



Institut de Recherche
sur les Phénomènes
Hors Équilibre

Contrôle des écoulements - la sténose -

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Post-Doc

Avec Stéphane Le Dizès, Thomas Leweke, Martin Griffith
"Primaires IRPHÉ"

pipe2d plotbase jolie figure w



Cursus

1) DEUG, licence, maîtrise:

Université Paul Sabatier, Toulouse
(Alessandro Bottaro, Christophe Airiau...)

2) Thèse de doctorat:

à KTH (Stockholm) avec *Dan Henningson*

"Stability and control of shear flows subject to stochastic excitations"

Cooperations avec Thomas Bewley (UCSD), Uwe Ehrenstein (Nice).

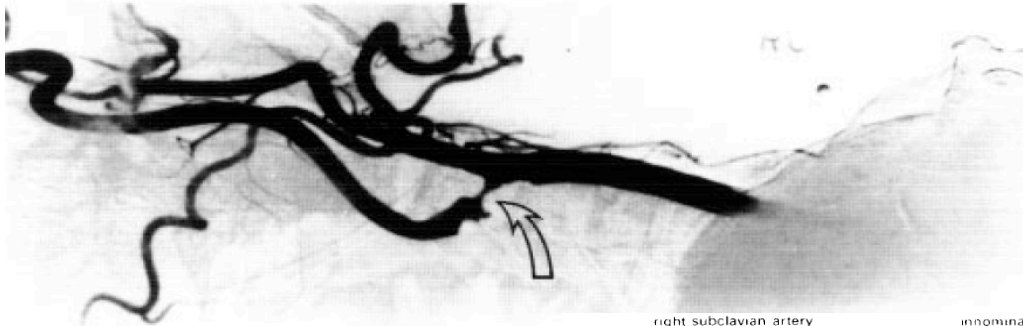
3) Thèmes récurrents:

- Étude de stabilité par méthodes d'optimisation (croissance transitoire...)
- Contrôle optimal
- Description stochastiques des perturbations
- Transition à la turbulence en couche limite

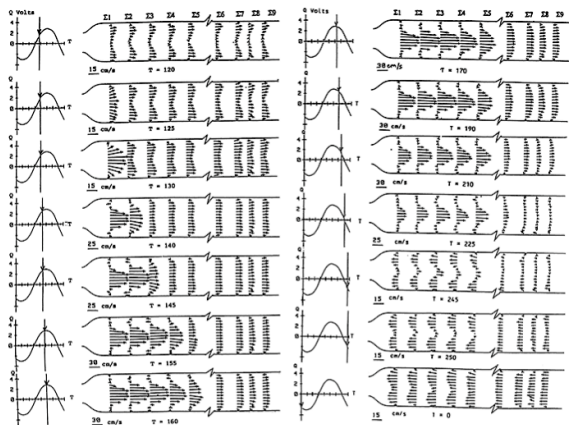
Sténose et manip

D. N. Ku,
Annu. Rev. Fluid Mech. 1997
Blood flow in arteries

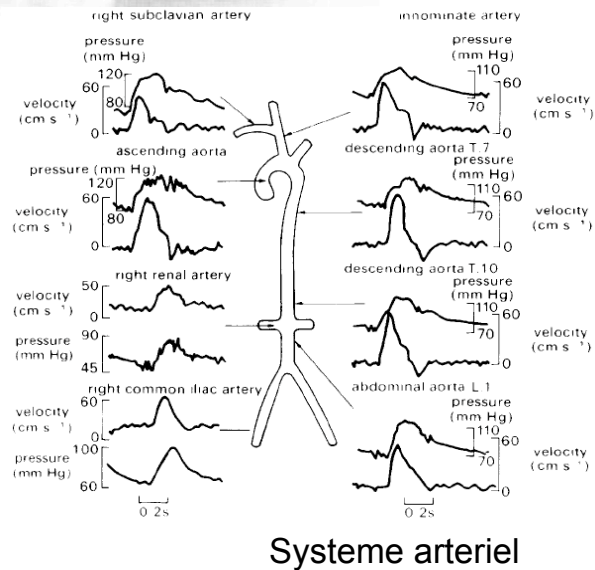
Sténose carotidienne:



Mesures de profil en sténose:



M. Siouffi, V. Deplano, R. Pelissier,
Experimental analysis of unsteady flows through a stenosis.
Journal of biomechanics, 1997.

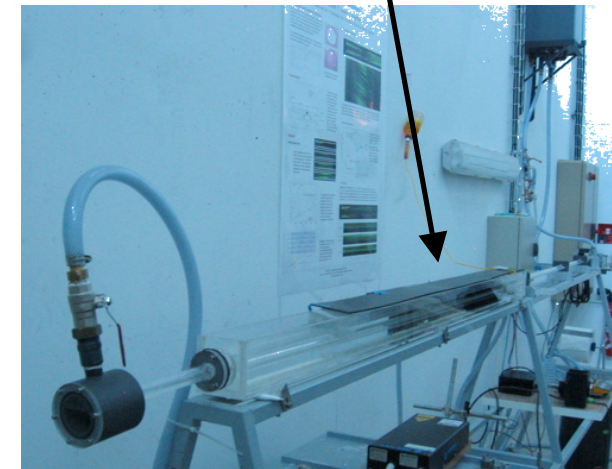


Systeme arteriel

Trois sténoses en alu:



Notre tuyau:



Présentation de l'écoulement

Faible Reynolds: bulle de recirculation stationnaire

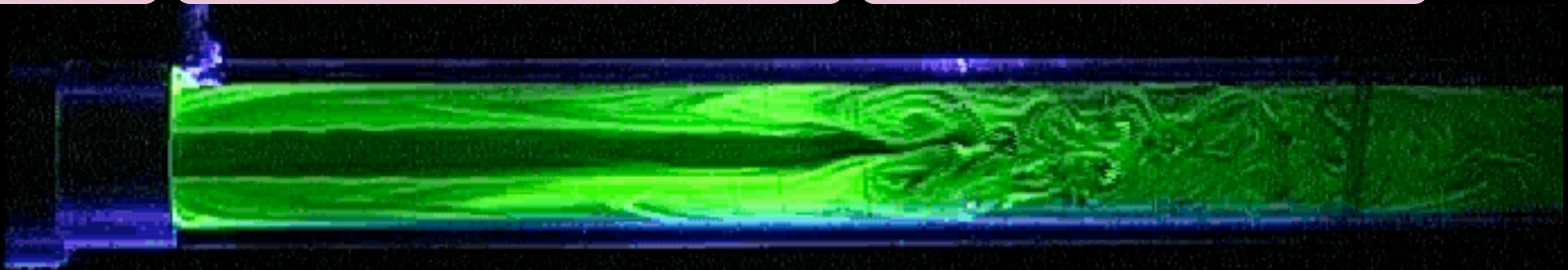


Reynolds modéré: cycle auto-entretenu

sténose

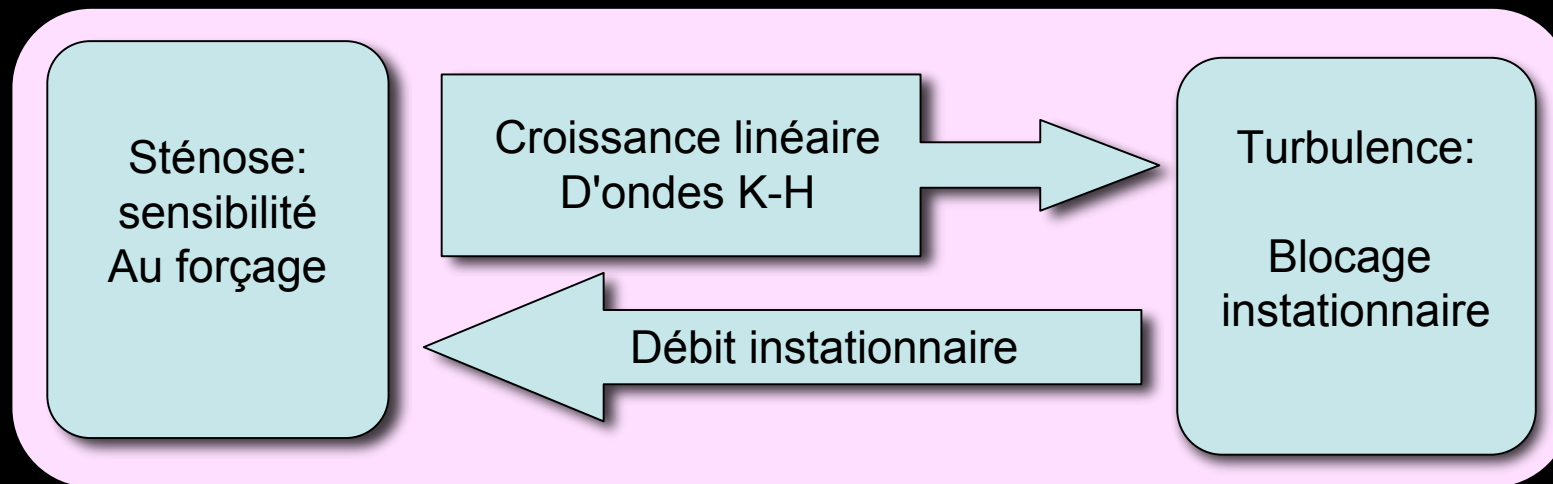
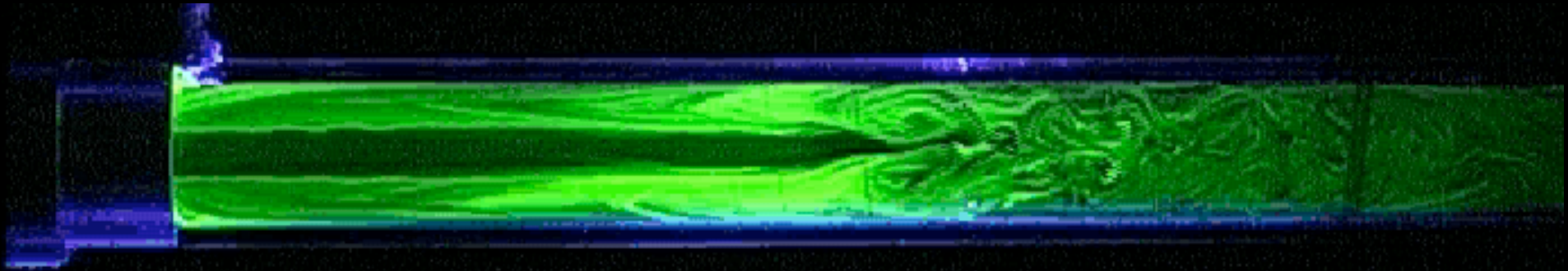
Kelvin-Helmholtz

turbulence



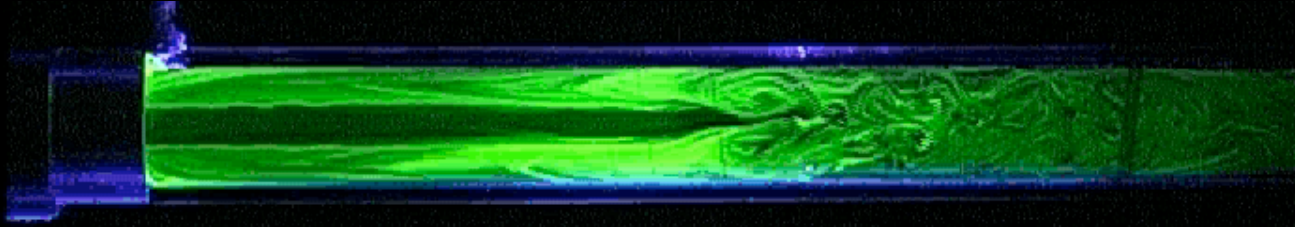
Reynolds élevé: Instabilité globale linéaire (Koanda)

Écoulement auto-entretenu



Cycle global nonlinéaire

Spatio-temporel

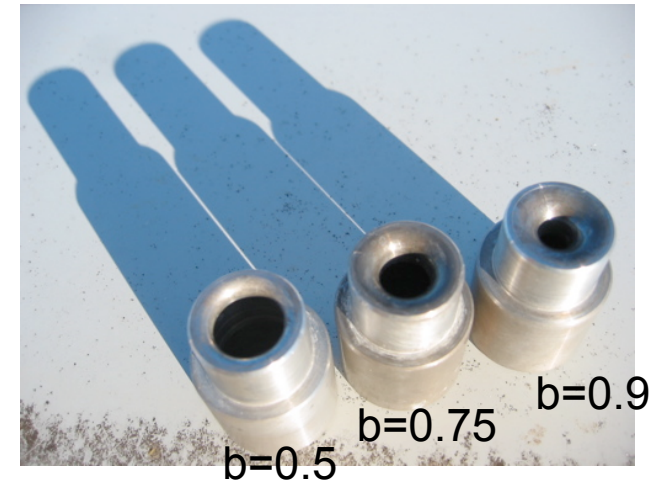
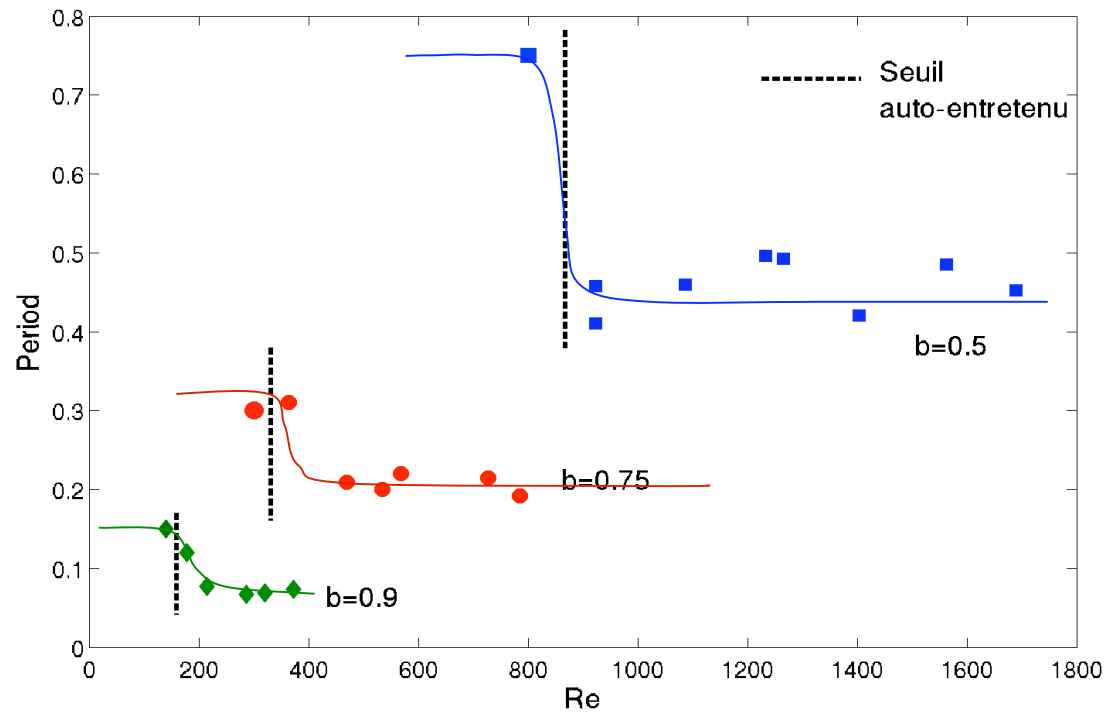


temps

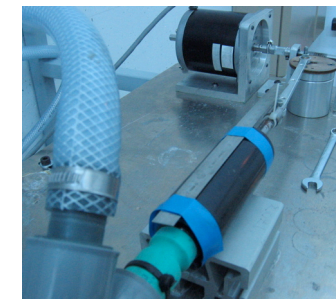


Oscillations

Avant le seuil: forçage
Après le seuil: oscillation

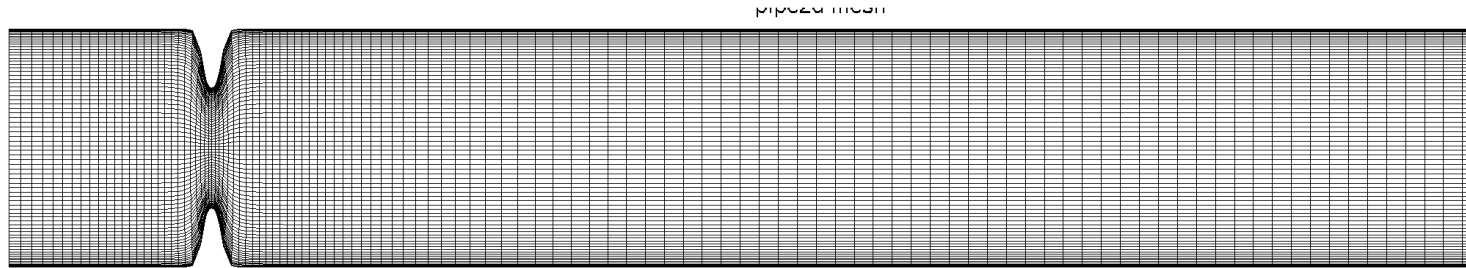


Période auto-entretenue
plus courte que
Période plus amplifiée



- 1) L'instabilité est excitée par la turbulence
- 2) Une large bande de fréquence est amplifiée
- 3) Les hautes fréquences croissent plus vite dans l'espace

Tuyau numérique

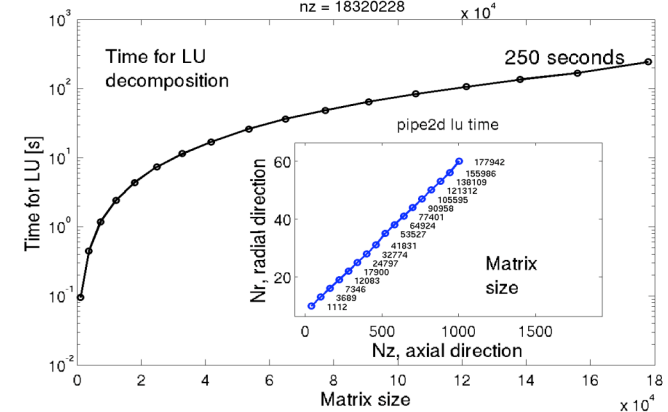
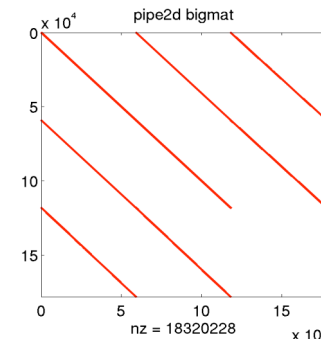


- 1) Chebyshev en radial, differences finies en axial
- 2) Transformation conforme pour la stenose
- 3) Variable primitives: u, w, p (eq. de cont. explicite)

$$\begin{pmatrix} \partial_t u \\ \partial_t w \\ 0 \end{pmatrix} = \begin{pmatrix} \dots & \dots & -\partial_r \\ \dots & \dots & -\partial_z \\ \partial_r + 1/r & \partial_z & 0 \end{pmatrix} \begin{pmatrix} u \\ w \\ p \end{pmatrix}$$

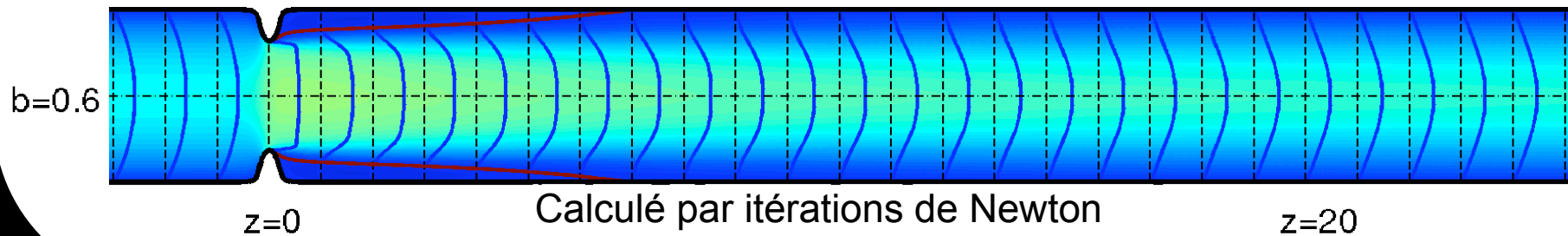
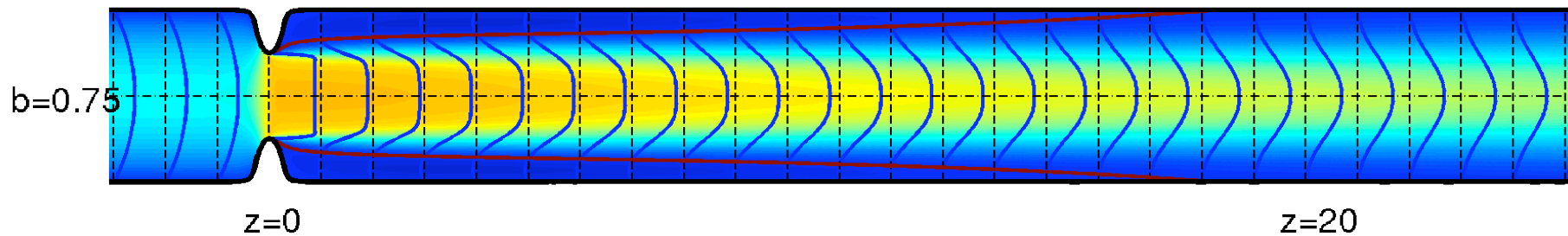
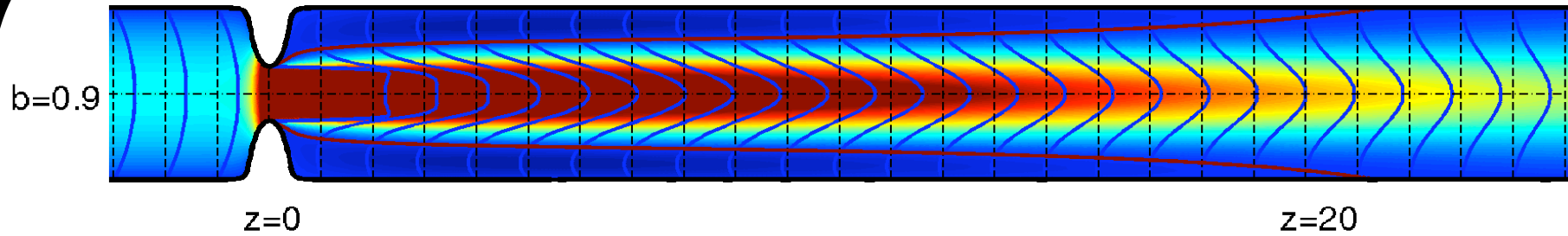
→ avantages : larges matrices creuses,
decomposition LU rapides

- 4) Newton pour l'écoulement de base (stationnaire)
- 5) Krylov pour les modes propres (U. Ehrenstein)



Résultats numériques

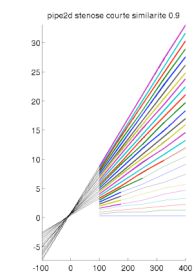
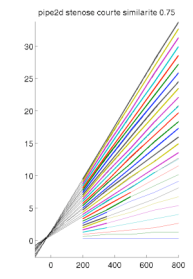
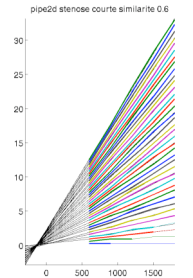
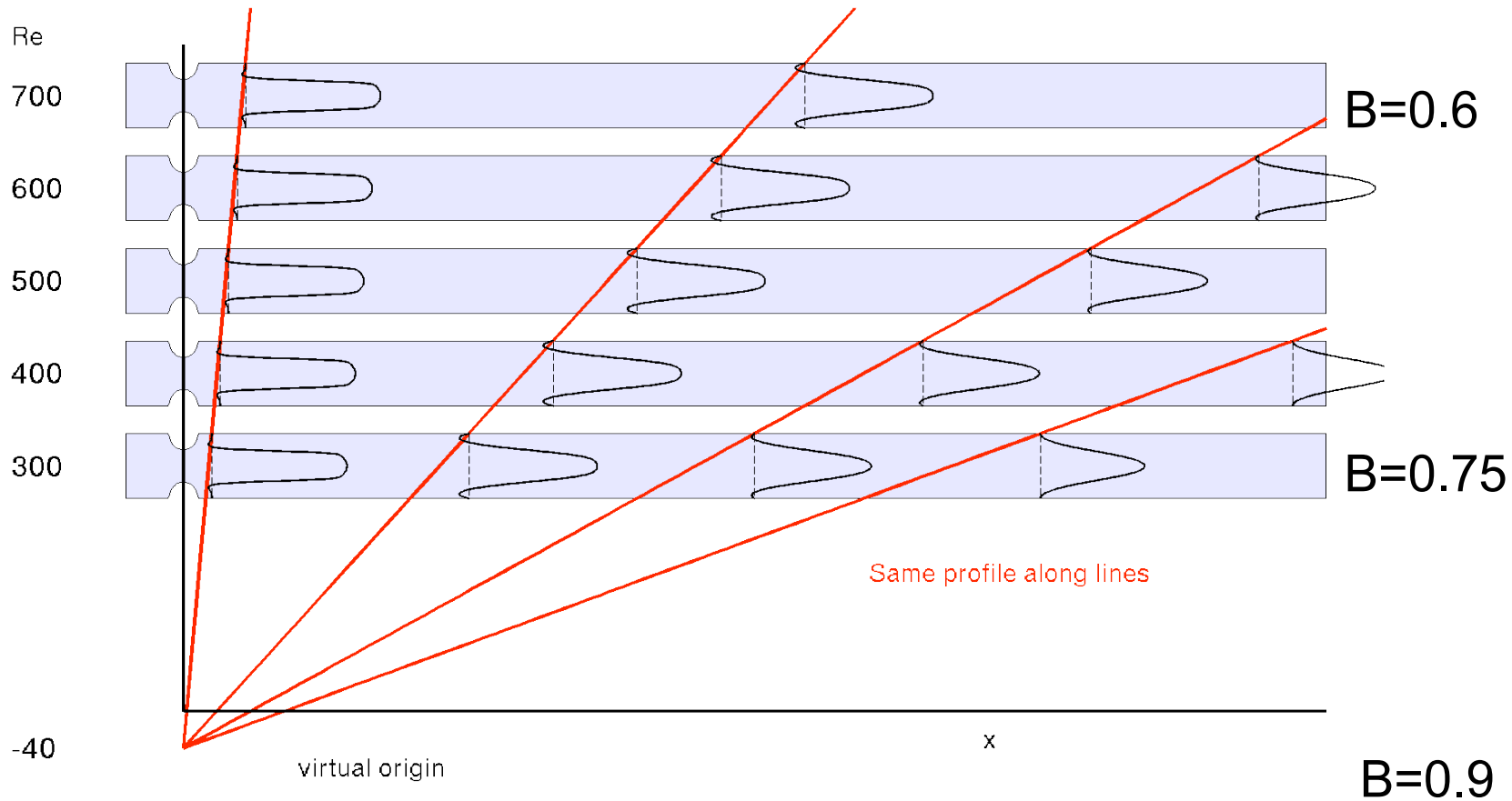
Ecoulement de base, vitesse axiale:



pipe2d base flow

Calculé par itérations de Newton

Similarité, écoulement de base



Il suffit de calculer l'écoulement de base pour un seul Reynolds

Forçage harmonique

Forçage à la sténose, sur vitesse axiale:

Fonction de forçage:

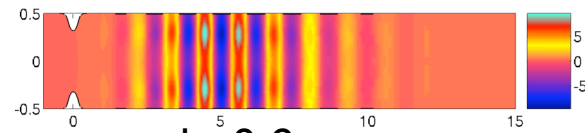
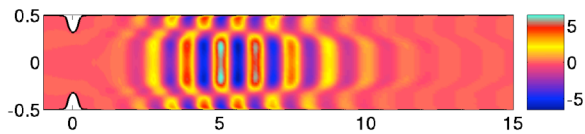
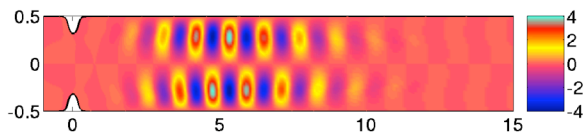


$$E\dot{q} = Aq + Bu,$$

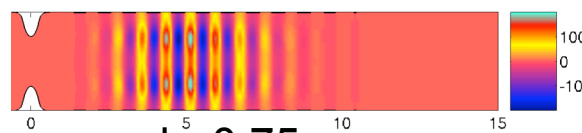
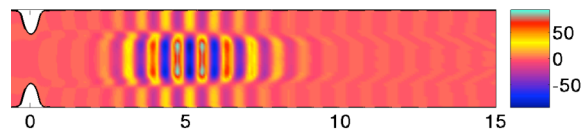
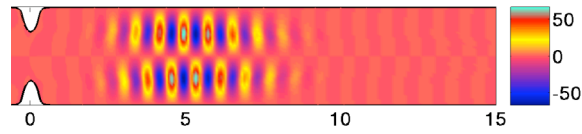
$$u = \hat{u}e^{i\omega t}, \quad q = \hat{q}e^{i\omega t},$$

$$\Rightarrow \hat{q} = \underbrace{(i\omega E - A)^{-1} B}_{\mathcal{R}} \hat{u}$$

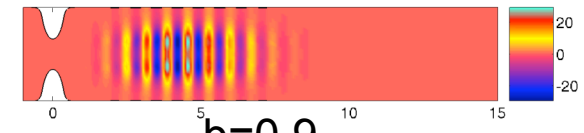
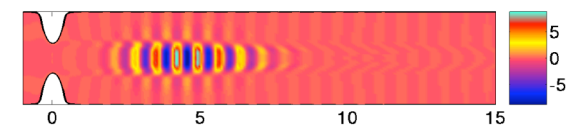
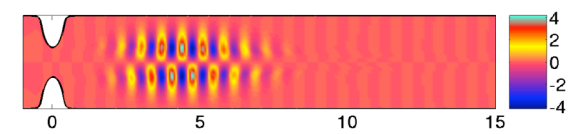
Vitesse radiale, axiale, pression:



$b=0.6$

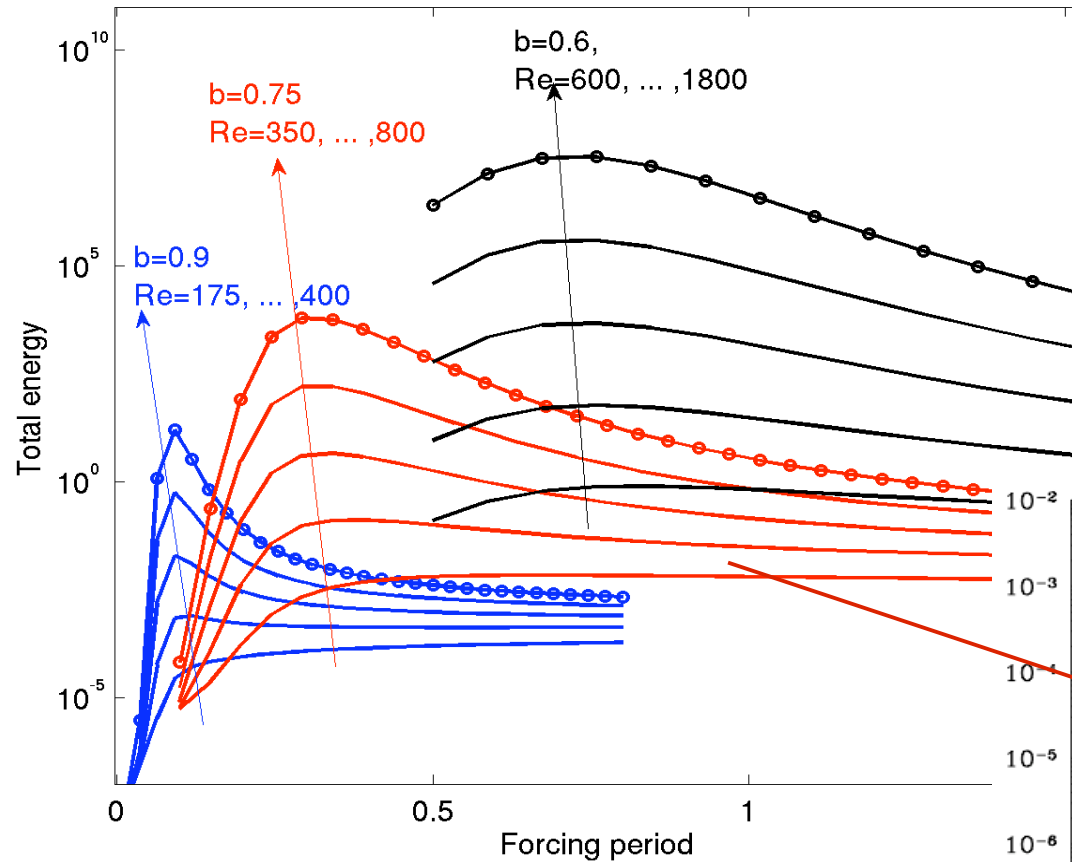


$b=0.75$



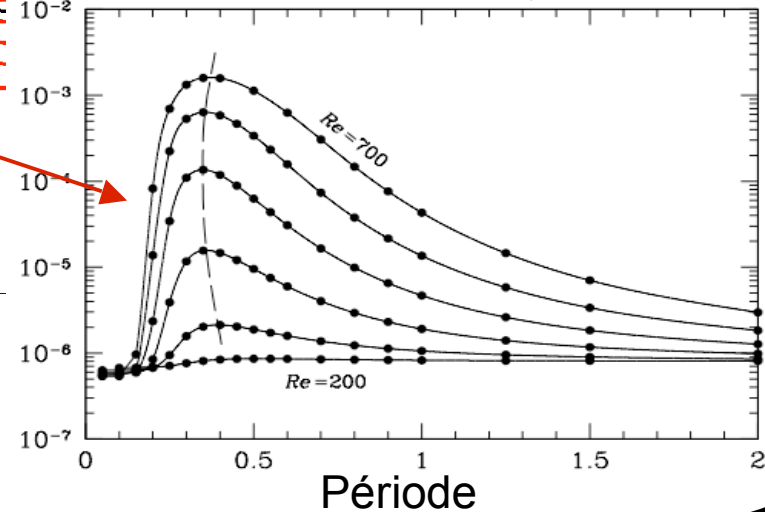
$b=0.9$

Réponse énergétique

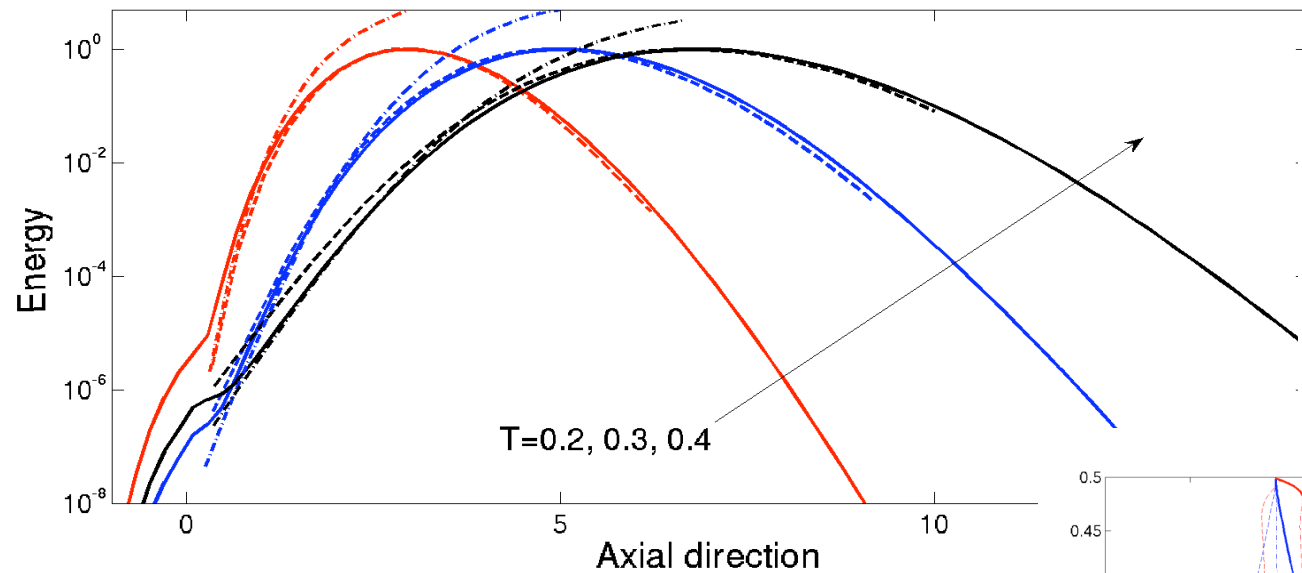


En variant la période de forçage, le Reynolds et la sténose

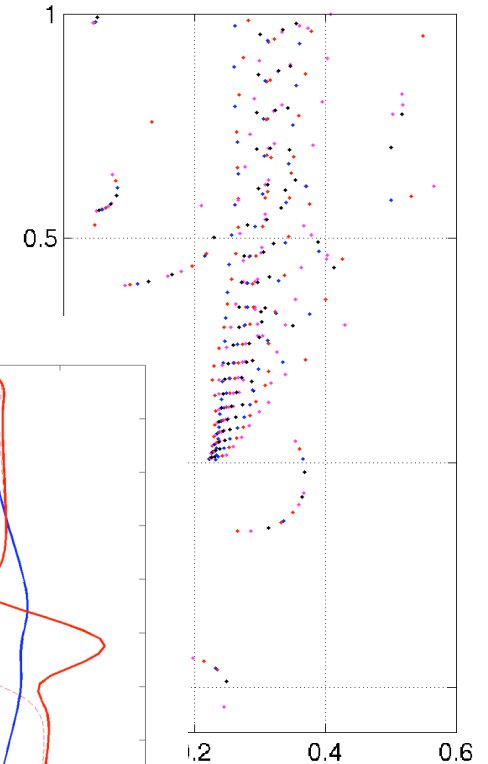
B=0.75, forçage piston, Sténose longue Sherwin&Blackburn JFM, 2006.



Instabilité convective



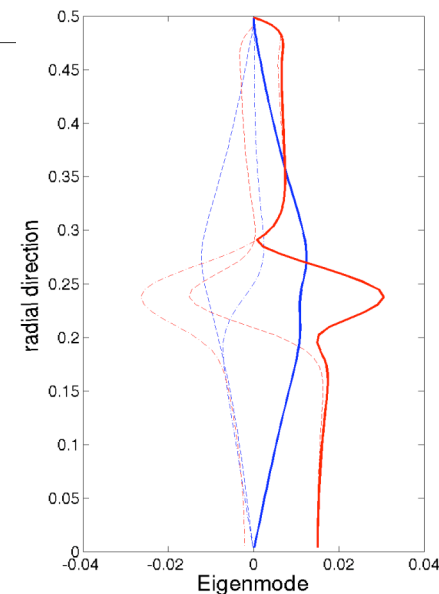
Spectre local,
En variant z:



Comparaison: forçage 2D,
analyse de stabilité

Continu: forçage 2D
Pointillet: stabilité locale
Pointillet mixte: local non-visqueux

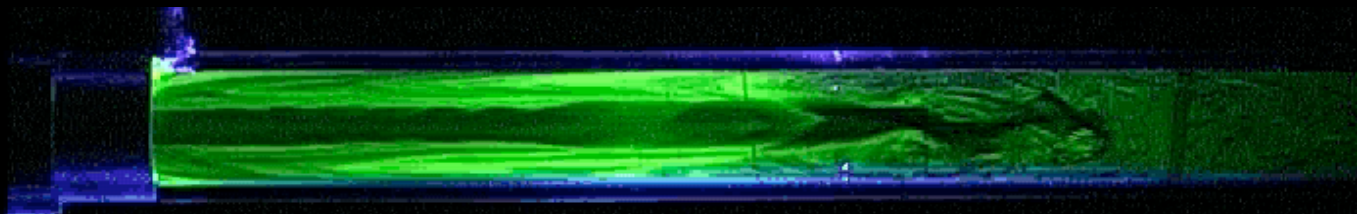
Mode propre:



Le contrôle,
ou, comment stabiliser notre écoulement?

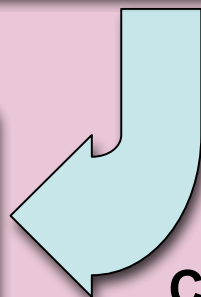
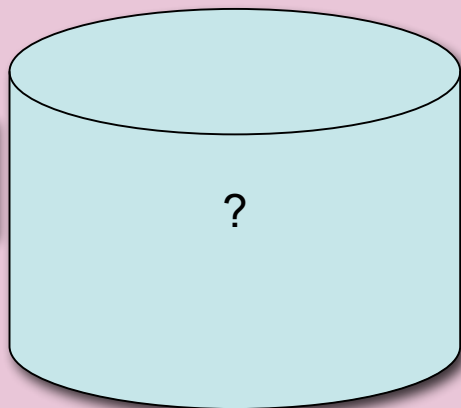
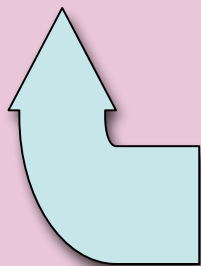
Perturbations

Contrôle



Un actionneur
Pour agir

Une sonde
Pour mesurer



CON
TRÔ
LEUR

Perturbations

Écoulement

CON
TRÔ
LEUR

Nouvelle
dynamique de
l'écoulement
contrôlé

Contrôle

L'état est connu,
quel est le meilleur contrôle?

$$\begin{cases} \dot{q} = Aq + B_1 w + B_2 u, \\ y = Cq + g \end{cases}$$

Feedback: $u = Kq$

Nouvelle dynamique:

$$\dot{q} = \underbrace{(A + B_2 K)}_{A_c} q + B_1 w$$

Eq de Lyapunov:

$$A_c^+ P + P A_c + B_1 W B_1^+ = 0$$

Mult. De Lagrange: Eq de Riccati:

$$\begin{cases} 0 = A^+ \Lambda + \Lambda A - \Lambda B B^+ \Lambda / \ell^2 + Q, \\ K = B^+ \Lambda / \ell^2. \end{cases}$$

Estimation

Une mesure est connue,
quel est l'état?

$$\begin{cases} \dot{\hat{q}} = A\hat{q} - L(y - \hat{y}), \\ \hat{y} = C\hat{q} \end{cases}$$

Erreur d'estimation: $\tilde{q} = q - \hat{q}$:

Nouvelle dynamique:

$$\dot{\tilde{q}} = \underbrace{(A + LC)}_{A_e} \tilde{q} + B_1 w - Lg$$

Eq de Lyapunov:

$$A_e \tilde{P} + \tilde{P} A_e^+ + B_1 W B_1^+ + LGL^+ = 0$$

Mult. De Lagrange: Eq de Riccati:

$$\begin{cases} 0 = A\tilde{P} + \tilde{P}A^+ - \tilde{P}C^+C\tilde{P}/\alpha^2 + W \\ L = -\tilde{P}C^+/\alpha^2. \end{cases}$$

Le contrôleur

Signal de
contrôle

u

Signal de
mesure

y

$$\begin{cases} \dot{\hat{q}} = \overbrace{(A + B_2K + LC)}^{A_0} \hat{q} + Ly, \\ u = K\hat{q} \end{cases}$$

CON
TRÔ
LEUR

$$\begin{aligned} \text{Système: } & \begin{cases} \dot{q} = Aq + B_1w + B_2u, \\ y = Cq + g \end{cases} \\ \text{Estimateur: } & \begin{cases} \dot{\hat{q}} = A\hat{q} - L(y - \hat{y}), \\ \hat{y} = C\hat{q} \end{cases} \end{aligned}$$

Utilisons l'état estimé pour

Le contrôle: $u = K\hat{q}$

Dynamique de l'écoulement contrôlé:

$$\begin{pmatrix} \dot{q} \\ \dot{\hat{q}} \end{pmatrix} = \begin{pmatrix} A & B_2K \\ -LC & A + B_2K + LC \end{pmatrix} \begin{pmatrix} q \\ \hat{q} \end{pmatrix} + \begin{pmatrix} B_1 & 0 \\ 0 & -L \end{pmatrix} \begin{pmatrix} w \\ g \end{pmatrix}$$

Le contrôleur:

$$\begin{cases} \dot{\hat{q}} = \overbrace{(A + B_2K + LC)}^{A_0} \hat{q} + Ly, \\ u = K\hat{q} \end{cases}$$

Réduction de modèle

Ce système doit être
Simulé en temps réel:

$$\begin{cases} \dot{\hat{q}} = \overbrace{(A + B_2K + LC)}^{A_0} \hat{q} + Ly, \\ u = K\hat{q} \end{cases} \quad \text{CON} \\ \text{TRÔ} \\ \text{LEUR}$$



Balanced truncation:

- 1) Excite le système **direct** par l'input **u**:
→ Sous espace énergétique
- 2) Excite le système **adjoint** par l'output **y**:
→ Sous espace sensible
- 3) **Projete** Navier-Stokes sur l'intersection
→ Modèle réduit

Il faut construire un modèle
réduit de la relation
entre **y** et **u**

(pas du système tout entier!)

Résolution de deux équations
de Lyapunov

Déjà fait sur le sujet

State estimation in wall-bounded flow systems. Part 1. Perturbed laminar flows

By JÉRÔME HËPFFNER¹, MATTIAS CHEVALIER^{1,2}, THOMAS R. BEWLEY³, AND DAN S. HENNINGSON^{1,2}

Estimation1: utilisation d'un modèle stochastique pour les perturbations, écoulement en canal, *laminaire*.

JFM, vol. 534, 2006.

State estimation in wall-bounded flow systems.
Part 2. Turbulent flows

By MATTIAS CHEVALIER^{1,2}, JÉRÔME HËPFFNER², THOMAS R. BEWLEY³ AND DAN S. HENNINGSON^{1,2}

Estimation2: construction d'un modèle stochastique des termes non-linéaire en canal *turbulent*.

JFM, vol. 552, 2006.

Linear feedback control and estimation applied to instabilities in spatially developing boundary layers

By MATTIAS CHEVALIER^{1,2}, JÉRÔME HËPFFNER², ESPEN ÅKERVIK² AND DAN S. HENNINGSON^{1,2}

Estimation et contrôle en couche limite: Extension aux écoulements *Faiblement non-parralleles*

JFM, en révision.

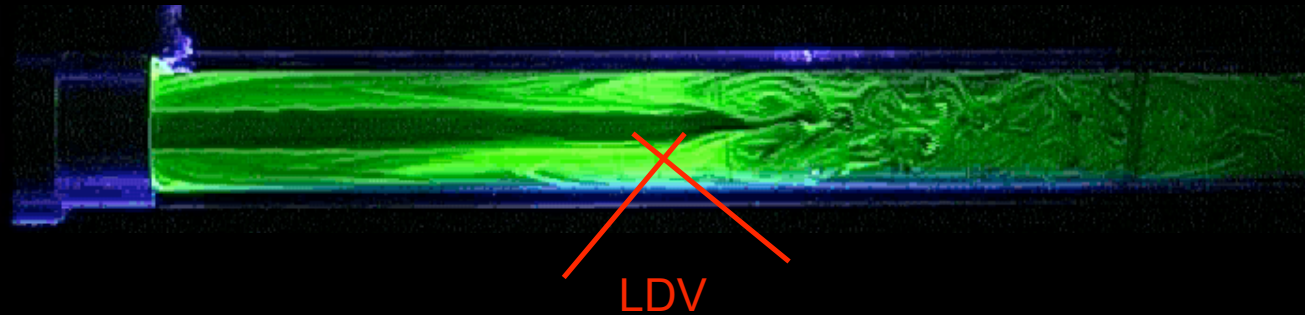
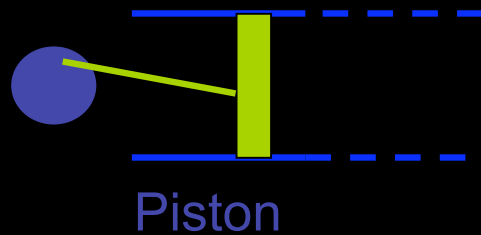
Optimal growth, model reduction and control in a separated boundary-layer flow using global eigenmodes

By ESPEN ÅKERVIK¹, JÉRÔME HËPFFNER¹, UWE EHRENSTEIN² AND DAN S. HENNINGSON¹

Contrôle et réduction de modèle: Écoulement *fortement non-parrallele*, Modèle réduit avec modes globaux 2D.

JFM, soumis.

Contrôle dans la sténose



Sonde: LDV, UDV

Actionneur: piston

+ ordinateur pour calcul

Procédure de contrôle

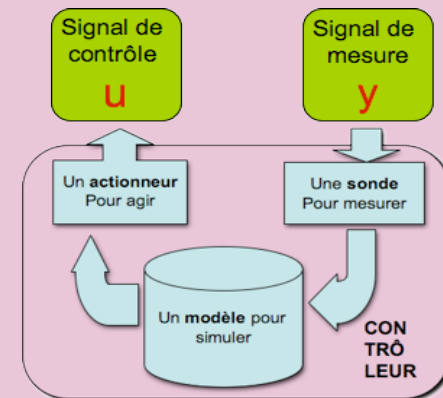
Numérique:

- 1) **Discrétisation** (→ Navier-Stokes, axisymétrique),
- 2) **Réduction** (→ 2 Lyapunov → projection)
- 3) **Optimisation** (→ 2 Riccati → gains de feedback)

$$\begin{cases} \dot{\hat{q}} = \overbrace{(A + B_2 K + LC)}^{A_0} \hat{q} + Ly, \\ u = K\hat{q} \end{cases} \quad \text{CON TRÔ LEUR}$$

Manip:

- 1) **Mesure** (→ LDV, en un point)
- 2) **Actionneur** (→ Piston, fréquences modérées)
- 3) **Simulation** (→ du modèle réduit, en temp réel)



Challenges

Numérique:

1) Equation de Lyapunov par **méthode itérative**

Balanced truncation pour système 2D: première fois

$$\begin{cases} \dot{\hat{q}} = \overbrace{(A + B_2K + LC)}^{A_0} \hat{q} + Ly, \\ u = K\hat{q} \end{cases}$$

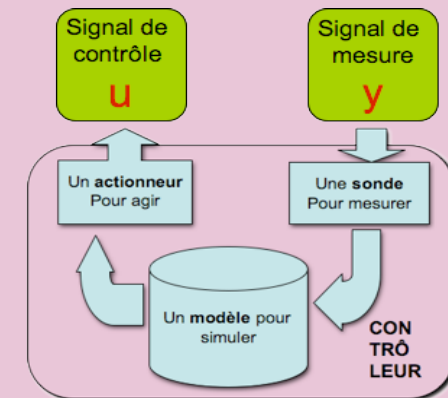
CON
TRÔ
LEUR

Manip:

1) Un **mécanisme de piston** rapide et précis

2) Un délai "**mesure-calcul-action**" court

Contrôle optimal avec rétroaction en manip: première fois



Conclusions

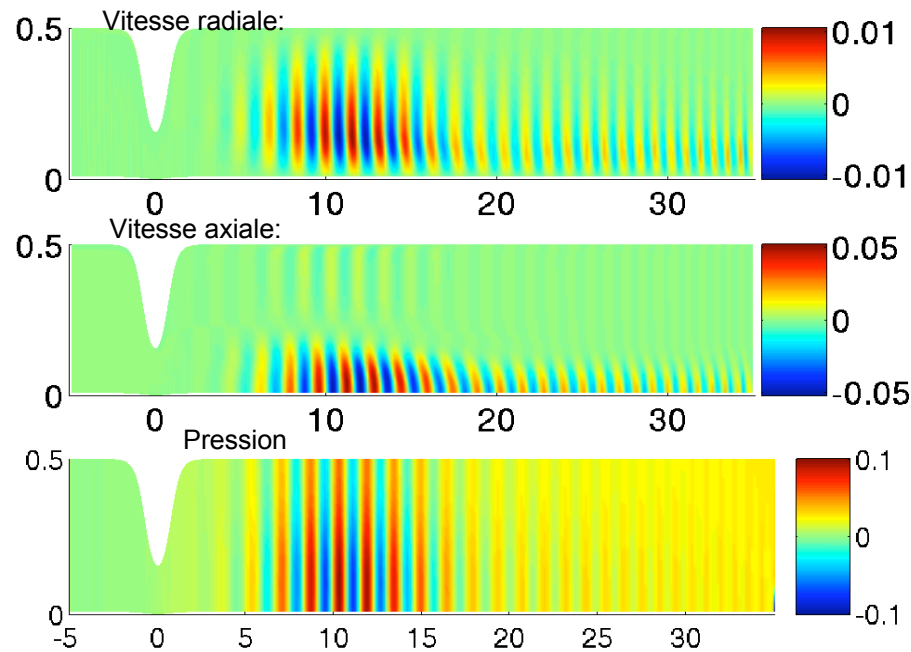
- 1) **Instabilité convective linéaire**, qui donne lieu a un écoulement auto-entretenu
- 2) **Modèle numérique flexible** pour la réduction et l'optimisation
- 3) **Contrôle avec rétroaction** avec
 - une mesure,
 - un calculateur
 - un actionneur
- 4) **Cas simple d'écoulement 2D** pour appliquer l'artillerie du contrôle

Merci de votre attention!

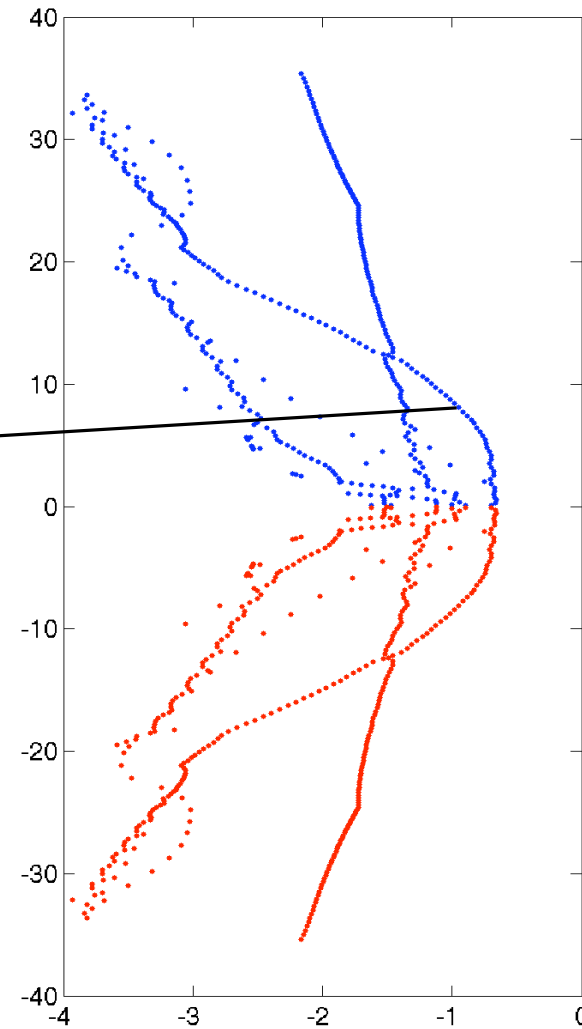
Transparents additionnels

Modes globaux 2D

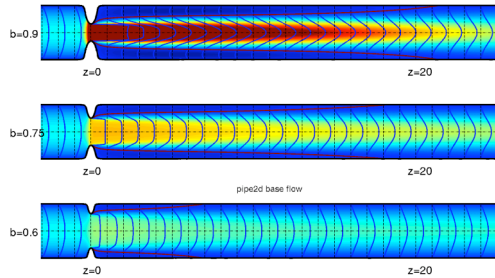
Un mode propre:



Partie du
spectre global:
 $b=0.75$, $Re=800$
Modes directs
Modes adjoints

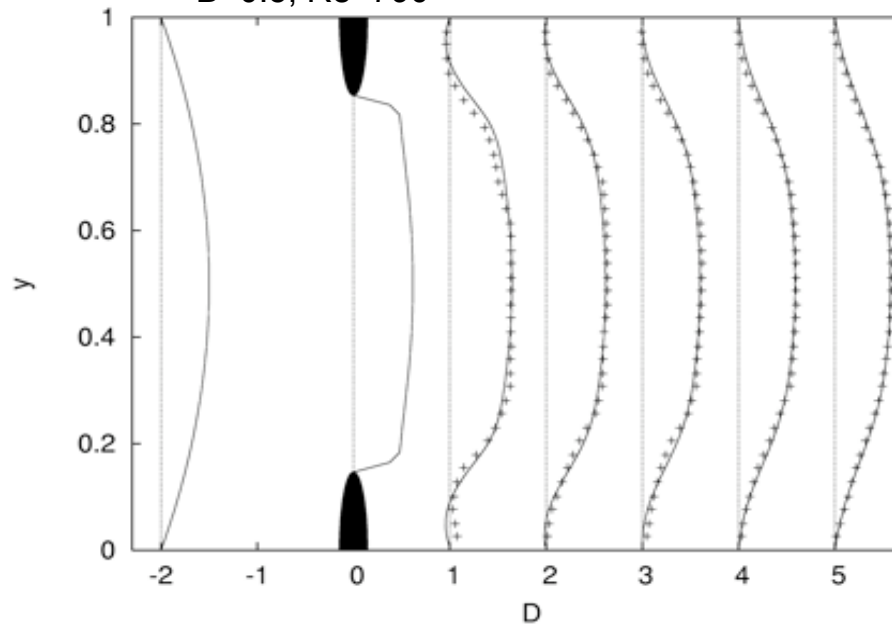


DNS / PIV / TuyauNum

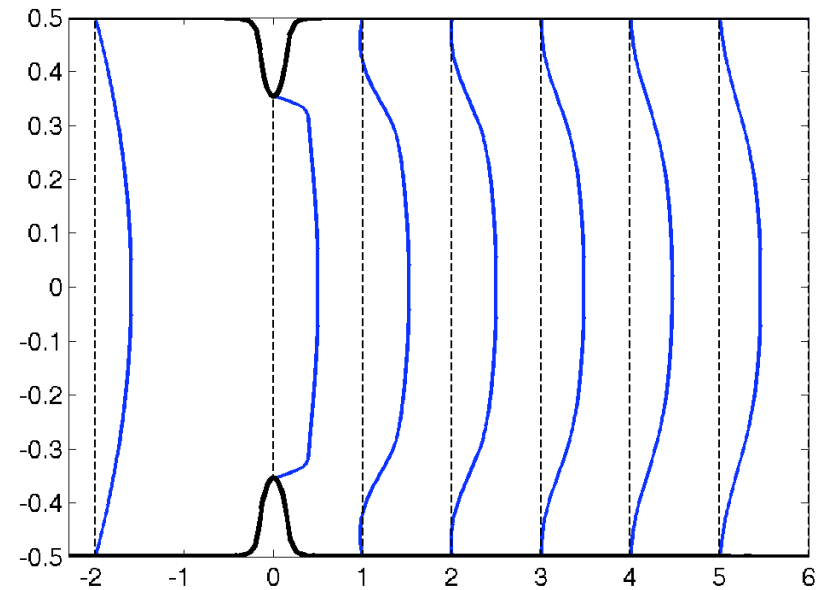


Comparaison des profils:
 $B=0.5$, $Re=790$

DNS/PIV, Griffith et. Al. 2006.
 $B=0.5$, $Re=790$



Mon calcul:



optimisation



Constrained minimisation → Lagrange multiplier Λ
 Minimax problem for Lagrangians \mathcal{L}_c and \mathcal{L}_e .

Control:

minimize

$$E(\underbrace{\|q\|^2 + \ell^2 \|u\|^2}_{\|Kq\|^2}) = \text{Tr}(PQ + \ell^2 KPK^+)$$

$$\mathcal{L}_c = \overbrace{\text{Tr}(PQ + KPK^+)}^{\text{Objective}} + \text{Tr}[\Lambda \overbrace{((A + BK)P + P(A + BK)^+ + W)}^{\text{Constraint}}]$$

$$\left. \begin{array}{l} \nabla_{\Lambda} \mathcal{L}_c = 0 \\ \nabla_P \mathcal{L}_c = 0 \\ \nabla_K \mathcal{L}_c = 0 \end{array} \right\} \Rightarrow \begin{cases} 0 = A^+ \Lambda + \Lambda A - \Lambda B B^+ \Lambda / \ell^2 + Q, \\ K = B^+ \Lambda / \ell^2. \end{cases}$$

Estimation:

minimize

$$E(\underbrace{\|q - \hat{q}\|^2}_{\|\hat{q}\|^2}) = \text{Tr}(\tilde{P})$$

$$\mathcal{L}_e = \overbrace{\text{Tr}(\tilde{P})}^{\text{Objective}} + \text{Tr}[\Lambda \overbrace{((A + LC)\tilde{P} + \tilde{P}(A + LC)^+ + \alpha^2 LL^+ + W)}^{\text{Constraint}}]$$

$$\left. \begin{array}{l} \nabla_{\Lambda} \mathcal{L}_e = 0 \\ \nabla_{\tilde{P}} \mathcal{L}_e = 0 \\ \nabla_L \mathcal{L}_e = 0 \end{array} \right\} \Rightarrow \begin{cases} 0 = A \tilde{P} + \tilde{P} A^+ - \tilde{P} C^+ C \tilde{P} / \alpha^2 + W \\ L = -\tilde{P} C^+ / \alpha^2. \end{cases}$$

Same structure for control and estimation → two **Riccati equations**

Lyapunov equation

Explicit state solution:

$$\dot{q} = Aq + w \quad \Rightarrow \quad q(t) = \int_{\tau=0}^{\infty} e^{A(t-\tau)} w(\tau) d\tau + e^{At} q_0$$

State covariance:

$$\begin{aligned} \underbrace{Eq(t)q(t)^H}_{P(t,t)} &= \int_0^{\infty} \int_0^{\infty} e^{A(t-\tau)} \overbrace{Ew(\tau)w(\tau')^H}^{W\delta(\tau-\tau')} e^{A^H(t-\tau')} d\tau d\tau' \\ &= \int_0^{\infty} e^{A(t-\tau)} W e^{A^H(t-\tau)} d\tau \end{aligned}$$

Differentiating this convolution integral:

$$\dot{P} = AP + PA^H + W$$

Numerical solution of the Lyapunov equation

Solve: $AX + XA^H + W = 0$

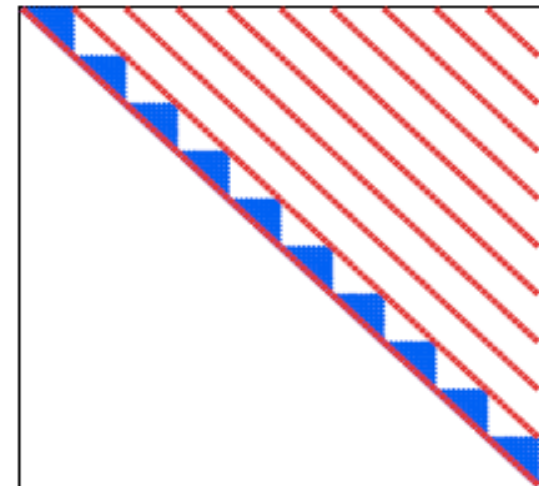
1. Schur decomposition $A = UA'U^H$, $\rightarrow A'$ upper diagonal, U orthogonal.

2. Resulting equation $A' \overbrace{U^H XU}^{X'} + \overbrace{U^H XU}^{X'} A'^H + \overbrace{U^H WU}^{W'} = 0$

3. Use Kronecker product \otimes

$$A \otimes B \triangleq \begin{pmatrix} a_{11}B & a_{12}B & \dots & a_{1m}B \\ a_{21}B & a_{22}B & \dots & a_{2m}B \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1}B & a_{n2}B & \dots & a_{nm}B \end{pmatrix}$$

$$\begin{aligned} \text{vec}(A'X' + X'A'^H + W') &= 0 \\ &= \underbrace{(I \otimes A' + \overline{A'} \otimes I)}_{\mathcal{F}} \text{vec}(X') + \text{vec}(W') \end{aligned}$$



\mathcal{F} has upper diagonal structure

Estimation

Sensor information

+

→ estimate full 3D flow state

Dynamic model

Case 1:

No disturbances,
Known initial condition
→ Need good model

$$\begin{array}{l} \text{Flow:} \\ \text{Estimator:} \end{array} \left\{ \begin{array}{l} \dot{q} = Aq \\ y = Cq \end{array} \right\}, \quad q(0) = q_0$$
$$\left\{ \begin{array}{l} \dot{\hat{q}} = A\hat{q} \\ \hat{y} = C\hat{q} \end{array} \right\}, \quad \hat{q}(0) = q_0$$

Case 2:

Disturbances,
Unknown initial condition
→ Need feedback

$$\begin{array}{l} \text{Flow:} \\ \text{Estimator:} \end{array} \left\{ \begin{array}{l} \dot{q} = Aq + w \\ y = Cq + g \end{array} \right\}, \quad q(0) = q_0$$
$$\left\{ \begin{array}{l} \dot{\hat{q}} = A\hat{q} - L(y - \hat{y}) \\ \hat{y} = C\hat{q} \end{array} \right\}, \quad \hat{q}(0) = 0$$

Numerical solution of Riccati equation

$$\text{Solve: } A^H X + XA + XBB^H X + Q = 0$$

1. Build Hamiltonian: $\mathcal{H} = \begin{pmatrix} A & BB^H \\ -Q & -A^H \end{pmatrix}$

2. Schur decomposition $\mathcal{H} = USU^H$, $\rightarrow S$ upper triangular, U orthogonal

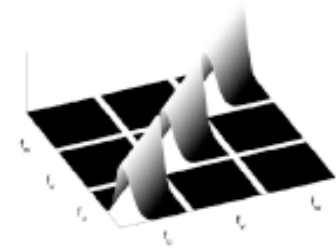
3. Order Schur decomposition to decompose stable/unstable subspaces:

$$S = \begin{pmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{pmatrix}, U = \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix}$$

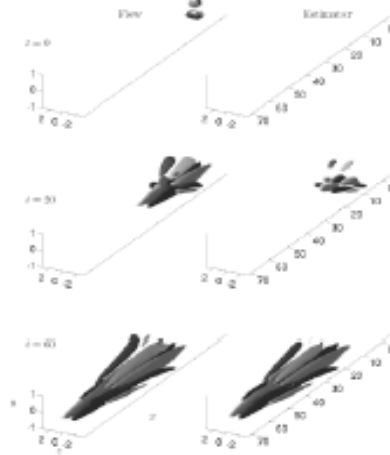
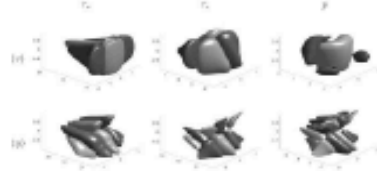
4. Solve $X = U_{21}U_{11}^{-1}$

Estimation in laminar channel flow

Simple covariance model:



Estimation convolution kernels:



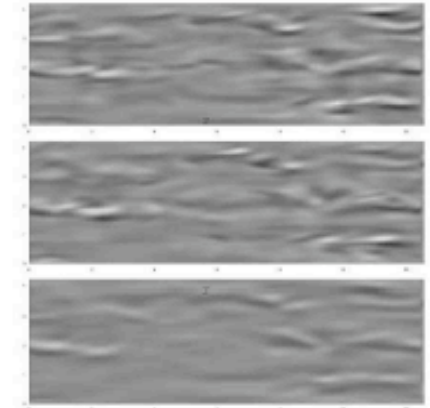
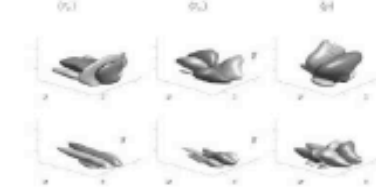
Estimation of initial condition

Estimation in turbulent channel flow

Covariance model:



Estimation convolution kernels:



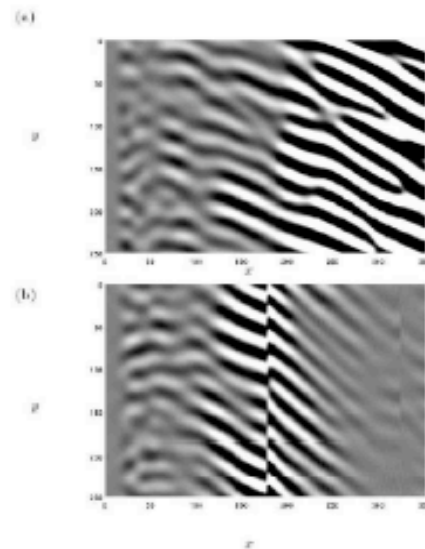
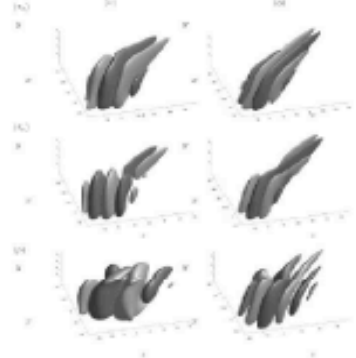
Snapshot of flow/estimated flow

Estimation/Control of swept boundary layer

Control convolution kernels

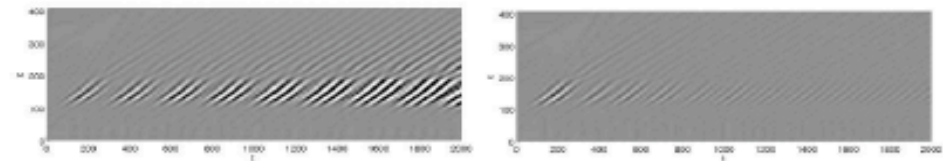


Estimation convolution kernels



Controlled cavity flow

Wall normal velocity: no control / control



Pressure: no control / control

