

IDENTIFICATION OF SOURCES OF SOUND IN LOW MACH NUMBER FLOWS BY THE USE OF FLOW FIELD EIGENMODES

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Abstract

We present a method to study sound generation processes in low Mach Number flows. Instead of the full flow field obtained from e.g. a DNS, we consider a base flow together with a time-dependent perturbation, where the perturbation satisfies the Navier-Stokes equations linearized around the base-flow. In a reduced model the perturbation is approximated by a linear combination of the eigenmodes of a corresponding eigenvalue problem. The behavior in time is determined by the corresponding eigenvalues. Curle's equation is used to calculate the acoustic field. By studying the source terms in Curle's equation, it is possible to identify mechanisms for sources of sound. This makes it possible to study how the different sources of sound depend on different structures of the flow field. We apply the methodology on a two-dimensional flow over a cavity with smoothed corners. Results of acoustic pressure in the far field and source strengths for different superpositions of eigenmodes are presented.

INTRODUCTION

One common method used to obtain source data to analyze for acoustical purposes is to solve the full Navier-Stokes equations, with the level of modeling of the viscous terms in the Navier-Stokes equations can range from RANS, LES/DES to DNS.

In DNS, the non-linear Navier-Stokes equations are solved numerically, yielding time-dependent data of e.g. the pressure and velocity field. This means that it can be difficult to clearly identify the sound generating structures in the solution. Also, performing DNS are computationally expensive, and the amount of raw data that is produced is large enough such that storage becomes an issue. Because of these large amounts of data, the dominating part of the computational time in solving Curle's equation will be spent on memory handling.

In this paper another method to obtain data for use in the source terms in Curle's equation is evaluated. It is also applied to a problem in the form of a low Mach number flow over an open cavity.

The theory of reduced models obtained from global modes has been developed especially for flow control purposes, see for example [7]. Although the analysis has been known for a long time, it is only in recent years that the calculations has been possible to carry out in two space dimensions, due to increasing computational performance. Here, however, we will use a reduced model in order to get a further understanding of sound generation mechanisms.

A REDUCED MODEL OF THE FLOW FIELD

The basic idea is to project the dynamics of the flow field on a subspace spanned by the eigenmodes of the linearized Navier-Stokes equations. The flow data used in an acoustic analogy can then in principle be reduced to only include the eigenmodes that are thought to be of interest for the sound generation mechanisms. The derivations below will be carried out for a two-dimensional problem, but are easily generalized to three space dimensions.

In this paper the non-dimensional incompressible Navier-Stokes equation

$$\begin{cases} \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u} \\ \nabla \cdot \mathbf{u} = 0 \end{cases} \quad (1)$$

is considered, since the method is to be applied to low Mach number flows. Here $\mathbf{u} = (u(x, y), v(x, y))$ is the velocity of the flow field, p is the hydrodynamic pressure and Re is the Reynolds number. When Eq. (1) is linearized around a base flow $\mathbf{U}(x, y) = (U(x, y), V(x, y))$, $P(x, y)$, the equations for a small perturbation (u', v', p') is obtained as, see [5]:

$$\begin{cases} \frac{\partial \mathbf{u}'}{\partial t} + (\mathbf{U} \cdot \nabla) \mathbf{u}' + (\mathbf{u}' \cdot \nabla) \mathbf{U} = -\nabla p' + \frac{1}{Re} \nabla^2 \mathbf{u}' \\ \nabla \cdot \mathbf{u}' = 0 \end{cases} \quad (2)$$

Assuming harmonic time dependence and that the perturbations are separated into the eigenmode solution and harmonic time dependence we let

$$\mathbf{q} = \hat{\mathbf{q}}(x, y) e^{-i\omega t} \quad (3)$$

where $\mathbf{q} = (u', v', p')^T$. The ansatz (3) implies that the space and time dependence is separated. Here $\hat{\mathbf{q}}$ and ω are complex-valued quantities.

With (3) in (2), we obtain, for two-dimensional flows, the following eigenvalue problem

$$A \hat{\mathbf{q}} = -i\omega \hat{\mathbf{q}} \quad (4)$$

where

$$A = \begin{pmatrix} -\mathbf{U} \cdot \nabla - \frac{\partial U}{\partial x} + \frac{1}{Re} \nabla^2 & -\frac{\partial U}{\partial y} & -\frac{\partial}{\partial x} \\ -\frac{\partial V}{\partial x} & -\mathbf{U} \cdot \nabla - \frac{\partial V}{\partial x} + \frac{1}{Re} \nabla^2 & -\frac{\partial}{\partial y} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0 \end{pmatrix} \quad (5)$$

For a discretized problem, the solution of the eigenvalue problem Eq. (4) is a finite set of eigenmodes $\hat{\mathbf{q}}_n = (\hat{u}_n, \hat{v}_n, \hat{p}_n)^T$ with corresponding eigenvalues ω_n . The number of elements M in these sets will be two times as many as the number of grid points that has been used to discretize the domain for a two-dimensional problem, and three times as many for a three-dimensional problem. The matrix (5) is typically too large for the system (4) to be solved by standard methods. Krylov subspace methods provide the possibility to solve these large eigenvalue problems by computing parts of the spectrum. Here we have used the Arnoldi method, [2], [4]. In a reduced model, only a subset $N \ll M$ of the eigenmodes is used.

The flow field perturbation can now be written as

$$\mathbf{q}(x, y, t) = \sum_{n=1}^N c_n \hat{\mathbf{q}}_n(x, y) e^{-i\omega_n t} \quad (6)$$

where c_n are constants determined by the initial condition. The benefit of writing the flow field on this form is that it is now possible to analyze the eigenmodes separately, or to analyze how different combinations of eigenmodes interact. From a computational point of view this form is also beneficial since only a set of eigenmodes are required to be saved as opposite to the whole flow field for every time step of the DNS.

We would like to stress that the reduced model is only valid when linearization is valid.

CURLE'S EQUATION

In order to calculate the sound that is generated by the flow field, we consider a non-dimensionalized formulation of Curle's equation, [1], with the scalings

$$x^* = \frac{x}{L}, \quad p^* = \frac{p}{\rho_0 U^2}, \quad u^* = \frac{u}{U}, \quad t^* = \frac{tU}{L} \quad (7)$$

where L is a length scale of the source domain. Since the flow is assumed to be of low Mach number, the volume sources and the shear stress sources are neglected, [5]. If the observation location is in the acoustical far-field, Curle's equation is reduced to (omitting the stars):

$$p'(\mathbf{x}, t) = \frac{1}{4\pi} \int_S l_i n_j \frac{M_\infty}{r} \frac{\partial}{\partial t} [p \delta_{ij}] dS \quad t \geq 0 \quad (8)$$

where $p'(\mathbf{x}, t)$ is the sound pressure at the observer location \mathbf{x} , S is the solid boundary of the source region, n_i is the outward-facing normal of the boundary, l_i are the direction cosines pointing from the source region to the observer, r is the distance from the source region to the observer, M_∞ is the free stream Mach number, p is the hydrodynamic pressure at the boundary of the source domain and the brackets denote that the pressure is evaluated at the retarded time.

We rewrite the hydrodynamic pressure using the pressure component of the eigenmodes in Eq. (6):

$$p(x, y, t) = P(x, y) + \sum_{n=1}^N c_n \hat{p}_n(x, y) e^{-i\omega_n t}. \quad (9)$$

and introduce (9) into (8). The angular frequency ω_n is on non-dimensional form from Eq. (4), but the retarded time $t - r/a_\infty$ needs to be non-dimensionalized. With the scalings above, the time-dependence can be formulated as $\omega_n \tau$ where $\tau = t - M_\infty r$. Summation over $i = 1, 2$, $j = 1, 2$ yields

$$p'(\mathbf{x}, t) = \frac{1}{4\pi} \int_S (l_x n_x + l_y n_y) \frac{M_\infty}{r} \frac{\partial}{\partial t} \left(P + \sum_{n=1}^N c_n \hat{p}_n e^{-i\omega_n \tau} \right) dS.$$

The temporal derivative is equivalent to a multiplication with $-i\omega_n$ due to the ansatz of the time-dependence of the eigenmodes, and the time independence of the base pressure, hence

$$p'(\mathbf{x}, t) = \frac{1}{4\pi} \int_S (l_x n_x + l_y n_y) \frac{M_\infty}{r} \sum_{n=1}^N c_n \hat{p}_n (-i\omega_n) e^{-i\omega_n \tau} dS.$$

which can be reformulated as

$$p'(\mathbf{x}, t) = \sum_{n=1}^N c_n e^{-i\omega_n t} \underbrace{\left[-\frac{1}{4\pi} \int_S (l_x n_x + l_y n_y) \frac{M_\infty}{r} \hat{p}_n i\omega_n e^{i\omega_n M_\infty r} dS \right]}_{\phi_n}.$$

for $t \geq M_\infty r_{max}$ where r_{max} is the distance from the observer to the point in the source region that is furthest away from the observer location.

Note that ϕ_n is independent of time. This means that the integral only needs to be evaluated once for every n and every observation location. We have

$$p'(\mathbf{x}, t) = \sum_{n=1}^N c_n \phi_n e^{-i\omega_n t}, \quad t \geq M_\infty r_{max}. \quad (10)$$

Writing (8) in the form of (10) is beneficial, since it makes it an extremely fast task to perform an analysis of a chosen initial condition. Also, since the equation is written as a sum of constant coefficients and complex exponential functions, the spectrum of the sound is in fact determined at the same time as the sound itself.

In this paper we only consider the far-field sound generated by the pressure fluctuations at the surface, but the same methodology applies to all the linear terms in the full Curle's equation. The non-linear terms have to be considered differently.

OPEN CAVITY

In contrast to many aeroacoustic studies on open cavities, where cavities of short length to depth ratio and sharp corners has been considered, see e.g. [8], [3], the cavity geometry used here have a very long length to depth ratio of about $L/D \approx 20$, with smooth corners. A sketch of the geometry of the cavity is given in Figure 1.

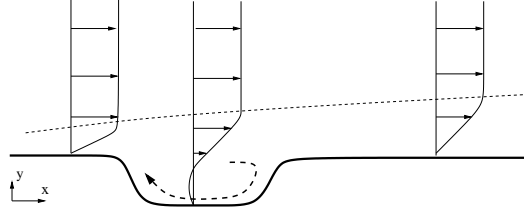


Figure 1: *Overview of the geometry of the cavity*

Calculations were performed using the non-dimensional form of the incompressible Navier-Stokes equations, at a Reynolds number of $Re_{\delta_0^*} = U_{\infty} \delta_0^* / \nu = 325$ based on the boundary layer thickness at the inflow of the domain, δ_0^* where

$$\delta_0^* = \int_0^{\infty} \left(1 - \frac{U(y)}{U(\infty)} \right) dy.$$

Here U_{∞} is the free stream velocity and ν is the kinematic viscosity. The boundary layer thickness at the inflow is also the length scale used in the scalings in (7).

A set of eigenvalues with corresponding eigenmodes were calculated, and a subset of the computed eigenvalues is shown in Figure 2. Since the eigenvalue problem is formulated using the non-dimensional Navier-Stokes equation, the eigenvalues and the eigenmodes are given on non-dimensional form. To relate the non-dimensional eigenvalues in Figure 2 to the corresponding dimensional eigenvalues, the scaling $\omega = U_{\infty} \omega^* / \delta_0^*$ can be used. Here ω and ω^* are the dimensional and the non-dimensional eigenvalues, respectively. As can be seen from (10), ω will also be the angular frequency of the generated sound.

Note that the damping of two of the eigenvalues actually is positive, i.e. they will grow in time. Hence, the corresponding eigenmodes will cause exponential growth and the dynamics is after a certain time not described by the linearized equations.

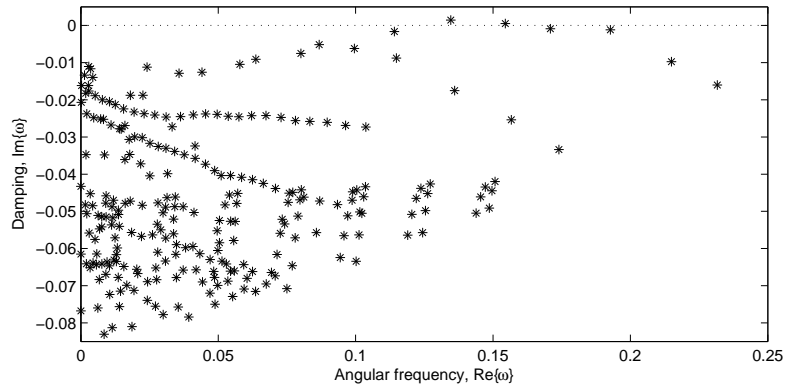


Figure 2: *Spectrum of the non-dimensional eigenvalue problem.*

In Figure 3 the real part of the velocity in the x -direction is shown for the most unstable eigenmode, that is, the eigenmode with the most positive imaginary part. The corresponding eigenvalue is $\omega = 0.1346 + 0.0014i$. If any or both of the unstable eigenmodes are triggered, they will grow exponentially and will eventually dominate the sound field.



Figure 3: *Overview of the real part of the u -component of the most unstable eigenmode.*

A Vortex In The Cavity

We apply the eigenmode decomposition technique on a $M_\infty = 0.1$ flow where the initial condition describes a vortex with the velocity field $\mathbf{u}(r, \theta) = (1 - e^{-r^2})/(50r) \mathbf{e}_\theta$ centered at $x = 120$, $y = 2$. This can be seen as a simple model of a perturbation propagation in the shear layer. Figure 4 shows the absolute value of the velocity of the initial perturbation of the initial condition.



Figure 4: *Absolute value of the velocity of the initial perturbation.*

In Figure 5 the generated sound generated measured at an observer located at $x = 50$, $y = 100$ is shown. The observer location was in the direction where most sound is

assumed to be radiated. In the left figure, the sound from the individual eigenmodes are plotted separately. As can be seen, at around $t = 400$ the sound field is dominated by the two unstable eigenmodes, which together creates a beating effect.

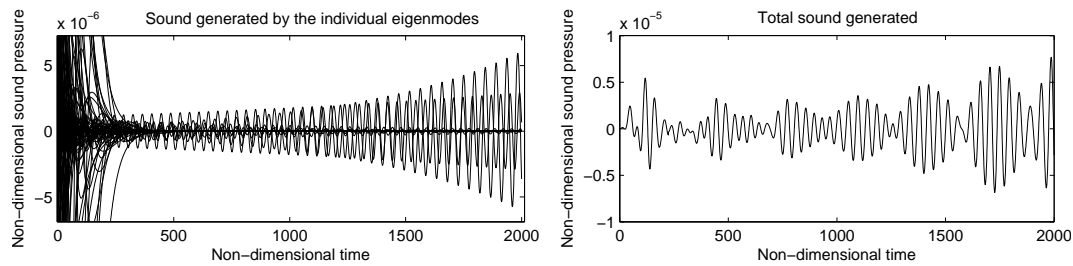


Figure 5: *Sound generated by the individual eigenmodes (left), and the total sound (right).*

In Figure 6 the surface pressure fluctuation is plotted for different times. The cavity is located in $50 < x < 100$. It is clearly seen that the sound generating mechanisms are at the trailing edge of the cavity.

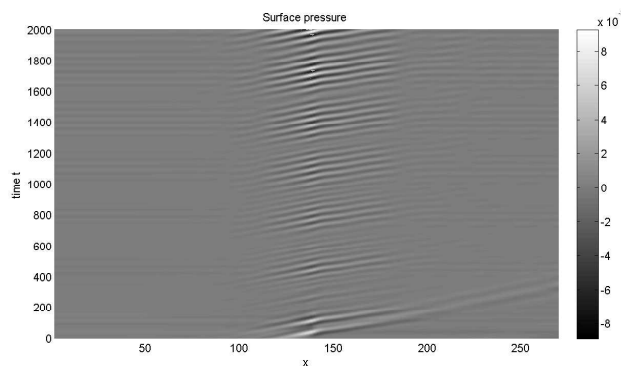


Figure 6: *Surface pressure fluctuations for different times*

The calculated directivity of the generated sound is governed by two effects. Firstly, the large pressure fluctuations at the trailing edge of the cavity generates sound that is close to omnidirectional, but with a slightly stronger radiation in the downstream direction. Secondly, the regions above and below the trailing edge seems to form a dipole in the stream-wise direction which yields a large dip vertically above the trailing edge.

Optimal Growth

As a second test case we used initial data, chosen such that an optimal growth is obtained, see [6]. For the cavity flow considered in this study, the optimal growth initial data is mainly governed by perturbations located in the upstream region of the shear layer. Calculations

showed that the generated sound is about three orders of magnitude larger than from the initial condition describing a vortex in the cavity, even though the perturbations are of the same order at $t = 0$. This clearly shows the strong dependence on the initial value of the sound generation.

SUMMARY

An alternative approach to using DNS as data for Curle's equation has been investigated. The dynamics of the flow field was decomposed into its global modes and a reduced model consisting of only a small part of the global modes have been used as source data. The technique is fairly new in the field of fluid dynamics and its usefulness to aeroacoustics therefore interesting to examine. The computational effort of solving Curle's equation is also drastically decreased using this approach. Although the method requires the dynamics to be within the linear regime, its strength is its ability to identify the sound generating structures or "acoustic hotspots".

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