Inviscid stability of a jet,
application to sound production in a recorder.
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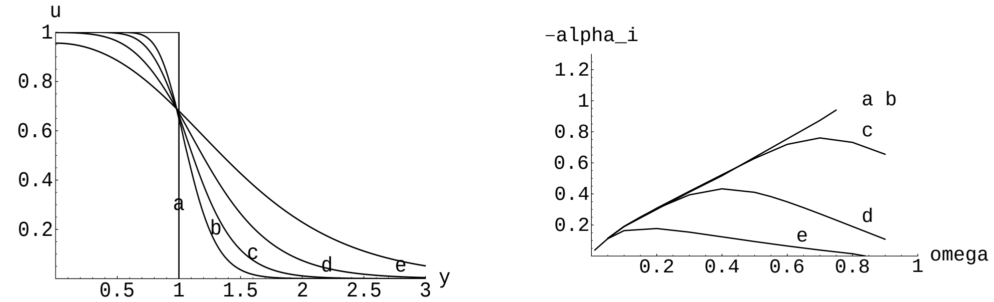
Abstract - In a recorder like instrument, sound is produced by interaction between an air jet and an acoustical field. Instability models of jets usually consider the velocity profile in the jet at the flue exit as the input for calculations of amplification factor and phase velocity of perturbations. These two factors are obtained by solving Rayleigh's vorticity conservation equation in a potential description. Therefore, the jet velocity profile is assumed to be constant along the jet axis. Here, we investigate the effect of the spreading of the jet on the unstable behavior.
First, the jet profiles are computed using the boundary layer approximation (stationary 2D, laminar incompressible Prandtl equations with constant pressure). As an example, starting from a flat profile at $x = 0$ (distance along the jet axis is adimensionalized by $h(hU_0/\nu)$ where $2h$ is the width of the slit), we see on the left figure some profiles. Enough downstream ($x > 0.5$) of the slit, the Bickley profile is obtained (as expected).



Example of some velocity profiles in the jet (left) and the associated curves of imaginary wave number ($-\alpha_i$) as a function of the (real) pulsation (ω) (right). Positions are: a: $x = 0$, b: $x = 0.01$, c: $x = 0.02$, d: $x = 0.05$, e: $x = 0.15$.

Second, to account for the convective instability of the jet, a spatial analysis is done (ω is real and given), the resulting complex wave number $\alpha = \alpha_r + i\alpha_i$ ($\alpha_i < 0$) is obtained. This analysis is justified by the fact that $(hU_0/\nu) >> 1$, and that the length wave scales with h, which is smaller than the spreading length of the jet $h(hU_0/\nu)$. The amplification factors corresponding to the previous family of profiles are ploted on the right figure.
Third, the global jet oscillation under acoustical perturbation will be determined by integration from the flue exit and compared to results obtained considering the profile at the flue exit. This will be compared to experimental amplification factors obtained by means of visualization of the flow.

References

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