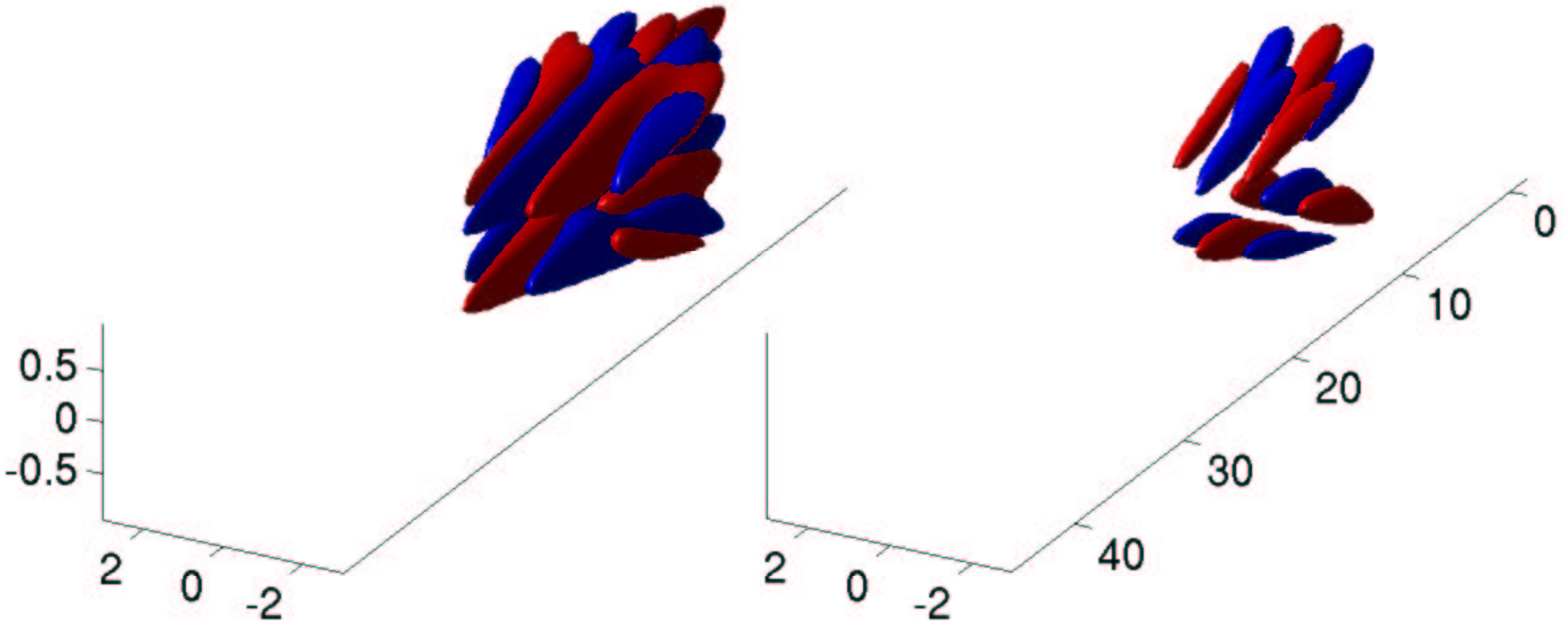
KUNGL TEKNISKA HÖGSKOLAN

Estimation of wall bounded shear flow



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Outline

- Flow control
- model based control
- linear compensation : control and estimation
- Perturbation model for the estimation
- Results on localized perturbation

Control theory

Mathematically well develloped
and central to many engineering applications

- Space ship - satellites trajectory
- Break control (ABS)
- Any automatic pilots
- etc ...

Linear theory pushed to an extremum,
nonlinear theory at its beginning

Flow control

One could like to :

- Postpone transition
- Relaminarise turbulence
- Increase mixing
- Avoid detachment

But also :

- Lock reatachment point
- Lock oscillatory behaviour
- Modify transition scenario
- Modify turbulence statistics

Flow control - continued

To achieve this :

- passive control
  - geometry design/optimization
  - roughness element
  - vortex generators
- active control
  - constant blowing or suction
  - wall temperature
  - Periodic blowing and suction
- reactive control (feedback)
  - sensors and actuators → introduce the estimation

Feedback

The control u is based on measurement y from the system state

$$\begin{cases} \dot{q} = Aq + B_1 u(y) + B_2 f, \quad q(0) = q_0, \\ y = Cq + g, \end{cases} \tag{1}$$

The system is subject to inital condition q_0,
volume forcing f,
and sensor noise g.

Model based control, and optimization

- The feedback law can be based of physical insight
- But as well on a model $\longrightarrow$ can be optimized

Note : Even with a model we need physical insight

1. What is a good model?
2. Which objective function?
3. Which actuation and sensing?

Those three problems are not independent!

Decoupling control–estimation

$$\text{Plant}\begin{cases}\dot{q} = Aq + B_1 u + B_2 f, \quad q(0) = q_0,\\ y = Cq + g,\end{cases}$$

$$\text{Estimator}\begin{cases}\dot{\hat{q}} = A\hat{q} + B_1 u - v, \quad \hat{q}(0) = \hat{q}_0,\\ \hat{y} = C\hat{q},\end{cases}$$

$$v = L\tilde{y} = L(y - \hat{y}).$$

The best feedback controller is composed of

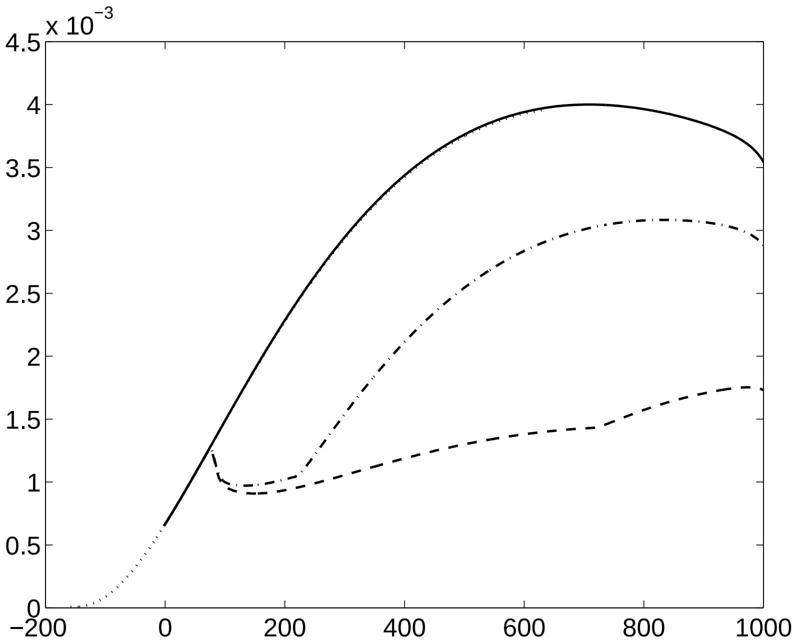
the best full information controller

and the best estimator.

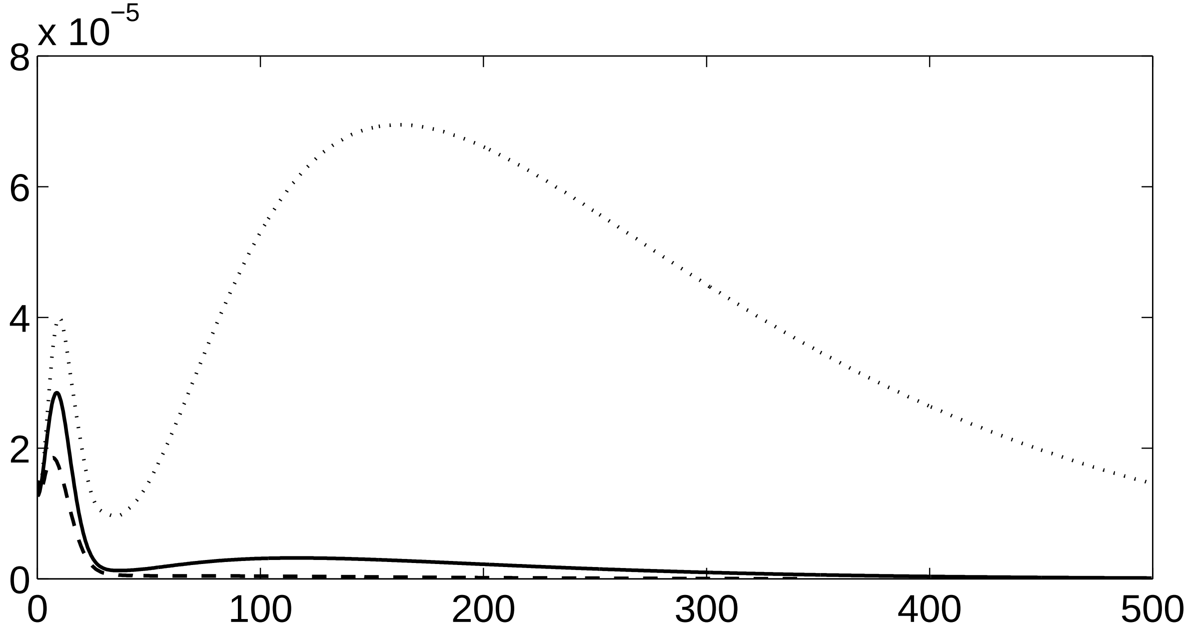
(for linear systems)

Previous achievements

State feedback for streaks



Measurement feedback for oblique wave



Objective function *vs* noise model

Control : act at T to affect the flow later

- We need a policy on how to act :
  objective function & dynamic model

Estimation : measure before T to know the flow now.

- We need a policy on how the information is provided :
  perturbation model & dynamic model

Why a stochastic model ?

Deterministic We know either everything or nothing

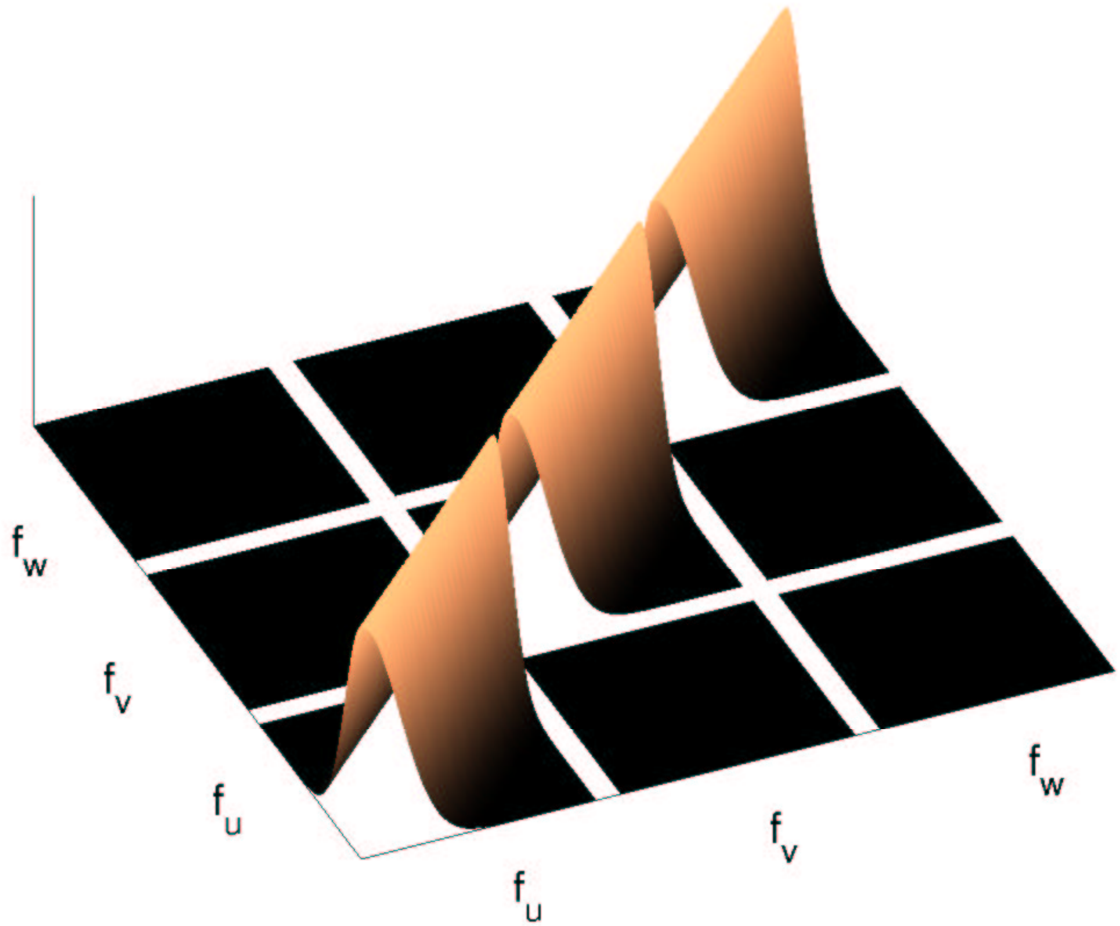
Stochastic We use the average behaviour to estimate the instantaneous state

- Average over initial condition
- Average over volume forcing

We optimize the performance
averaged over all initial condition
and all volume forcing

Correlation model for the volume forcing

y variation



Fourier space variation : exponential decay

Model for the initial conditions

k is the realisation number

$$q_0^{(k)} = \theta_1(k) \left(\theta_2(k) \underbrace{\frac{q_s}{||q_s||_E}}_{\textit{Specific}} + \sum_j \vartheta_j(k) \underbrace{\frac{r_0^j}{||r_0^j||_E}}_{\textit{Random}} \right),$$

The corresponding covariance becomes

$$P_0 = \lambda_1 \left(\lambda_2 \underbrace{\frac{E[q_s q_s^*]}{\mathrm{Tr}(E[q_s q_s^*])}}_{\textit{Specific}} + (1 - \lambda_2) \underbrace{\frac{E[r_0 r_0^*]}{\mathrm{Tr}(E[r_0 r_0^*])}}_{\textit{Random}} \right), \qquad (2)$$

The energy in Fourier space

$$\lambda_1(k_x, k_z) = v_1 k \mathrm{e}^{-s_\lambda k^2/2},$$

Results

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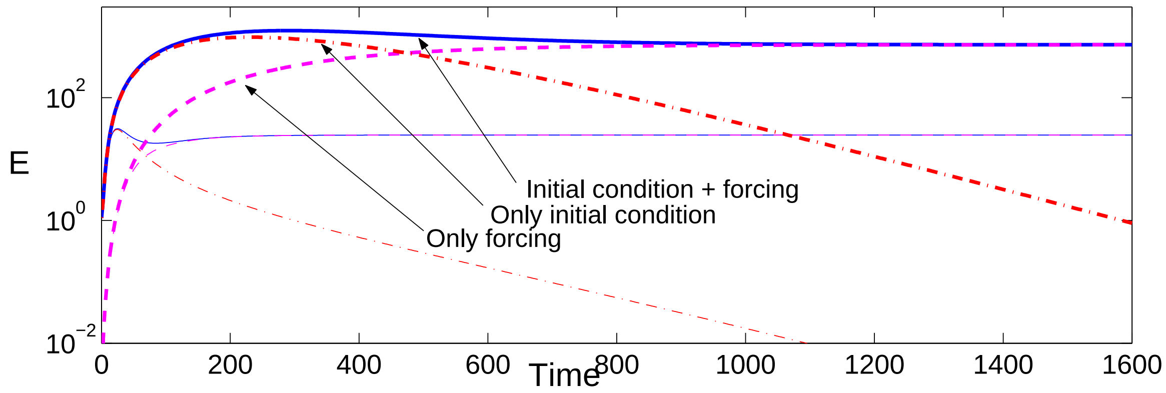
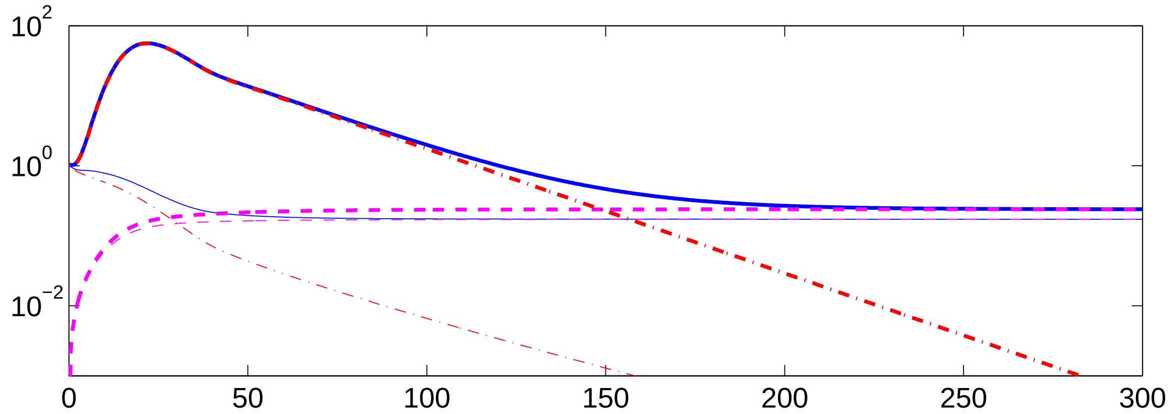
Three flow cases



(0,2)



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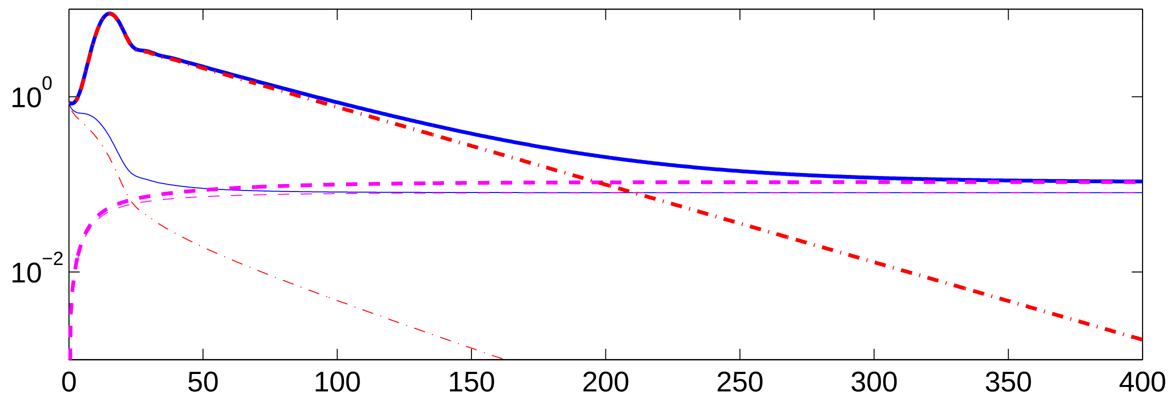
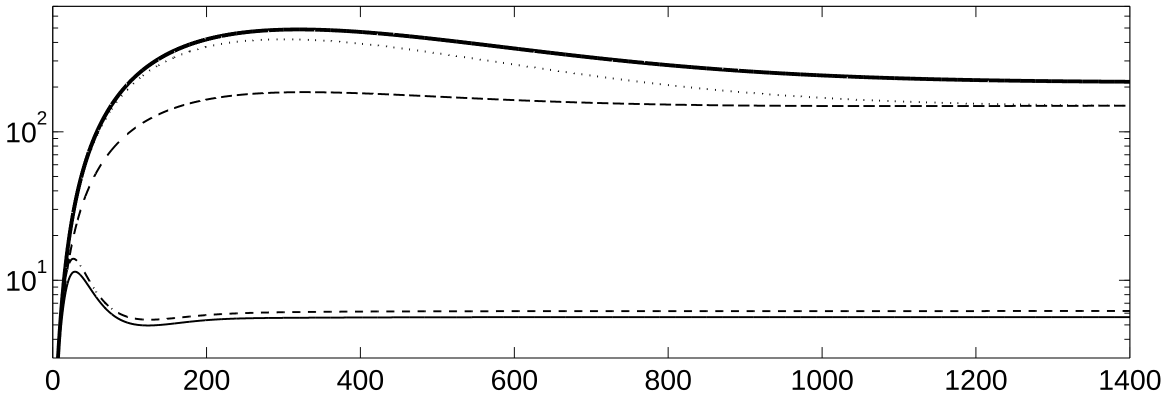


(1,0)

Measurements

(0,2)



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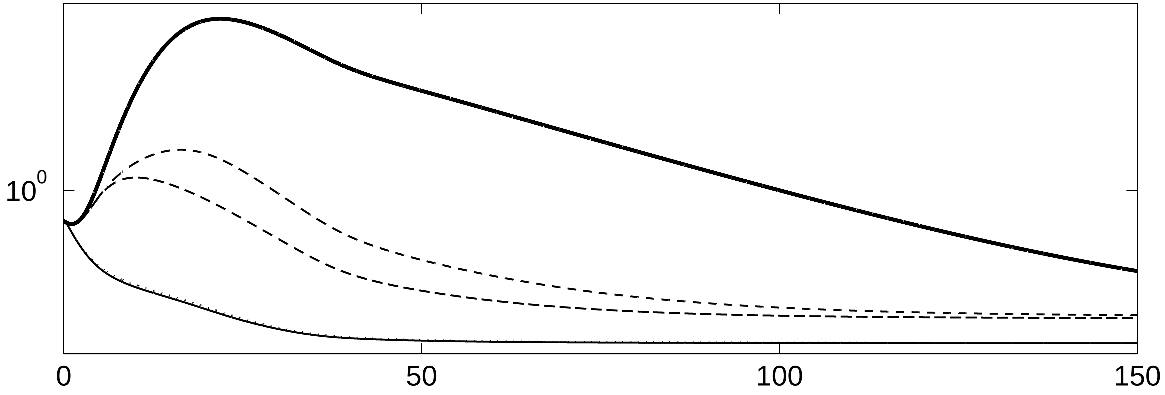

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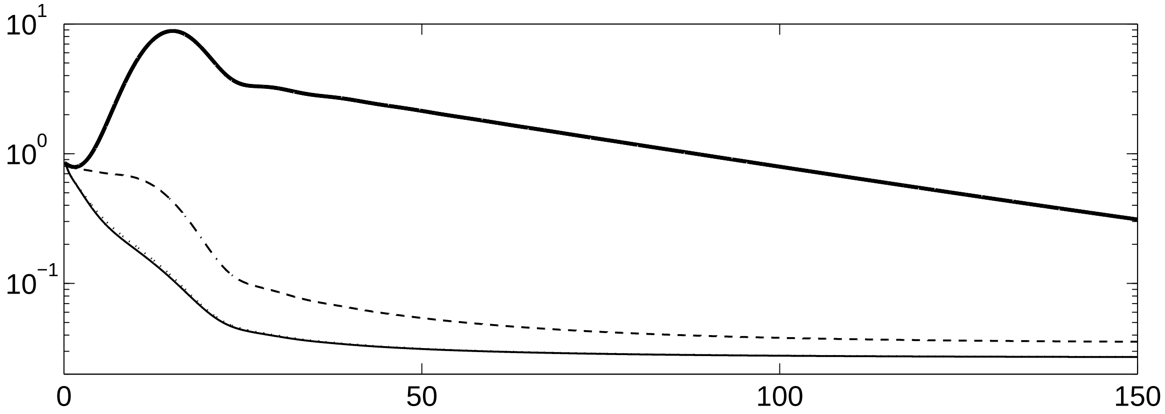
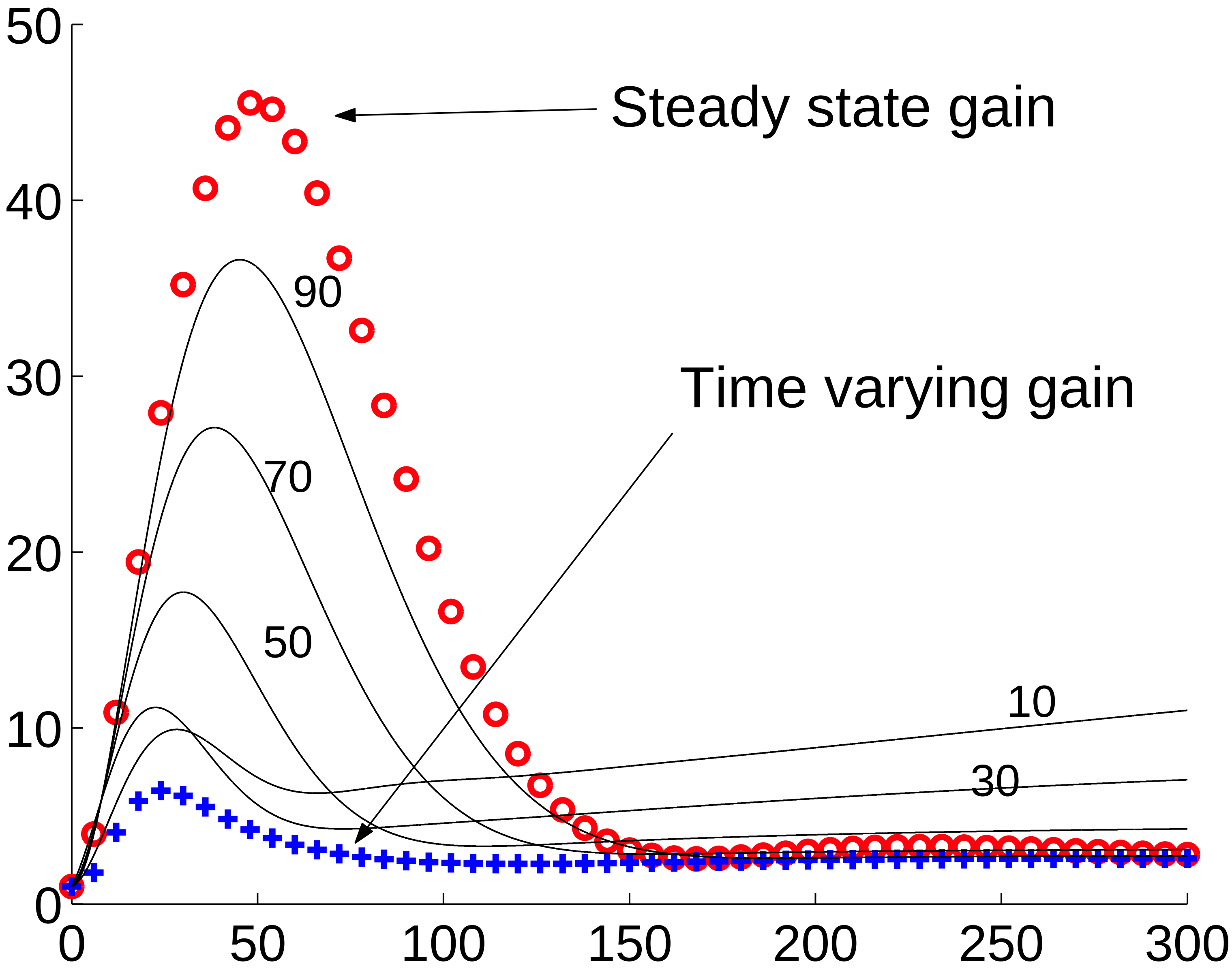
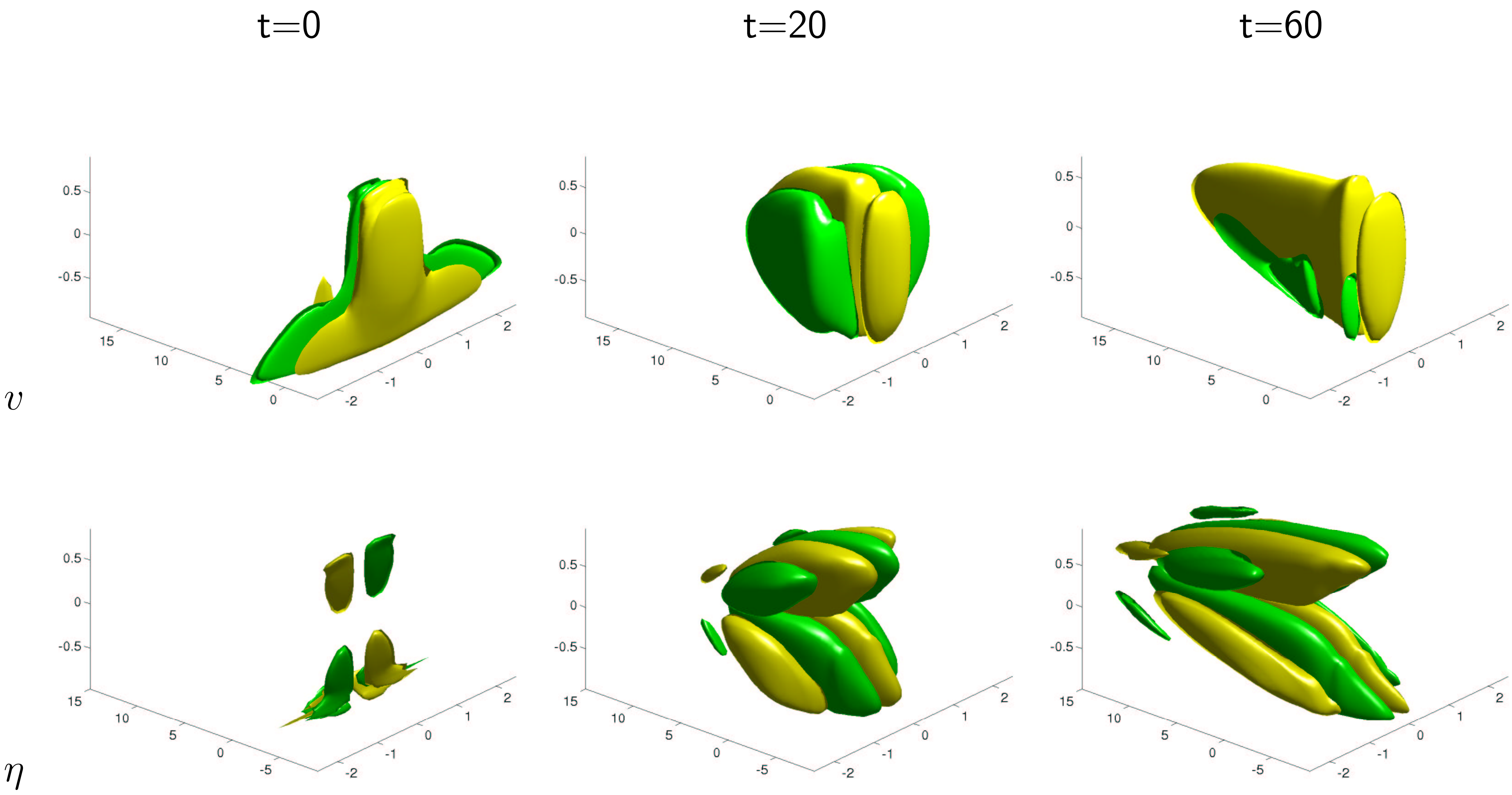


Gain scheduling

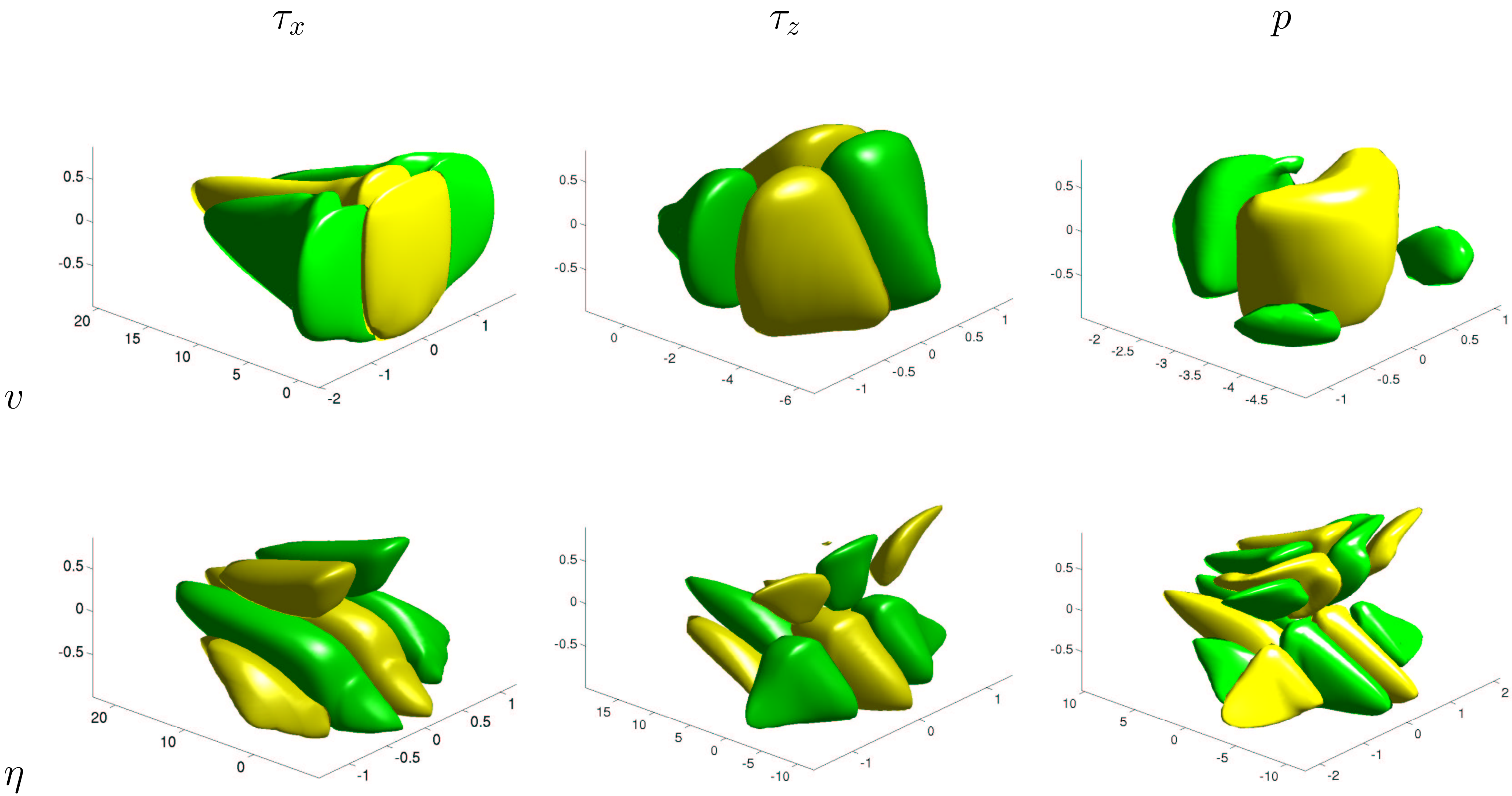
Pick a gain from time t and apply it all the time



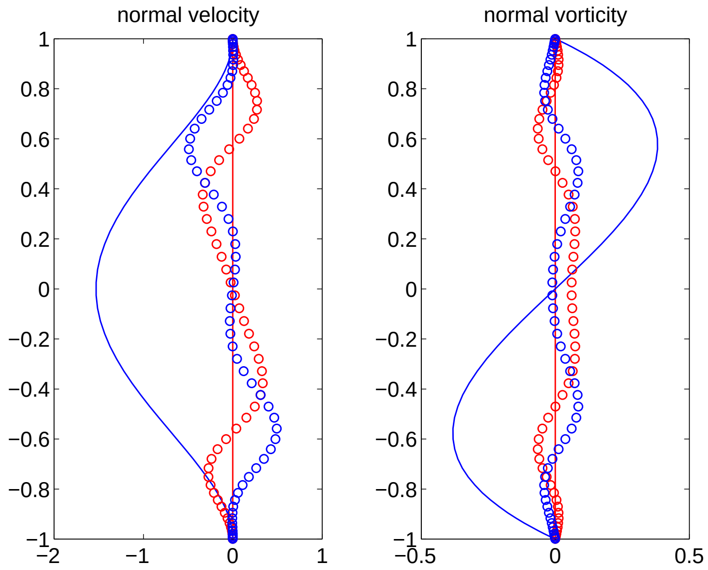
Time varying kernels

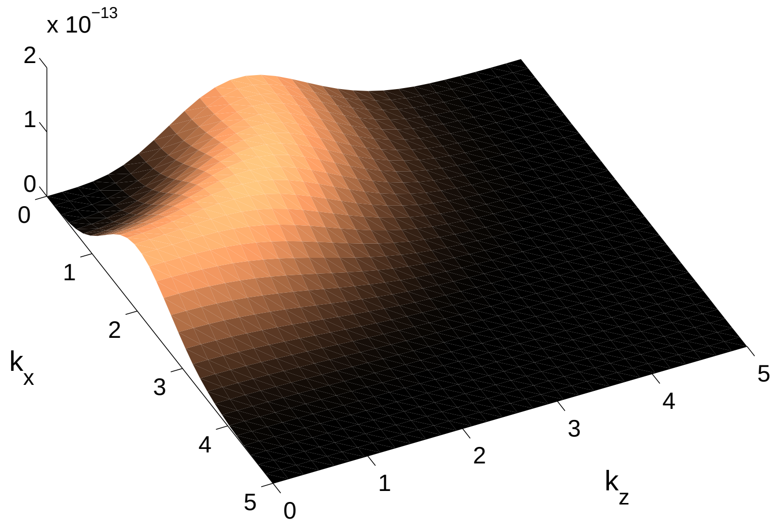


Steady state kernels

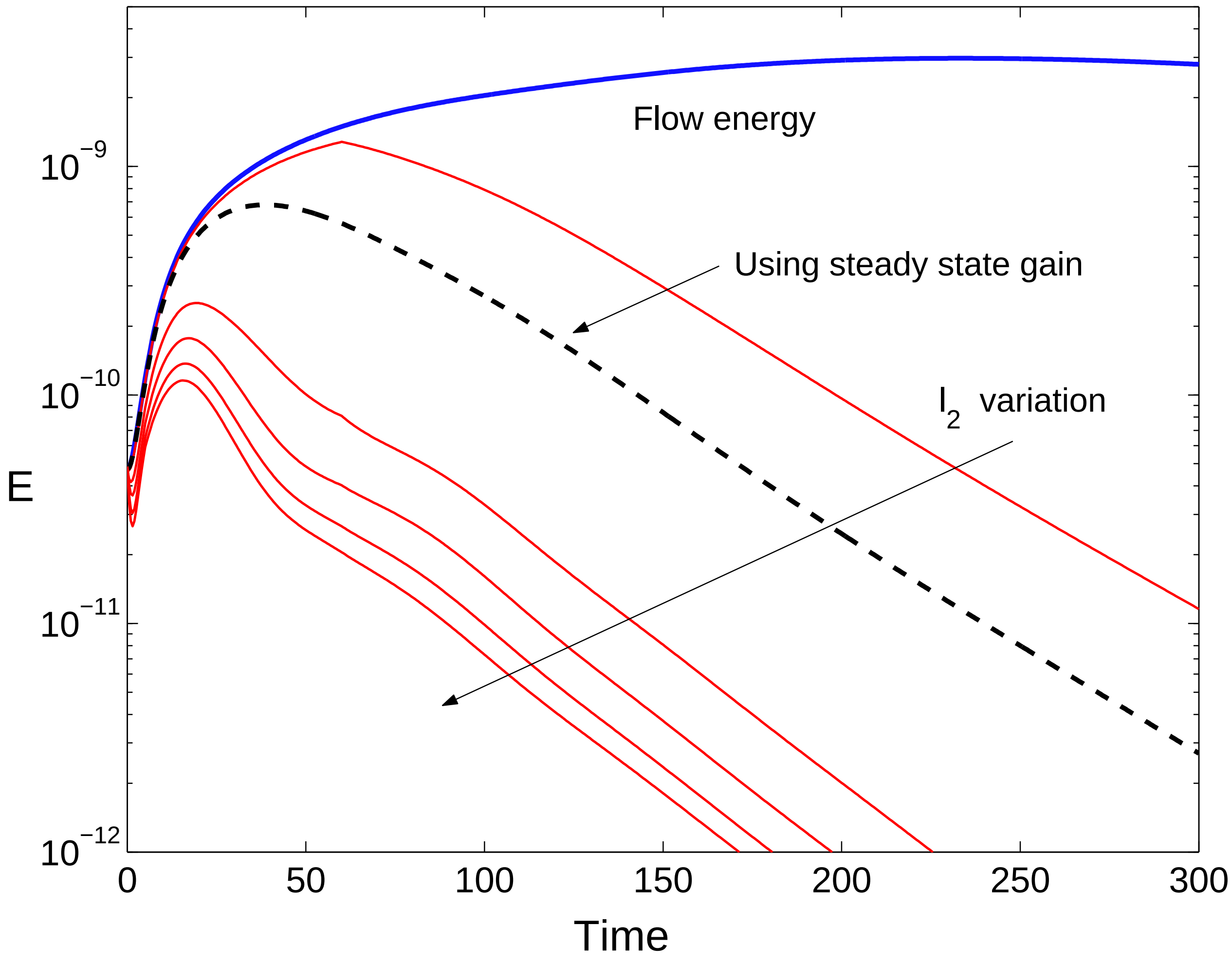


Localized perturbation

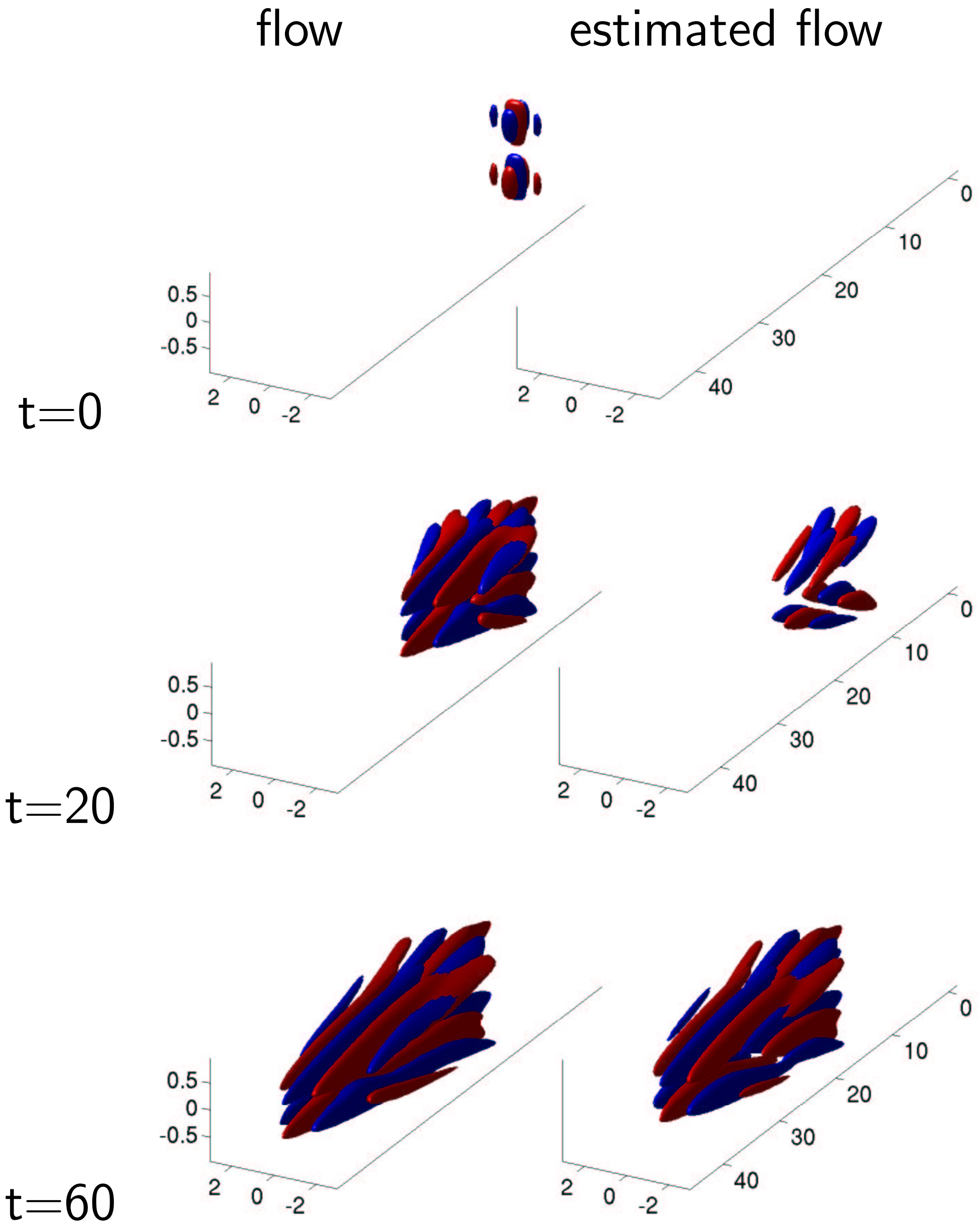




Estimation performance



Flow evolution



Conclusion

Was done

- A model for perturbations
- Choice of measurements
- Investigation of transient for estimation
- A sub-optimal procedure

To be done

- Transient for the control as well