# An Interacting Boundary Layer and a Triple Deck model of dune movement and ripple formation in water

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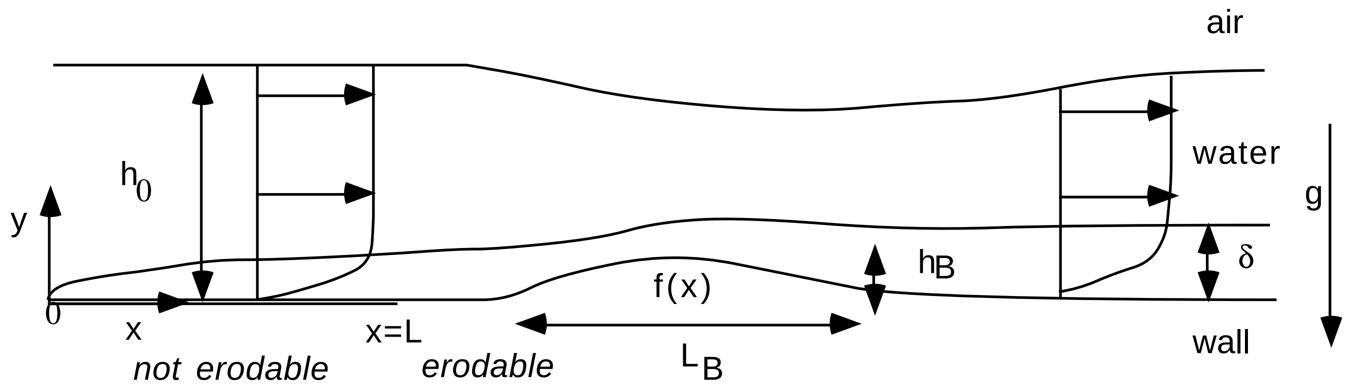



$Re = U_0 L/\nu$ (Re >> 1) $\delta = L\, Re^{-1/2}$ $\delta \ll h_0$

two cases:

* (*IBL*) Interacting Boundary Layer Theory

$L_B \sim L$ $\delta \sim h_B$

* (*TD*) Triple Deck Theory

$\delta \leq L_B \leq \delta\, Re^{1/8}$ $\delta \gg h_B$

*IBL 1*

**Asymptotic description:**

With scales:

$x^* = Lx,\ y^* = LRe^{-1/2}y\ ,\ u^* = U_0 u\ ...$

$\varepsilon = (L/h_0)Re^{-1/2} \qquad \varepsilon \ll 1,$

$Re = U_0 L/\nu \qquad Re \gg 1$

$Fr^2 = U_0^2/(gh_0) \qquad Fr = O(1) < 1$

we obtain the quasistatic Interacting Boundary Layer Problem:

$$\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0,$$

$$(u\frac{\partial}{\partial x}u + v\frac{\partial}{\partial y}u) = u_e\frac{du_e}{dx} + \frac{\partial^2}{\partial y^2}u.$$

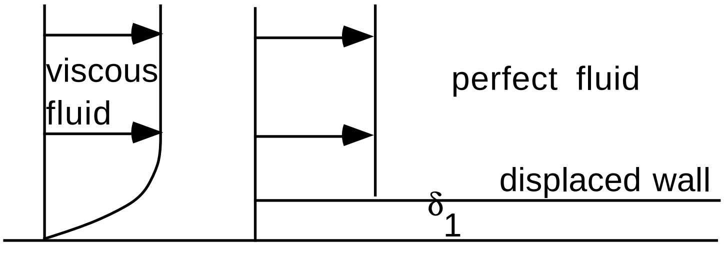
$$\delta_1 = \int_0^\infty (1 - \frac{u}{u_e})dy$$

* no slip condition on $y= f(x,t)$

* $y \to \infty$, $\qquad u \to u_e = 1 + \varepsilon \dfrac{\delta_1 + f(x)}{1-Fr^2}$

- BL equations + matching condition with the perfect Fluid.

- $(\delta_1)$, is the displacement thickness of the boundary layer




- The movement of the dune (f(x,t)) is slow

- There is a strong coupling $(\delta_1)$, boundary layer separation is possible

- Turbulence may be added (mixing length theory)

## Transport equation

Model of convective/ diffusive transport with a settling velocity $-V_f < 0$ (Boundary Layer scales):

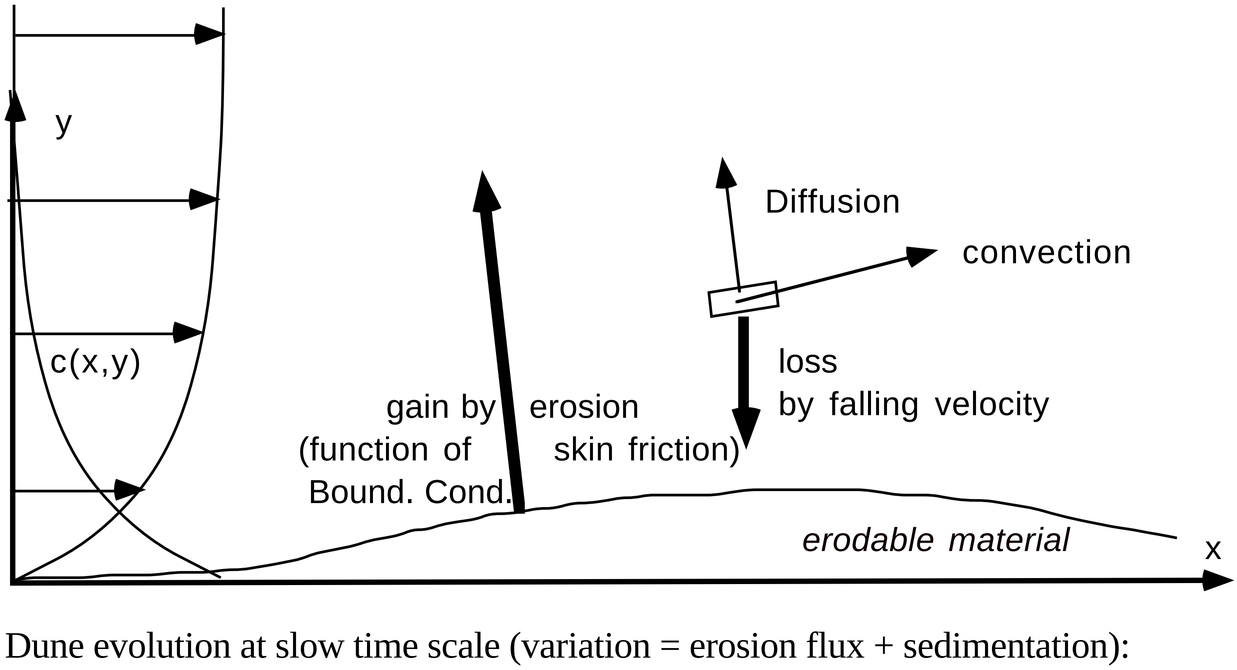
$$(u \frac{\partial}{\partial x} c + (v - V_f) \frac{\partial}{\partial y} c) = \frac{1}{S} \frac{\partial^2}{\partial y^2} c.$$

No income, and flux condition at the wall depending on the value of the skin friction:

$$\text{if } \frac{\partial}{\partial y} u(x,0) < \tau_w \quad \text{then } -\frac{\partial}{\partial y} c(x,0) = 0$$

$$\text{if } \frac{\partial}{\partial y} u(x,0) > \tau_w \quad \text{then } -\frac{\partial}{\partial y} c(x,0) = \beta \left( \frac{\partial}{\partial y} u(x,0) - \tau_w \right)^b$$

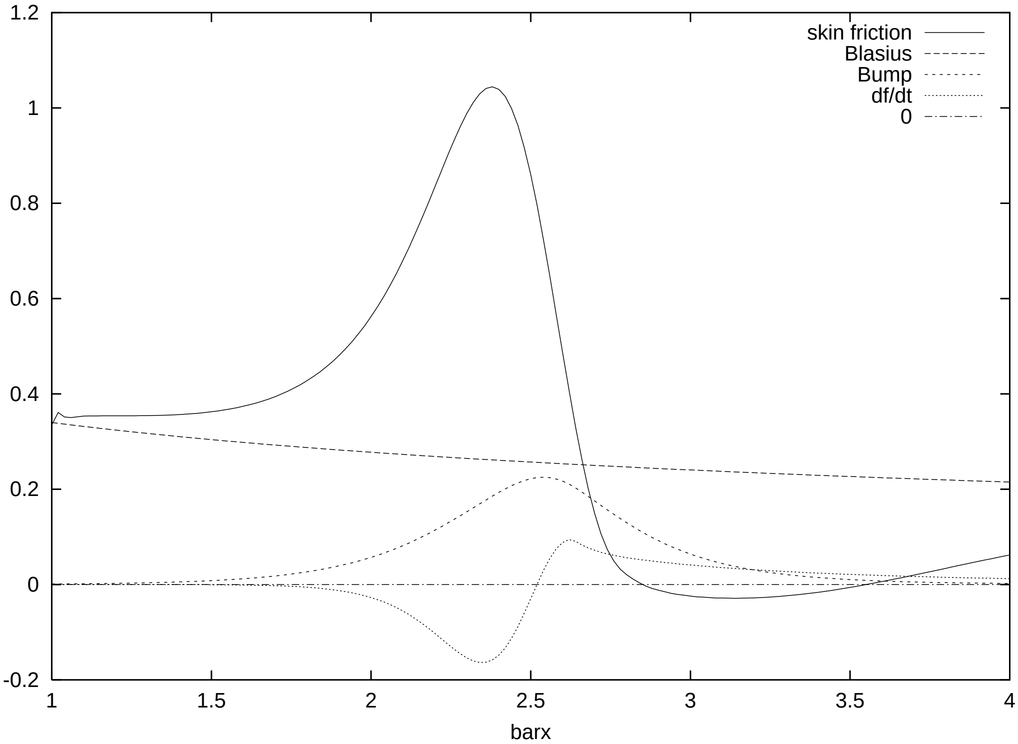
$b=3/2$, $\beta=O(1)$



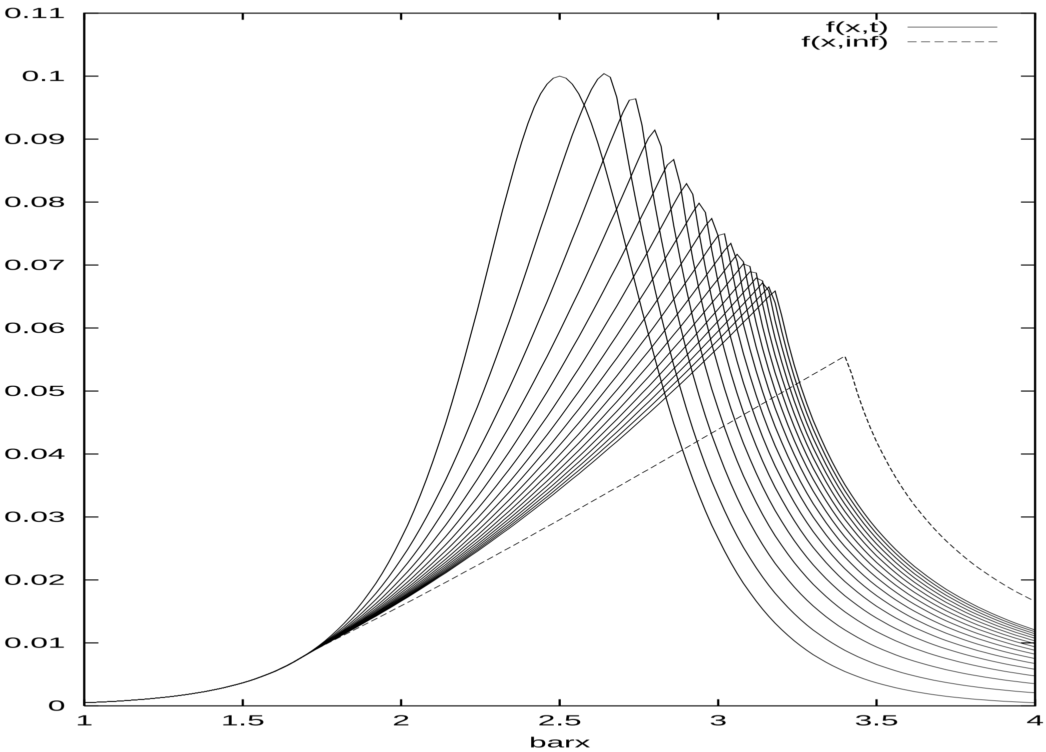

Dune evolution at slow time scale (variation = erosion flux + sedimentation):

$$\frac{\partial f(x,t)}{\partial t} = S^{-1} \frac{\partial}{\partial y} c(x,0) + V_f\, c(x,0).$$

*IBL 3*

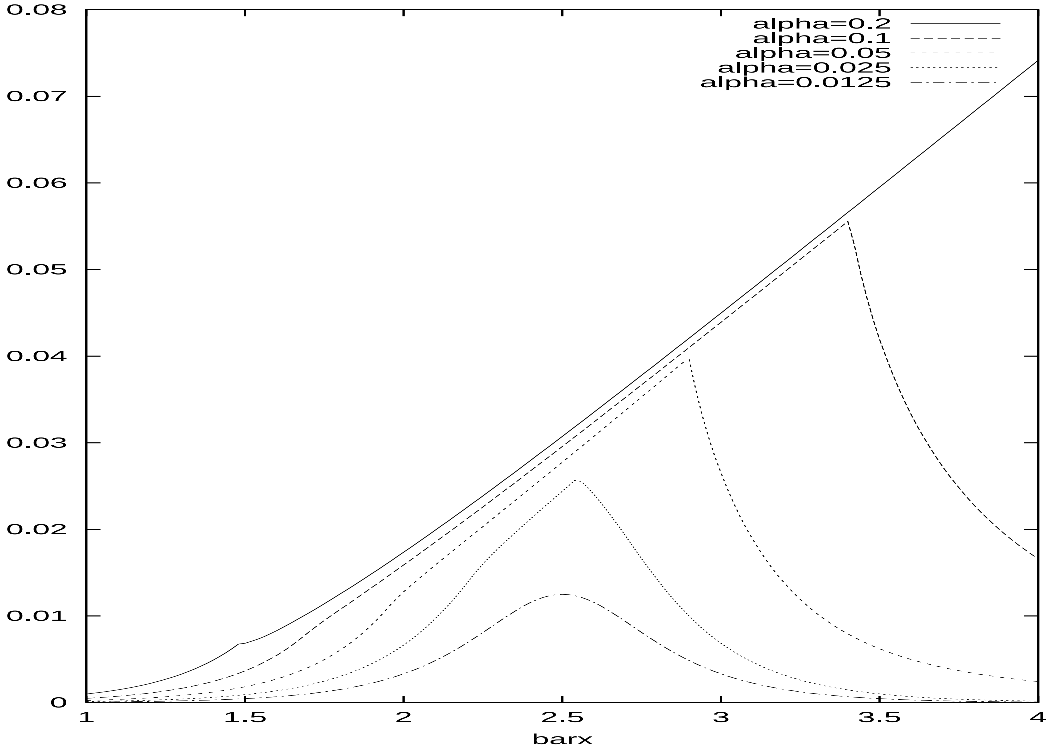



At initial time, the initial bump: f(x,t=0) and the associated computed skin friction at the wall




The dune shape as a function of time t=0, 1, 2, 3...,16 ,∞

*IBL 4*




Final dune shapes for different starting values of $\alpha$

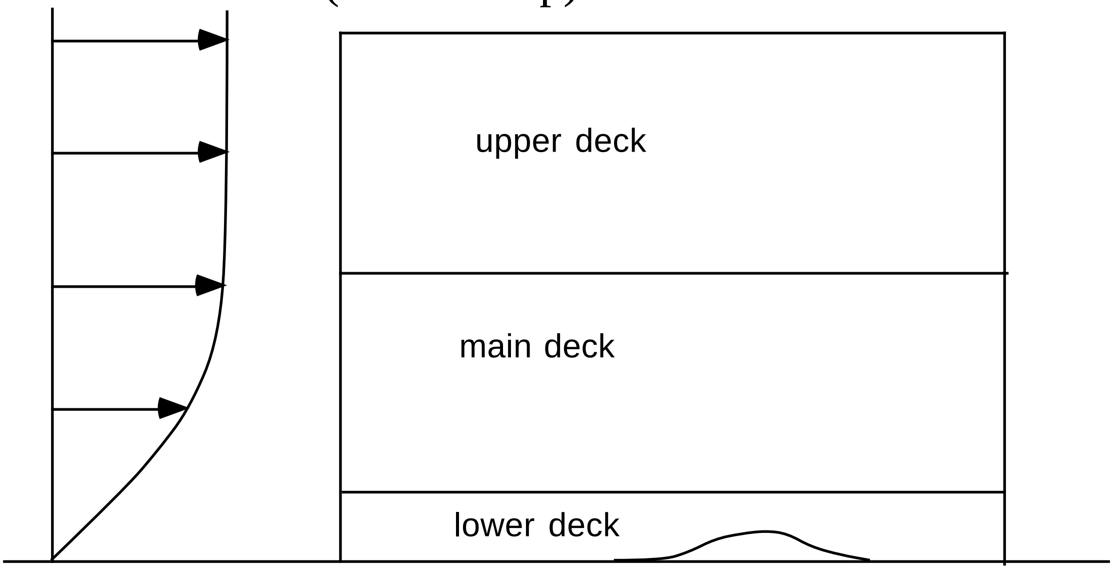
**Some conclusions**

- No displacement of the dune at those scales,
- Final flat portion of the dune (constant skin friction in a convergent channel)
- Need of a simplified model for even more rapid computations: what happens if the bump is small?... =>Triple Deck!

*TD 1*

## **Triple Deck description** (Gajjar Smith 83)

With *ad hoc* scales (small bump)




Viscous equations in the viscous lower layer ("lower deck")

$$\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0,$$

$$(u \frac{\partial}{\partial x}u + v \frac{\partial}{\partial y}u) = - \frac{dp}{dx} + \frac{\partial^2}{\partial y^2}u.$$

$y \to \infty \quad u \to y + A \qquad \& \quad u(x,f(x)) = v(x,f(x)) = 0.$

Displacement of the stream lines in the boundary layer: -A ("main deck")

and pressure deviation relation form perfect fluid ("upper deck")

| | | |
|---|---|---|
| $p = A$ | fluvial flow | (Fr<1) |
| $p = - A$ | supercritical flow | (Fr>1) |
| $p = \frac{1}{\pi} \int \frac{A'}{(x-\xi)} d\xi$ | infinite depth | |

$A = 0$, very small bump not perturbing the perfect fluid (or Couette)

## *TD 2*

# The linearized Fourier solution for a small bump:

Perfect fluid response ("Upper Deck")

$$\beta TF[p] = TF[A] \text{ , with } \beta = 1, -1, 1/|k|, 0$$

and "Lowed Deck" response:

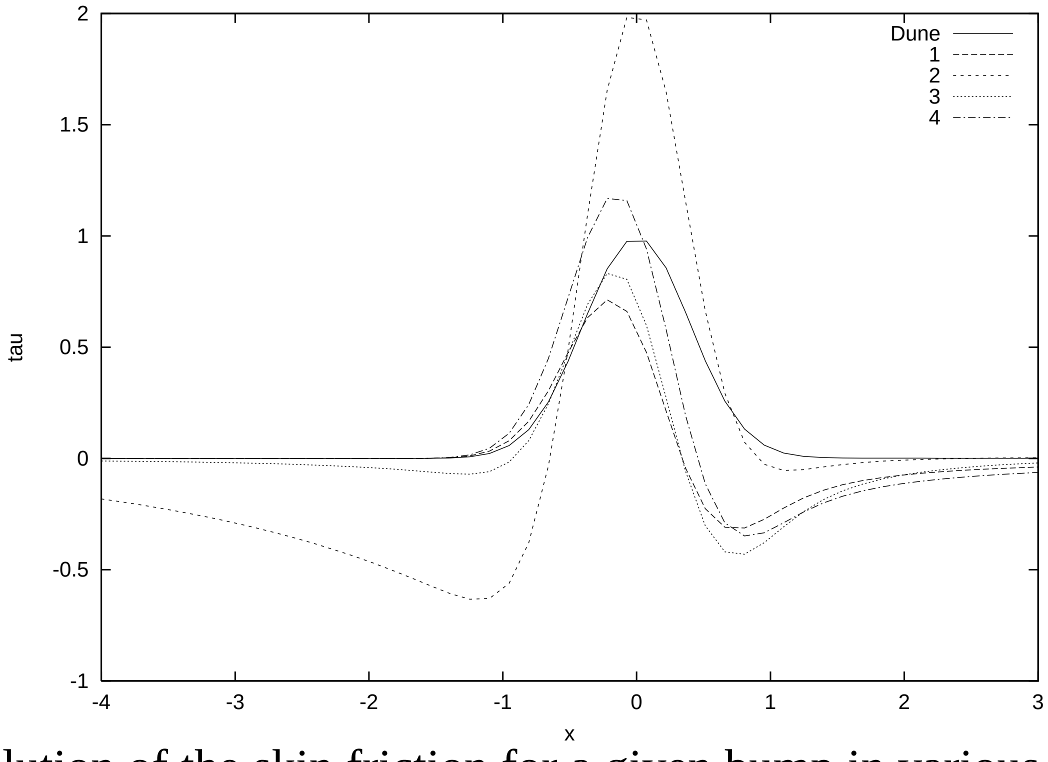
$$\beta^* \, TF[p] = (TF[A] + TF[f]), \text{ with } \beta^* = (-i\,k)^{1/3} /(3\, Ai'(0)),$$

perturbation of pressure is:

$$TF[p] = \frac{TF[f]}{\beta^* - \beta}$$

$\tau$ *perturbation* of skin friction is:

$$TF[\tau] = (i\,k)^{2/3} \, \frac{Ai(0)}{Ai'(0)} \, TF[p]$$




Evolution of the skin friction for a given bump in various flow régime:

1 subcritical flow ($\beta = 1$) no upstream influence

2 supercritical flow ($\beta = -1$) upstream influence

3 infinite depth ($\beta = 1/|k|$) global influence

4 no perturbation in the perfect fluid ($\beta = 0$) no upstream influence

***TD 3***

We are here near the limit $\tau_w$ (1 with the chosen scales)

The mass transport equation:

$$\frac{\partial}{\partial x}\left(\int_0^\infty cu\,dy\right) = -c(x,0)(V_f) - S^{-1}\frac{\partial}{\partial y}c(x,0) \qquad \text{with} \qquad q = \left(\int_0^\infty cu\,dy\right)$$

$$A_r = -S^{-1}\frac{\partial}{\partial y}c(x,0) \quad \text{is } (\tau - \tau_s)^b \qquad \text{if} \quad \tau > \tau_s$$

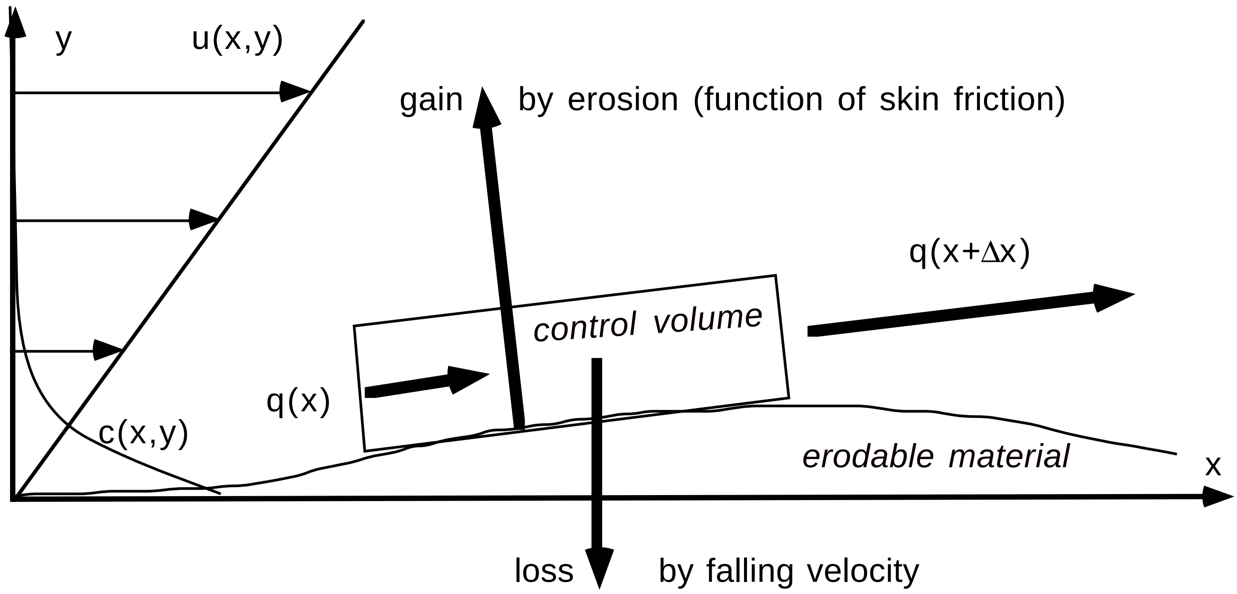
$$A_r = -S^{-1}\frac{\partial}{\partial y}c(x,0) \quad \text{is } 0 \qquad \text{if} \quad \tau < \tau_s$$

The final problem for the sediments and the flow is:

$$\tau = TF^{-1}\left[(i\,k)^{2/3}\,\frac{Ai(0)}{Ai'(0)}\;\frac{\beta^* f}{1 - \beta\beta^*}\right],$$

$$\frac{\partial}{\partial x} q = -V q + A_r \quad \text{and} \quad \frac{\partial}{\partial t} f = -\frac{\partial}{\partial x} q$$

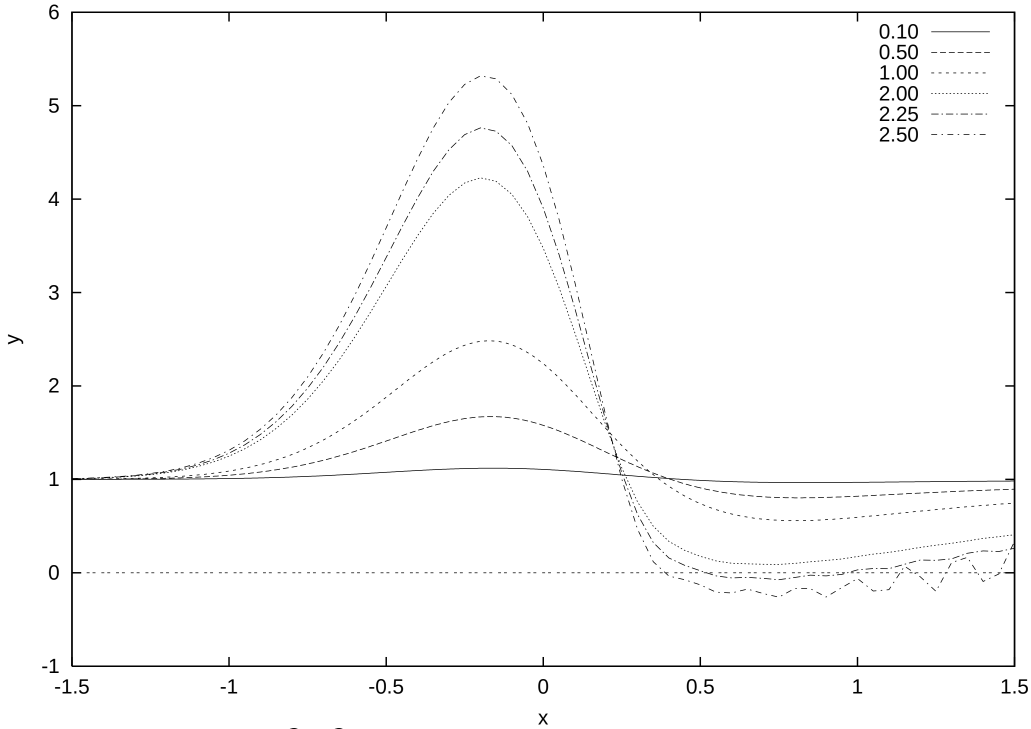
$$t=0,\; f(x,0) = \alpha \exp(-\pi x^2),\; q(x,0)=0.$$



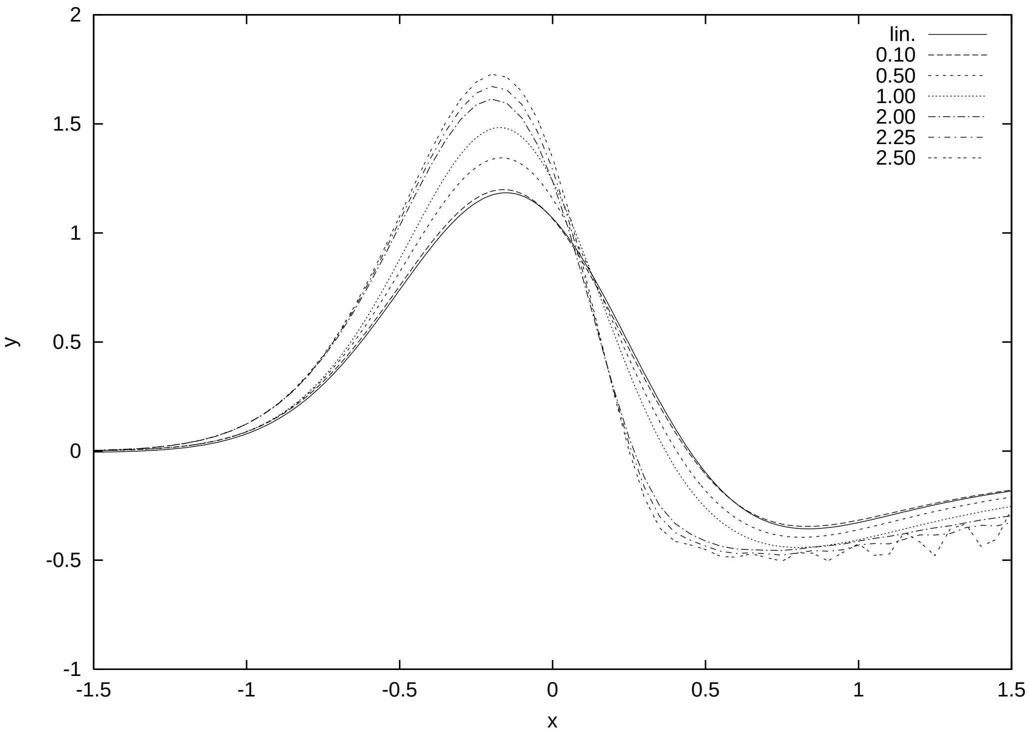
***TD 4***

# Notes on the linearisation,

comparisons with a nonlinear resolution of the flow equations



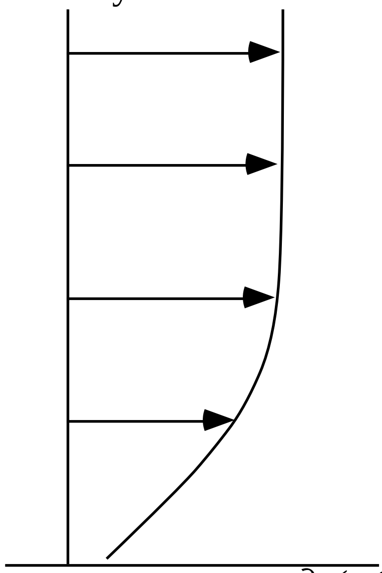

The wall shear stress ∂u/∂y-1 function of x as function of the bump height, non linear resolution. For α=0.10, α=0.5, α=1.0, α=2, α=2.25, α=2.50




The reduced wall shear stress $(\partial u/\partial y-1)\alpha^{-1}$ function of x as function of the bump height, non linear resolution compared to the linearized prediction.
For α=0.10, α=0.5, α=1.0, α=2, α=2.25, α=2.50

## *TD 5*

Remark, influence of the slip velocity



If with, Beavers and Joseph, we write $u(x,0) = \varepsilon \frac{\partial u(x,0)}{\partial y}$ at the bottom, $\varepsilon<<1$.

The development of u in powers of $\varepsilon$:

$$u(x,y) = u_0 + \varepsilon\, u_1(x,y) + \varepsilon^2\, u_2(x,y) + ...$$

gives

$$u_0(x,0) = 0,\ u_1(x,0) = \frac{\partial u_0(x,0)}{\partial y},\ u_2(x,0) = \frac{\partial u_1(x,0)}{\partial y}\ ...$$

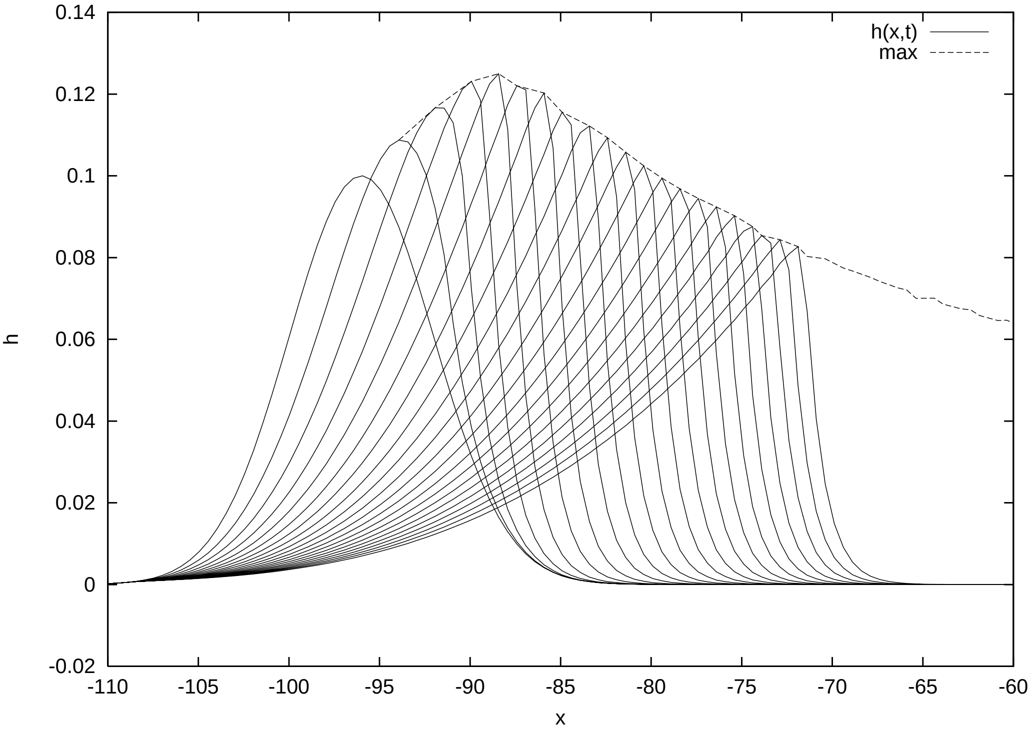
so,

$$\frac{\partial u(x,0)}{\partial y} = \frac{\partial u_0(x,0)}{\partial y} + \varepsilon \frac{\partial^2 u_0(x,0)}{\partial y^2} + O(\varepsilon^2)$$

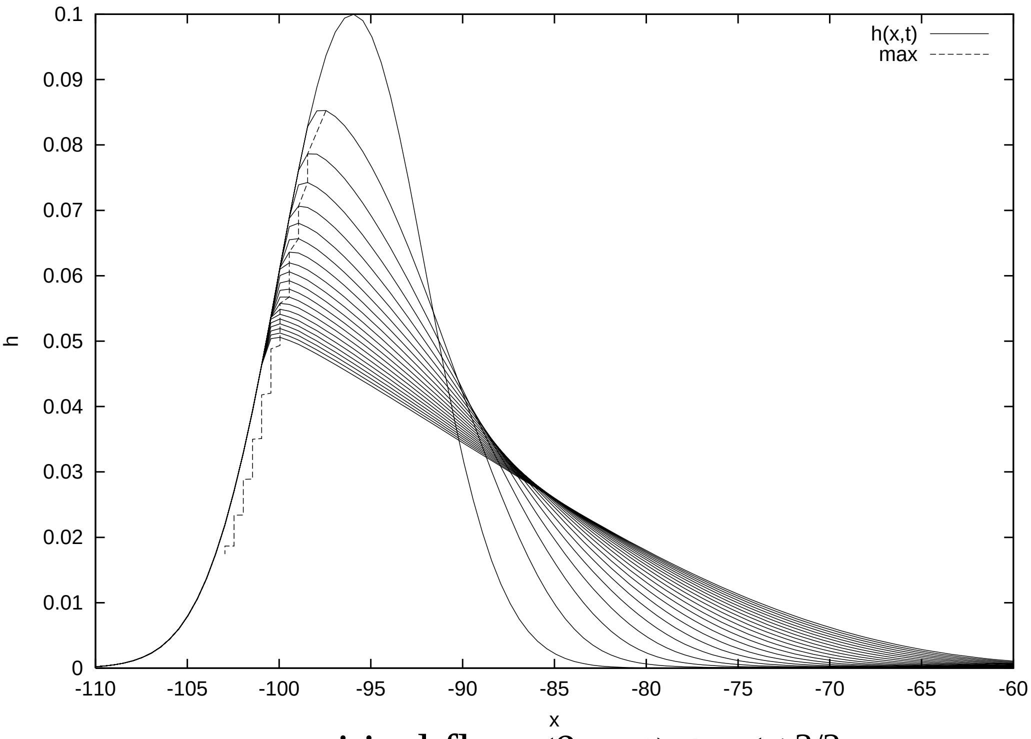
## *TD 6*

Evolution of an initial bump in various régimes, threshold = 0, V=1,

if $\tau>0$ ( $\frac{\partial}{\partial x} q = - V q + (\tau)^{3/2}$) else ( $\frac{\partial}{\partial x} q = - V q$)




flow ($\beta$ = 0) Ar=$(\tau)^{3/2}$

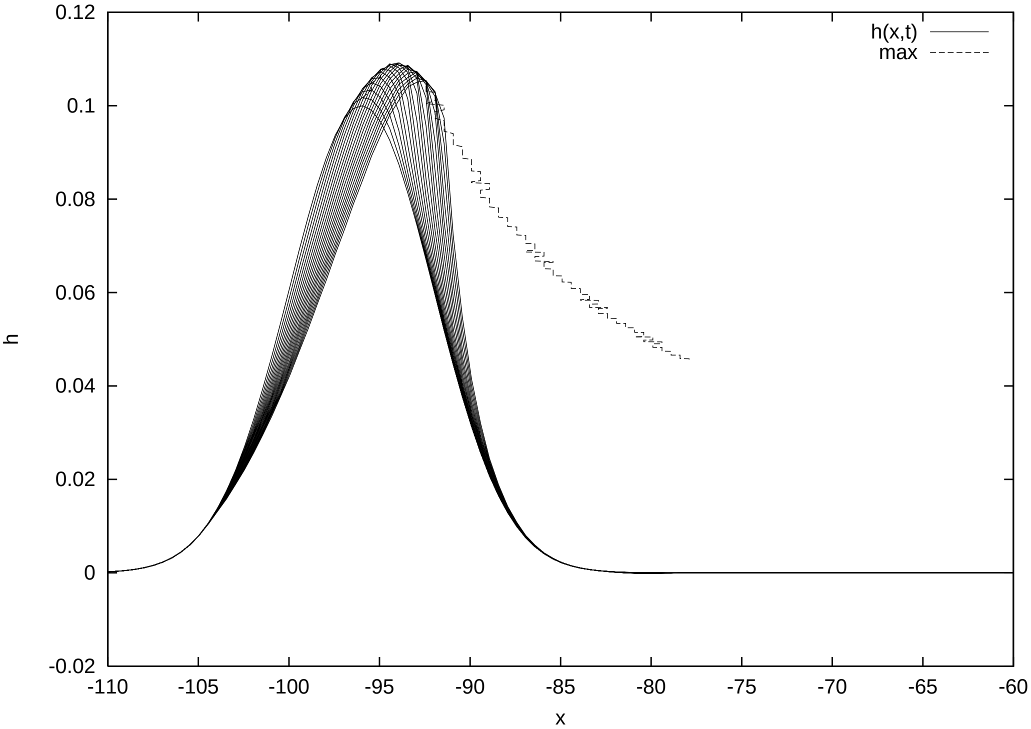



supercritical flow ($\beta$ = -1) Ar=$(\tau)^{3/2}$

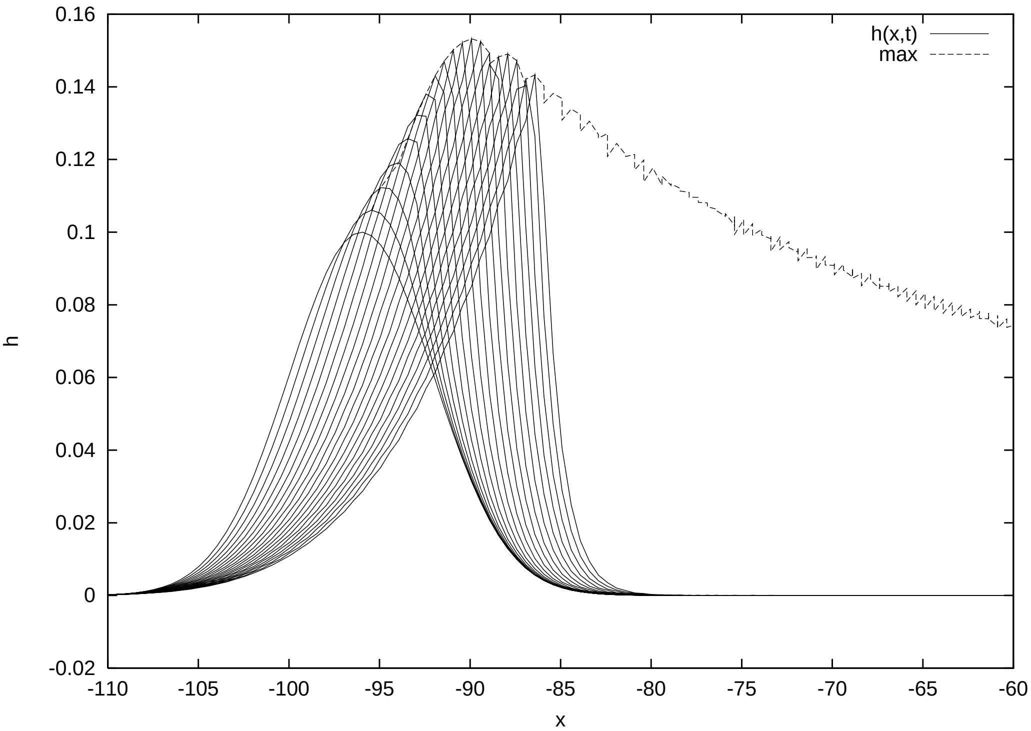
## *TD 7*

Evolution of an initial bump in various régimes, threshold = 0, V=1,

if $\tau>0$ ( $\frac{\partial}{\partial x} q = - V q + (\tau)^{3/2}$) else ( $\frac{\partial}{\partial x} q = - V q$)




infinite depth ($\beta = 1/|k|$) Ar=$(\tau)^{3/2}$

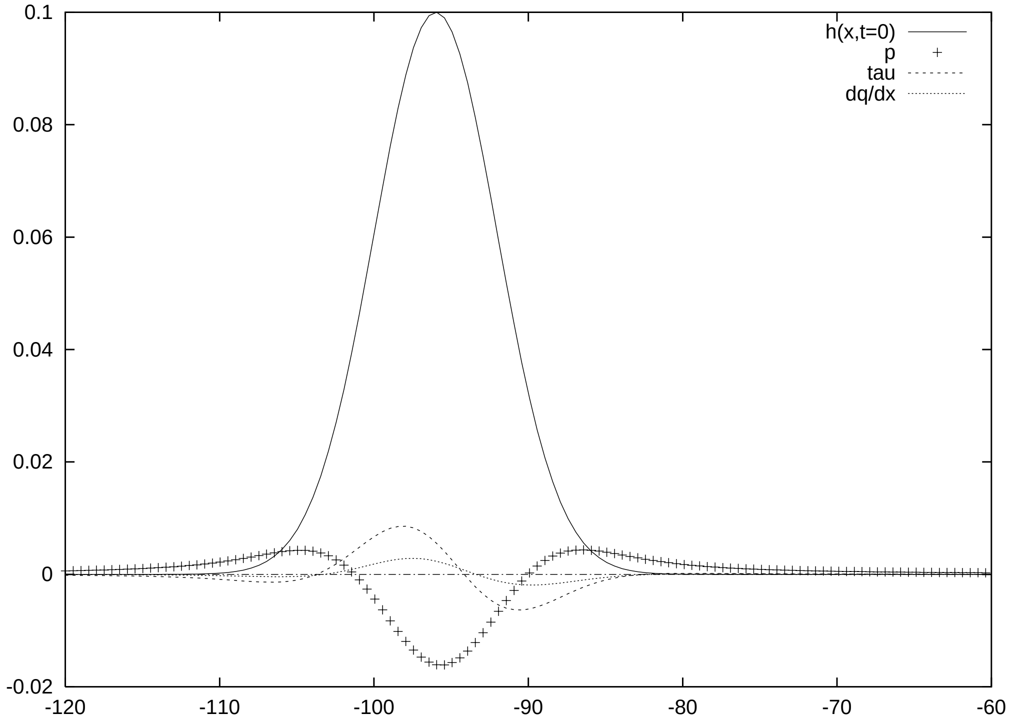



subcritical flow ($\beta$ = 1) Ar=$(\tau)^{3/2}$

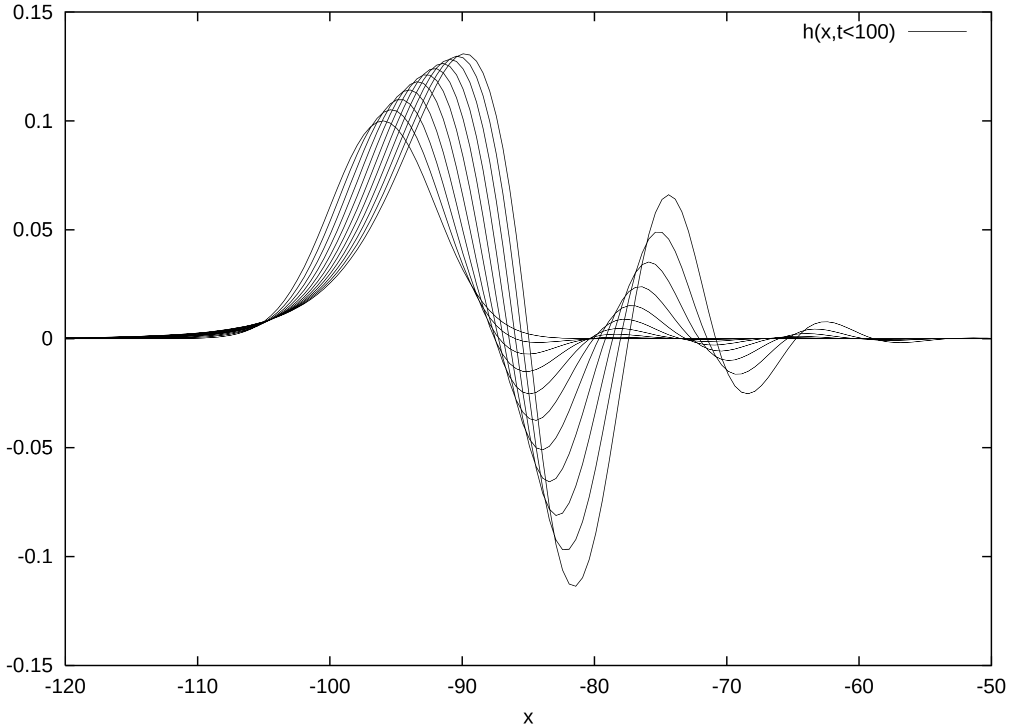
## *TD 8*

Evolution of an initial bump in the infinite depth régime, threshold = -0.05, V=1,

if $\tau>-0.05$ ( $\frac{\partial}{\partial x} q = - V q + (\tau-(-0.05))^{3/2}$ ) else ( $\frac{\partial}{\partial x} q = - V q$ )




**t=0, bump, skin friction, pressure, infinite depth case**

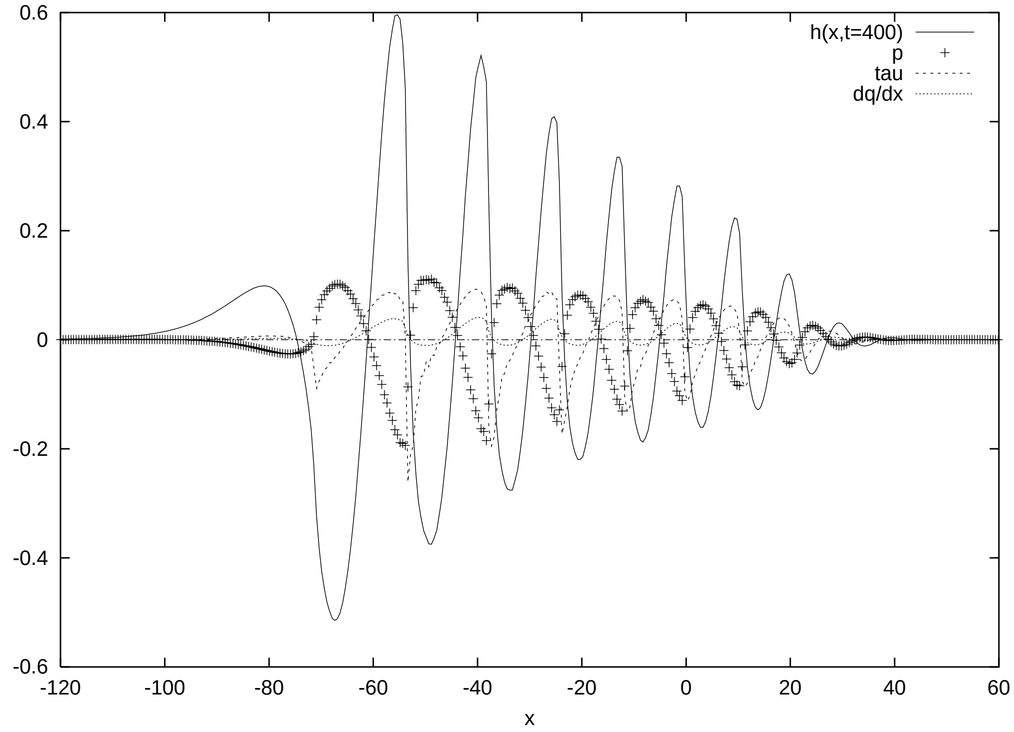



**t≤100, bump, infinite depth case, creation of ripples.**

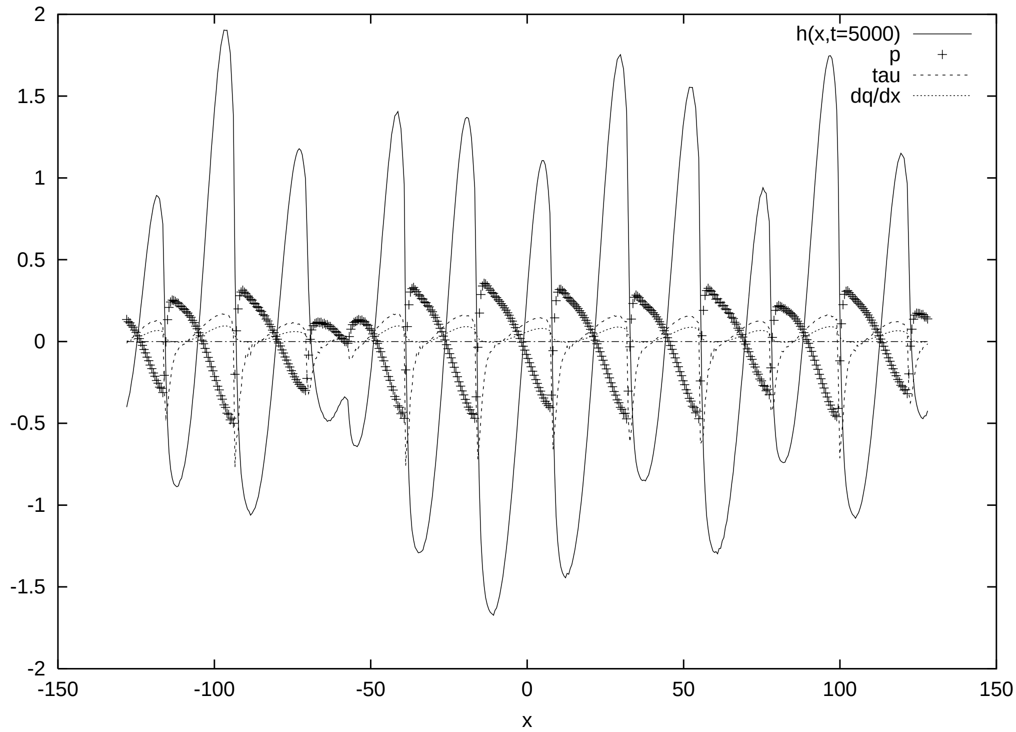
## *TD 9*

Evolution of an initial bump in the infinite depth régime, threshold = -0.05, V=1,

if $\tau$>-0.05 ( $\frac{\partial}{\partial x} q = - V q + (\tau-(-0.05))^{3/2}$ ) else ( $\frac{\partial}{\partial x} q = - V q$ )




**t=400, bump, skin friction, pressure, infinite depth case**

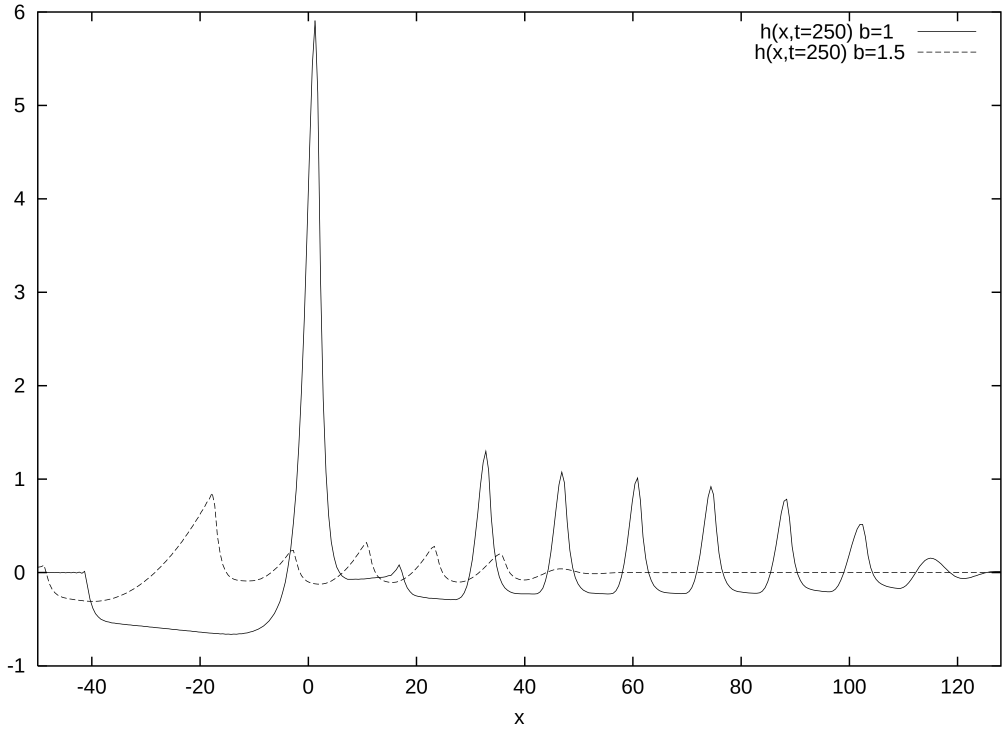



**t=5000, bump, skin friction, pressure, infinite depth**

## *TD 10*

Evolution of an initial bump in the subcritical régime, threshold = -0.05,
influence of the index of the pick up,

$$\text{if } \tau>-0.05 \left( \frac{\partial}{\partial x} q = - V q + (\tau-(-0.05))^b \right) \text{ else } \left( \frac{\partial}{\partial x} q = - V q \right)$$

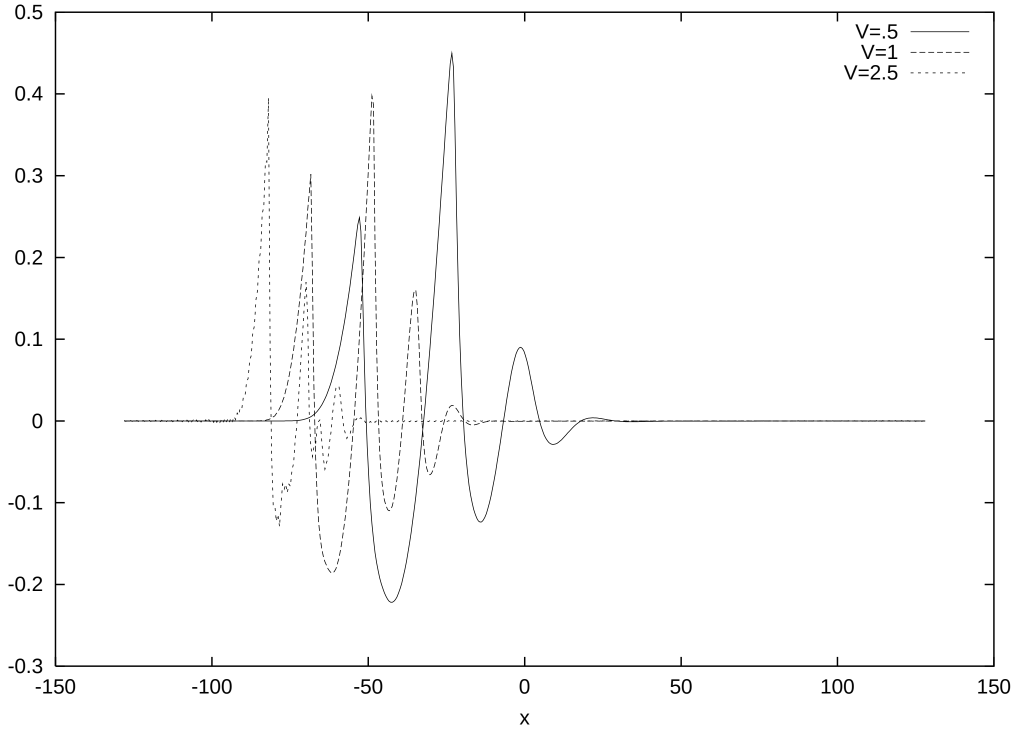



difference in the pick up function b=1 b=1.5

## *TD 11*

Evolution of an initial bump in the subcritical régime, threshold = -0.05, influence of the falling velocity,

if $\tau$>-0.05 ( $\frac{\partial}{\partial x} q = - V q + (\tau-(-0.05))^{1.5}$) else ( $\frac{\partial}{\partial x} q = - V q$)

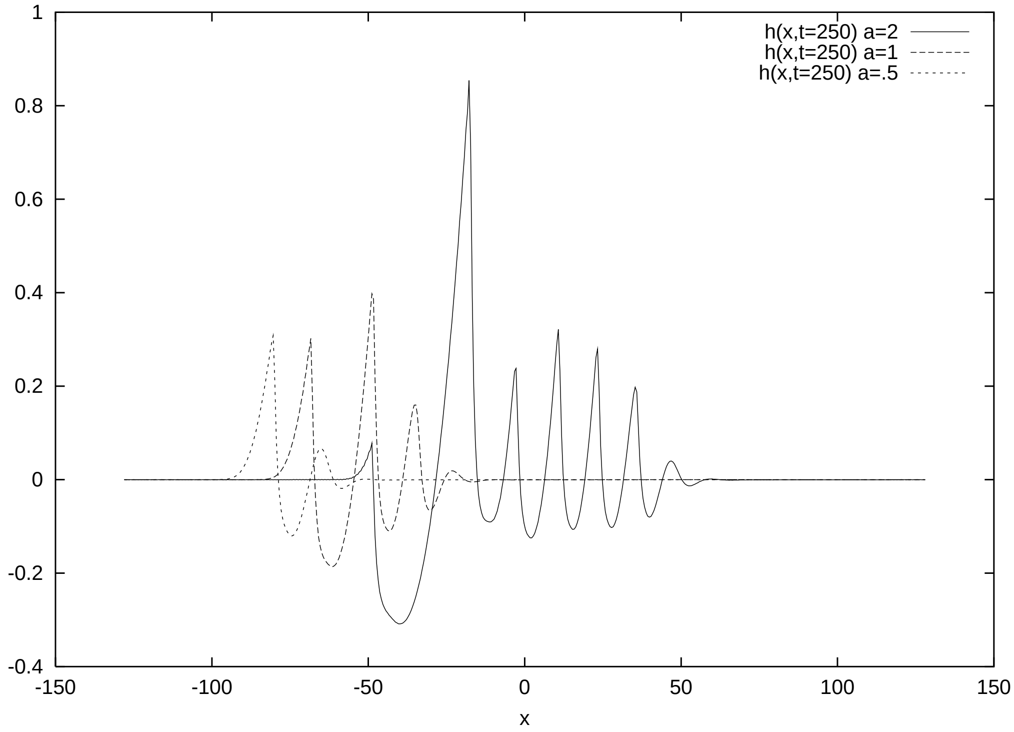



V=0.5, V=1, V=2.5

## *TD 12*

Evolution of an initial bump in the subcritical régime, threshold = -0.05, influence souce magnitude,

if $\tau$>-0.05 ( $\frac{\partial}{\partial x} q = - V q + a (\tau-(-0.05))^{1.5}$) else ( $\frac{\partial}{\partial x} q = - V q$)




a=1 a=1.5

# Conclusion

The advantage of this model is that a lot of hydrodynamical mecanisms have been put without usual integral simplifications.

The triple deck description allows the movement of the bump...

Of course, the first hypotheses to introduce in the model would be a turbulent stress viscosity and diffusivity and for the river bed

It would be interesting to introduce the slope limitation.

**Ref:**

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