KTH Mechanics

Control of instabilities in a cavity-driven separated boundary-layer flow

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Feedback control and estimation applied to boundary layers subject to free-stream turbulence

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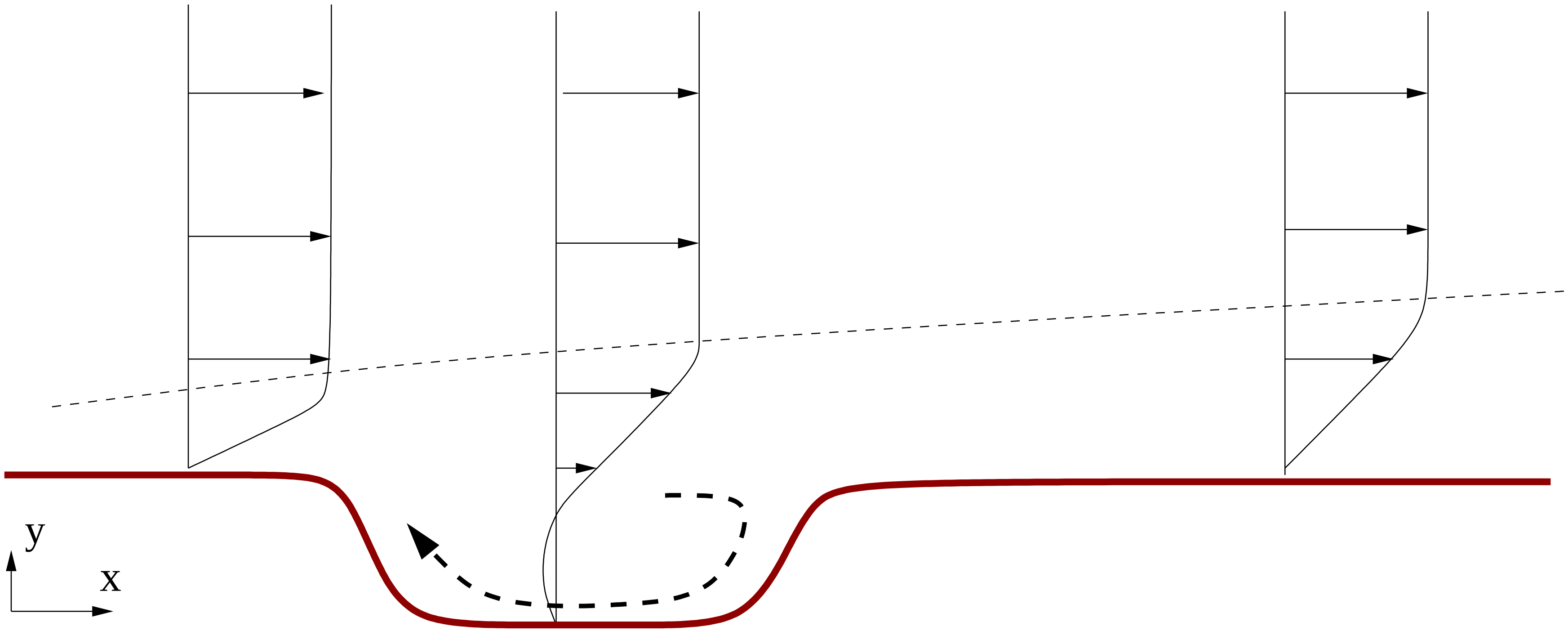
Outline

- The flow case
- Investigation tools
- Reduced dynamic model for feedback control
- Control performance

Boundary layer with cavity



2D flow over a smooth cavity

Inflow: Blasius profile

Reynolds number : 500

Aim of the project

Design of a feedback controller $\longrightarrow$ need of a dynamic model.

Preventing transition to **nonlinear oscillating regime** in the cavity,
or **turbulence** downstream of the cavity.

Investigation tools

DNS to compute the base flow:
Chebyshev in wall normal, finite difference in streamwise.

Stability analysis by computation of 2D eigenmodes:
Chebyshev/Chebyshev and Arnoldi

Control optimization by solution of two Riccati equations:
Using the reduced order model

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The eigensolver

2D Navier-Stokes + continuity

$$\begin{cases} -i\omega\hat{u} = -(U\cdot\nabla)\hat{u} - (\hat{\mathsf{u}}\cdot\nabla)U - \dfrac{\partial\hat{p}}{\partial x} + 1/Re\nabla^2\hat{u} \\ -i\omega\hat{v} = -(U\cdot\nabla)\hat{v} - (\hat{\mathsf{u}}\cdot\nabla)V - \dfrac{\partial\hat{p}}{\partial y} + 1/Re\nabla^2\hat{v} \\ 0 = \nabla\cdot\mathsf{u} \end{cases}$$

Generalized eigenproblem:

$$B\omega\mathsf{u} = A\mathsf{u}$$

To be rewritten

$$A^{-1}B\mathsf{u} = \frac{1}{\omega}\mathsf{u}$$

Solved by Arnoldi iterations.

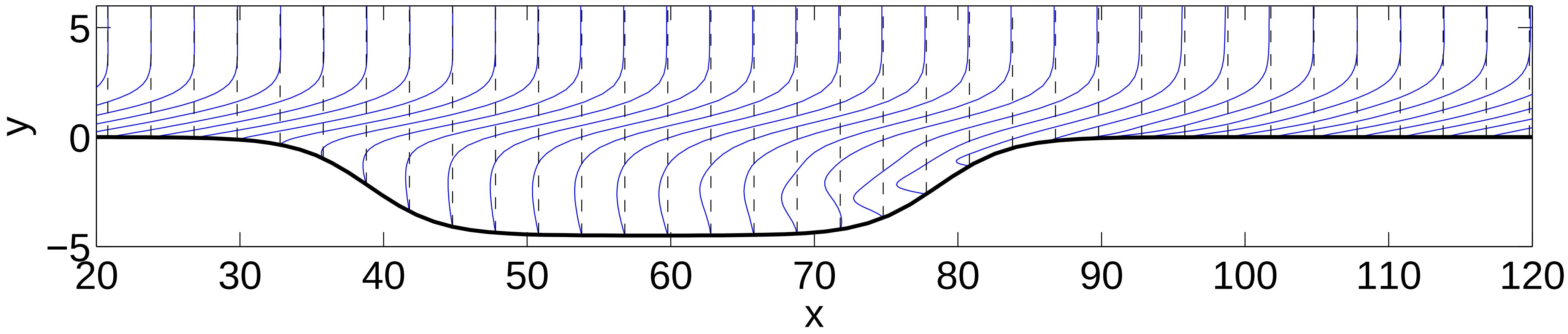
Matrix formulation:

$$\begin{pmatrix} -i\omega\hat{u} \\ -i\omega\hat{v} \\ 0 \end{pmatrix} = \begin{pmatrix} \dots & \dots & -\frac{\partial}{\partial x} \\ \dots & \dots & -\frac{\partial}{\partial y} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & C \end{pmatrix} \begin{pmatrix} \hat{u} \\ \hat{v} \\ p \end{pmatrix}$$

Additional constraints C

The base flow

Streamwise velocity profiles u:



Flow is composed of :

- Boundary layers (before and after the cavity)
- Shear-layer over the cavity
- Recirculating zone inside the cavity

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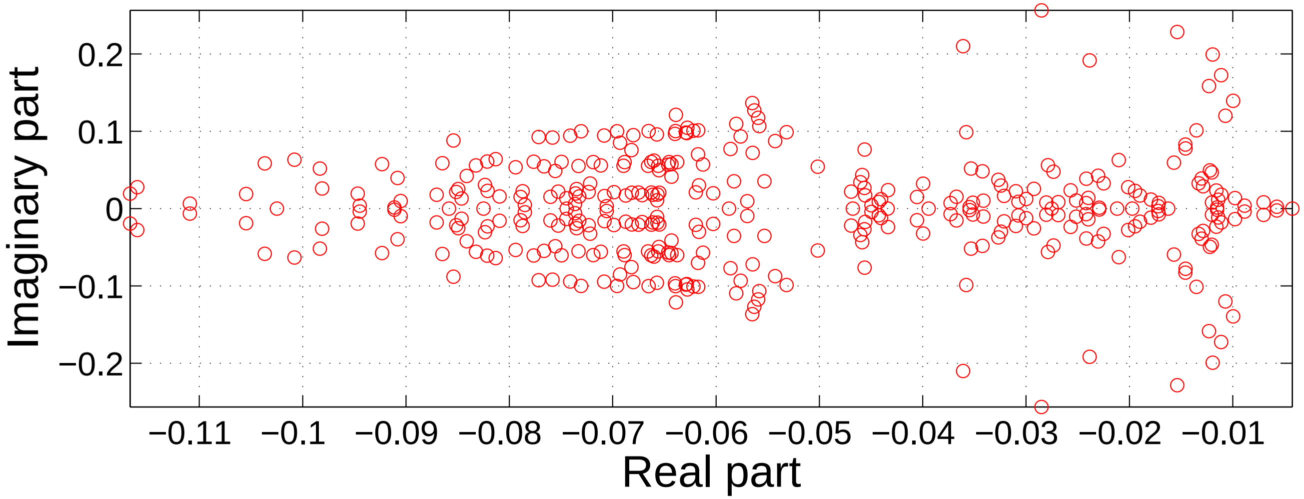
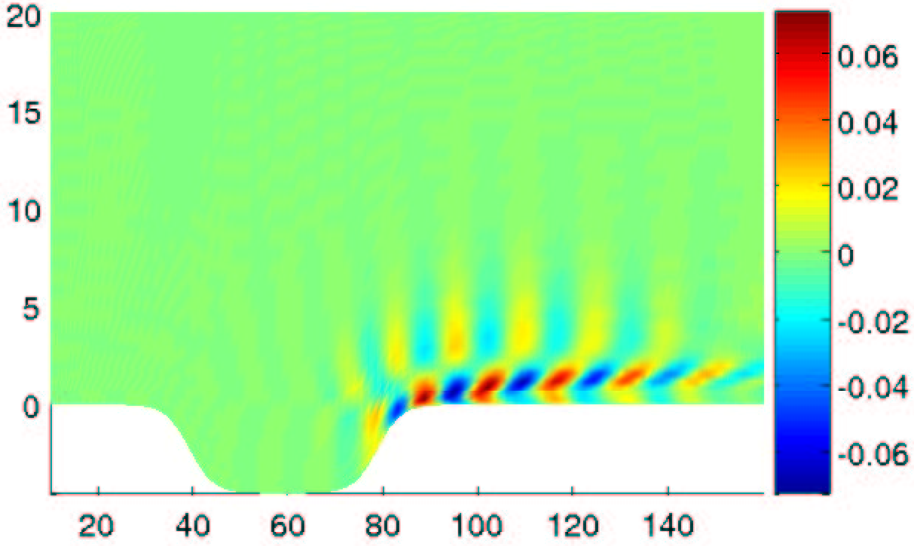
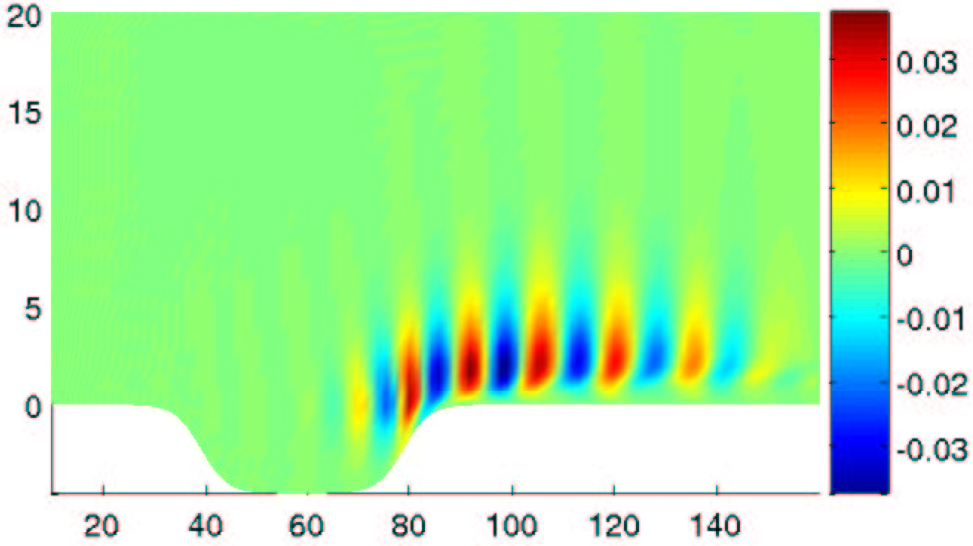
Spectra: shear layer modes







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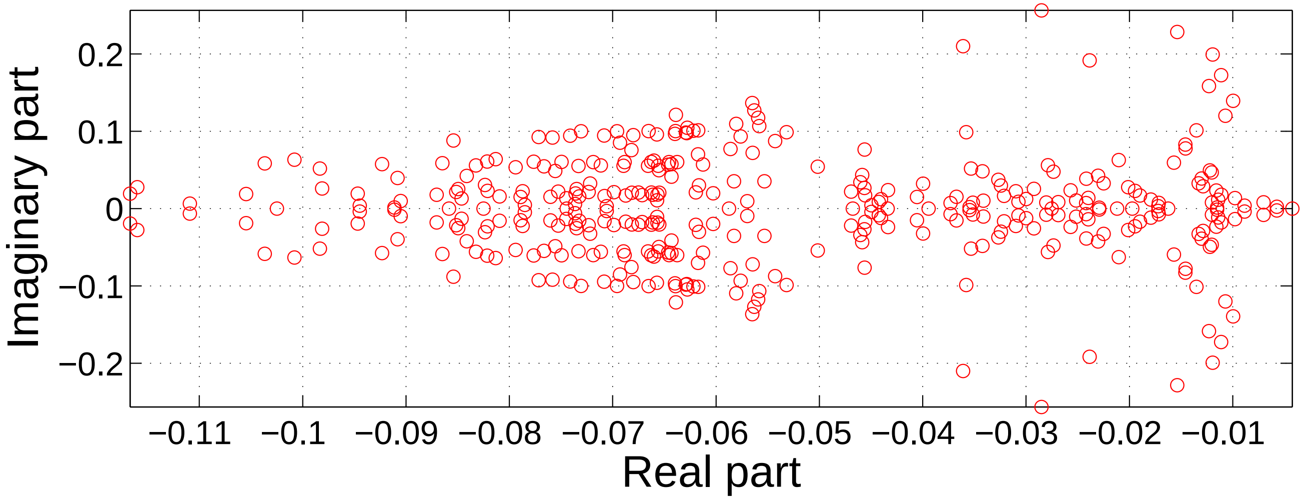
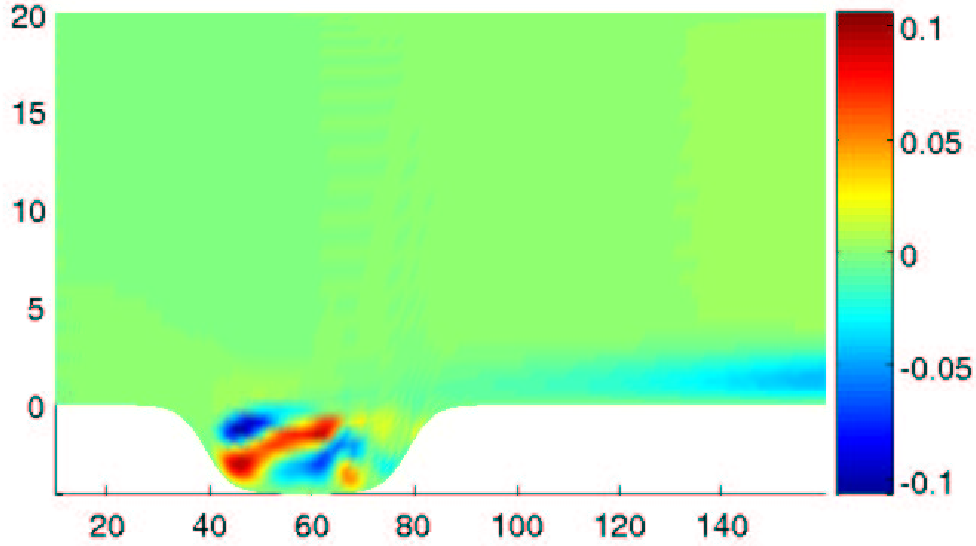
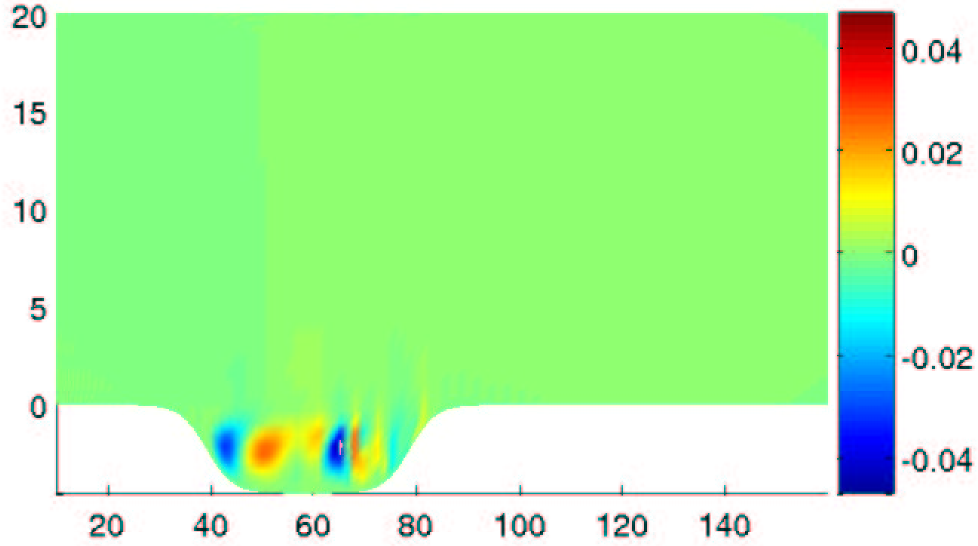
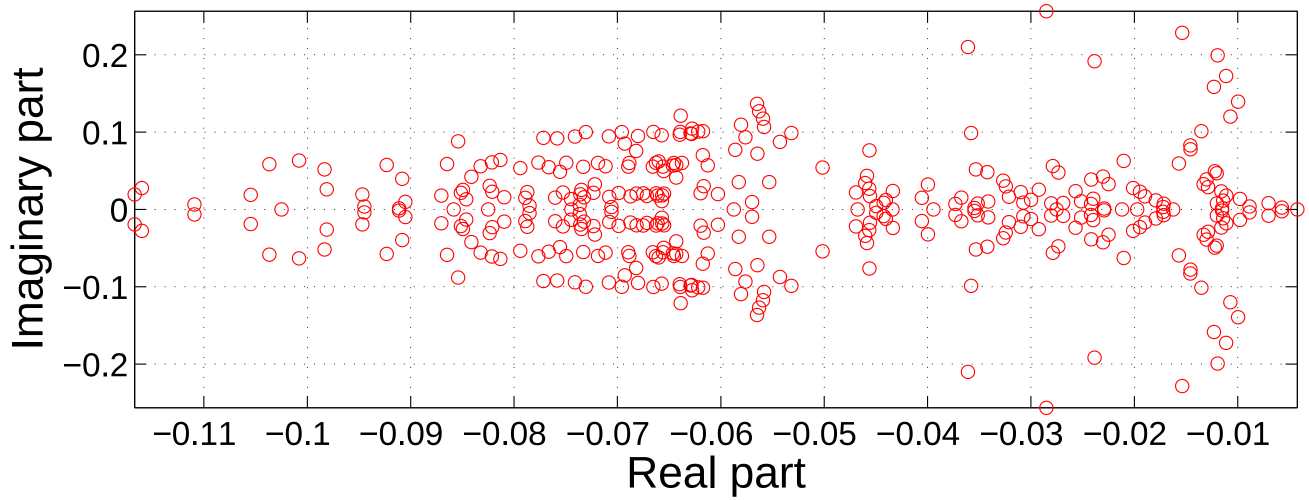
Spectra: cavity modes

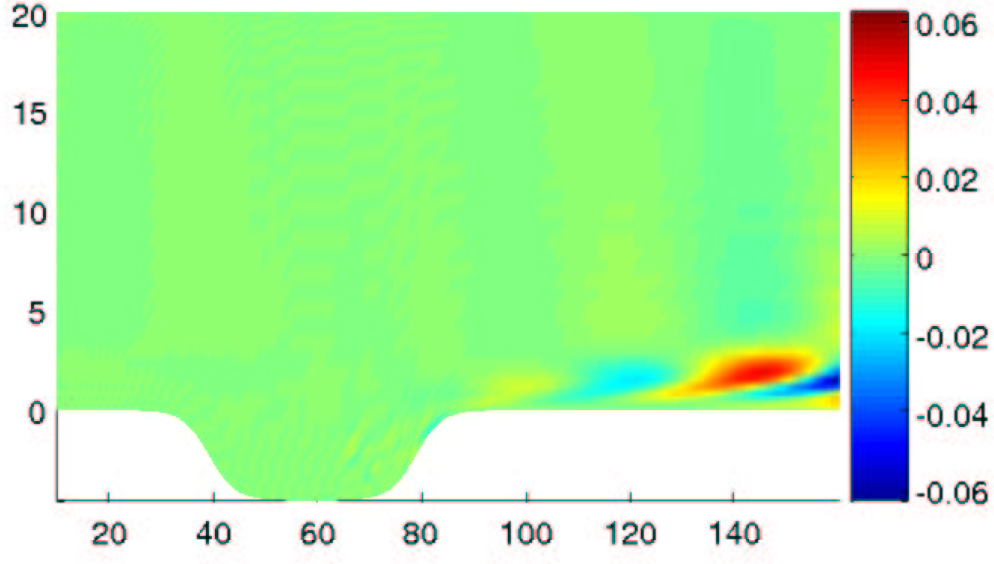


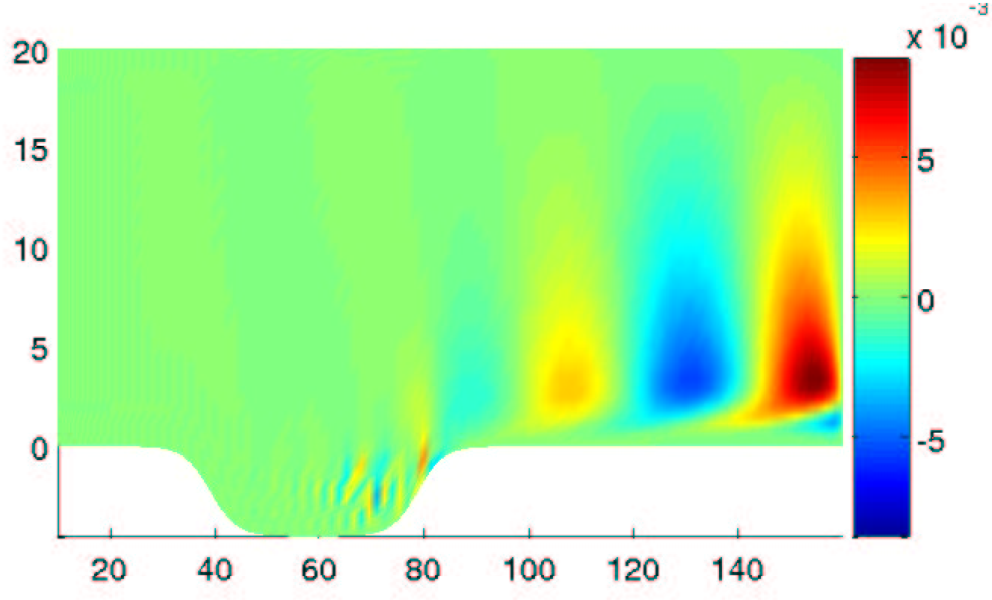




Spectra: outflow modes

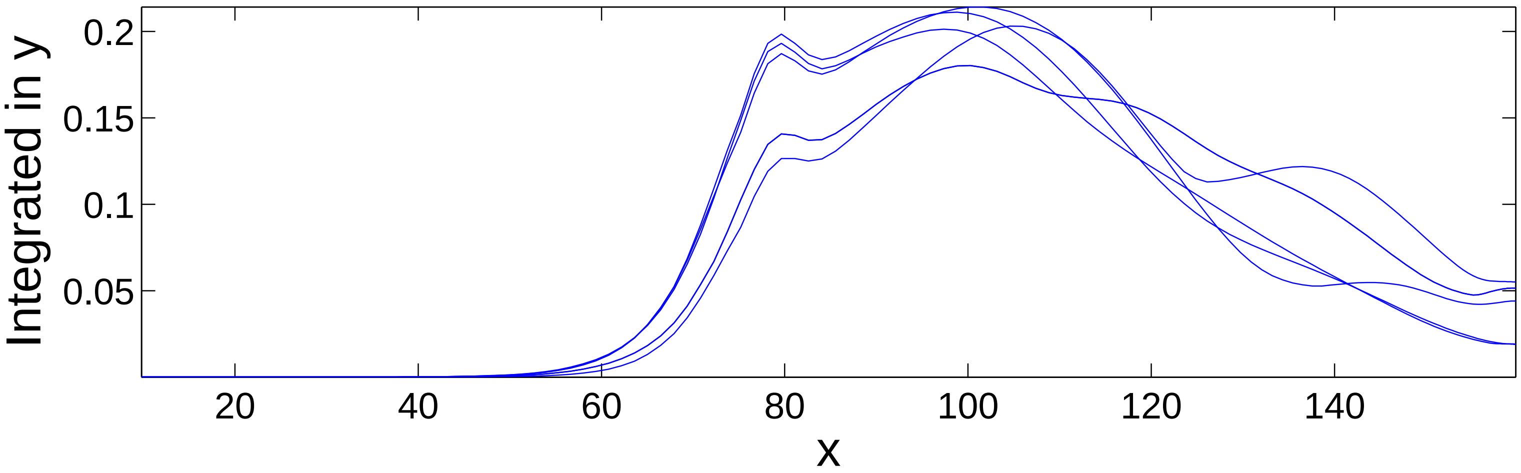


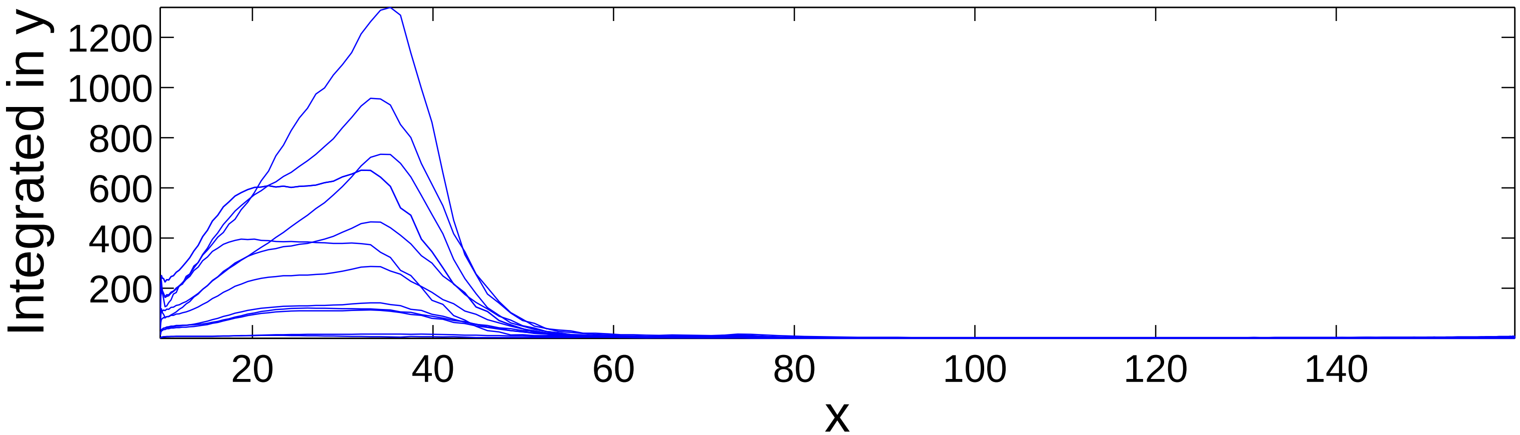




Shear layer modes and adjoints

Integrated in y:





Where are the modes localised and where are they sensitive ?

Feedback control

Using a dynamic model of the system:

$$\begin{cases} \dot{x} = Ax + Bu \\ r = Cx \end{cases}$$

One can optimize for the feedback

$$u = G(r)$$

- The model in 2D is too big for optimization $\longrightarrow$ reduced model .
- For reduction: project the dynamics on the least stable eigenmodes.
- Finally, couple the reduced controller and the flow system

Model reduction

Galerkin projection on least stable eigenmodes:

Physical space:

$$\begin{cases} \dot{x} = Ax + Bu \\ r = Cx \end{cases}$$

Eigenmode space:

$$\begin{cases} \underbrace{P\dot{x}}_{\dot{k}} = \underbrace{PAP^{-1}}_{A^M}\underbrace{Px}_{k} + \underbrace{PB}_{B^M}u \\ r = \underbrace{CP^{-1}}_{C^M}\underbrace{Px}_{k} \end{cases}$$

Projection on eigenmodes → **biorthogonal** set of vectors:

- Eigenmodes: q_i,
- Adjoint operator: $A^+ / < Ax_1, x_2 > = < x_1, A^+ x_2 >, \forall x_1, x_2$
- Adjoint eigenmodes: q_i^+,
- Biorthogonality: $\delta_{ij} = < q_i, q_j^+ >$, Projection: $k_i = < x, q_i^+ >$

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Control terminology

- **Estimation:** From sensor information, recover the instantaneous flow field.
- **Full information control:** From full knowledge of the flow state, apply control.
- **Compensation:** Close the loop by using the estimated flow state for control.
- **Model reduction:** Project the dynamics on a set of selected basis vectors.

- **Control penalty:** Penalisation of the actuation amplitude.
- **sensor noise:** Uncertainty in the measured signal.

- **Disturbances:** External forcing exciting the flow.
- **Objective function:** Function of the flow state to be minimized.

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Central elements of the design

1) From the sensors, estimate the flow state:

- Sensor location
- Sensor noise
- Disturbance model (here perturbations at the inflow)

2) Using the flow state information, apply control :

- Actuator location
- Control penalty
- Objective function

Optimization is done by solving two Riccati equations

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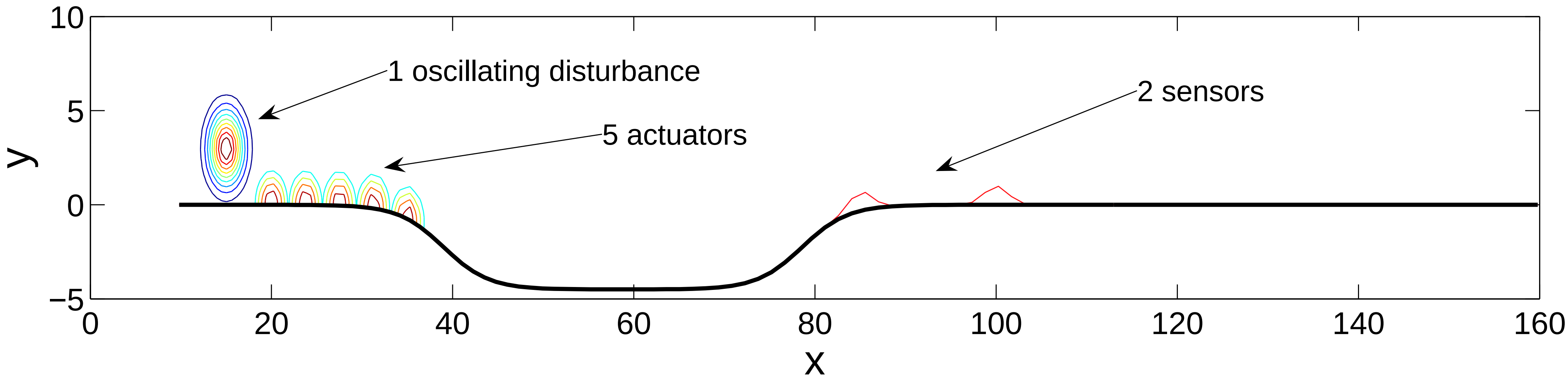
Sensors, actuators, disturbances



Disturbances at the **inflow**

Actuators **upstream** of the cavity: where sensitivity is high

Sensors **downstream** of the cavity: where energy is high

Testing procedure

1. Decide penalties, sensor noise, locations
2. Reduce the model by projection
3. Optimize for the feedback
4. Couple flow system and controller

   The reduced controller (75 states) is applied on the full system (20,000 states)

5. Compute energy of controlled flow

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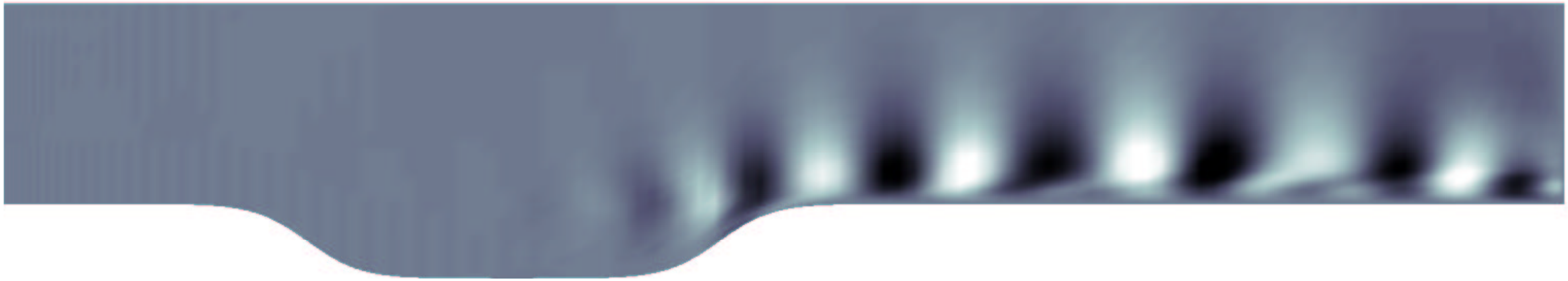
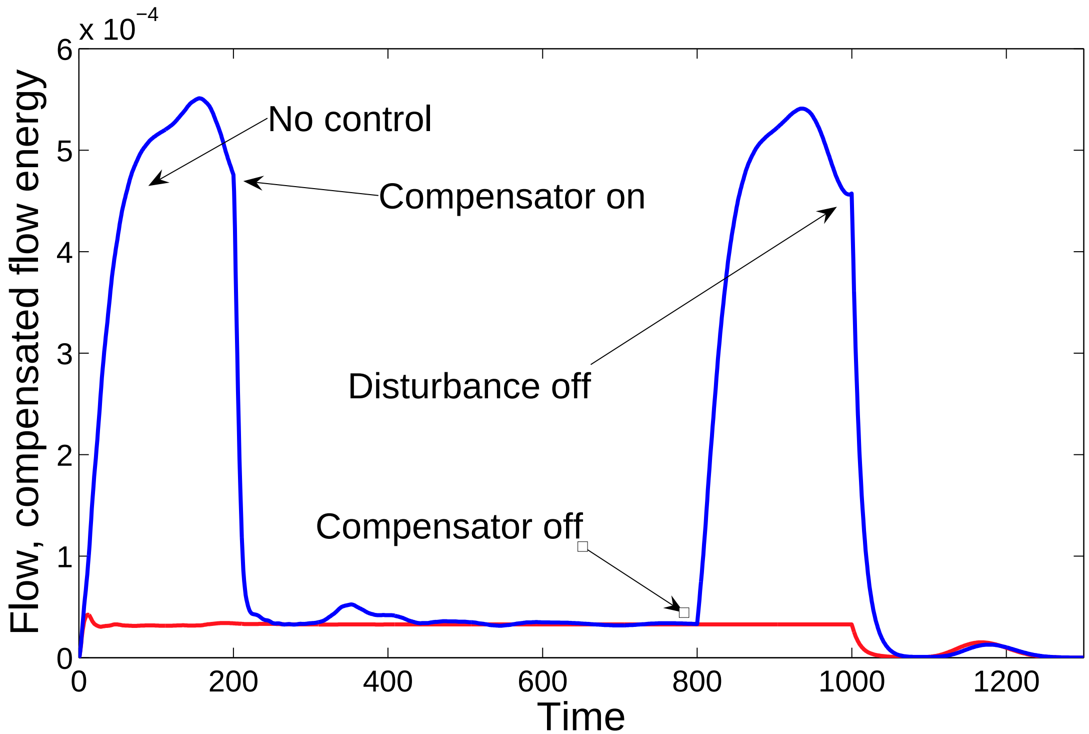
Flow animation

Normal velocity v:

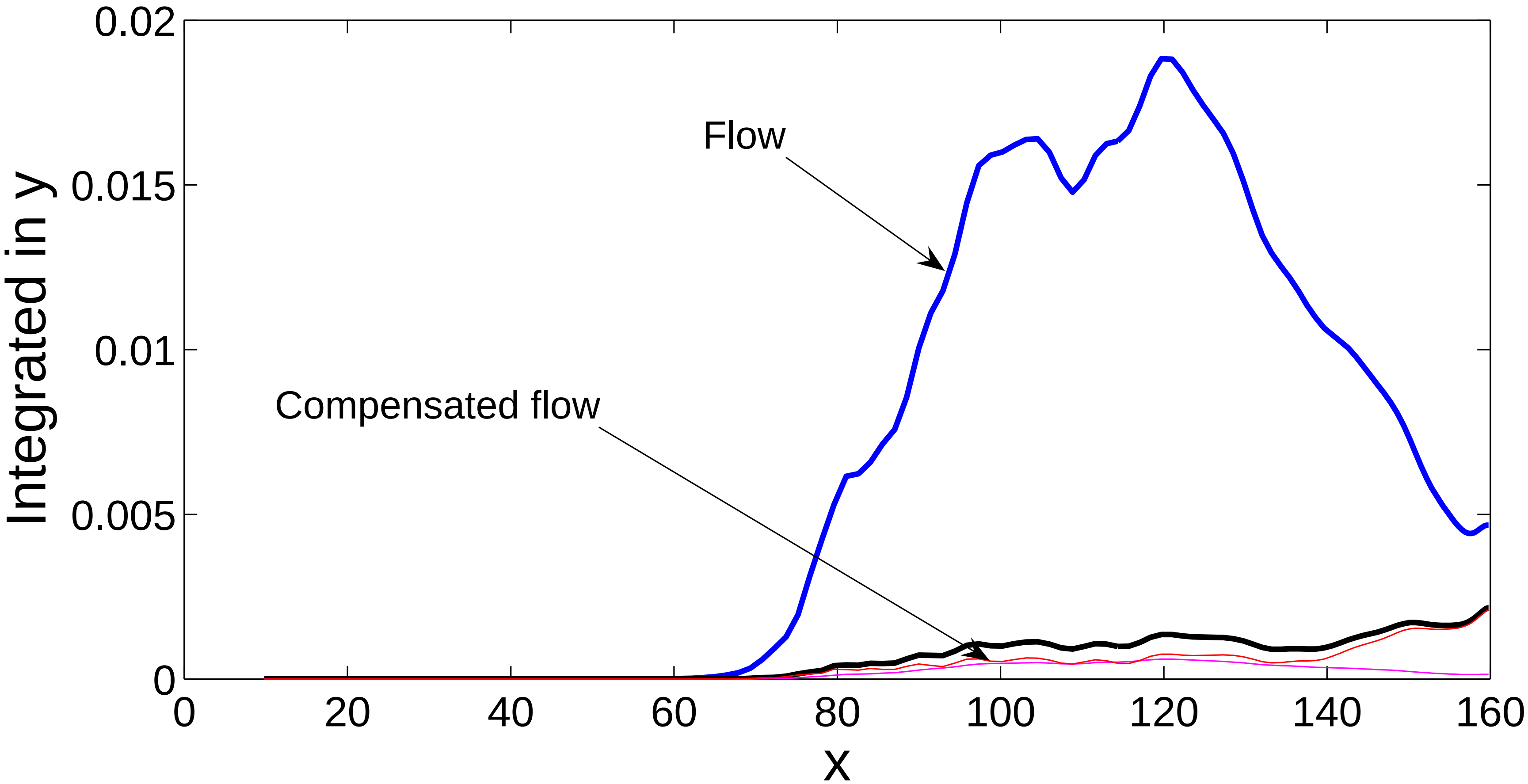


Energy evolution in time:



Estimation and control performances

Mean energy, integrated in y:



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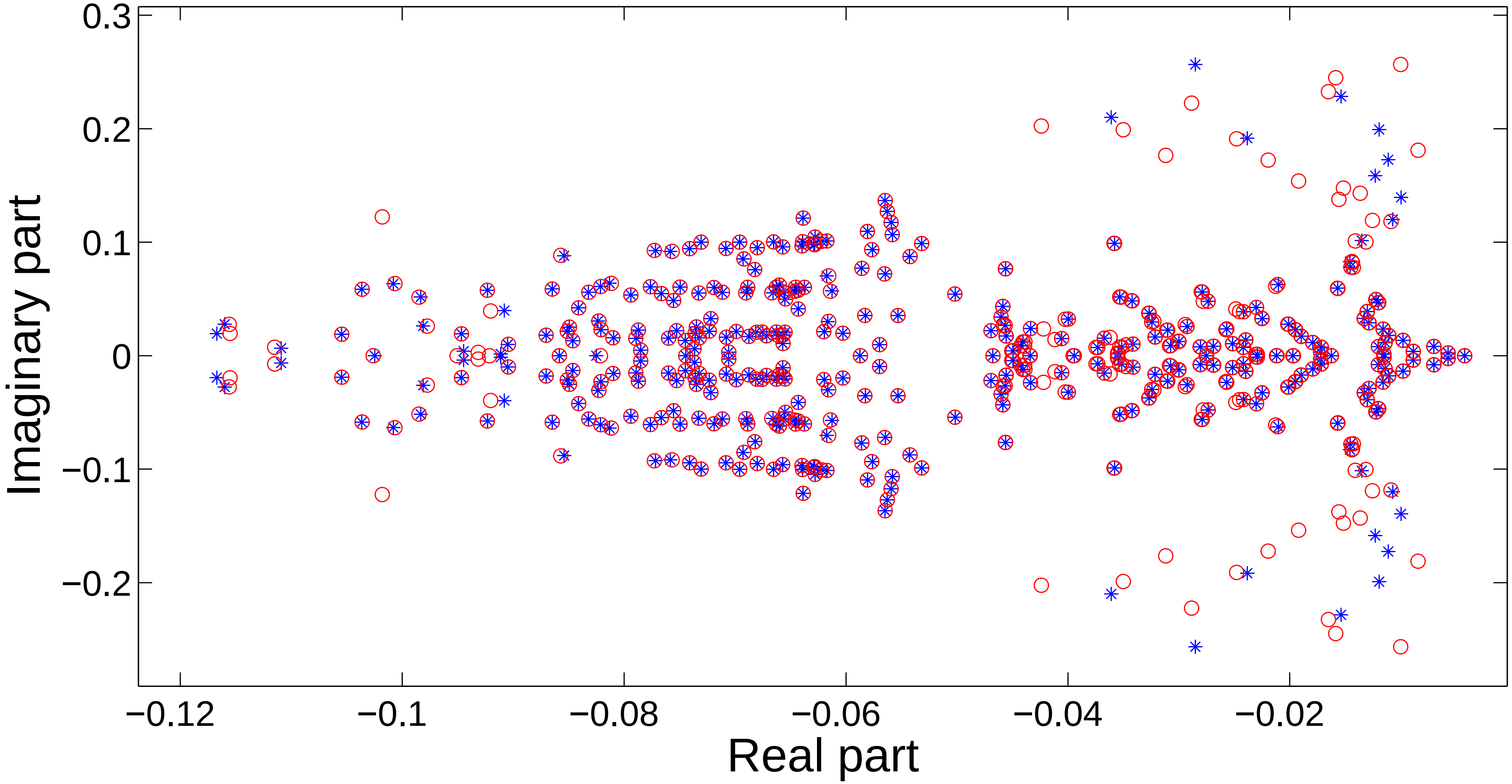
Dynamic distorsion

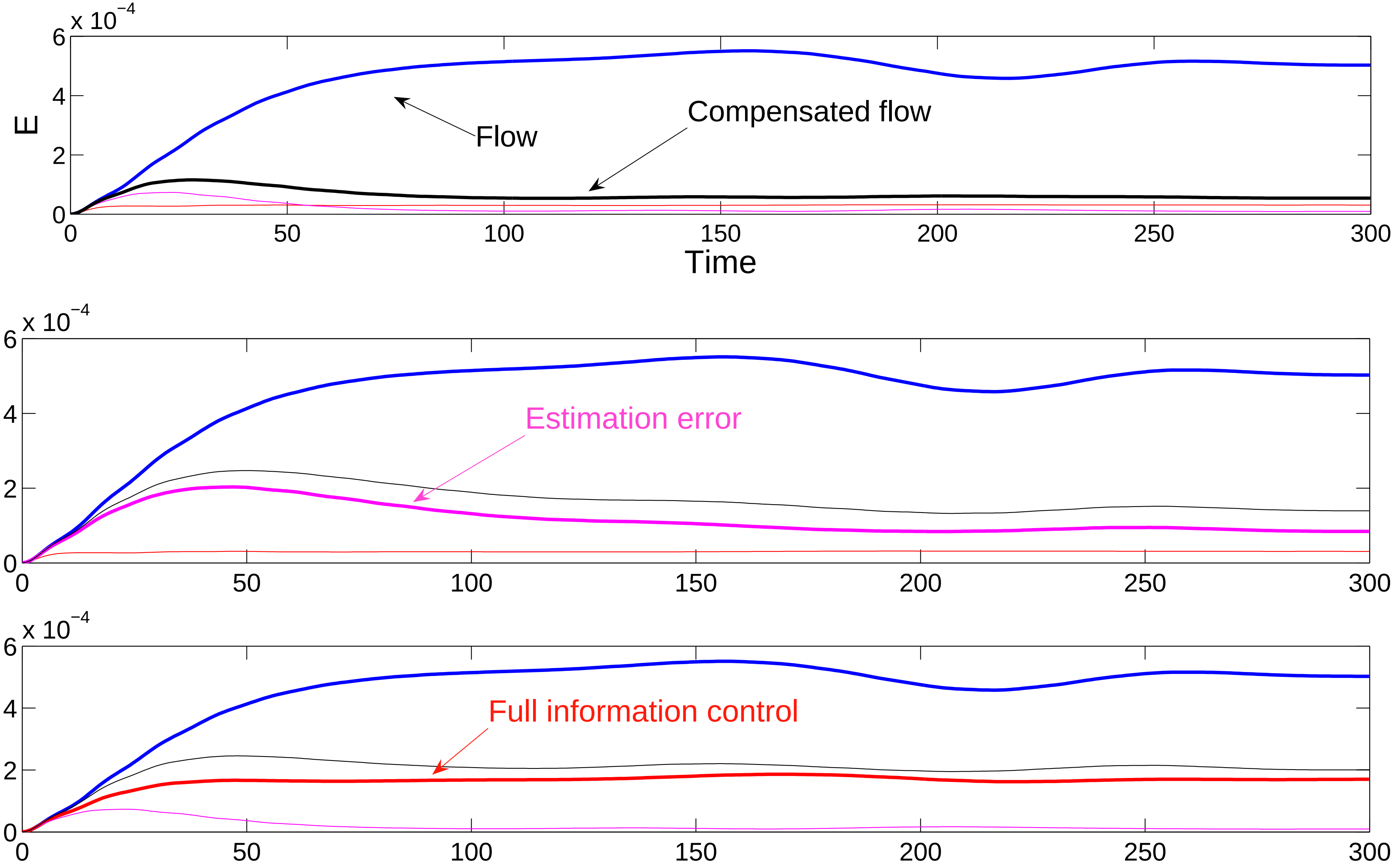
The compensated flow obeys new dynamics:



Blue: flow , red: Compensated system

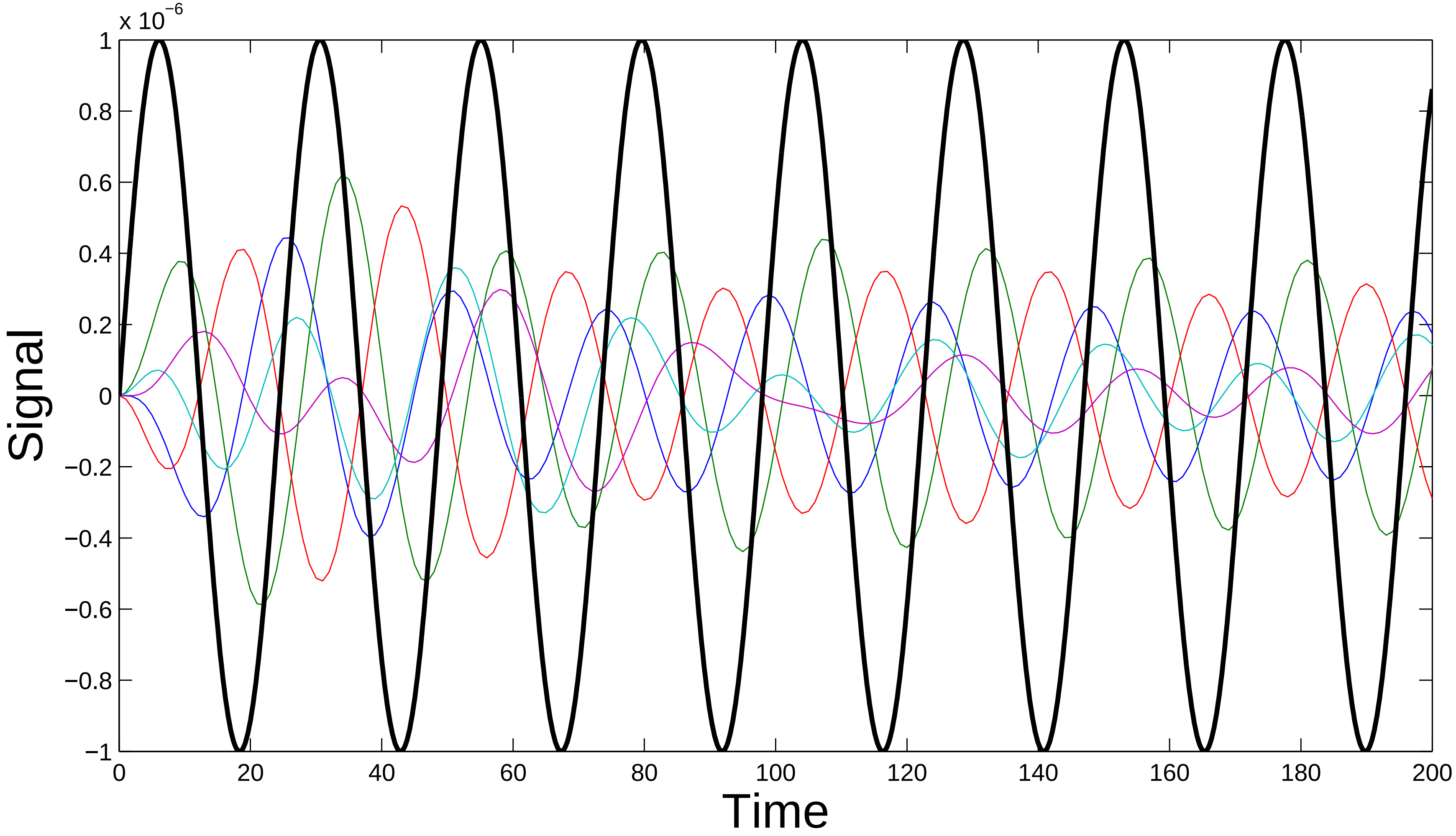
Energy evolution

Estimation or control can be the limiting factors:



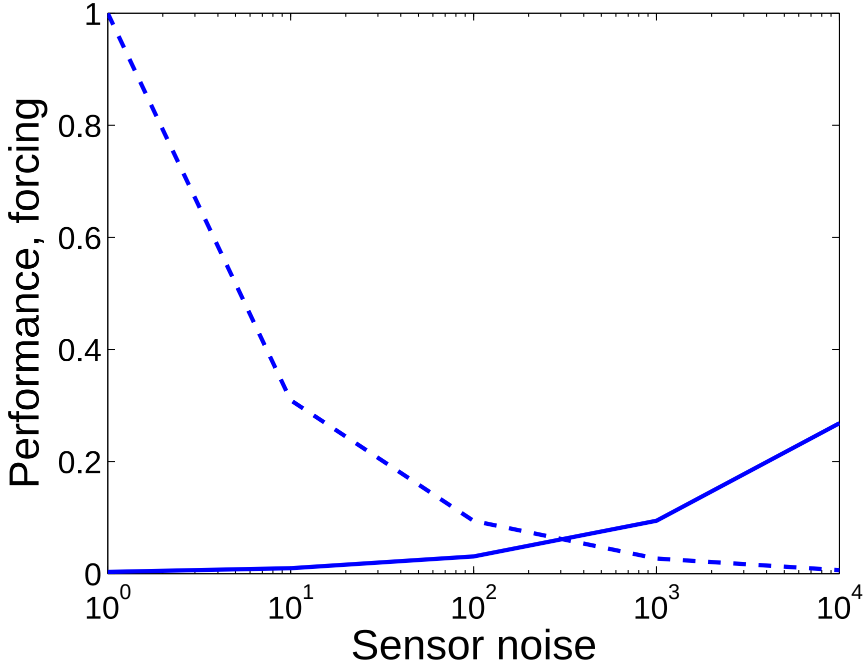
1)Reference case– 2)High sensor noise– 3)high control penalty–

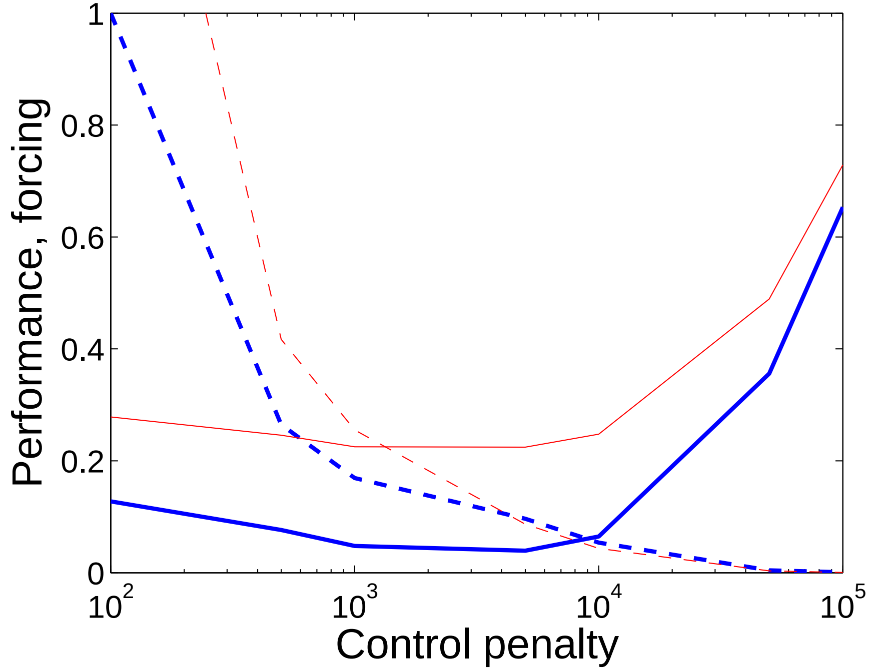
Actuator signals



Five actuators, compared with disturbance signal

Penalties and performance





lower sensor noise $\longrightarrow$ better performance but stronger forcing

lower actuator penalty $\longrightarrow$ better performance but stronger stronger

Conclusion

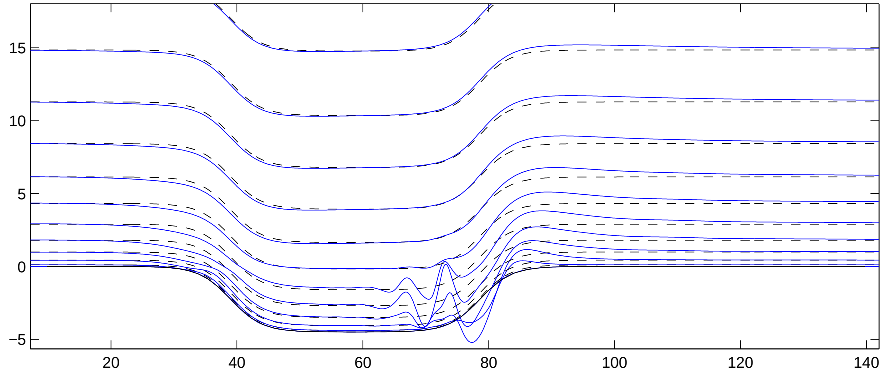
- Global eigenmodes can be used for model reduction.
- Model reduction allows optimal feedback design for large systems.
- Non-parallel effects/global instabilities can be treated.
- There is a balance to find between controller performance and forcing amplitude.

Future and ongoing work:

- Improve outflow boundary condition.
- Find parameter case with global instability.
- Apply the method to boundary layer subjected to free stream turbulence.

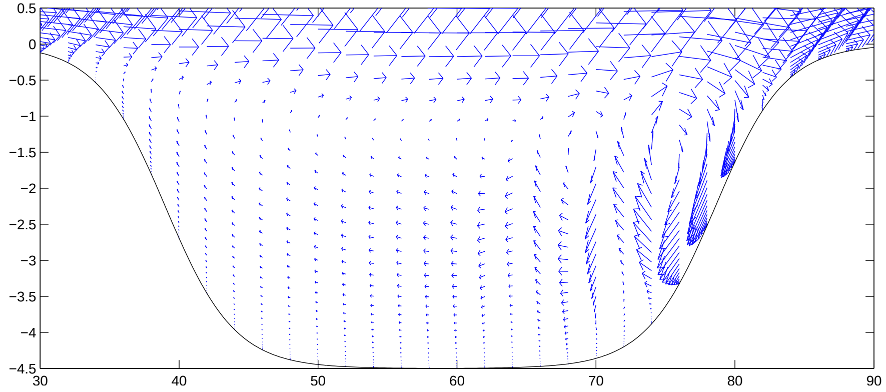
Extra slides

Base flow:

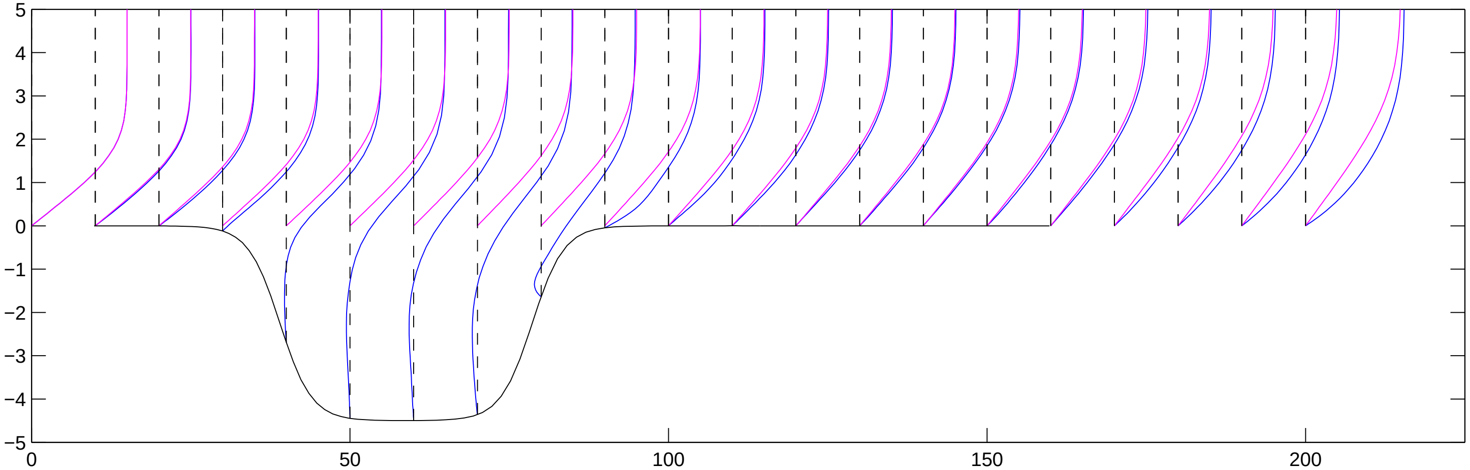


Normal velocity:

Quiver:



Re500/Blasius:



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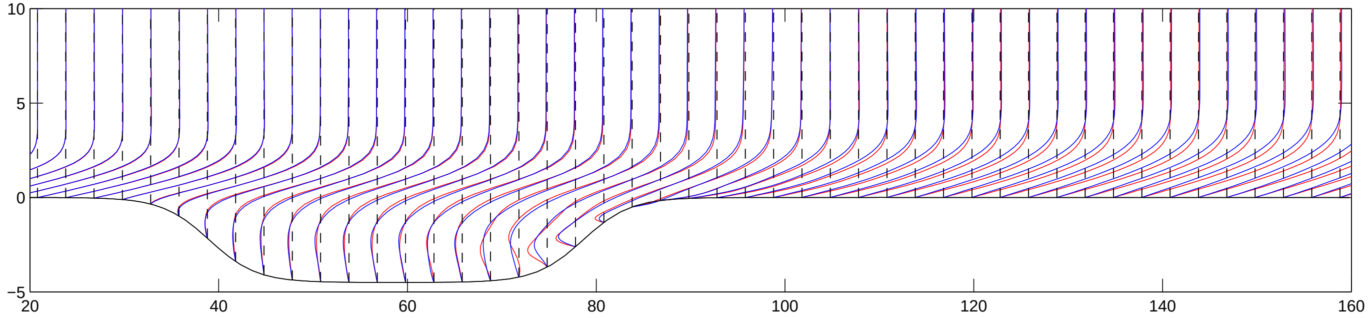
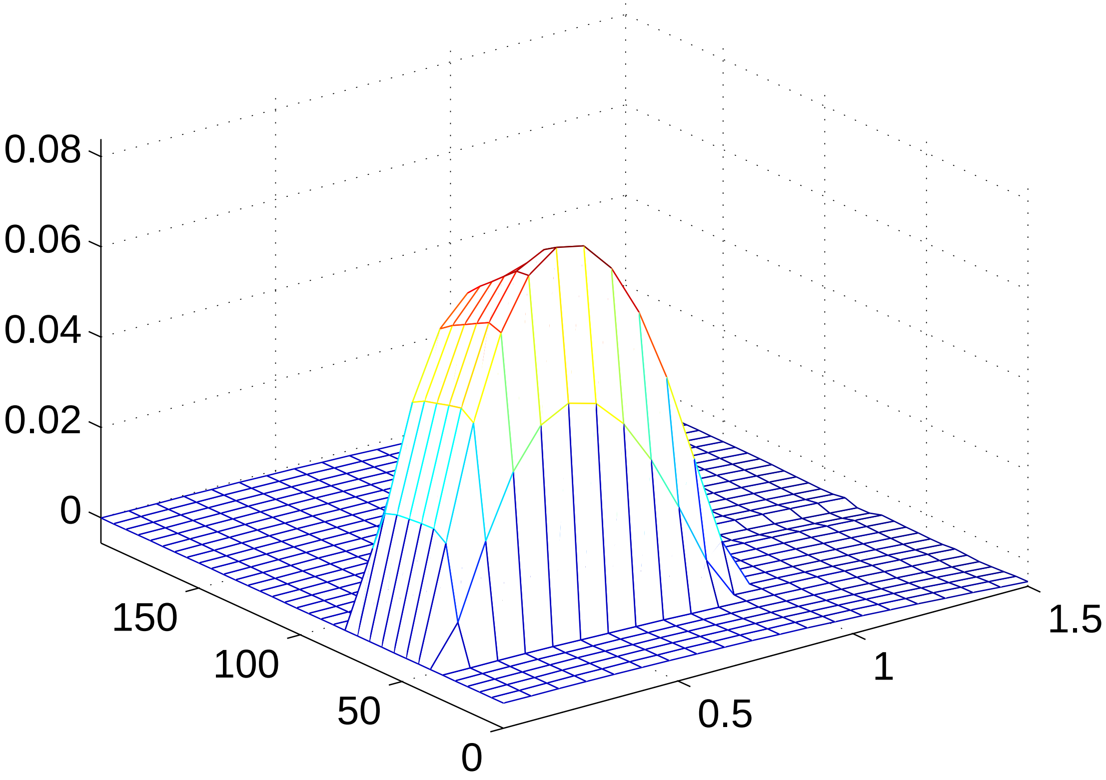
Re 300/Re 500:

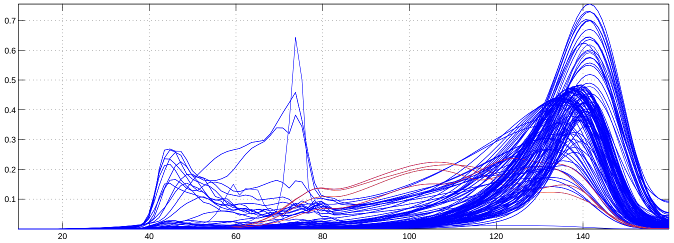


Local stability:

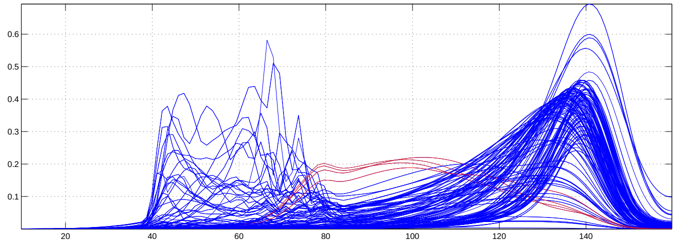


Outflow modes

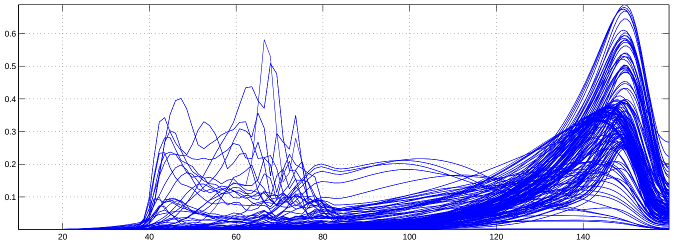
Re 300



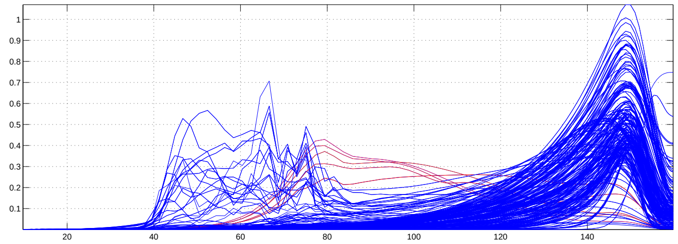
and re 500:



fringe 20

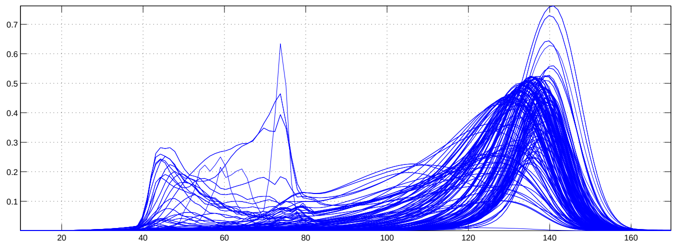


low resolution:



long and strong

fringe:



no fringe