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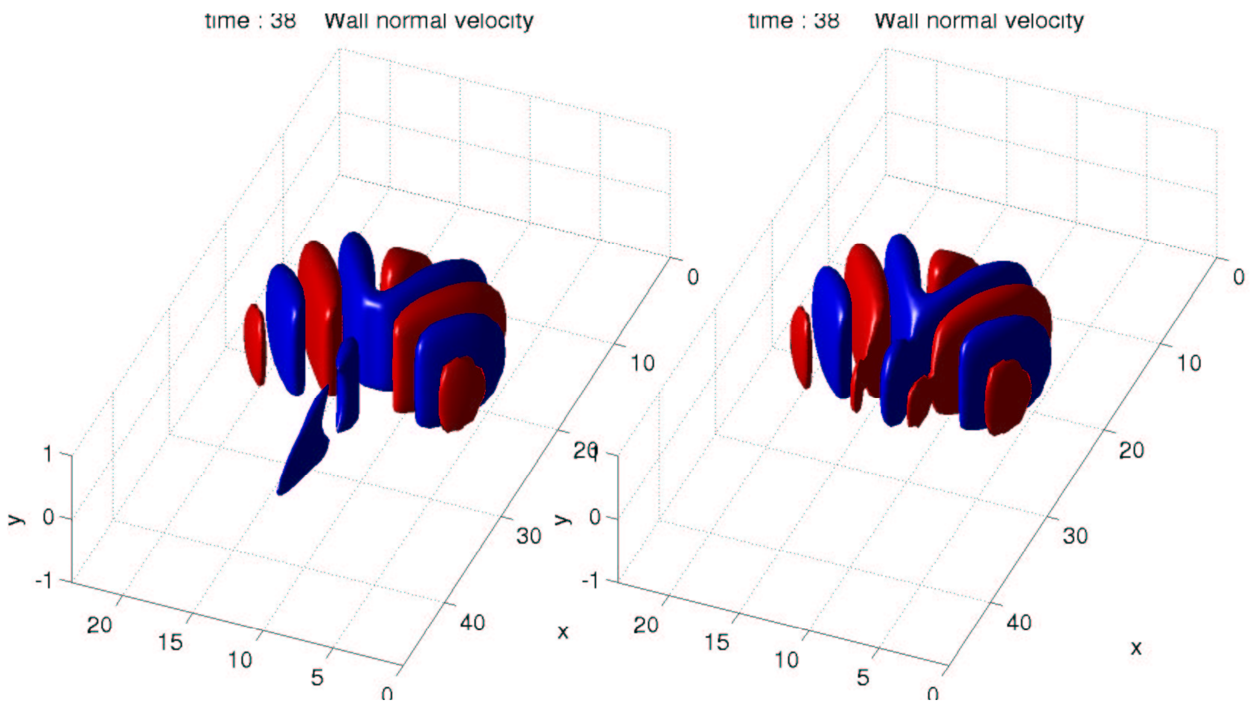
Optimal estimation applied to wall bounded shear flows



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Motivation

Reconstruct state from partial information The estimation problem

- Partial information of the state : wall sensors
- Incomplete information about the dynamics : linearized equations
- Statistical information on uncertainties

Flow control The compensation problem

- Promising results : numerical experiments of state feedback control (Högberg, Bewley, Henningson)

PART 1 : Estimation

from the stochastic point of view

The system

Addressed flows limiting assumptions

- Parallel shear flow : 2 homogeneous directions
- No nonlinear dynamic

Governing equation Incompressible Navier Stokes

$$\underbrace{\frac{d}{dt} M \hat{q}_{mn} + L \hat{q}_{mn}}_{Linear\ dynamic} = \underbrace{\sum_{k+i=m, l+j=n} N(\hat{q}_{kl}, \hat{q}_{ij})}_{Nonlinear\ coupling} + \underbrace{g_{mn}\hat{(}y,t)}_{External\ forcing}$$

$$\hat{q}_{mn} = \begin{pmatrix} \hat{v}_{mn} \\ \hat{\eta}_{mn} \end{pmatrix}, M = \begin{pmatrix} -D^2 + k_{mn}^2 & 0 \\ 0 & 1 \end{pmatrix}, L = \begin{pmatrix} \mathcal{L}_{OS} & 0 \\ \mathcal{L}_{C} & \mathcal{L}_{SQ} \end{pmatrix}$$

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$$\underbrace{\frac{d}{dt}M\hat{q}_{mn} + L\hat{q}_{mn}}_{Linear\ dynamic} = \underbrace{\hat{f}_{mn}(y,t)}_{Stochastic\ forcing}$$

At the boundaries No-slip

Information at the walls

Customary So far either skin friction or pressure at the wall

$$\tau_{xy} = \mu(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) \quad , \quad \tau_{zy} = \mu(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z})$$

Taylor expansion Each of these three measurements give additional information

In the optimization code Formulation :

Wall normal velocity $v(y)$

Wall normal vorticity $\eta(y)$

$\rightarrow$ convenient measurements are $\mu\ \eta_y$, $\mu\ v_{yy}$ and pressure

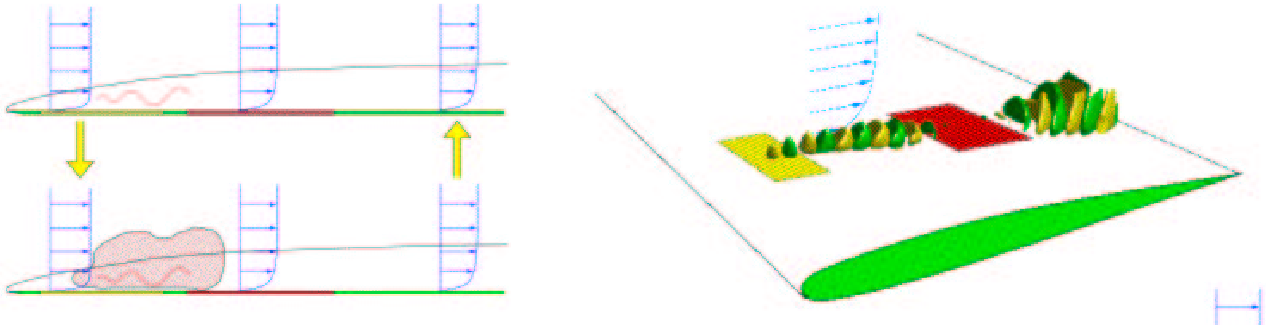
The filter

Available (affordable) knowledge :

- Noisy measurements at the wall
  $\longrightarrow$ known variance and covariance of the sensor noise
- Statistical description of the state disturbance
  $\longrightarrow$ Unmodeled dynamic and external forcing
- Modeled dynamic of the state
  $\longrightarrow$ Incompressible Navier-Stokes equation linearized about base profile

**the estimator is a numerical simulation of the experiment,
which is forced in order to converge to the true state**

The procedure



- measure flow quantities at the wall in the experiment : $y = C_2 x + B_1 w_1$
  $\longrightarrow$ Wind tunnel (DNS for the moment)
- measure flow quantities at the wall in the estimator : $\hat{y} = C_2 \hat{x}$
  $\longrightarrow$ DNS of full Navier Stokes (reduced order model in the future)
- Apply a volume forcing in the estimator from the measurement error $\delta y = y - \hat{y}$
  $\longrightarrow$ Through the estimation feedback kernels L

The plant and the estimator

The optimization is achieved at each wave number pair independently :

- The plant

$$\begin{cases} \dot{\mathsf{x}} = A\mathsf{x} + B_1 w_1 \quad , \quad \mathsf{x}(0) = \mathsf{x}_0 \\ y = C_2\mathsf{x} + D_{21} w_2 \end{cases}$$

- w_1 and w_2 uncorrelated white noise processes
- The estimator

$$\begin{cases} \dot{\tilde{\mathsf{x}}} = A\tilde{\mathsf{x}} - \tilde{v} \quad , \quad \tilde{\mathsf{x}}(0) = \tilde{\mathsf{x}}_0 \\ \tilde{y} = C_2\tilde{\mathsf{x}} \end{cases}$$

- The feedback law

$$\tilde{v} = L(y - \tilde{y}) = L\delta y$$

Optimality and covariances

Minimize the expected energy of the estimation error due to :

- Stochastic forcing of the state
- Error in the measurement
- Error in the initial condition

Estimation error covariance and dynamic

$$P = cov(\delta \mathsf{x}, \delta \mathsf{x}) \quad , \quad A_0 = A + LC_2 \quad , \quad P(0) = X_0$$

Lyapunov equation :

$$\frac{d}{dt}P = \underbrace{A_0 P + P A_0^H}_{Dynamic\ terms} + \underbrace{B_1 B_1^H}_{State\ disturbances} + \underbrace{(LD_{12})(LD_{12})^H}_{Measurement\ noise}$$

Then carry an optimization on L

Estimation gains

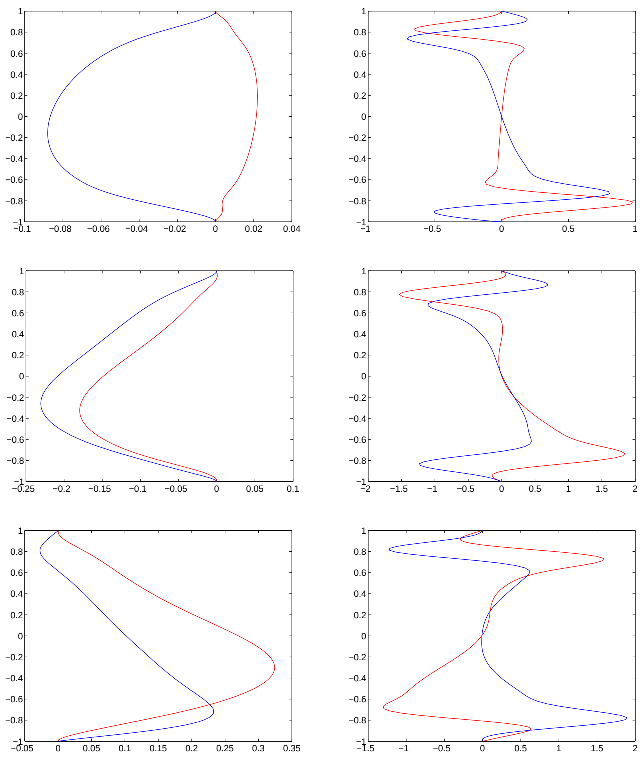
Multiplicative gains in Fourier space : $L_{mn}(y) \rightarrow$ Convolution kernels in physical space : $L(x, y, z)$

forcing on v | Forcing on η

η_y measurement

v_{yy} measurement

p measurement



Modal decoupling : tractability and physical relevance

Decoupling The dynamic equation and objective function are decoupled in Fourier space

Reconstruction But what about the physical structure of the estimation forcing?

What is the 'input knowledge' to the optimization that ties the gains together?

Localized perturbations

Physical argument : correlation decays with distance
$\rightarrow$ action of estimator, remote from sensor must decay as well

This argument is valid if the noise has itself a localized structure.

Assume a noise shaped in physical space by functions of the form :

$$R(r_x, r_y, r_z) = \exp^{-(s_x r_x)^2 - (s_y r_y)^2 - (s_z r_z)^2}$$

Corresponding shape in Fourier space

$$\hat{R}(y, \alpha, \beta) = \frac{v}{4\pi s^2} \exp^{-\frac{k^2}{4s^2}} \exp^{-(s_y r_y)^2}$$

$\rightarrow$ **specification of the input operator** B_1

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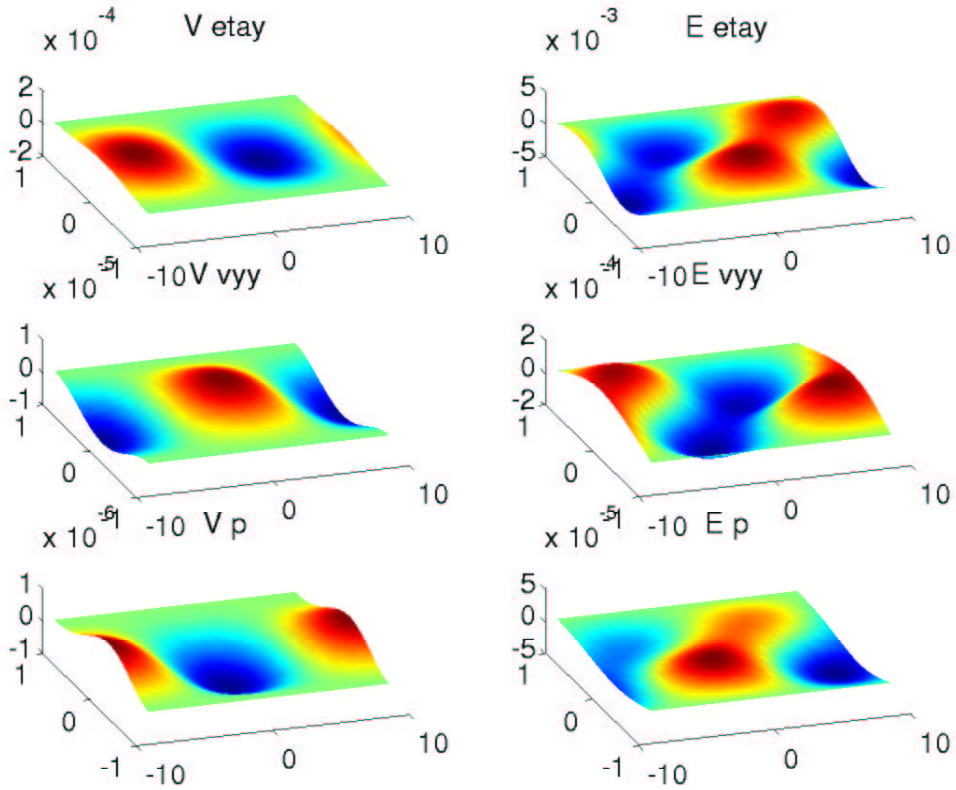
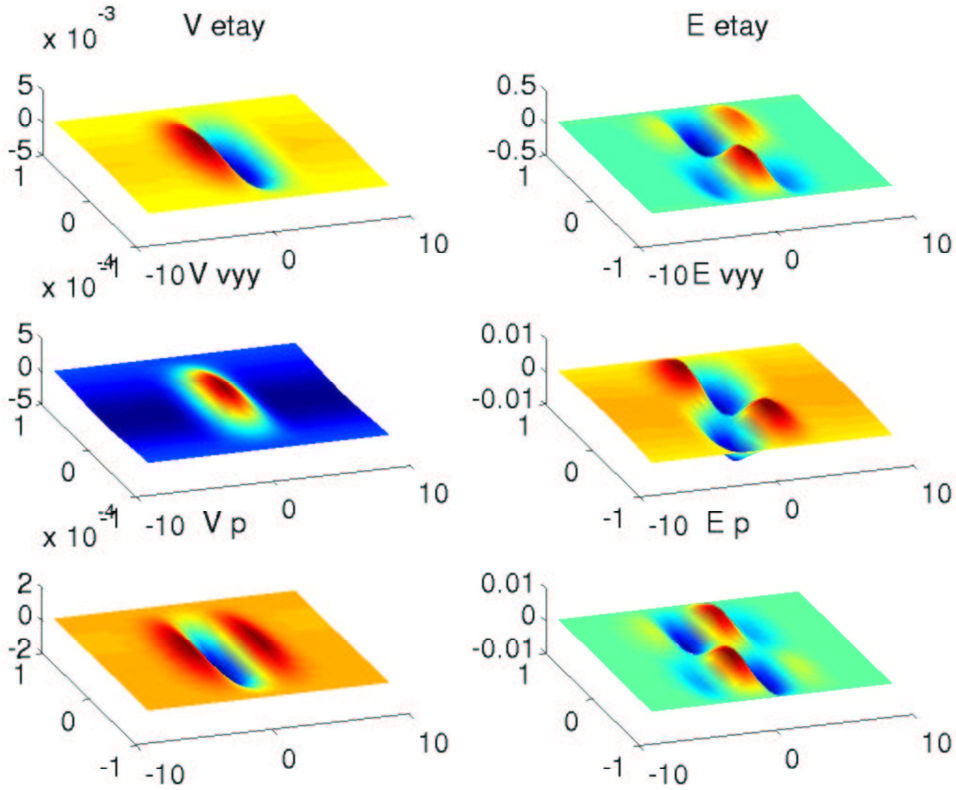
Localized action $s_x = s_z = 0.1$



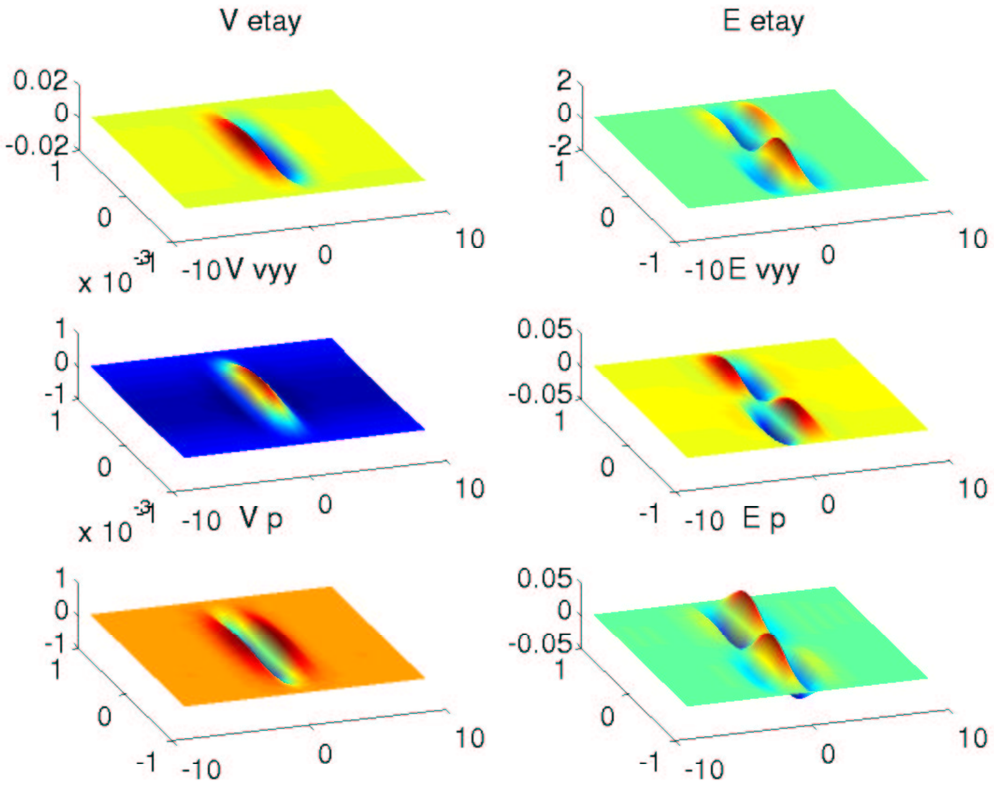
2D (z,y) Fourier mode of estimation kernels

Localized action $s_x = s_z = 0.5$



2D (z,y) Fourier mode of estimation kernels

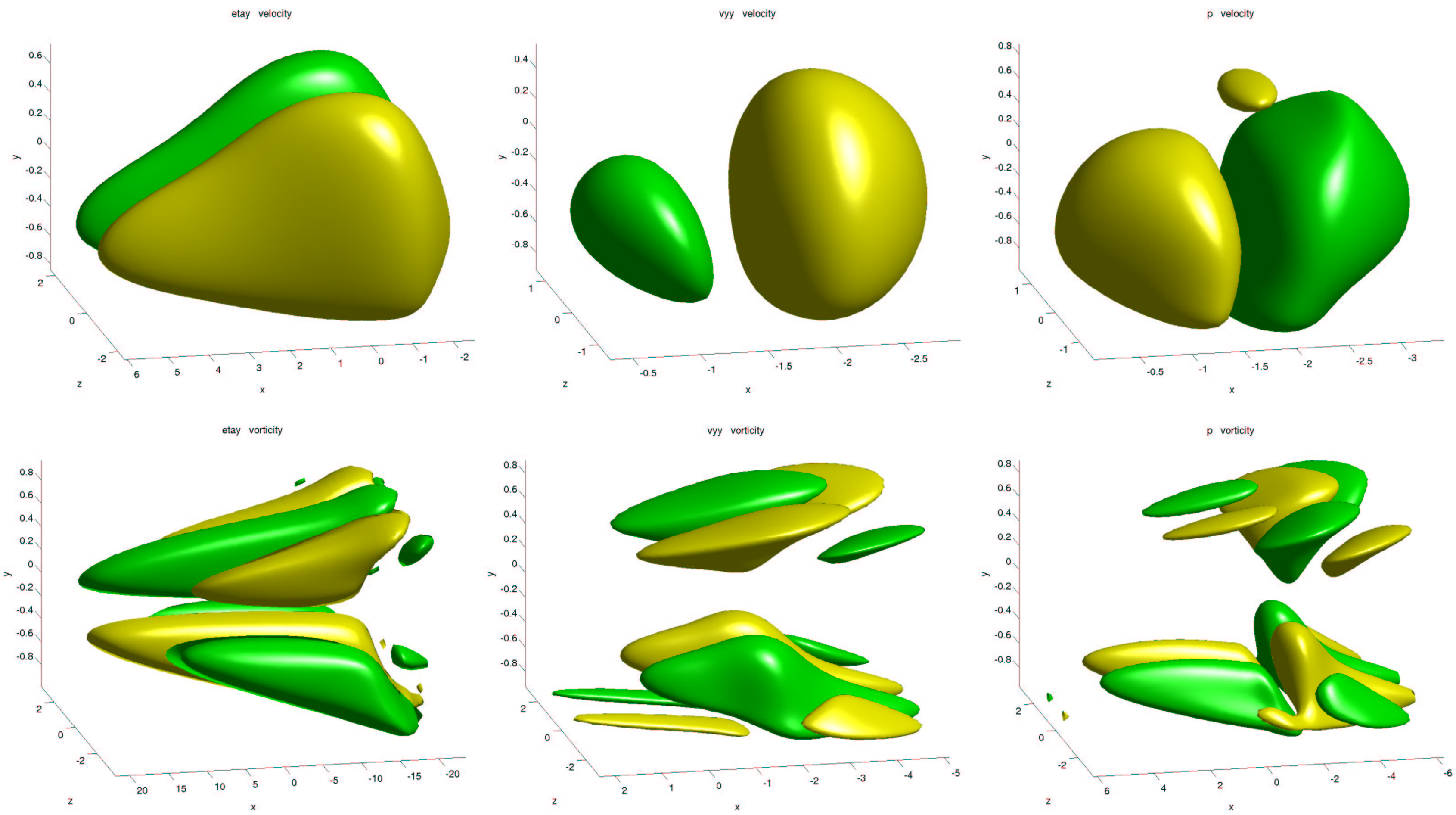
Localized action $s_x = s_z = 1$



2D (z,y) Fourier mode of estimation kernels

Convolution kernels

η_y v_{yy} p



PART 2 : Test on the flow

Direct Numerical Simulations

OPUS – incompressible Navier–Stokes equation solver

Spectral / finite-difference / spectral discretization

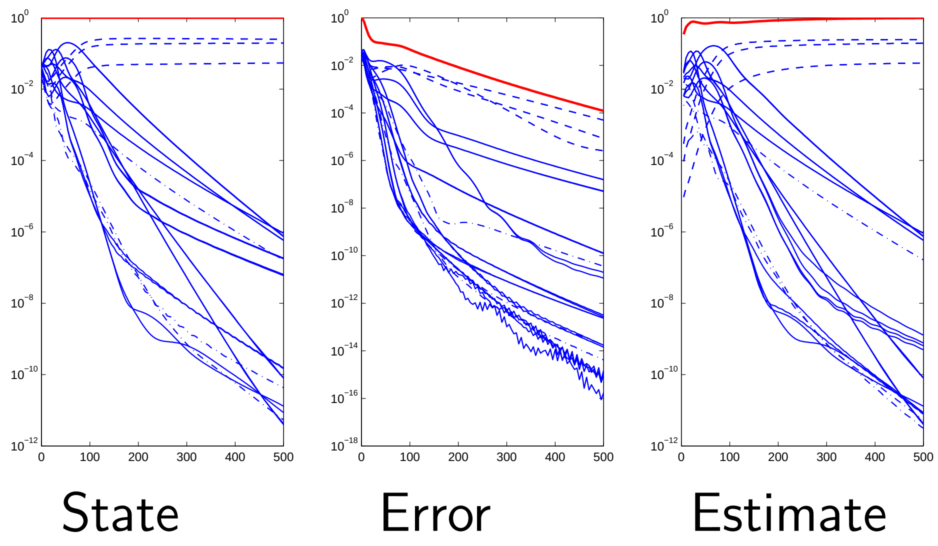
Constant-mass flux turbulent channel flow at $Re_{CL} = 3000$

A first test test case

- Put for each wavenumber pair the worst case linear initial disturbance and estimate the evolution of the flow
  - → rich behaviour of the flow
  - → low number of modes
  - → low amplitudes

- Test each of the measurement separately

Energy of each Fourier mode in time



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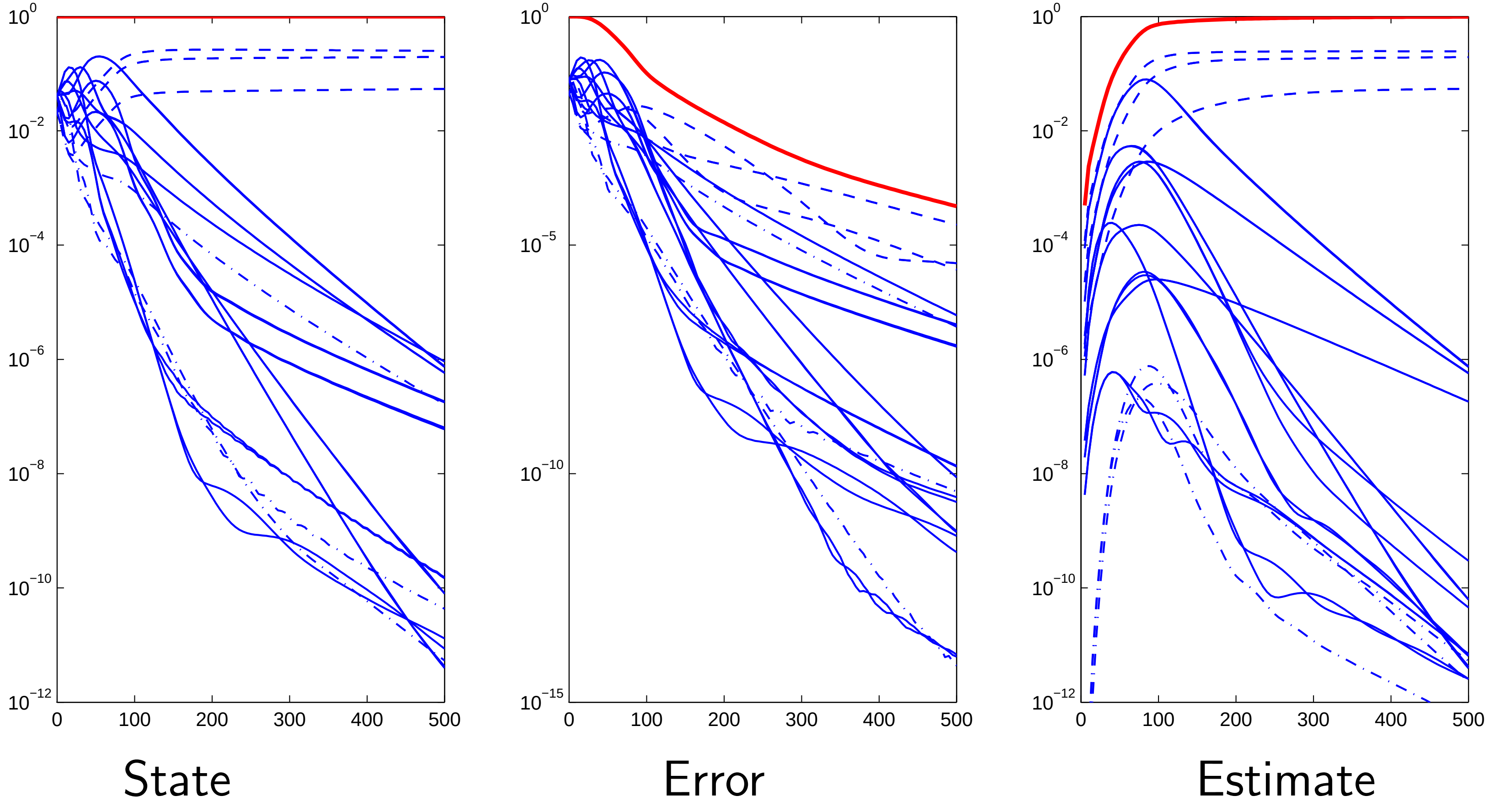
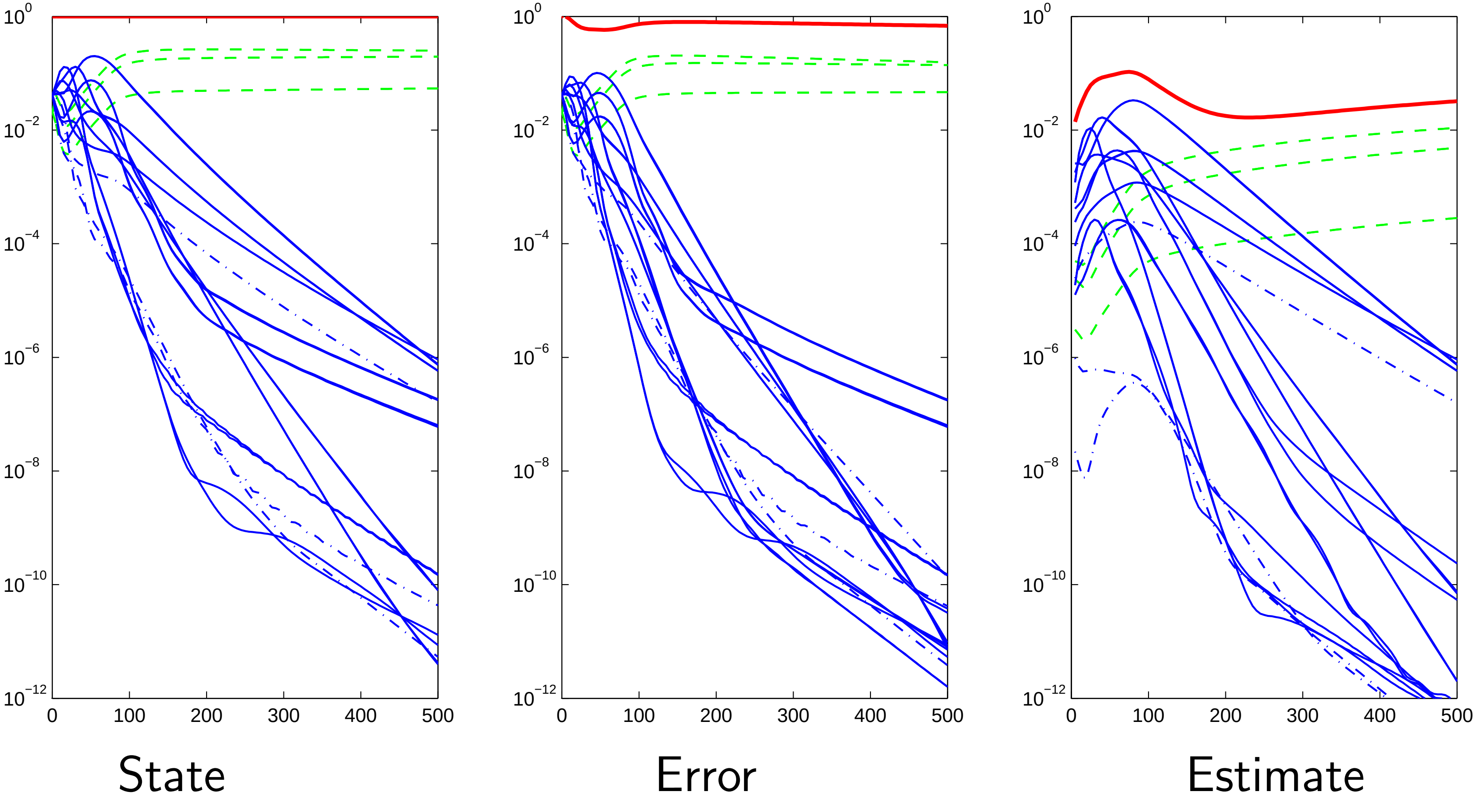
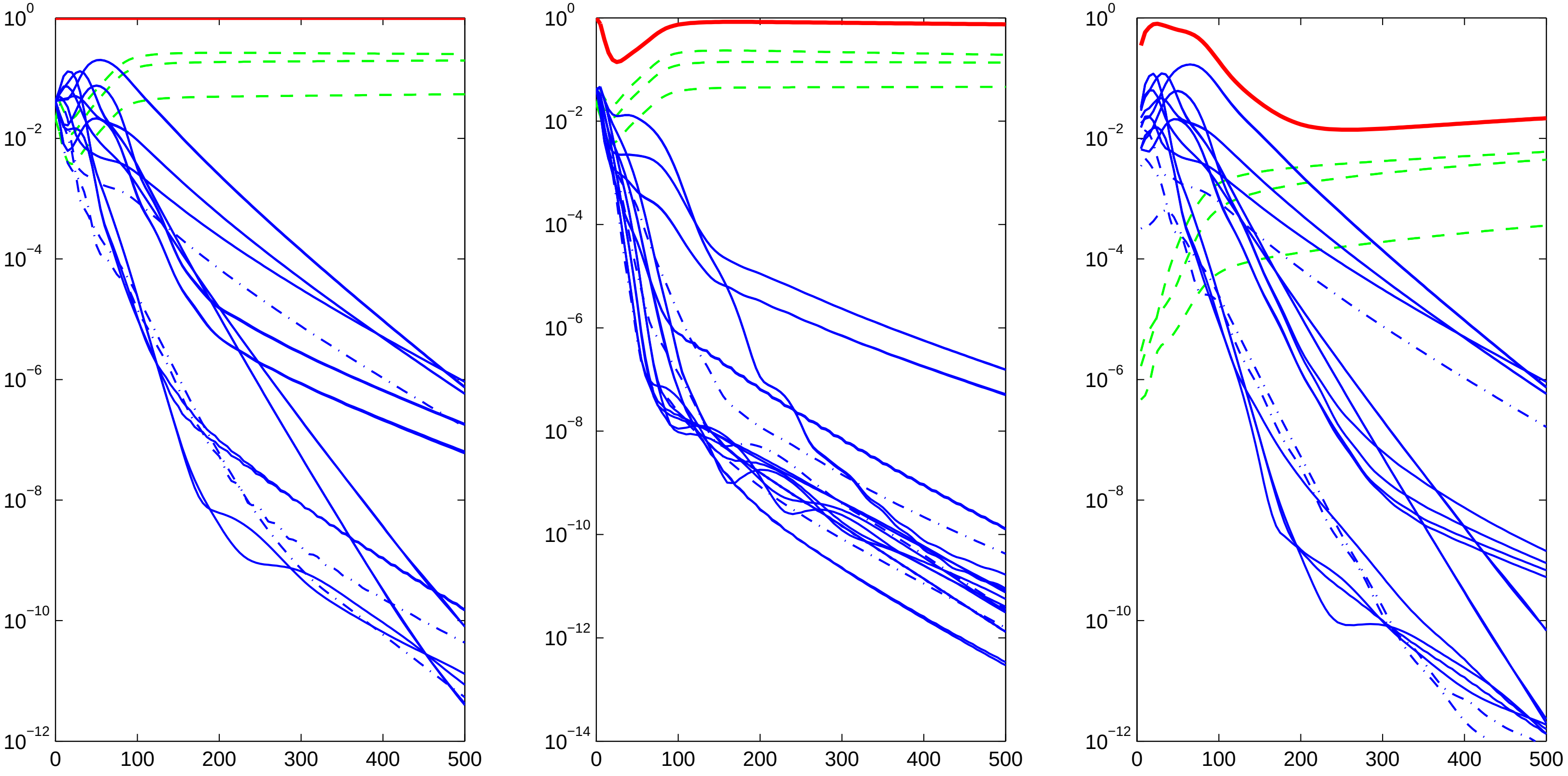
η_y measurement only



v_{yy} measurement only

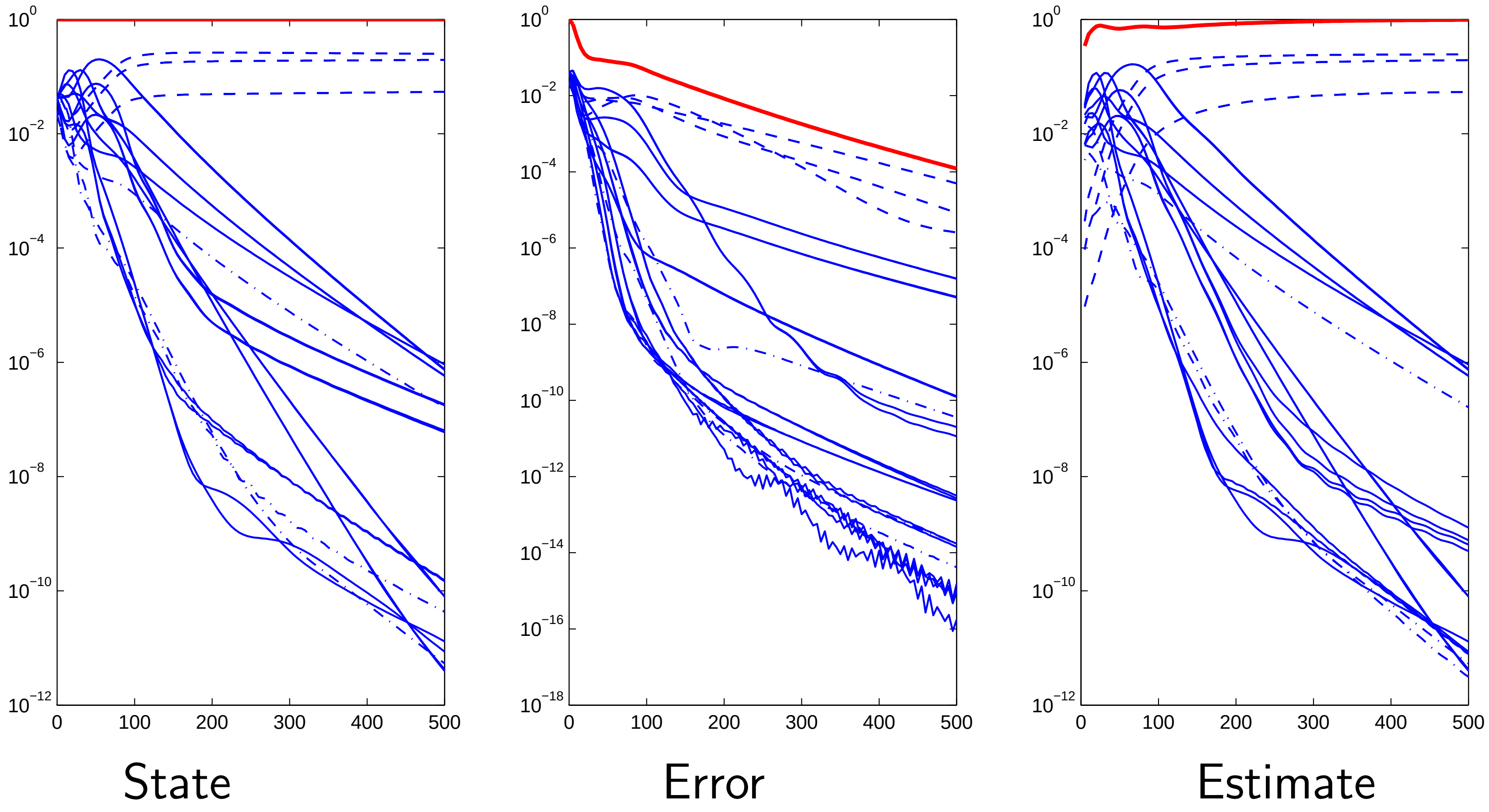


Pressure measurement only

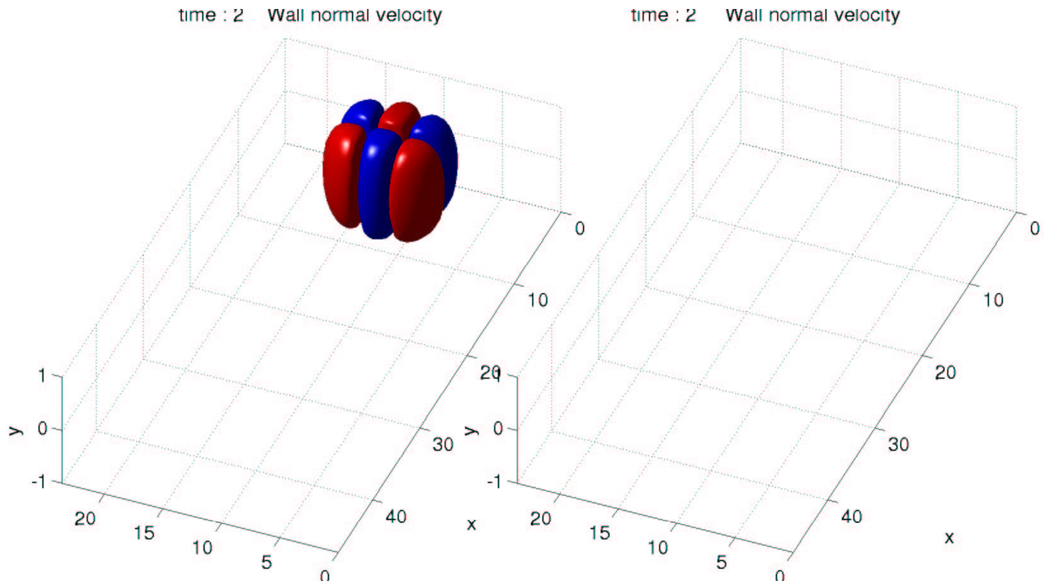


State Error Estimate

All three measurements

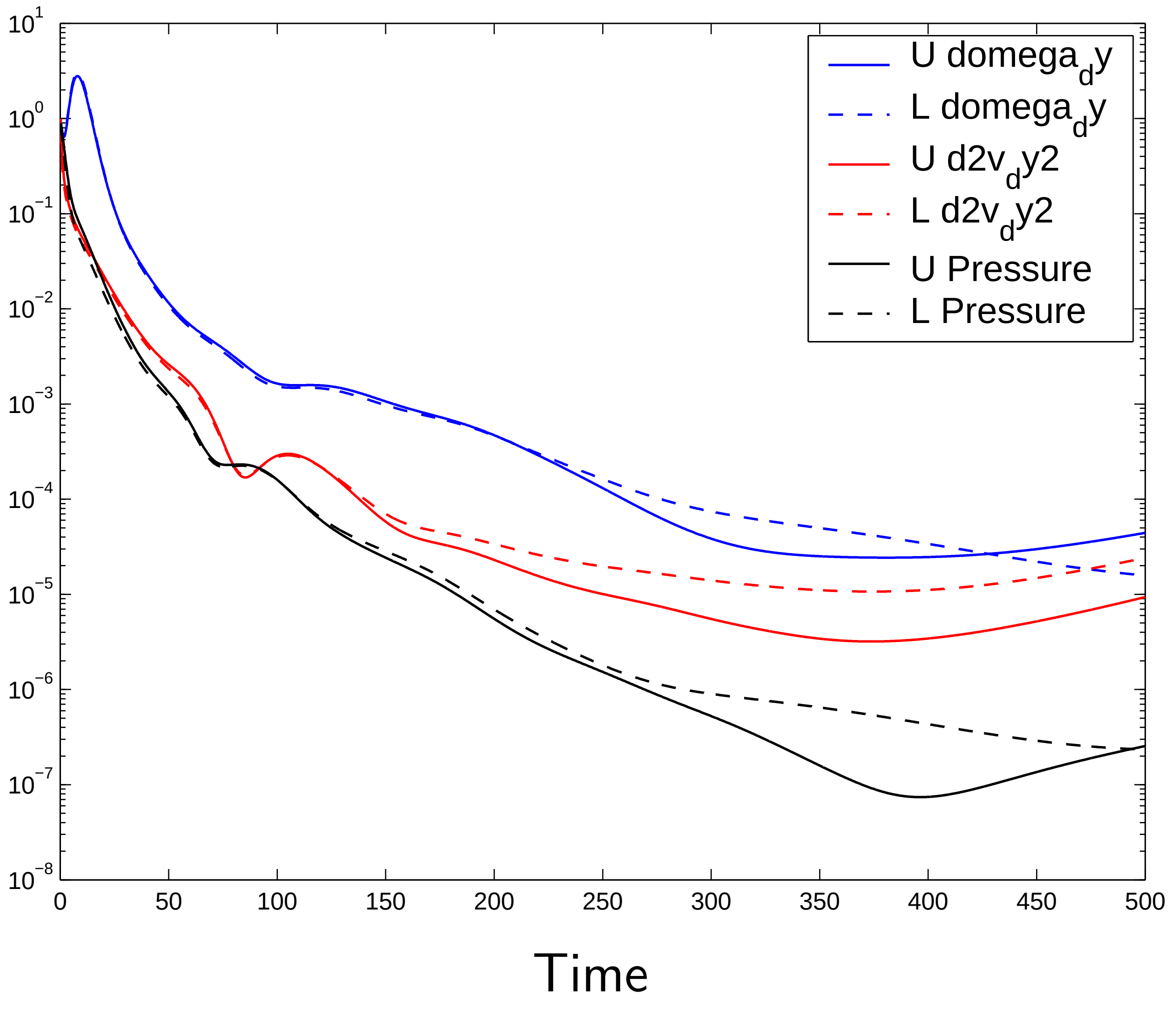


A realistic test case

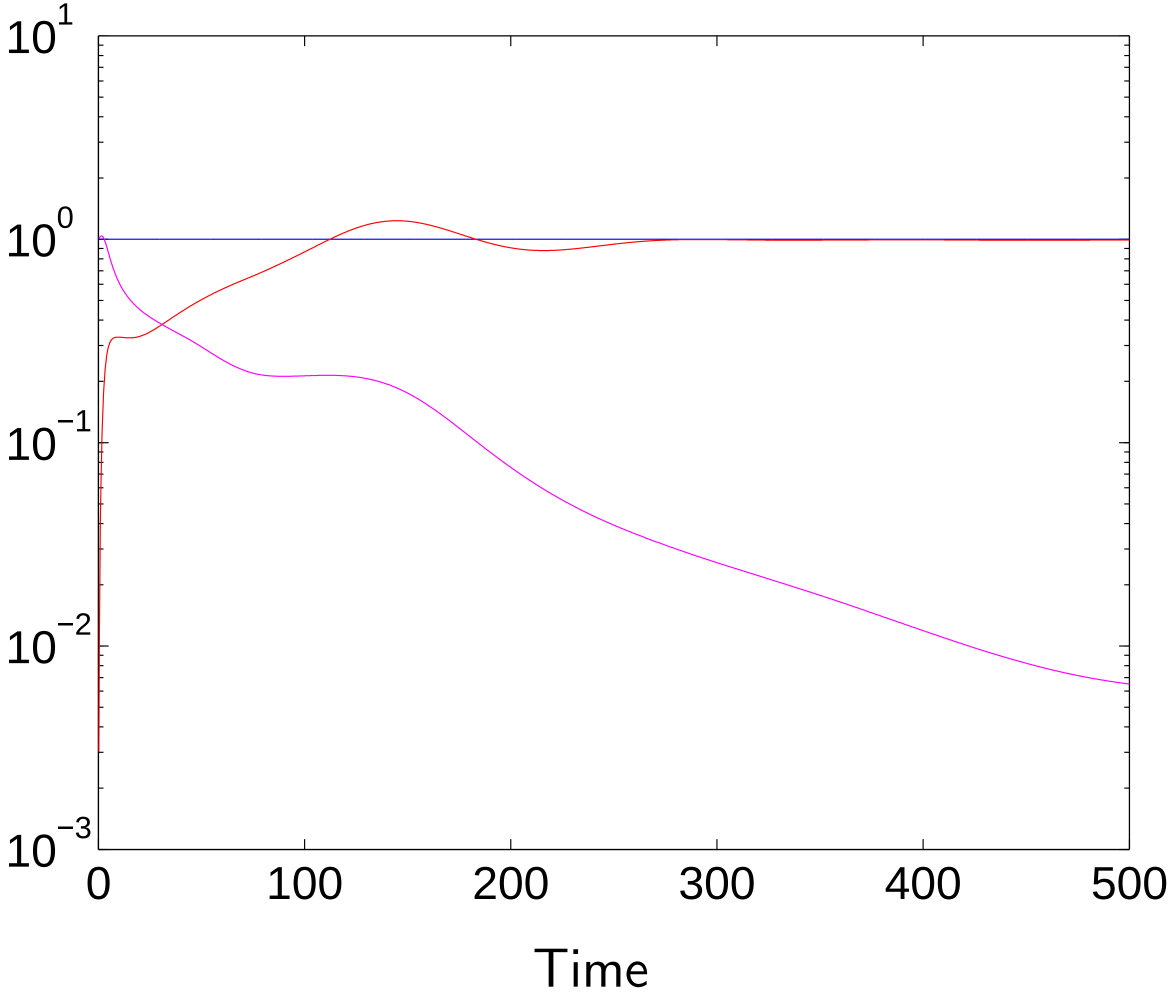


Small amplitude case of :
A mechanism for bypass transition from
localized disturbances in wall bounded shear flows (Henningson et. al. 1992)

Measurement error



Energy error



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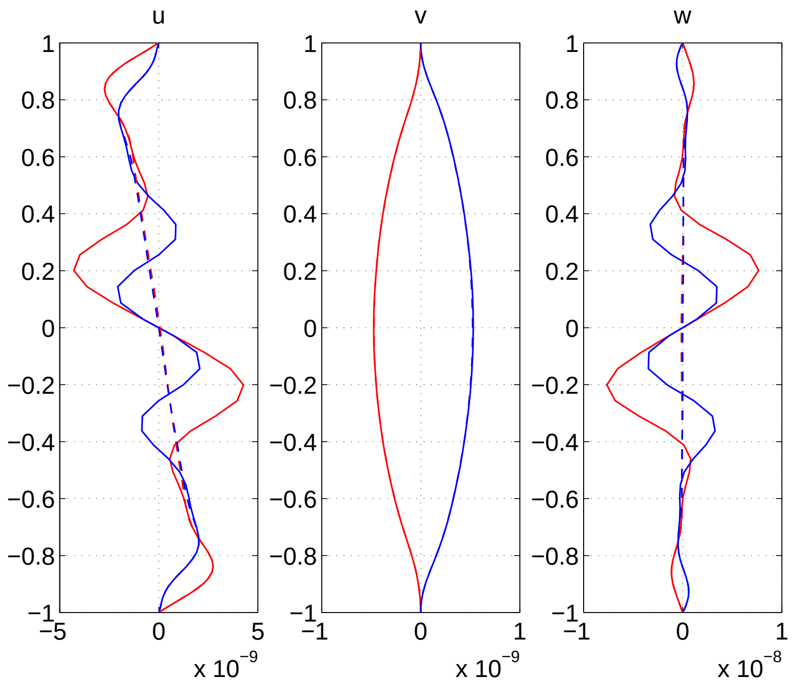
Convergence of the Fourier modes

At time 125



Blue : real part

Red : imaginary part

Dashed : estimate

Solid : state

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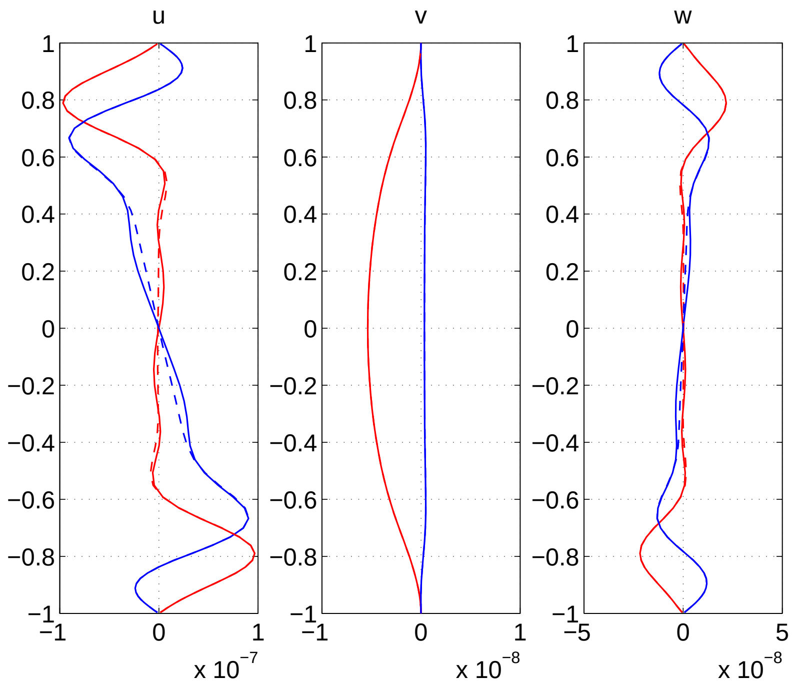
Convergence of the Fourier modes

At time 250



Blue : real part
Dashed : estimate

Red : imaginary part
Solid : state

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PART 3 : Turbulence estimation

Preliminary study and first results.

Direct Numerical Simulations

OPUS – incompressible Navier–Stokes equation solver

Spectral / finite-difference / spectral discretization

Constant-mass flux turbulent channel flow at $Re_\tau = 100$

Resolution : $42 \times 64 \times 42$

Plan to compute statistics also for $Re_\tau = 180$

Turbulent Flow Statistics

Compute statistical quantities of the physics missing in the dynamical model

Linearize incompressible Navier–Stokes equations and add forcing terms f_1, f_2, and f_3

$$\begin{aligned}
\frac{\partial \check{u}}{\partial t} + U\frac{\partial \check{u}}{\partial x} + \check{v}\frac{\partial U}{\partial y} &= -\frac{\partial \check{p}}{\partial x} + \frac{1}{Re}\Delta \check{u} + f_1 \\
\frac{\partial \check{v}}{\partial t} + U\frac{\partial \check{v}}{\partial x} &= -\frac{\partial \check{p}}{\partial y} + \frac{1}{Re}\Delta \check{v} + f_2 \\
\frac{\partial \check{w}}{\partial t} + U\frac{\partial \check{w}}{\partial x} &= -\frac{\partial \check{p}}{\partial z} + \frac{1}{Re}\Delta \check{w} + f_3
\end{aligned}$$

The flow variables are divided into mean (U) and fluctuating ($\check{u}$, $\check{v}$, $\check{w}$, and $\check{p}$) part

Forcing Terms

$$f_1 = -\check{u}\frac{\partial \check{u}}{\partial x} - \check{v}\frac{\partial \check{u}}{\partial y} - \check{w}\frac{\partial \check{u}}{\partial z} - \frac{\partial P}{\partial x} + \frac{1}{Re}\frac{\partial^2 U}{\partial y^2}$$

$$f_2 = -\check{u}\frac{\partial \check{v}}{\partial x} - \check{v}\frac{\partial \check{v}}{\partial y} - \check{w}\frac{\partial \check{v}}{\partial z}$$

$$f_3 = -\check{u}\frac{\partial \check{w}}{\partial x} - \check{v}\frac{\partial \check{w}}{\partial y} - \check{w}\frac{\partial \check{w}}{\partial z}$$

Two-point Correlation (R) and Covariance (Θ)

$$R_{ij}(r_x, y, y', r_z) = \langle \check{f}_i(x, y, z, t)\check{f}_j(x + r_x, y', z + r_z, t)\rangle$$

$\langle \quad \rangle$ denotes sample mean and $\check{f}_i = f_i - F_i$

$$\Theta_{ij}(k_x, y, y', k_z) = \langle \hat{\check{f}}_i(k_x, y, k_z, t)\hat{\check{f}}^*_j(k_x, y', k_z, t)\rangle$$

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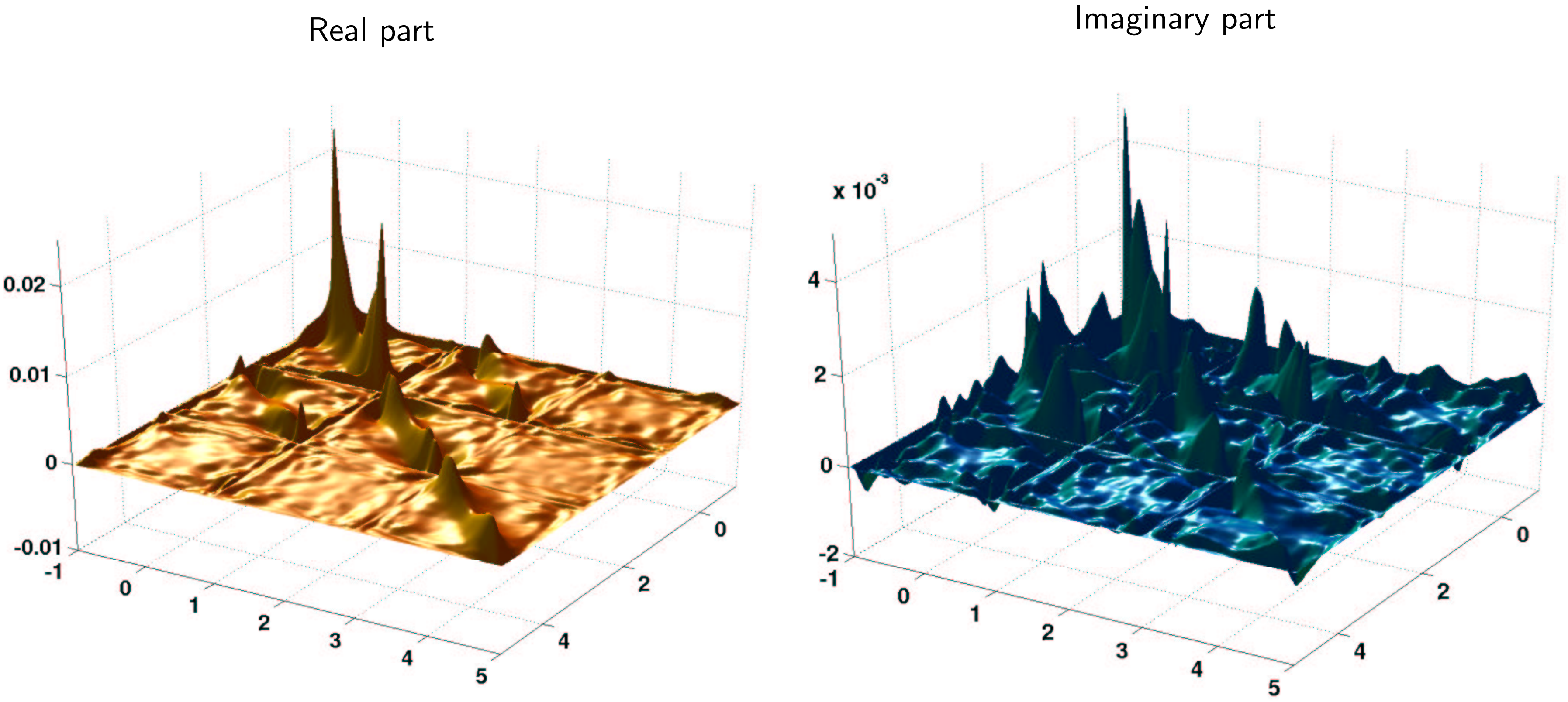
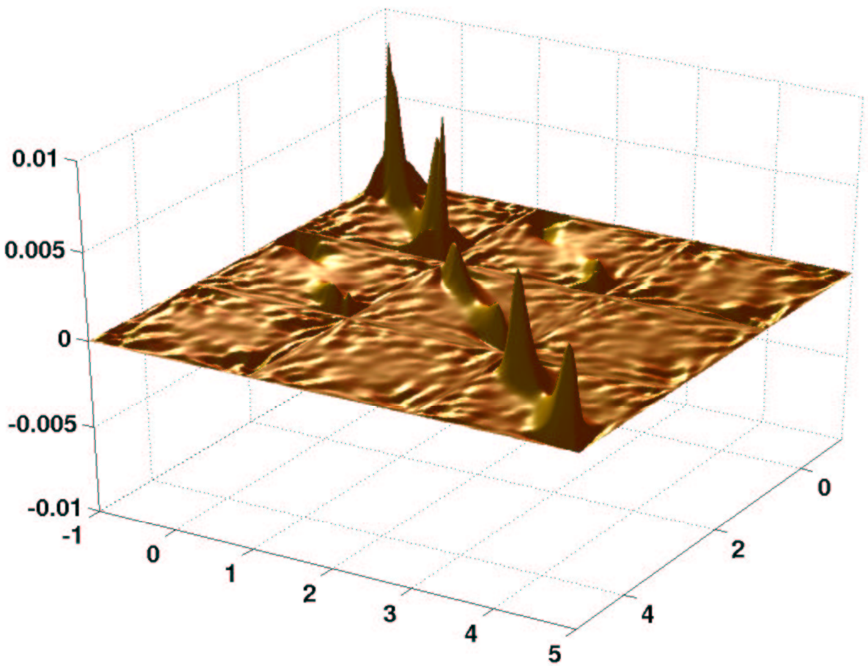
Covariance Data I



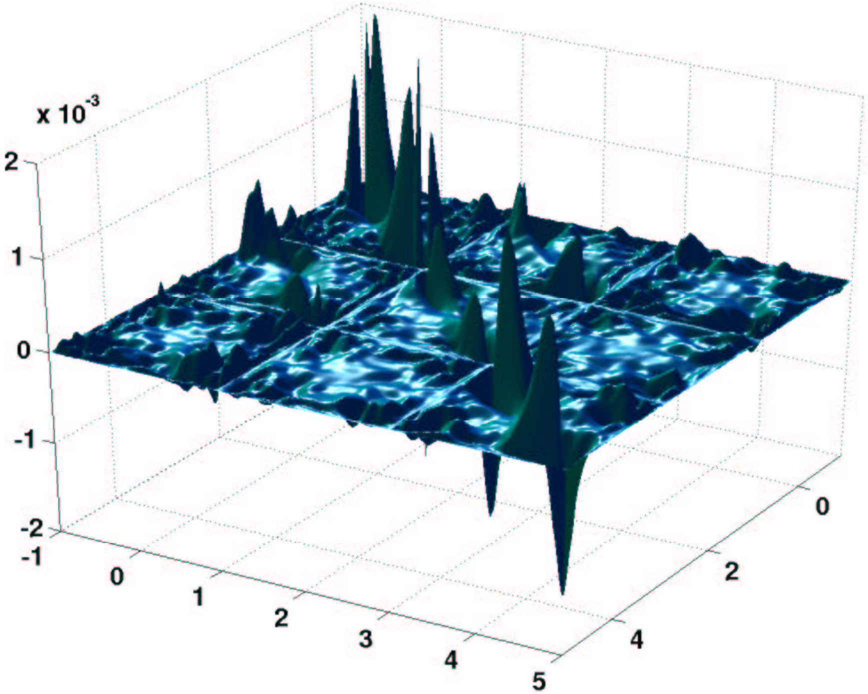
$k_x = 0.500$ and $k_z = 3.008$

Covariance Data II

Real part

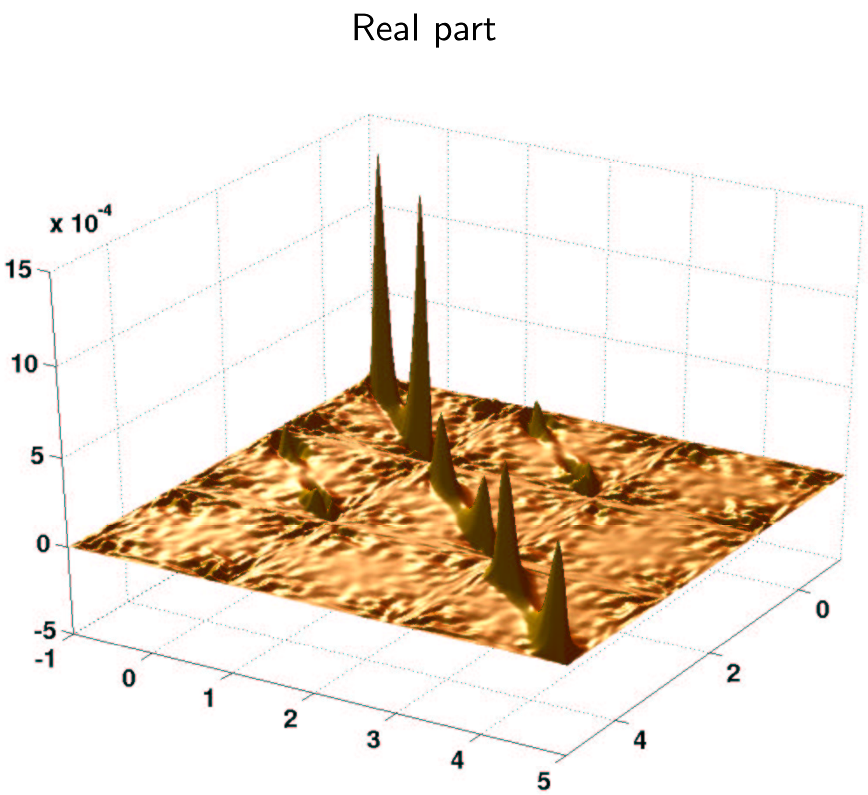


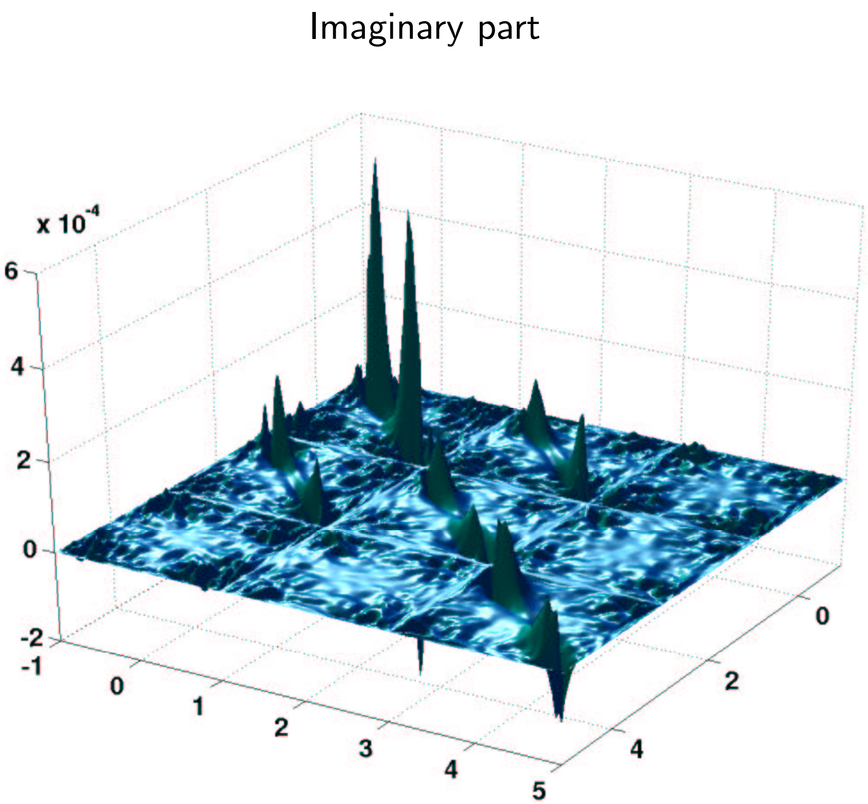
Imaginary part



$k_x = 3.500$ and $k_z = 10.5026$

Covariance Data III





$k_x = 7.500$ and $k_z = 22.5056$

Singular Value Decomposition

Extract noise shape functions from the covariance data through SVD

Approach related to the so-called "Proper Orthogonal Decomposition"

Pick only the few most energetic modes

Compute estimator gains based on extracted subset of shape functions

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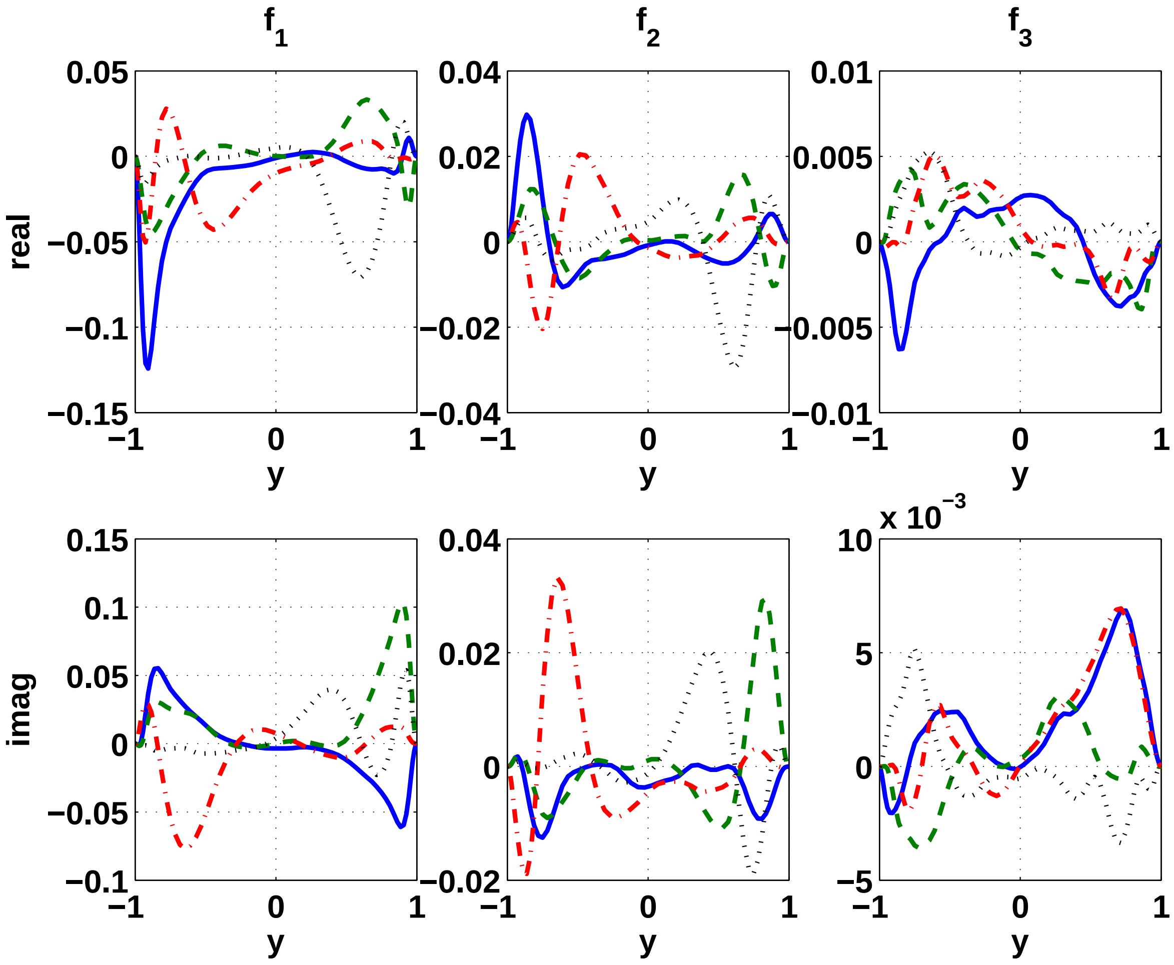
Shape Functions



Preliminary Results

Turbulent DNS for $Re_\tau = 100$

Estimator gains computed based on statistical data

Correlation between state (u) and estimator ($\check{u}$) for the streamwise flow component

$$\mathsf{corr}(u, \check{u}) = \frac{\int_0^{L_x}\int_0^{L_z} u\check{u}\,\mathsf{d}x\mathsf{d}z}{\sqrt{\int_0^{L_x}\int_0^{L_z} u^2\,\mathsf{d}x\mathsf{d}z}\sqrt{\int_0^{L_x}\int_0^{L_z} \check{u}^2\,\mathsf{d}x\mathsf{d}z}}$$

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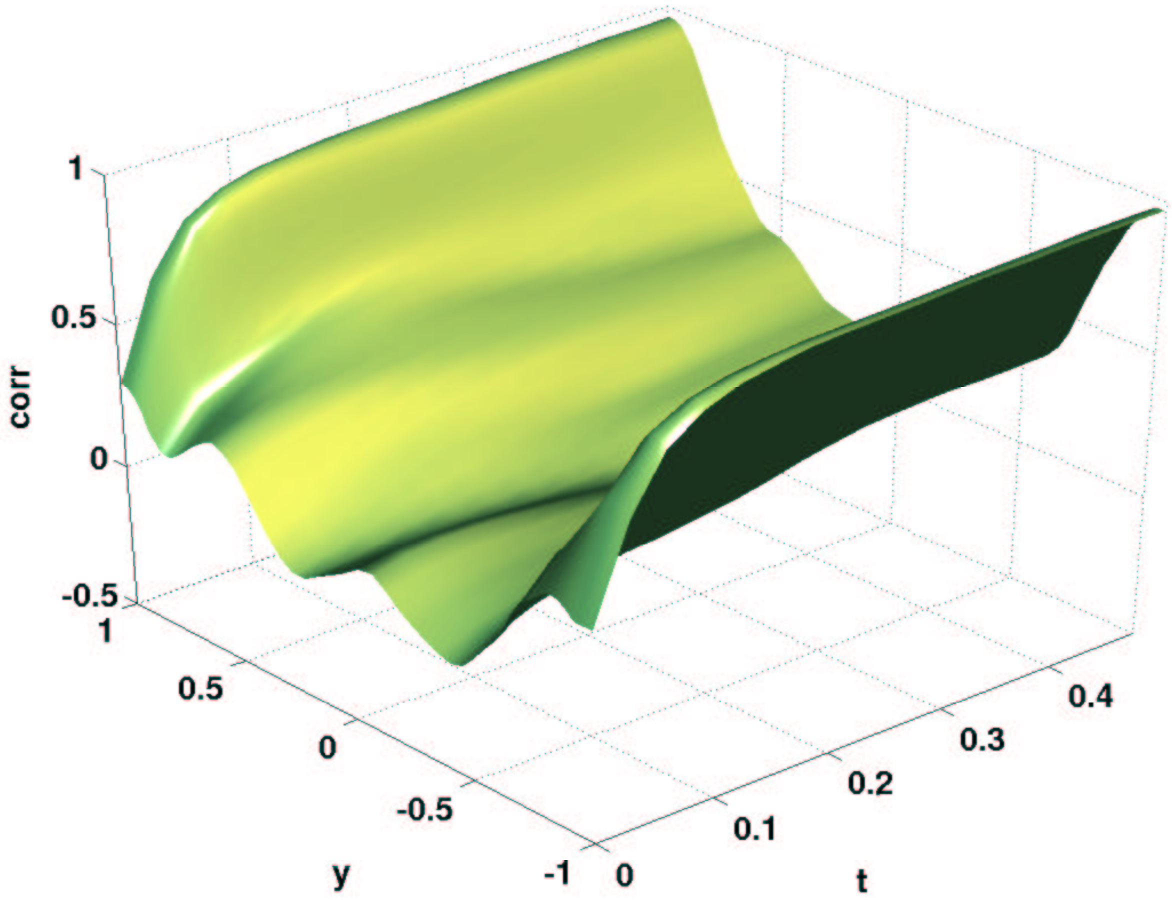
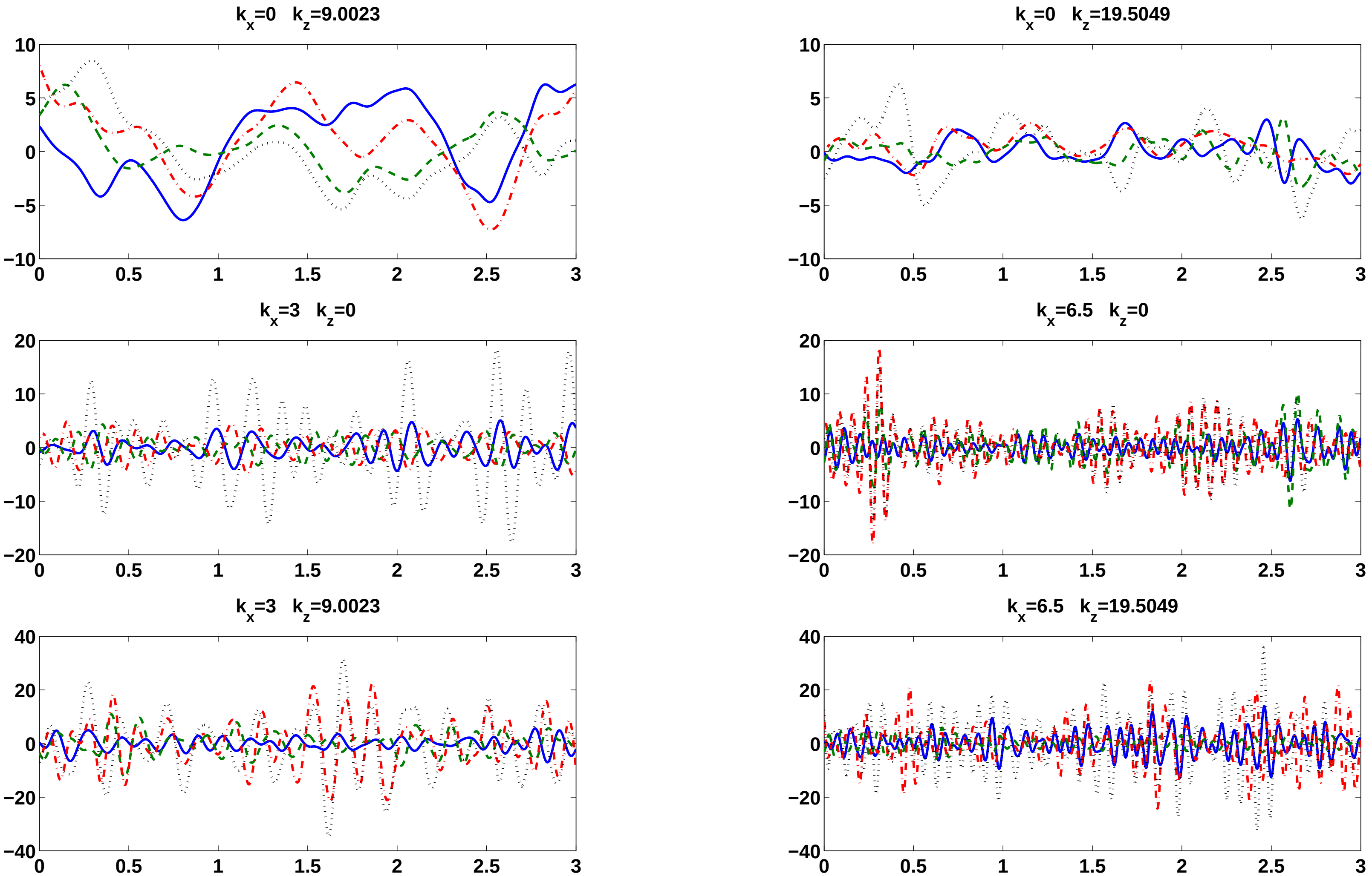
Correlation Data



Temporal Structures



Conclusion

- Three measurements
- The definition of the input
- Localization of the estimation
- Turbulence and nonlinear forcing

Ongoing work :

- frequency study
- Robustness
- Investigation on optimization norms
- Transient growth targeted directly (Generalized H_2)
- More flexibility in the measurement choice $\rightarrow$ towards the wind tunnel experiment
- Model reduction $\rightarrow$ towards the wind tunnel experiment