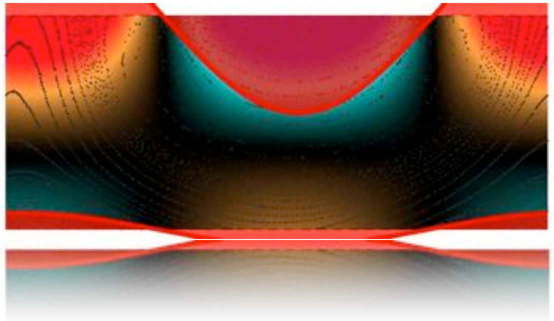
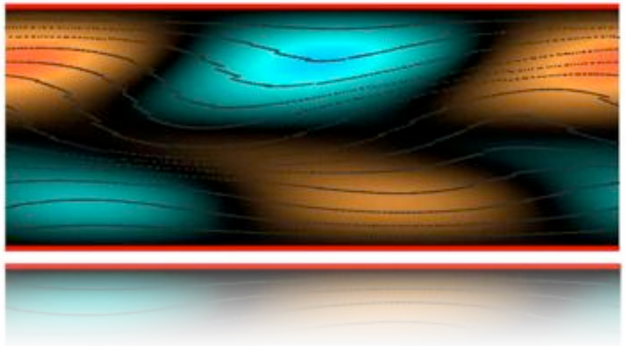


Energy growth in the compliant channel

Jérôme Hœpffner
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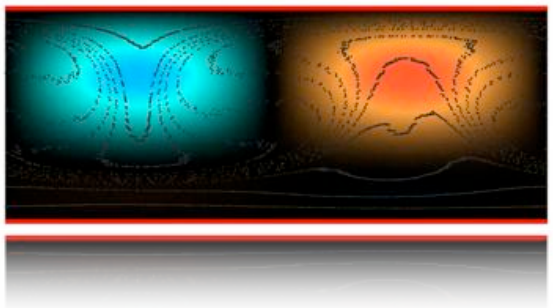
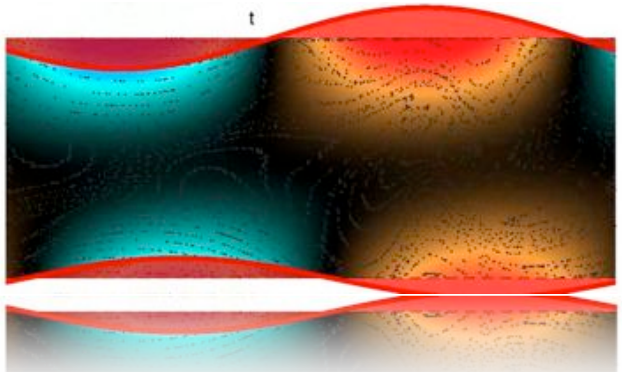
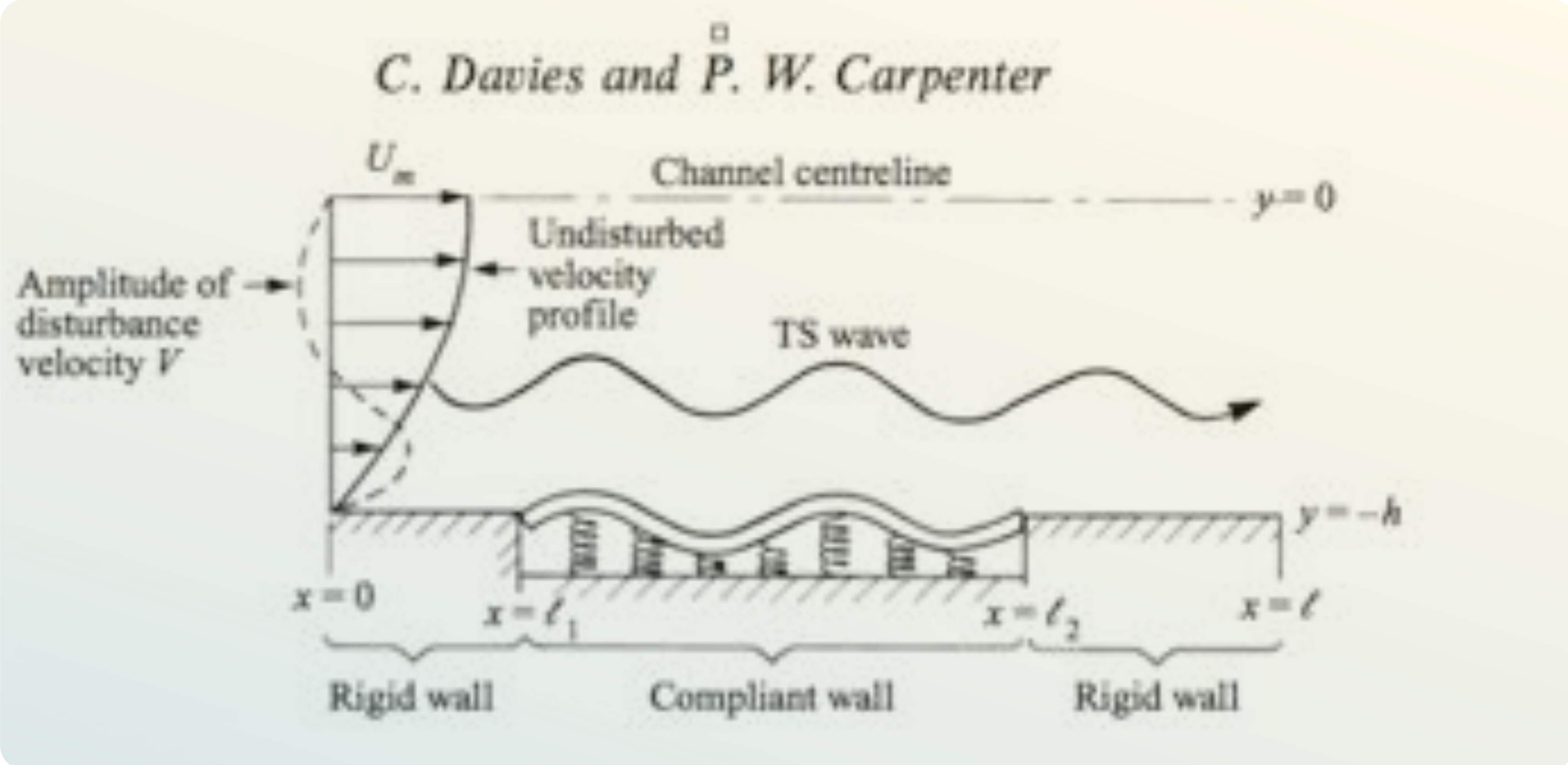
Julien Favier, Alessandro Bottaro
DICAT, Università di Genova.





Compliant surfaces



Compliant surfaces

Gray(30s): observation of dolphins
Kramer(50s): reproduce dolphins' skin

Classical question:
Can flow/walls coupled dynamics reduce instabilities?
Delay/prevent transition to turbulence

Sensitivity:
an alternative view on the problem

Wall dynamics

η: wall displacement:

$$m\eta_{tt} + \frac{d}{Re}\eta_t + \frac{B\Delta^2 - T\Delta + K}{Re^2}\eta = \pm p|_{\text{wall}},$$

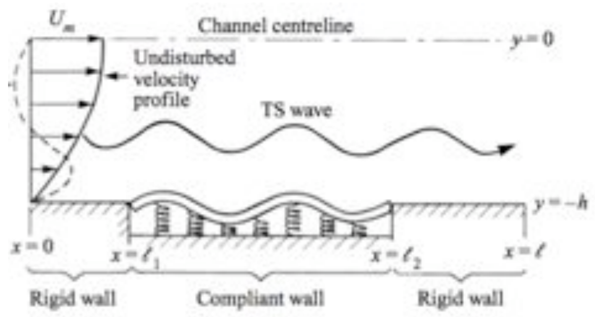
mass, acceleration

Damping

Tension T, rigidity B, spring stiffness K

Forcing by the pressure

Flexible plate forced by fluid pressure



Boundary conditions:
no slip at the moving wall

Coupled system: flow/walls

Flow

$$\begin{aligned} u_t + U u_x + U_y v &= -p_x + \Delta u/Re, \\ v_t + U v_x &= -p_y + \Delta v/Re, \\ w_t + U w_x &= -p_z + \Delta w/Re, \\ u_x + v_y + w_z &= 0. \end{aligned}$$

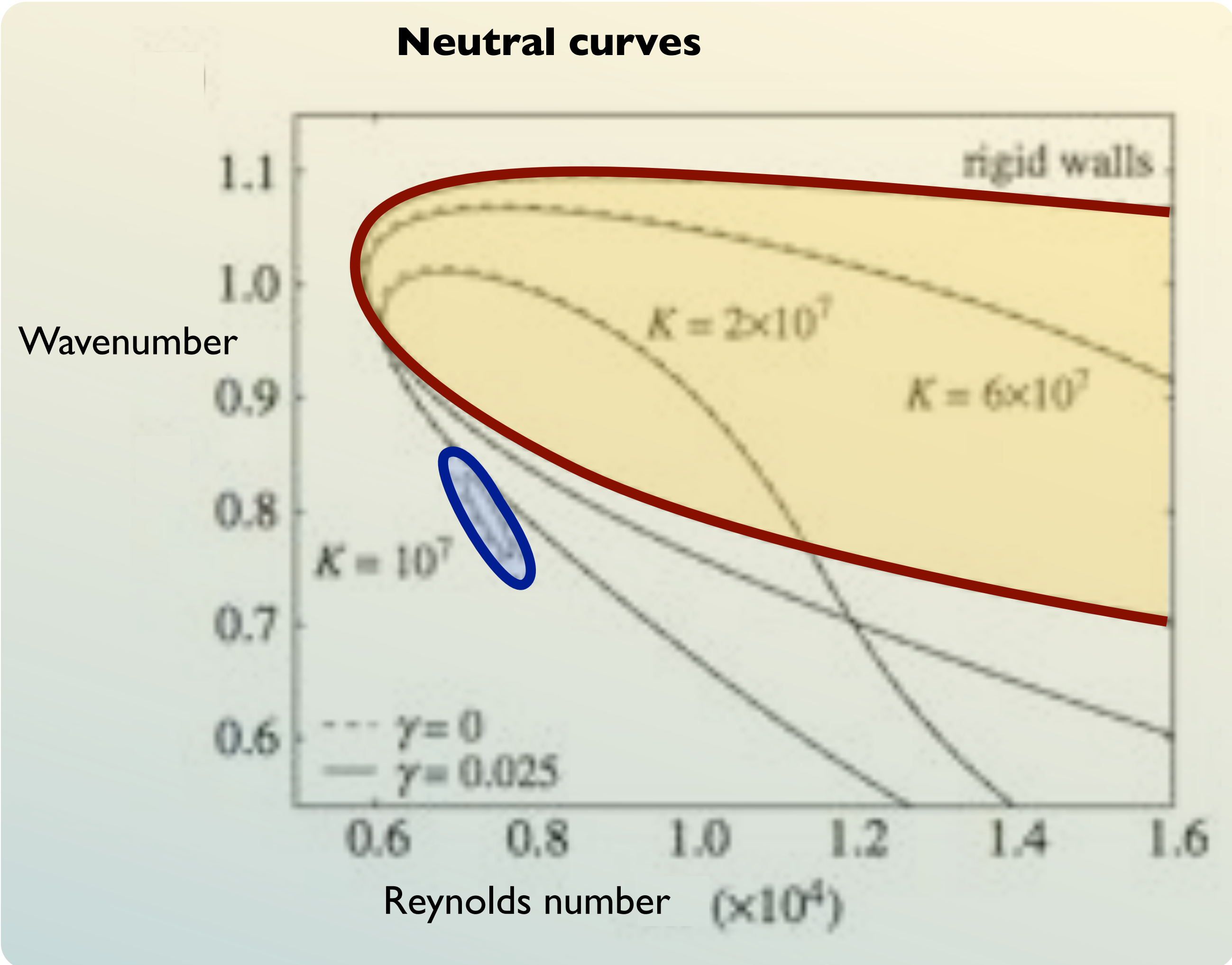
Navier-Stokes linearised about base flow profile: **stability analysis.**

Wall

$$m\eta_{tt} + \frac{d}{Re}\eta_t + \frac{B\triangle^2 - T\triangle + K}{Re^2}\eta = \pm p|_{\text{wall}},$$

The state vector: $(\hat{u}, \hat{v}, \hat{w}, \hat{p}, \hat{\gamma}_{top}, \hat{\eta}_{top}, \hat{\gamma}_{bot}, \hat{\eta}_{bot})^T$,

Stabilisation of Tollmien-Schlichting instability



Ann. Rev. Fluid Mech. 1988. 20 : 393–420

COMPLIANT COATINGS

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Other instabilities!

410 RILEY, GAD-EL-HAK & METCALFE

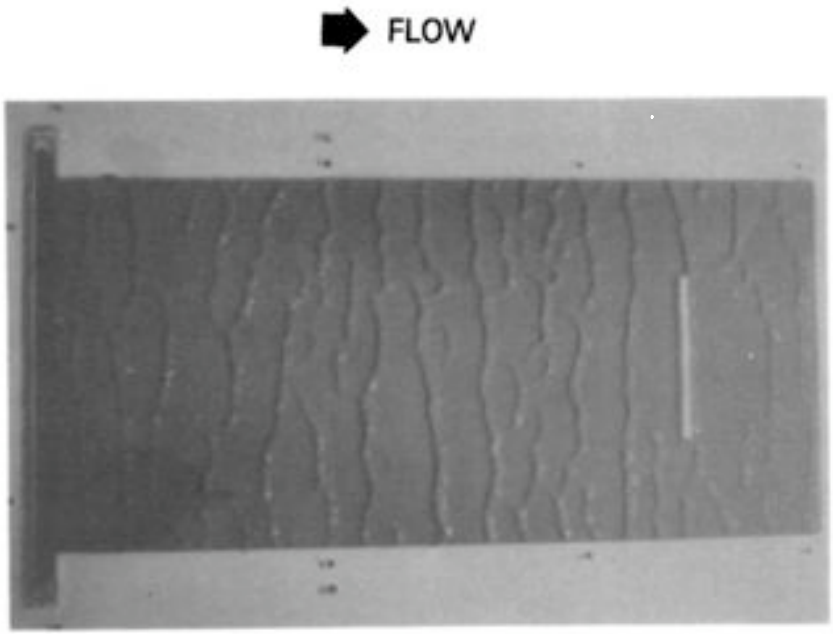


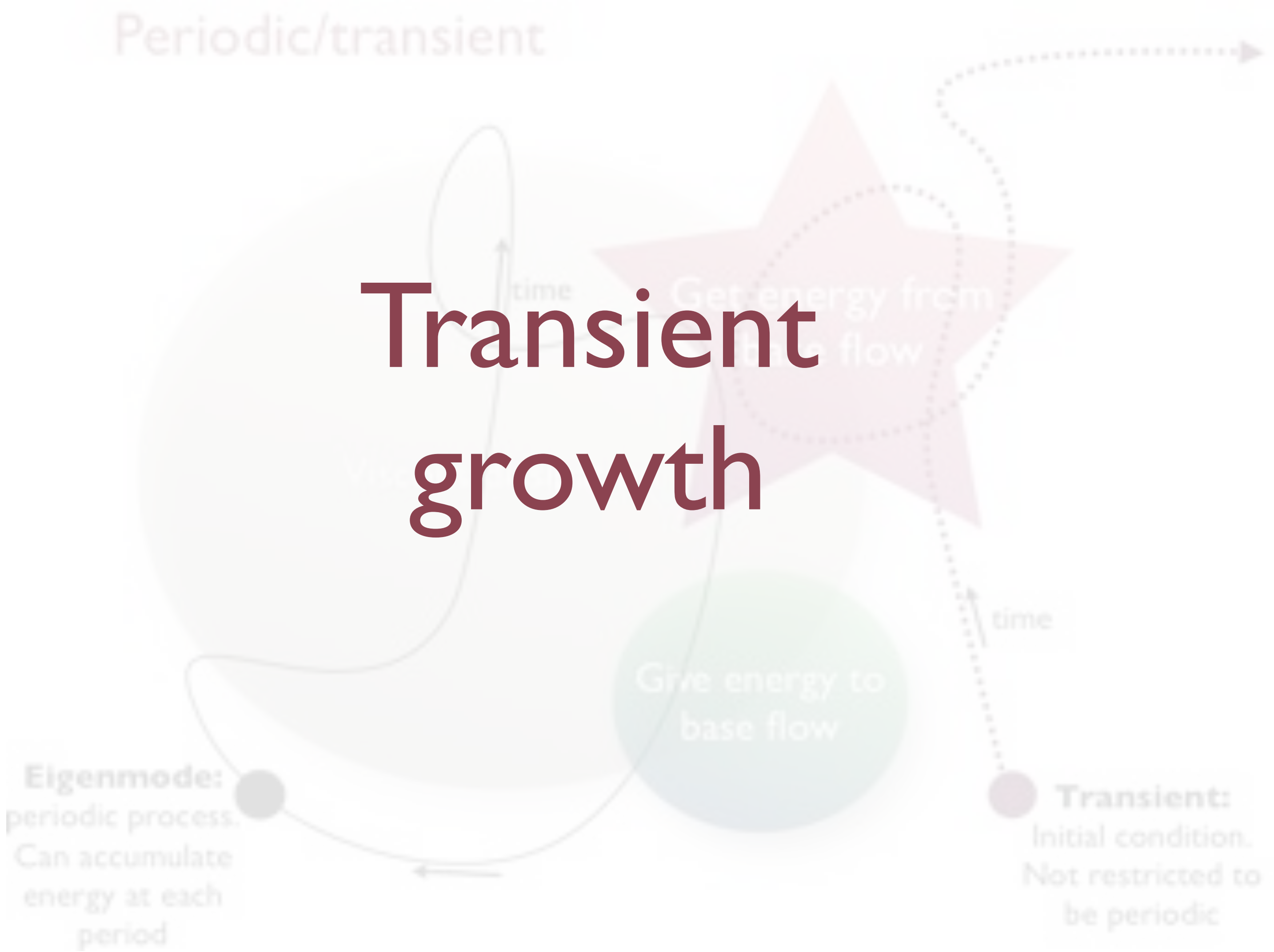
Figure 5 Static-divergence waves under a turbulent boundary layer (from Gad-el-Hak et al. 1984).

Turbulent boundary layer

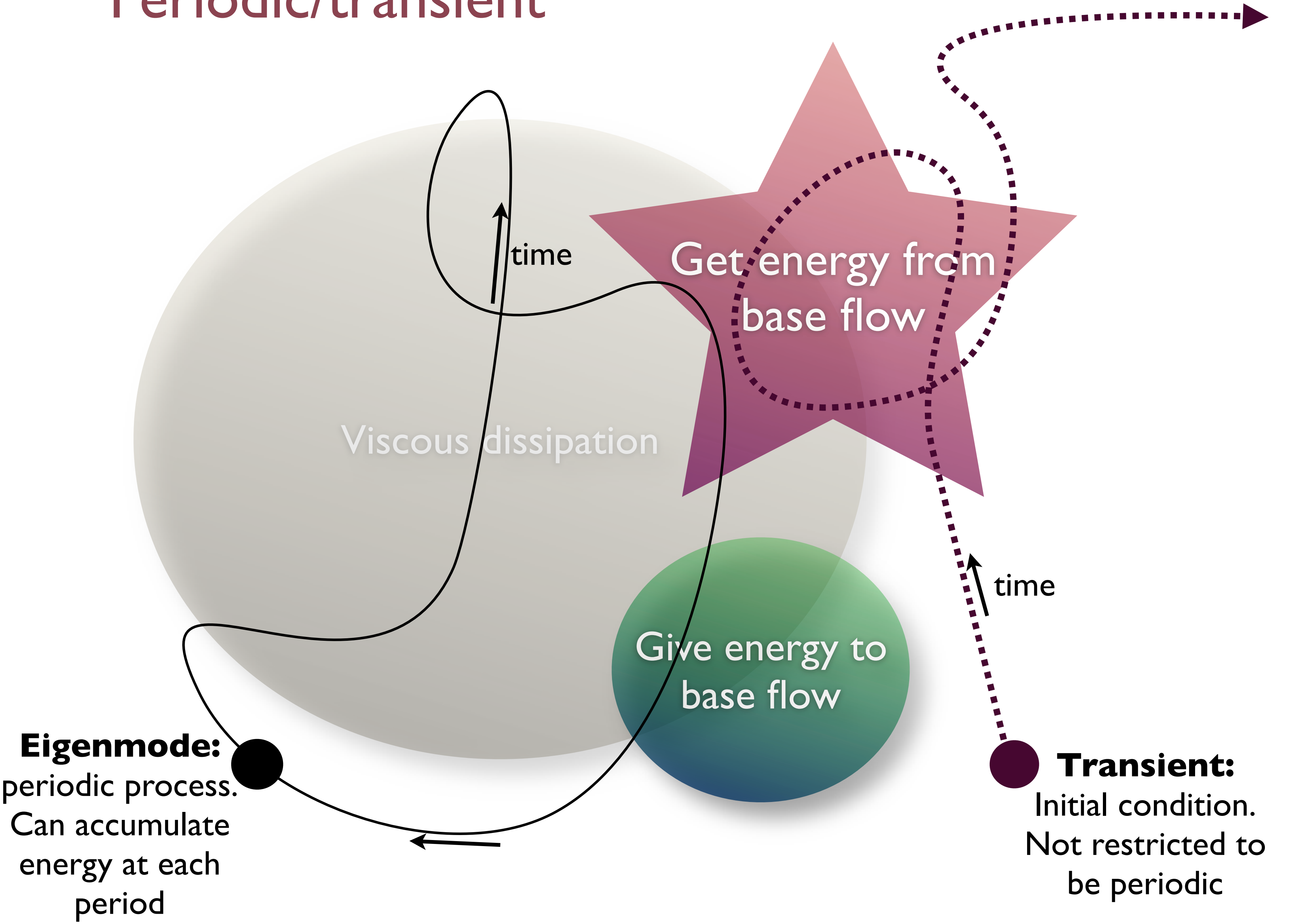
Static divergence

Aero-elastic instabilities

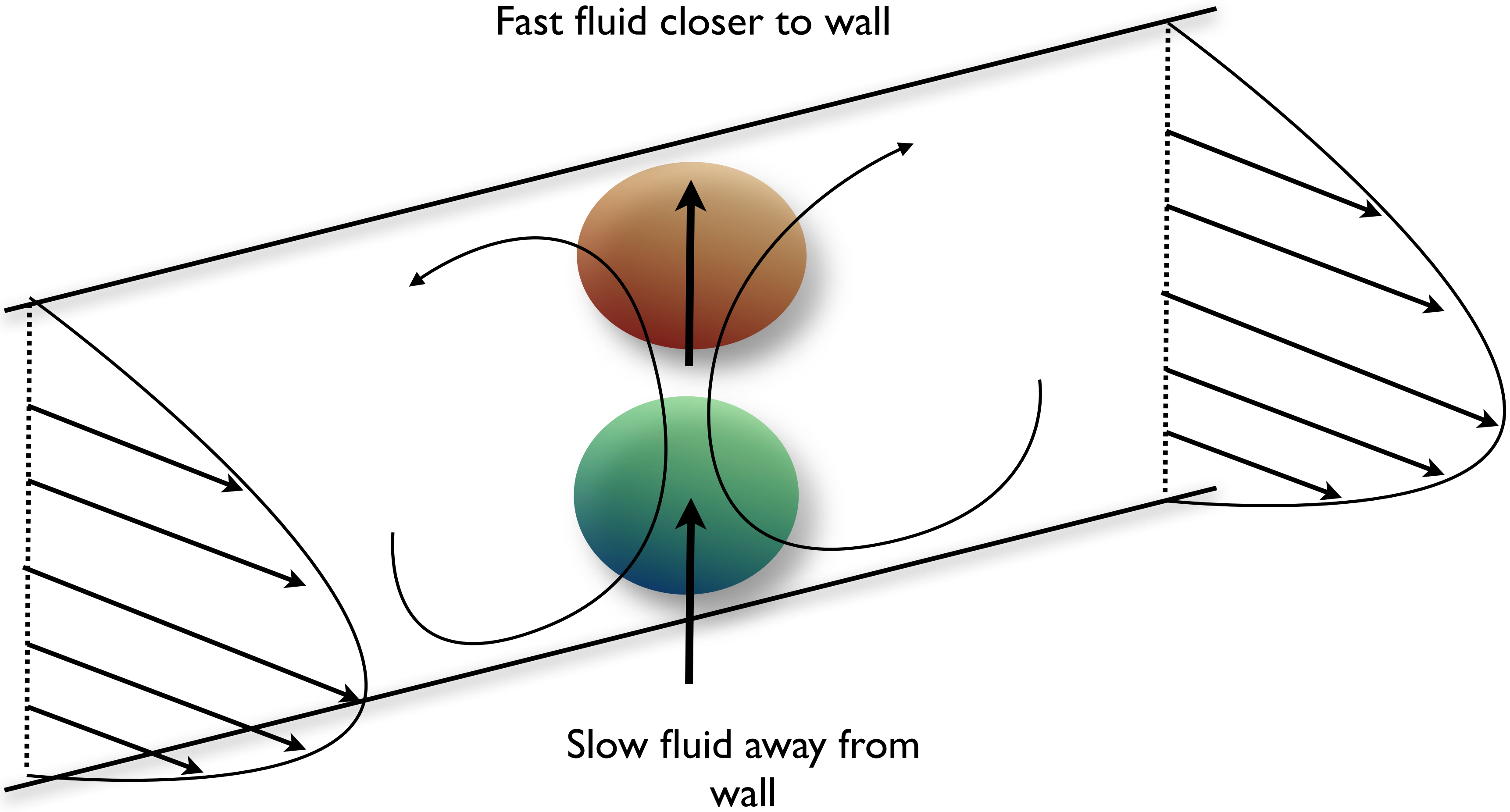
Transient growth



Periodic/transient



Lift-up



Lift-up

$$
\begin{aligned}
u_t + U u_x + U_y v &= -p_x + \Delta u / Re, \\
v_t + U v_x &= -p_y + \Delta v / Re, \\
w_t + U w_x &= -p_z + \Delta w / Re, \\
u_x + v_y + w_z &= 0.
\end{aligned}
$$

← Navier-Stokes linearized about parallel base flow

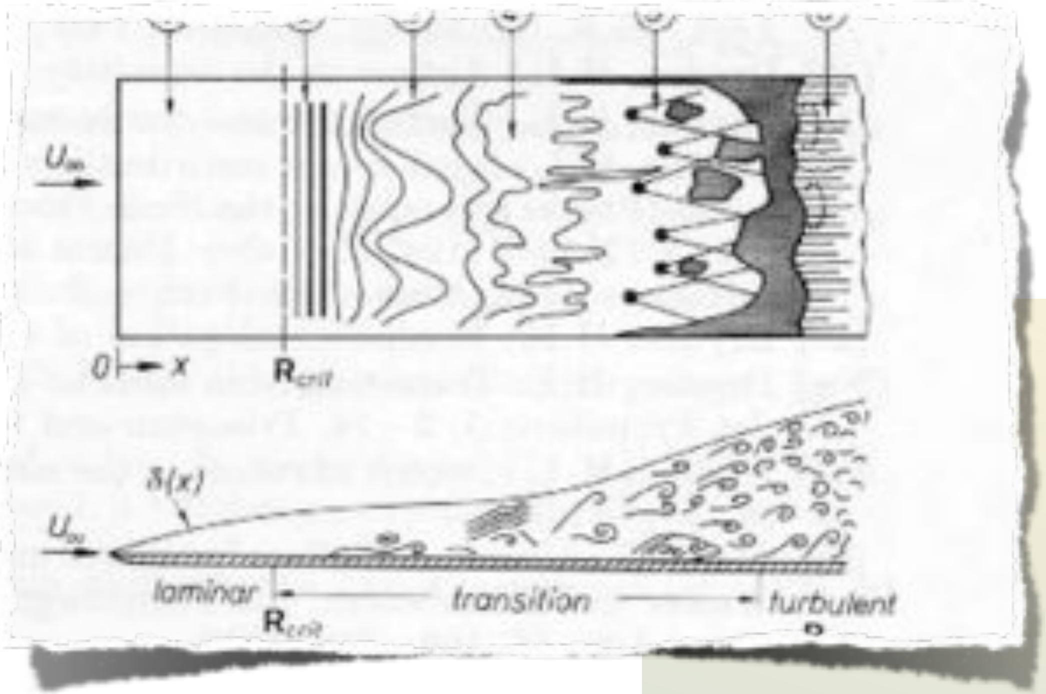
Low viscosity constant in streamwise →

$$
\begin{aligned}
u_t + U_y v &= 0, \\
v_t &= -p_y, \\
w_t &= -p_z, \\
v_y + w_z &= 0.
\end{aligned}
$$

$$u = -t U_y v,$$

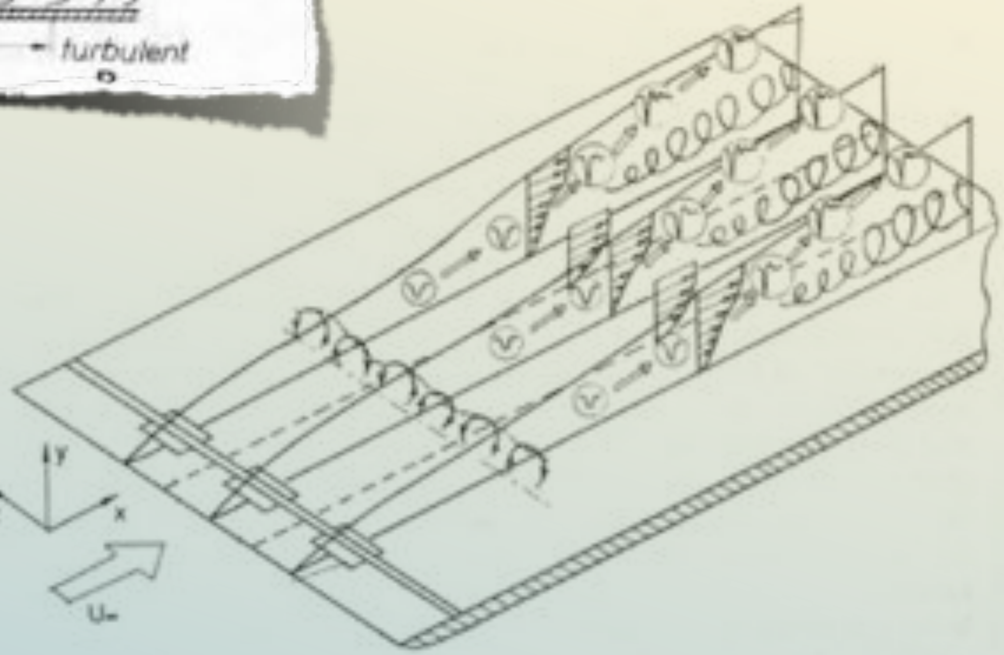
Most efficient mechanism to enhance transient amplification
α=0

Lift-up: Transition to turbulence

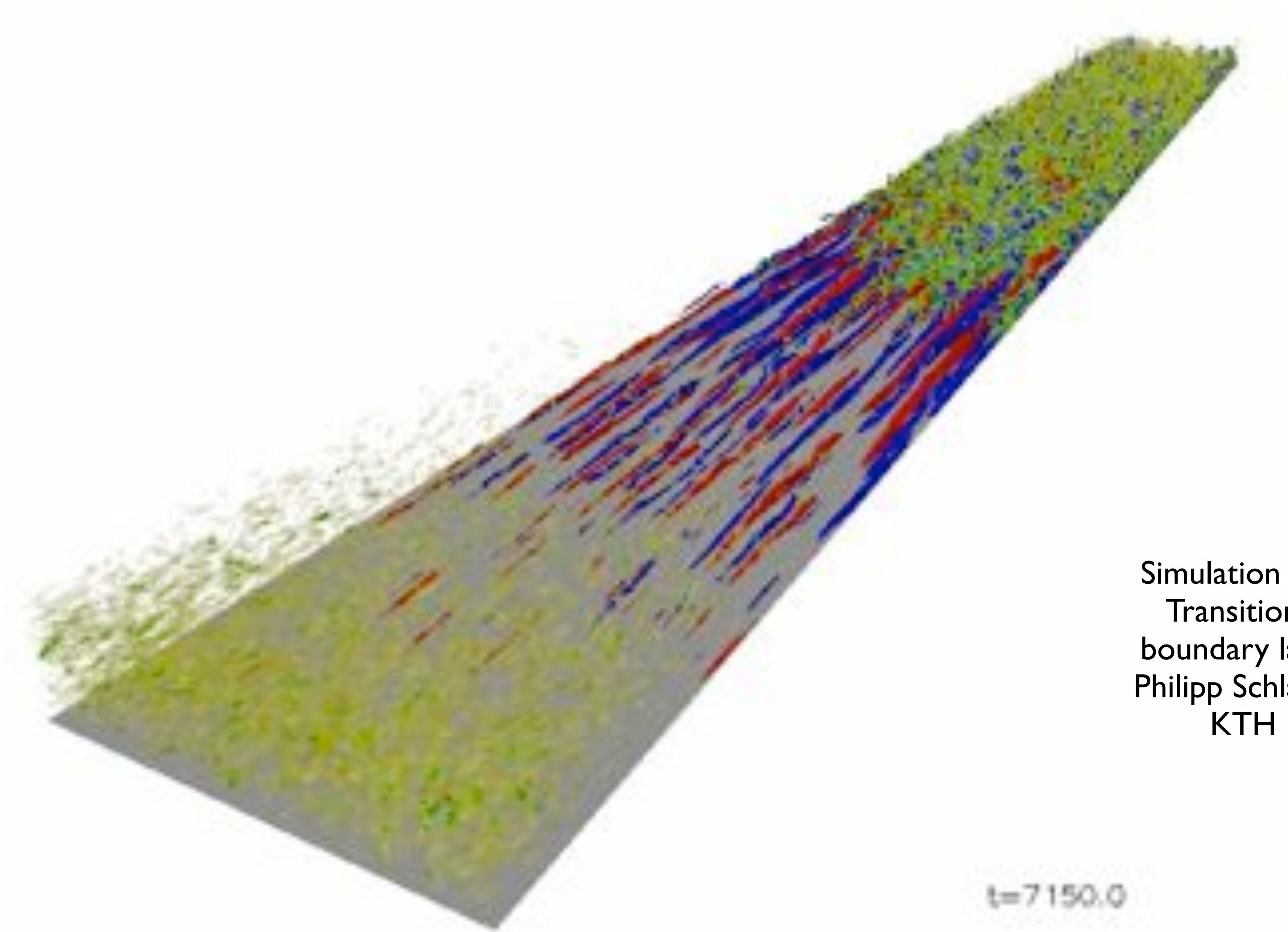


Classical mechanism:
Tollmien-Schlichting waves and secondary instabilities

Bypass transition:
free-stream turbulence/ streaks and secondary instability



Lift-up



Simulation LES.
Transitional boundary layer.
Philipp Schlatter.
KTH

Looking for “special things” in flows using optimization

Three-dimensional optimal perturbations in viscous shear flow

Kathryn M. Butler and Brian F. Farrell
Division of Applied Sciences, Harvard University, Cambridge, Massachusetts 02138

(Received 28 May 1991; accepted 6 April 1992)

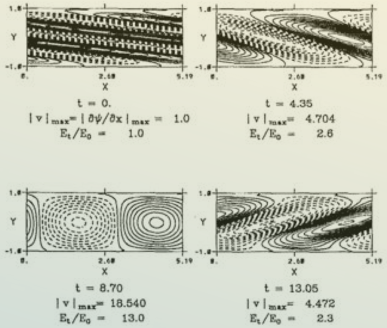


FIG. 2. Development of the perturbation streamfunction ψ for the best growing 2-D energy optimal in Couette flow with $R=1000$, located at $\alpha=1.21$, $\tau=8.7$. The streamfunction ψ is defined by $-\partial\psi/\partial y=u$ and $\partial\psi/\partial x=v$.

J. Fluid Mech. (2002), *vol.* 463, *pp.* 163–171. © 2002 Cambridge University Press 163
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On the stability of a falling liquid curtain

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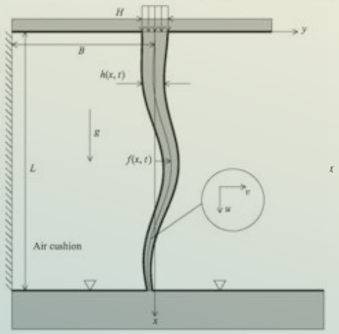


FIGURE 1. Sketch of the geometry for a falling liquid sheet.

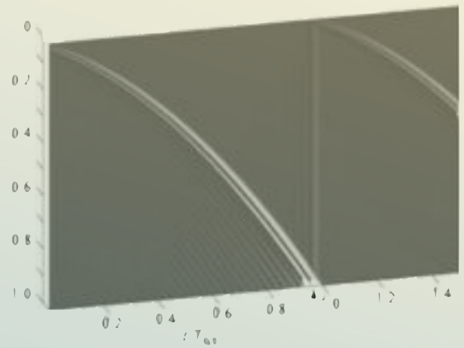
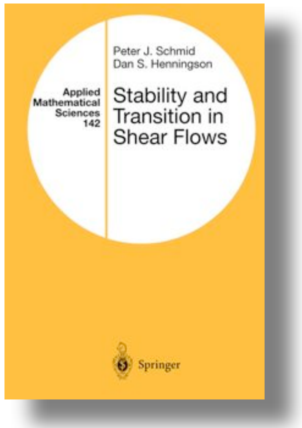


FIGURE 5. Curtain shape versus time for $\kappa = 5 \times 10^4$ and $U = 0.4$ starting with the optimal initial condition, i.e. the initial condition that results in the maximum energy amplification near $t = T_{60}$ in figure 4(*a*).



J. Fluid Mech. (2005), *vol.* 528, *pp.* 43–52. © 2005 Cambridge University Press 43
doi:10.1017/S0022112005003307 Printed in the United Kingdom

Transient growth in two-phase mixing layers

By P. YECKO[1,2] AND S. ZALESKI[3]

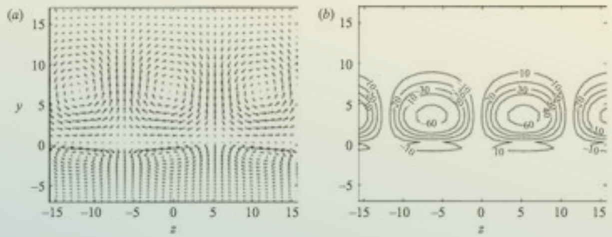


FIGURE 4. Optimal disturbance having $G_O = 13660$, $\beta_O = 0.25$, $t_O = 29.8$ at $Re = 100$, $We = 5.5$, $r = 0.0012$, $m = 0.018$: (*a*) $(0, v, w)$ field; (*b*) $(u, 0, 0)$ field.

Optimization of the initial conditions

Optimality:

$$G(t) = \max_{\kappa_0} \frac{\|\kappa(t)\|_Q}{\|\kappa_0\|_Q} = \max_{\kappa_0} \frac{\|e^{\Lambda t}\kappa_0\|_Q}{\|\kappa_0\|_Q} = \|e^{\Lambda t}\|_Q = \|\underbrace{F^{-1}e^{\Lambda t}F}_{\mathcal{H}}\|_2$$

Energy

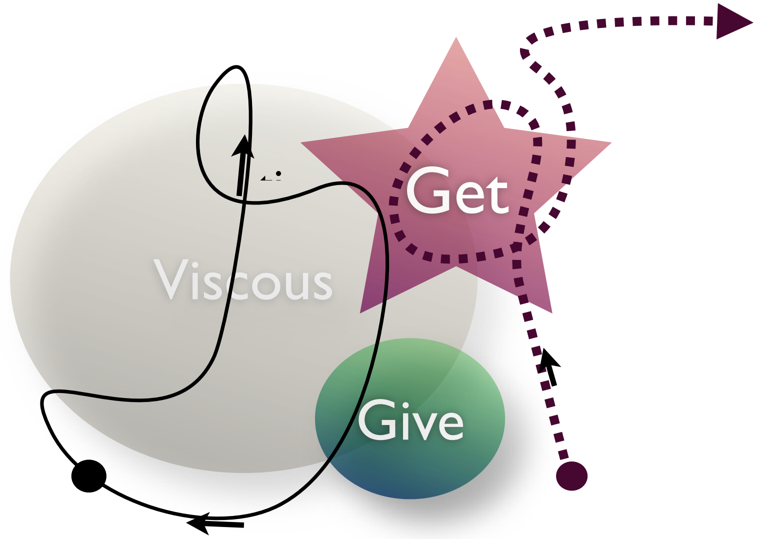
$$E \triangleq \underbrace{\frac{1}{2}\int_y \overline{u^2+v^2+w^2}\mathrm{dy}}_{\text{Flow}} + \underbrace{\sum_{\text{bot}}^{\text{top}} \frac{1}{2}\left(m\overline{\eta_t^2} + \frac{B\Delta_{2D}^2+T\Delta_{2D}+K}{Re^2}\overline{\eta^2}\right)}_{\text{Walls}}$$

Optimize:

Flow energy+wall kinetic and potential energy

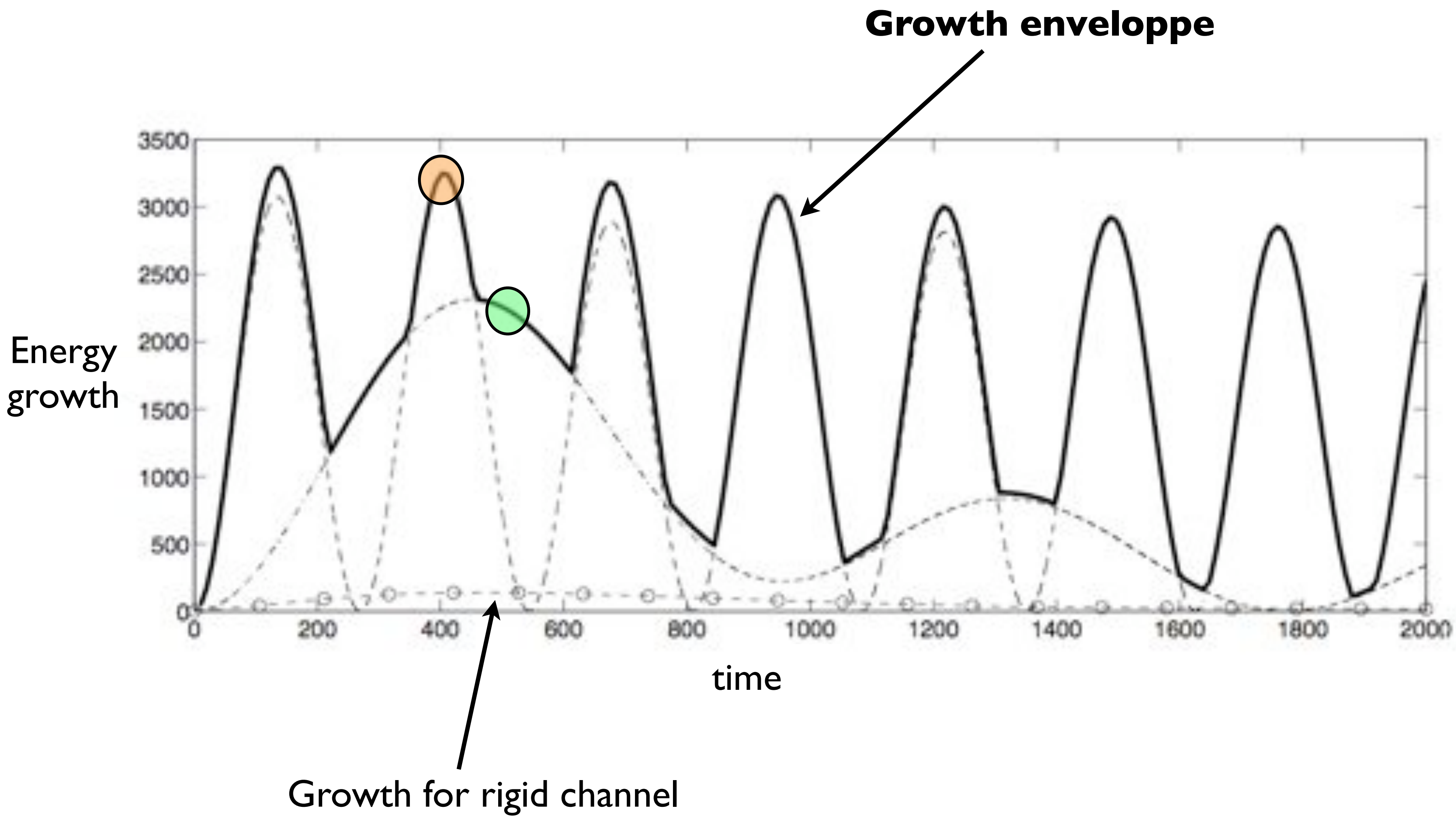
Energy exchange:

$$E_t = \underbrace{-\int_y U_y\overline{uv}\mathrm{dy} + \frac{1}{Re}\left[\overline{(u^2+v^2+w^2)_y}\right]_{\text{bot}}^{\text{top}}}_{\text{Energy exchange with base flow}} \underbrace{-\frac{1}{Re}\int_y \overline{\omega.\omega}\,\mathrm{dy} - \sum_{\text{bot}}^{\text{top}} \frac{d}{Re}\overline{\eta_t^2}}_{\text{Viscous damping}}.$$

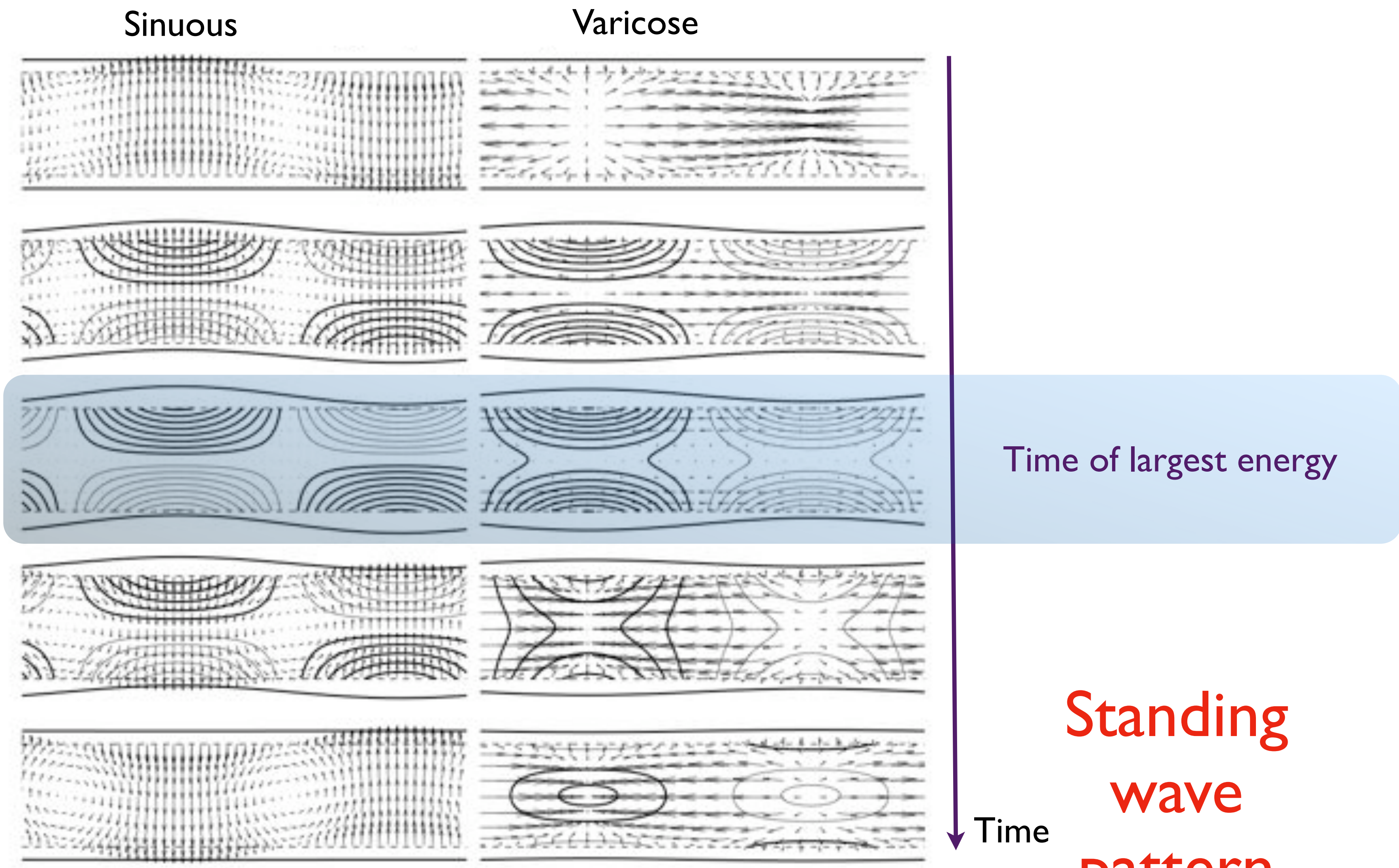


What is the effect of wall compliance?

Optimization results

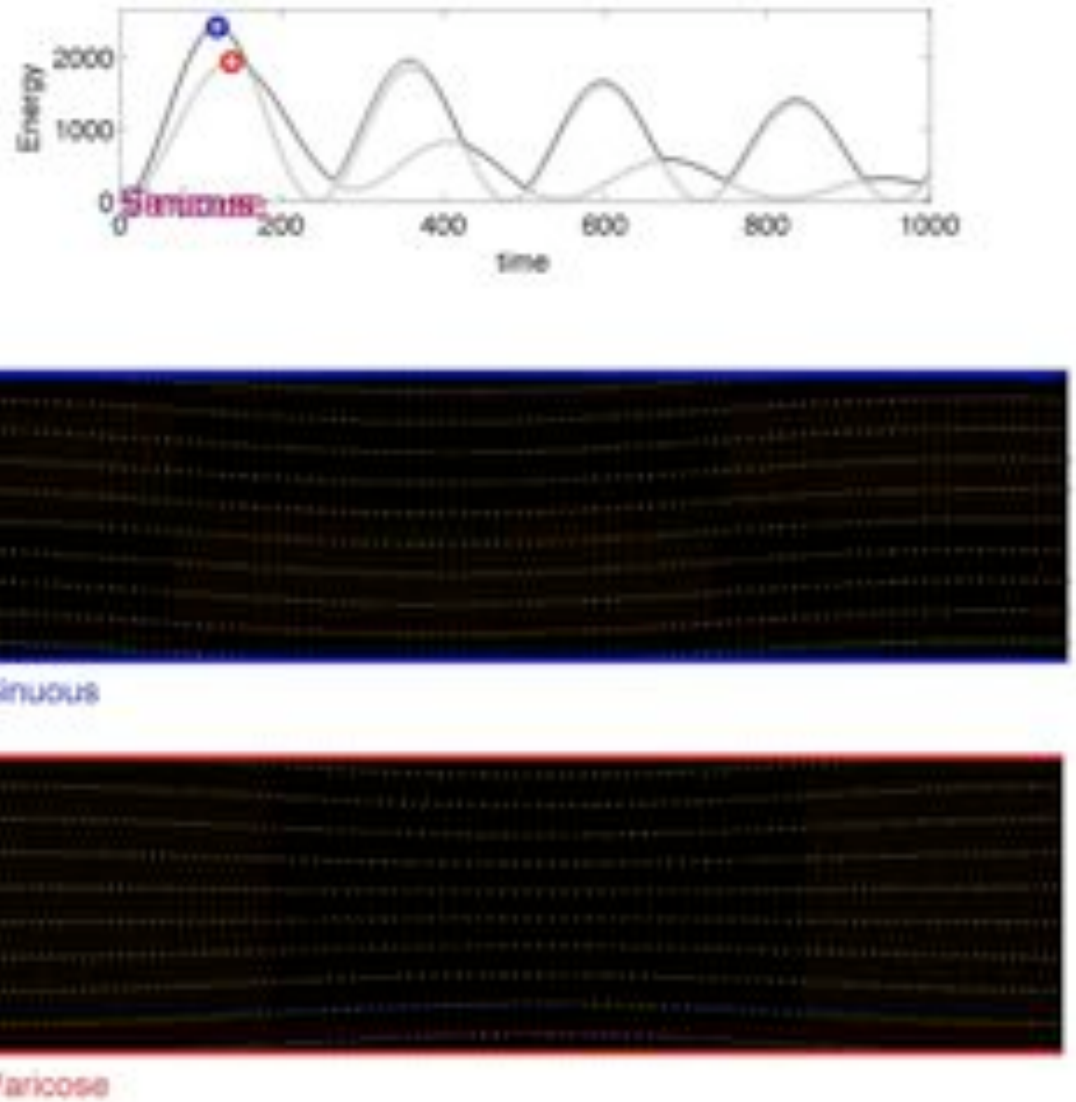


Optimization results

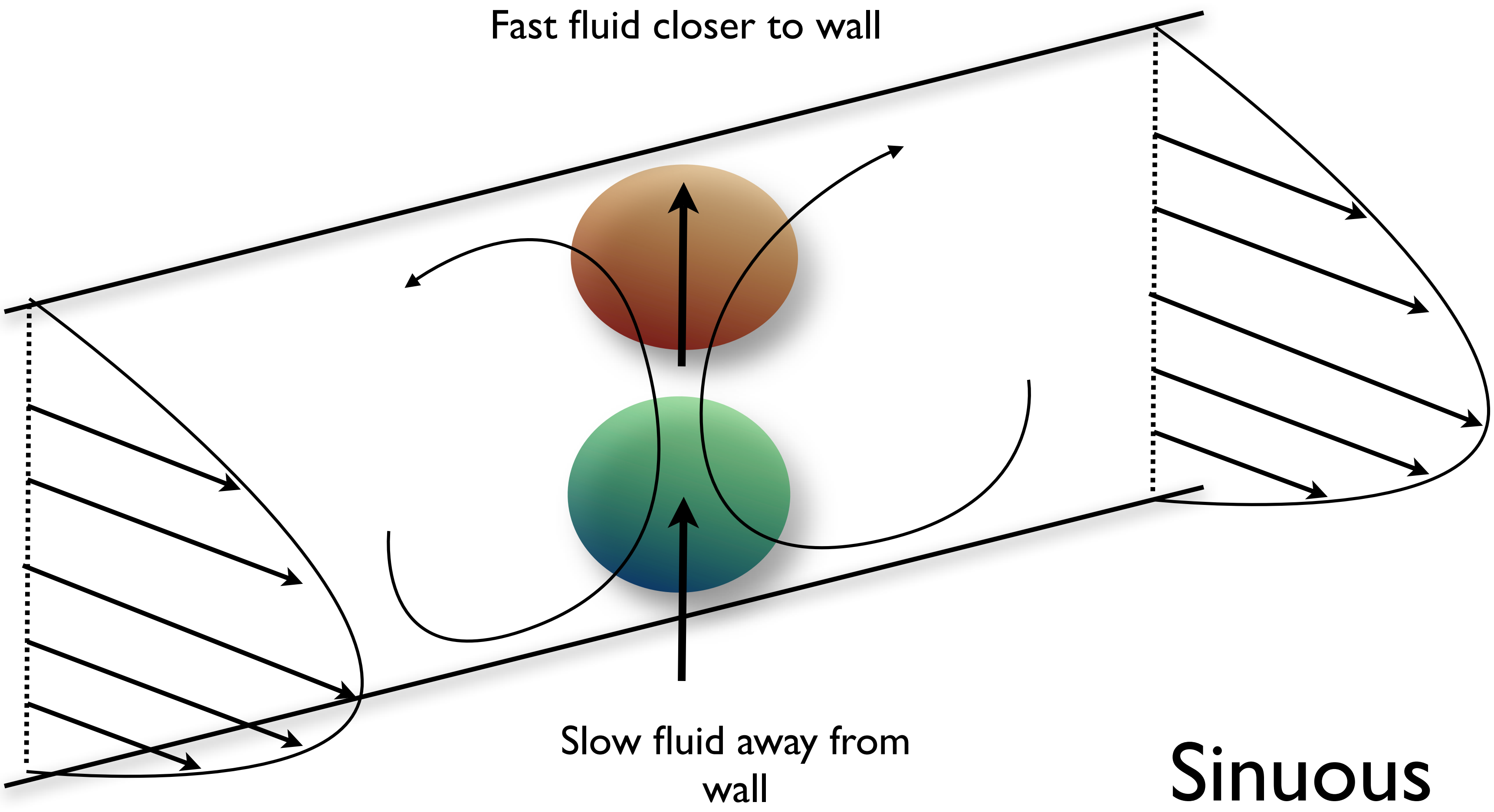


Standing wave pattern

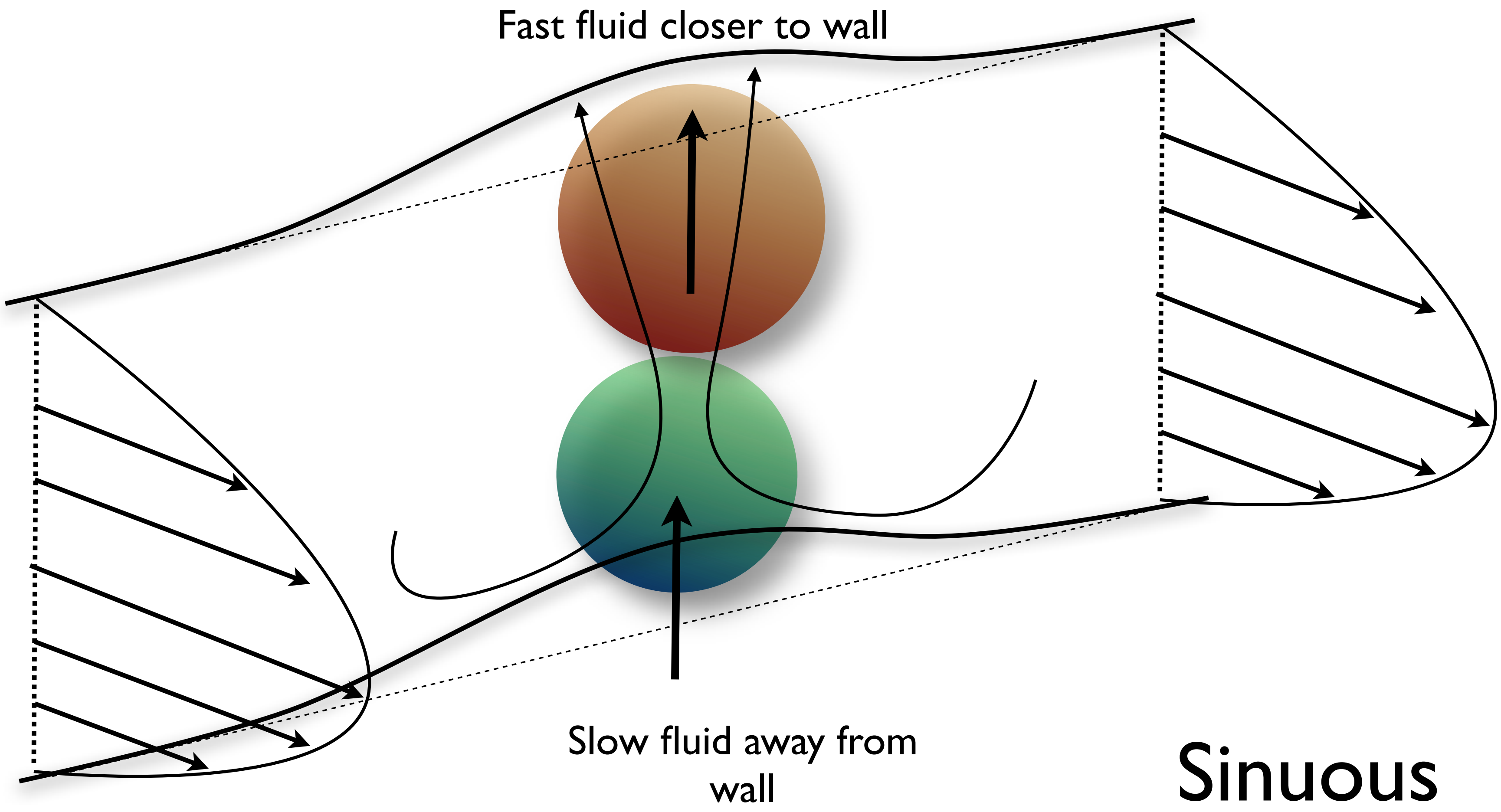
Optimal solutions



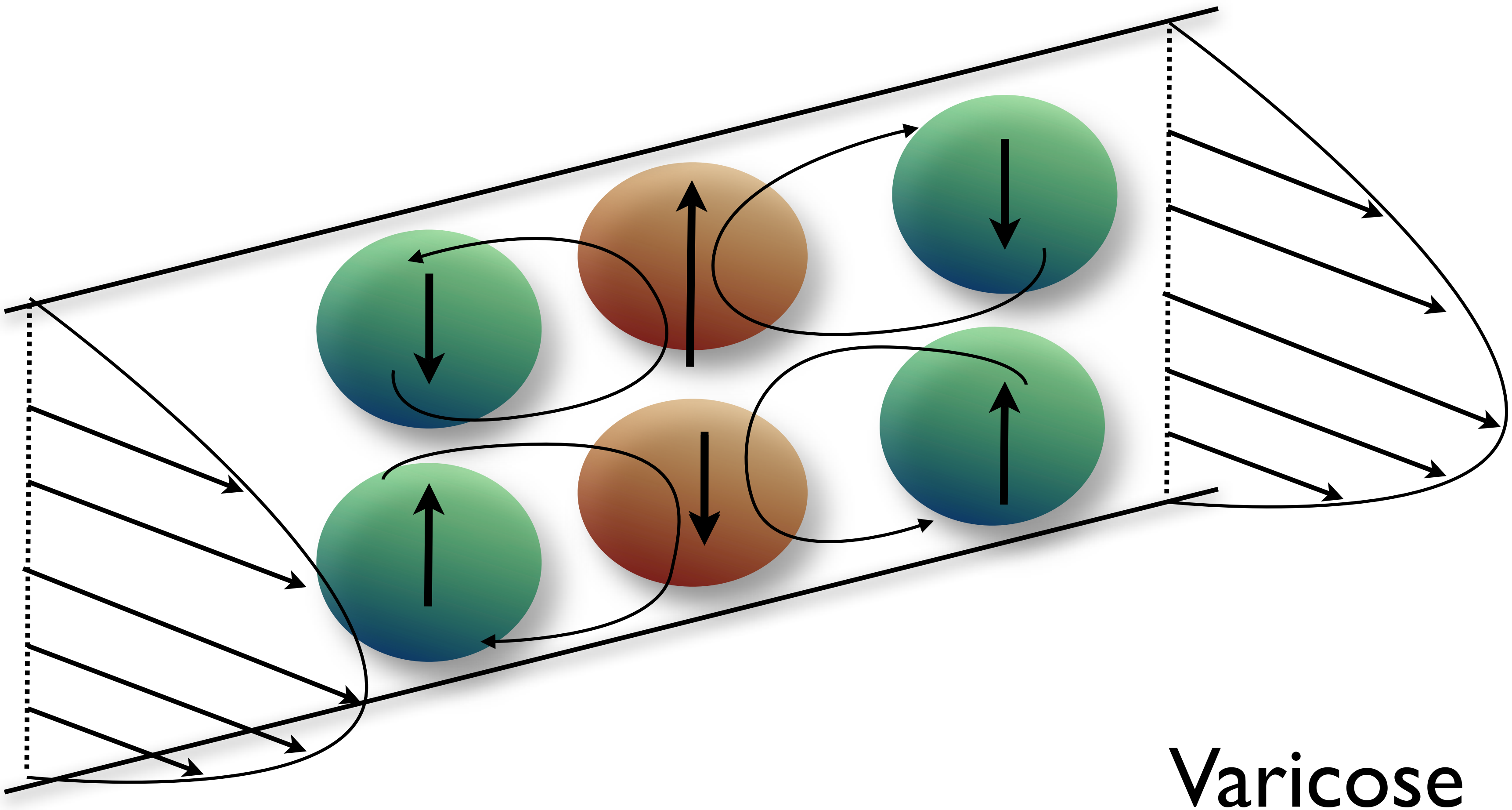
Lift-up: rigid walls



Lift-up: flexible walls

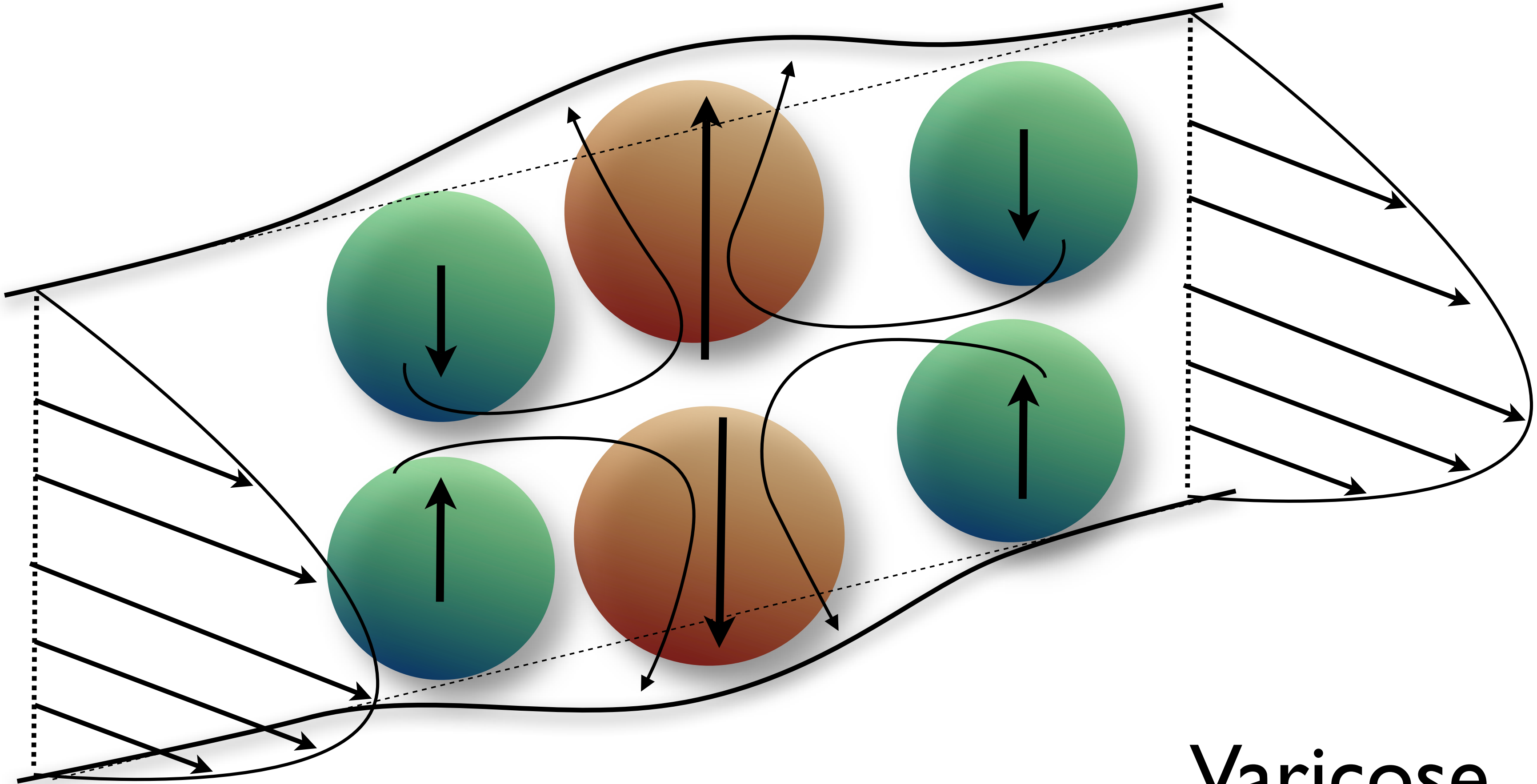


Lift-up: rigid walls



Varicose

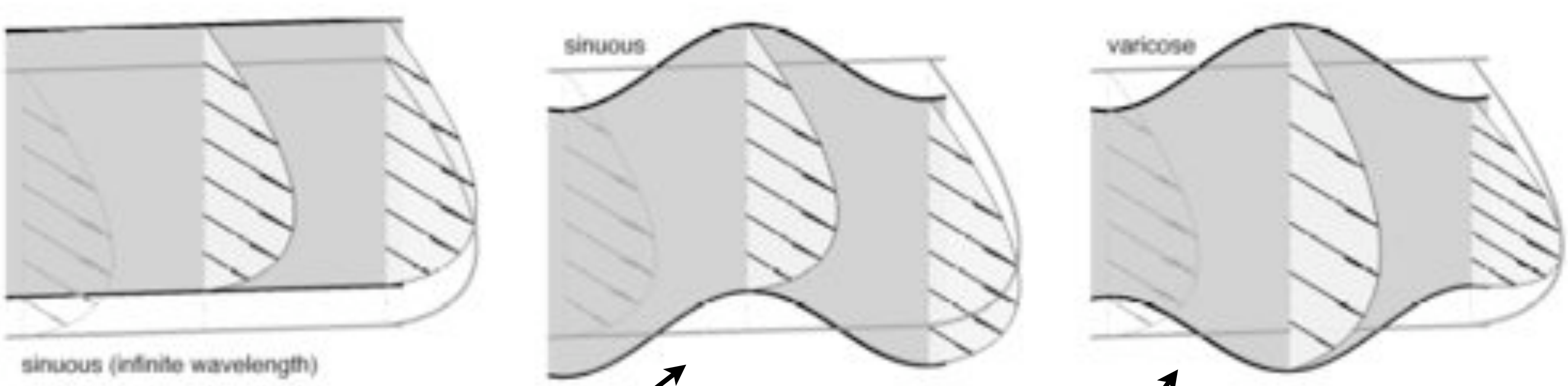
Lift-up: rigid walls



Varicose

Candidate mechanisms

Model:
We suppose a simple deformation of the base flow

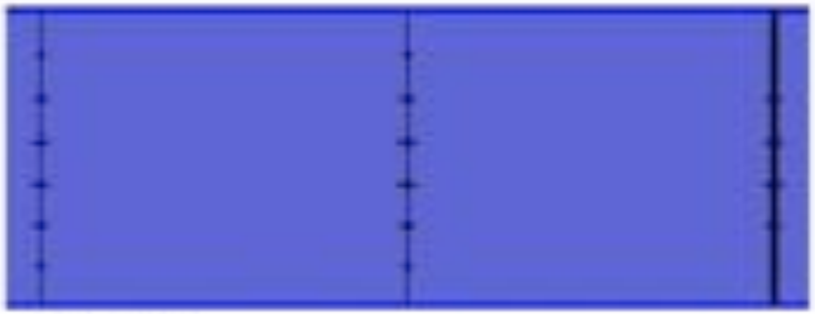


$$G^s = 1 + \frac{4}{3}\frac{Re^2}{Bk^4 + Tk^2 + K}$$

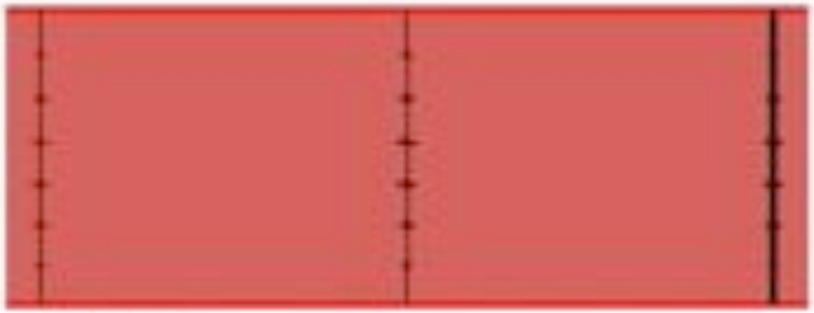
$$G^v = 1 + \frac{14}{15}\frac{Re^2}{Bk^4 + Tk^2 + K}$$

Compliant surfaces

Standing wave in the spanwise direction



sinuous

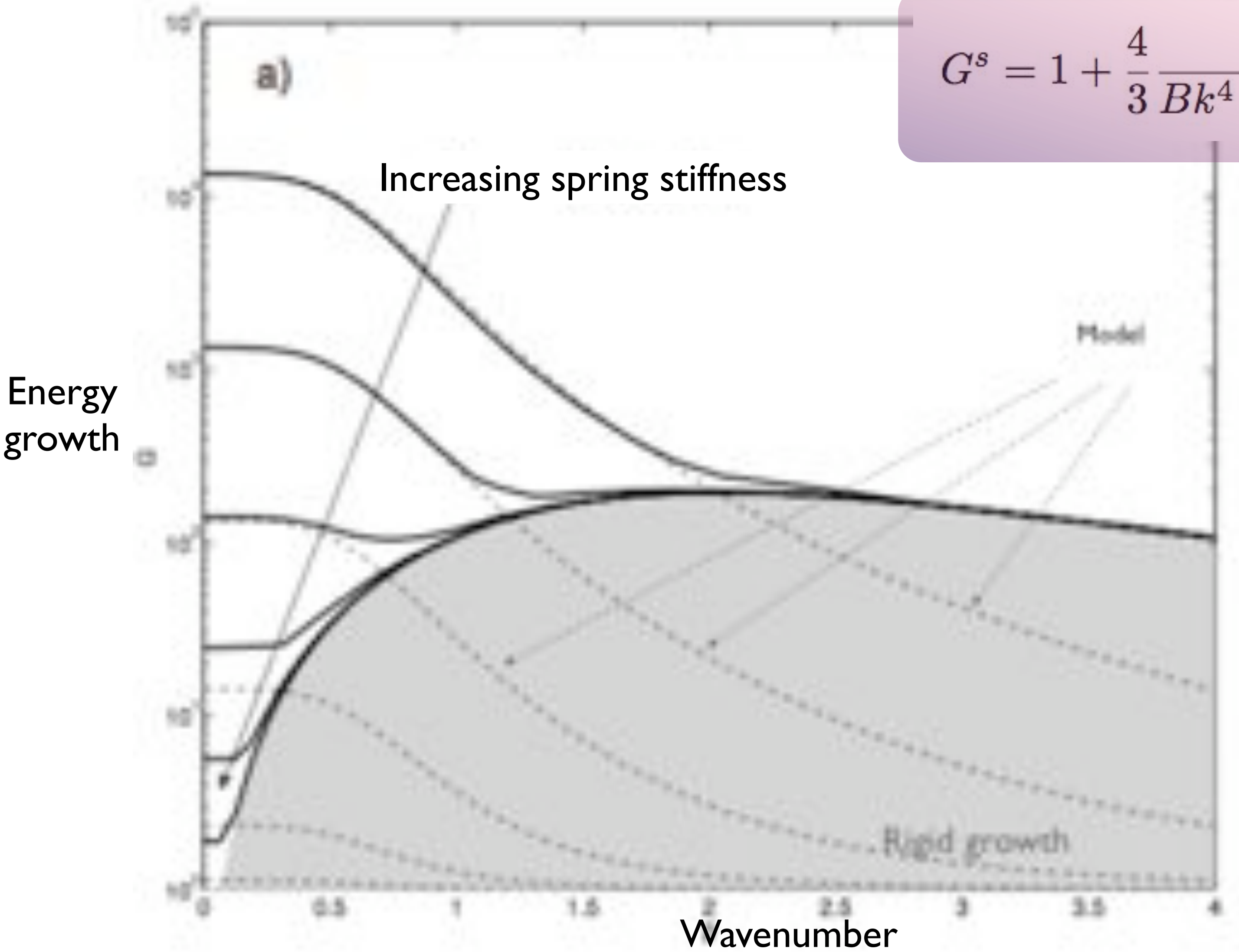


varicose

Up and down model

Sinuous

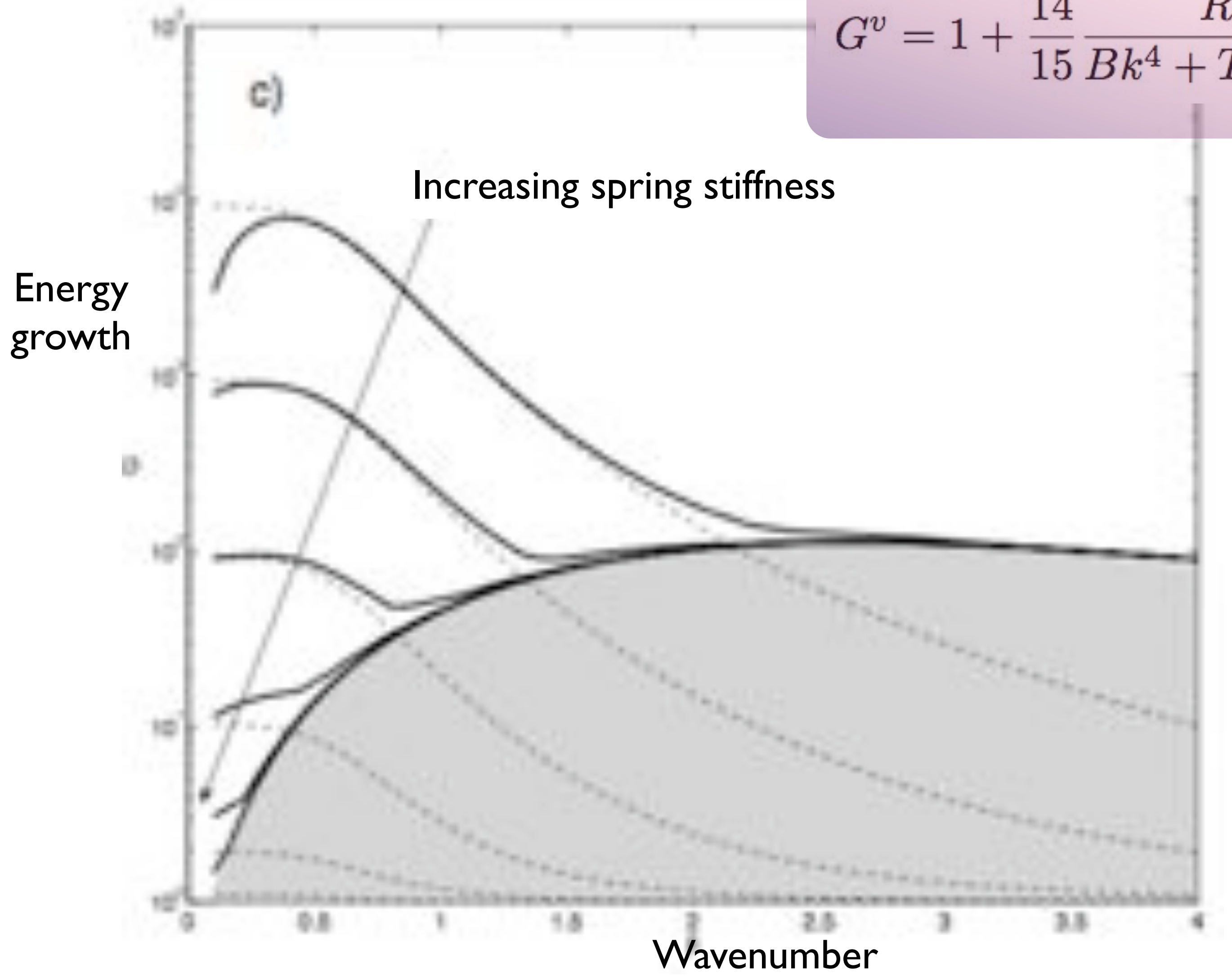
$$G^s = 1 + \frac{4}{3}\frac{Re^2}{Bk^4 + Tk^2 + K}$$



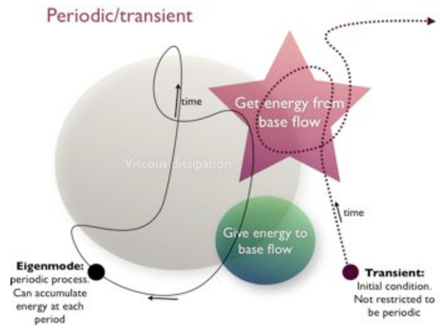
Up and down model

Varicose

$$G^v = 1 + \frac{14}{15}\frac{Re^2}{Bk^4 + Tk^2 + K}$$



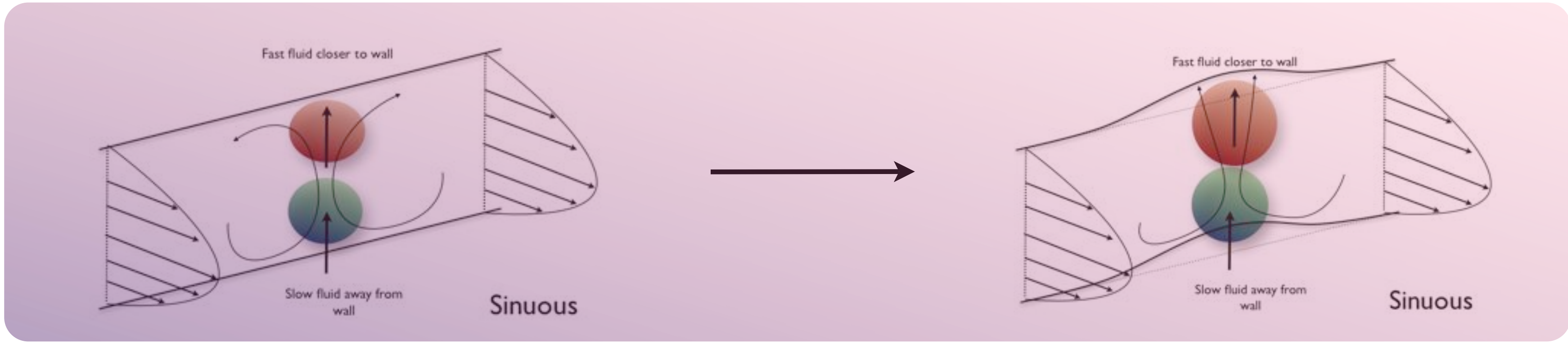
Conclusions



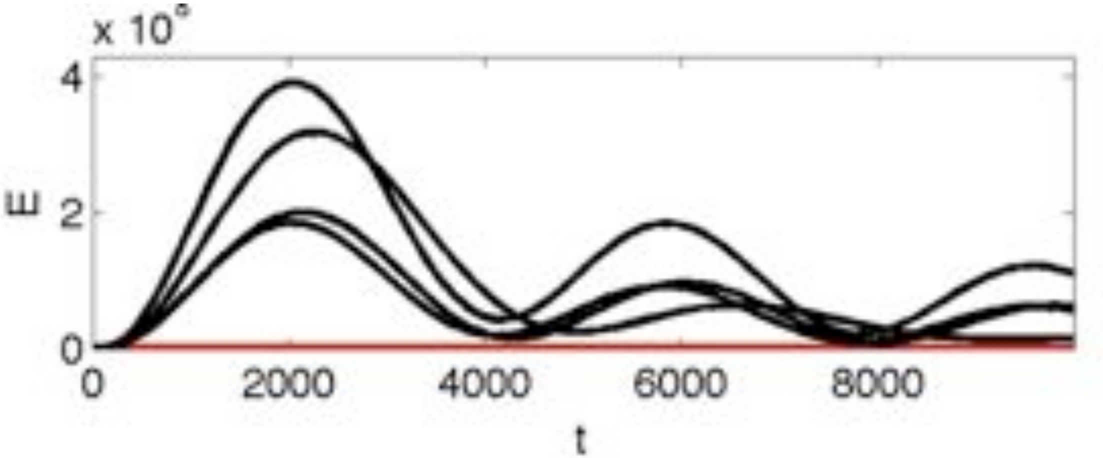
- Computation of the optimal initial conditions in channel flow with compliant walls: growth increases with wall compliance

- Transition likely to result from a competition of algebraic and exponential amplification mechanisms

- Random initial conditions excite sinuous and varicose mechanisms



Compliant surfaces



K=10^1

Flexible channel: slow oscillations

Large amplitude

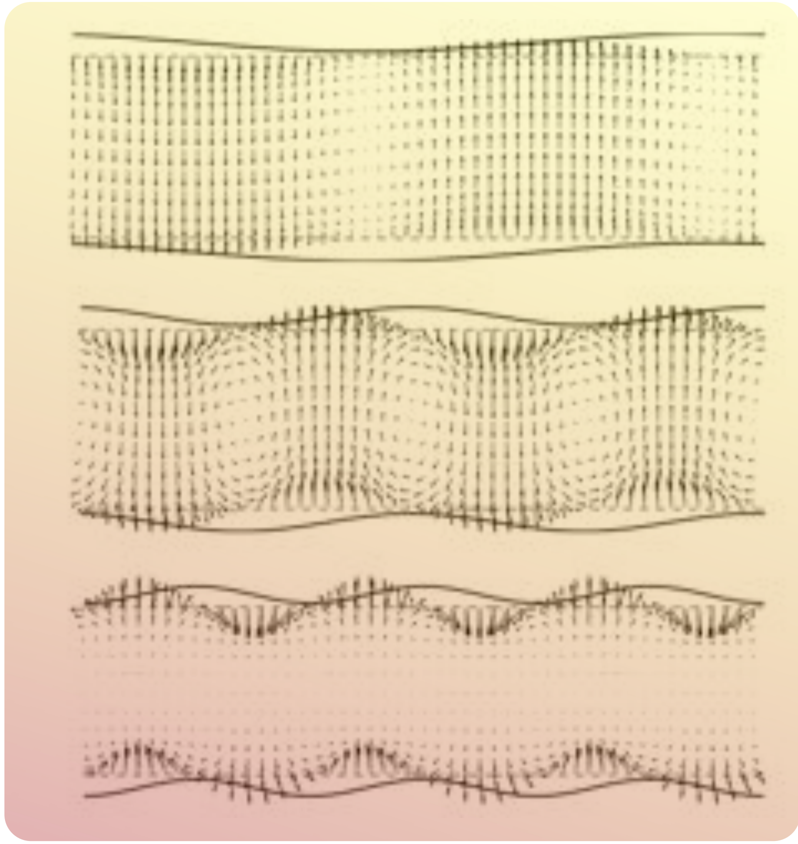
Wall waves

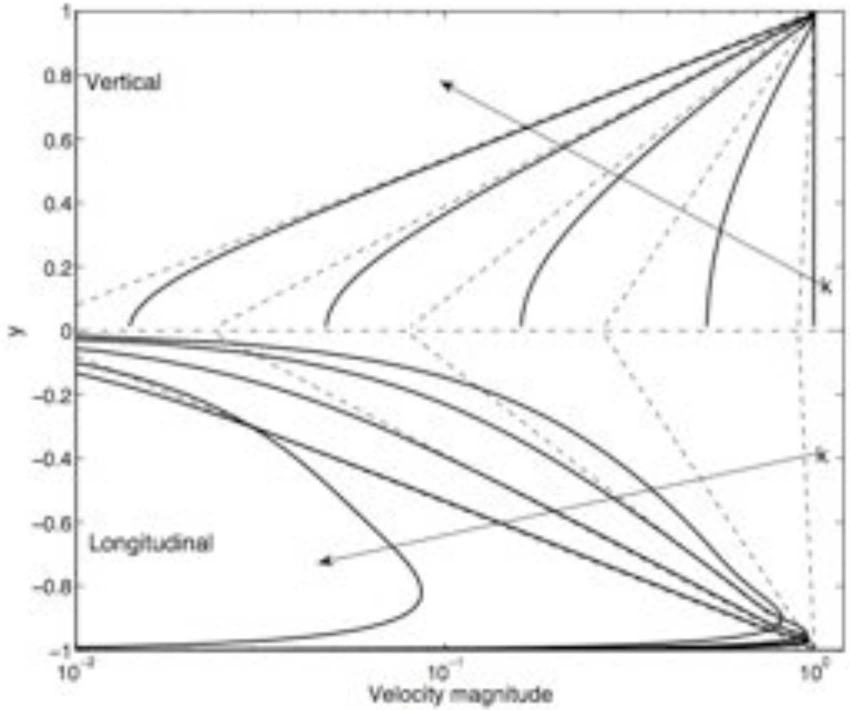
Pulsation of
the free wall:

$$m\omega^2 = \frac{Bk^4 + Tk^2 + K}{Re^2}$$

Wall waves

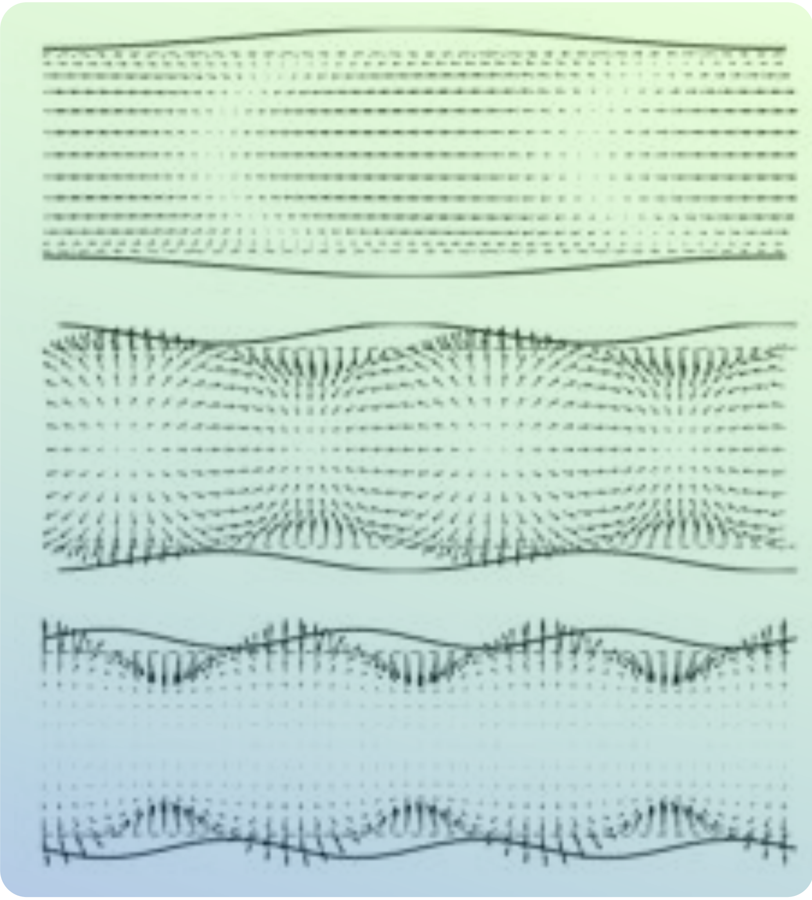
Sinuous:

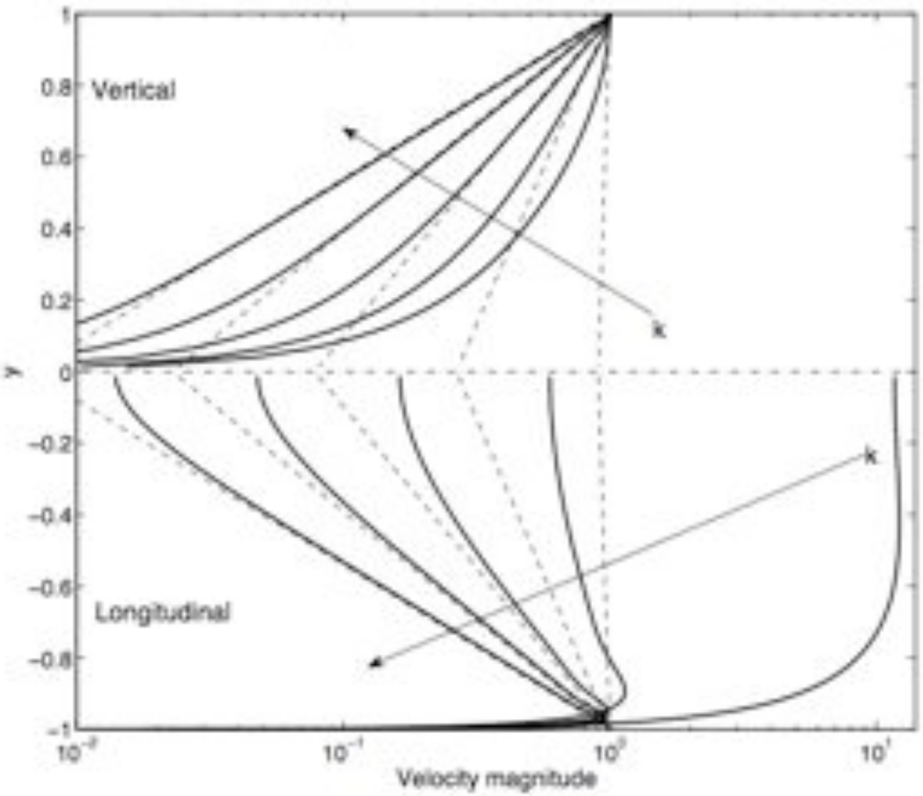




Wall waves

Varicose:

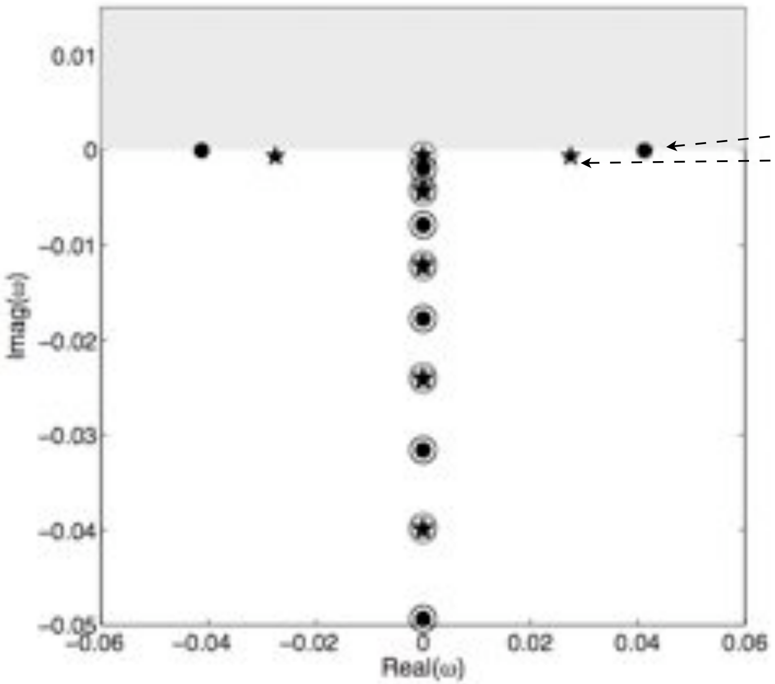




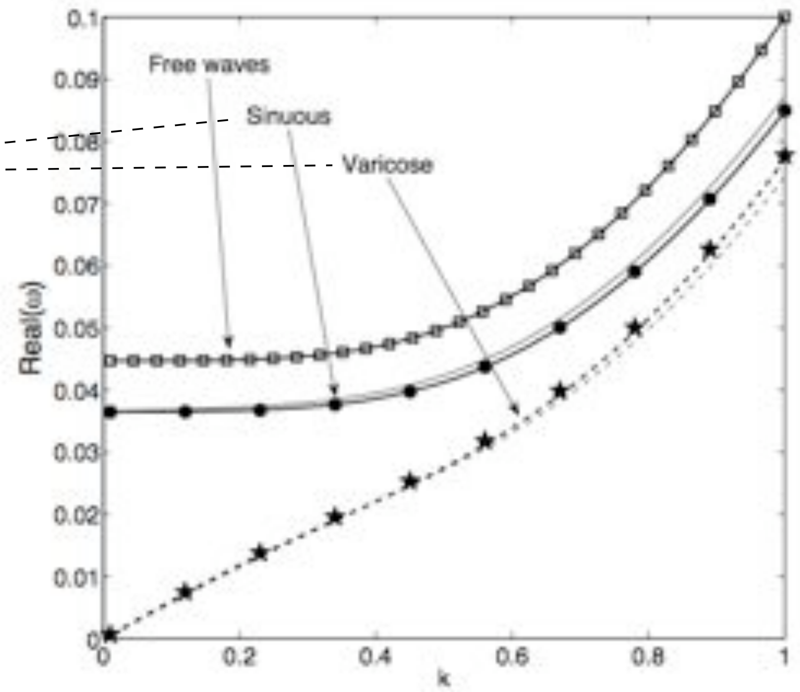
Fluid effect: added mass

sinuous: $m_a^s = (1 - e^{-k})/k$

varicose: $m_a^v = (1 - e^{-k})/k + 1/k^2$



Spectra



Pulsation:

free wall, computed eigenmodes, added mass model, inviscid

Energy budget

$$\overbrace{\left(\frac{1}{2}\int_y \overline{u^2+v^2+w^2}\mathrm{d}y\right)_t}^{(E_K^f)_t} + [\overline{vp}]_{\mathrm{bot}}^{\mathrm{top}} =$$

$$- \underbrace{\int_y U_y \overline{uv}\mathrm{d}y}_{\text{Reynolds stress term}} + \underbrace{\frac{1}{Re}\left[\overline{uu_y + vv_y}\right]_{\mathrm{bot}}^{\mathrm{top}}}_{\substack{\text{Extra-energy removal}\\ \text{and supply term}}} - \underbrace{\frac{1}{Re}\int_y \overline{\boldsymbol{\omega}.\boldsymbol{\omega}}\,\mathrm{d}y}_{\substack{\text{Fluid viscous}\\ \text{damping term}}},$$

$$[\overline{vp}]_{\mathrm{bot}}^{\mathrm{top}} = \sum_{\mathrm{bot}}^{\mathrm{top}} \overline{\eta_t\left(m\eta_{tt} + \frac{d}{Re}\eta_t + \frac{B\Delta^2 - T\Delta + K}{Re^2}\eta\right)},$$

$$= \sum_{\mathrm{bot}}^{\mathrm{top}} \left\{ \underbrace{m\left(\frac{\overline{\eta_t^2}}{2}\right)_t}_{(E_K^w)_t} + \underbrace{\frac{d}{Re}\overline{\eta_t^2}}_{\substack{\text{Viscous damping}\\ \text{term in the wall}}} + \underbrace{\left(\frac{B\overline{(\Delta\eta)^2} + T\overline{\nabla\eta\cdot\nabla\eta} + K\overline{\eta^2}}{2Re^2}\right)_t}_{(E_P^w)_t} \right\},$$

$$\overline{E} \triangleq \underbrace{\frac{1}{2}\int_y \overline{u^2+v^2+w^2}\mathrm{d}y}_{\text{Flow}} + \underbrace{\sum_{\mathrm{bot}}^{\mathrm{top}} \frac{1}{2}\left(m\overline{\eta_t^2} + \frac{B\overline{(\Delta\eta)^2} + T\overline{\nabla\eta\cdot\nabla\eta} + K\overline{\eta^2}}{Re^2}\right)}_{\text{Walls}}.$$

Optimization of the initial conditions

$$
\begin{aligned}
u_t + U_y v &= 0, \\
v_t &= -p_y, \\
w_t &= -p_z, \\
v_y + w_z = 0.
\end{aligned}
$$

$$i\omega\hat{u} + U_y\hat{v} = 0 \quad \Rightarrow \quad \|\hat{u}\| = \frac{T^2}{4\pi^2}\int_y |U_y\hat{v}|^2 \mathrm{d}y,$$

Energy growth scales like the square of the oscillation period

alpha non zero

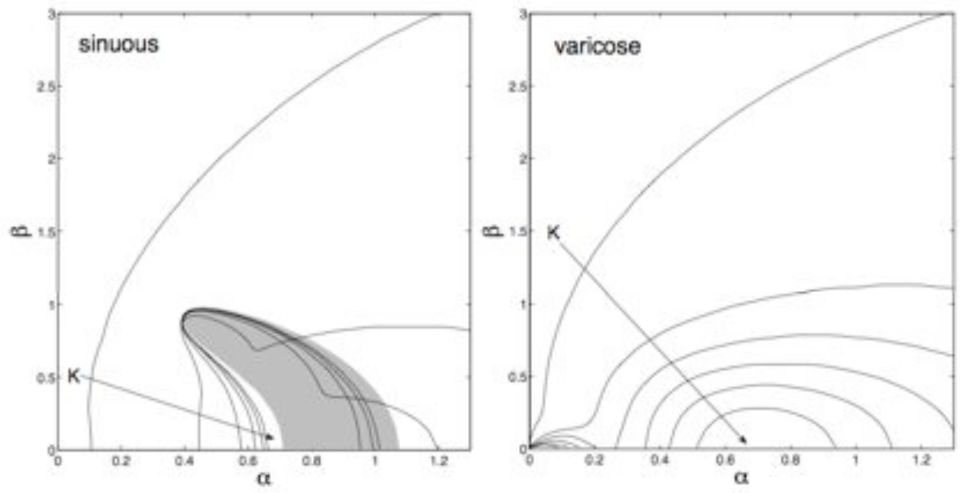


FIGURE 11. Neutral curves for spring stiffness K logarithmically spaced from 10^6 to 10^8 at $Re = 15000, d = 100$. The shaded area corresponds to unstable TS waves in the rigid channel.

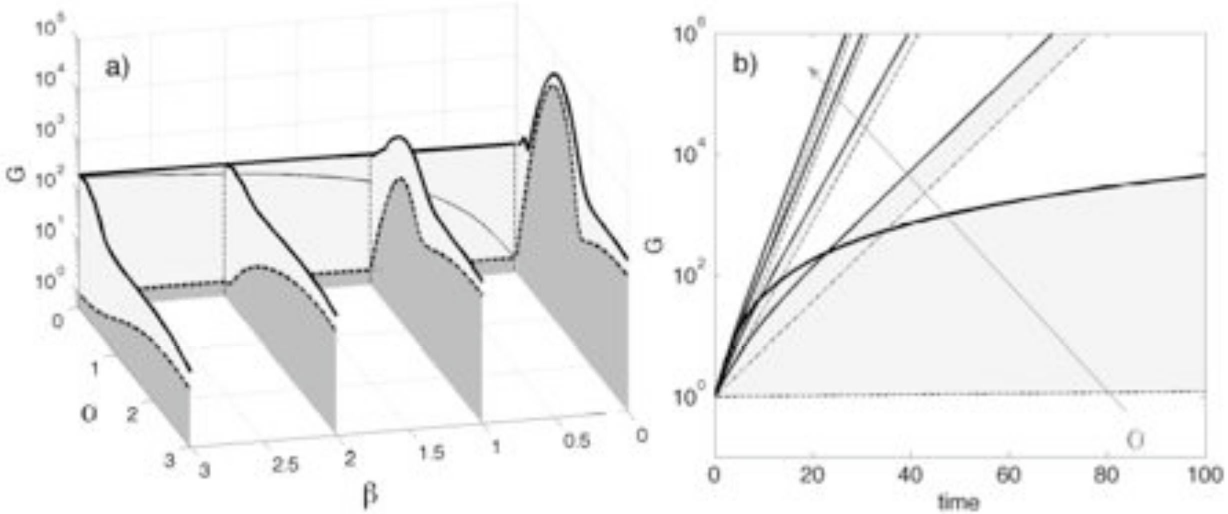
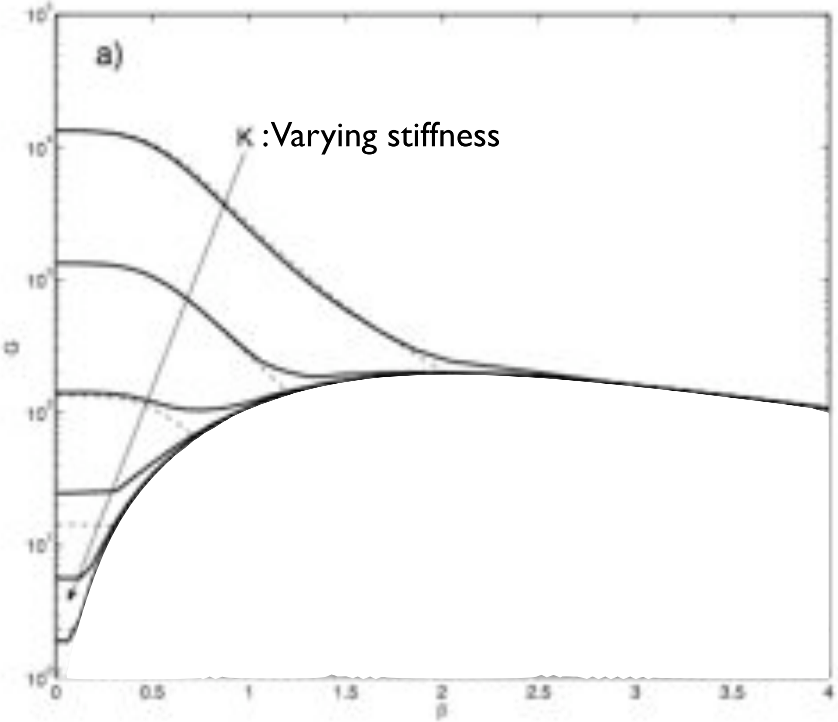
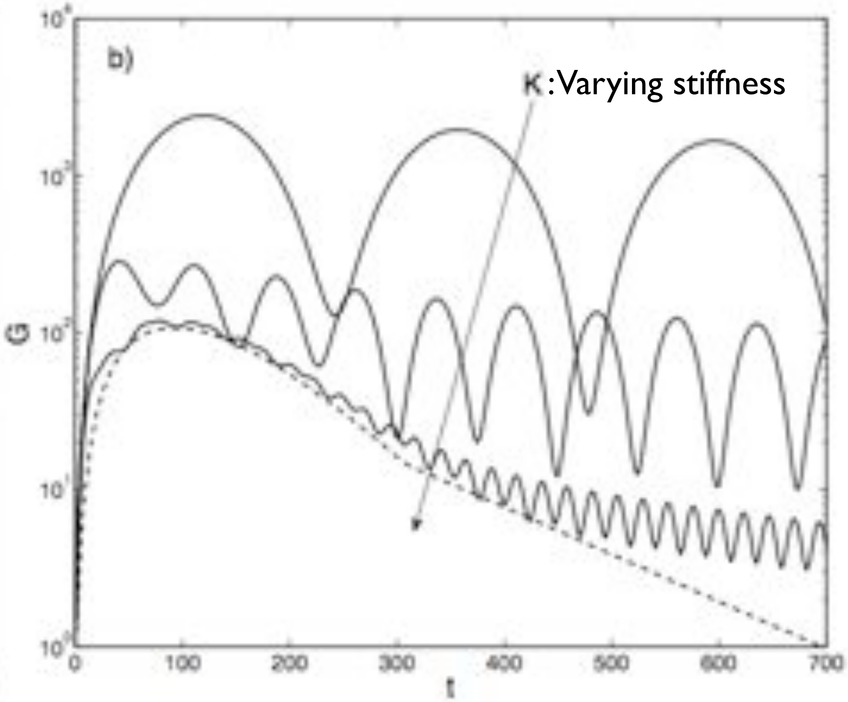


FIGURE 12. *a)* Optimal (thick solid) and exponential (dashed) growth at time 20, compared to optimal growth of the rigid-walls system at $\alpha = 0$ (thin solid) for $K = 1000, Re = 5000, d = 100$. *b)* Optimal (thick solid) and exponential growth (dashed) in time for $\beta = 0$ and α equispaced from 0.01 to 1.

Optimization results



Maximum growth, varying spanwise wavelength, compare to rigid growth



Envelopes in time, varying stiffness