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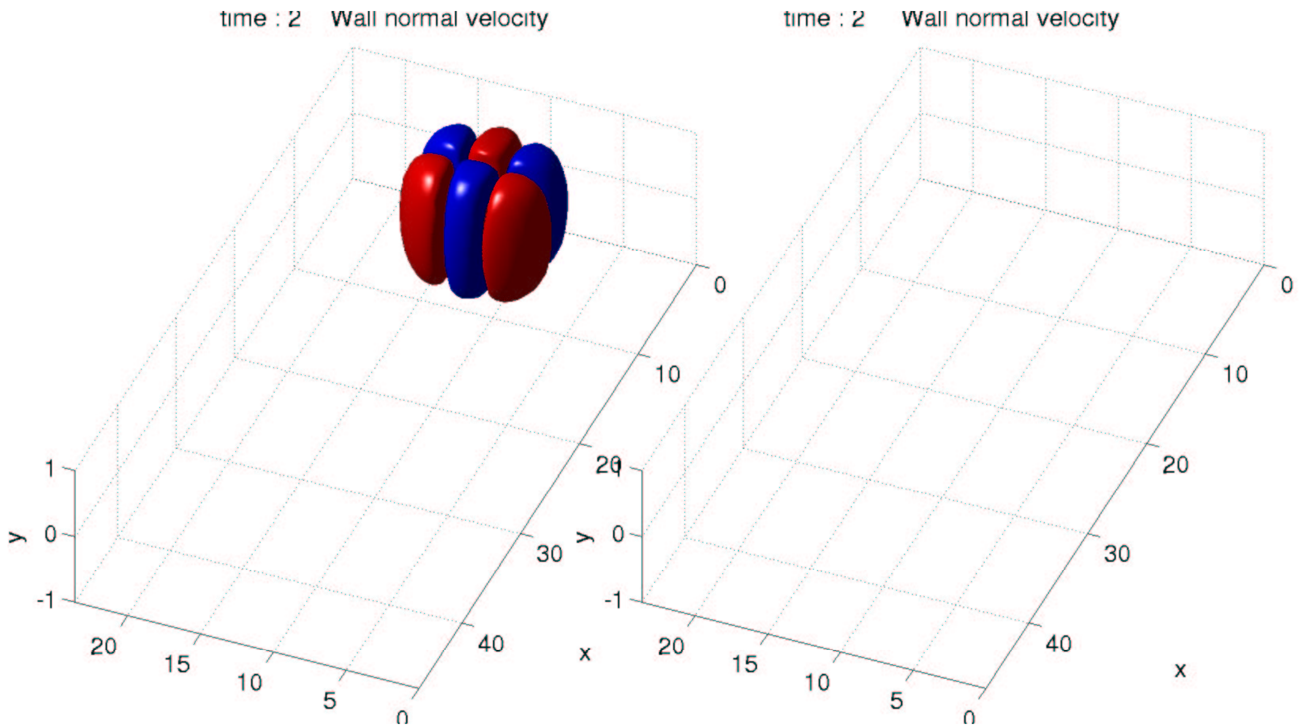
Estimation and wall bounded shear flows



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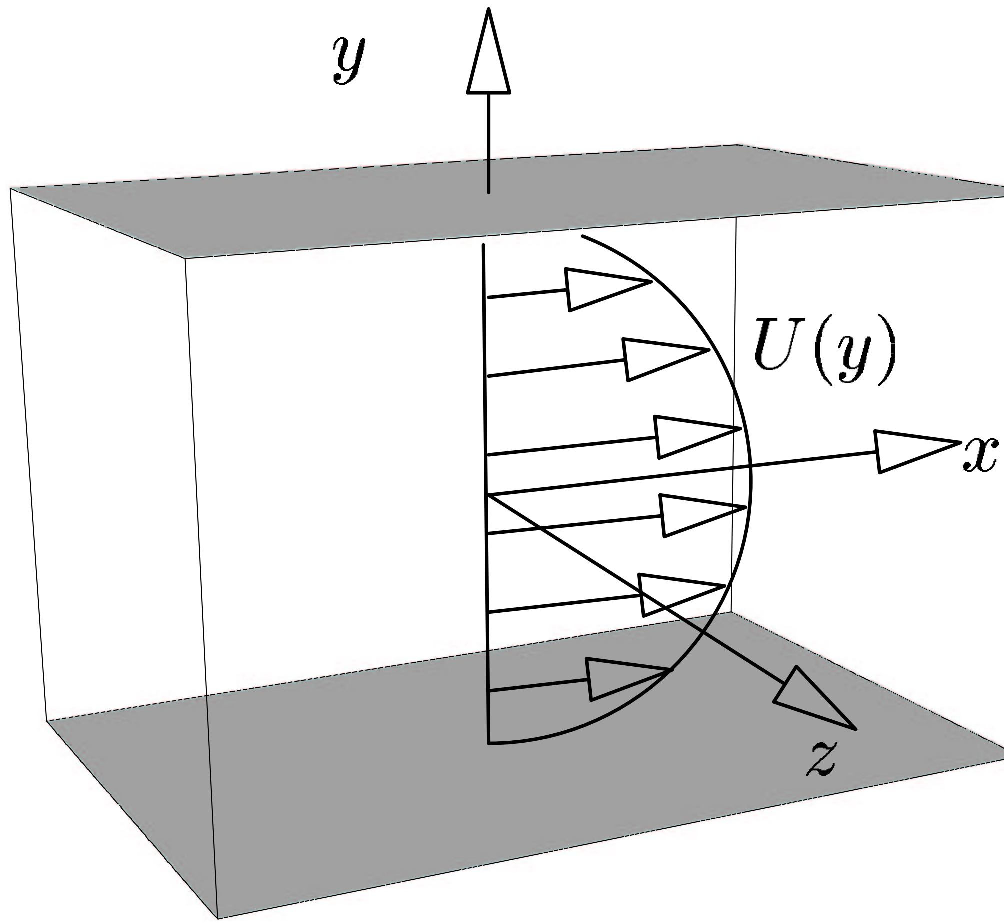
Outline

- Estimation
- Covariance and energy
- Disturbances
- Optimization
- Time varying gains
- Steady state kernels

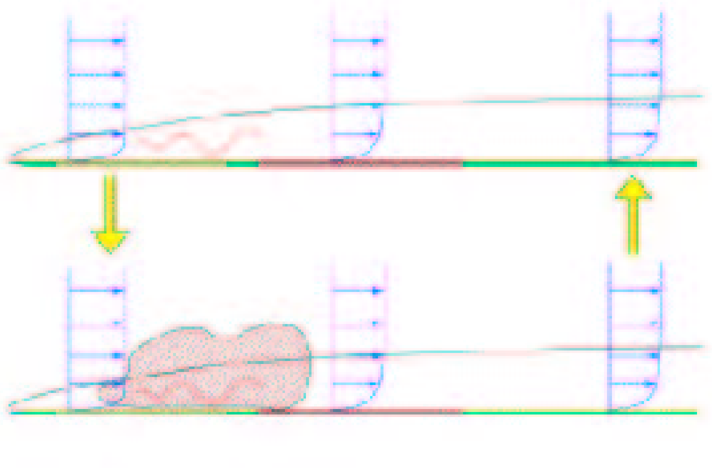
Flow addressed

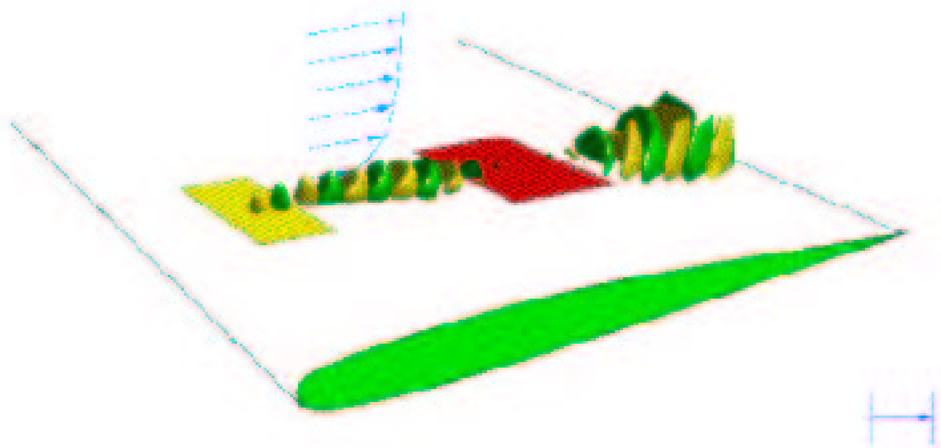


Parallel wall bounded flow

Estimation

Recover the state q from measurements y.



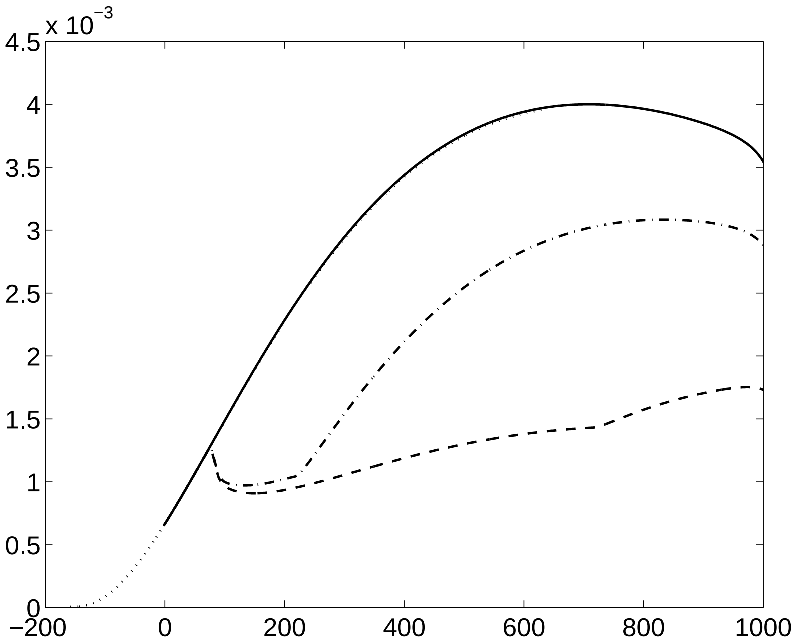


Why?

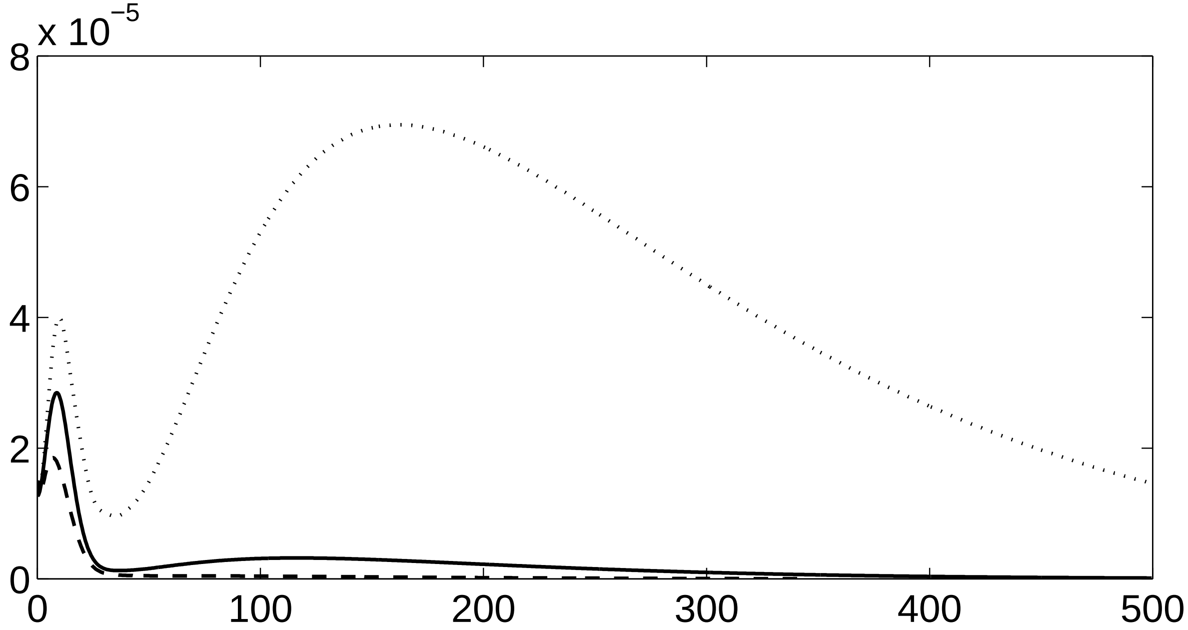
- Diagnosis
- Forecast
- Feedback control

Previous achievements

State feedback for streaks



Measurement feedback for oblique wave



The linear estimator

$$Plant \begin{cases} \dot{q} = Aq + Bf \quad , \quad q(0) = q_0 \\ y = Cq + g \end{cases}$$

$$Estimator \begin{cases} \dot{\hat{q}} = A\hat{q} - \hat{v}(y) \quad , \quad \hat{q}(0) = \hat{q}_0 \\ \hat{y} = C\hat{q} \end{cases}$$

$$Feedback \quad v = L\delta y = L(y - \hat{y})$$

Linear filtering

Propagation of the estimation error $\tilde{q}$

$$\dot{\tilde{q}} = \underbrace{(A - LC)}_{A_0}\tilde{q} + \underbrace{Bf + Lg}_{d} \quad , \quad \tilde{q}(0) = q_0 - \hat{q}_0$$

Lyapunov equation for $P(t) = E[x(t)x(t)^*]$

$$\dot{P}(t) = \underbrace{A_0P(t) + P(t)A_0^*}_{Dynamic\ terms} + \underbrace{BRB^*}_{State\ disturbances} + \underbrace{LGL^*}_{Measurement\ noise} \qquad P(0) = P_0$$

Process noise : A model

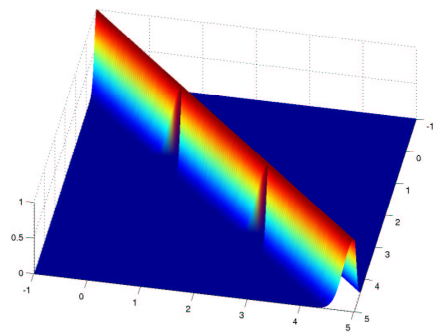
Two point correlation for the process noise

$$\Theta_{ij} = cov(f_i, f_j)$$

$$\Theta_{ij}(r_x, y, y', r_z) = v\ \delta_{ij} M^x(r_x) M^z(r_z) M^y(y, y')$$

Model

$$\begin{cases} M^x(r_x) &= \frac{1}{\sqrt{2\pi s_x}} e^{-\frac{r_x^2}{2s_x}} \\ M^z(r_z) &= \frac{1}{\sqrt{2\pi s_z}} e^{-\frac{r_z^2}{2s_z}} \\ M^y(y, y') &= \frac{1}{\sqrt{2\pi s_y}} e^{-\frac{(y-y')^2}{2s_y}} \end{cases}$$



Amplitude varying with wave number pair

Model parameters v, s_x, s_y, s_z.

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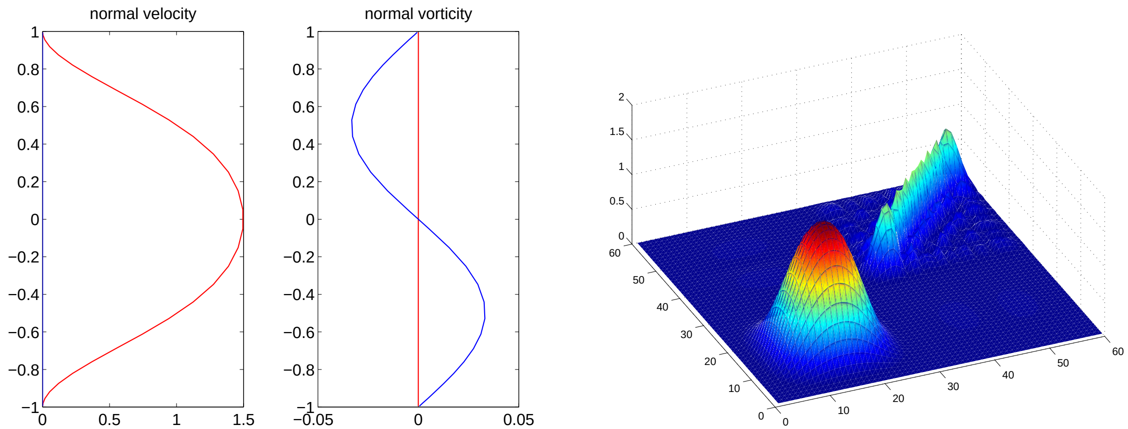
Initial condition : A model

Desired initial condition q_0 + uncertainty f_0

$$P_0 = r_1 \left(\underbrace{r_2 \frac{E[q_0 q_0^*]}{Tr(E[q_0 q_0^*])}}_{Known} + \underbrace{(1 - r_2) \frac{E[f_0 f_0^*]}{Tr(E[f_0 f_0^*])}}_{Uncertainty} \right)$$



Model parameters r_1 r_2

Optimisation : the Lagrange multiplier approach

Objective $\mathcal{J} = Tr(P(t))$

Constraint $\dot{P}(t) = A_0 P(t) + P(t) A_0^* + BRB^* + LGL^* \quad , P(0) = P_0$

Lagrangian $\mathcal{L}(t) = Tr(P(t)) + Tr\left[\Lambda(-\dot{P}(t) + A_0 P(t) + P(t) A_0^* + BRB^* + LGL^*)\right]$

Extremum of $\mathcal{J}(t)$:

$$\frac{\partial \mathcal{J}}{\partial \Gamma} = 0$$
$$\frac{\partial \mathcal{J}}{\partial P} = 0$$
$$\frac{\partial \mathcal{J}}{\partial L} = 0$$

Gives the Riccati equation :

$$\dot{P}(t) = AP(t) + P(t)A^* + BRB^* - P(t)C^* G^{-1} C P(t) \quad , P(0) = P_0$$

$$L(t) = -P(t)C^* G^{-1}$$

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Measurements

- Spanwise skin friction
- Streamwise skin friction
- Pressure

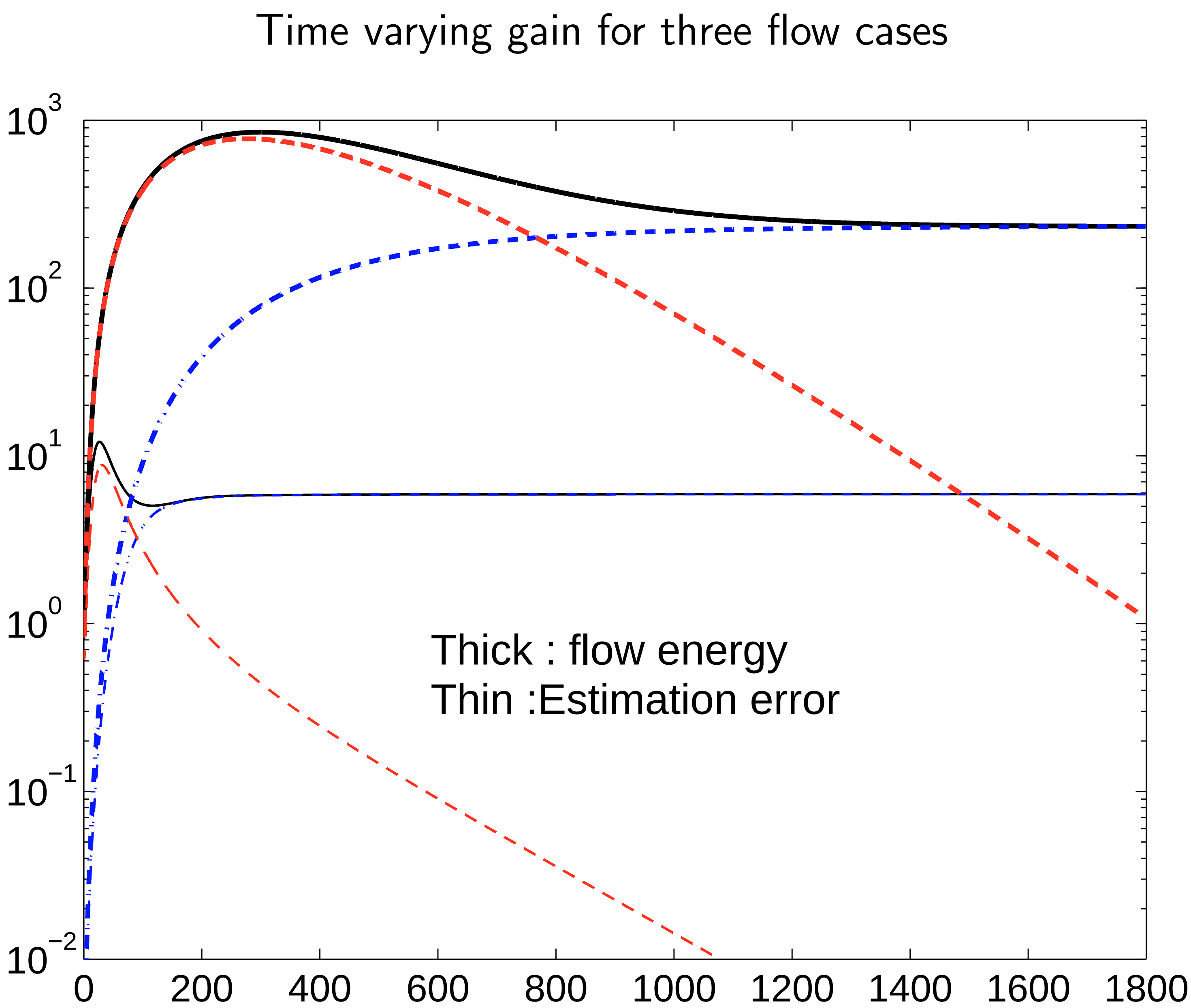
$$\begin{cases} m_1 &= \tau_{xy}|_{wall} = \mu\frac{\partial u}{\partial y}(y=0) = \frac{i\mu}{k^2}(\alpha D^2 v - \beta D\eta)|_{wall} \\ m_2 &= \tau_{zy}|_{wall} = \mu\frac{\partial w}{\partial y}(y=0) = \frac{i\mu}{k^2}(\beta D^2 v + \alpha D\eta)|_{wall} \\ m_3 &= p|_{wall} = \frac{\mu}{k^2}D^3 v \end{cases}$$

Measurement matrix

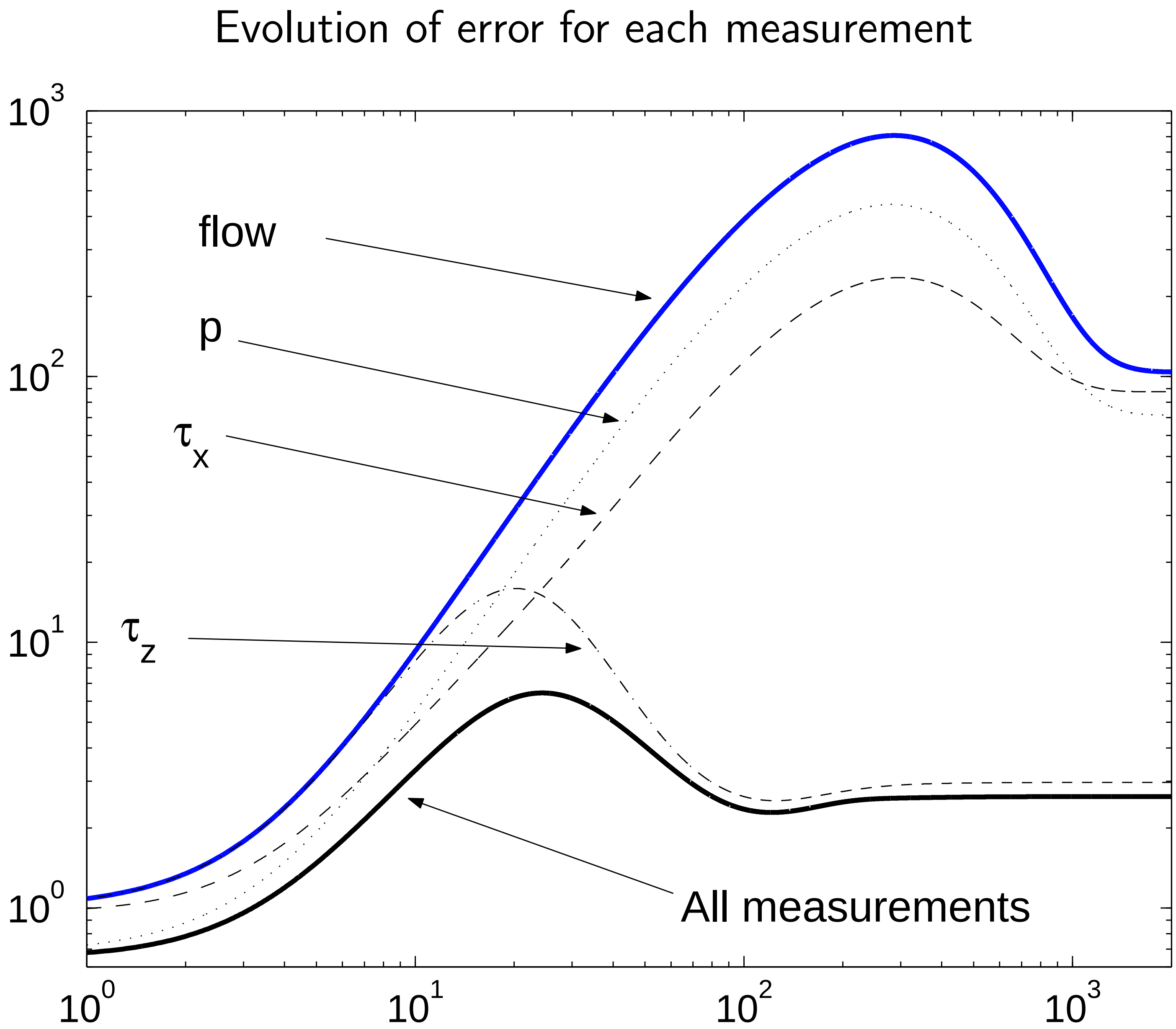
$$C = \frac{\mu}{k^2}\begin{pmatrix} i\alpha D^2 & -i\beta D \\ i\beta D^2 & i\alpha D \\ D^3 & 0 \end{pmatrix}$$

Results 1 : one wave number pair (0,1)

Flow energy, error energy

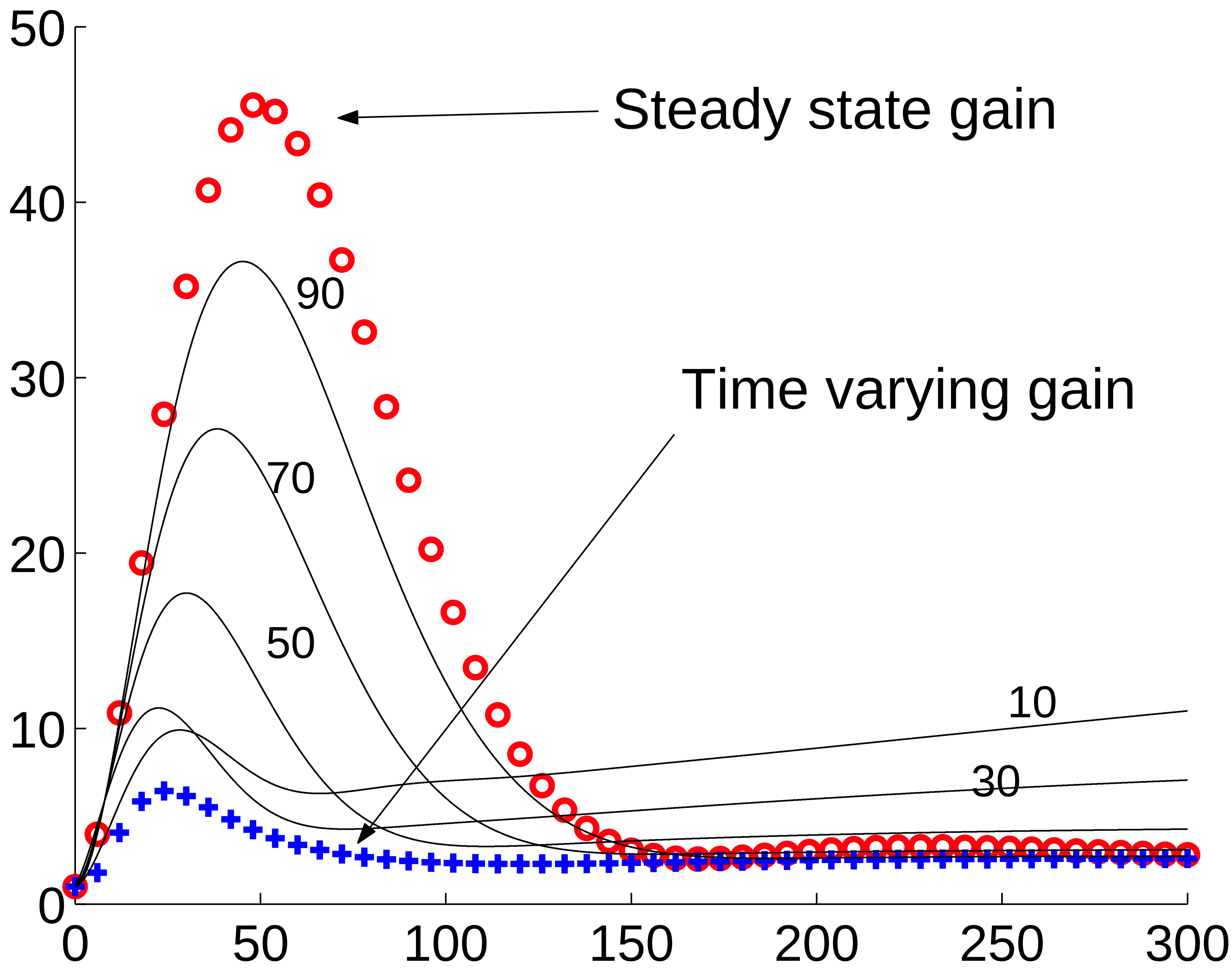


Different measurements



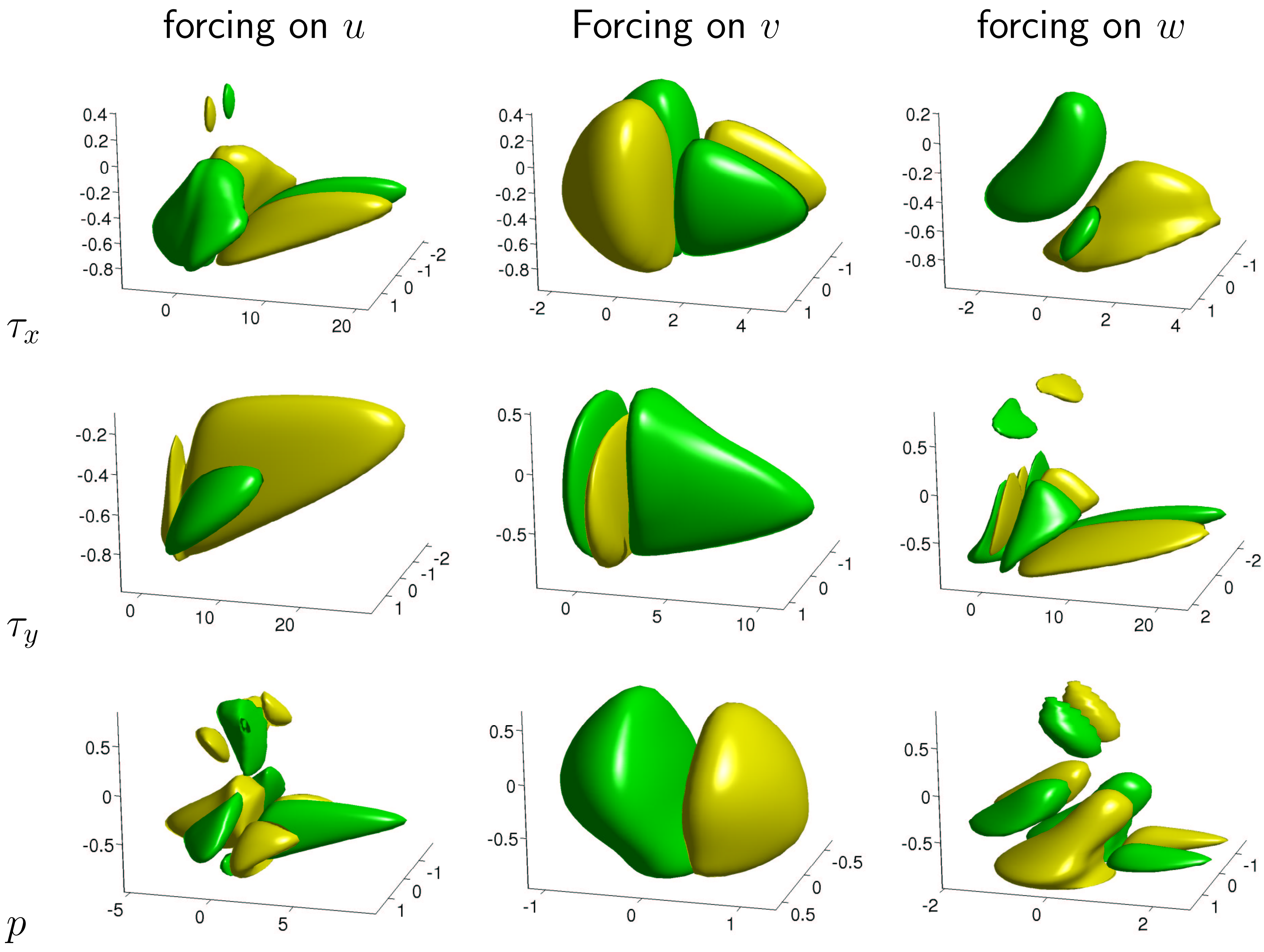
A sub-optimal procedure

Pick a gain from time t and apply it in $[0\ T]$

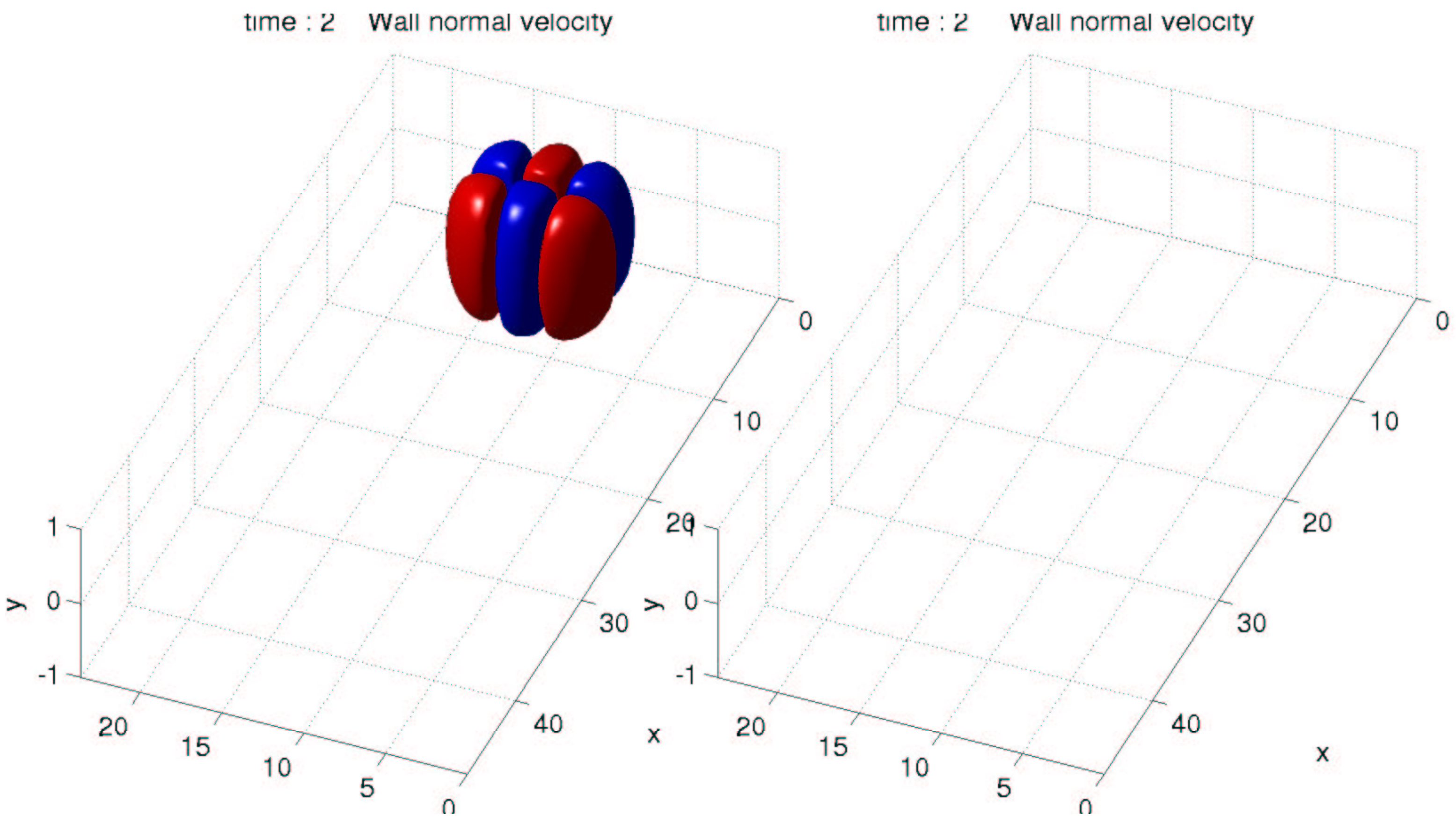


Results 2 : Steady state kernels

Steady state kernels

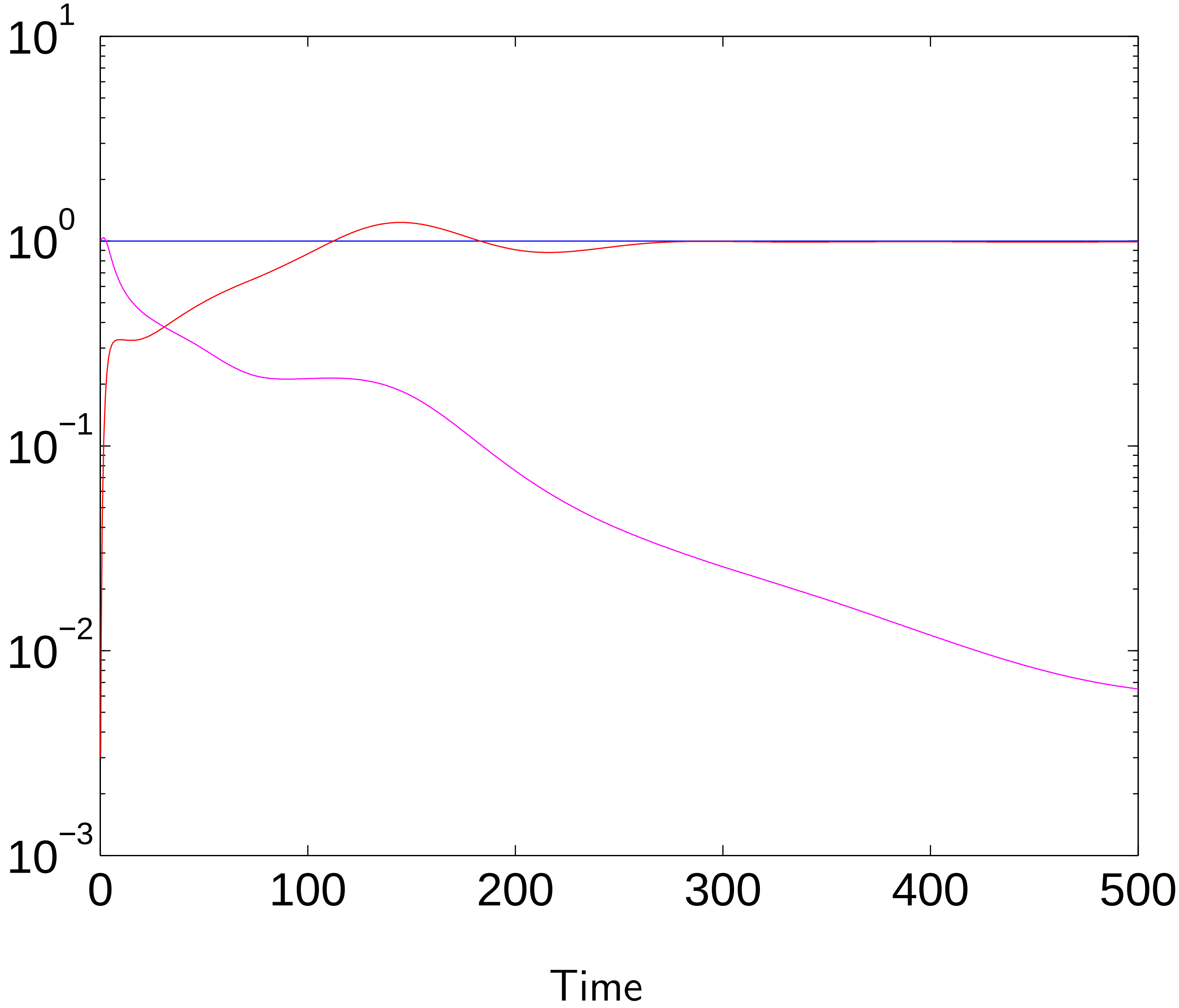


DNS with steady state kernels



Small amplitude case of :
A mechanism for bypass transition from
localised disturbances in wall bounded shear flows (Henningson et. al. 1992)

Normalised energy error



Conclusion

Was done

- A model for perturbations
- Choice of measurements
- Investigation of transient for estimation
- A sub-optimal procedure

To be done

- Transient for the control as well
- Apply those ideas on spatially evolving flows