# Habilitation à diriger les recherches

UPMC univ Paris 6

Spécialité: mécanique
Ecole doctorale de Sciences mécaniques, Acoustiques et Electronique de Paris

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En vue d'une soutenance publique le 15 septembre 2015

# Archetypes in mechanics

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# Contents

# Chapter 1

# Introduction

## 1.1 Academicism versus Impressionism

> — There too, I have a comparison, because I like to work with comparisons. I would say that this is a little bit the problem of painting at the time of Chassériaux or Ingres, and of impressionist painting. There was a time when we got lost in the perfect detail and we did not see that what was important was to get a large contrast, the fact that this shadow was blue and not black, things like that...
> — Yes, but this was partly induced by photography.
> — You are absolutely right! *Photography killed Ingres, in some way, and I would say that simulations play a similar role to photography when it comes to our subject; this is the same thing.* And in opposition, the impressionist point of view is really important, because on one hand it shows the central points which will be usable on various concepts, and on the other hand, it is easy to transmit to students. Because if you show to students a graph extremely complex, and you say:"I explain this by putting this and that, I chose this orbit and things like that", he cannot remember anything... If on the other hand you say "on this very long chain, I think that it moves in a way that resembles the motion of a snake, and things like that—and this implies some constraints, which will appear in the mathematical formalism—but there, he remembers something. So, as well from the point of view of culture, it is very important to go toward impressionism, and not to remain Ingres. (laughs...)

This is an excerpt from a conversation of Pierre-Gilles de Gennes with Sydney leach in [de Gennes and Leach, 2005]. This is from tune 7 "artisans et specialistes", starting at time 7.10.

In figure 1.1, I recall famous paintings relevant for this quote, and in figure 1.2 I put together a chronology of painting, science and computing. In the Oxford american dictionary we can read the definition of Impressionism:



> - A style or movement in painting originating in France in the 1860s, characterized by a concern with depicting the visual impression of the moment, esp. in terms of the shifting effect of light and color.
> -A literary or artistic style that seeks to capture a feeling or experience rather than to achieve accurate depiction.
> -Music: a style of composition (associated esp. with Debussy) in which clarity of structure and theme is subordinate to harmonic effects, characteristically using the whole-tone scale.
>
> The impressionist painters repudiated both the precise academic style and the emotional concerns of Romanticism, and their interest in objective representation, esp. of landscape, was influenced by early photography. Impressionism met at first with suspicion and scorn, but soon became deeply influential. Its chief exponents included Monet, Renoir, Pissarro, Cézanne, Degas and Sisley.
>
> Origin: from french *impressionisme*, from *impressioniste*, originally applied unfavorably with reference to Monet's painting *Impression: Soleil levant (1872).*

I did my research and teaching these last years with in mind the question: "what would it be to be impressionist in science?". I try to organize here examples that may help you find your own answer.

## 1.2 "Outdoorism": the flapping flag

Before it got its final name of impressionism, the new movement of painting was called "plain-airism" (outdoorism) because before them, painters worked in their studio where they made their own colors from powders. At the time of the impressionists, color tubes were being produced industrially, so they could paint outdoor and capture rapidly changing landscape appearances like for instance along the duration of a sunrise.

This term introduces the proper context for the work I did on the flapping flag. The instability of the flapping flag has a large literature, see for instance the review [Shelley and Zhang, 2011]. Most of these papers would focus on the academic representation of a flapping flag, following the original inspiration of Rayleigh in [Rayleigh, 1879]. The academic formulation of the question is: "why does a flexible surface start to flap in the wind". They would strive to find the simplest case in which this happens and describe it with the full power of the tools the scholar have at hand. There are many interesting results on that. I did something different: I followed the idea of outdoorism and watched flags as they naturally flap in the wind and saw things different from those I could read from the literature. On figure 1.3, the left flag is a scholarly flag, and the right one is an outdoor flag.

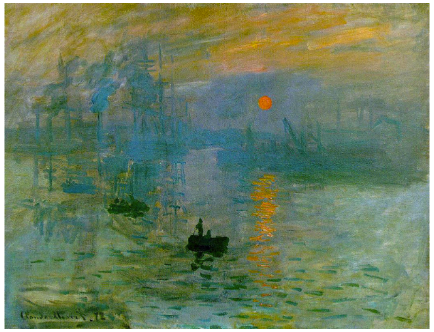
I had the chance that the authors of a paper on numerical simulation of a flag ([Huang and Sung, 2010]) would let me play with their data, and that I had a friend doing a post-doc in Lille working on a big wind-tunnel. Figure 1.5 shows the pattern of waves for different intensities of the wind. The colored

Jean-Auguste-Dominique Ingres «Napoleon premier sur le trône impérial», 1806, Musée de l'armée, Paris.

The master of romantic painting

Théodore Chassériaux «Andromède attachée au rocher par les néréides» 1840. Musée du Louvre, Paris.

Student of Ingres



Claude Monet «Impressions soleil levant» 1872. Musée Marmottant Monet, Paris.

The painting that gave its name to impressionism

Pierre Auguste Renoir «La grenouillère» 1869. Nationalmuseum, Stockholm.

Figure 1.1: Representative paintings for the Ingres-Impressionism transition. Source for the images: wikipedia.



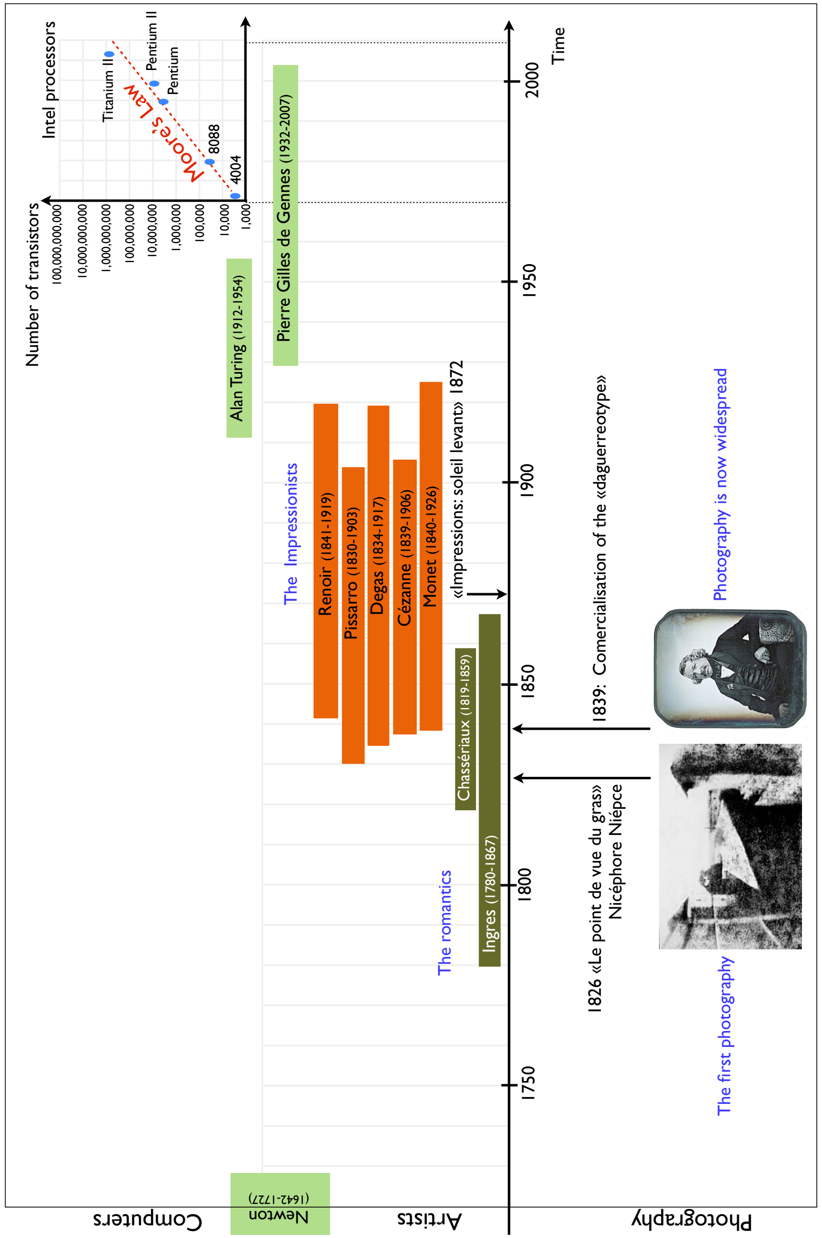



Figure 1.2: Chronology of painting, photography and computing. Sources: Wikipedia. This shows the widespread access to photography at about the middle of Ingres' life, and the widespread access to computing at about the middle of de Gennes' life.

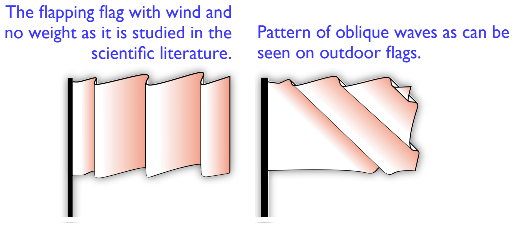



Figure 1.3: Wave patterns of the flapping flag.

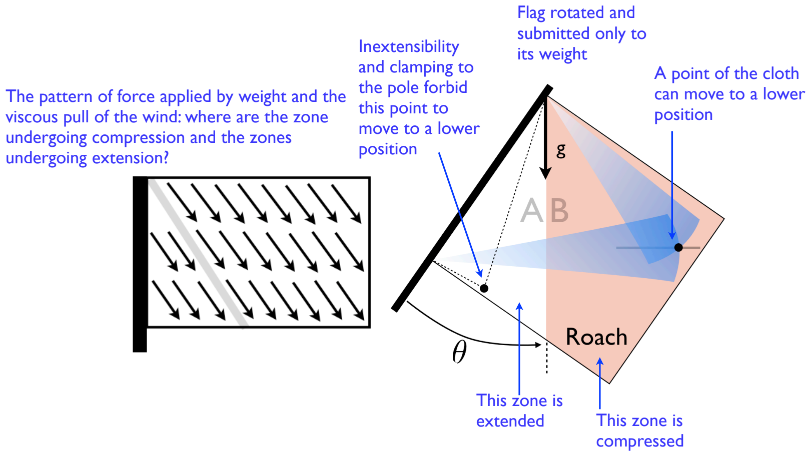



Figure 1.4: Pattern of forces on the flag.

flag is the numerical one and the gray flag is a 60cm square of silk in the wind tunnel. It is a lucky thing that the wave pattern on a square flag is much clearer than that on a rectangular one. You can see on the figure how the angle of the waves changes. I endeavored to find a model for this angle. You can already see on the figure the comparison of the model and data.

The model goes as follows. If we assume that the cloth is subject on one hand to its weight and on the other hand to an average horizontal traction from the viscous boundary layer, you get a pattern of applied force like depicted on figure 1.4. Now, the idea–putting aside all other considerations of fluid-structure interaction—is that if there is a zone of compression on the flag, this zone will buckle. We can think this in analogy with the way we build concrete buildings. Concrete is very good to resist compression but much worse at extension, so for complicated geometries with overhanging parts, it is central to avoid zones where the loading and clamping yields extension. This is just the opposite for cloth, which is good to resist to tension but cannot withhold compression. The



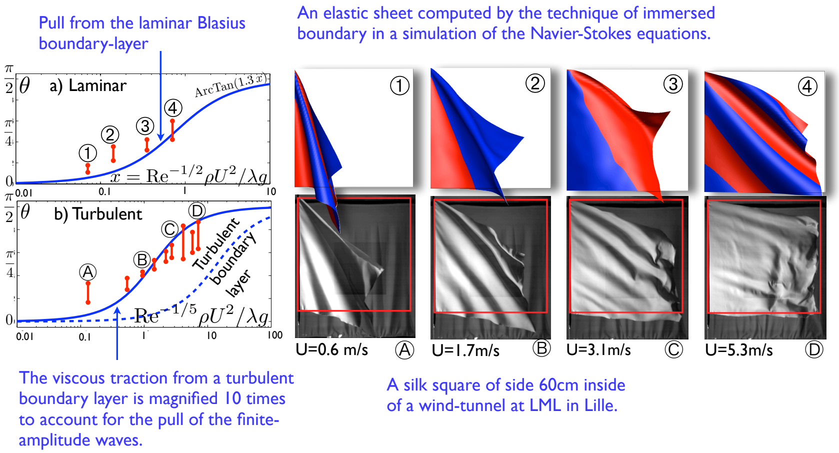



Figure 1.5: Numerical simulation of a flag flowing in the Navier-Stokes equations, and experiment with square of silk in LML's wind tunnel.

riddle is thus to guess directly from the pattern of forces where are the zones under compression and where are the zones under extension.

We could do a numerical simulation, but there is an easy way to get the answer, just by tilting the flag in order to replace the composite load of vertical weight and horizontal drag by just a weight put at an angle. Then it is clear that part B of the flag in figure 1.4 is entirely in compression and will buckle down. A way to rationalize this is to examine which points can move down and which cannot under the constraint of inextensibility and attachment to the pole (let yourself be guided by the blue circular sectors on the figure).

If the flag with this specific buckling and with this specific angle is now put back in the wind, the waves will be carried away downstream by the wind and we have the flapping flag. For the model in figure 1.5, we have two cases. For the laminar case (numerical) we assume a drag equal to the average on the surface of the Blasius boundary layer. And for the turbulent case (experimental) we need to take 10 times the average drag of the turbulent boundary layer. Indeed, the finite amplitude waves have been shown to induce a drag of a factor about 10 times larger in [Morris-Thomas and Steen, 2009].

The general discussion of academism and impressionism of this section gives me the opportunity to show some representations of flags, seen from the eye of the artist, see figure 1.6. If you go to the Louvres, you will see many flags, especially in paintings of sea battles. In some of them, the more attentive artist would have spotted the oblique waves. As a counter example, here an advertisement for the Eurostar tunnel, with Joan of Arc holding a british flag with perfect vertical waves. Below, you see a painting by Ingres "The source". A detailed look at the stream of water flowing from the Jar, shows that Ingres was not aware of the Rayleigh-Plateau instability segmenting liquid cylinders into droplets.

## 1.3 Pole vault

Christophe Clanet organized a conference about the physics of sports. He introduced me to the work of Joseph Keller on competitive running [Keller, 1974]. It impressed me as an expression of scientific thinking at its best and I was eager to follow him on this track. I had done some studies of beams in flexion with Sebastien Neukirch. The initiation of my interest in pole vaulting was linked to the question of how we can change the "direction of the kinetic energy", by using a spring, at theoretically zero energetical cost.

To introduce the topic, let us read the interview of Renaud Lavillénie (figure 1.7), just after his gold medal in the 2012 summer olympic:

> — When you jump 5.97m, you rejoice even before falling back down. Do you feel instantly that you have succeeded in your jump?
> — Most often, Damien (Innocencio, his trainer) doesn't even have to tell me what I have done. I have this feeling in me. I know for example when my run-up or my take-off are unexceptional. But I also know how to salvage a jump to go over. And this is an asset few people have. This enables me to make my jump more or less all the time and not to waste an attempt stupidly. This is also due to all the jumps I do in training, so I have a huge amount of markers.
> — To which other pole vaulter would you compare yourself?
> — To Thierry (Vigneron, last French world record in 1984 with 5.91m). French pole vaulting has more to do with feelings than other countries. But Thierry was always searching for the trick, he had the feeling. What he liked, for example, was jumping 5.70 with as many different poles as possible. And when you can do that, it means you have understood pole vaulting.
> — To be capable of always adapt?
> If you are able to adapt your technique to the gear you have, to the conditions, it is an immediate advantage. You're not limited to saying: "I can only use this pole if the weather is good, if I have the wind in my back, and if I am fit." After all, we are not asked to jump well or badly, but to get over the bar! Of course, to reach 5.90m or 6m, it's easier if you jump well (smile).
> — Do you feel yourself to be the heir to a long French pole tradition?
> Er... well, I don't know. One goes on carrying the torch, but I don't feel I'm following my predecessors. I do my bit in there and I carry on the tradition. What we have in common is this pole vaulting thing that enables us do be the first to do something significant. It's a kind of family. But we don't all have the same philosophy of pole vaulting, we don't train or jump the same way. We're all different.
> — So you don't jump à la française?



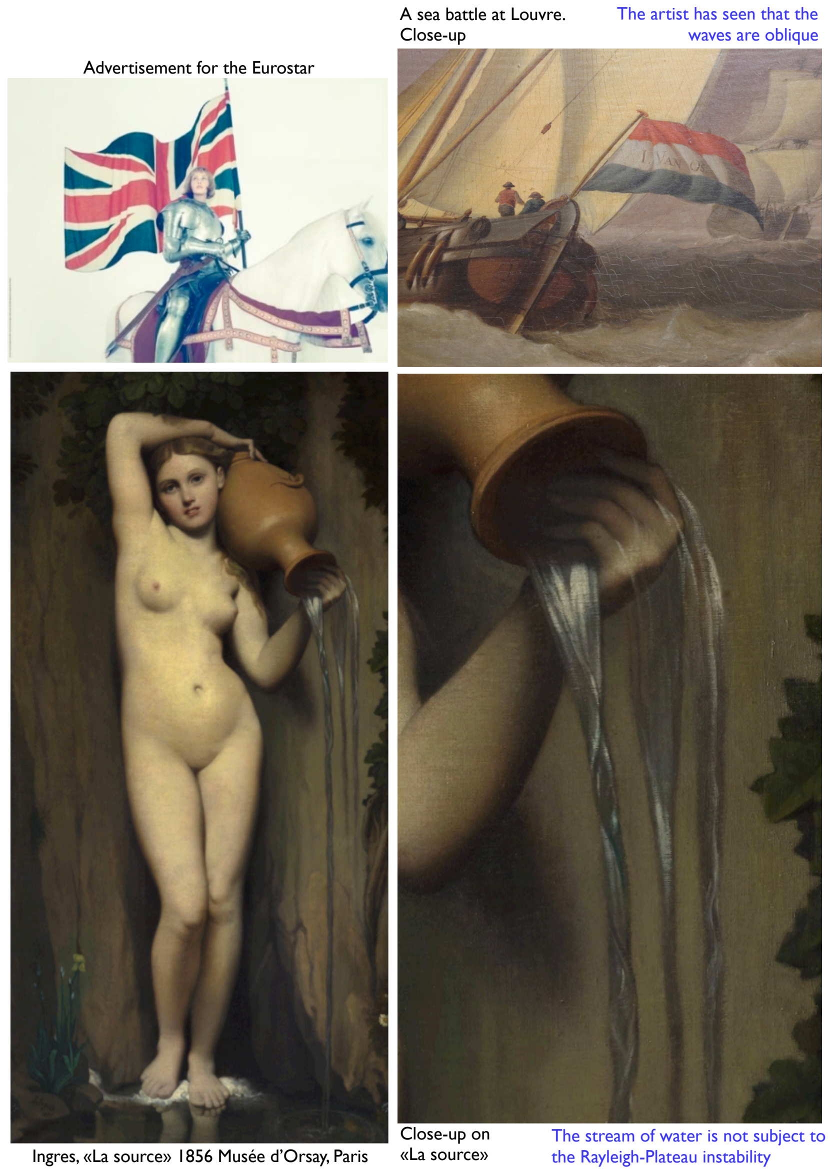



Figure 1.6: Artistic and commercial flags. Absence of the Rayleigh–Plateau instability in Ingres' "La source".


MONTPELLIER - TOULOUSE : 1-1

Le champion démarre au ralenti

(Page 20)

L'ÉQUIP

LE QUOTIDIEN DU SPORT ET DE L'AUTOMOBILE

À-HAUT

Figure 1.7: Front cover of french popular sport magazine "L'équipe", the day after Lavillenie's Olympic gold.

> This is a typical French problem. I was told not to jump like that, it wasn't beautiful, exactly as they said I was too small (1.75m) to ever hope jumping higher than 5.80m. I didn't give a damn! I knew I would pass 6m. I only wanted to be content as pole vaulting, that's what I like. We don't have to jump beautifully, but high. With Damien, we searched for a technique adapted to me and we found it.
> — It's no use copying Bubka...
> Certainly not! Bubka did 6.15m (inside) in a certain way, but if everyone tries to copy him, no one will do it. This is true in every domain. It's useless to try to do like Bolt, no one will manage it. You have to borrow from everyone and adapt it for yourself. Finally, my technique mixes various tricks, and the result is one of the best in the world. People used to say that a good sprinter is a small ball of nerves, if he measured 1.85m he should change sport. But Bolt (1.96m) changes everything; I change everything with a pole. This is the beauty of sport. Take Bubka, Hooker and me: we have completely different builds but similar results. This is probably why pole vaulting is possibly one of the most beautiful sport in athletics[1].

Following the interview, we have the analysis of Nicolas Herbelot:

> Glory! How is it reached? One or two millimetres only. The last attempt at 5.97m by Renaud Lavillenie, ultra favourite of a particularly random sport, raised him from bronze to gold, from disappointment to consecration. A perfect illustration of the uncertainty of pole vaulting. Clearly, luck has its place here, as ill luck had last year during the world championship of Daegu, when he went the other way. It is difficult to understand, when sitting in front of one's television screen, or even on the terrace of a stadium, what a mechanical high-precision sport is pole vaulting. No other athletic sport uses such equipment for its performance.
>
> Imagine Renaud's case: forty-five metres for the approach, twenty steps while holding a 5.10m pole weighing a few kilos that feel like tons. At the end of the approach, the radar of his coach, Damien Inocencio, indicated 34km/h. The adjustment of the approach is only one of the elements. The stiffness of the pole has its importance. Too flexible and it will not project you very far. Too stiff and it will not bend enough. Renaud has plenty of poles to chose from. And then, if he is happy with his pole and his approach but does not clear the bar, he can bring the standards backward or forward, this is an art.
>
> A mechanism finely tuned by training sessions during which Lavillenie may string together forty jumps, in every situation, with all the poles. Subtle impressions to compensate for an unusual build. He is only 1.77m for 70kg, while the other pole vaulters who passed 6m tend to be 1.88m high for 80.5kg. He is the proof that pole vaulting does not depend on sheer force but on technique. He is a little runt incapable of a 100kg bench-press but capable of running faster than everyone. And his take-off is a miracle. As King Serguei Bubka told him many times, its not size that matters, its technique, especially at take-off.
>
> This is where Lavillenie excels and flabbergasts his opponents. His last step is active. When the pole is planted, his feet have already left the ground. He throws himself under so as to better fly like a catapult. Once up there, having jumped so often, even in his garden when he was a kid, enables him to correct even the worst start. To pass. for one or two millimetres[2].

I take the occasion of this section on impressionism to discuss a model which can be for us a tangible tool to translate into graphics and numbers the content of this spontaneous expression of a historical performance. To make it as simple as possible, I do the following simplifications (The model system is sketched in figure 1.8):

- I replace the nonlinear elastic ingredient of the pole by a linear spring of length $L$. I thus neglect the fact that the pole has a critical Euler buckling load, the fact that it reacts differently to pure axial loading than to a combination of compression and torque. The fact that the planting of the pole is a strong shock on the pole linked with impact behavior.

[1] "L'équipe", saturday 11th of august 2012. Interview by Nicolas Herbelot and Marc Ventouillac. Translation Bernard Hoepffner.

[2] Translation Bernard Hoepffner.

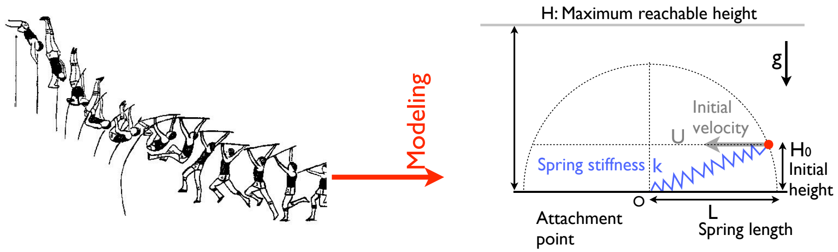



Figure 1.8: Modelling of the pole vault. Left: image from [Ganslen, 1961].

- I replace the athlete by a mass $m$ at initial height $H_0$. I thus neglect all muscular action; forces and torques applied at planting and during the flip. I neglect the fact that the center of mass of the athlete can move with respect to the tip of the pole. Still, we will see that I can account for the muscular effort by considering that the athlete can apply a finite power during the extent of the flight.
- I replace the run-up sequence by an initial horizontal velocity $U$. I thus neglect the fact that the athlete can jump before planting the pole (the typical take-off angle is 20 degrees).
- I replace all requirements on the sequence of passing the bar by the aim of a vertical velocity at time of recoil of the spring. I thus neglect the fact that the athlete still needs a horizontal velocity while passing the bar. I neglect the fact that he can pass the bar with his center of gravity in fact below the bar, just as for high jump.

This system is extremely simple compared to what is proposed in the scientific literature on pole Vaulting. Typically, published papers either seek to measure the features of real vaults (trajectories, forces, body motion…), or to reproduce the dynamic of the pole/athlete combined system as accurately as possible, using for instance finite elements on an flexible Euler beam, accounting for the shape of the body of the vaulter, with possibly several articulations and applied torques depending on time during the sequence of the vault.

For its jump, the athlete can chose its pole, and it has basically two parameters: how stiff? how long? To change the stiffness, he must switch pole, and to change the length, he can grip higher or lower a pole which is manufactured long enough to allow some freedom in that. As a central rule of the game, the hand cannot move up along the pole during the vault (forbidden to climb the pole).

Pole vault is a special discipline. Champions are typically much older than in other athletic disciplines because mastering requires a long experience of practice:

- it is a dangerous sport: falling from 5 meters high can interrupt a complete season of competition.



- The shock when planting the pole is violent (additional risk of injury).
- Each jump requires a lot of effort (run-up), so one session of training is not so many jumps.
- When after the run-up the athlete feels that he is ill-engaged, he will not complete the flip, so the effort of the run-up and planting the pole is lost without the compensation of gaining some practice in the most subtle part of the sequence.
- The athlete needs both a lot of strength for run-up and a lot of skill for flying.
- Since a progression of career is long, the selection of the future champions is delicate.

We use $k_0$ as a reference stiffness, the stiffness for which the initial kinetic and gravity potential energy is equal to the potential energy of complete compression of the spring. In figure 1.9, I show several trajectories for a given initial height. For each length of a pole, I show the best stiffness, that one yielding a vertical final velocity. For a short pole, the trajectory is quick, the stiffness is large (and the forces applied to the mass are large). To avoid these large forces, we increase the pole length and accordingly, we are able to decrease the pole stiffness. Doing this too much, we enter another regime when the pole is too long, and there exist no stiffness that yield vertical redirection. The spring pushes the mass down to the floor. We will see that the boundary of this regime is the most interesting feature of our model. These simulations correspond to a fixed height $H_0$ and initial velocity $U$, with the gravity $g$ chosen such that the initial energy corresponds to a height 1 in potential energy.

This graph is the quantitative equivalent for our model of the statement by Lavillenie that “Thierry was always searching for the trick, he had the feeling. What he liked, for example, was jumping 5.70 with as many different poles as possible. And when you can do that, it means you have understood pole vaulting.” This statement means that Lavillenie, just like Thierry Vigneron, have developed the ability to move on this one-parameter manifold displayed in the figure. When Lavillenie says that “French pole vaulting has more to do with feelings than other countries”, he means that instead of betting on strength: initial velocity and strong planting shock, french vaulters have the gymnastic ability required to take a longer and softer pole for which the flight is longer and the planting shock is a lesser impulsive loss of energy. A longer flight is much more dangerous. But Lavillenie says he is able to correct a jump during the flight: “I also know how to salvage a jump to go over. And this is an asset few people have. This enables me to make my jump more or less all the time and not to waste an attempt stupidly”. This means that he can compensate for his small size and run-up speed by his ability to control his trajectory while flying.

Indeed, the typical run-up velocity is 10m/s, which is equivalent to 5 meters in height. Since the center of mass is at about 1m high, this adds up to reach

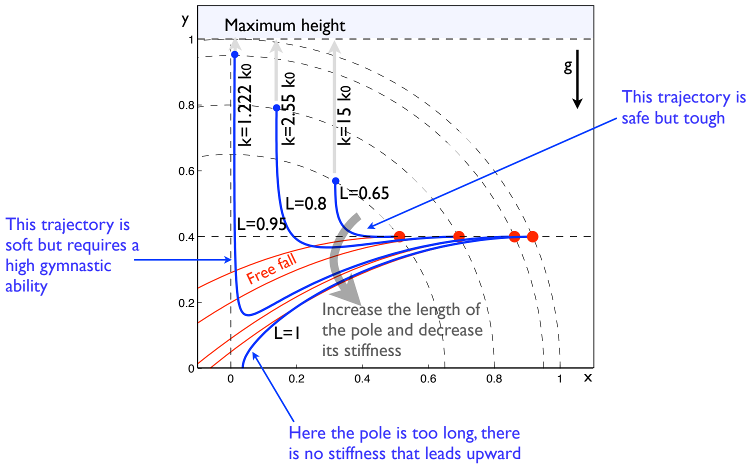



Figure 1.9: Several trajectories while varying the pole length and adjusting the stiffness for vertical reach.

the 6m of the record. But the shock of planting a stiff pole is received by the body in an inelastic way, yielding a loss of energy, the equivalent of about 2m/s of velocity. It means that the vaulter has to input this missing bit during the flight. If we consider that the vaulter has a given power, this means that the shock must be as soft as possible (meaning a pole as soft as possible), and the flight as long as possible (meaning the need of a high gymnastic ability).

I would like to represent more fully the phase diagram of this system. This is shown in figure 1.10 where we vary both the pole length and the pole stiffness and depict performance. I show four regimes:

- "Too soft", the spring is not able to redirect the mass and the vaulter fly below the bar.
- "Too stiff", the redirection is too quick and the vaulter is propelled back on track (the most dangerous regime).
- "Too long", when the vaulter falls onto the floor, pushed down by the elastic recoil of the spring.

I have drawn on the graph the red line of the optimal stiffness. This is the manifold on which Lavillenie moves when he tries to pass the bar with as many different poles as possible. Moving from right to left on this graph means that the pole becomes increasingly long. When getting to longer poles, you can see that the (light red) safe zone for good performance becomes increasingly thin, until a critical point where it is infinitely thin. This is the point of a *catastrophe*, the frontier between a region of parameter space where it is possible to use the pole to rise and a zone where this is nolonger possible.



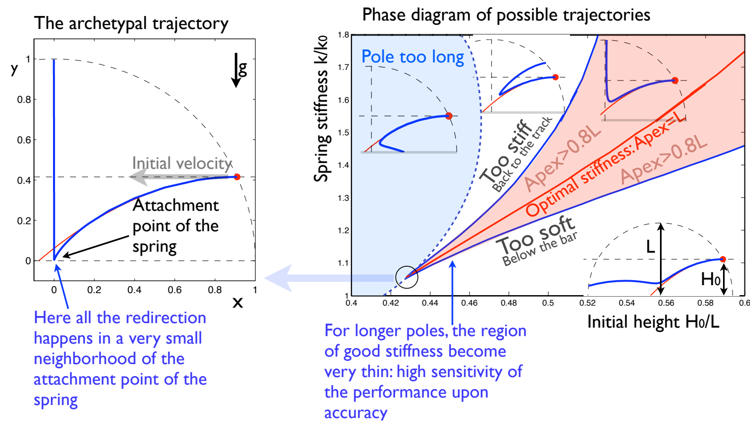



Figure 1.10: The phase diagram of possible trajectories, and the archetypal trajectory.

On the graph, I have drawn the example of a trajectory very close to this critical point. This corresponds to a stiffness near to $k_0$ but a little higher. For most of the trajectory duration, the mass follows the trajectory of free fall, and the combination of initial velocity and height is such that the trajectory comes in the very near proximity of the attachment point of the spring. There, it comes to a stop, then recoils vertically. We understand that this kind of trajectory would be impossible if the stiffness of the spring was a little less than $k_0$, that is, if the spring could not contain all of the initial mechanical energy.

In a way, this near-catastrophical trajectory is *archetypal* for the description of the system that we have at hand. This is the absolute most dangerous choice for a vaulter. The last possible choice. This catastrophe is the point of a qualitative change in the properties of the trajectory. It is a central structuring point of the phase diagram of the trajectories.

When Lavillenie practices the ability of jumping with as many poles as possible, he moves in parameter space between two archetypes of the possible trajectories. The first one is the very stiff pole which makes a strong shock and immediately redirect vertically the mass, and the second is this last trajectory. Off course, Lavillenie does not need to talk about these extreme cases. They are the mathematical objects that in their extremity embody the natures of the choices technically offered to the athlete.

In the article [Hoepffner, 2012], I follow this idea to find for a given athlete what would be the best trajectory. First I need to introduce the idea of imperfection of the jump: giving one of the parameters a stochastic uncertainty, for instance the initial velocity. Then, I quantify the advantage of a soft pole by giving the athlete a given power so that he is able to add energy to the vault during the flight: the longer the better. Then I vary the pole length while adjusting the stiffness at its optimal and record the expectation of reached height.

This is a tangible way to represent how a longer and softer pole is better for an athlete with good gymnastic ability (in the context of this model, gymnastic ability means a low variation of the initial velocity about its mean).

Off course, this simple model is a fiction of the mind, very far from the mechanical complexity of a real vault, and the human muscular and emotional involvement of the athlete. But interestingly, it is something light and tangible. The real vault is tangible, but hard to manipulate. The discussion of the model and its manipulations is a nice way to aggregate as much as possible from aspects of the athletic culture and history of the discipline. The comments on the features of this model is a nice alibi for talking about the real world. The two central parameters of the pole are its length and stiffness. For one value of the first, there is an optimal value of the other (at least, in the model, there is a clear definition of this optimality). This optimality leads to a single degree of freedom. Essentially a short and stiff pole gives a quick trajectory with a strong force. This trajectory is tough but safe. For a longer pole, the trajectory is long with a moderate force. This trajectory is soft but dangerous. In this sense, the model encompasses the dilemma which the vaulter is confronted with.



# Chapter 2

# "Propose a theory or suggest a new experiment"

> You are in a laboratory, you talk to a colleague, and suddenly, he shows to you a result which frankly is bizarre, and you don't see where this comes from. You go back home, you think and you try to understand. This is a great joy in some cases. Most often you come up with an explanation which is not correct; but *from time to time, you get one that works, or you suggest an other experiment* which will prolongate this process, which will tell us where we stand, and this is something absolutely extraordinary.

This is an excerpt of [de Gennes and Leach, 2005], tune 1 at time 3.33. In general, when you are shown an experiment and you want to make a model of it (a mathematical system that mimics the dependency between the parameters of the experiment). You are typically confronted to the following choice: on one end of the spectrum of possible models, you have a complicated model which reproduce accurately the data, and on the other end of the spectrum, you have a simple model which is rather more approximate. What you gain in simplicity (ease of understanding, clarity), you loose in precision. When qualifying a model, instead of talking of *simplicity*, I prefer to talk of *clarity*, because are clear things that you can "see through", things that the light can shine across.

In the following list of examples—just as suggested by de Gennes—I would like to point toward an alternative: instead of being stuck in the dilemma of accuracy versus clarity, we have the right to suggest a new experiment. This experiment will be chosen in such a way that a clear model will as well be accurate. If we call the experiment-model couple a *system.* Then we could call such a system an *archetype* of our phenomenon.

## 2.1 Gliding drop versus sliding coin

I start with an example of an archetype which de Gennes describes in the follow-up of the previous quote:

> I will tell you about a very factual example. It's not in a lab, but still, it's a bit the same. A colleague sends me a paper which he wants to submit to the PNAS, of the american academy. I leave this paper alone for a while, I don't like at all to work for PNAS because it asks for a huge editorial work. Then, almost by chance, I have a new look at it, and I see that he has a plate on which he vibrates horizontally a small droplet, and he uses non-symmetric vibrations—that is—instead of just being some simple wave, it has a peak, followed by a lesser trough. In such conditions, he observed that the drops would go to one side. Then, curiosity starts, and this curiosity is amplified by the fact that he really did not understand what was going on.
>
> He made a completely ludicrous explanation, and me, I was convinced that there was a simple explanation, based on the fact that when a drop shakes, it does not unpin immediately. There is some hysteresis in the wetting. From that moment on, we started to have a great fun. You see that initially, it comes from curiosity—there is something really surprising—and after that, you suggest some other experiment, and in this case, we suggested an experiment that was really much simpler.
>
> You put—that was really fun—a coin of one cent of euro on a plexiglass plate, you vibrate the plate, and the coin starts to move toward one direction. . . with laws which by the way are subtle; still we spent three months to understand, because this is what people call dry friction tribology, it's really nonlinear, you have a friction force which appear at some threshold, changes sign when the velocity changes sign. It's a really nonlinear phenomenon, not completely simple, but we got a lot of fun out of that, and I find that this is a good example for the different stages that we all know, somehow. That is: you see something strange, you try to understand, you propose to tackle it a new way, sometimes simpler, and finally you get some result.

Here, the phenomenon which is surprising is the fact that the excitation is periodic but the response has a nonzero mean. The essential ingredient from the drop comes from the contact line, which tends to remain pinned on solid surfaces when they are not extremely clean. First, the original author "had made up an explanation completely ludicrous", so there is clearly a call for something better. In addition to this, the flow inside of the drop is not at all trivial, the behavior of the contact line is complex and even the shape of the drop is changing, so it is a hard thing to make a model for this experiment as



it was originally designed. So the idea is to replace a complex object: the drop, with a simpler one: the coin. The coin does not change shape and there is no flow in it. The analogue to contact line hysteresis of the vibrated drop is the nonlinearity of the static/dynamic dry friction of the coin on its vibrated plate. A new experiment is suggested for which a clear model can be as well accurate.

## 2.2 Peristaltic pumping

During my post-doc in Japan, I did some work on the peristaltic pumping. This is the pumping happening for instance in the guts, induced by the motion of the walls in the form of a traveling wave. The original motivation was flow control. In [Bewley, 2001] Thomas Bewley (I spent four months working with him in San Diego during my PhD) proposed the conjecture that even though you try to act on a flow by blowing and suction at the walls, you will never be able to flow a fluid through a pipe with less pumping energy than needed for a Poiseuille flow. Blowing and suction at the walls is a nice way to control fluids because it is easy to do in numerical simulations by simply changing the wall boundary conditions. His hope was to say that the best thing you can do for a turbulent flow is to try to relaminarize it, you cannot do better than the laminar flow. This would have been good to him because he had run numerical simulations in which he succeeded to do a relaminarization.

Unfortunately, [Min et al., 2006] gave a counter example to his conjecture. They had a plane channel flow with a given pressure gradient and thus a Poiseuille flow. Then they actuate the system with wall blowing and suction in the form of a wave traveling upstream. They showed that they could this way increase the flux pumped through the channel. This left us with the question: what is the mechanism of this actuation, how come this type of actuation, against Bewley's intuition could induce a pumping effect?

The topic of my post-doc was to understand this pumping. [Luchini, 2006] already had the idea that this pumping effect was linked to peristalsis[1]. Here, instead of having wall deformation, there is wall blowing an suction, but by analogy one could try to pin down the mechanism. The most puzzling effect was the following: peristalsis induces a pumping in the direction of the wave whereas blowing and suction induces pumping in the direction *opposite* to the wave. This shows that the mechanism are different (opposite?). This is shown by the instantaneous velocity field on figure 2.1.

This opposite direction is paradoxical[2], it escapes intuition to such an extent that in the paper that exhibited the counterexample to Bewley's conjecture, we read:

> Finally, the current control scheme, consisting of surface blowing and suction in the form of traveling waves, is mathematically simple [. . .], yet it may not be straightforward to implement in real flows.

[1] "peri" for the periphery of the pipe, and "stalsis" for contraction.
[2] *para*:against, *doxa*: the common opinion.

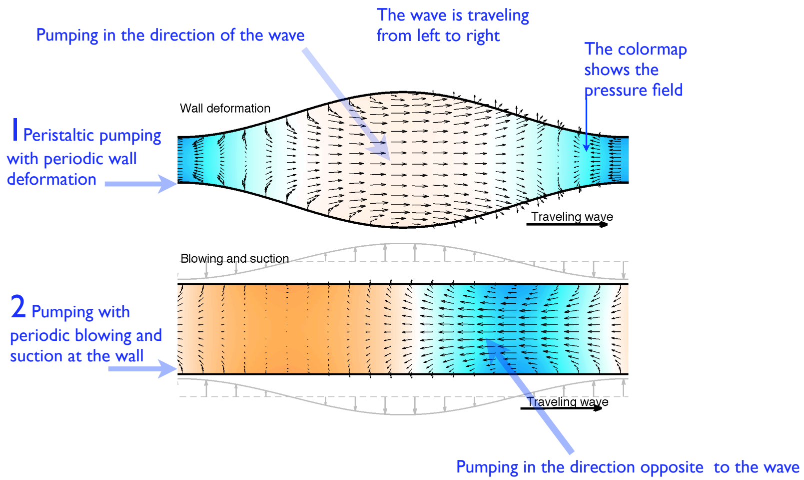



Figure 2.1: Comparison between the wall-deformation peristalsis and the blowing and suction pumping.

> [...] However, a moving surface with wavy motion would produce a similar effect, since wavy walls with small amplitudes can be approximated by surface blowing and suction. We plan to perform simulations over moving wavy walls.

So you see, we really need to make a clear model. Clear models are such stuff as intuition is made on[3].

> You want to look closer at an insect?
> - Take a magnifying glass.
> You want to look closer at a mechanism?
> - Make a model!

Here I will not extend overall upon our results, please see [Hoepffner and Fukagata, I just wish to exemplify "propose a theory or suggest a new experiment". I do this here for a model of the pumping induced by peristalsis in absence of an external pressure gradient: the flux induced by only the wall motion. First let us see what the mechanism of pumping is. The first step to understand what happens is to imagine a flexible pipe which you pinch between your fingers and slide the pinch along. Since you first chose to pinch hard, the pipe is now locally

[3]Our revels now are ended. These our actors, as I foretold you, were all spirits, and are melted into air, into thin air: and like the baseless fabric of this vision, The cloud-capped towers, the gorgeous palaces, the solemn temples, the great globe itself, yea, all which it inherit, shall dissolve, and, like this insubstantial pageant faded, leave not a rack behind. We are such stuff as dreams are made on; and our little life is rounded with a sleep.
Shakespeare, The Tempest, Act 4, scene 1



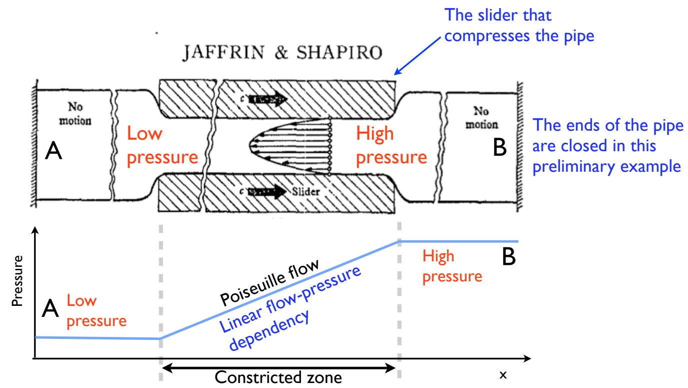



Figure 2.2: Sketch for the pumping mechanism of peristalsis. Image adapted from [Jaffrin and Shapiro, 1971].

occluded and the fluid is forced to move along as you slide your fingers. This is the extreme case of peristalsis. In this limit, it is easy to tell what the value of the flux is.

To understand what happens before this extreme case of complete occlusion, please see the sketch on figure 2.2. A flexible pipe is filled with a fluid at rest. A slider compresses the pipe to a lesser diameter. When you slide the slider, since occlusion is not complete, there is a back flow. If your slider is long enough and the fluid viscous enough, the flow will take the form of a Poiseuille flow: a parabolic velocity profile and a linear decrease of the pressure downstream. We can thus tell what will be the pressure jump between upstream and downstream of the slider. The pressure is large at the right and low at the left. If now the channel is not closed at his ends, this pressure will induce a flow in the non-constricted part of the channel, going in the same direction as the slider.

This is the mechanism as it is described in the literature. Then, once this description done, you want to get a formula for the flux as a function of the severity of occlusion, typically for the low Reynolds numbers of the fluids that are typically pumped using peristalsis. For this, the approach in the literature consist in a completely different approach: they take the Stokes equations, assume a varicose deformation of the walls in the shape of a sinus, assume a long wavelength for this wall deformation, and perform the algebraic manipulations to finally get the formula:

$$Q = 3\phi^2/(2+\phi^2),$$

where $\phi$ is the occlusion: $\phi = 0$ is a straight channel and $\phi = 1$ is an occluded channel. For complete occlusion, $\phi = 1$ and $Q = 1$. Here $Q$ is properly dimensioned with the channel height and the velocity of the wave.

I was not satisfied by this formula, because I felt a discontinuity in the process of its derivation. First, one gives a very clear account of the mechanisms

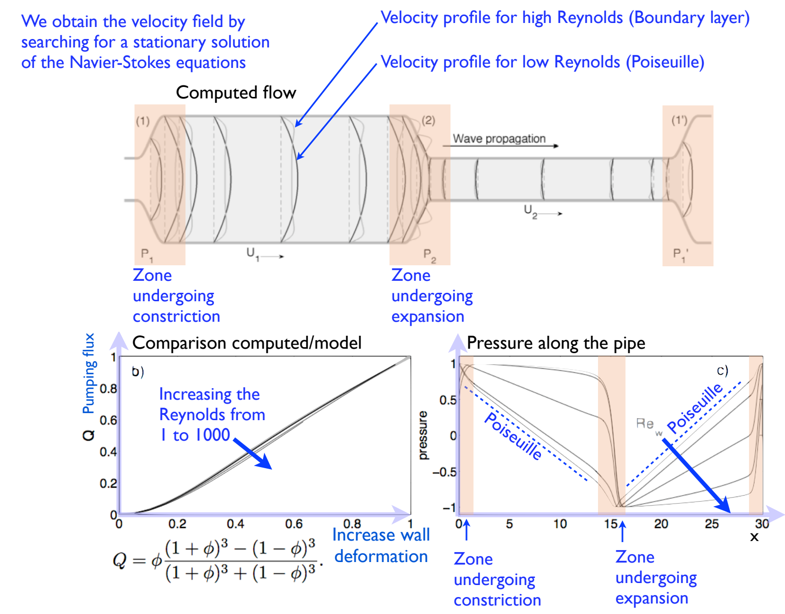



Figure 2.3: Quantitative validation of the piecewise-continuous channel for peristalsis.

for pumping in terms of a known and very classic example: the Poiseuille flow, then one suddenly jumps to the first principles of Newtonian mechanics, take a fundamental equation, modify the geometry in a way that simplifies the mathematics (a sinusoidal wave) and then get a formula. Doing so, the formula is disconnected from the mechanism—from the understanding of what is actually responsible for the pumping. Most importantly, the quantitative agreement between the formula and a numerical simulation or an experiment will not be a validation of the mechanism, it will be a validation of the Stokes equation.

In order to obtain a quantitative model to validate my understanding of the mechanism, I suggested to modify the experiment: a different geometry. This geometry would be very cumbersome if manipulating the Stokes equations, but it is very convenient for the steps that we will follow together. The new flow geometry is shown in figure 2.3.

Instead of assuming a wall deformation in the form of a sinus, I assume a wall deformation in the shape of two different channel heights, connected by two short converging and diverging sections. Then, this wall deformation is set into motion to the right. This is simply the slider configuration, made periodic. The modeling goes as follows. I assume that the flow in each straight section is a Poiseuille. The pressure thus varies linearly along the length of each section, and the slope of this pressure distribution is related to the fluid viscosity (which



is the same for each part of the channel), the local flux and the local pressure gradient. Then, we just need to account for the connection of the flux and the pressure between the different sections to get a quantitative account of the flow in this geometry. This gives the formula (please see the details of the derivation in [Hoepffner and Fukagata, 2009])

$$Q = \phi \frac{(1+\phi)^3 - (1-\phi)^3}{(1+\phi)^3 + (1-\phi)^3}.$$

In figure 2.3, I compare the formula to a numerical simulation of the Navier-Stokes equations (in the reference frame moving with the wave, the flow is steady if it is stable, and we compute the flow field and flux by converging iteratively with the Newton-Raphson method toward the steady solutions of the Navier-Stokes equations in this deformed domain). The agreement is good, even for large Reynolds numbers. We show the velocity profiles of the flow for Re=1000 (based on channel height and wave velocity). The flow profiles are no longer parabolas, but clearly wall boundary layers. This indicate that the same formula could be derived by accounting for the pressure variation using thin boundary layers along the walls of the straight sections instead of parabolas (yet to be done...). The nice agreement between the formula and the numerical simulations also shows that the details of the velocity/pressure fields in the diverging and converging section is either not important, or cancels out.

Here, I have taken the freedom to change the experiment. Not because the original model was not accurate nor because it was complicated. I did so because I wanted to use the elements through which I got my understanding of the phenomenon as the building steps for my model. The quantitative agreement between data and a formula is a validation of the understanding. In most of the cases, we nolonger need to validate the Stokes equation nor the Navier–Stokes equations. But on the other hand, my understanding in terms of the slider and the two Poiseuille flows: one flowing backward and one flowing forward, needs to be validated. In §3, I discuss in more detail the use of quantitative comparison, commenting Rutherford's statement that *qualitative is nothing but low quality quantitative.*

## 2.3 Liquid segmentation

Later in §4, I will discuss in length atomization: *the process of transformations from a liquid body to a cloud of droplet.* Now, I would like to have a first look at the question of liquid segmentation. Surface tension is a force acting at the interface between immiscible fluids like air and water. This surface is such that a liquid drop behaves to some extent like an elastic balloon. The force is that of a *tension*, so we may at first believe that it will always gather fluid. But since the liquid body is a 3D volume and its tensed boundary a 2D surface, the tension may as well induce segmentation.

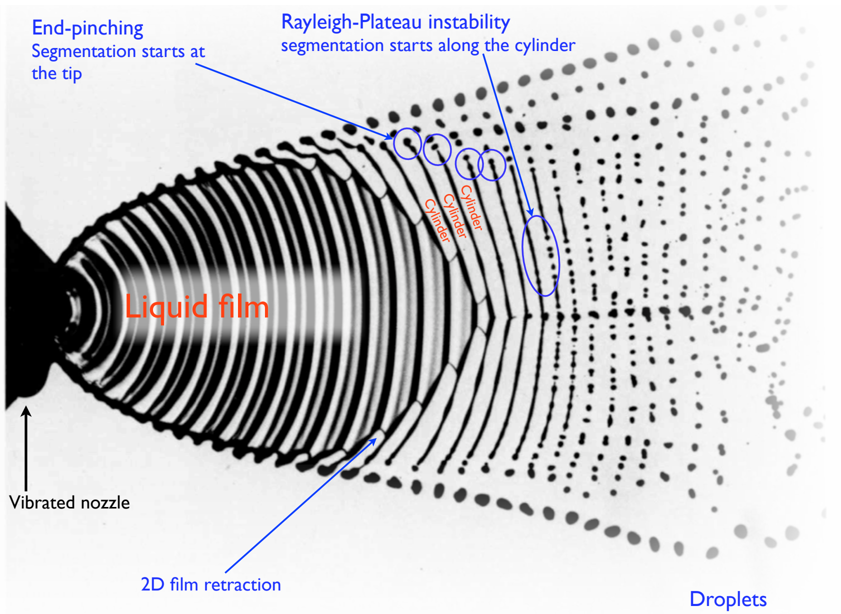



Figure 2.4: The ways to make drops from liquid cylinders exemplified on a vibrated film atomizer. Image adapted from [VanDyke, 1982].



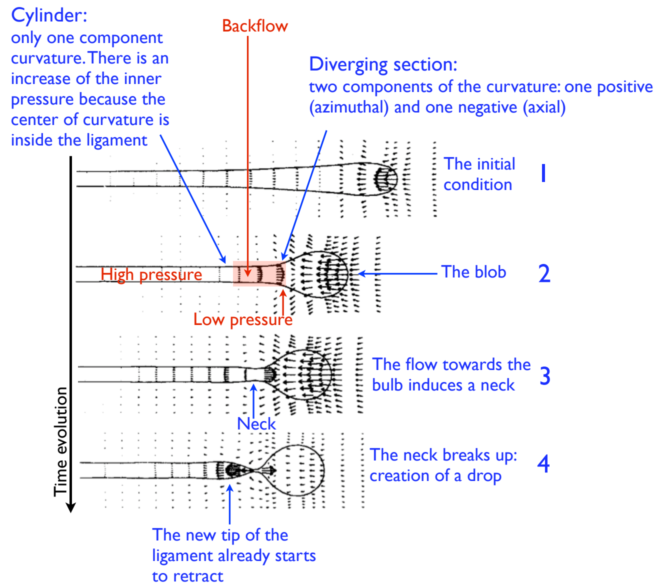



Figure 2.5: The end-pinching mechanism for creation of the neck. Image adapted from [Stone and Leal, 1989].

To probe for the cohabitation of two such opposite behaviors we seek the simplest system for which we can draw a boundary between gathering and segmenting. We take a sphere: pure gathering. If we “elongate” the sphere, we approach the cylinder for which there is already an archetype of segmentation. It is the Rayleigh-Plateau instability: an infinite cylinder of water is unstable to varicose deformation whose wavelength exceed its perimeter. Let us see what archetype can be inserted between the sphere and the cylinder. Figure 2.4 shows an atomizer where we see spheres, cylinders and the recessing tips of cylinders.

### 2.3.1 Retracting ligament and end-pinching.

We call the “elongated sphere” a ligament. At the tip of the ligament, surface tension induces a recession of the tip. The liquid of the ligament is progressively gathered in a blob of the recessing tip. This experiment in the context of viscous fluids was one of the famous experiment of G. I. Taylor. He was interested in understanding the processes of creation of an emulsion (mixing vinegar and oil). His system and experiment was to let a drop be extended in an extensional flow produced by rotating four cylinders in a bath of viscous fluid. When the flow is started, the drop first extends, and finally segments. What happens next was the central topic of the PhD thesis of Howard Stone. He was rather interested in the recession of the tips after segmentation and his experiment was to extend a drop using the four row mill, and then stop the extensional flow and look at the recession in a fluid at rest. He observed that segmentation would happen at the tip of the recessing ligament rather than along the cylinder. He did a surprising observation: the blob of recessing fluid is connected to the yet untouched ligament by a neck. This means that before reaching the blob, the flow has first to accelerate at the neck then decelerate.

He gives a nice explanation for the presence of this neck, this is described in figure 2.5. Essentially, the flow is dictated by the pressure variation along the ligament, which itself is enforced by the capillary pressure jump through the tensed interface. Where the ligament is cylindrical, there is only one component of curvature of the interface: $1/R$ where $R$ is the ligament radius. Where the cylinder meets the blob, there is a second component of curvature, with a center of curvature located outside of the liquid, which means that this induces a decrease of the local pressure. Since the pressure is less there than in the cylinder, fluid is sucked toward the opening of the blob, which means that there will be less fluid in the cylinder, which makes a neck. This neck is the good zone then for the Rayleigh-Plateau mechanism to act and segment the blob from the cylinder. Stone has coined this mechanism "end-pinching", showing that it was more relevant than the Rayleigh-Plateau instability itself to predict wether a finite ligament will segment or not. Segmentation will preferentially happen near the tips.

We have performed numerical simulations of this process for fluids of low viscosity in a range relevant for atomization and we found something unexpected. For some range of flow viscosity, just at the time when the process described above would have led to segmentation, we observed that the flow through the neck would detach into a jet and the neck would reopen. The sequence is described on figure 2.6. The retraction of a semi-infinite liquid ligament is entirely described by the Ohnesorge number

$$Oh = \mu/\sqrt{\rho\sigma R}$$

with $\mu$ the viscosity, $\sigma$ the surface tension, $\rho$ the fluid density and $R$ the radius of the ligament. The Ohnesorge number is the inverse of a Reynolds number based on the velocity of retraction of the tip (known as the Taylor-Culick velocity)

$$U_{tip} = \sqrt{\sigma/\rho R}.$$

This observation was the subject of a paper, [Hoepffner and Paré, 2013]. Our understanding for the escape from segmentation was the following:

1. The retraction creates a blob which gathers the liquid.
2. Because of Stone's effect, there is the creation of a neck
3. The flow has to accelerate in the converging part of the neck and decelerate in the diverging part of the neck.
4. The acceleration of the fluid decreases the pressure right at the neck, which increases the rate of closing of the neck (even more than just Rayleigh-Plateau).



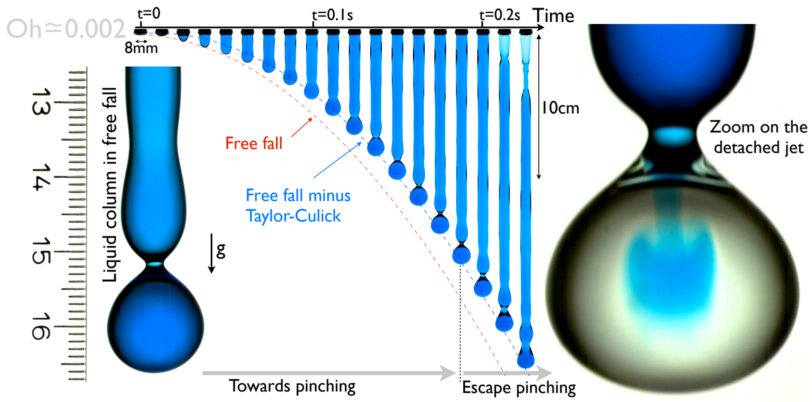



Figure 2.6: Free-falling liquid column experiment showing the escape of pinch-off.

5. The jet has a thin viscous boundary layer along the free surface (the familiar boundary layer is along a solid wall where the velocity is zero; here, this is a boundary layer along a free surface where the *stress* is zero).
6. This boundary layer can detach from the free-surface, just like the usual boundary layer can detach from a solid obstacle.
7. The high pressure downstream of the neck cannot be recovered (head loss).
8. The inner pressure downstream of the neck can nolonger counteract the pressure of the tensed interface around it.
9. The fluid from downstream of the neck is expelled back to the neck which thus reopens,

and this leads to escape from segmentation.

We performed some experiments where we wanted to demonstrate the reality of the jet of boundary layer detachment. We found a way by stratifying the liquid column with clear and dyed water, such that a jet of blue water would detach into the blob filled with clear water right at the time of reopening of the neck; please see figure 2.6.

Our paper was mostly observational. We have run a series of simulations varying the Ohnesorge number and showed that there was a critical value of it below which this mechanism is too weak to prevent the segmentation ($Oh_{crit} \approx 0.0025$) and we showed as well that this process can happen several successive times, leading to a periodic oscillation of the radius of the neck for viscous fluids. Please see the paper for more details [Hoepffner and Paré, 2013].

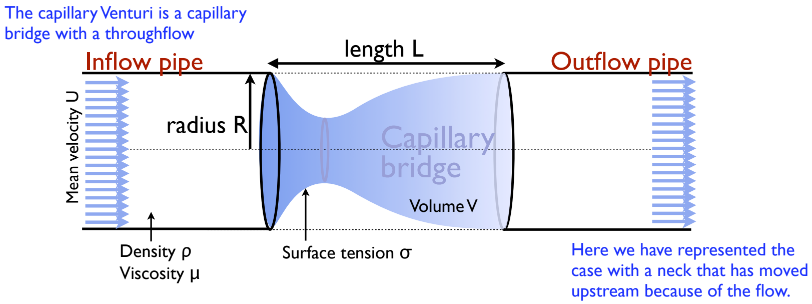



Figure 2.7: Sketch of the capillary Venturi.

### 2.3.2 The capillary Venturi

This system of the retracting ligament was too complex and our efforts to make a simple model failed, so we chose to change the system into something that could lead to a quantitative description. We thought that the essential ingredient for the mechanism was that the flow has to accelerate down the neck and then decelerate. This gave us the idea of the *capillary Venturi*.

The Venturi flow is a famous flow: pump a fluid through a pipe with a local constriction. Since the velocity is high at the neck, the pressure is low. You can use this to make for instance a vacuum pump, by connecting a pipe to the neck which will be depressurized. This is used for instance in old perfume vaporizers (the kind that my grandmother used). The perfumed is sucked up from its container into the neck, and from here atomized into the jet of air (so you see, it is an other link with atomization ;-).

To study the impact of the Venturi flow on the capillary surface, we took the classical system of the capillary bridge and instead of having the bridge between two discs like Plateau did in his historical experiment [Plateau, 1873], we had the capillary bridge between the ends of two pipes, and we would run fluid through it. When the bridge has a volume less than the cylinder occupied between the two pipes, there is a neck, and this is a capillary Venturi. The flow configuration is sketched in figure 2.7.

We wrote a paper on this flow, [Paré and Hoepffner, 2015]. And this indeed turned out to be a useful simplification because we could do several degrees of modeling to reproduce its behavior. There are two relevant parameters for this system. First the aspect ratio of the bridge; this is important because we know from the Rayleigh-Plateau instability, that the cylindrical bridge will become unstable if its length is larger than its perimeter. The second parameter is the volume of liquid compared to the volume of the cylinder that encloses it, $V/V_0$. When this volume is progressively reduced, for instance by sucking fluid through one of the ends using a syringe, the bridge reaches a minimum volume below which there is no longer a static solution. The nonlinear branch of the static solution has a fold bifurcation.



In the retracting ligament, the escape from breakup is due to the detachment of the jet that goes through the neck in a sudden event that disrupts the pressure balance. One of the original aims of jumping from the retracting ligament to the capillary Venturi was to be able to observe a static detached jet through the neck. We thought that the presence of the two pipes upstream and downstream of the capillary surface would overall make this system more rigid and that we could observe the same detachment as in the solid pipe Venturi flow. Indeed we have observed such stable configuration in some preliminary experiments, but we have observed several other things that we first took time to dwell upon before going to the detailed study of our original motivation.

These interesting behavior are showed on figure 2.8. We show two cases of a bridge with aspect ratio (length over radius) equal to 4, and run simulations of the Navier-Stokes equation while increasing quasistatically the throughflow. If the volume is low, the bridge breaks at a given critical velocity, and on the other hand if the volume ratio is close to 1, instead of breaking, the bridge shows a nonlinear orbit and breaks up at a larger critical velocity. To exhume the skeleton of this flow, we have computed using nonlinear continuation, the bifurcation diagram of a 1D model of this flow, the system of equation from [Eggers and Dupont, 1994]. This model is obtained like the Saint-Venant equation for water waves: assuming a long wavelength, but keeping the exact expression for the pressure jump through the curved interface:

$$\begin{array}{l} u_t = -uu_x - \frac{p_x}{\rho} + \frac{3\nu(r^2 u_x)_x}{r^2} \\ p = \sigma\left[\frac{1}{r(1+r_x^2)^{1/2}} - \frac{r_{xx}}{(1+r_x^2)^{3/2}}\right] \\ r_t = -ur_x - \frac{1}{2}u_x r \\ u(0) = U, r(0) = R, r(L) = R. \end{array} \tag{2.1}$$

There is a very nice agreement between this bifurcation diagram and the simulations. The model assumes a long wavelength so the agreement deteriorates for shorter bridges (for instance $L/R = 2$).

In this study, we did two observations that are of primary interest to get more understanding of the escape. First, for lengths 3 and 4 and volumes close to 1, just at the time when we would think that breakup is going to happen, suddenly, there is an escape and the neck reopens, and the downstream pipe ingests gas. Secondly, I think that we can explain the nonlinear orbit thanks to the mechanism of escape: the branch followed by the simulations is Hopf-unstable but the simulation has a finite orbit, so what is the saturation mechanism? The saturation is the intensification of the viscous boundary layer along the free surface and nearly detachment at the neck. Because the instability is oscillatory, the neck then shrinks and reopens. Just as for the escape of the retracting ligament, we have the detachment playing the role of reopening the neck. Here this is done periodically and takes the shape of a nonlinear self-sustained oscillation. In figure 2.9, you see two cases where breakup seems finally unavoidable. The left one indeed breaks, and the right one escapes.

To be more convincing with this scenario of the importance of the escape for the nonlinear oscillation, we currently develop with Pierre-Yves Lagrée a

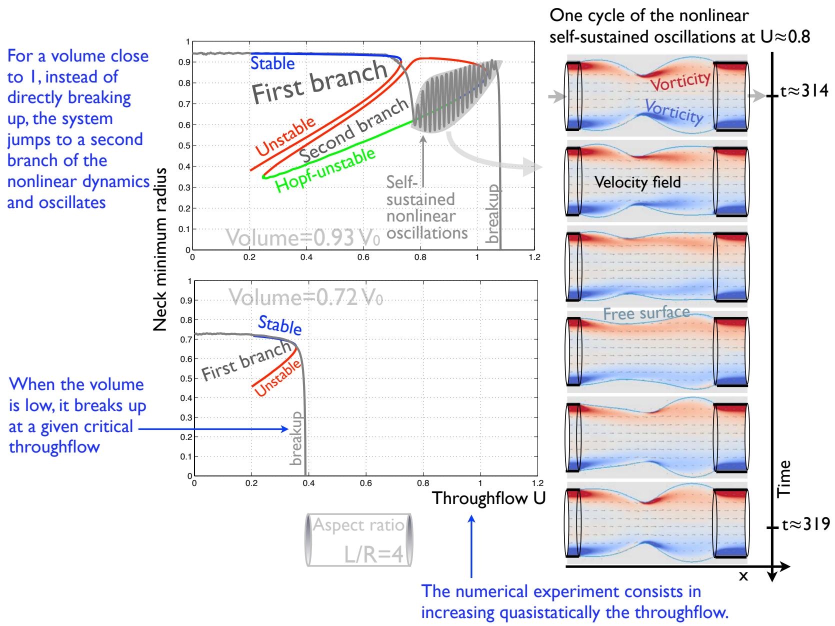



Figure 2.8: Capillary Venturi: comparison of the numerical simulations and the nonlinear branches.

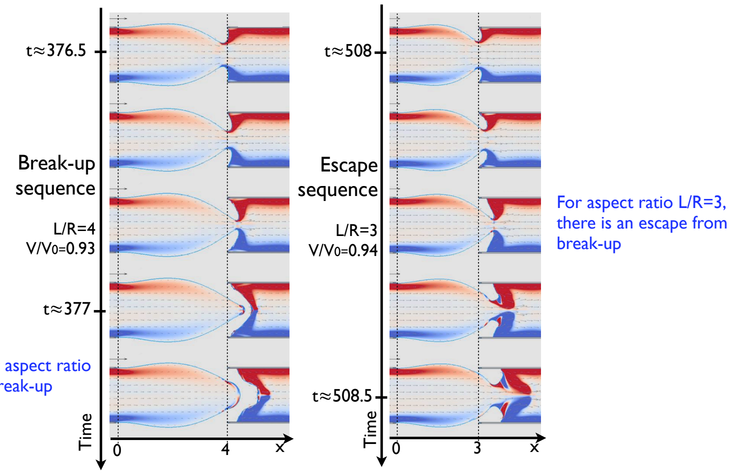



Figure 2.9: Final breakup of the capillary Venturi, and escape from this breakup.



composite model of the flow with boundary layers. The core of the flow is inviscid (described by a potential) and the flow near the free surface is described by a viscous boundary layer. This system will be triply interactive. Considering for instance the bottom interface:

1. the upper condition on the boundary layer will be set by the core flow and
2. the potential flow region is bounded by the displacement thickness of the boundary layer (these two are the classical ingredients of the interactive boundary layer).
3. The geometry is set by the free surface, in a way which is sensitive to the pressure along the axis of the bridge (and thus sensitive upon the potential flow).

Some preliminary tests show that the third ingredient can lead to unexpected behavior, for instance, the dramatic increase of the surface curvature right at the place where the detachment of the boundary layer happens.

Here I talked about the capillary Venturi from the aspects which illustrate the idea of "propose a theory or suggest a new experiment". In the retracting ligament, the Venturi was changing in time. The capillary Venturi is a more controlled geometry. As said, we did not yet go all the way toward the original motivation of this experiment. Instead, we spent some time on these observation. Indeed we should always welcome unexpected interesting results.

## 2.4 Archetypes

To escape from the dilemma of accuracy versus clarity, we may come back and change the original experiment. Once you have got entangled in the steps of building a model for a given experiment, you start to get a feeling of what are the reasons that make your model complicated. You are thus in a position to suggest modifications that will prevent these difficulties. During this process, which is typically iterative, you may converge to a system for which your model is both accurate and clear. In this process of refinement, off course, you should not have modified the system such as to loose the phenomenon of interest[4]. The process is iterative and tortuous, but can be successful[5].

For instance for the gliding drop, the phenomenon is the periodic forcing resulting surprisingly in a non zero mean displacement of the drop. This is kept when replacing the drop with a solid coin. For the peristaltic pumping, we have replaced a sinusoidal wave with a piecewise-constant deformation. The mechanism for pumping, originating from the fact that it is more difficult to flow (upstream) in a thinner channel, is kept in the new geometry. For the end-pinching, we have replaced the neck of the retracting ligament with a capillary

[4] En français: "perdre le bébé avec l'eau du bain".

[5] Success consists of going from failure to failure without losing enthusiasm (Winston Churchill).

bridge with throughflow, and we show that the escape mechanism was still an active part of the system.

The fact that we need to modify the questions we encounter is probably why the work of a scientist is to "research" and not to "find":

> — What do you mean?
> — That most of the questions that you ask yourself at the start of a research lead you to dead ends, and that you end up realizing that the question is too complicated or simply ill-posed and does not have an answer! Then you have to reconsider it, modify its elements, again and again, until you emerge on a path that takes you to the goal. That would be a possible definition of science: "the art of transforming questions until they have an answer"[6].

We find another expression with Carl-Gustav Jung, that of *overtaking* rather than *solving*:

> Indeed, I learned on the way that the most vital problems are in fact all unsolvable, and they must be so, because they express the necessary and intrinsic polarity to any self-regulating system. They can never be solved, but only overtaken[7].

Off course, the idea of modifying the original experiment is not new. For instance Rayleigh gives a vivid metaphor for the risk of "blind experiments":

> Experimenters on this, as on other subjects, have too often observed and measured blindly, without taking sufficient care to simplify the conditions of their experiment, so as to attack as few difficulties as possible at a time. The result has been vast accumulations of isolated facts and measurements which lie as a sort of dead weight on the scientific stomach, and which must remain undigested until theory supplies a more powerful solvent that any now at our command[8].

We also know the famous quote from Einstein that "a model should be as simple as possible, but not simpler". Here again the notion of simplicity, but with a new idea: the warning against an excess of it. In the Oxford american dictionary, we find a definition for simplicity:

> —The quality or condition of being easy to understand or do: *for the sake of simplicity, this chapter will concentrate on one theory.*
> — the quality or condition of being plain or natural: *the grandeur and simplicity of Roman architecture.*
> — a thing that is plain, natural, or easy to understand: *the simplicities of pastoral living.*

[6] [Lévy-Leblond, 2014].
[7] [Jung, 1979] page 32.
[8] [Rayleigh, 2009] page 33.



Maybe, when it comes to Euclid's geometry and Newton's mechanics, it is the analogy with Roman architecture that would be the least inappropriate, but then why fearing an excess?[9] The risk with "simplicity" is identification with *easiness* which is inappropriate for a theory. Recall by the way that the "most simple" does not necessarily have to *be* simple. The one dimensional (vertical) axis from easy to difficult is not the proper scale at which to measure the appropriateness of a system as representative of a class of phenomena.

The idea of an *archetype* relieves us from the qualification of simplicity. In our iterations between an experiment and a model, we have converged to a "triple point". The archetype is the meeting point for technical simplicity, cognitive clarity, and phenomenological generality.

The Cartesian point of view on the description of the world is that we should cut it down into the shortest list of most general principles, called *first principles.* One of these first principles for Newtonian mechanics is that for a material point, the acceleration times the mass is equal to the applied forces. To describe the gliding drop, the pumping and the escape mechanism, we could have gone back to this first principle to obtain the quantitative description of the phenomenon. Instead, we chose to refer to "secondary principles", like for instance the Poiseuille flow and the static capillary bridge. Indeed, the Poiseuille flow and the capillary bridge are archetypes is their own right. And in fact, the material point itself, in addition to being a first principle, is also an archetype:

> According to Newton, physical phenomena must be interpreted as the motion of material points in space; motion which is controlled by laws. The material point: this is the exclusive representative of reality, whatever nature's versatility. Clearly, the perceptible bodies have given birth to the concept of material point; people would think of the material point as an analogue to mobile bodies, by suppressing in these bodies all attributes of spatial extent, shape, orientation, that is, all "intrinsic" characteristics. You would keep inertia, translation, and then you add the concept of a force. Material bodies, *psychologically transformed* by the creation of the concept of a "material point", must from then on themselves be conceived like systems of material points. Then, this theoretical system, in its fundamental structure, can be seen as an atomistic and mechanical system. Thus, all phenomena must be conceived from the mechanistic point of view, that is, simple motions of material points, which follow Newton's law of motion[10].

A model should be a quantitative description of our phenomenon. This is discussed in §3. But it should as well be *clear.* The need for this clarity is expressed by the idea of *psychological transformation* from the quote above. If any mechanical system can be transformed (digested) in the mind of the

[9] and what of "pastoral living" for Einstein's relativity? probably the fact that it happens in *fields*;-)
[10] [Einstein, 2009], page 220

observer into a system of material points, it implies that the model should be expressed in terms that are clear to the mind: can be *assimilated*. This does not mean that the model should be "easy". For instance, our native language and our writing system are perfectly adapted to our cognitive system, but require a long apprenticeship. It is more straightforward to define technical simplicity than cognitive clarity, but I believe that the scientist, in its daily intercourse with obscurity builds for himself a very acute sense of the relief associated with clarity.

Note by the way, that in fact for the description of our fluid flows, we did in fact *come back to the first principles of mechanics*, we could do this thanks to the computers. Indeed we have solved the Navier-Stokes equations for comparison with our models. Coming back to de Gennes' analogy in §1.1 between photography at the time of the impressionists and the computer today, the material point of Newton is the pixel of our photograph of these system. To be able to claim that our models are accurate, we have compared them to the photographs of the systems. If we continue to follow the analogy with impressionism, this would mean that as painters, we have worked both by going outdoor and watching the landscape (outdoorism) and also have painted indoor in our workshop, based on photographs of these landscapes.



# Chapter 3

# "Qualitative is nothing but low quality quantitative"

This quote by Rutherford, cited from [Thom, 1984] seems a provocation, in a thesis where I try to define scientific impressionism. But the added freedom from impressionism shall not take us away from data. On the contrary.

> I realized that one gets nowhere unless one talks to people about the things they know. The naïve person does not appreciate what an insult it is to talk to one's fellows about anything that is unknown to them. They pardon such ruthless behavior only to a writer, journalist or poet. I came to see that a new idea, or even just an unusual aspect of an old one, can be communicated only by facts. Facts remain and cannot be brushed aside; sooner or later someone will come upon them and know what he has found[1].

## 3.1 The breakup of the static capillary bridge

The behavior of the capillary bridge is very rich as you can see on figure 3.1. Thus, we need to select some particular cases for which something simple can be said, and then we will claim we understand it once we have patched together the well explored spots of parameter space.

Here are the three basic questions for the breakup of the capillary bridge:

1. Why does the static bridge break when we reduce the volume?
2. Why does the capillary Venturi break for a larger volume than the static bridge?
3. Why does the cylindrical bridge break when we increase the throughflow?

[1] [Jung and Jaffe, 1989]

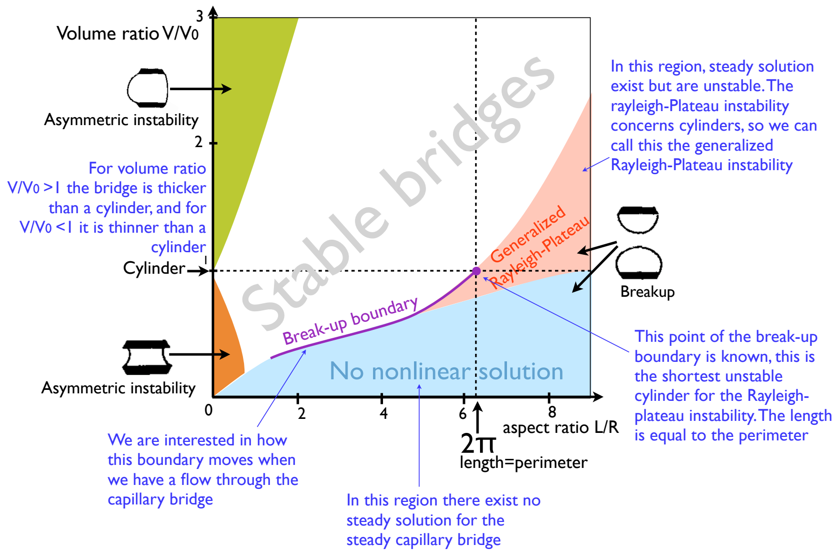



Figure 3.1: Overview of the properties of the static capillary bridge (no gravity).

For question 2 and 3 which are concerned with the effect of the throughflow, please see [Paré and Hoepffner, 2015]. We will dwell upon the first one because it illustrates well something that tends to happen when the number of degrees of freedom of a model tends to zero.

The first question comes from the original experiment by Plateau [Plateau, 1873]. For instance, have a drop of water between the ends of two pipes, and connect one of the pipes to a syringe. Start with enough water to have a cylindrical shape and then proceed to slowly pump fluid out. As you do so, the volume of the bridge will progressively reduce, inducing a smaller and smaller radius at the middle of the bridge: the neck. Then, the bridge will suddenly break, well before the neck radius has vanished. Question one is: why?

Let us propose in words the scenario that would be the hidden map behind this question:

> The inner pressure of a static bridge is uniform. Since the jump from atmospheric pressure depends on the total curvature of the surface, this total curvature must as well be uniform. The curvature has two components: the azimuthal curvature and the axial curvature. When we reduce progressively the volume of the bridge, the neck radius $r$ will decrease. The pressure jump through the interface at the center of the bridge due to the azimuthal curvature is $\sigma/r$. This increase must be compensated for by the increase in negative axial curvature. When $r$ is small, $1/r$ grows faster than it is possible to grow for the axial curvature, the balance can no longer be satisfied



> and the bridge breaks.

This answers question 1. Now addressing question 2:

> The situation is even worse when there is a throughflow: because of the acceleration of the fluid through the neck of reduced radius, the inner pressure at the neck decreases (Bernoulli), adding its contribution to that of the azimuthal curvature.

Stated in words as here, there are several ambiguous statement that urge the call for a well defined model.

- "When we reduce progressively the volume of the bridge, the neck radius $r$ will decrease": how will it decrease?
- "When $r$ is small, $1/r$ grows faster than it is possible to grow for the axial curvature": how does the radial curvature grow?

For quantification, instead of describing as accurately as possible the shape of the free surface, let us make a bold simplification and see what numbers would come out if it had the shape of a parabola

$$y = r + (R - r)\left(\frac{x}{L/2}\right)^2.$$

A parabola is nice because it is left-right symmetric like the bridge. This parabola has height $R$ at both ends of the bridge at $x = \pm L/2$.

We can now see under which circumstances this parabola obeys the balance asked of the capillary bridge. We cannot impose a curvature constant everywhere, so lets enforce that the curvature at the ends and at the middle are the same. At the middle, the curvature is the sum of the azimuthal curvature $1/r$ and the axial curvature, which is simply the second derivative of the parabola $8(R - r)/L^2$. Thus equal total curvature yields

$$\frac{1}{R} = \frac{1}{r} - \frac{8(R - r)}{L^2}.$$

In this expression we have neglected the axial curvature at the ends of the bridge, this seems reasonable since the farther from the middle, the lesser is the curvature of the parabola. We will see later that we have an other reason to neglect this term, which does not have to do with accuracy.

The pressure balance is depicted in figure 3.2. Now, instead of looking at this figure as a graph, let us look at it as a *mechanism*: as we change the aspect ratio of the bridge $L/R$, the black straight line rotates about the fixed pivot at $(1, 1)$. This black lever meets the blue line $1/r$ at the neck radius of a static bridge. When we decrease the bridge length, the lever rotates clockwise and the bridge has a very thin neck, when we increase the bridge length, the lever rotates counter-clockwise and the neck tends to $R$.

You can now appreciate the reason for neglecting the change of curvature at the end of the bridge when changing $r$. With this simplification, varying $L$

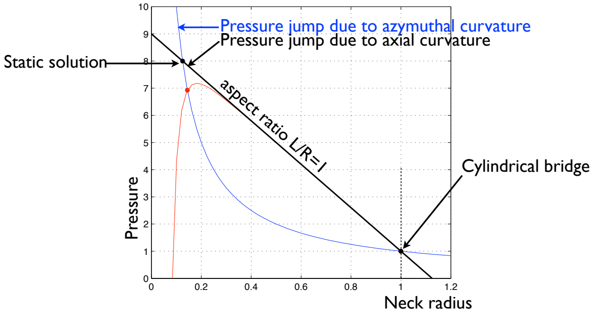



Figure 3.2: The graphical mechanism that can be used as an analogue to the physical mechanism for breakup of the static liquid bridge.

only rotates the black lever, so it is clear how changing the physical parameter changes the solution. Including this effect does not change the fact that (1,1) is fixed, and does not change the fact that $1/r$ rises quickly when $r$ becomes small (when the volume decreases).

This is fine, but does this answer our question? Certainly not. The experience of Plateau shows that for a given length, there is a possible range of given volumes, bounded below by a minimum volume under which the bridge breaks. Our model instead does not have a continuum of solutions, but just two discrete solutions: the cylinder and a small volume. In a way, we can say that the solution has changed qualitatively from the original system: our model is *degenerated*. To understand this, we need to add a degree of freedom. So now, instead of having a bridge in the shape of an $x^2$, let's have as well an $x^4$

$$y = r + (R - r)\left[\lambda\left(\frac{x}{L/2}\right)^2 + (1-\lambda)\left(\frac{x}{L/2}\right)^4\right],$$

with $\lambda$ our second degree of freedom. By varying $\lambda$ away from 1 we can continuously switch away from a parabola. $x^4$ has no curvature at the neck, thus the neck curvature remains that of the parabola $\lambda 8(R-r)/a^2$. On the other hand, $\lambda$ has an effect on the bridge volume. It is clear that adding this second degree of freedom, we have recovered the continuous distribution of solutions of the original system.

We show this on figure 3.3. On the right we have the different shapes of the bridge when we keep the volume constant and vary $\lambda$. These are all the possible shapes, given the two degrees of freedom $r$ and $\lambda$ for a given volume. If amongst this family of shapes, there exist one with equal curvature at the end and at the middle, then the bridge can sustain this volume. In fact, for moderate volumes, there are two such solutions. This again is a recovery of the real system. The solution with the lower $\lambda$ is stable, and the other one is unstable. As the volume is reduced, these two solutions move progressively



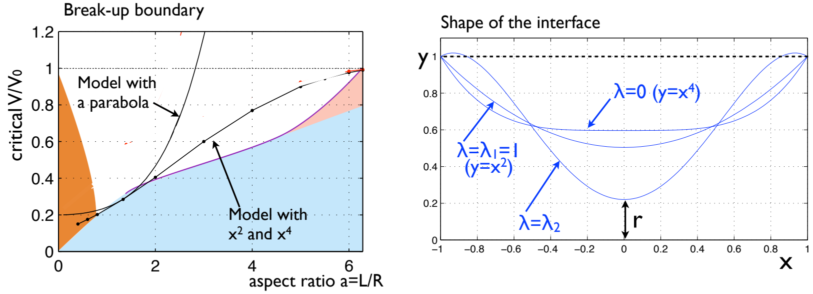



Figure 3.3: Left: Comparison of the breakup boundary with that predicted by the one degree of freedom model and the two degrees of freedom model. Right: Shapes of the liquid bridge with different values of the second degree of freedom $\lambda$.

toward each-other, then meet, then there is nolonger a solution. This is the typical sequence of a fold bifurcation of nonlinear systems, see [Strogatz, 2000].

On the left subfigure, we compare the results from these two models with the breakup boundary of the full system. This break-up boundary is for a given length of the bridge, the minimum volume that it can sustain. For the model with $x^2$ and $x^4$ we have as well neglected the curvature at the end.

## 3.2 Discussion on degenerated models

I like this example of modeling because it shows the kind of adventures you get caught in when the number of degrees of freedom tends to zero. I introduced this thesis with the transition from Ingres and Chassériaux to the impressionists, synchronous with the appearance of photography. There is nolonger a point of being very precise in representations, since photography does this very well. In the quote, de Gennes suggests that computers in science play the same role as cameras for pictorial art.

The numerical methods for describing systems with differential equations is to discretize the unknown functions, decompose them in small pieces put together, which become the degrees of freedom of the model. For instance with finite difference disretization, we approximate the function by straight segments or pieces of parabolas, or in general portions of polynomials. The quality of the discretization is quantified by the *order* of the numerical method: what is the polynomial rate of decrease of the approximation error when the number of degrees of freedom tends to infinity. This is the objective for the computer, just as it is for the camera, increasing the size of the light-sensitive film or decreasing the size of the light-sensitive grains (increasing the ISO for film-based photography and decreasing the pixel size for numeric). If you look at the painting "La grenouillère" on figure 1.1, you see that on the contrary, Monet deliberately used large pixels.

For the description of my systems, I am indeed interested in the limit of infinite numbers of degrees of freedom, and I actually performed it: it gave me the "photography" against which I validated my model. When it comes to understanding, I have looked for the other limit: what happens when the number of degrees of freedom instead tends to zero. With the parabola, I started with one single pixel. This gave us a mathematical structure—still not entirely trivial— which gave two solutions instead of a range of solutions. In this sense our model was degenerated.

Is degeneration a case for disqualification? Out of the two solutions, we *identified* the low-volume solution with the lower boundary of the range of solutions of the original system. Thus, the price of this degeneracy is the need for an identification. Drawing the quantitative line on the boundary obtained from a precise model (finite differences with many degrees of freedom), we recovered the main features of the boundary, and found even that it was rather accurate for short bridges. Considering the model with $x^2$ and $x^4$, we say that the new parameter $\lambda$ gave us some continuous freedom about the discrete solution. This additional freedom let us think that the discrete solution was some kind of *skeleton* underlying and supporting the flesh of the richer representation. This is a hint that when we are confident enough to let go with accuracy, we gain structure for our models—a structure readily identifiable. This is a model with more *clarity* (you can see through it).

Doing one *simplification* and one *identification*, we have gained a clear view on the behavior of the breakup of the bridge: seeing the graph as a mechanism: the rotation of a beam about a pivot, we have given ourselves a graphical mechanism analogue to the physical mechanism at play in the experiment. It is in the sense of this graphical analogy that we can say that this model is *clear*. On the other hand, we have failed to exhibit an archetype of liquid breakup, since even though the model is clear, it is not accurate.

We removed the degeneracy by taking the first step toward a more precise model: adding a second degree of freedom. We can be confident that this successive complexification of the interpolant of the bridge's shape will converge. Adding more and more terms of the Taylor expansion of the shape of the bridge is rationally natural—this is a spectral method—for which a computer would be the perfect tool in the search for accuracy. You can see already a big improvement of the approximation of the breakup boundary of the bridge between the model with one degrees of freedom and the model with two. We will see another encounter with degeneracy in §4 in the context of dimensional analysis.

## 3.3 Discussion on accuracy and modeling

The arrival of photography triggered a revolution for painting. There was nolonger a point in working very hard to develop a virtuosity in the capacity to represent with high precision the details of the scene[2]. Off course, there is much

[2]Except for the "hyperrealists" in the 1950s-1960s.



more to the painting of Ingres that merely his skill in representation. The arrival of a new technique does not change the ability of artists to express emotions, but on the other hand it can induce a change in their way of life. The time of Ingres was characteristic of the academic school who ruled the art market and the artistic life. Impressionists painted fast and could capture changing lights and atmospheric transitions, they did not belong to academies; they were rejected by academies.

We have played with the analogy to try to imagine what it would be to be impressionist as a scientist. Another game would be to try to imagine what would be the artistic equivalent of the following mismatch between Galileo and Aristotle:

> Aristotle tells that a hundred-pound cannonball falling from a height of hundred feet touches the ground earlier than a one-pound cannonball has fallen one feet. I tell that they arrive at the same time. You observe, when doing the experiment, that the bigger cannonball overtakes the smaller by two inches. Behind these two inches, you want to hide the 99 feet of Aristotle. You only talk about the small mistake that I made, and keep silent on his mistake, which is gigantic[3].

Probably, the artistic alter-ego of this quote's Aristotle would be Dali. This reminds us that comparing predictions to experiments has not always been a required step of modeling.

The parrallel that is drawn by de Gennes between the arrival of photography in pictorial art and the arrival of computing in science is interesting. This analogy asks the question related to the title of a book by René Thom: "Predicting is not explaining"[4]. Just like painters could have said: "reproducing is not expressing". Rapidly, photography has been used by artists and became an original means of artistic expression, and certainly in parrallel, computing has been used by scientist and became an authentic tool of explanation. The question is not to ask which technique is best, the question is: what does this new tool change in the way we think of the description and explanation of nature? Yet another question which is suggested by the analogy is: are scientists going to find a way to do their work in a more inventive way, farther from the heavy demand of an austere skill with its school system and hierarchy?

To overtake the question of accuracy of models, there is the idea of the archetype. The possibility that a model be both "expressive" (clear to the mind) and accurate. But first, archetypes are not found everyday, and second it takes a process of modeling to progressively get to an archetype, this is the time it takes to really understand what is it that causes the technical and cognitive troubles in the way the question is asked, in the way the experiment is designed. So we will not escape from digging into the causes of inaccuracy. So it means that we need to find another way to think of accuracy.

[3]Galileo, "the discourse about two new sciences"
[4][Thom, 2009]

In the example of the capillary bridge in §3.1, I started with the expression in words of what I thought was the mechanism of the breakup. This expression may be convincing at first sight or may not. In the spirit of Rutherford's quote as the title of this chapter, this wording of my understanding was a *qualitative* description of the mechanism. There was not a bijection between these words and numbers that can be compared to the numbers we get from the observation of a real capillary bridge. To remove this ambiguity, we had to make some arbitrary choice, we supposed a given shape of the bridge. This choice is *ad hoc*, and is only motivated by the technical ease of it. But on the other hand, once this choice made, ambiguity is removed. So there is maybe still an ambiguity remaining: what would have happened if I had chosen another shape like for instance an arc of circle? To relieve our anxiety concerning this particular ambiguity, we have this quote by Francis Bacon:

> Truth emerges more readily from error than from confusion[5].

Once all these ambiguities cleared by making the choices that help putting numbers on words, we obtain a formula which can be seen as the quantitative essence of our understanding. If the process of the successive transformations from the words to the formulas is clear and well assimilated, then we know exactly what the line of its graphical representation means. This summary of our understanding into a line is possible because of the immediateness of technical simplicity and cognitive clarity. Note by the way that cognitive clarity is also a question of fashion[6]:

> For the greeks, a *good* solution was the one which only employs ruler and compass; after, it became the one obtained by the extraction of radicals, then the one built only with algebraic or logarithmic functions[7].

Now the question is: the experimental data points, are they far or close to this line? here we can more or less replace "experimental data points" by "computational data points" with not much loss of our meaning if we trust the first principles and technicalities mixed in the computational "photograph" of the system.

The distance between the line and the point is a quantitative measure of our ignorance. If it is big, then it means that we lack some big aspects of our mechanism, which means in turn that the comparison teaches us important new things. On the other hand, if the distance is small, it means that we know most of the things and that the graph will not teach us anything more. Expressed in different words: a model is a magnifying glass which allows us to zoom on details of a phenomenon to the extent of its accuracy. You will not be able to talk about subtleties of mechanisms whose quantitative impact are lesser than the distance between your line and your points. We tend to use graphs as proofs of our understanding, and forget that they are as well tools toward understanding. The idea of the archetype recasts this duality in a new perspective. The graph is nolonger a proof of success, but a motivation and an information on how to modify the original experiment to converge toward both clarity and accuracy.

The computer is a great tool for prediction, which is the scientific analog to artistic *depiction*. The skill of depiction is now being increasingly externalized. Hardware development is externalized to industry and dedicated laboratories, and the coming of internet and the utopia of sharing resources has lead to the widespread accessibility of open source software. For instance, I used intensively the solver Gerris [Popinet, 2009] (with a constantly renewed feeling of gratitude toward its developers). It becomes increasingly clear that predicting is a tool for explaining. This leaves room for much freedom.

[5] Cited by Yves Pomeau during a summer school in Peyresc.
[6] . . . and fashionable things are things that get out of fashion.
[7] [Poincaré, 2012]

# Chapter 4

# "Be content with the knowledge of some special cases"

> Since a general solution must be judged impossible from wants of analysis, we must be content with the knowledge of some special cases, and that all the more, since the development of various cases seems to be the only way of bringing us at last to a more perfect knowledge[1].

In this section, I discuss a "special case" that I have explored in several contexts.

## 4.1 Atomization

Atomization is "how a body of liquid can be transformed into a cloud of droplets". See figure 4.1. It happens in the Vulcain II thruster of Ariane 5. Here is the count-down of the modeling steps:

1. A thruster with 566 coaxial injector elements. A thrust of 1359 Newtons. High pressure, combustion.
2. We extract from the thruster a single nozzle ejecting a fast outer gas stream which destabilizes a slow core jet of liquid. No more combustion. We can look at the formation of the waves, liquid films and liquid ligaments, finally droplets.
3. We remove the nozzle to keep an infinite cylinder of liquid at rest, surrounded with air moving to the right.

[1] [Euler, 1954], cited from [Craik, 1988].



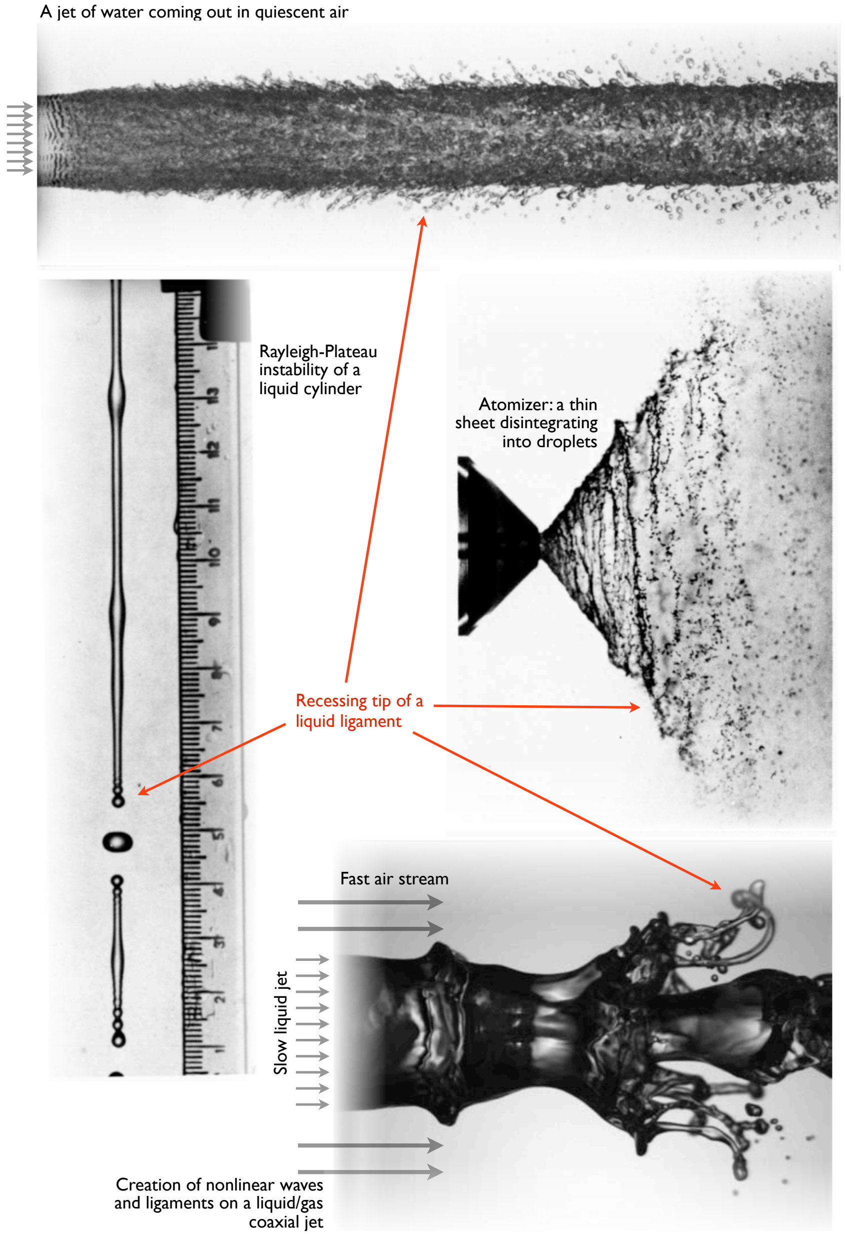



Figure 4.1: Images of creating droplets from a liquid body: atomization. All images come from [VanDyke, 1982] except the coaxial jet which comes from [Marmottant and Villermaux, 2004].

4. We make this cylindrical configuration into a plane configuration: an imaginary horizontal line with liquid at rest below and gas flowing to the right above. This line has a thickness that corresponds to the zone of progressive adaptation of the velocity from the liquid to the gas.

5. We remove the phase difference: same density and viscosity above and below the line. No more surface tension.

6. Now the line has zero thickness.

We have arrived at the single phase vorticity sheet (a "surface of division"), studied by Helmholtz and Kelvin. This is the original archetype of instability in fluid mechanics, Helmholtz wrote in [Helmholtz, 1868] that

> The stationary forms of the surfaces of division are distinguished, as experiment and theory alike indicate, by a remarkably high degree of alterability when subjected to the least disturbance, so that they comport themselves in some degree like bodies in unstable equilibrium. [...] Theory points out that, wherever an irregularity is formed on the surface of an otherwise stationary current, this must give rise to a progressive spiral unrolling of the corresponding part of the surface.

The classical approach to understand and quantify the sensitivity of the shear layer consists in evaluating its response to a perturbation in the shape of a low amplitude (linear) and periodic excitation. This is the work of Kelvin in [Thomson, 1871]. The original intuition on the other hand was that of Helmholtz in [Helmholtz, 1868], who describes the response of the shear layer to a localized perturbation yielding the response of a spiral. On our numerical simulations, we have studied this original point of view. The comparison is shown in figure 4.2.

When starting with Stephane Zaleski our study on atomization, we had simulations performed by Daniel Fuster showing the creation of large waves at the interface of a two-phase shear layer flowing passed a splitting plate. Because of intermittency, waves were being shed from the splitting plate with a low frequency, such that the successive waves were evolving more or less independently from each other. Our idea was then to just look at one of these wave. We took a large periodic box, removed the splitting plate, and initiated the perturbation using a localized forcing. What we observed from the very first simulation with Ralf Blumenthal was the creation of a single wave, which was growing in size with very little change in shape. It quickly appeared that this growth was self-similar. Stephane had the intuition that this could be easily justified on the basis of dimensional analysis.

The evolution of this wave is shown on figure 4.3, with three values of the density ratio $r$. On the left, $r = 1$, both fluids have the same density. The response of the shear layer is center-symmetric, structured as a couple of vortices, translating to the right at the average velocity of the two fluids. At the time,



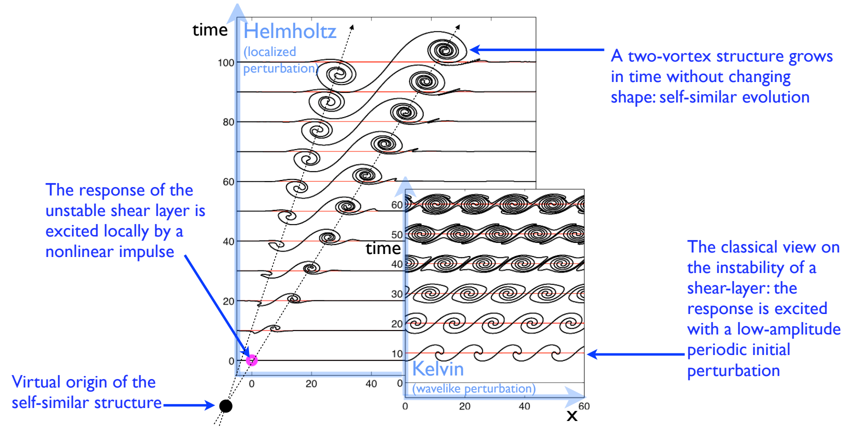



Figure 4.2: Helmholtz' spiral and the Kelvin periodic row of vortices. These are the response of the unstable shear layer to two different types of excitations: periodic and low amplitude, or localized and large amplitude.

we were still thinking in terms of the atomization problem, so the bottom fluid would be a liquid at rest and the top one the gas flowing to the right.

These results are described in [Hoepffner et al., 2011]. The argument for self-similarity is the following. According to the way Barenblatt formulates it ([Barenblatt, 2006]), a self-similar solution is the "degenerated evolution" of a system whose relevant scales can be distinguished into two groups, the small scales, and the large scales, and when the small scales and the larges scales are far apart, widely separated. In these conditions, there is the possibility for an "intermediate asymptotic" solution. Here the small length scales are the ones related to viscosity, surface tension and shear-layer thickness. A wave will be fragile to these scales when it is still small. Then we have the large scales, they are related to the size of the flow domain (confinement) and the gravity which prevents the wave to grow to a large height.

If we find a flow configuration where there is a large gap between the small scales and the large scales, this gap can be the place for the living and growing of a self similar wave. In this "no man's land", or rather a "no-scale land", the only relevant physical parameters are the velocity difference between the two streams $\Delta U$, and time $t$. So the wave locks onto this scale $L = \Delta U t$ growing in time, and thus its size increases algebraically in time. How do we tell to the wave that it should grow in this gap? We do this using the initial condition. The wave is initiated from a localized forcing, the size of which is the first reference of the wave. It will grow onward from this size.

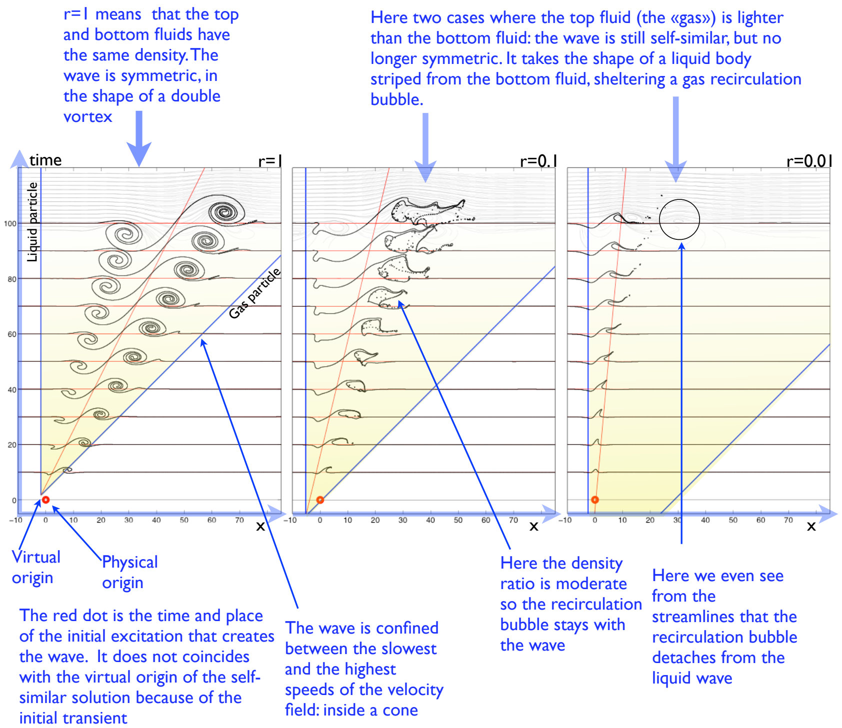



Figure 4.3: The self-similar response of the shear layer to a local excitation for three different values of the density ratio $r$.



## 4.2 Intermediate asymptotics

Figure 4.4 illustrates from an other point of view the notion of intermediate asymptotics dear to Barenblatt. We perform a simulation of the shear layer using the Gerris flow solver [Popinet, 2009]. To simplify as much as possible, this is again a periodic box in the horizontal direction, but instead of simulating the Navier-Stoke equations, this is the Euler equation in the vorticity-streamfunction formulation. There is no viscosity, and no surface tension, but still, the shear layer has a finite thickness. For the initial condition, in this vorticity formulation, it is convenient to have just a localized defect of vorticity at the origin. Because of this defect, we can think the initial condition as being two semi-infinite shear layers next to each other. Just like semi-infinite shear layers usually do, they roll-up. These are the two spirals that we see forming symmetrically to the origo (0,0). The panel on the right shows the same evolution but recast in the frame that grows with the similarity scale. Ideally in this frame, everything should be stationnary. We see indeed that there exist an intermediate time during which the two spirals appear to not change in time. This intermediate window during which time seems suspended is squeezed between the time of the initial transient: the time during which the size of the spiral is close to the thickness of the shear layer. And it is squeezed from above by a longer time, the time that its takes for the numerical noise to excite the shear layer into the nucleation of an array of vortices. This is for me the depiction of "intermediate asymptotics".

## 4.3 Self-similar solution of the shear-layer

Dimensional analysis shows that there is room for a self similar solution, but does not proves that it can exist, nor does it give information about its stability. Let us try to make a model of this flow by stripping away everything that is not completely necessary. There are two important things: the presence of the yet untouched shear layer, and the part of the shear layer which is rolled up into a spiral. We do the simplification showed on figure 4.5.

- We remove the spiral and replace it with a point vortex,
- then we say that the semi-infinite shear layers are flat and with constant amplitude.
- Also, we say that the intensity of the point vortex is equal to the amount of shear layer that has disappeared (has been "eaten" by the vortex). This is motivated by the conservation of vorticity. Vorticity can only rearrange, it cannot disappear (when viscosity is low).

Doing this, we get an evolution equation of the position of the vortices. We assume central symmetry for simplicity, so that we need to describe the position of only one vortex. This is a model with two degrees of freedom: the distance $\ell$ and the height $h$ as described on the figure. The system will evolve

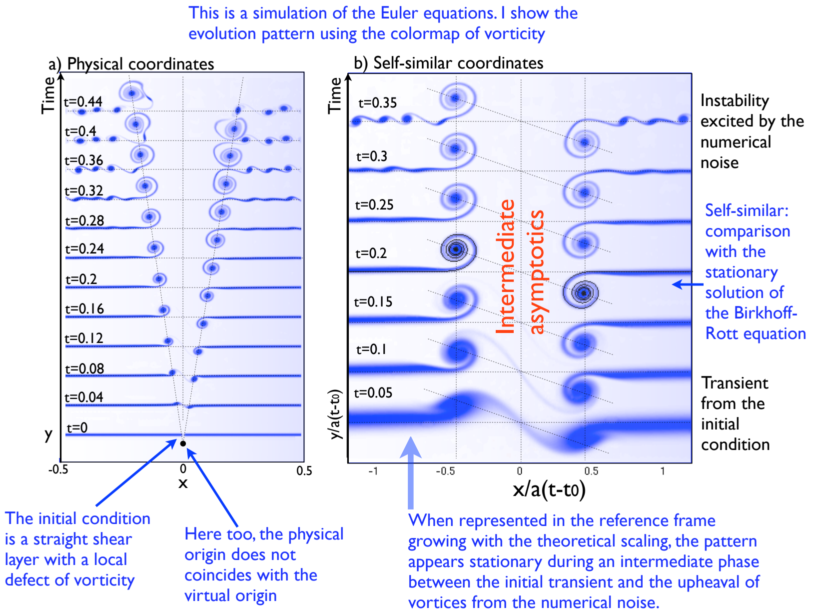



Figure 4.4: Rescaling of the frame to show the intermediate asymptotics of the self-similar response to a localized excitation of the shear layer.

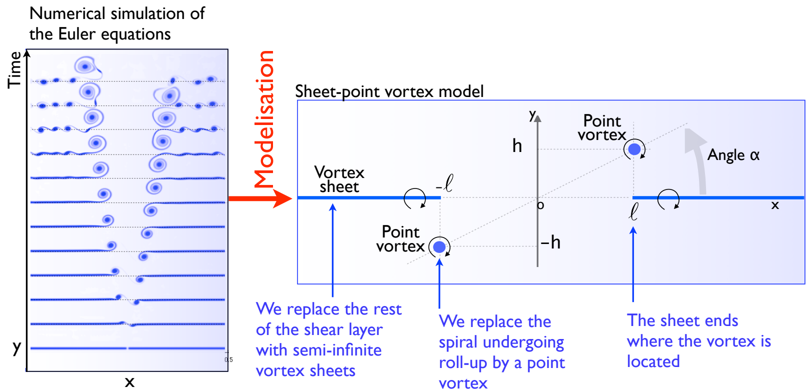



Figure 4.5: A vortex sheet-point vortex model for the self-similar structure.



as follows: the vortex is advected by the induction of the vorticity contained in the two semi-infinite shear layers and by induction from his mirror vortex. Since each vortex is assumed a point vortex, the induced velocity field is known (complex potential theory, see [Guyon et al., 2001]) and the velocity induced by the two semi-infinite shear layers can be calculated analytically by substracting the velocity induced by a segment of vorticity to the velocity induced by an infinite straight line. The velocity field induced by the infinite straight line of vorticity is just a discontinuity of horizontal velocity: $+U$ above the line and $-U$ below. The velocity induced by a segment is obtained by an integration in complex plane, see [Hoepffner and Fontelos, 2013] for the details.

Now that we have a mathematical description of the initial condition problem for this system, we can see what are its particular solutions. Especially, we are interested to know whether there exists a self-similar solution. We can reformulate the problem as follows: If there exist an angle $\alpha$ (see figure) for the position of the vortex where the velocity induced by the two semi-infinite shear layers and the mirror vortex has the same angle, then this is angle is a self-similar solution because the law of advection conserves the shape of the solution.

We do the calculation using the integrals and we draw on figure 4.6 for all choices of the angle $\alpha$, the angle $\beta$ of the advection velocity at the position of the vortex. We see that indeed there exist a self similar solution.

Is this solution stable? This is readily told by the graph: if a small perturbation displaces upward our vortex, we see that the angle $\beta$ of advection velocity will decrease, which will pull the vortex back on its particular solution. Also if the vortex is displaced down, $\beta$ increases and lead it back on track. The self-solution is asymptotically stable, and you can see using the same argument that it is globally stable (at least for positive $\alpha$).

There are two degrees of freedom, either the length $\ell$ and the height $h$, or the angle $\alpha$ and the distance $d$ from origo. We now consider this $d$. $d$ is a witness on how fast the self-similar solution grows in time. The intensity of the advection velocity for the angle $\alpha_{ss}$ depends only on the size of the solution $d$, and in a very simple way, since the position where the shear layer ends is set by the position of the vortex which is being advected. Since the geometry does not change, the growth must be a power law. Getting the value from the complex integrals and normalizing by the expected power law (given by dimensional analysis) gives the *size* of the self similar solution. This is shown on figure 4.6, on the right. We see that our model overestimates the rate of growth of the self-similar solution but predicts accurately its angle.

Basically our simplification amounts to localizing all the vorticity of the spiral into a point. It is not yet clear to me why this should lead to a stronger advection. A referees of the article claims that we make a mistake in the cut of the integral in the complex plane by not accounting properly for the fact that the intensity of the point vortex increases in time. This is something I will need to check.

We did the same analysis for another self-similar solution of a vorticity sheet: the well known rollup of the wingtip vortex. This mathematical solution is a

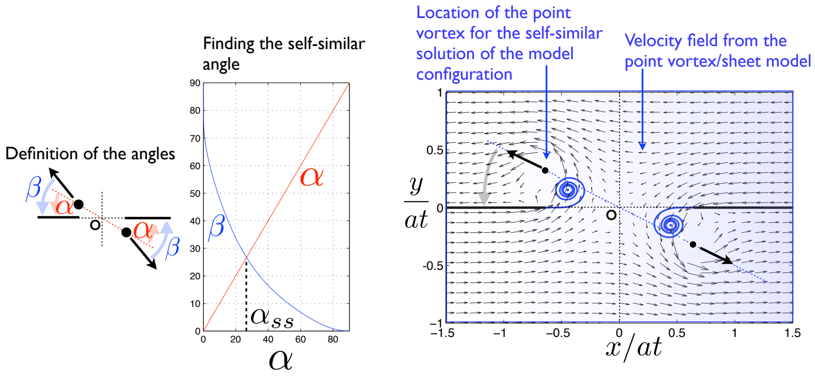



Figure 4.6: Results of the point-vortex model for the impulse response of the shear-layer.

model for the large vortical trail of the tip of airplane wings. You can see them in the sky behind commercial planes because the vortices trap the exhaust of the reactors. This was one of the early success of airplane aerodynamics at the time of Prandtl, known as the *Kaden spiral*. Kaden predicted the possibility of a self-similar rollup with a given temporal law, if the vorticity sheet had the intensity variation like produced by an elliptically loaded wing. This solution was as well simulated by Pullin in [Pullin and Phillips, 1981], solving for a steady solution of the Birkhoff-Rott equations once properly scaled in time and space. The Birkoff-Rott equation is an integro-differential equation describing in the complex plane the self-advection of a vortex sheet. In figure 4.7, I show Pullin's numerical simulation and our solution, replacing the spiral by a point vortex and doing the same analysis as above for the local impulse response of the shear layer.

Our model shows again in a very intuitive way that there is a self-similar solution, that this solution is globally stable (for positive $\alpha$) and here the rate of growth is nicely quite close to that of Pullin's computation. For this flow, the advection of the point vortex does not depend on its own intensity (unlike the double vortex solution above), so the good agreement here does not contradicts our reviewer's comment that we account incorrectly for the variation in time of the vortex intensity.

Spending some time on this literature and looking it with the fresh eyes of our aim of modeling, lead us to an overview of the self-similar solutions of vorticity sheets, see figure 4.8. There are two parameters on this graph, the value of $p$ describes the distribution of vorticity along the sheet: $p = 1$ is a uniform vorticity (straight shear layer for instance), and $p = 1/2$ corresponds to the elliptically loaded wing of the wingtip vortex rollup.

This overview might be useful, if we think of the efforts that are made in the modeling of transition to turbulence. For Poiseuille flow in a cylindrical pipe for instance, we know that the flow is asymptotically stable at all Reynolds numbers,



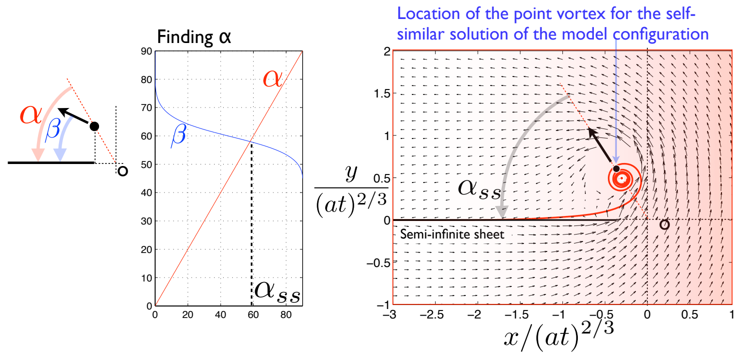



Figure 4.7: Results of the point-vortex model for the rollup of the wingtip vortex.

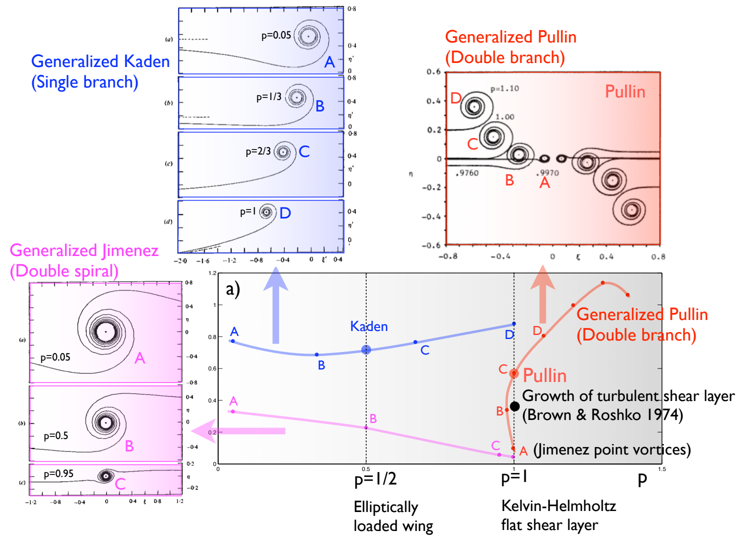



Figure 4.8: General families of self-similar solutions on a vortex sheet of varying intensity.

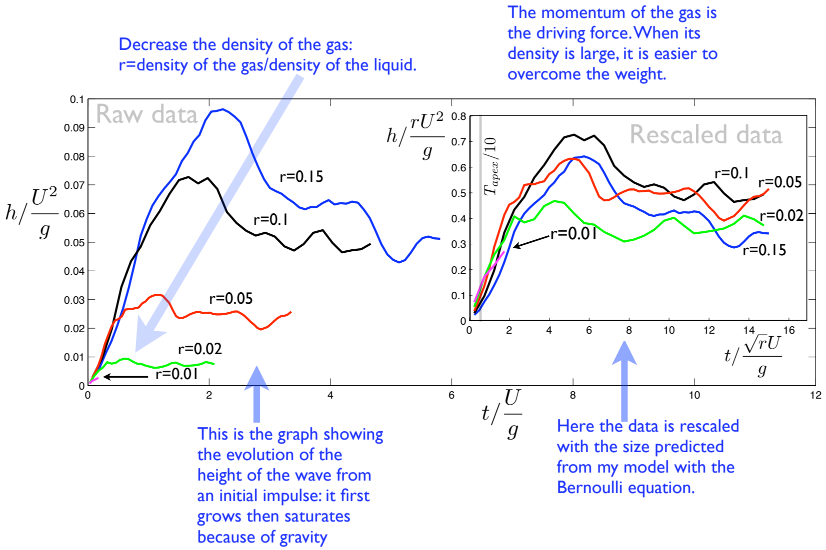



Figure 4.9: The evolution with time and parameters of the height of the self-similar wave, when it is perturbed by gravity.

but we observe transition to turbulence at about Reynolds=2000. People found (see [Eckhardt et al., 2007]) that we can start to find nonlinear traveling waves, self-sustained and periodic, and that their number starts to augment quite much around $Re = 2000$. They provide the rich nonlinear structure to the space of evolution that can lead to chaotic behavior of this flow. The diagram of self-similar solutions that I show above for the shear layer could play a similar role for the chaotic behavior of free shear layers. For the moment, this is just an idea. . .

## 4.4 Self-similar wave and gravity

Talking about large scales, we had also an idea to see what would be the effect of the gravity on this wave. Of course, for gravity to be relevant, it means that we should have a difference in density between the two fluids of the shear layer. This, we investigated in [Orazzo and Hoepffner, 2012]. Of course, in addition to the self-similar length-scale, we have now the gravity length $L_g = U^2/g$. As soon as two scales compete, dimensional analysis looses its power, so we have to come to a geometrical analysis. In our first paper on the self-similar wave [Hoepffner et al., 2011], we had used already a geometrical analysis to quantify the impact of the density ratio on the prefactor to the algebraic rule. It told that the wave should grow according to the square root of the density ratio $r$ between the two fluids. Adding to this force balance the effect of gravity,



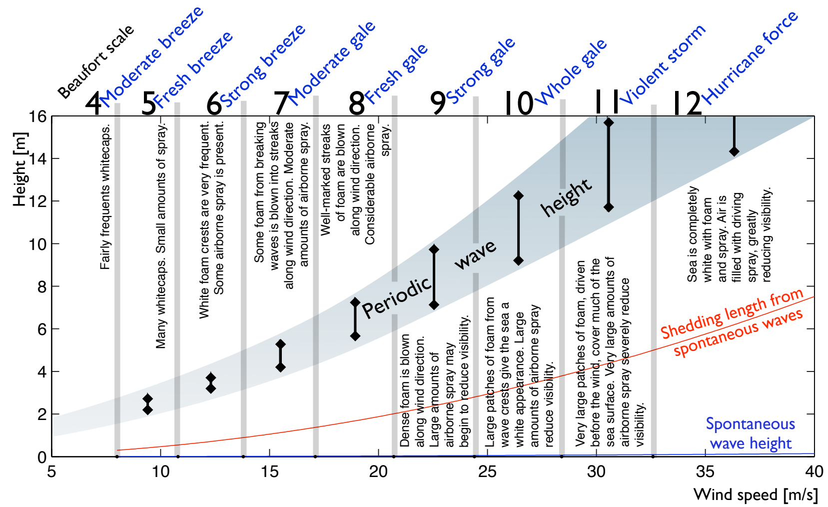



Figure 4.10: Comparison of the size of the self-similar structure and the shedding length of droplets with indications of the Beaufort wind scale.

we saw that gravity was putting an end to the driving power of the pressure drop as soon as the hydrostatic pressure along the wave height balances the aerodynamic pressure drop. This gives a law proportional to the density ratio $r = \rho_{gas}/\rho_{liq}$ for the maximum possible height of the wave, and a scaling like the square root of $r$ for the time of this maximum size. This is shown in figure 4.9.

On the oceans when the wind is blowing hard for a long time, large and long waves build up. These waves are dangerous to sailors once their size become of the order of the size or their ships. The self-similar wave with a wind of 100 km/s saturates at a heigh of about 5cm. This wave is not dangerous. On the other hand it grows quickly, and most of its liquid body is atomized with drops carried a long way by the wind. These droplets play a large role in the thermal and chemical exchanges between the ocean and the atmosphere. The duality between the large and slow Kelvin waves and the small and quick saturated self-similar waves encourages me to call these later ones “spontaneous storm waves”. Figure 4.10 reminds the Beaufort scale for wind strength used to describe and predict the state of the sea. I have drawn on this graph the height of the spontaneous wave (ridiculously small) and the length of the flight of its atomized drops. you can compare these data with the qualitative description of foam and spray of the scale.

## 4.5 The mechanism to catapult droplets

Now we consider yet another property of these waves. This is an old observation when looking at coflowing shear layer atomization, that there are some unexpectedly large ejection angles for the droplets. We were wondering together with the experimental team at LEGI, Grenoble (Jean-Philippe Matas and his students), wether the self-similar wave could be a good study case for a specific ejection mechanism. This study is described in [Jerome et al., 2013].

When the two fluids have the same density, the self-similar solution takes the shape of two well-behaved vortical spirals. When on the other hand we start to reduce the density of the top fluid, the solution becomes asymmetric. You can see this clearly on figure 4.3. The top spiral becomes a liquid wave with a gas recirculation bubble in its wake. Since the growth of the wave is like the square root of the density ratio, the wave grows slowly when the gas is made light. Since this wave does not "propagate" with a given phase speed, but just grows, the fact that it grows slowly means that it advances slowly. Thus the liquid wave behaves in a way like an obstacle for the gas stream when the density ratio is large. This means that beyond a given critical density ratio, the recirculation bubble behind the wave begins to shed vortices periodically. This vortex shedding is a strong forcing back on the wave, and we observe that the liquid tongue of the wave is stretched and torn periodically.

There are different regimes depending on the period of the shedding on one hand and on the other hand the time scale of the evolution of the wave's liquid tongue. There is an intermediate value of the density ratio for which these two resonate. This gives birth to the mechanism of "droplet catapult", displayed in figure 4.11.

Here the system is very fragile, and the process we would like to document happens on the top of many other processes, related to the initial transient, and the chaotic response to small perturbations, so the figures are quite dirty. This is why we have put in the center of the figure, a sketch of what is happening on the two sides. The left side is from the experiment at LEGI, and the right side comes from a Gerris simulation. The sketch comes from having seen many such events, with all their particularities, and seen the repeatable aspect of all of them. The sketch should be used as a guide to the eye when looking at the data.

The sequence of events goes as follow:

1. There is a recirculation bubble behind a liquid wave,
2. the vortex is shed and a liquid tongue is stretched,
3. since the vortex is gone, the streamlines are momentarily reattached,
4. the attached streamlines pushes the tongue down,
5. a new vortex is nucleated,
6. this new vortex blows the liquid tongue upward.



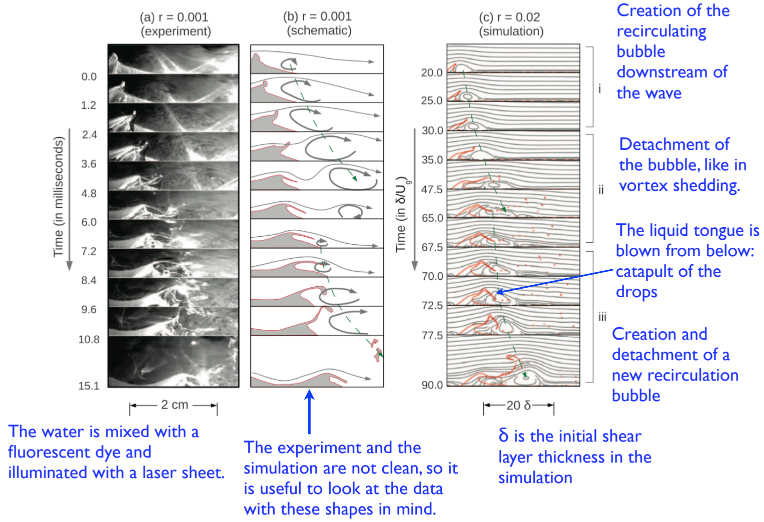



Figure 4.11: Experiments and numerical simulation for the catapult mechanism of droplets by the self-similar wave.

You can look at videos on my web page or together with the published article. The surprising thing when viewing the video is the fact that the liquid film is blown from below. The experimental video show both the liquid motion using a laser sheet and fluorescent dye, and the gas motion using smoke. The liquid film is stretched so it is thin, and it is blown from below so the droplets are catapulted upward. The blowing up of the liquid film bears many similarities with the bag breakup event in [Villermaux and Bossa, 2009].

## 4.6 The particular solution put back into context

The figure below is a summary of the all situations that we have explored regarding the self-similar wave. I want to show how the characteristic scales of the system can be close together, or far apart in a way to allow for scale degeneracy (self-similarity). The different scales are: shear layer thickness, viscous scale, capillary scale, gravity scale and domain scale. They are drawn as a function of only the velocity difference in the shear layer $\Delta U$. This is interesting because for fixed physical properties, the scales change depending on the velocity but with different scalings. The shear layer thickness and the domain size are the two only fixed scales. I have chosen here the prefactors in such a way that every physical effect will at some point be the limiting effect for the existence of self-similarity. The scales are:

- Gravity $L_{grav} = U^2/g$.
- Capillarity $L_{cap} = \sigma/\rho U^2$.
- Viscosity $L_{visc} = \mu/\rho U$.

I show them in a logarithmic plot to transform the power laws of $\Delta U$ into straight lines with different slopes.

The self-similar wave is a particular case of the response of the shear layer. Since the shear-layer is unstable and typical environments are noisy, we usually can view it under a complex chaotic behavior with large and small scales competing and interacting. You saw indeed that the two-phase atomization experiment in Grenoble is complex. Unprepared eyes would deem it purely chaotic and leave it to statistics. On the other hand, with eyes prepared by prior knowledge of a particular case (the self-similar solution), we have been able to extract from the apparently erratic atomization process, some well-defined events: the catapult mechanism.

It is somewhat similar to tossing a coin: it is an archetype of random processes. But indeed the rotation of the coin is deterministic. Magicians and swindlers who have an advantage in this misconception have dwelled into the physical mechanism of the toss to challenge the law of probability (see [Mahadevan and Yong, 2011].

Most of the classical archetypes of fluid instabilities assume periodicity in at least one of the spatial directions. Assuming periodicity of the response means exciting the instability with something periodic. This is a choice that led to many interesting results, easy technically and clear conceptually. Repetition is the easiest pattern to spot when looking at a random process: the eye is by default well trained for its detection. Periodicity is the simplest pattern; this is why when you see banded clouds patterns you are tempted to exclaim: Kelvin-Helmholtz!

The self-similar wave is not periodic, so the eye needs a specific training. You have by now acquired this training so you can look for yourself at figures 4.13 and 4.14.



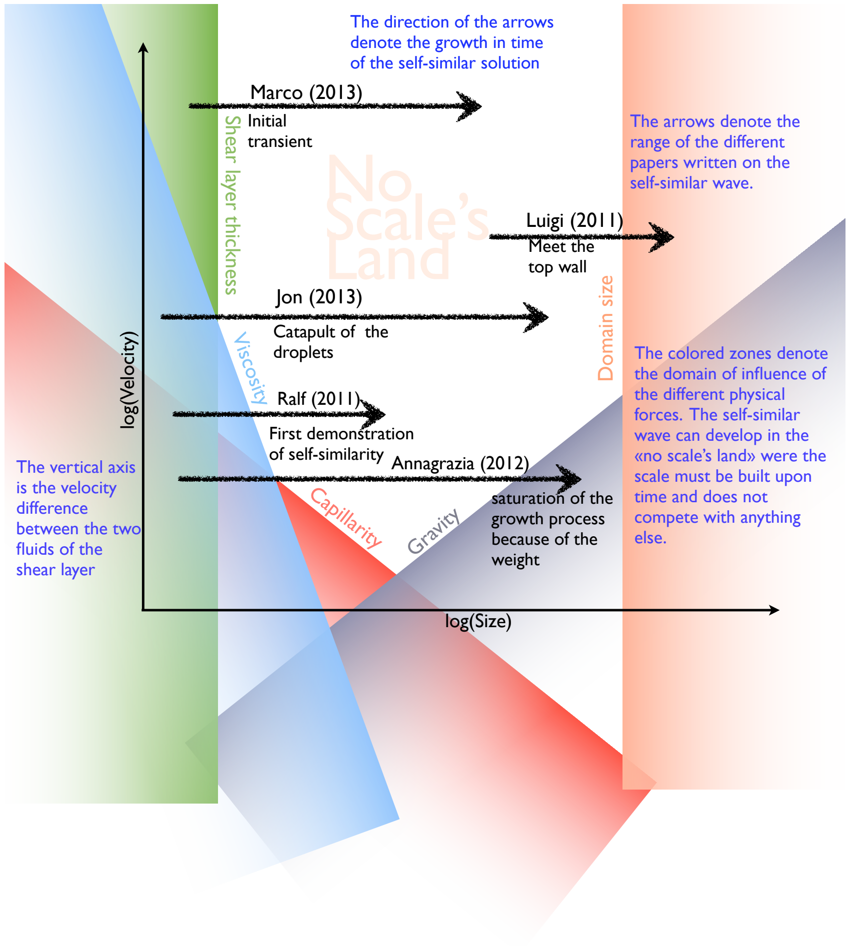



Figure 4.12: Overview of the papers on the self-similar wave. Ralf: [Hoepffner et al., 2011], Luigi: [Orazzo et al., 2011], Annagrazia: [Orazzo and Hoepffner, 2012], Jon: [Jerome et al., 2013], Marco: [Hoepffner and Fontelos, 2013].

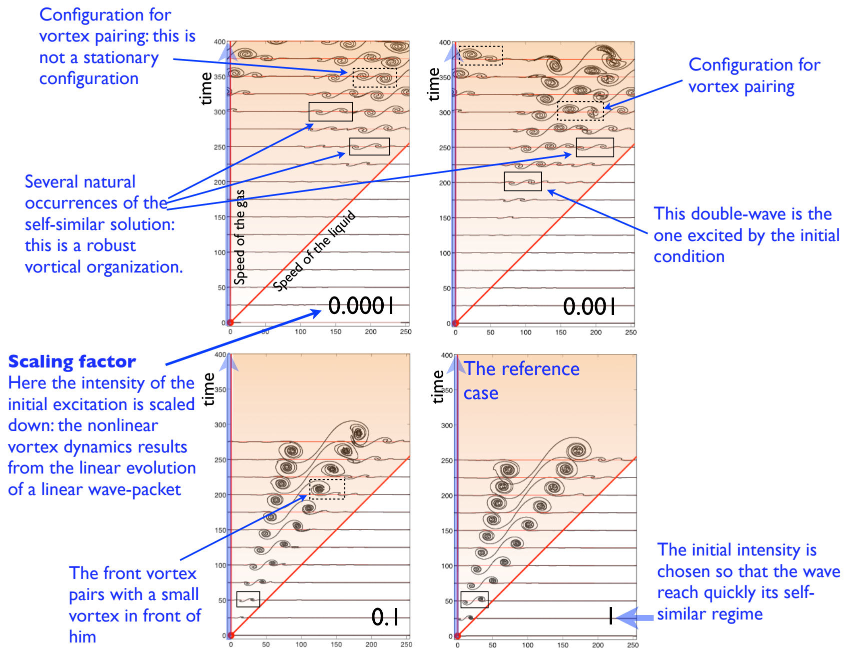



Figure 4.13: Impulse response of the shear-layer, with scaling of the initial intensity.

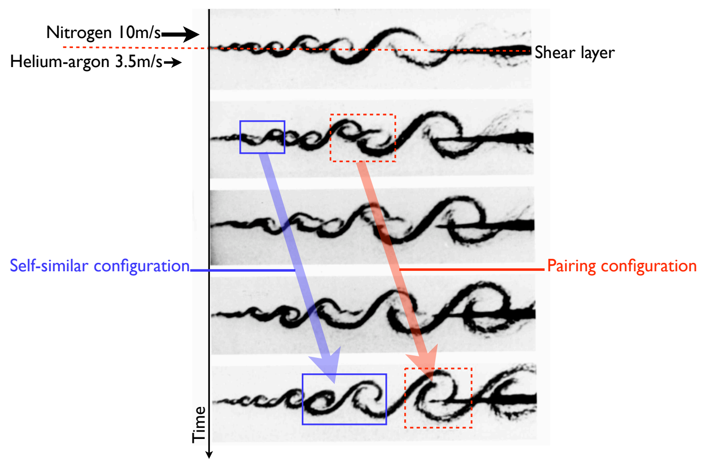



Figure 4.14: A shear layer at pressure 8 atmospheres and Reynolds 850,000. Image from [VanDyke, 1982].



# Chapter 5

# “Mentalities trudge along but technologies gallop”

“Les mentalitées cheminent mais les technologies gallopent”, said by Michel Cassé in [Cassé, 2005]. If it is true that technology goes a quick pace, should we run after it? Let us instead try to reach a state of mind from which we can understand the galloping. In this last chapter of the thesis, I play the game of opposing the classical axiomatic view of science and the work of the scientist seen as “creating archetypes”. I base this discussion on two excerpts of Matthew Crawford’s “The case for working with your hands”, [Crawford, 2010] and on quotes from Einstein’s “The world as I see it” [Einstein, 2009].

## 5.1 Cognitive dissonance and seasickness

> Working on my car without guidance, I felt constantly thwarted. Corroded nuts and bolts routinely broke or would round off; I came to be surprised when they simply loosened. Intermittent electrical gremlins eluded diagnosis. [. . . ] A lot seemed to depend on the weather. The car mocked my efforts to get a handle on it, as though it obeyed some evil genius rather than rational principles.
>
> Meanwhile, I was getting reacquainted with my father, living with him after six years away in the commune and another year living with my mother. A physicist, he would sometimes proffer some bit of scientific knowledge that was meant to be helpful as I sat on the ground in front of my lifeless engine. These nuggets rarely seemed to pan out. One day as I came into the house filthy, frustrated, and reeking of gasoline, my dad looked up from his chair and said to me, out of the blue, “Did you know you can always untie a shoelace just by pulling on one end, even if it’s in a double knot?” I didn’t really know what to do with this information. It seemed to be coming from a different universe than the one I was grappling

> with.
>
> Thinking about that posited shoelace now, it occurs to me maybe you can and maybe you can't untie it at a stroke—it depends. If the shoelace is rough and spongy, and the knot is tight, it will be a lot harder to undo if the knot is loose and the shoelace is made of something slick and incompressible, like silk ribbon. The shoelace might well break before it comes undone. He was speaking of a *mathematical string*, which is an idealized shoelace, but the idealization seemed to have replaced any actual shoelace in his mind as he got wrapped up in some theoretical problem. As a teenager, this substitution wasn't yet clear to me as such. But it began to dawn on me that my father's habit of mind, as a mathematical physicist, were ill suited to the reality I was dealing with in an old Volkswagen.
>
> Yet he seemed to know what he was doing as a scientist. This seemed like a contradiction. Weren't we dealing with the same physical reality? The dissonance between his utterances and my experiences planted the seed of a philosophical reflection that would come to fruition only twenty years later. The immediate effect was that I started to become a bit of a fatalist. [. . . ] As I groped my way toward a modus vivendi with the bug[1], I took my new fatalism to be a stinging rebuke to the pretense of easy intellectual mastery that my father was offering. So my own sense of impotence was weirdly delicious; it was based on a truer self-awareness than my father possessed, as I saw it.

This excerpt gives us a feeling of the mismatch between the experience of a phenomenon, and the mental representation that we have of this phenomenon. In his adolescence, Matthew did not have friends to give him guidance with the fixing of his car, and the only theoretical help he could get was from his father. We will see below in the second excerpt that the situation will change later, after meeting his friend Chas.

This mismatch is to *cognitive dissonance*. According to wikipedia, it is "the mental stress or discomfort experienced by an individual who holds two or more contradictory beliefs, ideas, or values at the same time, or is confronted by new information that conflicts with existing beliefs, ideas, or values". *Seasickness* is an extreme case of cognitive dissonance; according to wikipedia,

> it is a condition in which a disagreement exists between visually perceived movement and the vestibular system's sense of movement. [. . . ] The most common hypothesis for the cause of motion sickness is that it functions as a psychological defense mechanism against neurotoxins. The area postrema in the brain is responsible for inducing vomiting when poisons are detected, and for resolving conflicts between vision and balance. When feeling motion but not seeing it (for example, in a ship with no windows), the inner ear transmits to the brain that it senses motion, but the eyes tell the brain that everything is still. As a result of the discordance, the brain will come to the conclusion that the individual is hallucinating and further conclude that the hallucination is due to poison ingestion. The brain responds by inducing vomiting, to clear the supposed toxin.

[1] the car



We may say that seasickness is the *archetype* of cognitive dissonance (remember earlier in §2.4, Rayleigh mentioning the "dead weight on the scientific stomach"). Matthew Crawford did not vomit; he escaped the risk for intoxication by a long standing aversion toward mathematics:

> To repeat a point I made earlier, modern science adopts an otherworldly ideal of how we come to know nature: through mental constructions that are more intellectually tractable than material reality, and in particular amenable to mathematical representation. Through such renderings we become masters of nature. Yet the kind of thinking that begins from idealizations such as the frictionless surface and the perfect vacuum sometimes fails us (as my dad's advice failed me), because it isn't sufficiently involved with the particulars. Precisely because such thinking enjoys all the credit and authority of science, however, *when it fails us we may be tempted to see obscurity and unreason everywhere*, and even take pleasure in such obscurity. This reactionary tendency is a natural response to the pretense of modern reason. The reaction has an adolescent quality to it; there is a secret kinship between modernism and anti-modernism that just happens to mirror my relationship with my father. [. . . ]
>
> Some arts reliably attain their object—for example, the art of building. If the building falls down, one can say in retrospect that the builder didn't know what he was doing. [. . . ] Fixing things, wether cars or human bodies, is very different from building things from scratch. The mechanic and the doctor deal with failure every day, even if they are experts, whereas the builder does not. This is because the things they fix are *not of their own making*, and are therefore never known in a comprehensive or absolute way. This experience of failure tempers the conceit of mastery; the doctor and the mechanic have daily intercourse with the world as something independent, and a vivid awareness of the difference between self and nonself. Fixing things may be a cure for narcissism.
>
> Like building houses, mathematics is *constructive*; every element is fully within one's view, and subject to deliberate placement. In a sense, then, a mathematical representation of the world renders the world as something of our own making. Substituting mathematical strings for shoelaces entails a bit of self-absorption, and skepticism, too; the world is interesting and intelligible only insofar as we can reproduce it in ideal form, as a projection from our selves.

Mankind was not always aware in the possibility of these "constructive" activities. Here written by Albert Einstein in [Einstein, 2009]:

> We admire ancient Greece because it gave birth to western science. There, for the first time, was invented this masterpiece of human thinking, a logical system, that is, where propositions can be deduced from each other with an exactness such that demonstrations do not raise a single doubt. This is the system of Euclid's geometry. This admirable creation of human reasoning accredits the mind to acquire its self confidence for any new activity. And if someone, in the birth of his intelligence, failed to enjoy enthusiasm for such an architecture, then never shall he really penetrate theoretical research.

When we build a model to mimic the behavior of a physical system, we have a lot of freedom of being constructive as defined in Crawford's quote above. This freedom is offered to us by the use of mathematics. But on the other hand, we are bound by the reality of measurements: the experimental results.

> Reason is the structure of the system. Experimental results and their mutual imbrications can find their place thanks to deductive propositions. And it is in the possibility of such a representation that can be found exclusively the meaning and the logic of all the system, and more specifically, the concepts and principles that are at its root. Also, these concepts and principles are discovered as spontaneous creation of the human mind. They cannot be justified a priori by the structure of the human brain, nor, we must admit, by reason.

For Euclid's geometry, it is not straightforward to tell where does the "experimental result" come in. The theories of physics on the other hand are built as two parts, one the first hand a set of first principles, and then a tree of deductions:

> A mathematician friend recently told me, half-jokingly: "A mathematician actually knows something, but not exactly what he is asked at a given occasion". Often, the theoretical physicist finds himself in this situation when an experimental physicist consults him. What would be the cause for this characteristic lack of adaptation?
>
> The method of the theoretician implies that he uses as basis in all cases what we call first principles, from which he can deduce consequences. His activity is thus essentially divided in two parts. First he must seek the principles and then develop their inherent consequences. For this second part of the job, he is taught at school an excellent set of tools. If the first part of the job is already done in a given domain, he will certainly succeed by a determined work and reasoning. But the first key, that is the job of establishing principles that will serve for the deduction as a basis, has an entirely different aspect. Because then, there is no method that you can learn, or



> systematically apply to reach your goal. The researcher must rather spy, if one can say so, in nature these general facts, while he extracts from great ensembles of experimental facts some general and sure aspects, that can be clearly explicited.

So, the theoretician is trying, following the call of Descartes, to find the axiomatic basis, as concise as possible, that would be the root of a tree spanning the most salient observed facts from nature. And it is useful to stress the fact that the two activities: "seek the principles", and "develop their inherent consequences" are essentially different. Going from the established principles to the experimental facts is an automatic procedure, but going from the observed facts to the principles is an act of creation:

> These first principles, these fundamental laws, when they cannot be any more reduced by strict logic, uncover the unavoidable part of the theory. Because the essential goal of every theory is to dig for these fundamental and irreducible elements, as obvious and as few as possible, without forgetting about the adequate account of all possible experiment. An achieved system of theoretical physics has an ensemble of concepts, fundamental laws to be applied to these concepts, and logical propositions that can be deduced from it. These propositions where deduction is being used are exactly our daily experiences; *this is the deep reason why, in a theoretical book, deduction fills the entire book.*

A satirical cartoon of this classic view of science could be drawn as follows: great heroes[2] like Euclid, Galileo, Newton and Einstein do the work of spontaneous creation that provides mankind with a purified shortlist of principles. Then, the rather more common human beings will take over, and for some time do the tedious work of pushing as many as possible deductions that will span the world of observed phenomena.

## 5.2 The model as a looking glass

The scene takes place in Chas' workshop, several years after the "shoelace" excerpt. Now, Matthew is on the verge of being admitted in the elitist club of knowledgeable craftsmen.

> After some time in the solvent tank and some elbow grease, followed by hot soapy water, the internals of the torn-down motor were spotlessly clean. Chas then looked for signs of galling and discoloration that would indicate excessive heat, hence inadequate lubrication, or some other source of unacceptable friction. In fact there was some galling on the cam lobes, and the task now was to identify the root cause.

[2]Galileo: "Great scientists are very rare and, like eagles, scarcely audible; whereas well organized flocks of starlings kick up a racket", cited from [Arnold, 2007].

Root causes manifest as coherent *patterns* of wear, and knowledge of these patterns disciplines the perception of an engine builder; his attentiveness has a certain direction to it. He is not just passively receptive to data, but actively seeks it out. Pursuing an hypothesis, Chas looked for mushrooming at the tips of the valve stems, which bear on the cam lobes via rocker arms, push rods and lifters. Sure enough, some of the valves stems were slightly bulged out at their tips. Previously, as we were cleaning parts, I had held one of these valves in my hand and examined it naïvely, but had not noticed the mushrooming. Now I saw it. Countless times since that day, a more experienced mechanic has pointed out to me something that was right in front of my face, but which I lacked the knowledge to see. It is an uncanny experience; the raw sensual data reaching my eye before and after are the same, but without the pertinent framework of meaning, the features in question are invisible. Once they have been pointed out, it seems impossible that I should not have seen them before.

We see here emerging from this description a use of models that would be quite different from Einstein's quotes above. The classical attitude consist in taking some distance from facts, and find these first principles and relate them to experimental observations by using deductions. Now we see that the preconception that we have of the world, allow us to see things, or on the contrary, the misconceptions that we have of the world forbid us to access even its raw facts "right in front of our face".

We can pose the question of building models from this point of view. Instead of wanting to look for the shortlist of principles, we would like to build a representation of the world that would let us *see* most of the things that happen. These are clearly two different problems.

The quote above let us think that the representation that we have of the world would play the role of some "software for you head". This is the title of a book written by computer programmers who realized that it was not the individual technical capability of programmers that was the weak link in a teamwork, but rather their communication abilities, [McCarthy and McCarthy, 2001]. So, as genuine programmers, they coded a set of communication protocols, supposed to be loaded "in your head". We may formulate the question of modeling as follows. If, instead of putting all the emphasis on the laconicism of the list of first principles, we start to require *interactivity* with the world of phenomena, and *spontaneity*, what would be the structure of such a "software"?

## 5.3 Axiomatism, reductionism and emergence

Einstein, instead of talking about "software", talks about "image of the world[3]:

[3]In german: Weltbild. This is the title of Einstein's book: "Mein Weltbild".



Among all possible images of the world, what place should we give to that of the theoretical physicist? It involves the greatest demands, for rigor and accuracy, as only the use of mathematical language allows. But the physicist must, in concrete terms, restrain himself to account only for the most obvious phenomena accessible to our experience. Indeed, all the most complex phenomena cannot be reconstituted by human mind with the subtle precision and spirit required by the theoretical physicist. Extreme sharpness, clarity and certainty can only be learned at the cost of great sacrifice: the loss of an overview. But then what would be the seduction of understanding accurately only such a small piece of the universe and give up all that is more subtle and more complex? Is this shyness or lack of courage? The result of such a resigned exercise, could it wear the bold name of an "image of the world"?

I think that it indeed deserves this name. Because the general laws, basis of the intellectual architecture of theoretical physics, have the ambition to be valid for all events of nature. And thanks to these laws, using pure logical deduction, we should be able to find the *image*, that is, the theory of all phenomena in nature, including those of life, if this deduction process would not exceed to a large extent the ability of human thinking. This abdication to a physical image of the world in its entirety is not an abdication by principle, it is a choice, a method.

Einstein tells that there is indeed a choice. The attitude of believing that there is no choice to be made is called "reductionism". According to wikipedia,

in the sciences, application of methodological reductionism attempts explanation of entire systems in terms of their individual, constituent parts and their interactions. [...] In a very simplified and sometimes contested form, such reductionism is said to imply that a system is nothing but the sum of its parts. However, a more nuanced view is that a system is composed entirely of its parts, but the system will have features that none of the parts have. The point of mechanistic explanations is usually showing how the higher level features arise from the parts. [...]

Reductionism strongly reflects a certain perspective on causality. In a reductionist framework, the phenomena that can be explained completely in terms of relations between other more fundamental phenomena, are called epiphenomena. Often there is an implication that the epiphenomenon exerts no causal agency on the fundamental phenomena that explain it. The epiphenomena are sometimes said to be "nothing but" the outcome of the workings of the fundamental phenomena, although the epiphenomena might be more clearly and efficiently described in very different terms. *There is a tendency to avoid taking an epiphenomenon as being important in its own right.* This attitude may extend to cases where the fundamentals are not clearly able to explain the epiphenomena, but are expected to by the speaker. In this way, for example, morality can be deemed to be "nothing but" evolutionary adaptation, and consciousness can be considered "nothing but" the outcome of neurobiological processes.

> Reductionism does not preclude the existence of what might be called emergent phenomena, but it does imply the ability to understand those phenomena completely in terms of the processes from which they are composed. This reductionist understanding is very different from emergentism, which intends that what emerges in 'emergence' is more than the sum of the processes from which it emerges.

My idea is not so much wether we can or cannot "understand those phenomena completely in terms of the processes from which they are composed", I am mostly interested in "how spontaneously" can we go from an observation to a conclusion, and "how spontaneously" an adequate theoretical preconception can lead us to see new raw information. What "image of the world" will be most effective such as "not [being] just passively receptive to data, but actively [seeking] it out" like Chas did for mushrooming.

We find nevertheless an idea of *spontaneity* in physics. This is the "Fermi question". Enrico Fermi[4] considered that a proper scientist should be able to answer quantitatively any kind of question within a short notice. According to wikipedia:

> In physics or engineering education, a Fermi problem, Fermi quiz, Fermi question, Fermi estimate, or Order estimation is an estimation problem designed to teach dimensional analysis, approximation, and the importance of clearly identifying one's assumptions. The solution of such a problem is usually a back-of-the-envelope calculation. The estimation technique is named after physicist Enrico Fermi as he was known for his ability to make good approximate calculations with little or no actual data. Fermi problems typically involve making justified guesses about quantities and their variance or lower and upper bounds. [...] The classic Fermi problem, generally attributed to Fermi, is "How many piano tuners are there in Chicago?" A typical solution to this problem involves multiplying a series of estimates that yield the correct answer if the estimates are correct.

The axiomatic attitude is widespread but has not completely overtaken. Here an excerpt from the biographic recollections of mathematician Vladimir Arnold [Arnold, 2007] p17:

> After many generations of mathematical ancestors I also became a mathematician though our teacher, Anna Fedorovna, explained to my mother that I would not be able to finish the second year of school because I did not learn by heart the table of multiplication (and

[4](1901-1954), 1938 Nobel prize in physics.



> consequently I do not possess the mathematical ability necessary for arithmetic). "When I ask him what is four by seven, I see that he does not know this by heart, but very quickly adds in his mind".
>
> After that, my grandmother Vera Stepanovna taught me the multiplication table for ever. She made a pack of cards and wrote on one side of each card a question ("seven by eight", say), and one the other side, the answer ("fifty six"). The game was very simple: answer a question and turn the card; one wins if the answer is correct—this card is removed from the pack—and looses if the answer is wrong—the card goes back in the pack. The pack decreases quickly, and in an hour of such a game only three or four cards are left; correct answers for them one learns by heart very easily. Games teach better than punishments.
>
> I faced real difficulty with school mathematics several years after the multiplication table: it was necessary to learn that "minus multiplied by minus is plus". I wanted to know the *proof* of this rule; I have never been able to learn *by heart* what is not properly understood. I asked my father to explain the reason why $(-1) \times (-1) = (+1)$. He, being a student of great algebraists, S. O. Shatunovsky and E. Noether, gave the following "scientific explanation": "the point is," he said, "that numbers form a *field* such that the distributive law $(x + y)z = xz + yz$ holds. And if the product of minus by minus had not been plus, this law would be broken".
>
> However, for me this "deductive" (actually juridical) explanation did not prove anything [...]. Since that day I have preserved the healthy aversion of a naturalist to the axiomatic method with its non-motivated axioms.

In this example, we start to feel how the axiomatic attitude of building a shortlist can be far from the way our cognition works. In fact, again, it is not a question of moral, "is it good or is it bad?". It is a question of speed. Would the shortlist attitude help Matthew Crawford in front of his valve stem?

## 5.4 The axiomatic melancholy

The idea of segmenting an "image of the world" into a shortlist of axioms and long logical deduction is something that may have a cost. Indeed,

> At present, what situation is made in the social body of humanity to the man of science? To some extent, he can be proud that the work of his contemporaries, even very indirectly, has radically altered the economic life of men because it has virtually eliminated muscular work. But he is also discouraged since the results of his research have caused a terrible threat to humanity. Because the results of his investigations were overtaken by representatives of political power, these morally blind men. He also realizes the terrible

evidence of the phenomenal economic concentration caused by the technical methods from his research. He then discovers that political power, established on this basis, belongs to tiny minorities who run at will anonymous crowds, increasingly deprived of any reaction. More terrible still: the concentration of political and economic power around so few people not only causes the outer material dependence of the scientist, it threatens at the same time his profound existence. Indeed, the development of sophisticated techniques to direct intellectual and moral pressure prohibits the introduction of new generations of valuable human beings, but independent.

> The scientist today really has a tragic fate. He wants and desires truth and deep independence. But, with these almost surhuman efforts, he fathered the same means which reduce him externally in slavery and who annihilate his inner self. [...] He is so deeply deteriorated that he continues, obeying orders, to perfect the means for annihilation of his fellows.

Off course, the melancholy that we can feel in these world has also to do with the atom bomb; but not only. Man, viewing the world of phenomena from afar, has lost its immediate connection to a paradise which seems now irremediably lost. We recognize this feeling in a letter by Paul Ehrenfest[5]:

> The older we get, the more industrialized does physics become, and the more the worries of the world increase. So it becomes harder to escape into your work. In response, we start doing things we had been neglecting, for example rediscovering nature, traveling, reaching out to art and extending our circle of contacts. Amazed, we realize that such activities lend color to an otherwise grey life[6].

Here, the paradise is lost because of "industrialization". Contemporary scientists are lost in an axiomatic structure which no one can tell if it is a Babel tower on the verge of collapse or a labyrinth that is reaching its critical entanglement. Industrialization requires specialization: each and everyone digging its own corridor of the great maze, carefully adjusted with square angles; using the scale and compass of rational deduction. Here a quote from biologist Jacques Monod[7]:

> It took millennia before objective knowledge appeared as the only source of authentic truth. This austere and frigid idea which imposes an ascetic renouncement to any other spiritual nourishment, cannot ease our inborn anguish: on the contrary, it inflates it... and is not yet generally accepted. Nevertheless, it established thanks to its *prodigious power of performance*. In three centuries, science

[5](1880-1933), student of Boltzman and succeeded to Lorentz at the chair for theoretical physics at Leyden university.

[6]Physics today, january 2014, page 43.

[7](1910—1976), 1965 medicine Nobel prize.



> conquered its omnipotence: in practice but not in the souls. Modern societies are built on science. They owe to science their wealth, domination and faith that even mightier powers will tomorrow come within man's reach[8].

## 5.5 Constellations of archetypes

We need models because our intercourses with the world of phenomena happens through preconceptions. These preconceptions, or "image of the world", or again "software for your head", are not imposed to us forcefully: we inherit them from the growth and mutations of culture. The *prodigious power of performance* of science as it has become, gives us the feeling that there is no alternative.

But, as soon as we become conscious that we need preconceptions, we realize that we have as well the power to contrive these preconceptions. The formulation "software for your head" implies the power of the mind to self-engineer. *Let us self-engineer ourselves into scientific impressionists!* ;-).

In the body of this thesis, I expose several examples of my work. When I tried to find the common point between these results, I was led to the idea of the archetype: the particular case of a system which can serve as a meeting point for technical simplicity, cognitive clarity and phenomenological generality. It is not the same thing as a first principle, because the essence of a first principle is that it cannot be segmented into more fundamental constituents. A reductionist would call an archetype an "epiphenomenon". Why would we not segment any more each of our archetypes?

For peristaltic flow in §2.2 we could have cut down the building block of the Poiseuille law into the interaction of a continuous medium of material points. We would not segment any more our archetype because in gaining for instance for phenomenological generality and possibly technical simplicity, we would have lost in clarity. More specifically, we would have lost in "spontaneity". Replacing a connection of two Poiseuille flows with the Navier–Stokes equations, we give way to a long manipulation of first principles, for which the computer is the good tool, not the mind.

"All models are wrong but some are useful"[9]. If now we claim for ourselves the full freedom in forging our own image of the world, we should first decide what this image of the world should be useful for. Recall that

> The mathematician actually knows something, but not exactly what he is asked at a given occasion. Often, the theoretical physicist finds himself in the same situation when he is consulted by an experimentalist. What would be the cause for this characteristic lack of adaptability?[10]

[8]Jacques Monod "le hasard et la nécessité", page 185.

[9]quote by British mathematician George E.P. Box (1919—2013).

[10]Einstein, quoted in §5.1.

The axiomatic preconception is the fruit of an optimality problem seeking power of performance, and laconicism. Power of performance is the darwinist cause of its spreading and laconicism is the trigger for Kepler's euphoria in hearing the "Musica Universalis"[11]. Now, if we rather care for "mushrooming", we should optimize for spontaneity.

When in need to span the world of phenomenon, the cartesian tendency is to use a *sieve*. It is a grid that will help us to locate and distinguish the different elements on the surface of possibilities. The sieve is typically a square grid, and this squareness is chosen for its laconism, the strongest appeal to the axiomatic mind. The shape of this grid is motivated by an idea of the mind, but not by the structure of the surface that it is supposed to span.

It means that maybe it would be nice to find a structure that fits with the nature of our mind but also with the structure of the thing we are trying to span. Young cities on flat ground have a cartesian grid of streets, but it would be ill-advised to build a network of roads and tunnels that disregard rivers and hills. The adequate structure has an history (morphogenetic constraints) and is subject to the *particulars* of the landscape. There is in science an ambiguity between *discovery* and *creation*. According to Einstein's quote in §5.1, the fundamental principles of a theory are a spontaneous creation of the mind, but they are as well constrained by experimental facts. Richard Feymann phrases it like that: "the game I play is interesting, it's imagination in a straightjacket"[12]. A creation constrained by stringent facts is also a discovery.

The constellations that span the night sky are the meeting structures between preexisting stars and the mind of the one who is watching. The vertices are given, but we are free to chose the lines that connect these vertices. More generally, it is the realization of a culture to chose which stars should be connected and which one not. See figure 5.1 for a Pawnee night sky chart. So perhaps, the missing word that combines both discovery and creation could be *constellation*. In this sense, an archetype is a constellation.

We saw that great minds in their instant of inspiration can come up with first principles. After that, the work to link them to experimental observation is pure logical deduction. The axiomatic approach seeks to have as few as possible first principles, made once for all, but this system of preconception is not adjusted for spontaneity. Now, if we say that we would like to span the world of phenomena with archetypes, and that they articulate as stars into constellations to become a dynamic structure of our preconceptions, then it means that the role of the "hero", the great genius of scientific history, would also be offered to the daily scientist with his daily experiment. There is indeed room for "spontaneous creation of the mind" at all levels of the scientific description of reality, not only in the axiomatic depth of its roots.

Maybe, instead of being the stars of a constellation, archetypes could rather be pillars of a temple?

> Nature is a temple in which living pillars

[11] the harmony of the spheres

[12] Nova documentary "The best mind since Einstein" 1993.



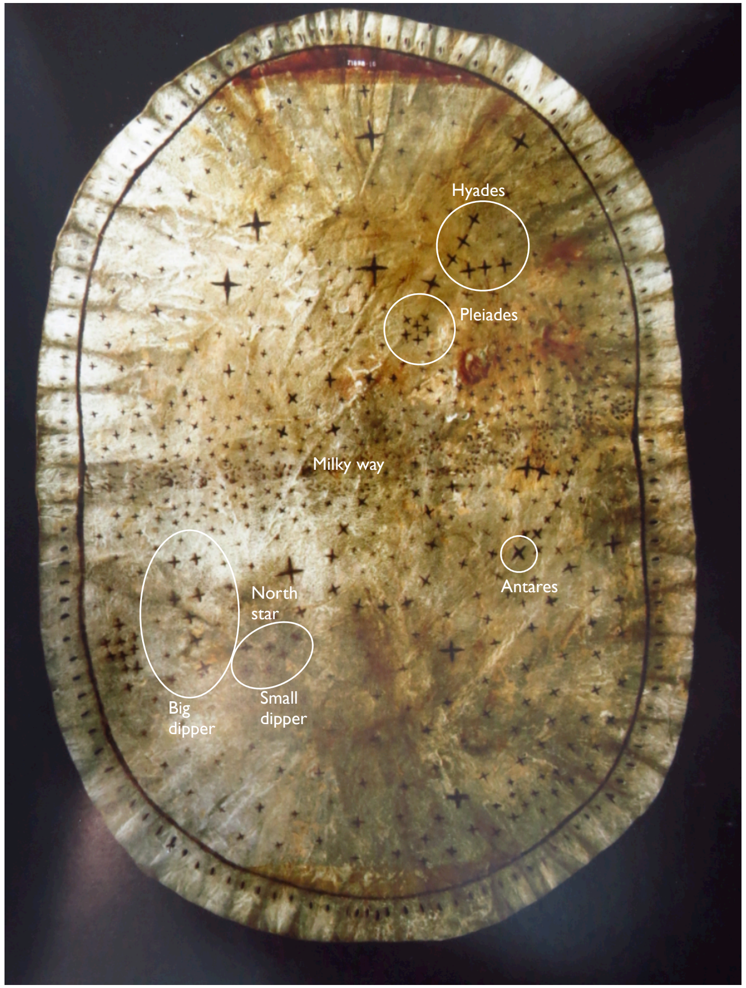



Figure 5.1: Pawnee map of the sky.

Sometimes give voice to confused words;
Man passes there through forests of symbols
Which look at him with understanding eyes.

Like prolonged echoes mingling in the distance
In a deep and tenebrous unity,
Vast as the dark of night and as the light of day,
Perfumes, sounds, and colors correspond.

There are perfumes as cool as the flesh of children,
Sweet as oboes, green as meadows
— And others are corrupt, and rich, triumphant,

With power to expand into infinity,
Like amber and incense, musk, benzoin,
That sing the ecstasy of the soul and senses[13].

## .1 A scientists' symposium

The story takes place at a time when Nasreddin is in favour. A famous Persian scientist arrives at the court of Timour the Lame as ambassador and, during his stay, asks his interpreter whether he can meet a Turkish scientist to compare the extent of their respective knowledge. The place chosen is the garden of the Emperor and Nasreddin is elected to the debate with him in front of all the dignitaries.

Both men start by standing in front of each other for a long time, then the Persian draws a circle on the ground with his cane. Immediately, Nasreddin draws a line dividing the circle in two. They stare again at each other, and Nasreddin draws another line, vertical this time, dividing the figure in four. He then makes a gesture with his hands to indicate that he takes the three quarters and he pushes away the remaining quarter. The Persian scientist answers by lifting one arm in the air, he then lowers it brutally, to which Nasreddin reacts by raising his fist high up. His opponent starts walking on the circle before running around it. Nasreddin takes an egg out of his pocket and exhibits it in front of everyone.

The Persian scientist probably thinks that their exchange of knowledge is at an end for, after saluting his colleague, he retires with his suite.
"This Turkish scientist is very learned", he confides to his assistants. "He knows as much as I do and never have I had such a pleasant exchange. I will tell our shah that with such a man, Timour the Lame is invulnerable." "Master", his disciples ask, "what have you said to each other? We haven't understood anything.

"Of course, this is a level you haven't reached yet. The subject I had chosen for our debate was the creation of the world. To start with, I asked him, 'Do you know that the Earth is round?' He answered with a line: 'Indeed, and this is the Equator.' Then, with another line, he told me, 'Remember that three quarters

[13]Baudelaire, "les fleurs du mal", translation William Aggeler [Aggeler, 1954].



of it are occupied by the sea, and one quarter by the land.' I then continued the dialogue by telling him that the land is nevertheless watered by the rain. 'And by the springs gushing forth', he said. I was so happy that of our understanding that I invited him finally to rejoice in this marvelous creation enabling millions of animals to frolic. 'Do not forget the birds!' he said to conclude. And all that without a word, my friends, What intelligence!"

Nasreddin is also well commended. Timour the Lame is proud to see how his buffoon distinguished himself.
"Tell us, Nasreddin. Don't make a secret of your exchange."
"Oh, Lord! This man is an impostor and I didn't take long to get rid of him. He started by ordering me: 'Hey, the Turk, bring me a plate of börek' I answered, "Yes, but we'll share it.' He looked furious. So, I added, 'In that case, I shall have three quarters and you will have to make do with the rest.' At this point, he raised his fist, and I threatened him: 'Watch it! I shall smash you face.' He then proceeded to insult me by calling me a dog, a jackal, a pig. I wasn't going to let myself be insulted. I screamed, loud enough to be heard by everyone: 'Go back to your country. You're a cowardly hen!'" [14]

[14]Sublime Words and Nonsense of Nasreddin, [Maunoury, 2002] translation Bernard Hoepffner.

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