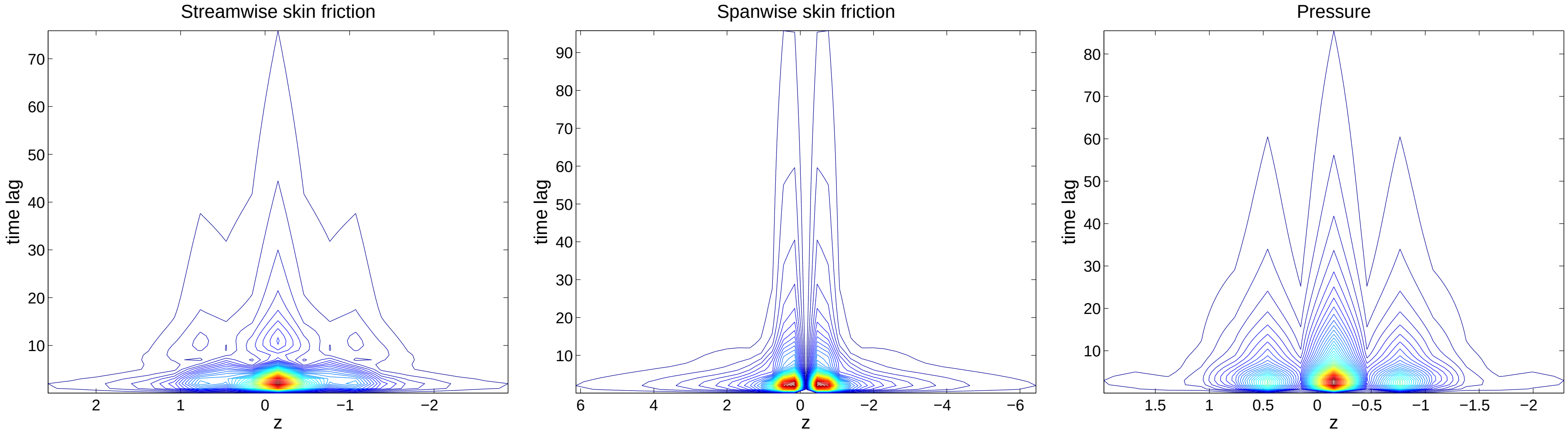


Convected information

Integrated in streamwise direction



Integrated in spanwise direction



Conclusions

- The transfer function is a natural formulation for control with spatially distributed sensing and actuation
- The transfer function is potentially unstable, even though it stabilises the flow
- This instability is due to the coupling between the input y and the output u