PERISTALSIS AND HYDRODYNAMIC INSTABILITIES

Jérôme Hœpffner and Koji Fukagata
Dept. of Mech. Eng., Keio university, 3-14-1 Hiyoshi, Kohoku-Ku, Yokohama 223-8522, Japan.

Summary Peristaltic pumping is considered in view of early nonlinear mechanisms in hydrodynamic instabilities. A propagating wall deformation generates pressure gradients in the flow, which act together with viscous friction to induce a mean flux. This process is analyzed in a plane channel, and a model is derived which agrees well with computed flow solutions. For the considered wall deformation, the pumping effect is found independent of the Reynolds number. Implications for instability in flows bounded by compliant walls are discussed.

"Peristaltic pumping is a form of fluid transport that occurs when a progressive wave of area contraction or expansion propagates along the length of a distensible tube containing liquid", see [1]. It is used for instance physiologically by the body to propel and mix the content of the gastro-intestinal tract, or in roller pumps when it is undesirable that the fluid come into contact with the mechanical parts of the pumping device. In view of the biological applications at hand, it was intensively studied assuming low inertia effects. The domain of validity of asymptotic expansion analyses in the Reynolds number and finite wavelength was studied computationally in [2]. In [3] the interaction of peristalsis with externally imposed Poiseuille flow is analyzed. Peristalsis have much features in common with steady streaming, see [4]. In [5], peristalsis/streaming is considered as a mean to transform high frequency vibrational motion into rotation.

At high Reynolds number, waves of finite amplitude resulting from hydrodynamical instabilities may induce mean flow deformation with "pumping" characteristics. Peristalsis is considered here in this context. We wish to study it in a configuration where the impact of inertia can be clearly isolated. We chose a plane channel flow with a steep wall deformation – as opposed to the common sinusoidal one. This choice was motivated by the need of distinguishing two regions: an expanded one and a constricted one, connected by short expansions.

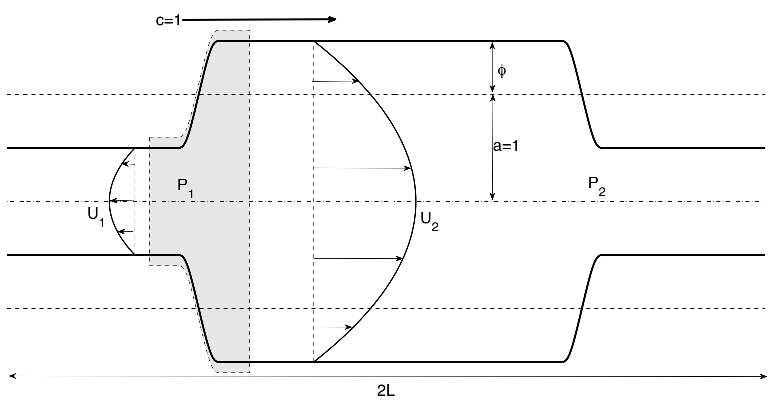

Figure 1. Flow geometry with main parameters

A sketch of the considered geometry is presented in figure 1: the plane channel has its height reduced as if locally constricted by a cuff, sliding towards the right. The wall motion is constrained in the vertical direction only, the wall acting on the flow through the propagation of this deformation to the right at speed c. We consider a configuration periodic in the streamwise direction. The parameters are the channel half height a, the compression ratio ϕ, the wavelength $2L$, the average velocities in the constricted and expanded regions U_1 and U_2, and the propagation speed c. The Reynolds number is based on channel height and propagation speed $Re = ac/\nu$.

The pumping effect has a viscous origin: due to the propagation of the constricted region, a high pressure is generated in P_1 where the fluid is pushed down, whereas a low pressure is generated inP_2 where the flow is sucked up by the vertical displacement of the wall. Flows along the channel are induced to balance this periodic pressure gradient: to the right in the expanded region and to the left in the constricted region. The larger viscous friction in the constricted section will induce an asymmetry in favor to the rightward flow, thus a mean flux in the direction of propagation of the wave.

To quantify this mechanism, we note that the pressure gradient for a parallel flow in a channel is that of a Poiseuille flow with same flux. Combining this with the conservation of volume – for instance in the shaded area of figure 1 – we obtain two equations linking U_1, U_2 to the flow parameters. We can then derive an expression for the resulting mean flux

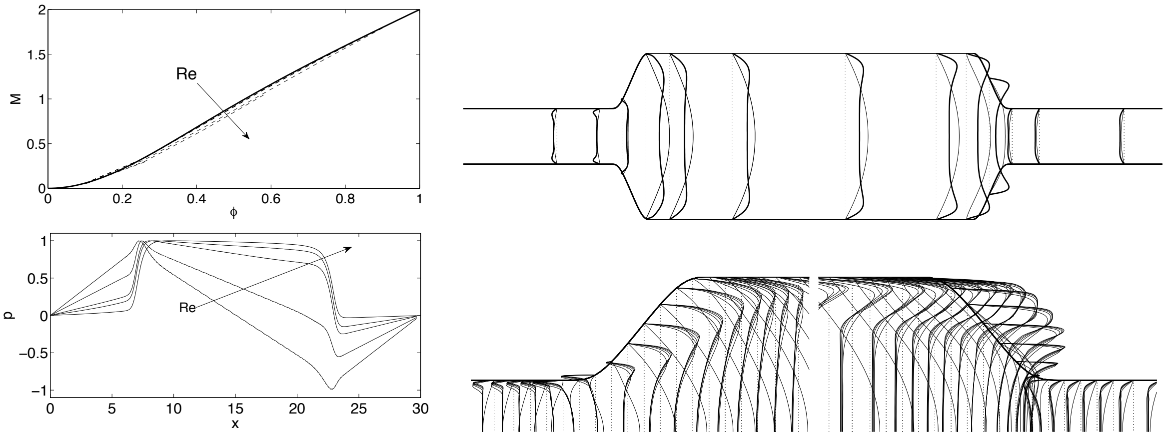

Figure 2. *Top-left*: Comparison of the computed mean flux (thin-dashed) to the model (thick-solid) when varying the constriction severity ϕ. Computed results are presented for $Re = 0.1, 1, 10, 100, 1000, 5000, 10000$. *Top-right*: Computed streamwise velocity profiles in the domain for $Re = 0.1$ (thin solid) and $Re = 1000$ (thick solid) for semi-obstruction $\phi = 0.5$. *Bottom-right*: close-up on the streamwise velocity profiles about the front and rear edges of the constriction at $\phi = 0.5$ for Reynolds numbers $Re = 10, 2000, 4000, \ldots, 10000$. *Bottom-left*: Centerline pressure along the channel, normalized to unit maximum value, for $\phi = 0.5$ and Re from 0.1 to 100

$$M = 2\phi \frac{(1+\phi)^3 - (1-\phi)^3}{(1+\phi)^3 + (1-\phi)^3}. \quad (1)$$

It vanishes for straight channel ($\phi = 0$) and has maximum value $M = 2$ for complete channel obstruction ($\phi = 1$). This functional dependency is depicted in figure 2. There is a quadratic behavior at small constriction ratio, followed by a nearly linear range. This expression obtained for steep wall deformation can be compared to that obtained from the Stokes equation at infinite wavelength for a sinusoidal constriction $M = 3\phi^2/(2+\phi^2)$, see [1]. Note that (1) is independent of the Reynolds number, since on one the hand the details of the velocity profile in the parallel regions does not affect the pressure gradient and on the second hand the viscous effects in the two regions oppose each other.

We compare the model (1) with the mean flux obtained from the computed flow fields for a wide range of Reynolds number and constriction ratii, for a long wavelength $2L = 30$. The Navier-Stokes equations in primitive variables are discretized in space using the fourth order finite difference scheme, assuming periodicity in the streamwise direction, and symmetry about the channel centreline. The domain deformation and mesh point clustering in the region of high shear are implemented using a domain mapping ([6]). In the reference frame travelling with the wave, the flow is steady and can be computed using the Newton iterations.
The results are gathered in figure 2. Despite the complexity of the computed velocity profiles, the obtained mean fluxes agree well with the model for all considered Reynolds number (0.1 to 10000) and for all constriction ratii. At low Reynolds, the flow in the straight regions have Poiseuille profiles. At larger Re, the profiles are flat about the centerline, with large overshoots towards the wall where the pressure gradient is steepest due to wall motion, and with thin boundary layers at the walls. The pressure along the channel centerline has the expected linear dependency in the straight sections. The pressure shows jumps of increasing amplitude about steep expansion regions as the Reynolds becomes large, with relatively slow evolution in the straight regions – this is a tendency toward to the inviscid limit, in which the pressure is constant in the straight regions, and the large gradients in the regions of change of channel height compensate each other exactly, with no pumping effect.

In the case of a channel or a boundary layer bounded by compliant walls, we suspect that the propagating wall deformation resulting from shear instability will induce a pumping effect by the mechanism described in this paper. The possible impact on stability evolution should be investigated.

References

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