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On the Influence of the Q Factor on the Oscillating Frequency of Flue Organ Pipes

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Summary
The exact prediction of the oscillating frequency of flutes and organ pipes is of great interest for instrument makers. For the time being, the prediction of this frequency relies on the calculation of the passive resonance of the pipe. However, the oscillating frequency is often found to be slightly higher than the resonance frequency, in order to enhance the spectral content of the sound. The aim of the present research is to include the influence of the blowing conditions in the prediction of the oscillating frequency. Following previous descriptions, we develop a model of flute as a loop system. The mechanism of sound generation is split into different lumped parts: the jet, the aero-acoustics source and the resonator. The linear study of this model shows that the frequency is defined with respect to the phase of the loop gain: the delay due to the jet (which convects the acoustics vibration from the flue exit to the edge) is balanced by the phase shift of the resonator. This model shows the influence of the frequency dependance of the resonator phase. An experimental setup is proposed to modify the propagation losses of a pipe in order to appreciate the influence of Q factor on the frequency dependency toward the jet velocity.

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1. Introduction

One part of the making of organ pipes consists in adjusting parameters to settle the note and the timbre. Some rules result from experiences through centuries and have been preserved and written down by organ makers [7]. Others have been partially explained by the acoustics and have been taken over by makers. For instance, the relation between the frequency f of a note and the length of a flue organ pipe is sometimes given by $L = c/2f - 5/3\varnothing$ where c is the speed of sound in the air, $\varnothing$ is the diameter of the pipe [20]. This formula, easy to use, is based on a *passive* description of the pipe acoustics. It includes correction such as the end correction of radiation.

However it is well known by makers that frequency also depends on the supply pressure or the jet velocity U_j. The frequency depends on the excitation parameters or more precisely on all the parameters which enable the sound production: the frequency also depends on an *active* description of the behaviour.

This paper investigates the influence of a passive parameter of the pipe, the quality factor (Q factor), on the active solution, the dependence of the oscillating frequency f toward the jet velocity U_j. In the first section, a simplified model is quickly presented including the passive description of the resonator. The second section describes the experimental variation of Q which is used in an active configuration in the third section.

2. Theoretical background

The model commonly used to describe auto-oscillation of flute-like instruments is based on a three elements loop system [19, 10]. The difference of pressure between the reservoir and the flue exit creates a flow. At the flue exit, the jet is sensitive to the acoustic field while it crosses the window to the edge. The jet-edge interaction creates an aeroacoustic source. The source is coupled with the resonator which amplifies the acoustic field. We present here the different parts of the model based on [2] and the results obtained by linear analysis of the system.

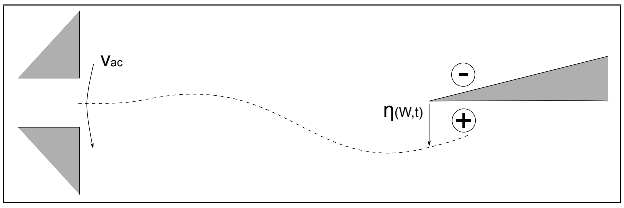

Figure 1. Active end of the pipe. The jet is sensitive to the acoustic velocity v_{ac} near the flue exit. While the air is moving, the perturbation made by the acoustic field grows. At the labium it alternately injects air in each side of the edge.

Jet

Based on works of Rayleigh [15], perturbations on the jet are amplified while convected downstream. The amplification is described, on a linear framework, by $e^{\alpha_i x}$ where α_i is the coefficient of amplification and x the distance from the perturbation.

The initial perturbation by the acoustic velocity, called the receptivity, has been studied by De la Cuadra [8]. From visualisation of a jet in presence of an acoustic field, he proposed the expression of the initial perturbation $\eta_0 = hV_{ac}/U_j$, where h is the height of the flue exit, V_{ac} the amplitude of the acoustic velocity and U_j the jet central velocity. Gathering the amplification term due to the development of the jet and the receptivity η_0 in a same expression leads to the transverse displacement of the jet at a distance x from the flue exit.

$$\eta(x,t) = \frac{h}{U_j} e^{\alpha_i x} v_{ac}(t - \frac{x}{c_p}), \tag{1}$$

where v_{ac} is the acoustic velocity at the flue exit and c_p is the phase velocity of perturbations on the jet. That is, the jet perturbation $\eta(t) \equiv \eta(W,t)$ which reaches the edge at a distance W from the flue exit corresponds to the acoustic velocity amplified by a term $he^{\alpha_i W}/U_j$ and delayed by a time $\tau = W/c_p$. De la Cuadra proposed experimental value of $\alpha_i \simeq 0.4/h$ and $c_p \simeq 0.3U_j$.

Aero-acoustic source

The interaction jet/edge is fundamental in flute-like instrument acoustics since it is the source of the acoustic vibration. Therefore it has been studied in detail by several authors. We adopt here the jet-drive description proposed by Coltman [3, 4, 5] and developped by Verge [18, 17]. Work by Dequand [9] indicates that this model is accurate at low values of the Strouhal number $Str = fW/U_j$ with f the frequency. The injection of air from both side of the edge creates of pressure difference

$$\Delta p = -\frac{\rho \delta_d}{S_m} \frac{dQ_1}{dt}, \tag{2}$$

where Q_1 is the oscillating part of the flow entering in the pipe, ρ is the air density, $S_m = WH$ the mouth section, H the mouth width and δ_d the distance between the two sources. Verge considers each source is about one jet height behind the edge of the labium: $\delta_d \simeq 2h$. As a first approximation, the spreading and associated slowing down fo the jet [16] is neglected and the jet velocity U is invariant along the mouth width, the flow Q_1 is given by

$$Q_1 = \left\langle H \int_{-\infty}^{y_0 - \eta(t)} U(y) dy \right\rangle. \tag{3}$$

The bell shape of the jet is assimilated to a Bickley profile $U(y) = U_j \sec^2(\frac{y}{b})$, where the jet velocity U_j and width b are constant. Finally the resonator is then driven by :

$$\Delta p(t) = \frac{\rho \delta_d b U_j}{W} \frac{d}{dt} \left[\tanh \frac{\eta(t) - y_0}{b} \right]. \tag{4}$$

Resonator

The resonator is a passive element of the loop system. However it has a main function in the sound generation : it enables to select and amplify the acoustic vibration near its eigen frequencies. Since the instrument is driven by a source pressure (4) we look for a passive description as an admittance $Y = V_{ac}/\Delta P$ where V_{ac} is the complex amplitude of the acoustic velocity and ΔP is the source in the frequency domain (it can be seen as the Fourier transform of (4)).

The admittance is calculated by considering the propagation in the pipe, with losses, and the radiation impedances from the mouth Z_m and the passive end Z_l. The two last are taken from [18] and [6] respectively. Special attention to the losses is needed and so a term of losses due to constriction of the pipe is add to the mouth radiation [13].

The admittance is calculated at the entrance of the pipe, where the acoustic field acts on the jet. Using the notation $\eta_i = \arg\tanh(Z_i/Z_c)$ with $i = l, m$ we write [2]:

$$Y = Y_c \frac{\cosh(\Gamma l + \eta_l) \cos \eta_m}{\sinh(\Gamma l + \eta_l + \eta_m)}, \tag{5}$$

where $Y_c = S/\rho c S_m$, l the length of the pipe, Γ is the complex wave number. For our geometry, the wave number is simplified with the wide duct approximation :

$$\Gamma = \alpha + j \frac{\omega}{v_\phi}, \tag{6}$$

where

$$\alpha = \frac{\omega}{c} \left[\frac{\alpha_1}{r_v} + \frac{\alpha_2}{r_v^2} \right] \text{ and } v_\phi = c \left[1 + \frac{\alpha_1}{r_v} \right]^{-1}, \tag{7}$$

with $\alpha_1 = 1.044$, $\alpha_2 = 1.080$, $r_v = \sqrt{2}R/\delta_v$ and $\delta_v = \sqrt{2\mu/\omega\rho}$ the viscous boundary layer thickness. The expression (6) includes viscous and thermal losses: they are both accounted for by r_v dependency. Thus, damping and dispersion that occurs during propagation are entirely described by the ratio between the radius of the pipe and the boundary layer thickness.

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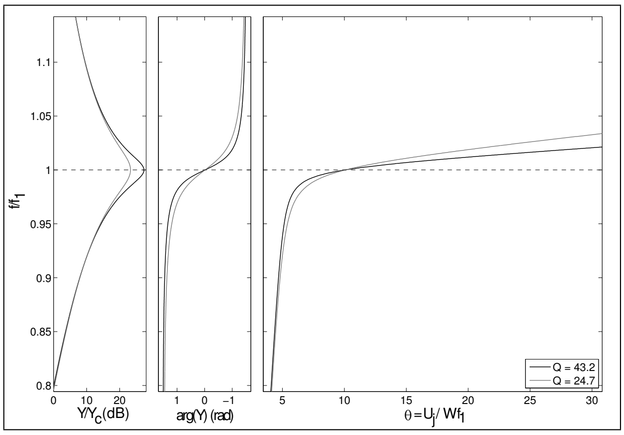

Figure 2. Left: Dimensionless amplitude and phase of two admittances Y whit different Q factors versus dimensionless frequency (axes are switched). Right: Dimensionless frequency f/f_1, solution of (9) with the two previous admittances, versus dimensionless jet velocity $\theta = U_j/Wf_1$. Dotted line represents the first resonance frequency.

Linear Analysis

Whereas the determination of oscillation thresholds seems to be predictible with a full numeric resolution of the non-linear system, a linearised solution still gives major result about oscillating regime. This section presents the method of linear analysis of the looped system.

The linearisation of equation (4) coupled to linear equations (1) and (5) and written in the frequency domain leads to the loop gain

$$G = \mu Y(\omega) e^{-i\omega\tau} i\omega, \qquad (8)$$

where $\mu = he^{\alpha_i W}\rho\delta_d/W$. As Powell showed in 1961 [14], the study of G provides a necessary but not sufficient criterion for the establishement of oscillation. First, assuming the phase shift around the loop must be an integer mutliple of 2π yields :

$$-\omega_n\tau + \frac{\pi}{2} + \arg(Y(\omega_n)) = 2n\pi. \qquad (9)$$

The solution ω_n of this equation depends on the delay τ, the phase of the resonator $\arg(Y)$ and the integer n. This last represents the hydrodynamic modes available of the jet. The case $n = 0$ represents the basic situation where the instrument usually sounds. Aeolian generation, obtained at lower jet velocity, corresponds to higher value of n.

For each ω_n the oscillation can start if $|G(\omega_{n,s})|$ is strictly higher than the unity, that means a perturbation of the zero solution will exponentially increase till it reaches an amplitude large enough to induce non-linear mechanisms which will ensure the saturation of the oscillation. The case $|G| = 1$ corresponds to a transition between a state where the oscillation can or can not grow. However, depending on non-linearities, an oscillation could be sustained or damped in regions where $|G| < 1$ or $|G| > 1$, respectively. Only the complete numerical resolution of the system will provide these informations.

Figure 2 shows the solution of (9) for different dimensionless jet velocity $\theta = U_j/Wf_1 = \sqrt{2P_{bl}/\rho}/Wf_1$ and for two pipes with different Q factors. As explain above, the solution results from the balance between the phase shift due to the jet and the phase of the resonator : the shape of the solution is quite similar to the shape of the phase response of the resonator. The way the frequency depends on jet velocity is directly linked with the parameters which drive the phase response, including the Q factor.

This model provides solutions for all frequency and for a large range of θ (from 0 to 350). This paper only focuses on the behaviour near the first resonance with a realistic range of θ (from 0 to 30). Note by the way that the computation of the loop gain modulus shows that for some θ, perturbations are able to grow on several pipe resonances. Thus, the linear analysis, though incomplete, provides a first prediction of the existence of hysteresis behaviour.

3. Acoustic behaviour of the modified pipe

Blanc *et. al.* [1] showed that the variation of the Q factor is a balance between radiation and propagation losses. Reducing the pipe diameter enables to reduce the radiation, but it also modifies the resonance frequency. Experimentally, it is easier to increase the propagation losses without significantly changing the resonance frequency. For $r_v \gg 1$, decreasing r_v in equation (7) enables to rise $\alpha \propto \omega\alpha_1/cr_v$ while keeping $v_\phi \propto c(1 - \alpha_1/r_v)$ almost constant. The experimental setup consist in local modification of the propagation by artificially reducing r_v.

Experimental setup

Measurements are made on a organ pipe, provided by an organ maker, in which we introduced a honeycomb consisting of several straws. The alteration of the pipe is experimentally characterized by its impedance $Z = P/U$ measured by a "Z-sensor" used in [12]. As the geometry of this sensor is plane, it is more convenient to measured the impedance at the passive end. The organ pipe being an instrument open at its both ends, frequencies which will be amplified are near the minima of the impedance or the maxima of the admittance $Y = 1/Z$. Furthermore, a correction should be add to take into account the radiation from the passive end which is closed by the measuring device.

The honeycomb is made of $N_s = 19$ plastic straws of radius $R_s = 4mm$ stack together in compact conformation with adhesive rubber. We use three honeycombs of length $d = 6cm$, $13cm$ and $19cm$. They are centred at the quarter wave length $\lambda/4$ of the first

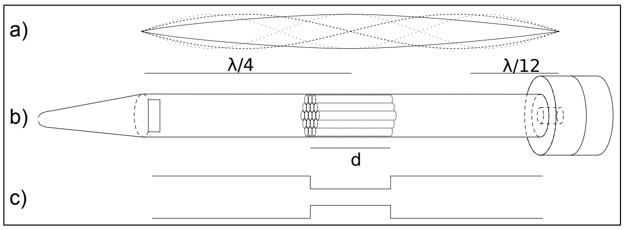

Figure 3. a) Shape of the pressure field for the first three modes of the pipe. b) Experimental setup to characterize the admittance of the pipe. The honeycomb consisting of straws is at a quarter wave length from the end of the pipe. The admittance is measured with the "Z-sensor" put at the passive end. c) Equivalent pipes used for the model of honeycomb.

resonance frequency (figure 3). This last is estimated with a first measurement without honeycomb and corrected with the end correction $\Delta l = 0.6R$ at the passive end.

Qualitative model

The division of the pipe in several smaller tubes can be described as a single pipe with equivalent radius R_{eq} and section S_{eq} as shown on figure 3. The equivalent radius account for the losses through the ratio $r_{v,eq} = R_{eq}/\delta_v$. The conservation of the flow is ensured by the ratio S/S_{eq}. We also assume the acoustic pressure is homogeneous near the change section on the pressure wave ($S/S_{eq} \sim 1$), that comes to neglect the effect of change in section. Thereby the impedances p/u at the change section are linked by:

$$Z^- = \frac{S_{eq}}{S} Z^+, \tag{10}$$

where Z^- and Z^+ are the impedances just before and after the change section, respectively. There is the same equation with inverse ratio of sections at the other side of the honeycomb. The admittance is just given by replacing in (5) l by l_{in} and η_l by:

$$\eta_{eq} = \arg\tanh\Big(SS_{eq}^{-1}\tanh\Big(\Gamma_{eq}d + \arg\tanh\left(S^{-1}S_{eq}\tanh(\Gamma l_r + \eta_l)\right)\Big)\Big), \tag{11}$$

Table I. Approximation of the theoretical and experimental resonator parameters obtained by (12) for different honeycomb's length d. The frequencies have to be adjusted with the end correction Δl for further uses. σ_{max} is the maximum relative error to the case without honeycomb.

	Y_1/Y_c		f_1 (Hz)		Q_1	
	Exp.	App.	Exp.	App.	Exp.	App.
–	1126	1174	291.7	292.0	43.2	42.9
$6cm$	1118	1174	292.8	293.3	37.2	34.1
$13cm$	1117	1171	293.6	294.5	31.2	27.3
$19cm$	1114	1164	293.4	295.0	24.7	23.3
σ_{max} (%)	1.7	0.9	0.7	1.0	43	46

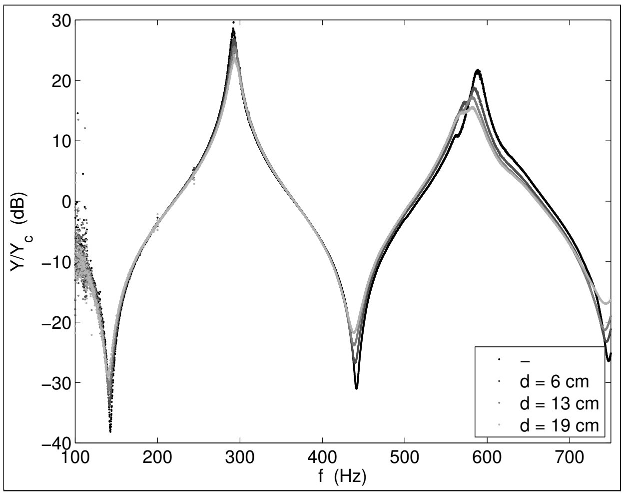

Figure 4. Admittance measured at the passive end of the organ pipe for different lattices of length d = 6, 13 and $19cm$.

where l_{in} and l_r are distances from honeycomb to active end and from honeycomb to passive end, respectively, Γ_{eq} is the modified wave number which includes the new ratio $r_{v,eq} = R_{eq}/\delta_v$.

The equivalent radius is just the radius of a straw R_s. The equivalent section is calculated by subtracting the section of the lattice $S_{eq} = S - N_s\pi\left[(R_s + e_s)^2 - R_s^2\right]$ with e_s the thickness of a straw. We neglected the propagation through the interstices.

Q variation

The presence of the honeycomb affects all the modes of the pipe. The experimental admittance is fitted with :

$$Y = \sum_{n=1}^{\infty} \frac{j\omega Y_n}{\omega_n^2 - \omega^2 + jQ_n^{-1}\omega_n\omega}, \tag{12}$$

where Y_n is the amplitude, ω_n the pulsation and Q_n the Q factor. The table I summarizez the variation of the three parameters for each mode. Only the Q factor is qualitatively modified.

The second mode of the pipe is modified by an additional resonance. This last has been identified as the foot acoustic response : several measurements have been made with different foot conditions (empty, fill with damping material).

The parameters of the model are extracted by identifying (11) with (12).Comparison with the model shows good qualitative agreements. The trends for the three parameters are correctly predicted. However, an accurate description accounting for propagation through several ducts should enable to refine the prediction of Q variation.

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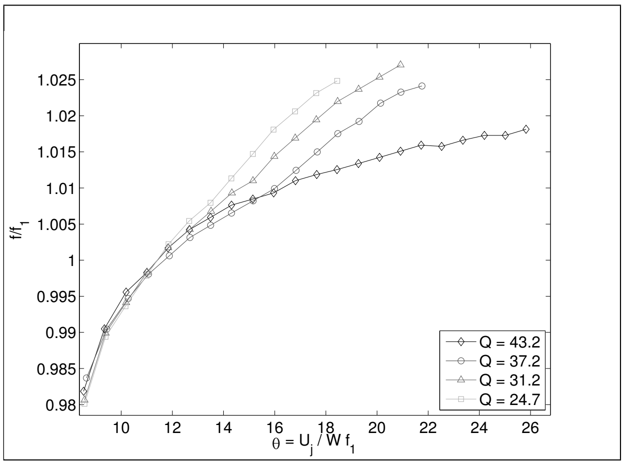

Figure 5. Dimensionless oscillating frequency f/f_1 versus dimensionless jet velocity $\theta = U_j/Wf_1$ where f_1 is the frequency of the first mode (different for each Q, table I)

4. Dependence with the blowing pressure

Experimental setup

The organ pipe is pluged in a reservoir supplied by a compressed N_2O_2 cylinder. The flow is regulated with an air mass flow controller Brooks 5851S and the pressure in the foot of the pipe is measured with a manometer Digitron 2020P. The acoustic field is gathered with a microphone B&K supplied with Nexus conditioner. In order to be sensitive to at least the first five modes of the pipe, the microphone is put at distance $l_m = \lambda/12$ where λ is the wave length of the first resonance as shown on figure 3. The signal is processed by a Stanford Research System spectrum analyser. Once the steady state is established, *i.e.* the pressure is stabilised in the foot pipe, the power spectrum density (psd) is averaged over five samples of 4 seconds with a frequency resolution $\Delta f = 0.25Hz$. The frequencies of the two first harmonics are estimated by taking the maximum of the psd in windows centred around first and second resonances of the pipe. The regime change occurs when the amplitude at one pipe mode resonance become greater than the amplitude of the other. This is accompanied by a slight shift in frequency since the mode-locking sound is harmonic and the modes of the pipe are not strictly harmonic. To avoid the effect of temperature variations during the measurements, this one is gathered with a thermometer Digitron 4140T. The frequency is corrected with a zero order modification $f_{corr} = f_{meas}\sqrt{T_{ref}/T_{meas}}$.

Effect of Q variation

As predicted by the linear analysis of the model, the variation of Q modifies the dependence of the oscillating frequency f toward the jet velocity U_j. At low jet velocity, it affects the presence of aeolian regimes and so the start oscillation threshold.

The variation of Q also affects the transition between the two first regimes. Two main behaviours have been observed. First, for the case $Q = 24.7$, the regime change presents an hysteresis as the non-modified case $Q = 43.2$. However, the thresholds of stability of each regime are modified. Particularly, the end of the branch of the first regime is reduced from 1360 Pa to 915 Pa.

For $Q = 37.2$ and $Q = 31.2$, the transition is no longer obvious. The amplitudes of the two first pipe modes smoothly increases and decreases while rising the supply pressure. The hysteresis in not observed any more, and the transition between first and second regimes gives rise to a rolling sound.

5. Discussion and Conclusion

The honeycomb method enables to modify the Q factor of the pipe without significantly changing other parameters. The position of the lattice, though not developed in this paper, is fundamental. A more accurate model, which includes the effect of change in section and the propagation through pipes of different radius, should bring elements of understanding about the position dependency. The description for higher modes, not presented in this paper, also showed good qualitative agreement and would be refined with such a model.

The simplified model enables to find the jet velocity which makes the pipe sounds at the resonance frequency. Taking the phase shift of the resonator to zero in (9) leads to the delay of resonance $\tau_1 = \pi/2\omega_1$. The associated jet velocity is $U_{j,1} = 2W\omega_1/0.3\pi$.

Looking for a simplified expression that leads to a rule of thumb that could be useful for organ makers, it is possible to estimate the variation of frequency for a slight variation of jet velocity around $U_{j,1}$. Taking a modal expression of the admittance like (12), the phase near a resonance is $\arg(Y) = \arctan(Q_1(\omega_1^2 - \omega^2)/\omega\omega_1)$. The solution of (9) with linearisation of $\arg(Y)$ near the resonance ω_1 is (with $n = 0$):

$$\omega = \omega_1 \frac{2Q_1 + \pi/2}{2Q_1 + \omega_1\tau}. \tag{13}$$

The pulsation can be seen as a function of the jet velocity and the developpment of (13) in Taylor's series yields:

$$\frac{\omega(U_j) - \omega_1}{\omega_1} = \frac{U_{j,1}}{f_1} \left.\frac{\partial\omega}{\partial U_j}\right|_{U_{j,1}} \frac{U_j - U_{j,1}}{U_{j,1}}. \tag{14}$$

The relative variation of frequency is proportional to the relative variation of the jet velocity with the coefficient:

$$\beta = \frac{U_{j,1}}{f_1} \left.\frac{\partial\omega}{\partial U_j}\right|_{U_{j,1}} = \pi/(4Q_1 + \pi). \tag{15}$$

For high Q value, observed for instance for principal organ stops, β is small and the solution remains close to the resonance frequency while rising the jet velocity. The larger the Q, the better the accuracy of the prediction of sounding frequency with a passive formula, such as $L = c/2f - 5/3\emptyset$. However for low Q value, such as gamba or salicional organ stops, the frequency is allowed to move far from the resonance frequency. Besides, as soon as the jet velocity is far from $U_{j,0}$, the linearisation of arctan in (13) and the Taylor expansion leading to (14) are no longer valids.

Typical values of Q factors for organ pipes are between 20 and 40. Application of equation (15) allows to grasp the influence of the Q factor on the frequency sensitivity relative to the jet velocity on a musical scale : the pitch rise for doubling the jet velocity ($(U_j - U_{j,1})/U_{j,1} = 1$) is approximately 64 cents for $Q = 20$ and 33 cents for $Q = 40$.

The experimental setup showed particular behaviours. The modification of the pipe allowed multiphonics. The measurements are made on a particular organ pipe and further studies on several pipes or similar physical system should confirm the effect of Q on the smooth transitions. Besides, care must be taken on the reservoir. Admittance measurements showed the effect of the foot on the second mode. Acoustic coupling between the pipe and the foot could modify the global behaviour. Because of the mutltiphonics and aeolian sounds, the estimation of thresholds is difficult. The prediction of the model is based on a linear analysis and should be numericaly studied in detail. The model also requires refinement: where the linear analysis is able to provide reliable informations, the model is actually not so accurate. At low supply pressure, the jet splits into vortices and it would be preferable to use a high Strouhal description as proposed by Dequand [9]. Moreover several points have been omitted such as the slowing down and spreading of the jet [16] or vortex shedding that occurs at the labium [11].

Finaly, the variations of the oscillating frequency while rising the jet velocity considerably depends on the quality factor. The linear analysis enables to provide a good estimation of the variation of the frequency, at least near the resonance, for a given jet velocity variation.

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