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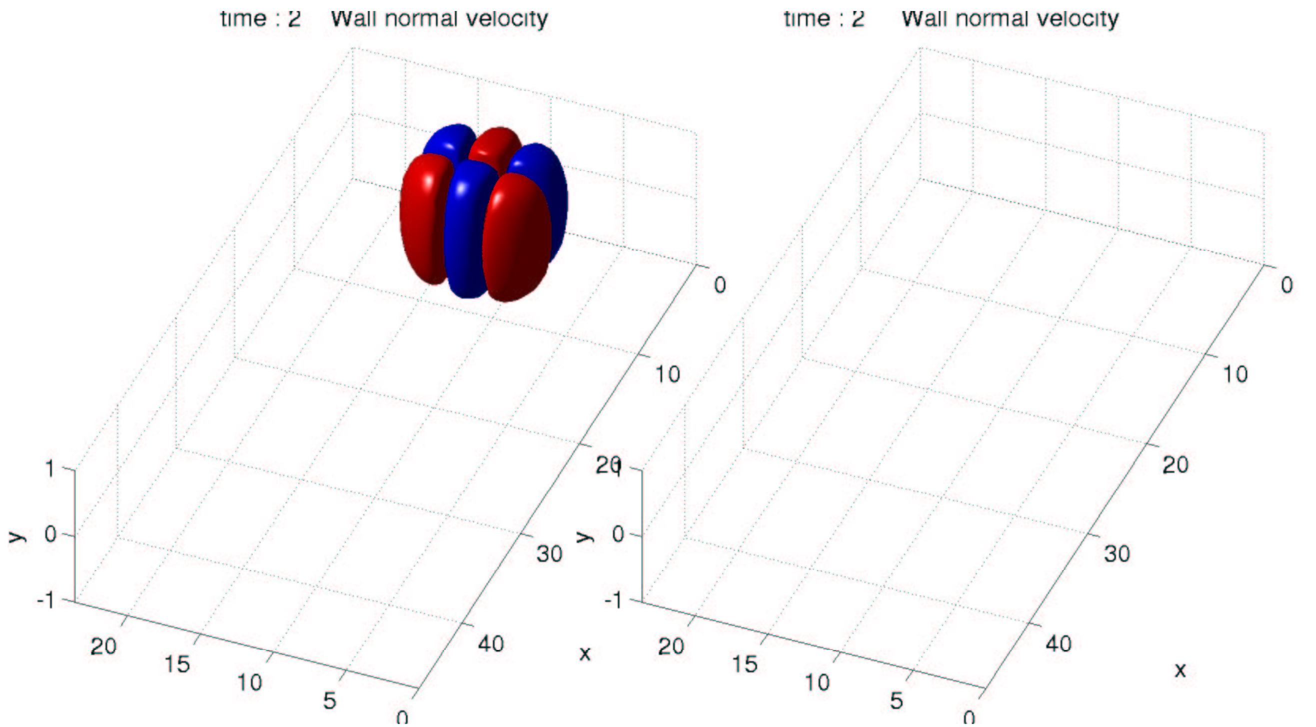
Estimation and wall bounded shear flows



Jérôme Hœpffner, Mattias Chevalier

Supervisor Dan Henningson

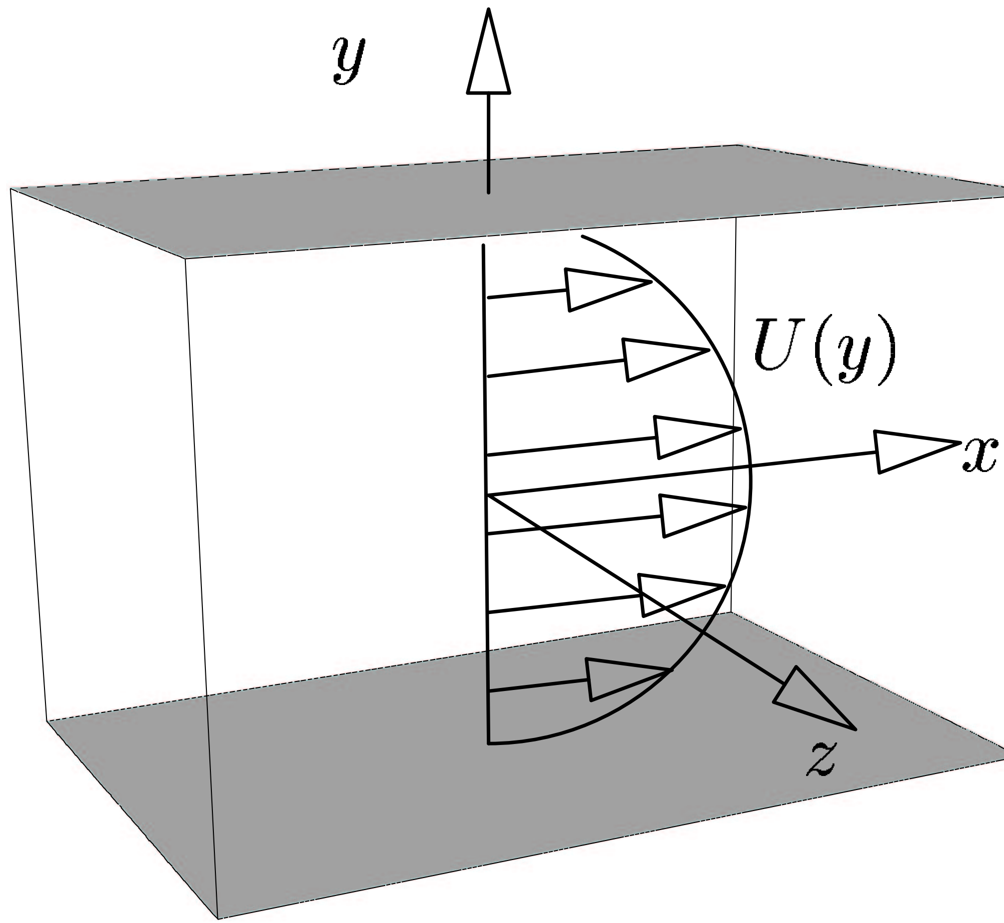
In cooperation with Thomas Bewley

Royal Institute technology , Sweden

Outline

- Estimation
- Covariance and energy
- Disturbances
- Optimization
- Time varying gains
- Steady state kernels

Flow addressed



Parallel wall bounded flow

Estimation

Recover the state q from measurements y.

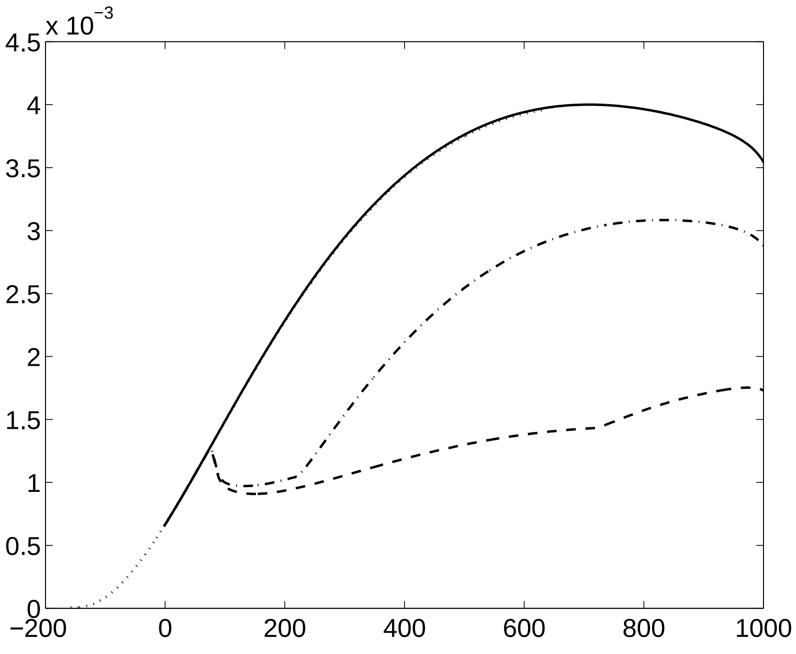
$$\begin{cases} \dot{q} = Aq + F(q) + Bf \quad , \quad q(0) = q_0 \\ y = C(q) + g \end{cases}$$

Why?

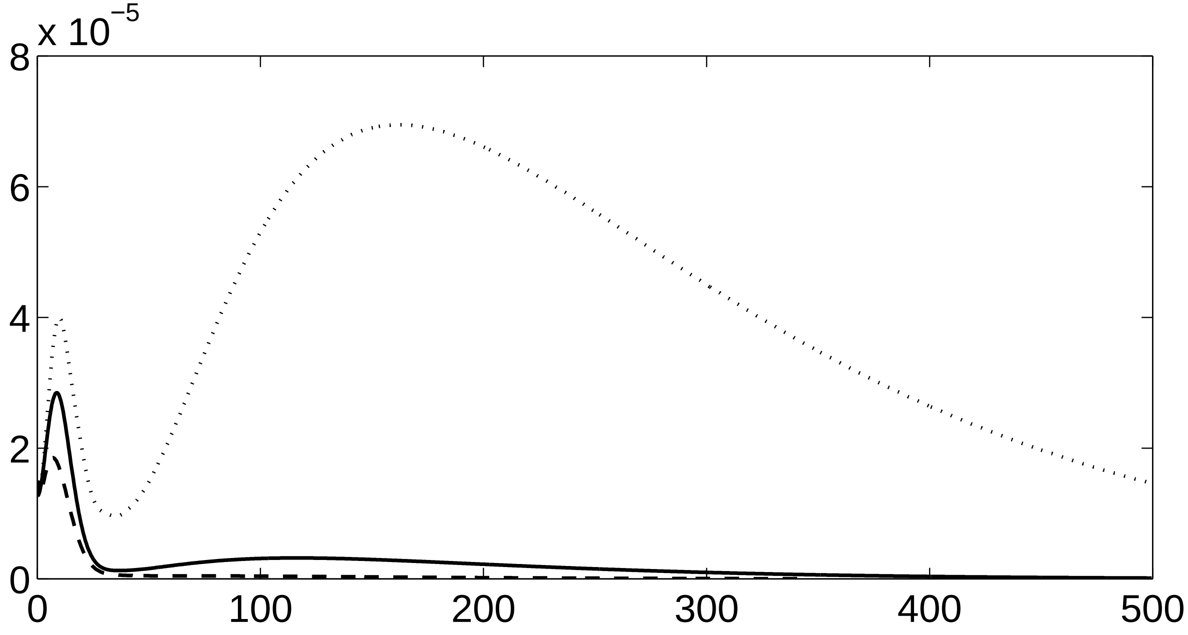
- Diagnosis
- Forecast
- Feedback control

Previous achievements

State feedback for streaks



Measurement feedback for oblique wave



Flow in the channel

The evolution for one Fourier mode q_{mn}:

$$\underbrace{\frac{d}{dt}Mq_{mn} + Lq_{mn}}_{Linear\ \ dynamics} = \underbrace{\sum_{k+i=m,l+j=n} N(q_{kl}, q_{ij})}_{Nonlinear\ \ coupling} + \underbrace{e_{mn}(y,t)}_{External\ \ forcing}$$

$$q_{mn} = \begin{pmatrix} \hat{v}_{mn} \\ \hat{\eta}_{mn} \end{pmatrix}, M = \begin{pmatrix} -\Delta & 0 \\ 0 & I \end{pmatrix}, L = \begin{pmatrix} \mathcal{L}_{OS} & 0 \\ \mathcal{L}_C & \mathcal{L}_{SQ} \end{pmatrix}$$

State space formulation

f_1, f_2, f_3 stochastic forcing on u, v, w

$$\frac{d}{dt}M\hat{q} + L\hat{q} = T f(y,t)$$

Basis transformation operator :

$$T = \begin{pmatrix} i\alpha D & k^2 & i\beta D \\ i\beta & 0 & -i\alpha \end{pmatrix}$$

Evolution form :

$$\dot{q} = \underbrace{-M^{-1}L}_{A} q + \underbrace{M^{-1}T}_{B} f$$

The linear estimator

$$Plant \begin{cases} \dot{q} = Aq + Bf \quad , \quad q(0) = q_0 \\ y = Cq + g \end{cases}$$

$$Estimator \begin{cases} \dot{\hat{q}} = A\hat{q} - \hat{v}(y) \quad , \quad \hat{q}(0) = \hat{q}_0 \\ \hat{y} = C\hat{q} \end{cases}$$

$$Feedback \quad v = L\delta y = L(y - \hat{y})$$

Stochastic processes

Infinite dimensional random process

$$\forall(\xi_1, \xi_2) \in H_1 \times H_2, \quad E[\langle q_1, \xi_1\rangle_1 \langle q_2, \xi_2\rangle_2] = \langle C\xi_1, \xi_2\rangle_2$$

Notation

$$C = cov(q_1, q_2) = E[q_1 q_2^*]$$

Manipulations

$$cov(\mathcal{H} f) = \mathcal{H} cov(f) \mathcal{H}^*$$

Energy

$$\boxed{E[\mathcal{E}(q(t))] = Tr(E[q(t)q(t)^*]}$$

Linear filtering

Propagation of the estimation error $\tilde{q}$

$$\dot{\tilde{q}} = \underbrace{(A - LC)}_{A_0}\tilde{q} + \underbrace{Bf + Lg}_{d} \quad , \quad \tilde{q}(0) = q_0 - \hat{q}_0$$

Lyapunov equation for $P(t) = E[x(t)x(t)^*]$

$$\dot{P}(t) = \underbrace{A_0P(t) + P(t)A_0^*}_{Dynamic\ terms} + \underbrace{BRB^*}_{State\ disturbances} + \underbrace{LGL^*}_{Measurement\ noise} \qquad P(0) = P_0$$

Process noise : A model

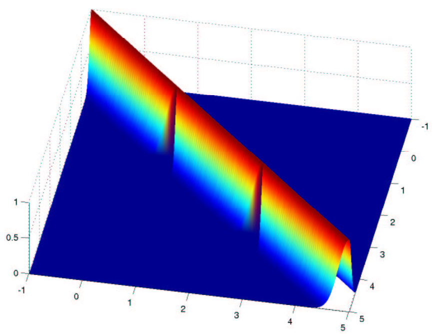
Two point correlation for the process noise

$$\Theta_{ij} = cov(f_i, f_j)$$

$$\Theta_{ij}(r_x, y, y', r_z) = v\ \delta_{ij} M^x(r_x) M^z(r_z) M^y(y, y')$$

Model

$$\begin{cases} M^x(r_x) &= \frac{1}{\sqrt{2\pi s_x}} e^{-\frac{r_x^2}{2s_x}} \\ M^z(r_z) &= \frac{1}{\sqrt{2\pi s_z}} e^{-\frac{r_z^2}{2s_z}} \\ M^y(y, y') &= \frac{1}{\sqrt{2\pi s_y}} e^{-\frac{(y-y')^2}{2s_y}} \end{cases}$$



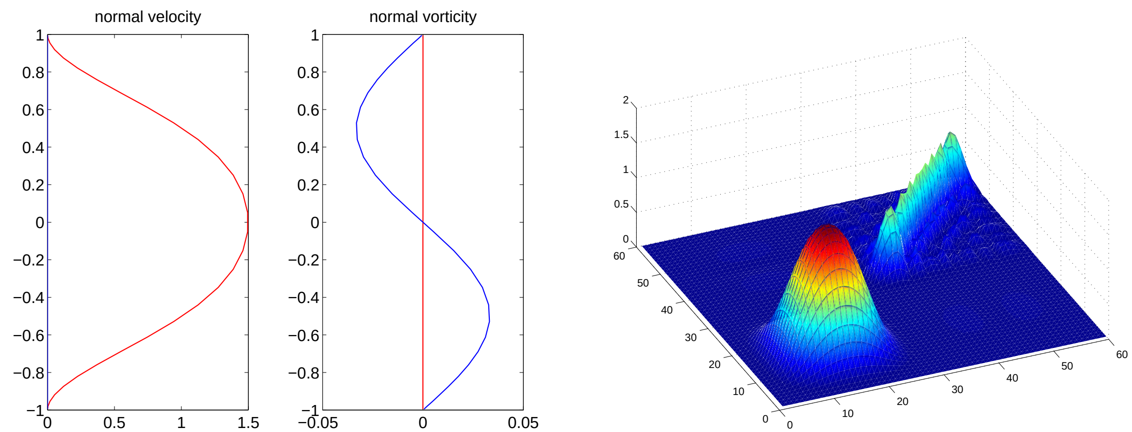
Amplitude varying with wave number pair

Model parameters v, s_x, s_y, s_z.

Initial condition : A model

Desired initial condition q_0 + uncertainty f_0

$$P_0 = r_1 \left(\underbrace{r_2 \frac{E[q_0 q_0^*]}{Tr(E[q_0 q_0^*])}}_{Known} + \underbrace{(1 - r_2) \frac{E[f_0 f_0^*]}{Tr(E[f_0 f_0^*])}}_{Uncertainty} \right)$$



Model parameters r_1 r_2

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Optimisation : the Lagrange multiplier approach

Objective $\mathcal{J} = Tr(P(t))$

Constraint $\dot{P}(t) = A_0 P(t) + P(t) A_0^* + BRB^* + LGL^* \quad , P(0) = P_0$

Lagrangian $\mathcal{L}(t) = Tr(P(t)) + Tr\left[\Lambda(-\dot{P}(t) + A_0 P(t) + P(t) A_0^* + BRB^* + LGL^*)\right]$

Extremum of $\mathcal{J}(t)$:

$$\begin{aligned} \frac{\partial \mathcal{J}}{\partial \Gamma} &= 0 \\ \frac{\partial \mathcal{J}}{\partial P} &= 0 \\ \frac{\partial \mathcal{J}}{\partial L} &= 0 \end{aligned}$$

Gives the Riccati equation :

$$\dot{P}(t) = AP(t) + P(t)A^* + BRB^* - P(t)C^* G^{-1} C P(t) \quad , P(0) = P_0$$

$$L(t) = -P(t) C^* G^{-1}$$

Stochastic interpretation of the optimal estimator

The best estimate is the conditional mean

$$\hat{q} = E[q|Y^t]$$

The Kalman filter is the propagator of the conditional mean

Assumption :

- Gaussian noise
- Linear system

Measurements

- Spanwise skin friction
- Streamwise skin friction
- Pressure

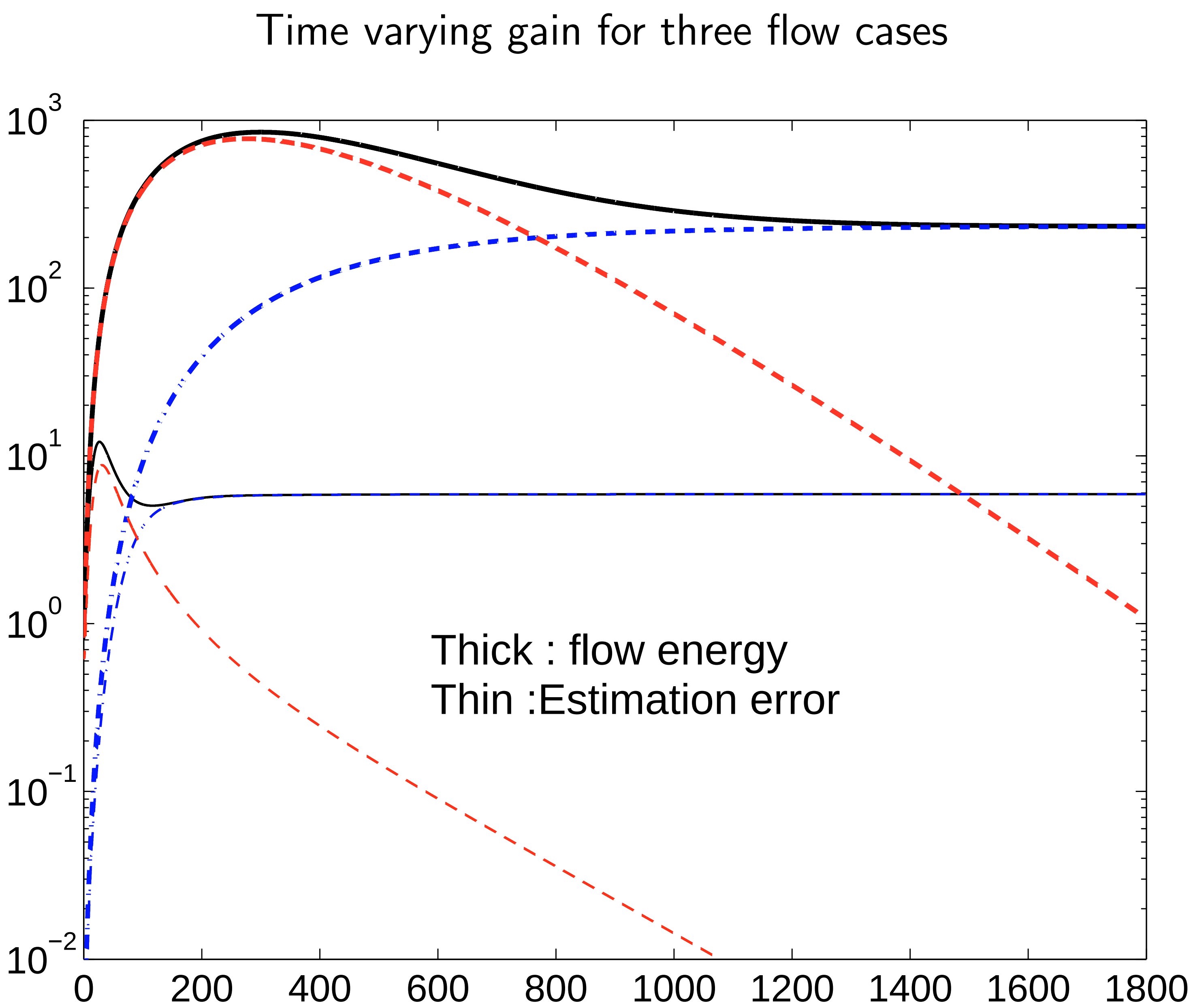
$$\begin{cases} m_1 &= \tau_{xy}|_{wall} = \mu \frac{\partial u}{\partial y}(y=0) = \frac{i\mu}{k^2}(\alpha D^2 v - \beta D\eta)|_{wall} \\ m_2 &= \tau_{zy}|_{wall} = \mu \frac{\partial w}{\partial y}(y=0) = \frac{i\mu}{k^2}(\beta D^2 v + \alpha D\eta)|_{wall} \\ m_3 &= p|_{wall} = \frac{\mu}{k^2} D^3 v \end{cases}$$

Measurement matrix

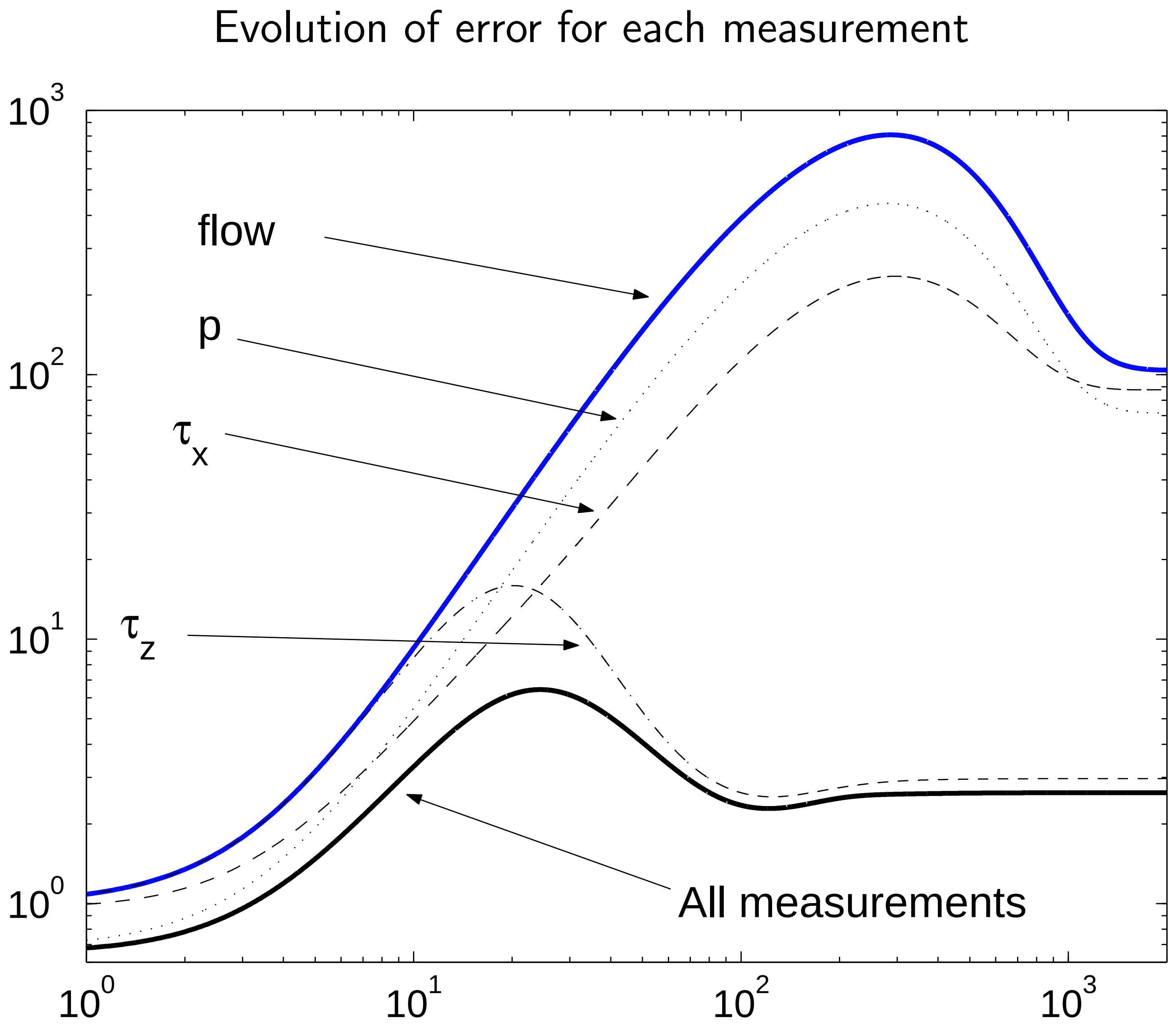
$$C = \frac{\mu}{k^2} \begin{pmatrix} i\alpha D^2 & -i\beta D \\ i\beta D^2 & i\alpha D \\ D^3 & 0 \end{pmatrix}$$

Results 1 : one wave number pair (0,1)

Flow energy, error energy



Different measurements



Time varying gains : control and estimation

- Control : **Final transient**

  Objective function

$$\mathcal{J} = E\left[q(T)^*Q_0 q(T) + \int_{t=t_0}^{t=T} (q(t)^* Q q(t) + l^2 u(t)^* u(t))dt\right]$$

- Estimation : **Initial transient**

  Description of the noise :

  – Initial conditions P_0

  – Process noise R

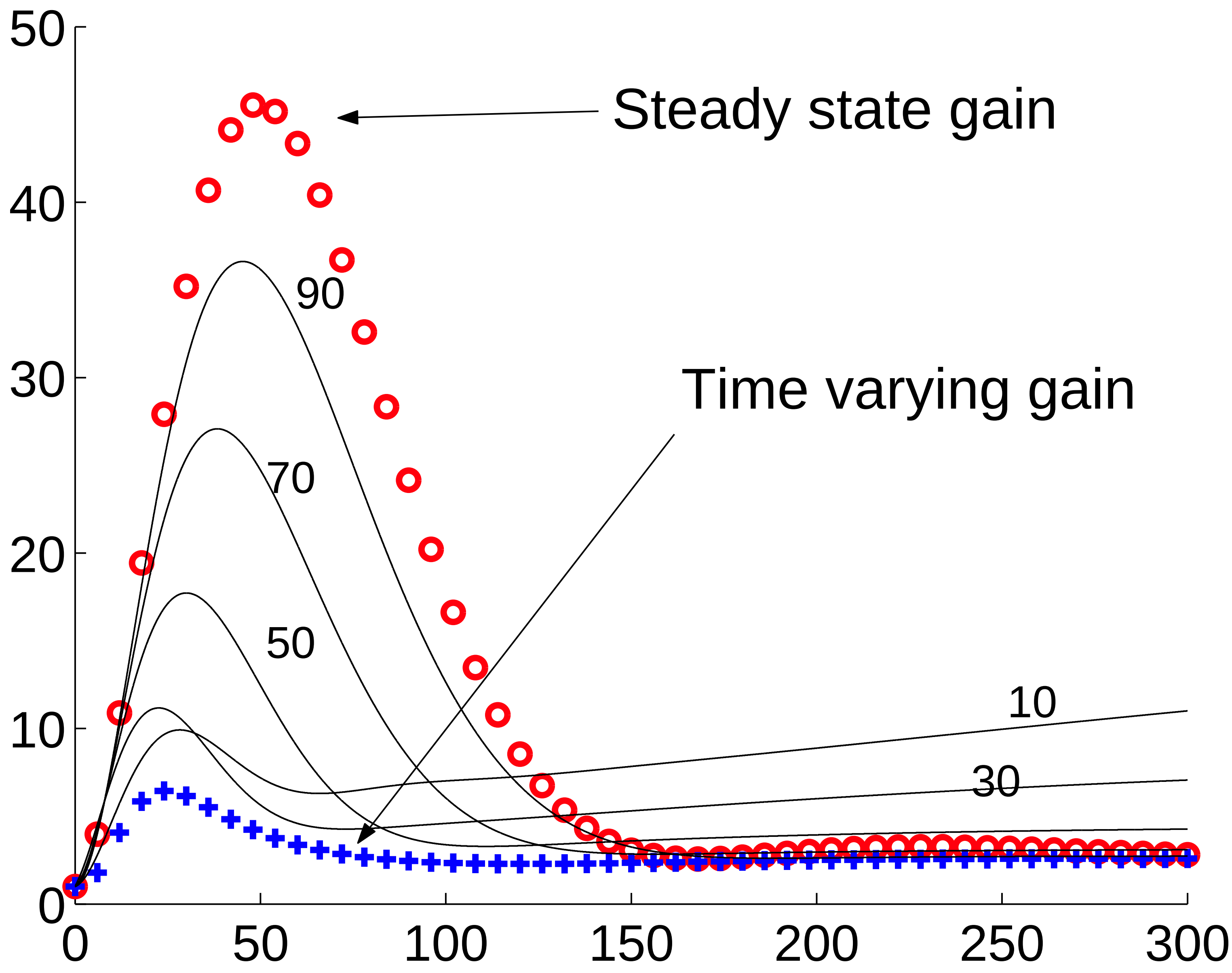
  – Measurement noise G

- Duality : **Backward in time**

$$A \to A^* \qquad Q_0 \to P_0 \qquad Q \to R \qquad l^2 \to G$$

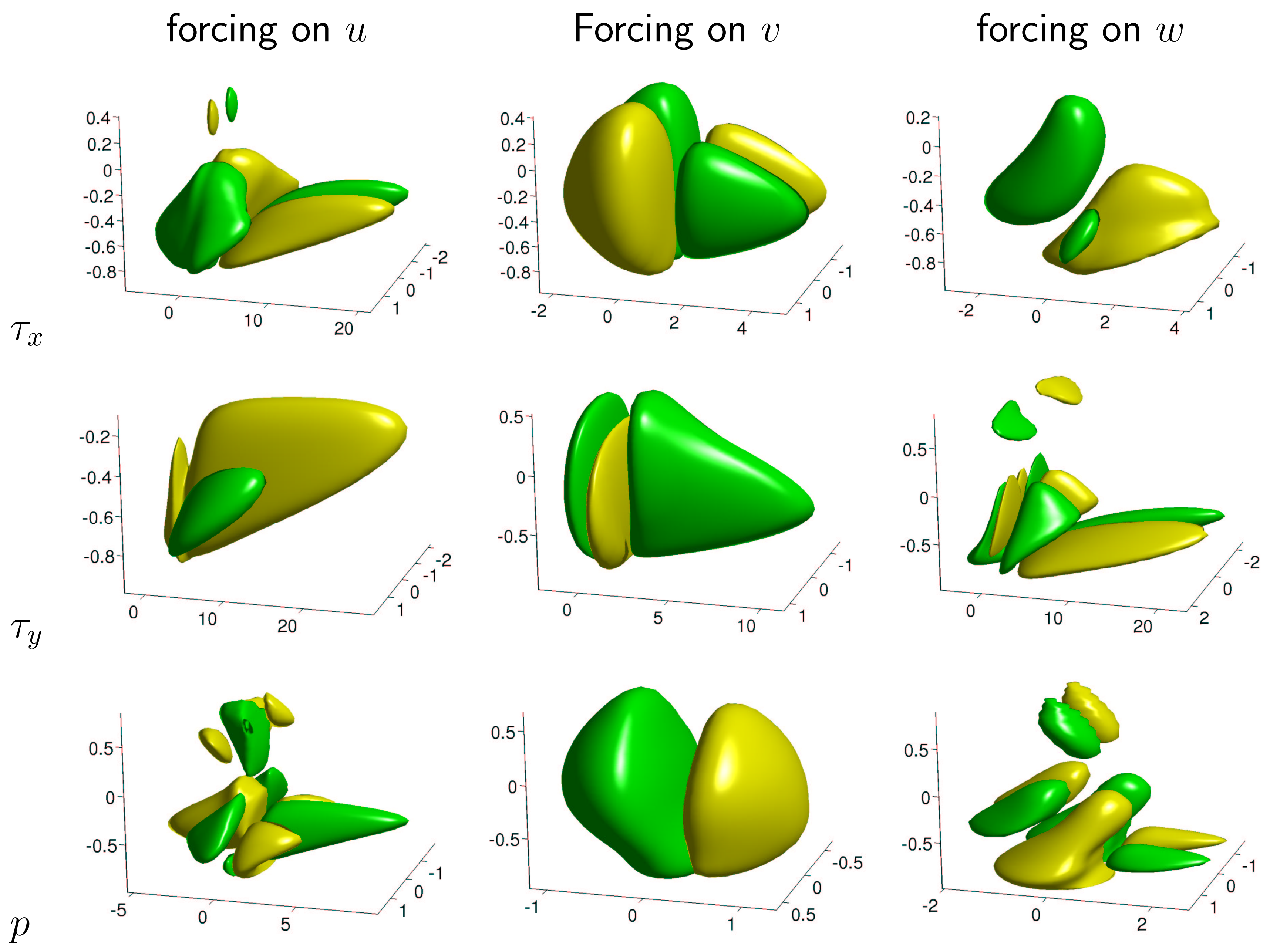
A sub-optimal procedure

Pick a gain from time t and apply it in $[0 \ T]$



Results 2 : Steady state kernels

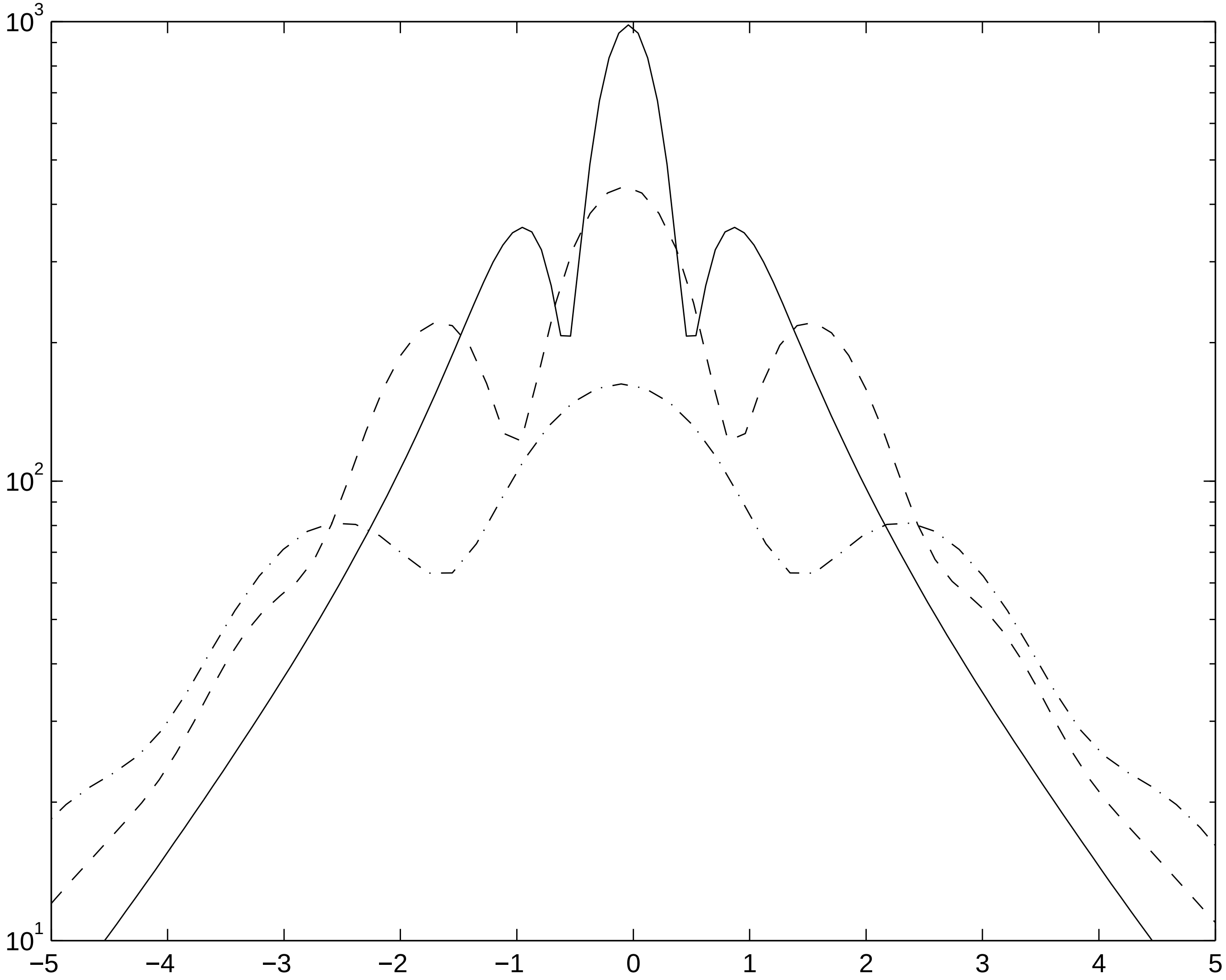
Steady state kernels



Spanwise extent of the kernel

Kernel K_{pu} integrated in x and y for three different spreading s_z

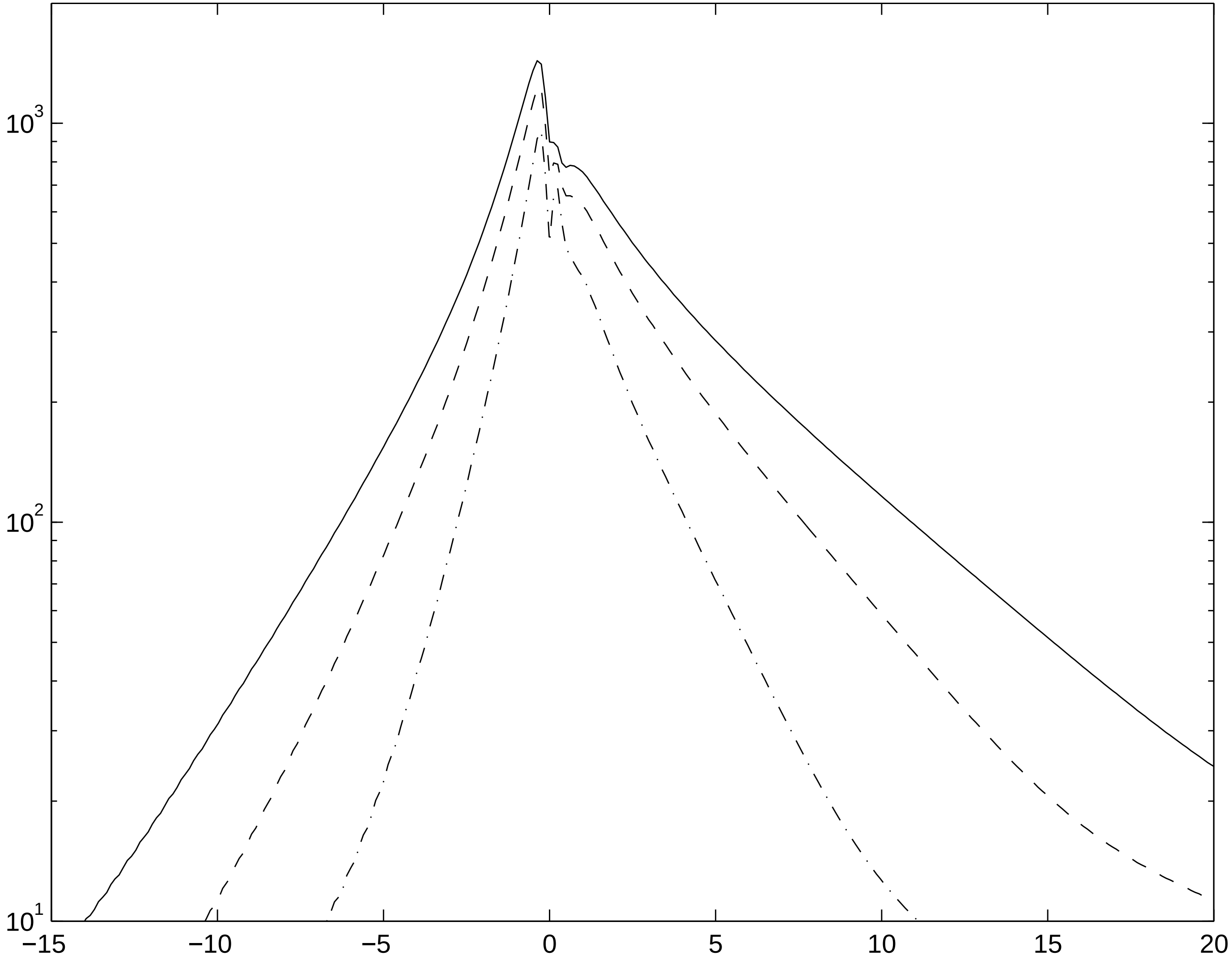
$$s_z = 0.2, 0.7, 1.3$$



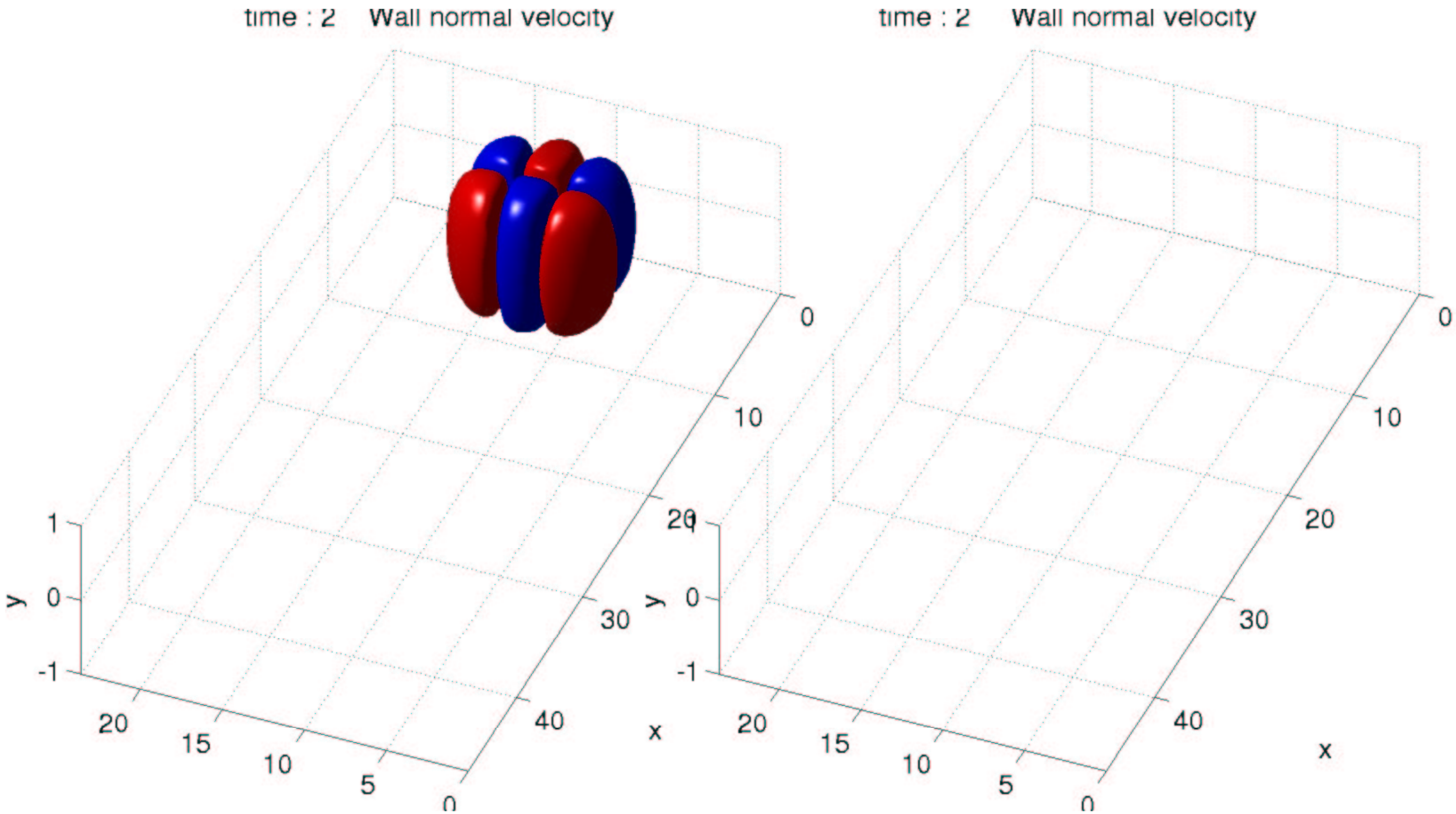
Streamwise extent of the kernel

Kernel K_{pu} integrated in z and y for three different Reynolds number

$$Re_{CL} = 1000, 2000, 3000$$



DNS with steady state kernels



Small amplitude case of :
A mechanism for bypass transition from
localised disturbances in wall bounded shear flows (Henningson et. al. 1992)

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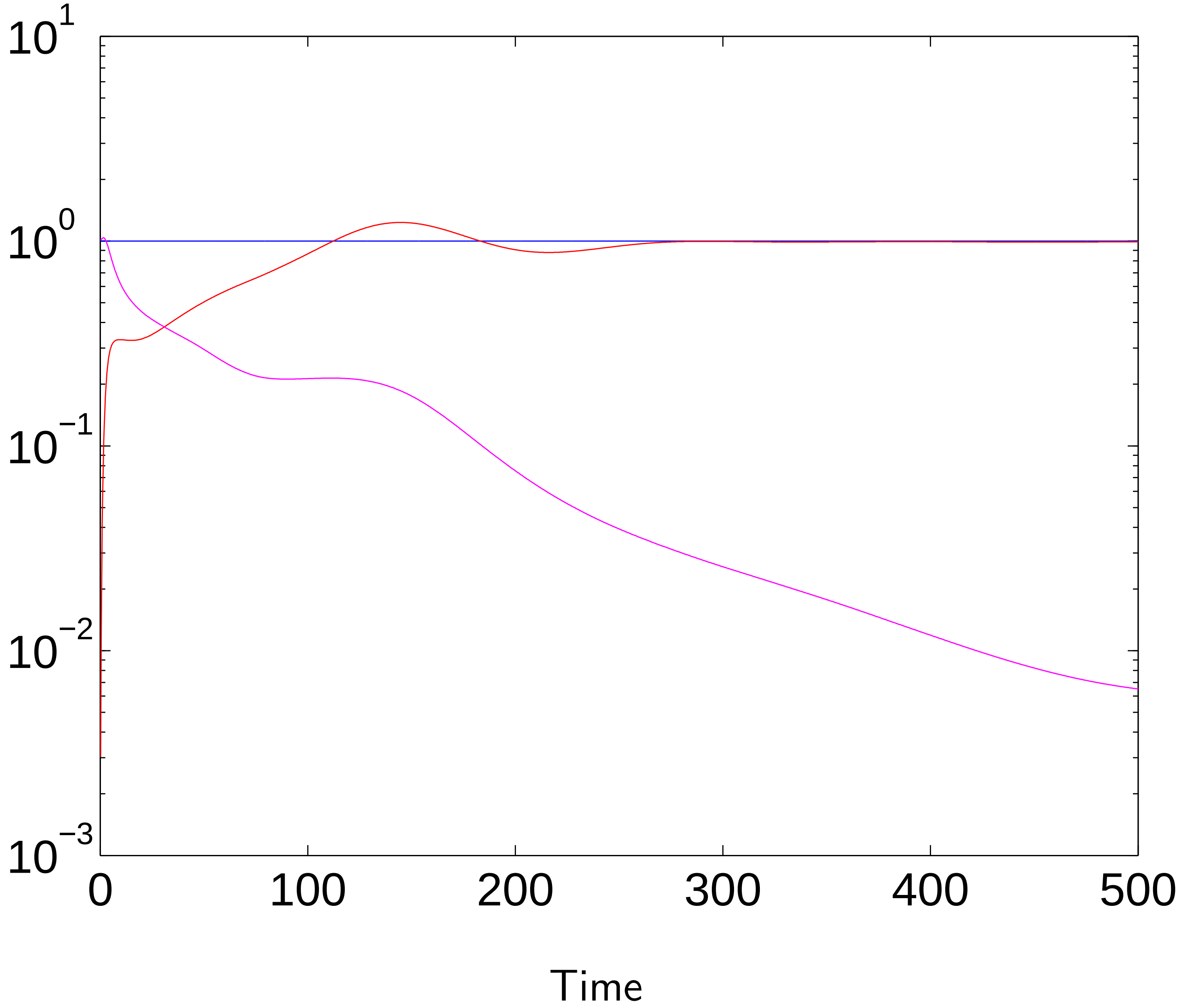
Normalised energy error



Conclusion

Was done

- A model for perturbations
- Choice of measurements
- Investigation of transient for estimation
- A sub-optimal procedure

To be done

- Transient for the control as well
- Apply those ideas on spatially evolving flows