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Transient growth on boundary layer streaks

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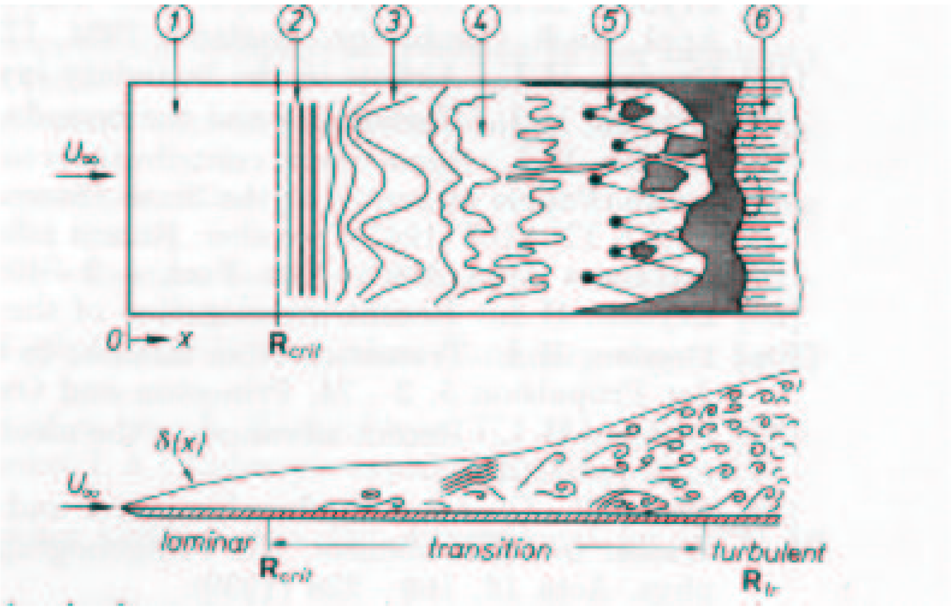
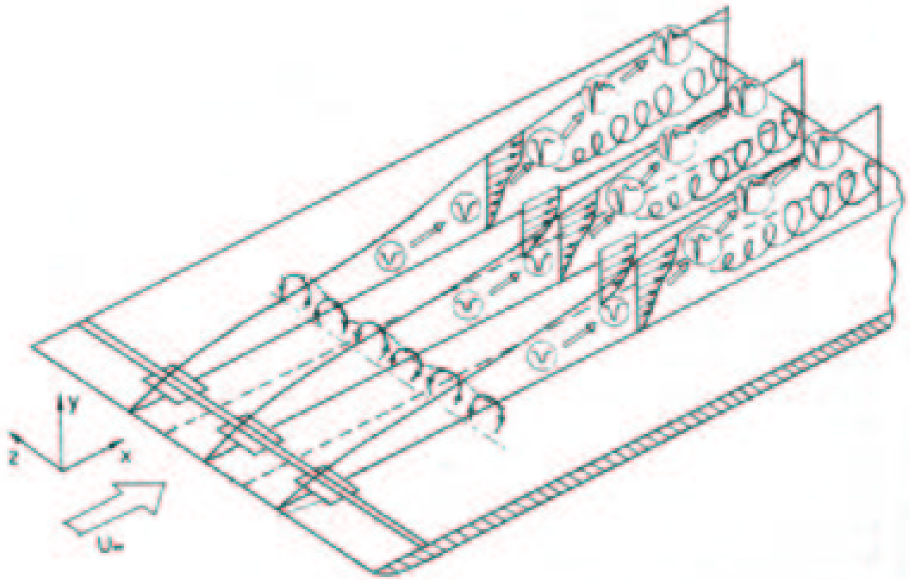
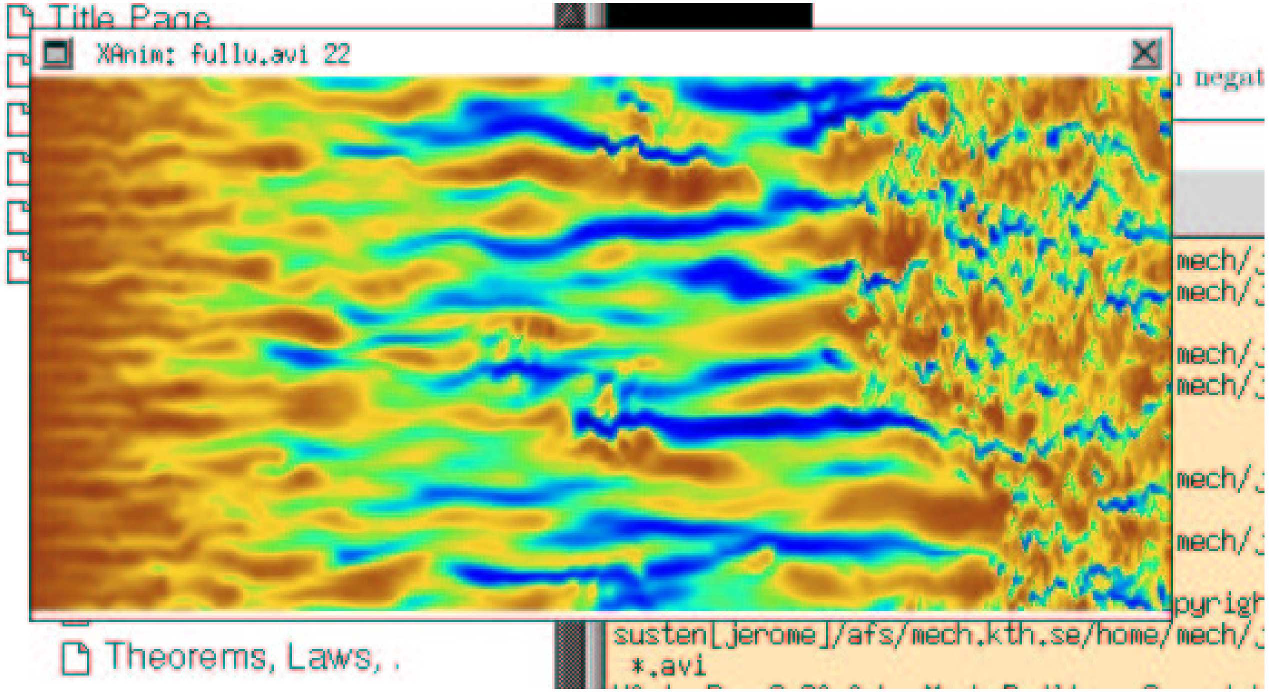
Primary instability: TS vs TG



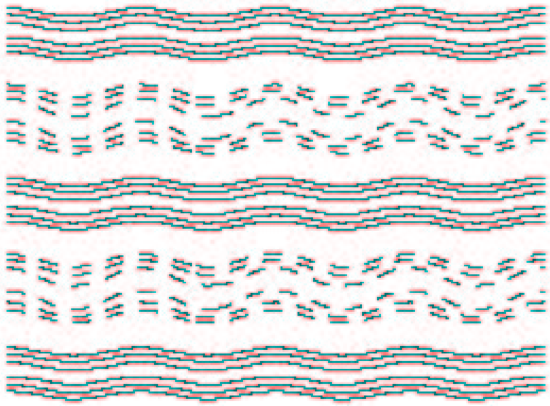


Secondary instability of the streaks

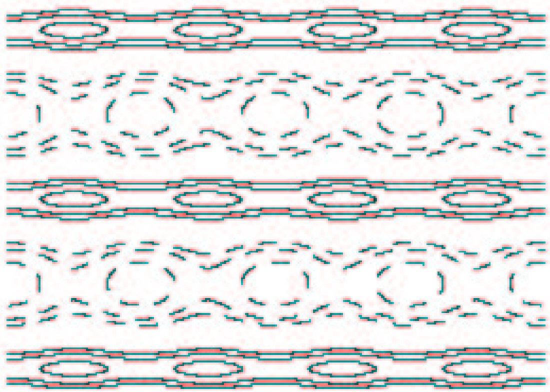


Movie of streaks breakdown (Luca Brandt)

Fundamental sinuous

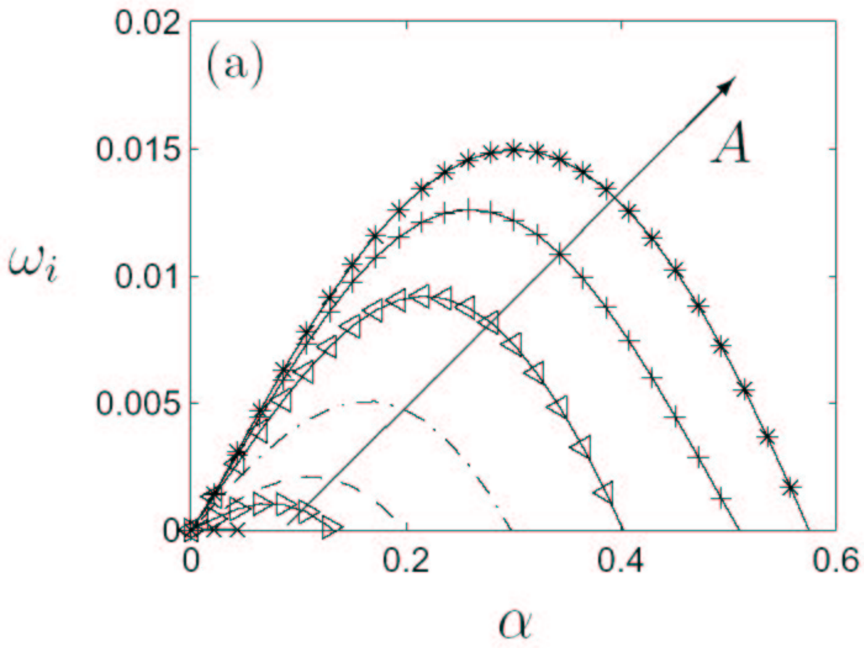


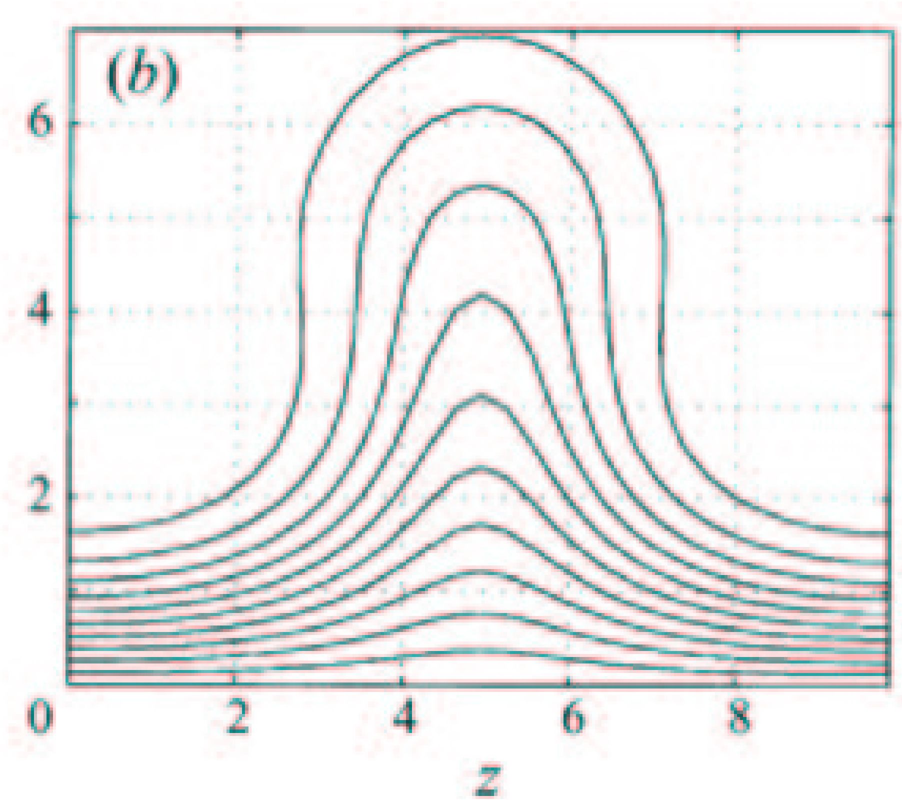
Fundamental varicose



Onset for streaks instability is at amplitude 26% for sinuous disturbances, and even larger for varicose.

Unstable sinuous eigenmode





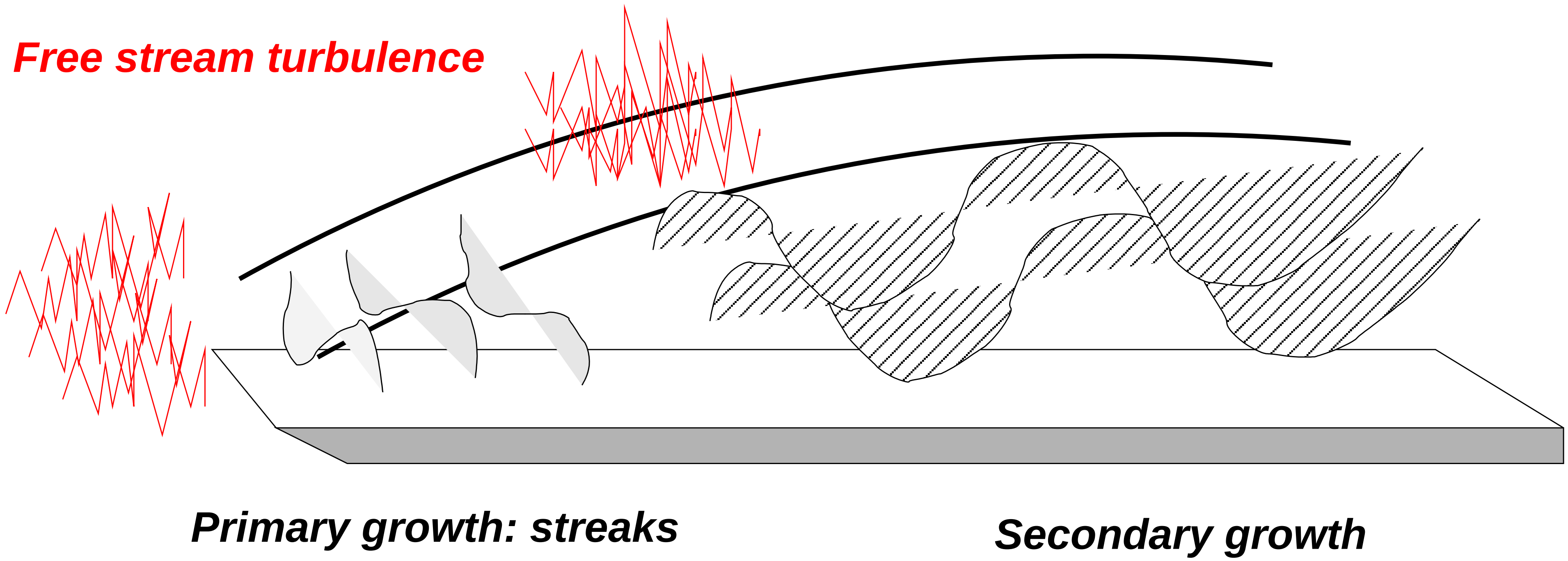
(From PhD thesis of Luca Brandt)

Temporal growth for unstable sinuous disturbance:

amplitude A=0.28, 0.32, 0.34, 0.36.

Secondary transient growth

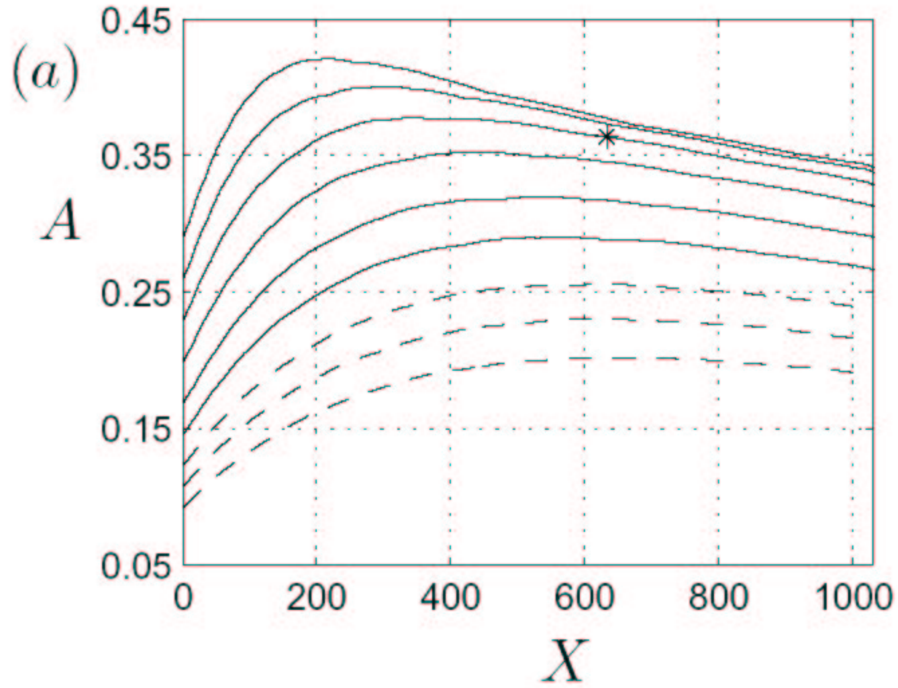
Maybe transient growth plays a role in secondary instability?



Conjectured by Schoppa&Hussain(2002) and Lundell(2004).

Base flow for stability

- Inflow condition from optimal vortices on Blasius flow
- Streaks are generated and grow downstream
- Extract Base flow +streaks at downstream location of maximum amplitude
- Freeze this flow and assume invariance in x $\rightarrow U(y,z)$
- Use $U(y,z)$ as base flow for linear stability analysis.
- floquet formulation for the spanwise periodic base flow $U(y,z)$



From Anderson et al (2001)

Eigenvalues tell about asymtoptic stability

Singular values tell about transient behaviour

Stability equations

Perturbation (v, η) on the base flow $U(y, z)$

Wavelike behaviour in the streamwise direction:

$$[v, \eta] = [\widehat{v}(y, z, t), \widehat{\eta}(y, z, t)]\ e^{i\alpha x} + c.c.$$

Derivation similar to the Orr–Sommerfeld/Squire equation:

$$\begin{cases} \Delta v_t + U\Delta v_x + U_{zz}v_x + 2U_z v_{xz} - U_{yy}v_x - 2U_z w_{xy} - 2U_{yz}w_x = \frac{1}{Re}\Delta\Delta v, \\ \eta_t + U\eta_x - U_z v_y + U_{yz}v + U_y v_z + U_{zz}w = \frac{1}{Re}\Delta\eta. \\ (\text{with} \quad w_{xx} + w_{zz} = -\eta_x - v_{yz}) \end{cases}$$

+ Floquet analysis: base flow and disturbance are periodic in spanwise direction.

Look only at fundamental modes

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Floquet analysis

- Base flow is spanwise-periodic
- Fundamental mode and its harmonics are coupled
- Detuned modes are decoupled to each other
- We look at fundamental modes in this study

Transient growth

- Dynamic system with initial condition:

$$\dot{q} = Lq, \quad q(0) = q_0$$

- Input-output operator $\mathcal{H}_\tau$:

$$q(t) = \mathcal{H}_\tau(q_0)$$

- Maximum possible growth:

$$G(\tau) = \max_q \frac{||\mathcal{H}_\tau q||_E}{||q||_E} = \max_q \frac{(\mathcal{H}_\tau q, \mathcal{H}_\tau q)}{(q, q)} \triangleq \max_q \frac{(q, \mathcal{H}_\tau^+ \mathcal{H}_\tau q)}{(q, q)},$$

- with adjoint operator:

$$(\mathcal{H} q_1, q_2) = (q_1, \mathcal{H}^+ q_2), \quad \forall q_1, q_2$$

Max $G(\tau)$ is the largest eigenvalue of operator $\mathcal{H}_\tau^+ \mathcal{H}_\tau$

Computation by power iterations

Power Iteration:

- Consider initial guess $q^0(0)$
- March forward in time with dynamic equation : $q^0(\tau) = \mathcal{H}_\tau q^0(0)$
- March backward in time with adjoint equation: $q^1(0) = \mathcal{H}_\tau^+ q^0(\tau)$
- Renormalize energy

Each of these power iteration magnifies
the component of the initial guess on the optimal initial condition.

Convergence in less than 20 iterations $\rightarrow$ well separated eigenvalues

Model limitations

Infinitely elongated and frozen streak:

- Disturbance wavelength small compared to streak evolution in x
  $\rightarrow \alpha > 0$
- Disturbance should be quick to reach high energy

Look at amplification for short times
and streamwise dependent perturbations

Results

Introduction to energy envelope

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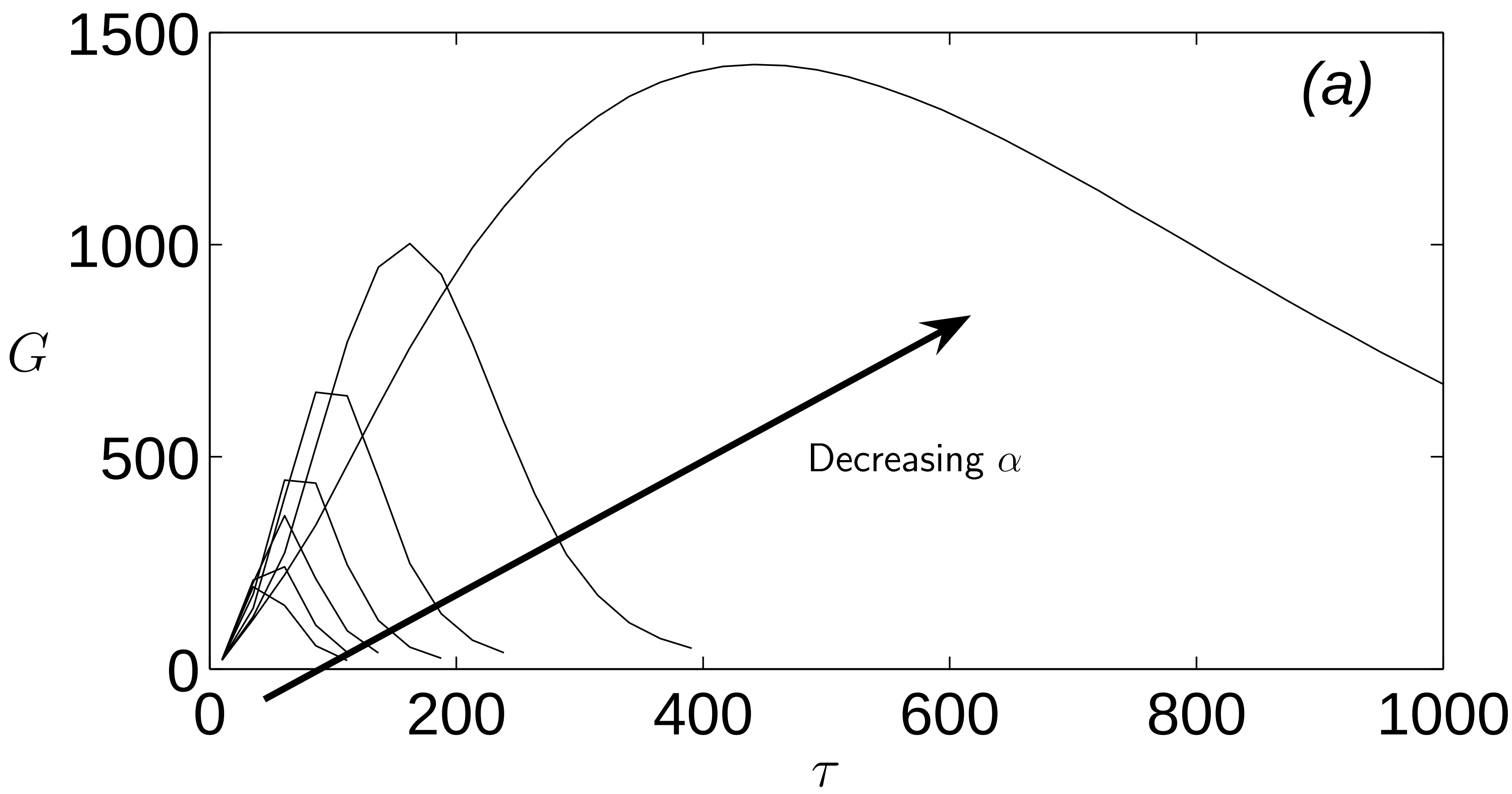
Energy envelope for several α.

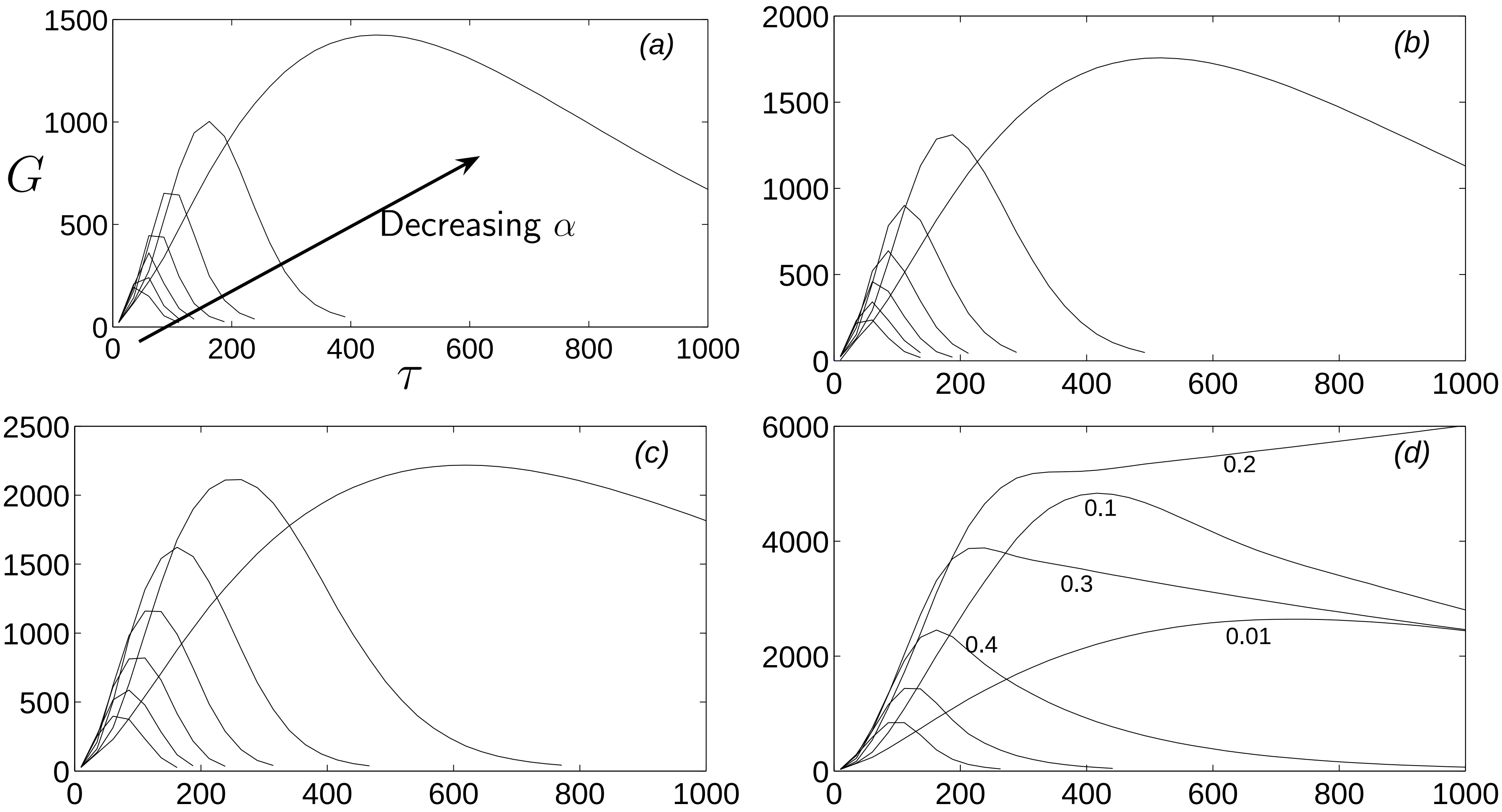


α =0.01, 0.1, 0.2, ..., 0.6

The envelope shows for each time the greatest reachable energy

Sinuous energy evolution

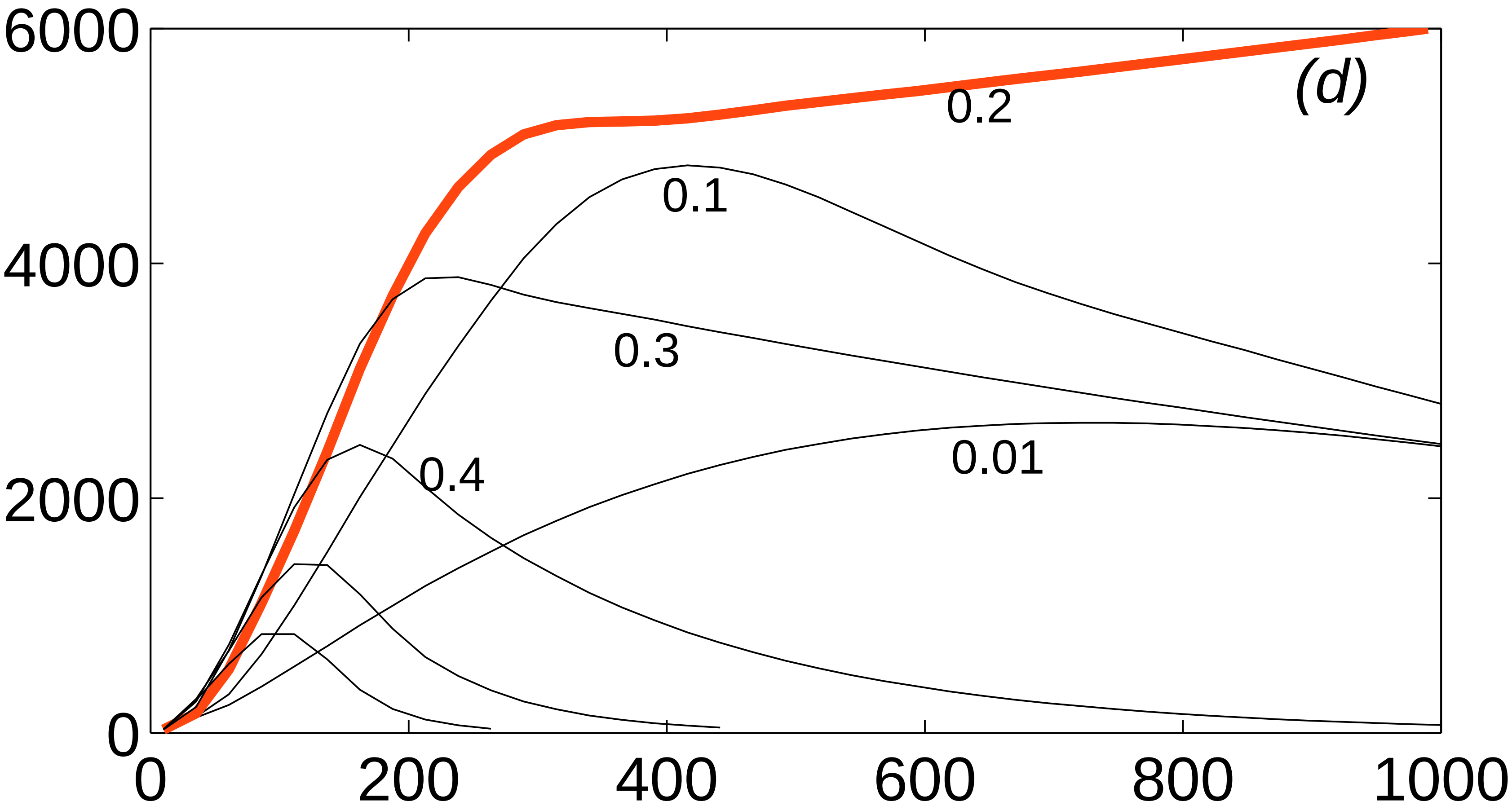
For 4 different streak amplitudes: energy envelope for several α.



α =0.01, 0.1, 0.2, ..., 0.6

(a),(b),(c),(d): Amplitudes: A=0.14, 0.2, 0.25, 0.29.

Sinous: onset of instability for large streak amplitude

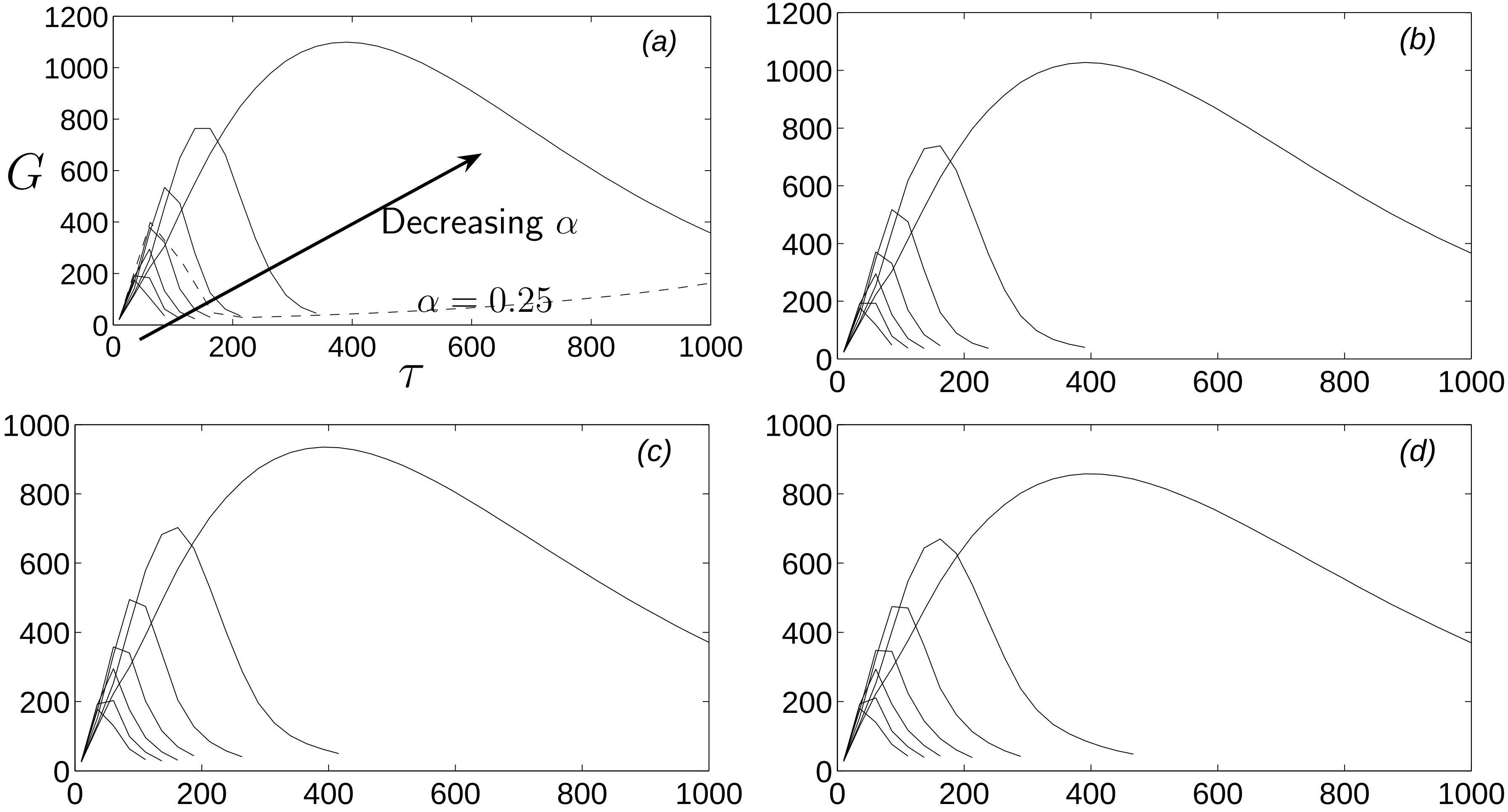


wavenumber $\alpha = 0.2$ is linearly unstable,

but can reach quickly high energy due to transient growth.

Varicose energy evolution

For 4 different streak amplitudes: energy envelope for several α.



α =0.01, 0.1, 0.2, ..., 0.6

(a),(b),(c),(d): Amplitudes: A=0.14, 0.2, 0.25, 0.29.

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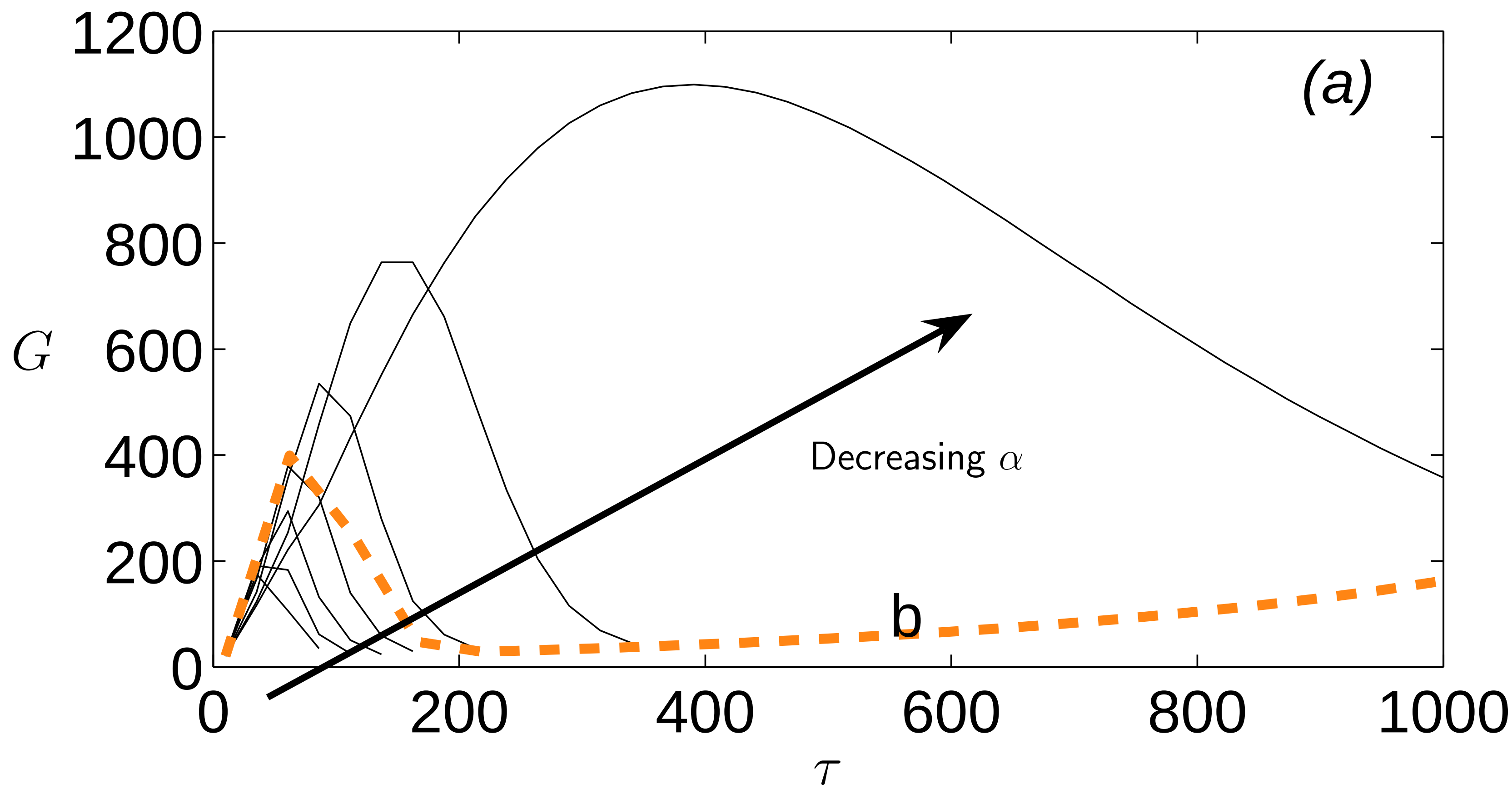
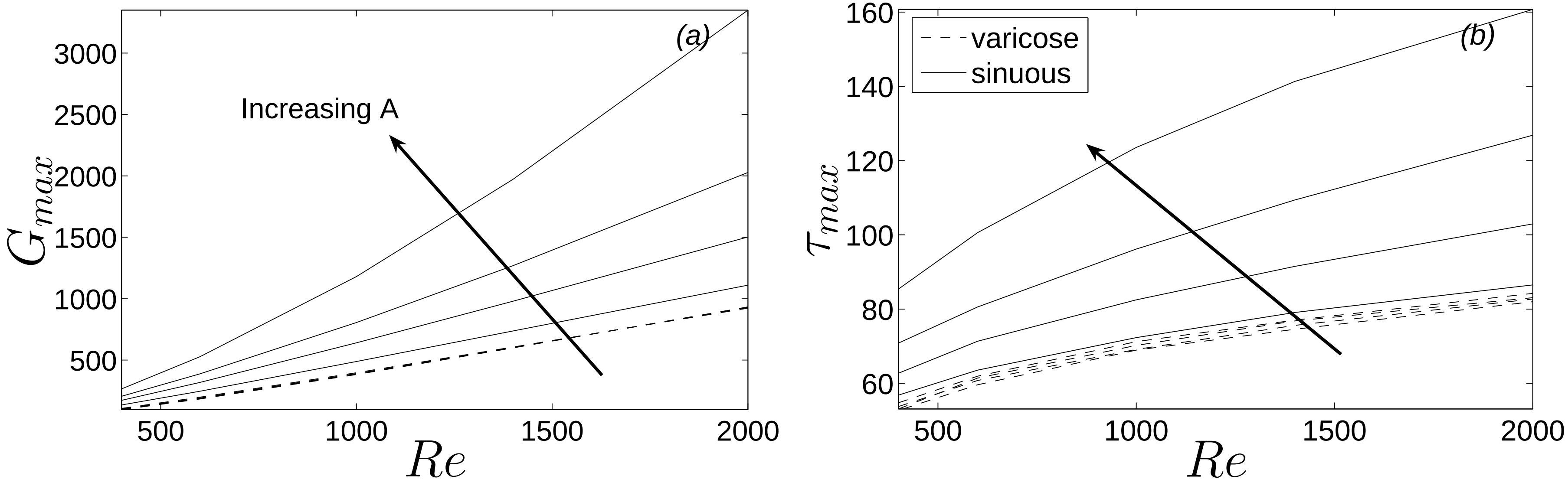
Varicose: TS-like instability for low amplitude



TS-like instability progressively disapear for streaky base flow...

(*cf* Cossu&Brandt 2003)

Amplitude and Reynolds number



Amplitudes: A=0.14, 0.2, 0.25, 0.29.

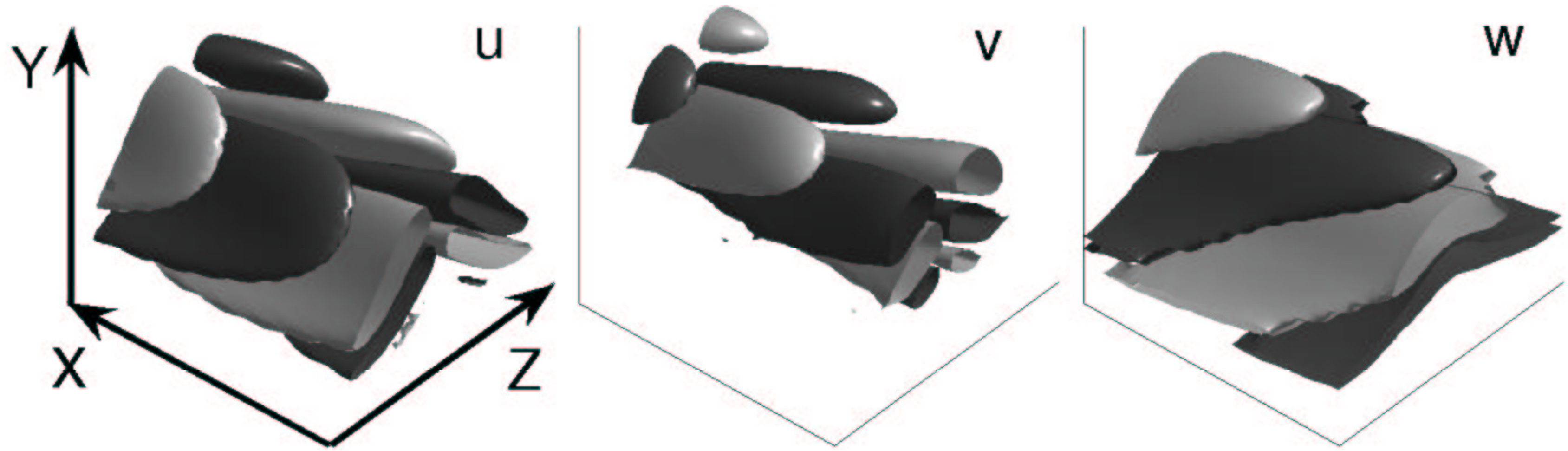
Maximum growth increases with increasing streak amplitude and Reynolds number

Time for maximum increases for increasing amplitude and Reynolds number

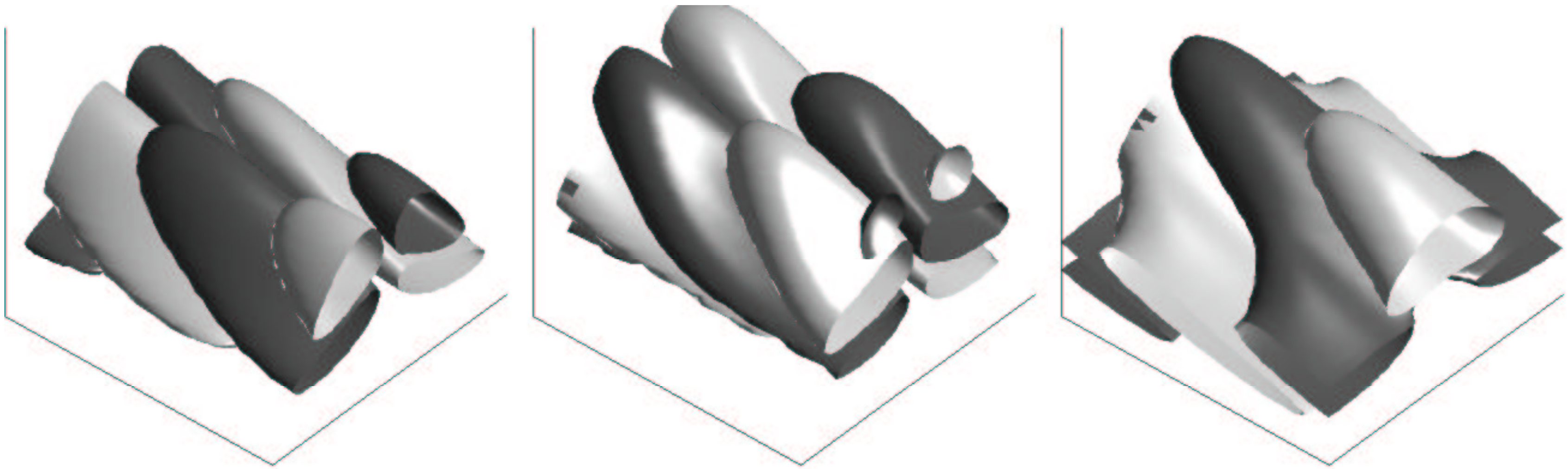
But no obvious scaling (we will see later)

Flow structures: Sinuous

Optimal disturbance:



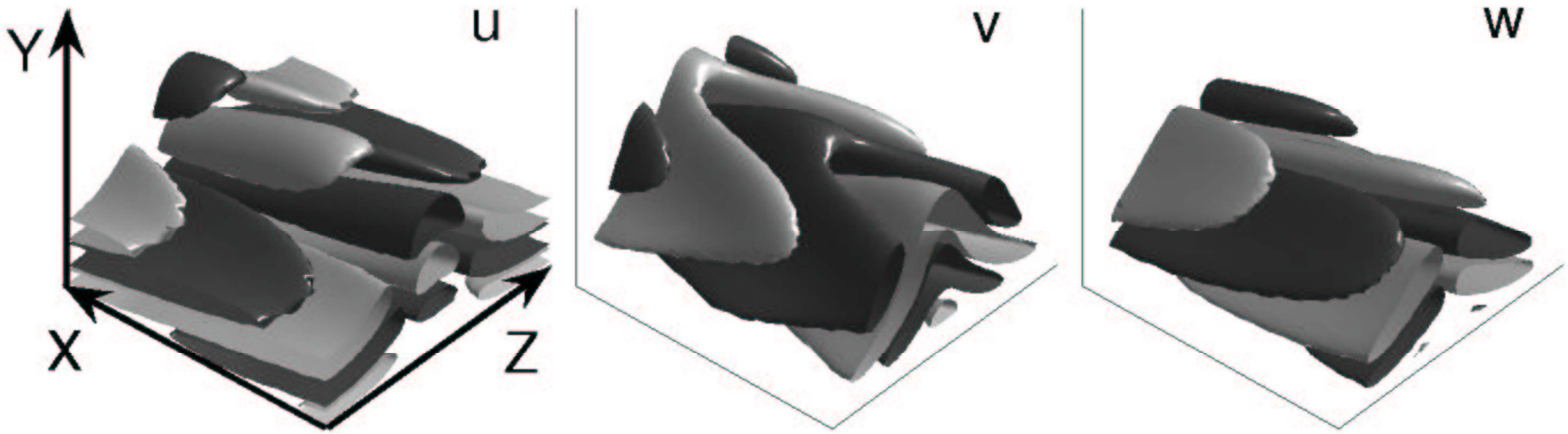
Optimal response



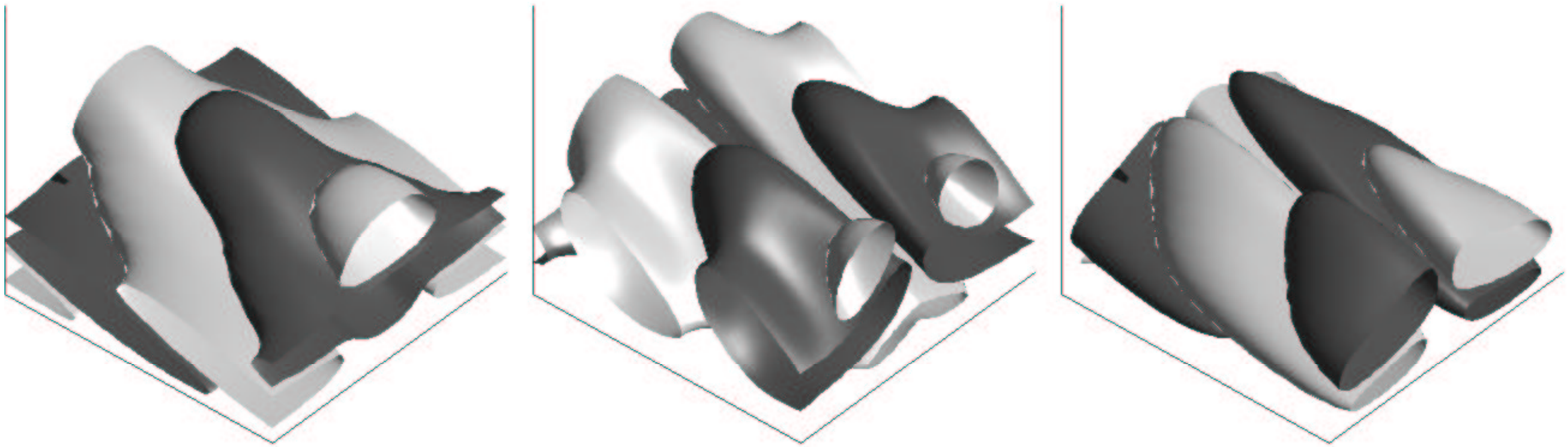
(Isosurface of constant velocity=20% of maximum)

Flow structures: Varicose

Optimal disturbance:

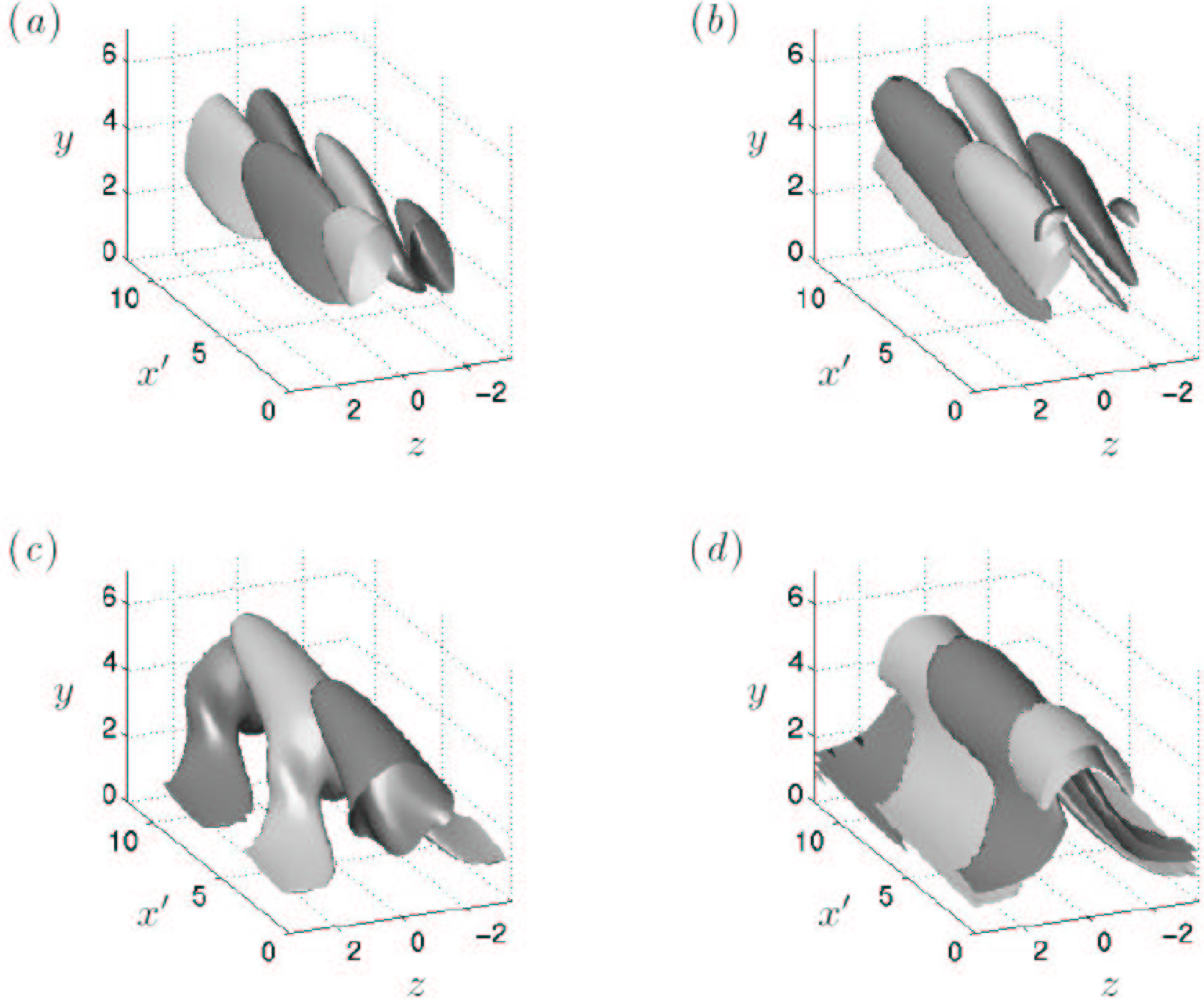


Optimal response



(Isosurface of constant velocity=20% of maximum)

Unstable sinuous eigenmode



(From PhD thesis of Luca Brandt)

Similarity in structure with our optimal response!

Analysis

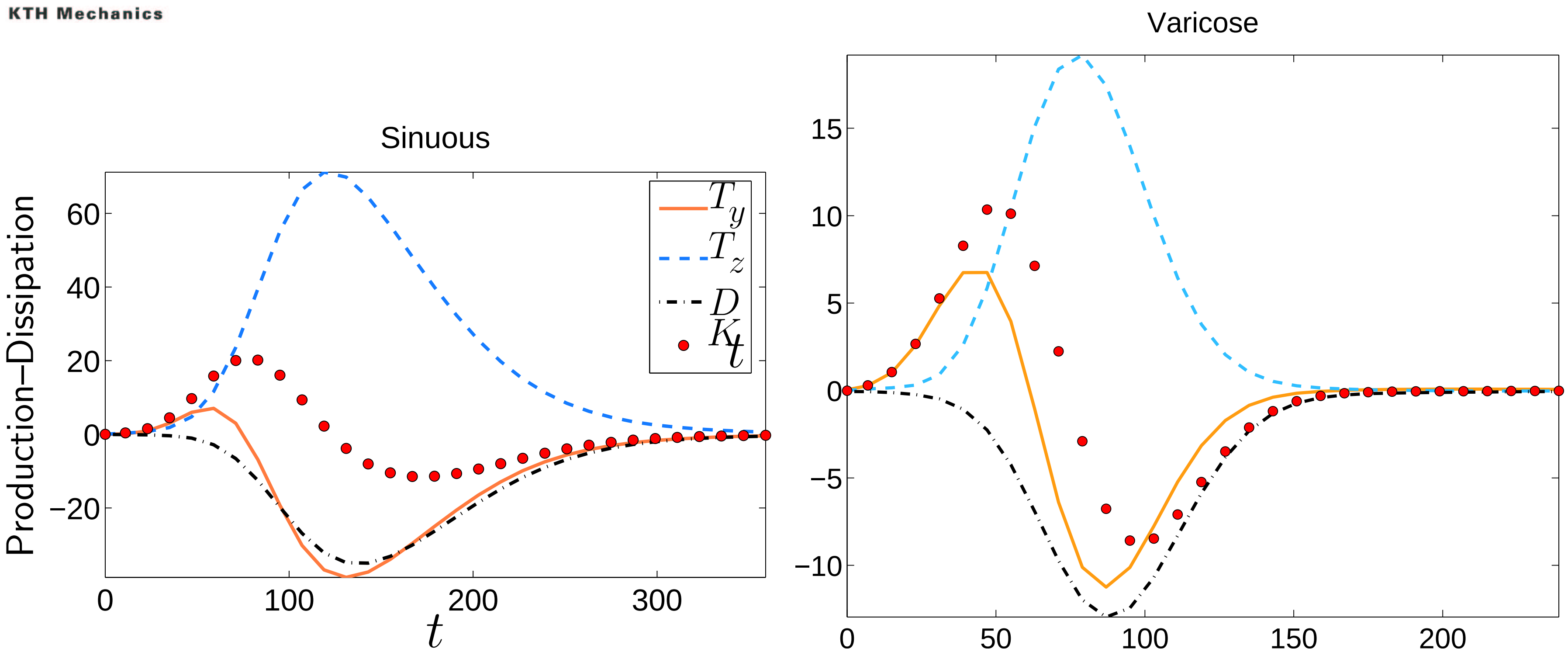
where does the energy come from?

Energy balance

$$K_t = \int (\underbrace{-uv\,U_y}_{T_y} \underbrace{-uw\,U_z}_{T_z} \underbrace{-\boldsymbol{\omega}\cdot\boldsymbol{\omega}/\mathrm{Re}}_{D})\,dy\,dz\,dx,$$

- K_t: time variation of kinetic energy
- T_y: production due to interaction with wall normal mean shear
- T_z: production due to interaction with spanwise mean shear
- D: dissipation due to viscosity

Production and dissipation



- Spanwise shear allways contributes to energy growth
- Wall-normal shear gives then takes: Orr mechanism related to structure tilting

Disturbance can gain energy

by interaction with <u>both</u> mean shear

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Conclusion

- Possibility of energy growth of $\mathcal{O}(1000)$ before onset of instability
- Time to reach peak is small (compared to streak evolution time scale)
- Optimal response resembles the unstable mode
- Two production mechanisms acting together: “lift up”+Orr

$\rightarrow$ **Observed transition from streaks may be a TG mechanism**

Remaining: how likely are those initial excitation in a boundary layer?

Submitted

Transient growth on boundary layer streaks

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The linear perturbations evolving on streamwise boundary layer streaks which yield maximum energy growth are computed. The steady and spanwise periodic streaks arising from the nonlinear saturation of optimally growing streamwise vortices are considered as base flow. It is shown that significant transient growth may occur for both sinuous antisymmetric perturbations and for varicose symmetric modes. The energy growth is observed at amplitudes significantly below the threshold beyond which the streaks become linearly unstable and is largest for sinuous perturbations, to which the base flow considered first become unstable. The optimal initial condition consists of velocity perturbations localised in the regions of highest shear of the streak base flow, tilted upstream from the wall. The optimal response is still localised in the areas of largest shear but it is tilted in the flow direction. The most amplified perturbations closely resemble the unstable eigenfunctions obtained for streaks of higher amplitudes. The results suggest the possibility of a transition scenario characterised by the non-modal growth of primary perturbations, the streaks, followed by the secondary transient growth of higher frequency perturbations. Implication for turbulent flow is also discussed.

1. Introduction

Eigenvalue analysis is traditionally performed to investigate the linear stability of a given flow configuration. The least stable among the exponentially decaying eigensolutions to the linearised disturbance equations provides information about the flow behaviour at large times. However, initial conditions which give transient energy growth may exist, a possibility related to the non-normality of the governing operator. This transient energy amplification is also referred to as non-modal since it is not due to the behaviour of a single eigenmode but it is caused by the superposition of several of them. In some cases the energy growth can be large enough to trigger nonlinear interactions and take the flow into a new configuration. The initial disturbance able to induce the largest perturbation at a given time is called *optimal* and can be computed applying optimisation techniques.

Here we apply this analysis to investigate the behaviour of small amplitude perturbations developing on boundary layer streamwise streaks. These elongated structures and their breakdown are found to be key factors both in transition in boundary layers subject to high levels of free-stream turbulence (Matsubara & Alfredsson 2001) and in the near wall region in turbulent flows (e.g. Kim, Kline & Reynolds 1971). The motivation for this study comes from the observation that the breakdown may occur also for asymptotically stable streaks. In the case of near-wall turbulence, it was noted by Schoppa & Hussain (2002) that only 20% of the streaks in the buffer layer exceed the amplitude threshold for instability. By choosing an *ad hoc* initial condition these authors were able to identify