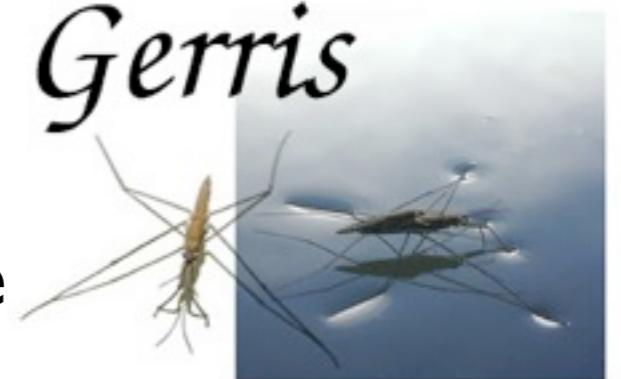


$\mu(I)$ rheology for granular flows with *Gerris* applications for avalanches and column collapse



Pierre-Yves Lagrée*,

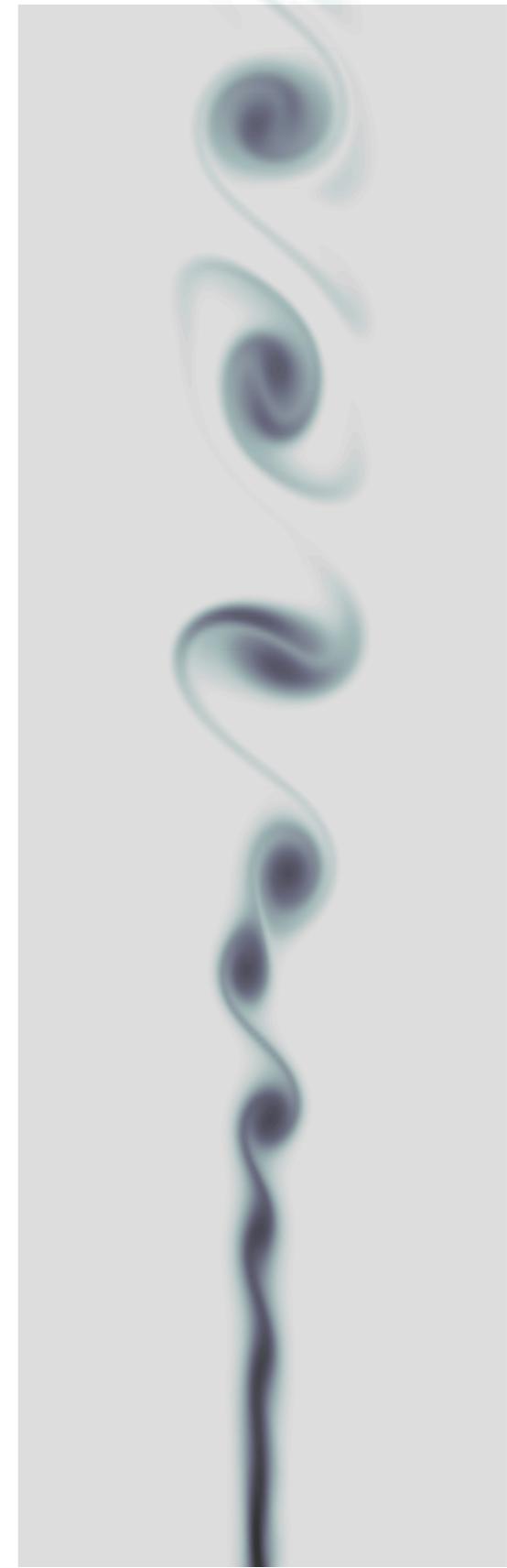
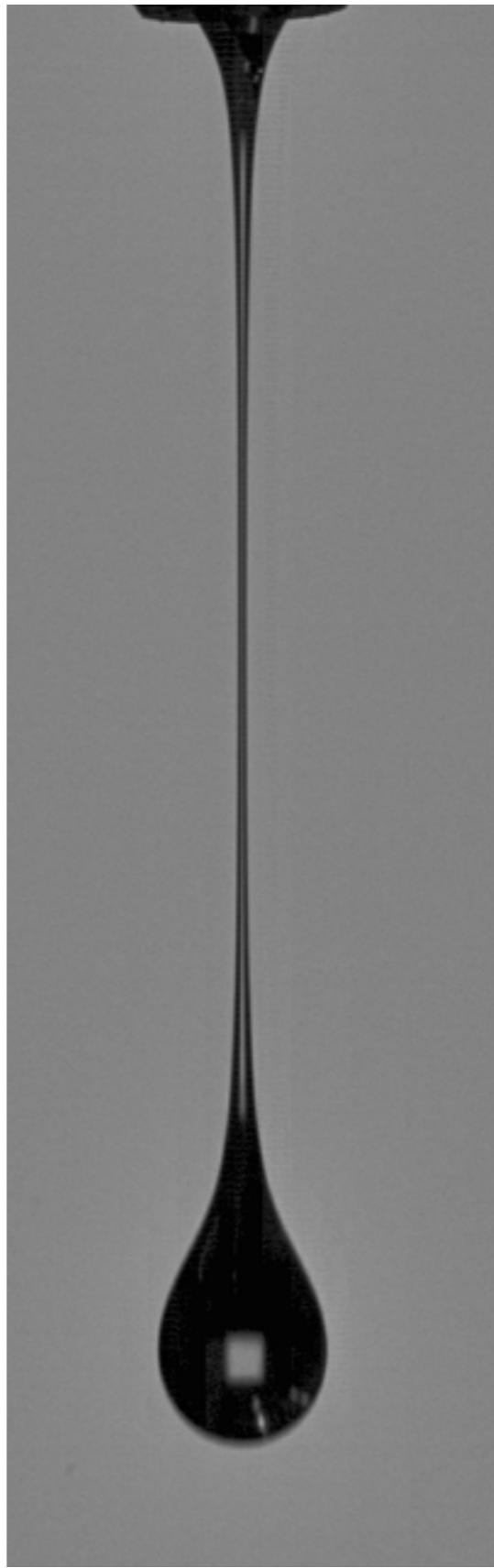
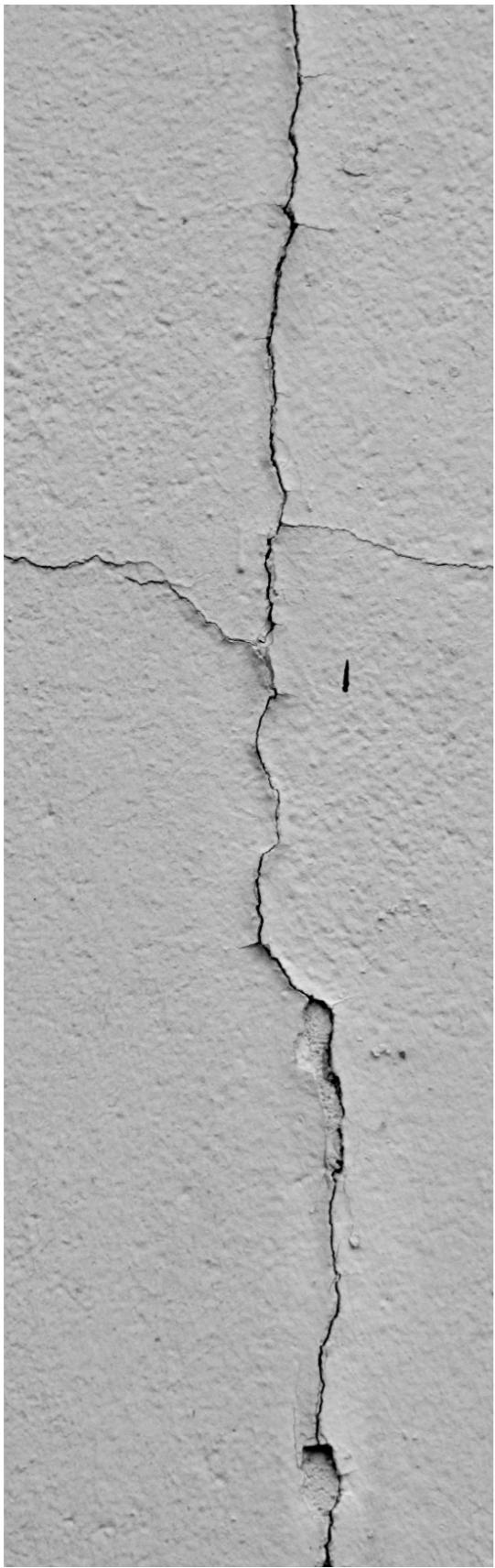
Lydie Staron*, Stéphane Popinet *o, (Daniel Lhuillier *, Christophe Josserand *)

*Institut Jean le Rond d'Alembert, CNRS, Université Pierre & Marie Curie, 4 place Jussieu, Paris, France

o National Institute of Water and Atmospheric Research, PO Box 14-901 Kilbirnie, Wellington, New Zealand

Saint Gobain 05/04/12

Institut Jean Le Rond d'Alembert





outline

- what is a granular fluid? some images
- the $\mu(I)$ friction law obtained from experiments and discrete simulation
- the viscosity associated to the $\mu(I)$ friction law
- the Saint Venant Savage Hutter Hyperbolic model
- implementing the $\mu(I)$ friction law in Navier Stokes
- Examples of flows: focusing on the granular column collapse and the Hour Glass



- What is a granular media?
- size > 100µm
- grains of sand, small rocks, glass beads, animal feed pellet, medicines, cereals, wheat, sugar, rice...
- 50 % of the traded products



FIG. 1.2 – Les milieux granulaires forment une famille extrêmement vaste.

PHYSIQUE
LES MILIEUX GRANULAIRES
ENTRE FLUIDE ET SOLIDE

Sable, riz, sucre, neige, ciment... Bien qu' omniprésents dans notre vie quotidienne, les milieux granulaires constituent de très nombreux systèmes fascinants le chercheur et d'intérêt pour l'ingénieur. Pourquoi le sable est-il tantôt un solide pour former un talus, tantôt le siège d'un mouvement, tout dépend du contexte comme un liquide, lors d'une avalanche ou dans un sable ? Pourquoi est-il difficile de compacter ou de mélanger des grains ? Comment le vent sculpte-t-il les ridges de sable sur la plage et les dunes dans le désert ? Longtemps l'apanage des ingénieurs et des géologues, l'étude des milieux granulaires constitue aujourd'hui un sujet de recherche actif à la frontière de nombreuses disciplines — physique, mécanique, sciences de l'environnement, géophysique et sciences de l'ingénieur.

Cet ouvrage s'attache à dresser l'état des connaissances sur les milieux granulaires et à présenter les avancées récentes du domaine. Issu de cours de Master et d'école d'ingénieur, il s'adresse aux étudiants et aux chercheurs en physique et en ingénierie, qui trouveront dans ce livre la présentation des propriétés fondamentales des milieux granulaires. Interaction entre grains, comportement solide, liquide et gazeux, couplage avec un fluide, applications au transport de sédiments et à la formation de structures géologiques). La description des phénomènes met arguments qualitatifs et formels, permettant de pénétrer des domaines aussi variés que l'élasticité, la plasticité, la physique statistique, la mécanique des fluides ou la géomorphologie. De nombreux encadrés permettent d'approfondir certains phénomènes et illustrent les propriétés singulières des milieux granulaires au travers de leurs manifestations les plus spectaculaires (chant des dunes, sables mouvants, avalanches de neige...).

Bruno Andreotti est professeur à l'université Paris VII et effectue ses recherches à l'ESPCI. Ses recherches portent sur l'hydrodynamique, le mouillage et la géomorphogénèse.

Yoël Forterre, est chercheur CNRS au laboratoire IUSTI à Marseille. Il travaille sur le comportement des fluides complexes, des milieux granulaires et sur la biomécanique des plantes.

Olivier Pouliquen, est chercheur CNRS au laboratoire IUSTI à Marseille. Ses travaux portent sur les matériaux granulaires, les suspensions et fluides complexes.

SAVOIRS ACTUELS

Série Physique et collection dirigée par Michèle LEDUC

**BRUNO ANDREOTTI
YOËL FORTERRE ET
OLIVIER POUILQUEN**

CNRS ÉDITIONS
www.cnrseditions.fr

Création graphique : Béatrice Coardel

ISBN EDP Sciences 978-2-7598-0097-1
ISBN CNRS ÉDITIONS 978-2-271-07059-0

00 €

EDP SCIENCES

PHYSIQUE
LES MILIEUX GRANULAIRES
ENTRE FLUIDE ET SOLIDE

SAVOIRS **PHYSIQUE** **ACTUELS**

LES MILIEUX GRANULAIRES
ENTRE FLUIDE ET SOLIDE



**BRUNO ANDREOTTI
YOËL FORTERRE ET
OLIVIER POUILQUEN**

CNRS ÉDITIONS

EDP SCIENCES



pyl

spoil tip (boney pile, gob pile, bing or pit heap), «terril» in french



pyl



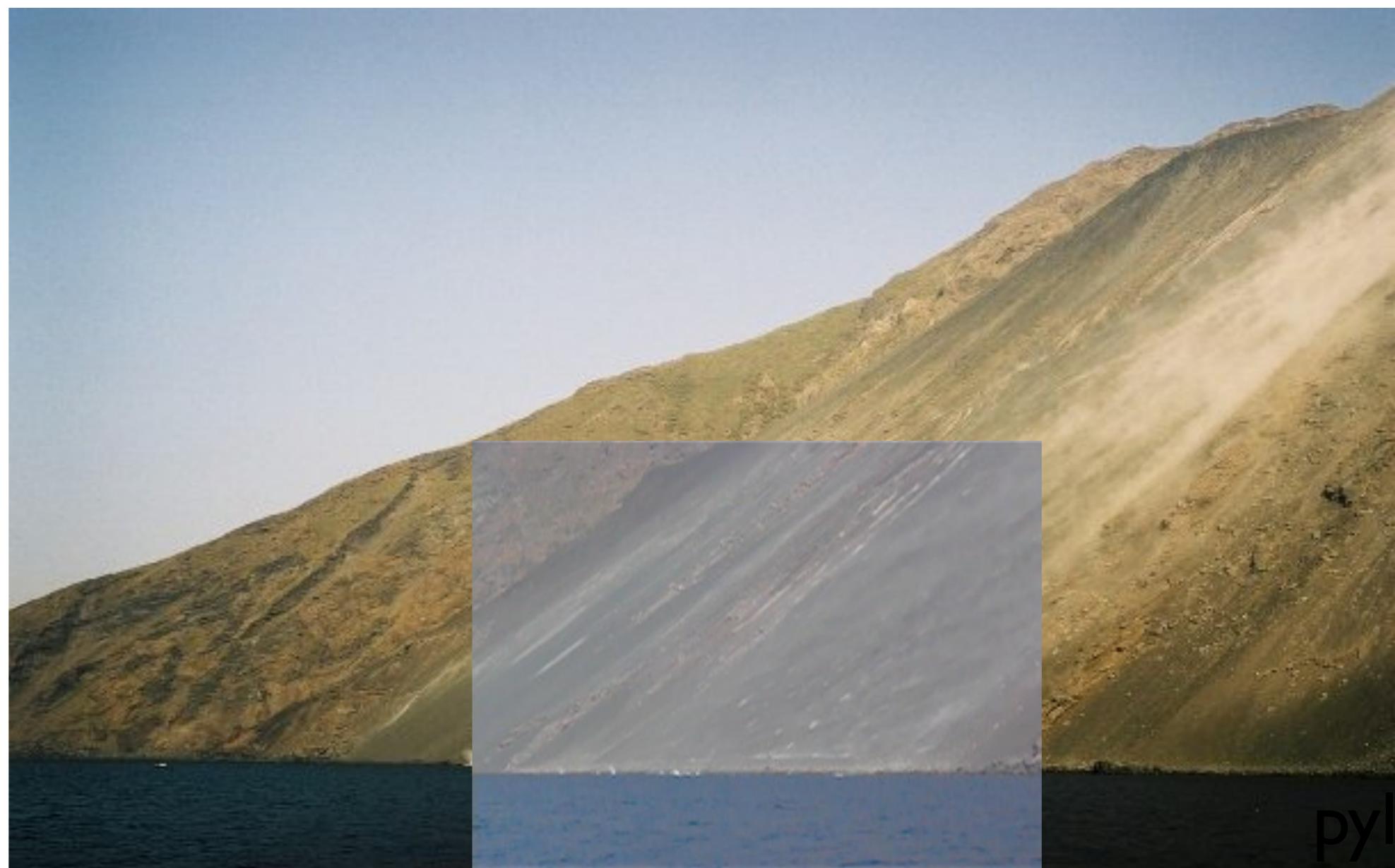
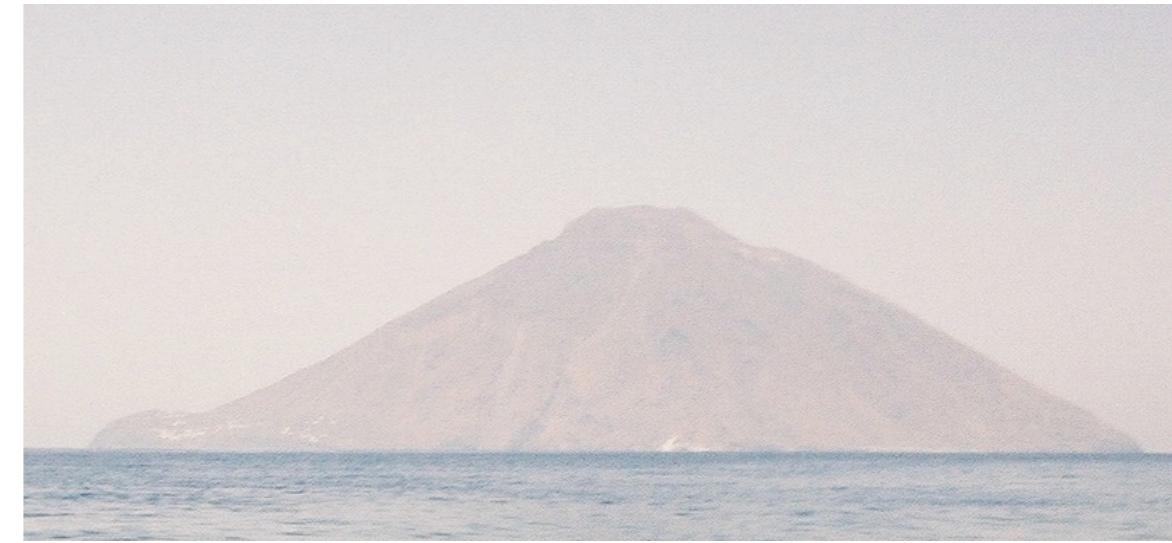
pyl



PyI



pyl





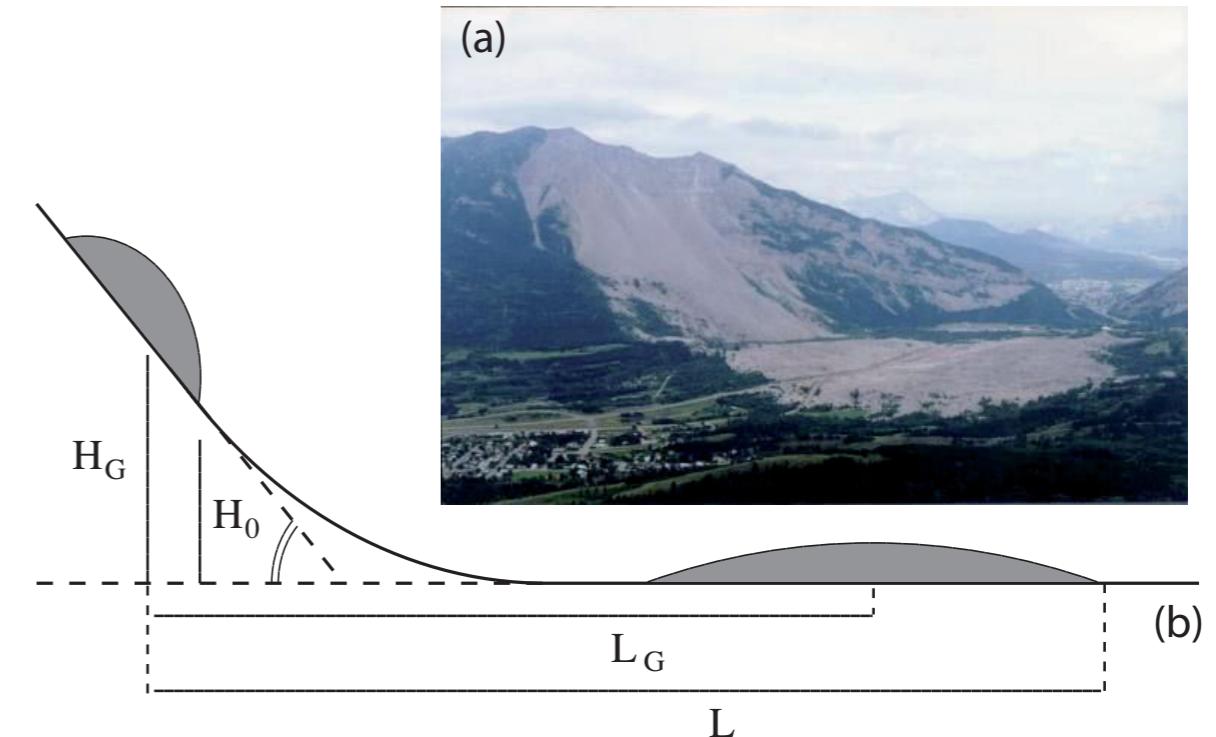
- the Saint Venant Savage Hutter Hyperbolic model



Fig. 20. Frank slide.

Environmental Modelling & Software xx (2006) 1e18
www.elsevier.com/locate/envsoft

The effect of the earth pressure coefficients on the runout of granular material
Marina Pirulli ^{a,*}, Marie-Odile Bristeau ^b, Anne Mangeney ^c, Claudio Scavia



$$H \times L = 1000m \times 2500m$$

Staron

1903, Alberta Canada
90 morts



<http://www.pbase.com/image/63044602>

2006 Gary Hebert

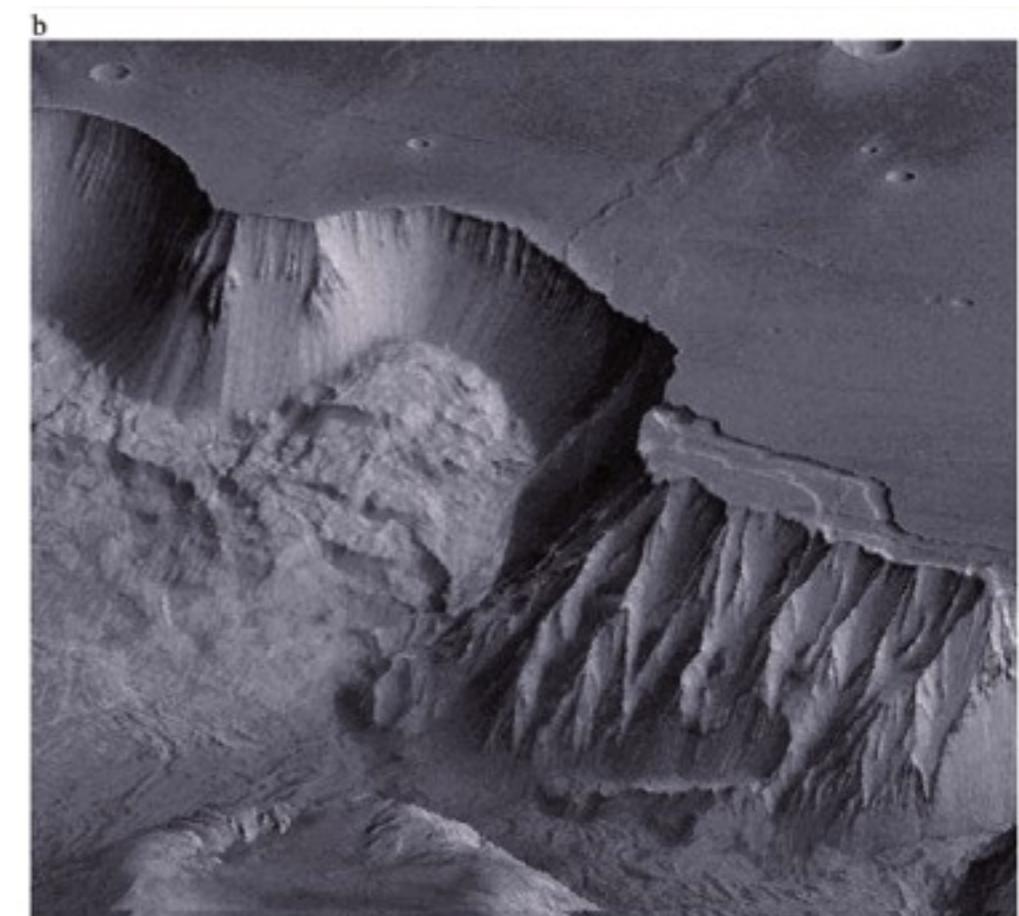
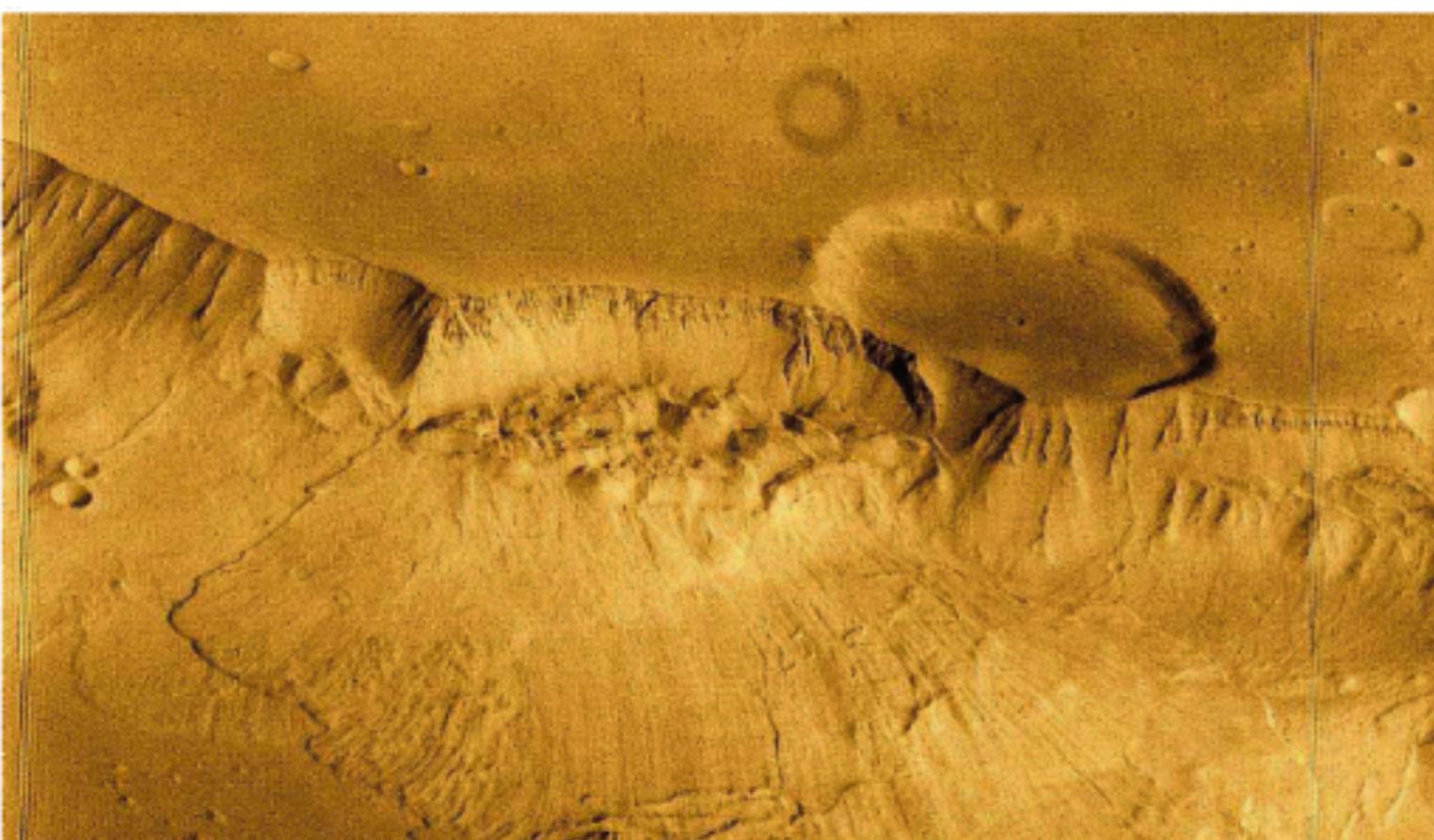


Lofoten Norway

12

photo PYL





http://books.google.fr/books?id=HY6Z5od4-E4C&pg=PA49&dq=granular+flow&hl=fr&ei=lamtTaa_NYyVOoToldcL&sa=X&oi=book_result&ct=result&resnum=10&ved=0CFkQ6AEwCTgK#v=onepage&q&f=true



http://www.cieletespace.fr/image-du-jour/5126_la-saison-des-avalanches-sur-mars



Granular Column Collapse

A model for avalanches



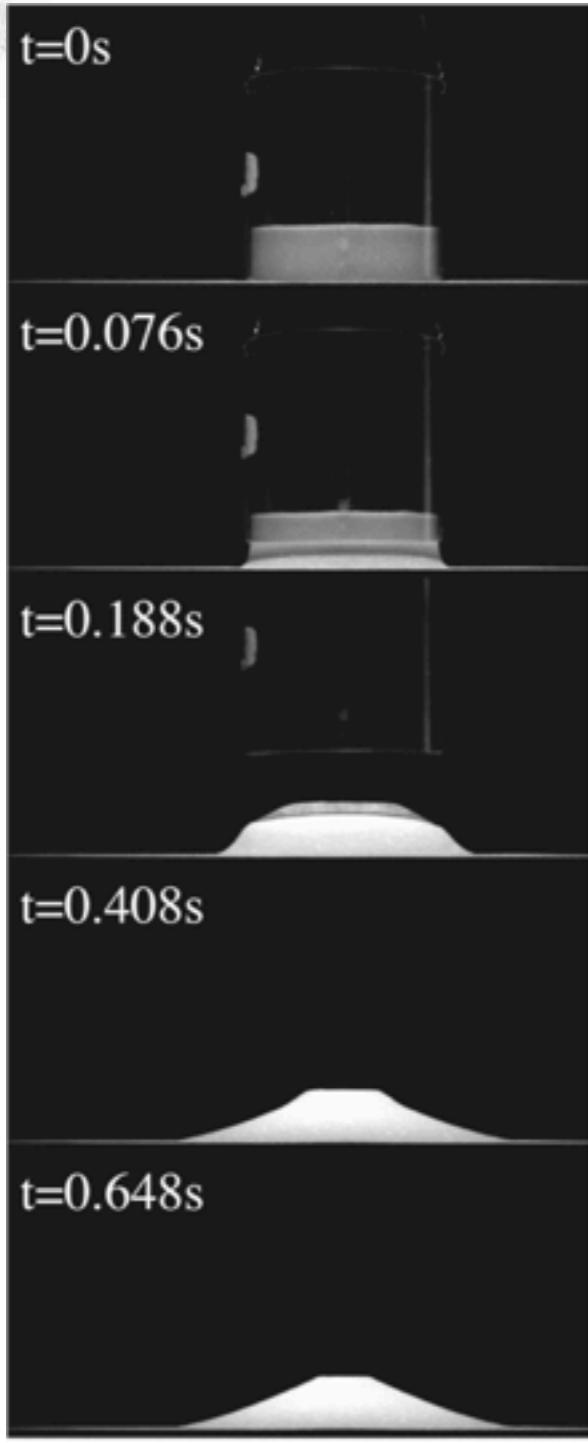
The sand pit problem: quickly remove the bucket of sand



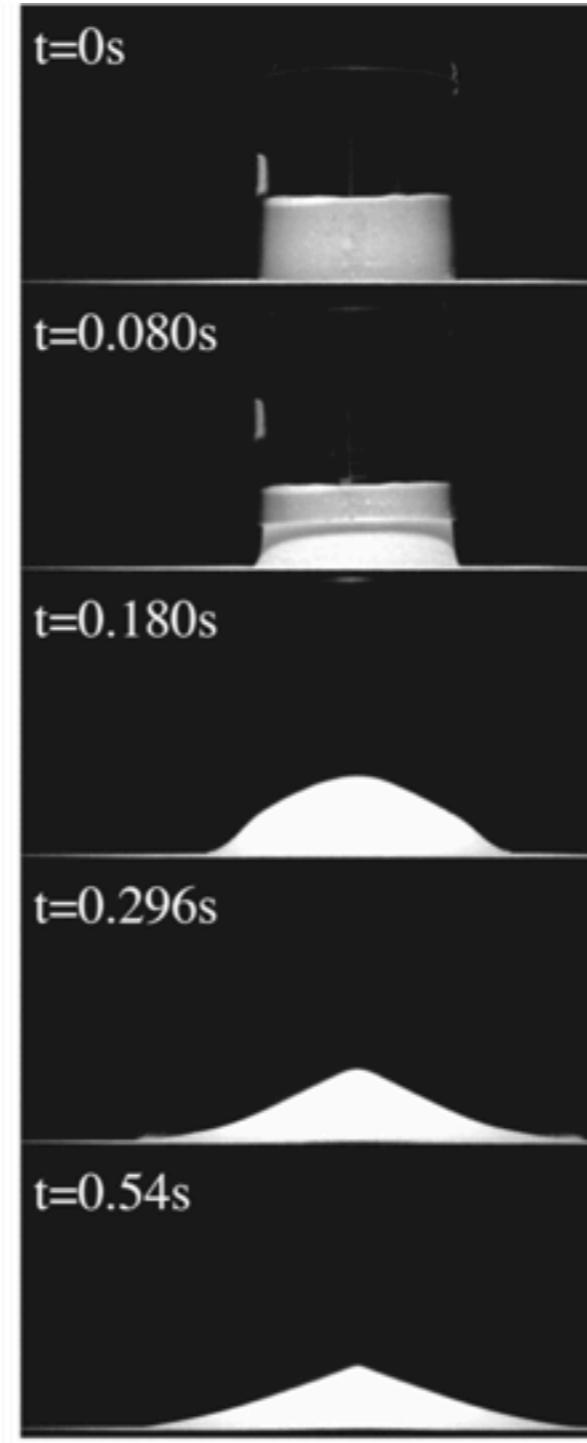


Granular Column Collapse

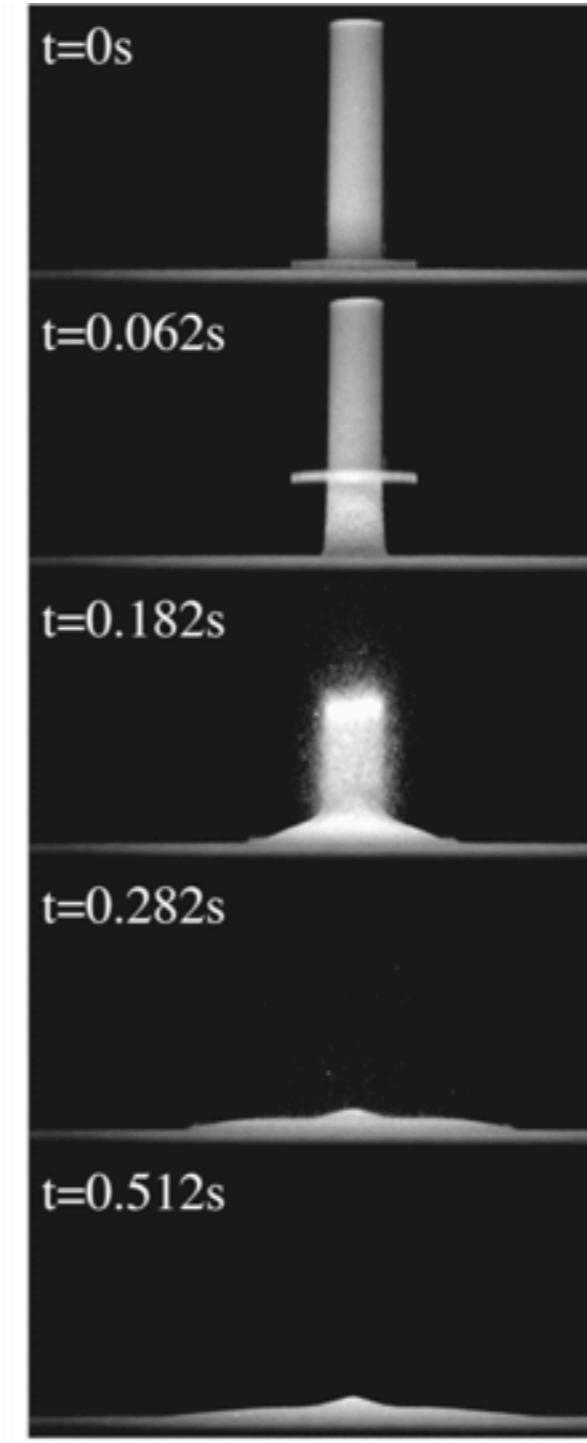
E. Lajeunesse A. Mangeney-Castelnau and J. P. Vilotte PoF 2004



(a)



(b)



(c)

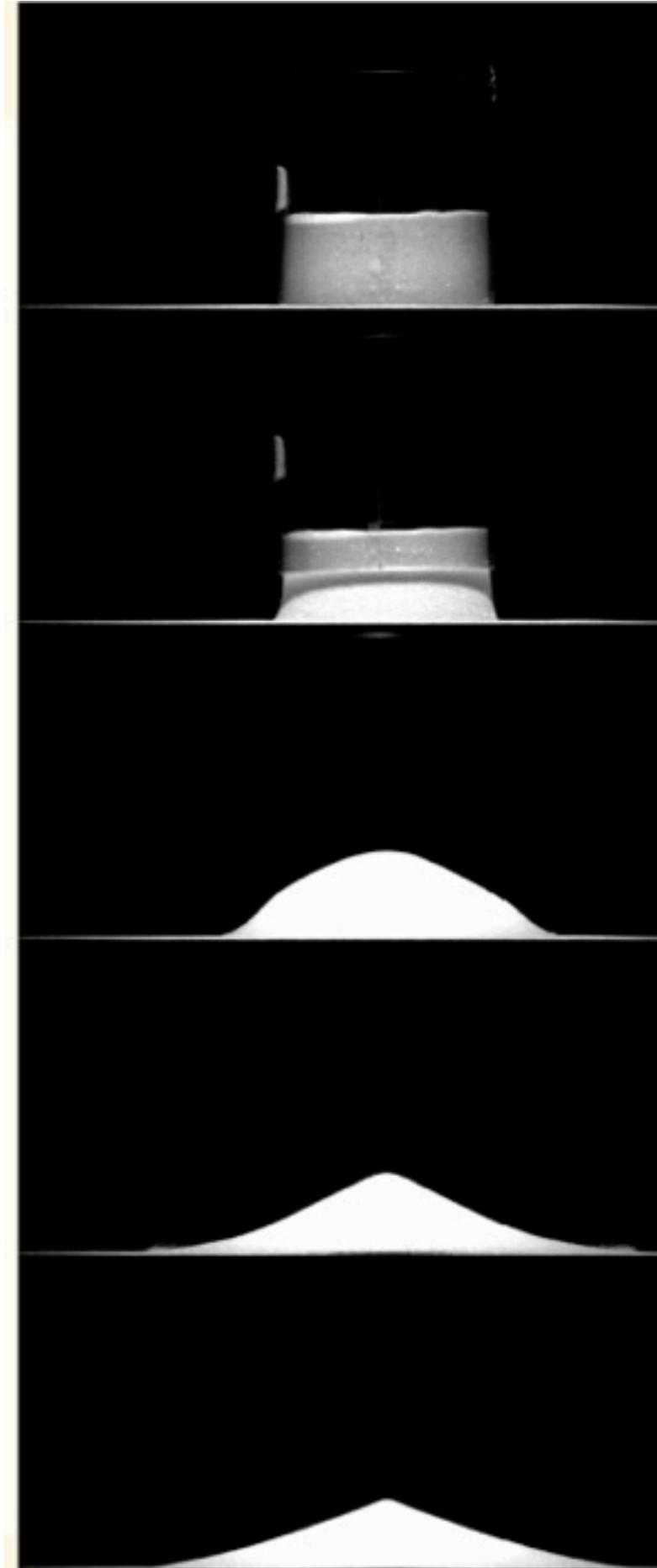
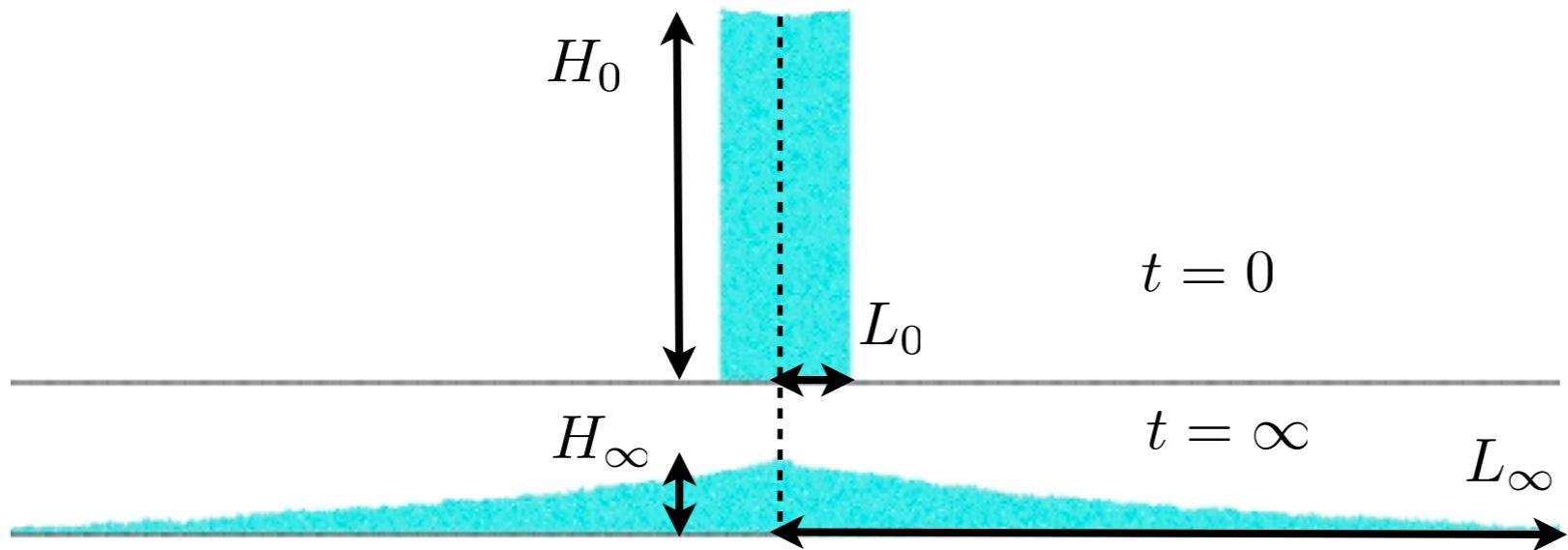


The sand pit problem: quickly remove the bucket of sand



Granular Column Collapse

aspect ratio $a = H_0/R_0 = H_0/L_0$



The sand pit problem: quickly remove the bucket of sand

Lajeunesse et al., 2004

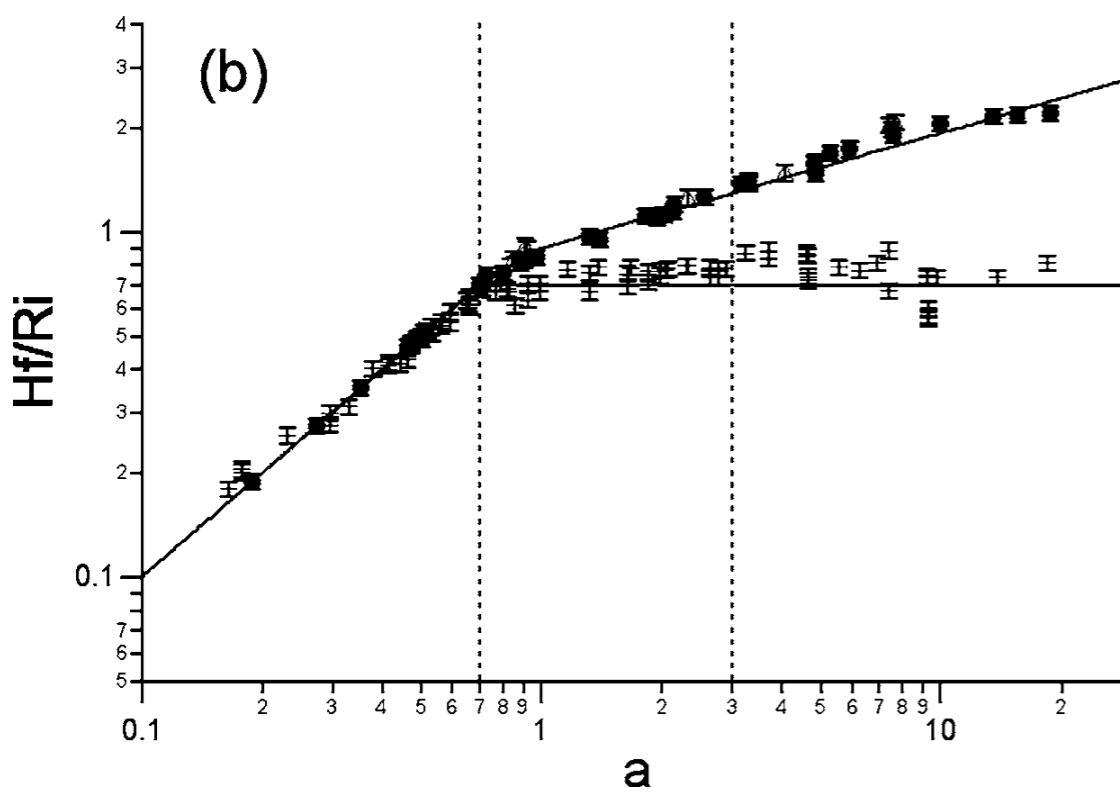
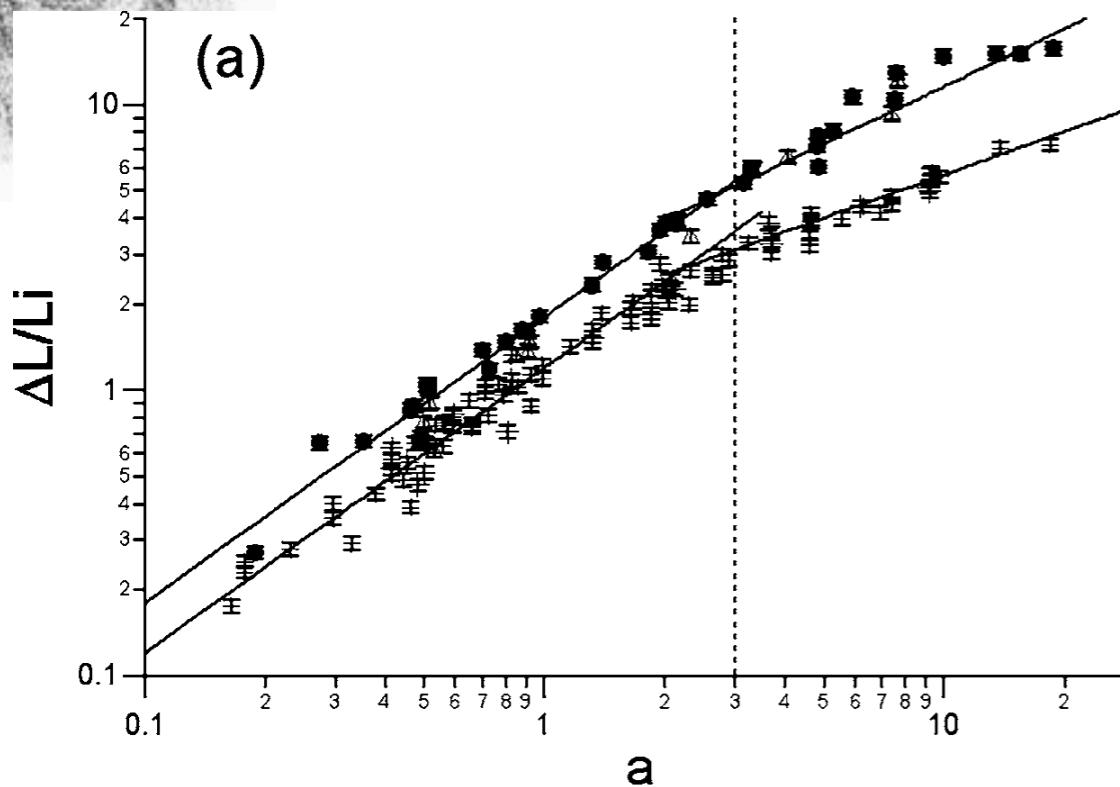


Granular Column Collapse



A possible experimental set up is a container filled by sand (left), the aspect ratio (height/length) is a . At initial time, the gate is opened quickly. After the avalanche, the grains stop, the final configuration is at rest (right). We compare results from Discrete Contact Method Simulations (simulation of the displacement of each grain) to a continuum Navier Stokes simulation with the $\mu(I)$ rheology *Gerris*.

The sand pit problem: quickly remove the bucket of sand



- In the axisymmetric geometry
- In the rectangular channel:

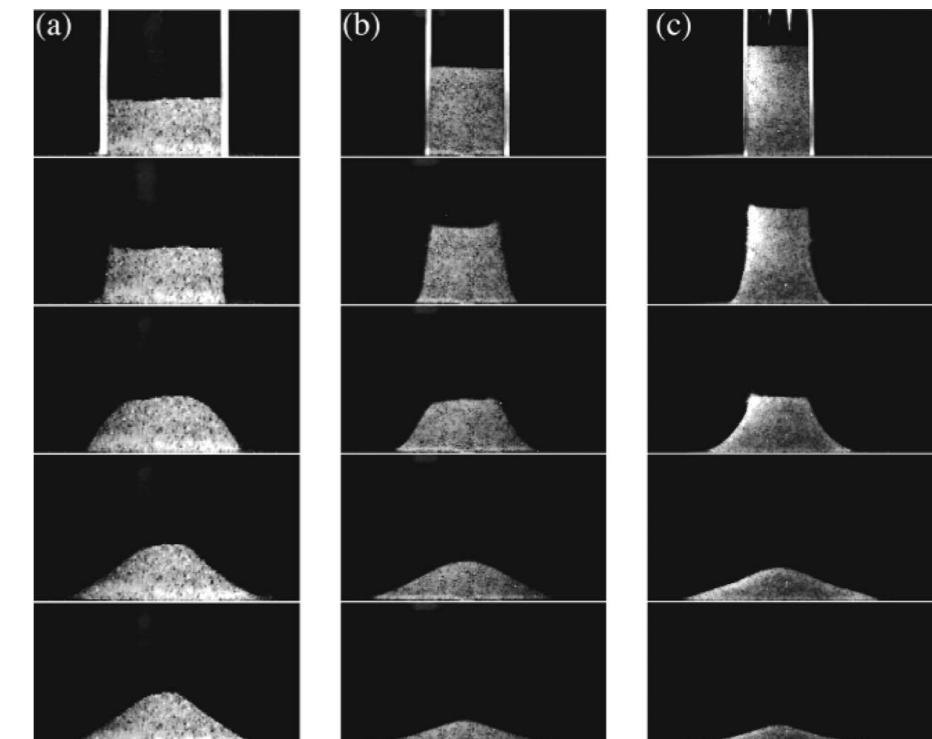
$$\frac{H_f}{L_i} = \begin{cases} a & a \leq 0.74, \\ 0.74 & a \geq 0.74, \end{cases}$$

$$\frac{\Delta L}{L_i} \propto \begin{cases} a & a \leq 3, \\ a^{1/2} & a \geq 3. \end{cases}$$

$$\frac{H_f}{L_i} \propto \begin{cases} a & a \leq 0.7, \\ a^{1/3} & a \geq 0.7, \end{cases}$$

$$\frac{\Delta L}{L_i} \propto \begin{cases} a & a \leq 3, \\ a^{2/3} & a \geq 3. \end{cases}$$

FIG. 6. Scaled runout $\Delta L/L_i$ (a) and scaled deposit height H_f/L_i (b) as functions of a . Circles and triangles correspond to experiments performed in the 2D channel working respectively with glass beads of diameter $d = 1.15$ mm or $d = 3$ mm. Crosses correspond to the data set of axisymmetric collapses from Lajeunesse *et al.* (Ref. 10).



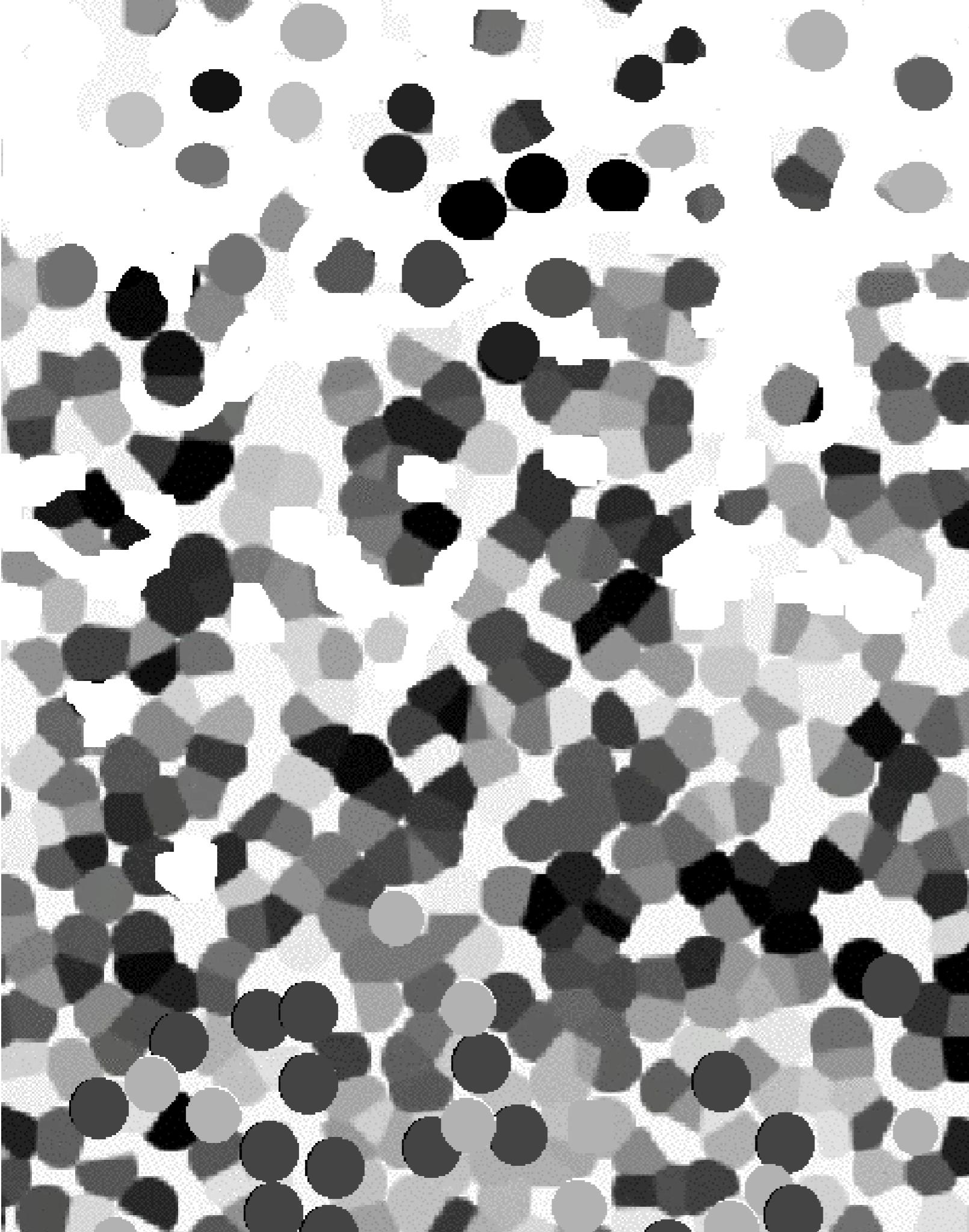
- Silo, hopper, hourglass





outline

- what is a granular fluid? some images
- the $\mu(I)$ friction law obtained from experiments and discrete simulation
- the viscosity associated to the $\mu(I)$ friction law
- the Saint Venant Savage Hutter Hyperbolic model
- implementing the $\mu(I)$ friction law in Navier Stokes
- Examples of flows: focusing on the granular column collapse and the Hour Glass



grains

gaz

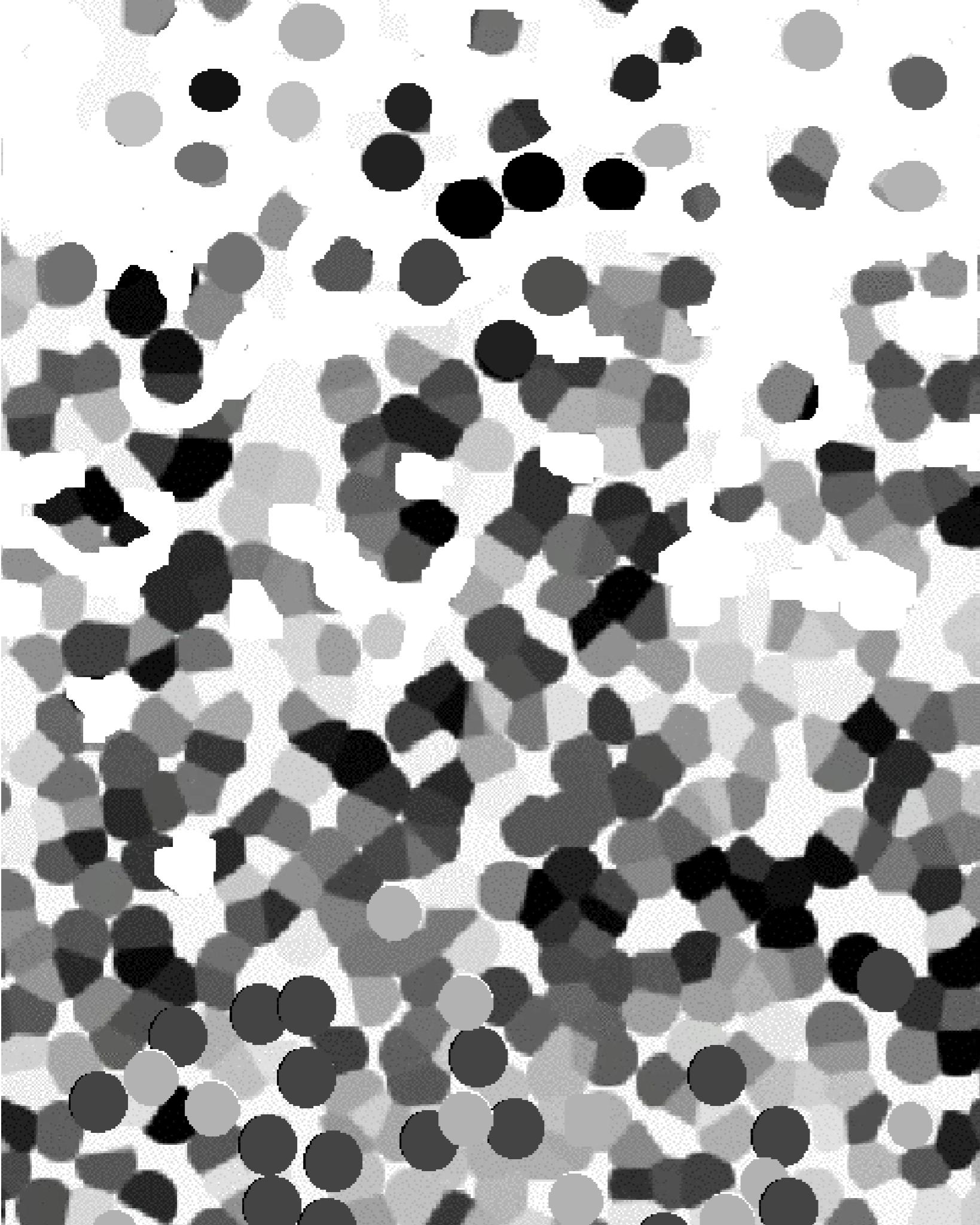
impacts:
suspension

granular media
contacts

liquid

like a solid:

from the grains to the fluid



grains

ϕ_{min}

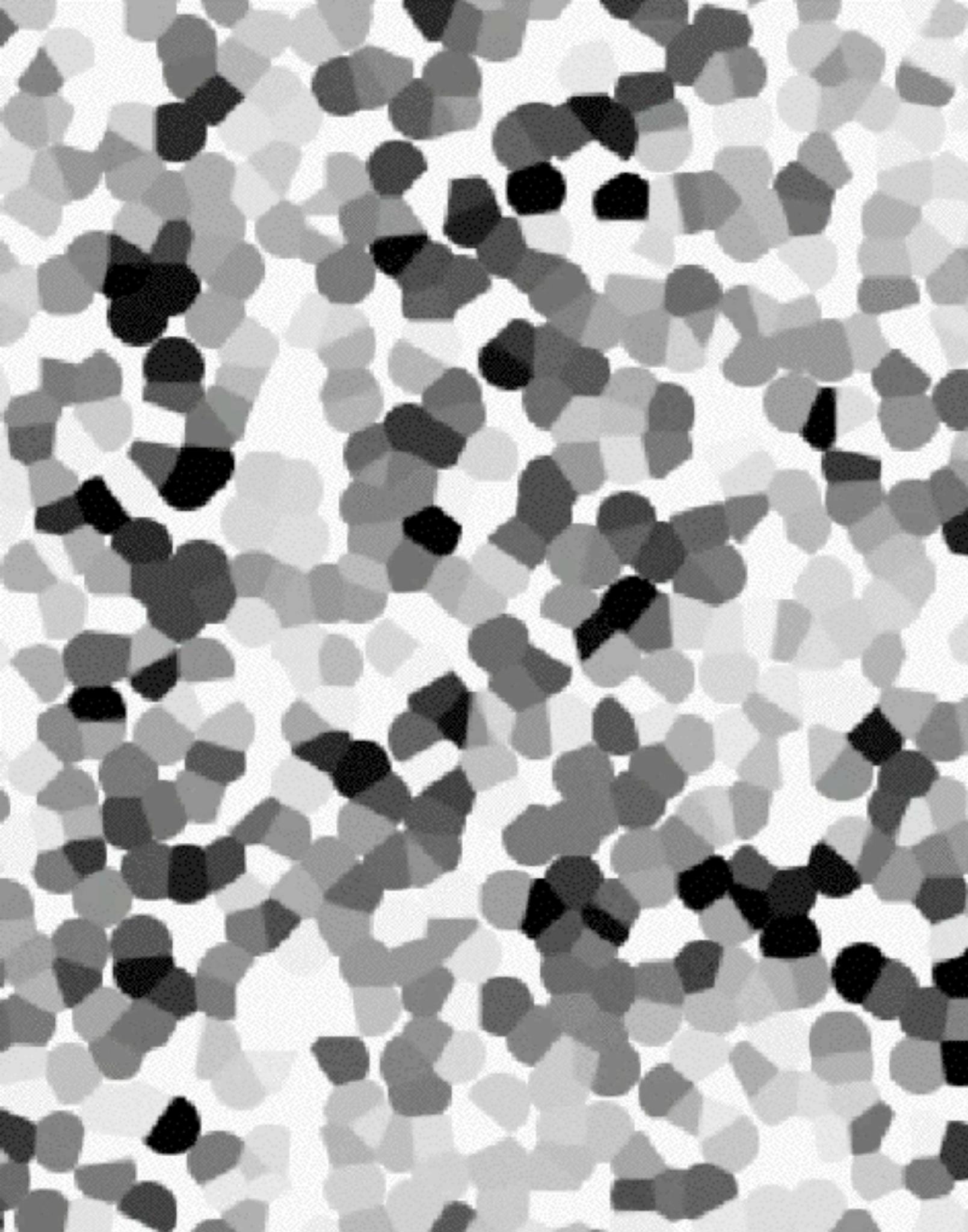
0.5 (2D) 0.55 (3D)

$\phi_{min} < \phi < \phi_{Max}$

ϕ_{max}

0.8 (2D) 0.65 (3D)

from the grains to the fluid



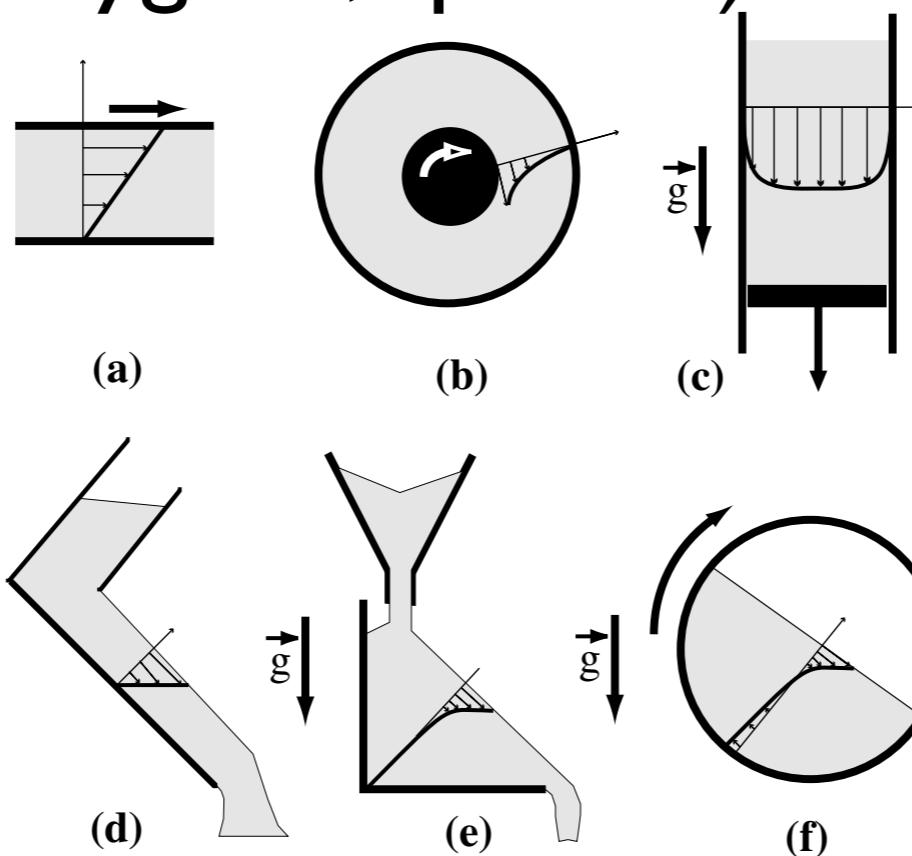
~~grains~~

continuum media
hypothesis

from the grains to the fluid



- Looking for a continuum description
- Lot of recent experiments in simple configurations: shear/ inclined plane, with model material (glass beads, sand...)
- Simulations with Contact Dynamics (disks, polygons, spheres)



GDR MiDi EPJ E 04

Fig. 1. The six configurations of granular flows: (a) plane shear, (b) annular shear, (c) vertical-chute flows, (d) inclined plane, (e) heap flow, (f) rotating drum.



- Looking for a continuum description
- Lot of recent experiments in simple configurations:
shear/ inclined plane,
with model material (glass beads, sand...)
- Simulations with Contact Dynamics
(disks, polygona, spheres)
- Defining a «viscosity»
- Implement it in the Navier Stokes solver *Gerris*
- Test on exact «Bagnold» avalanche solution
- Test on granular collapse and hourglass

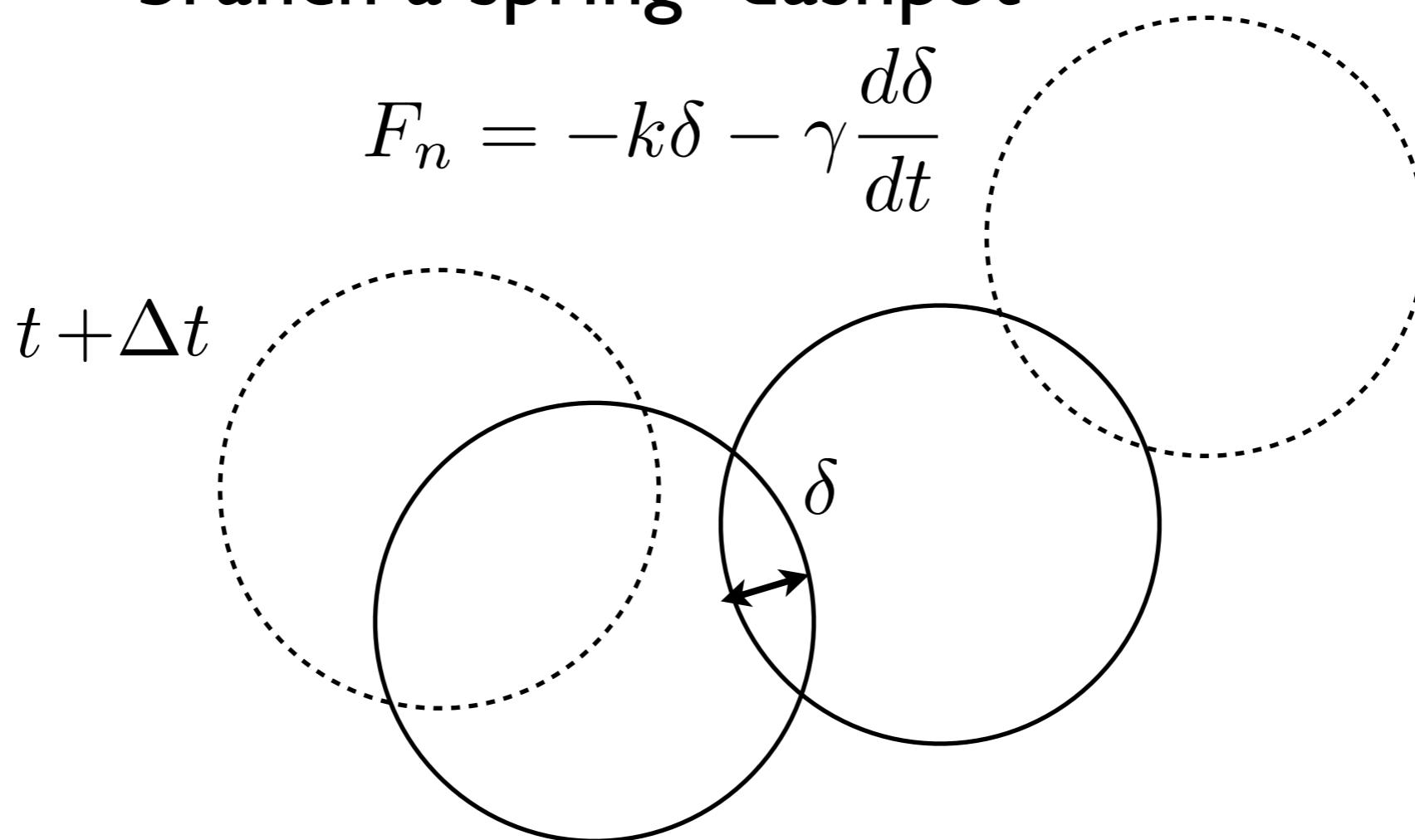


Molecular Dynamics:

$$m \frac{d}{dt} \vec{U} = \vec{F} + \vec{F}_n + \vec{F}_t \quad \text{Newton's equations}$$

branch a spring -dashpot

$$F_n = -k\delta - \gamma \frac{d\delta}{dt}$$



tangential Coulombic Friction $F_t < \mu F_n$



Contact Dynamics de (Moreau 1988)

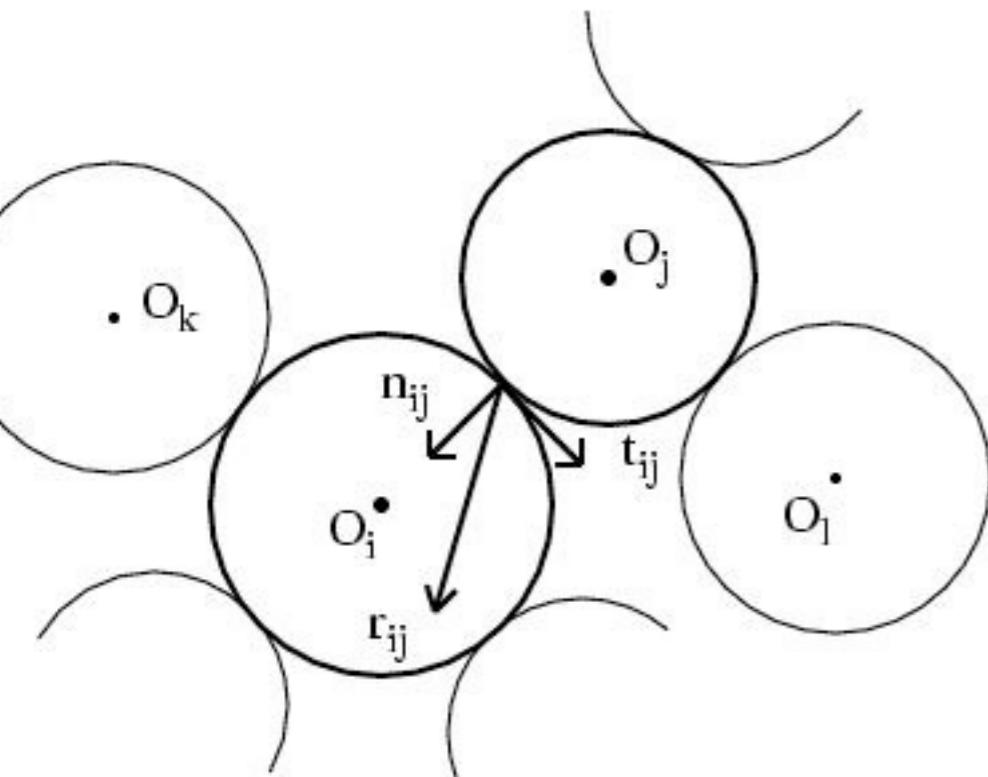
rigid grains

coefficient of friction μ

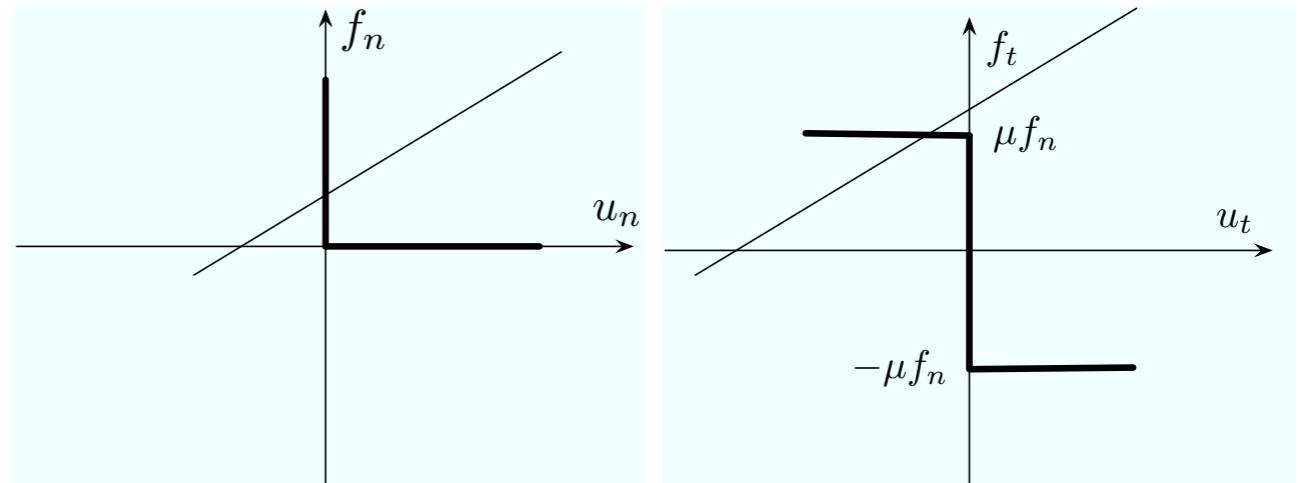
$$m(\vec{U}^+ - \vec{U}^-) = \vec{F} \delta t \quad \text{Newton's equations}$$

take the form of an equality between the change of momenta
and the average impulse during δt .

written for each grain at the contact

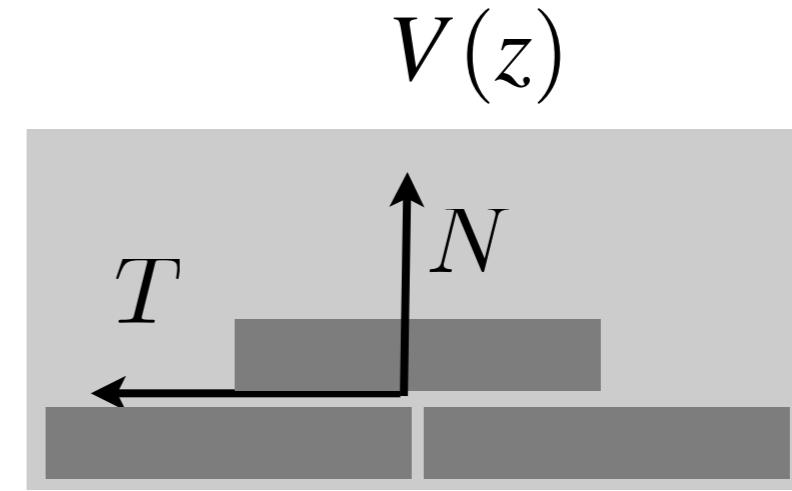
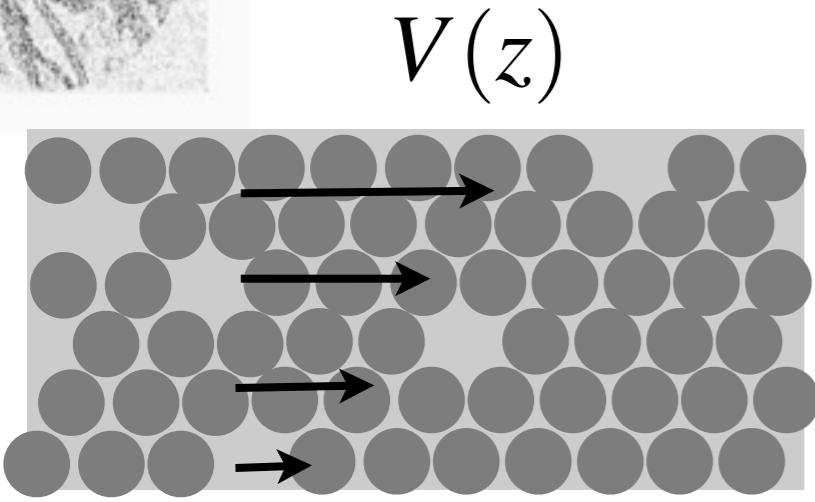


u_n, u_t



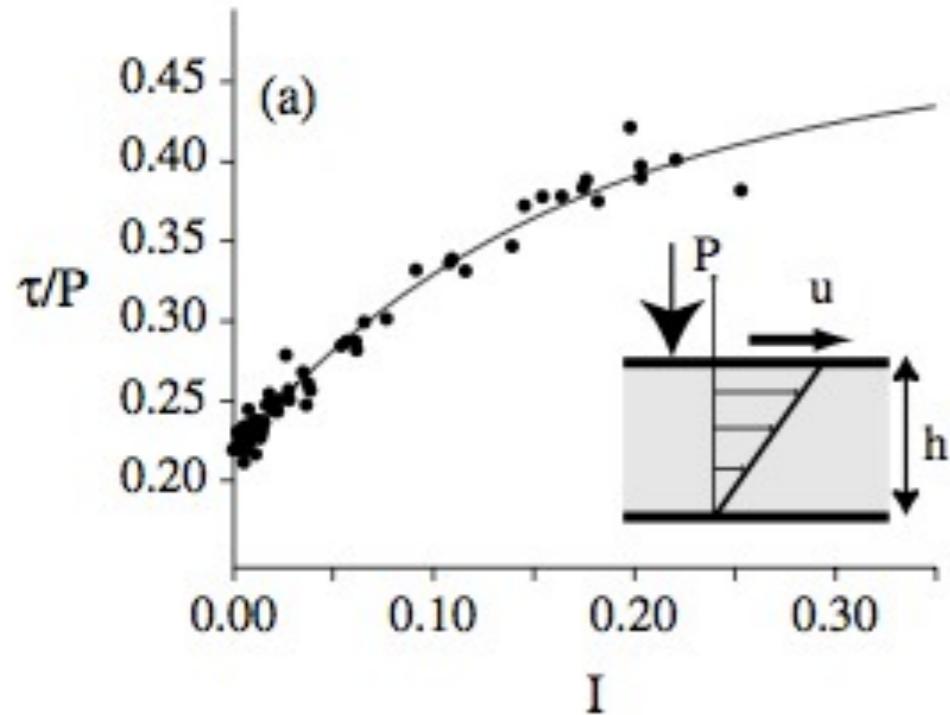


The $\mu(I)$ -rheology



$$T = \mu N$$

constitutive law?

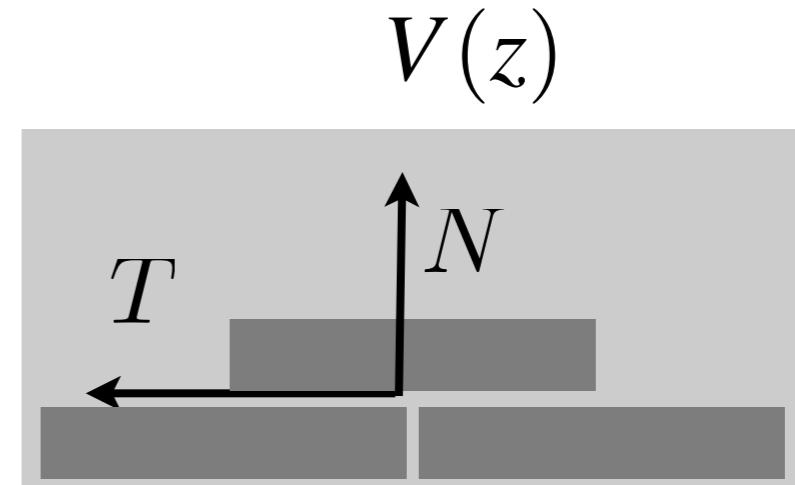
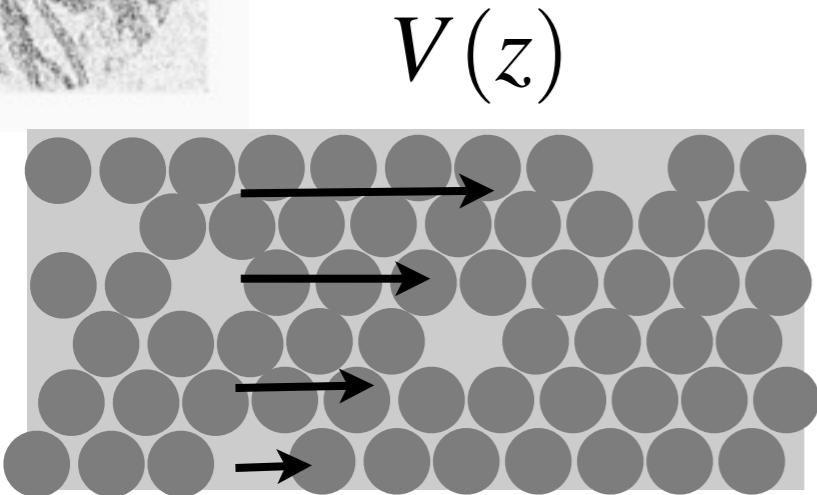


Coulomb dry friction
Coulomb friction law

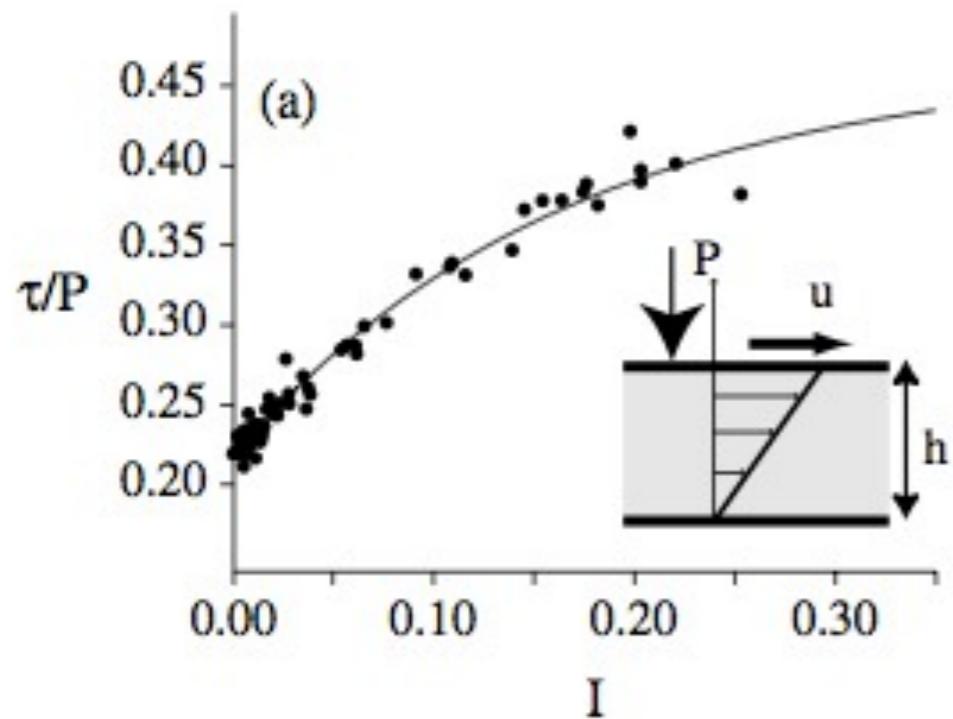
$$\tau = \mu P$$



The $\mu(I)$ -rheology



$$T = \mu N$$



$$I = \frac{d \frac{\partial u}{\partial y}}{\sqrt{P/\rho}}$$

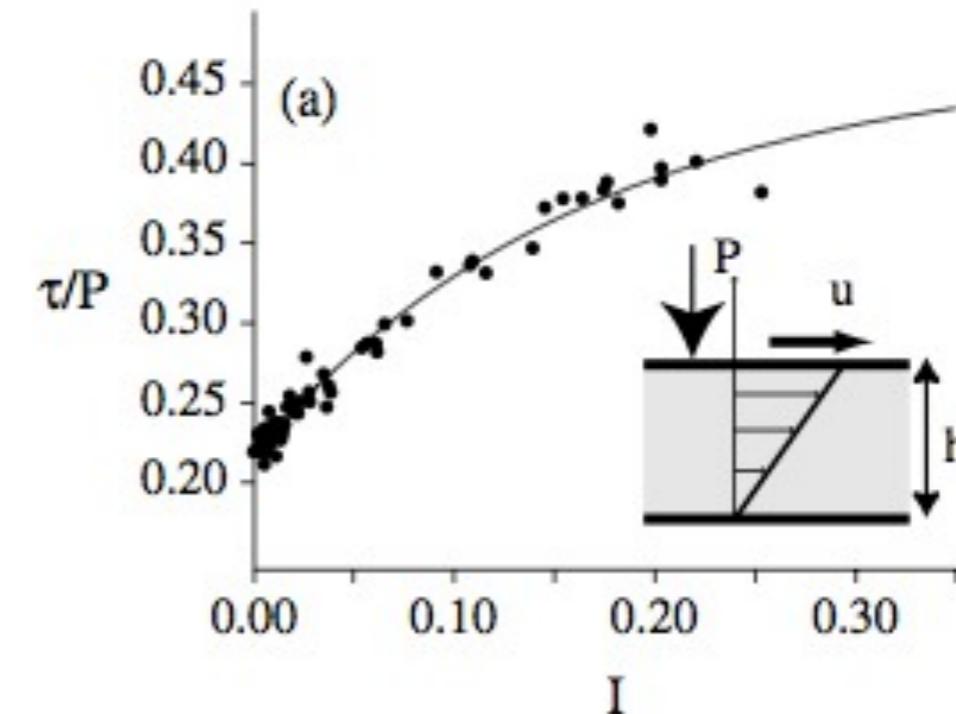
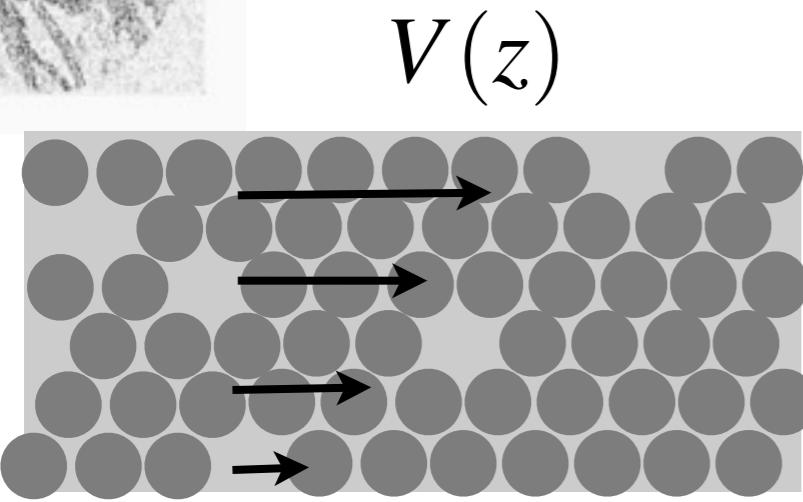
Coulomb friction law

$$\tau = \mu(I)P$$

non dimensional number: «Froude»
local «Inertial Number» (Da Cruz 04-05)



The $\mu(I)$ -rheology



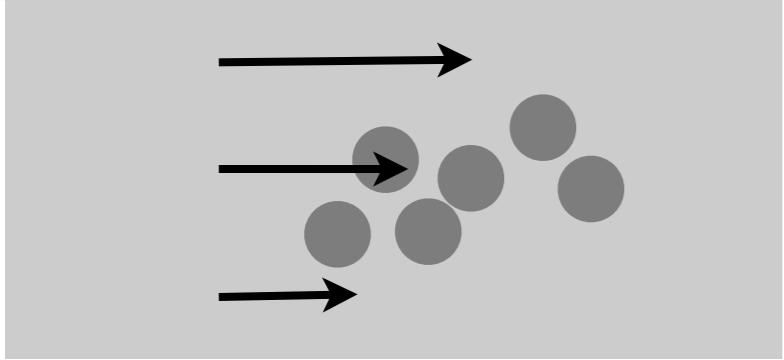
falling time
displacement time

$$I = \frac{d \frac{\partial u}{\partial y}}{\sqrt{P/\rho}}$$

non dimensional number: «Froude»
local «Inertial Number» (Da Cruz 04-05)



The $\mu(I)$ -rheology



$$\frac{dx}{dt} = d \frac{\partial u}{\partial y} \quad t = 1 / \frac{\partial u}{\partial y}$$

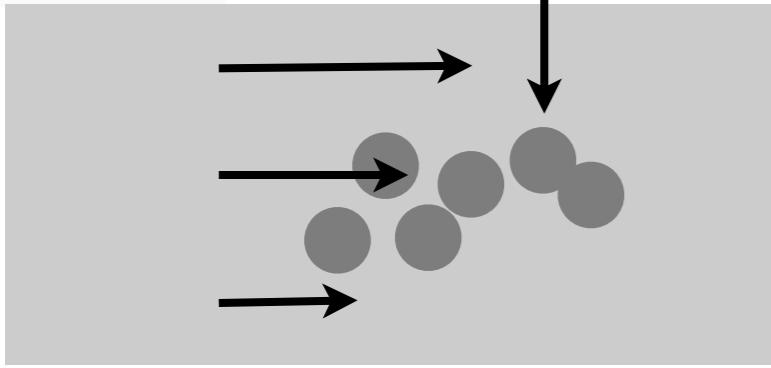
falling time

displacement time

$$I = \frac{d \frac{\partial u}{\partial y}}{\sqrt{P/\rho}}$$



The $\mu(I)$ -rheology



A diagram showing a sphere falling vertically through a fluid. The fluid is represented by a grey rectangular area with three horizontal arrows pointing to the right, indicating a uniform flow. A vertical arrow points downwards from the top of the sphere, labeled P , representing pressure or force. The sphere is shown in three stages of its fall, with arrows indicating its downward motion.

$$md^2y/dt^2 = Pd^2$$
$$\uparrow t^2 = \rho d^2/(P)$$
$$t = 1/\frac{\partial u}{\partial y}$$

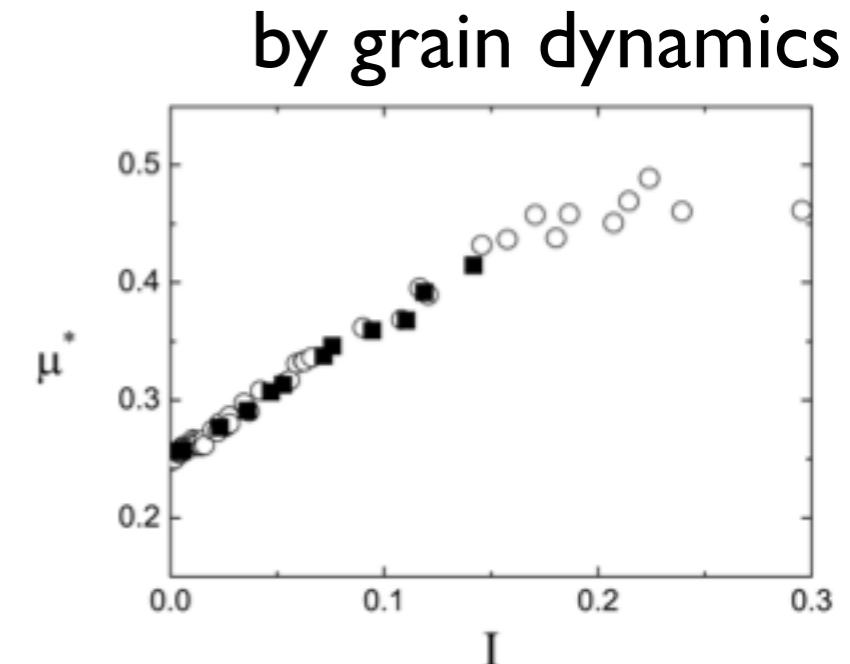
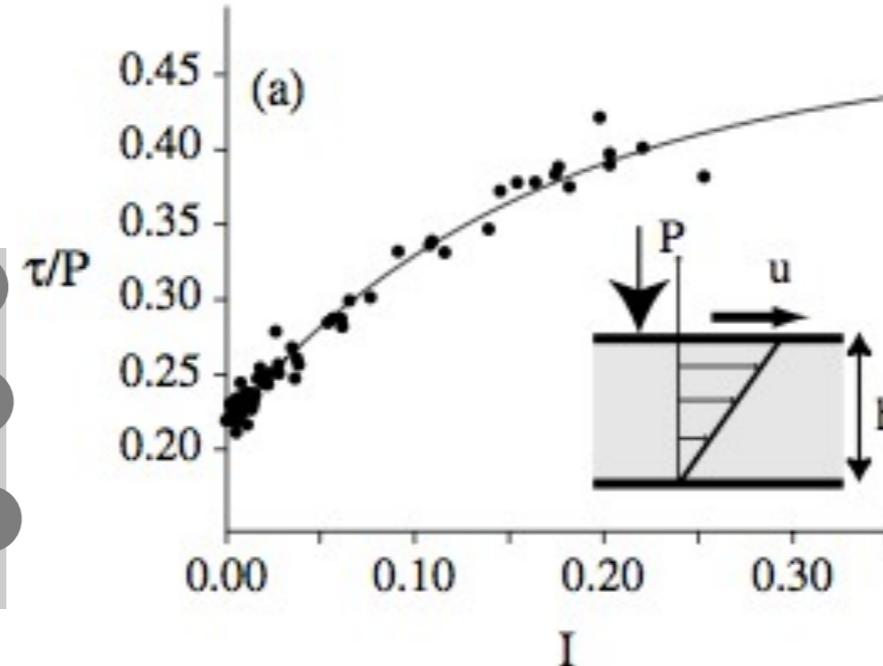
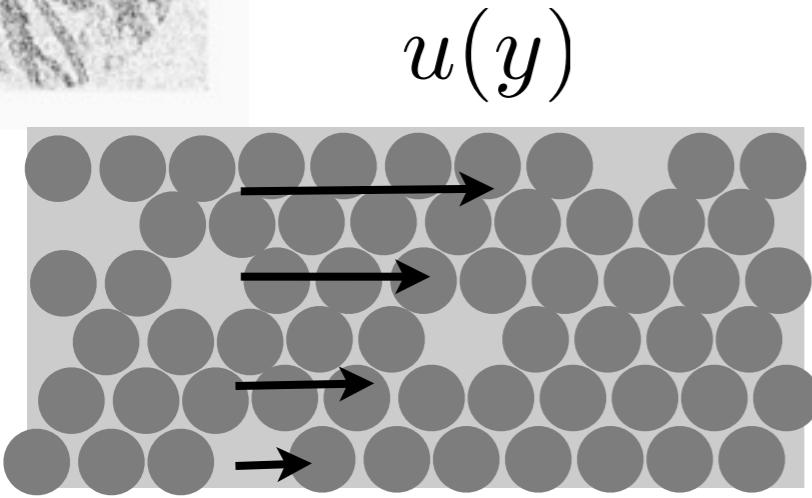
falling time

displacement time

$$I = \frac{d\frac{\partial u}{\partial y}}{\sqrt{P/\rho}}$$



The $\mu(I)$ -rheology



Da Cruz PRE 05

Coulomb friction law

$$\tau = \mu(I)P$$

falling time
displacement time

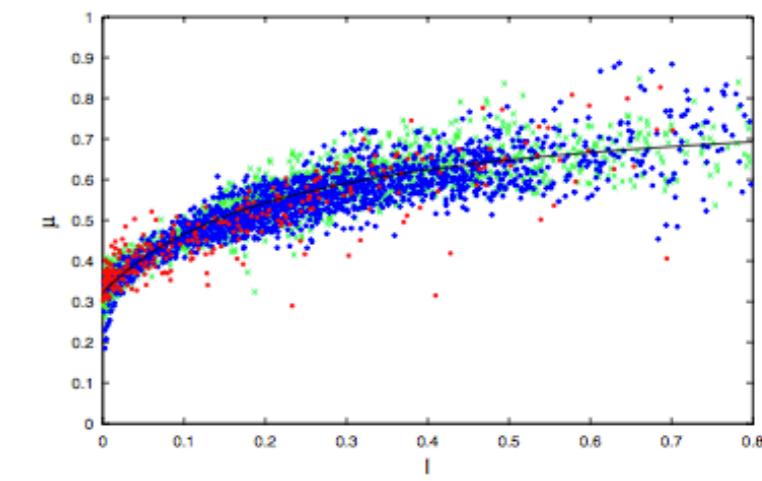
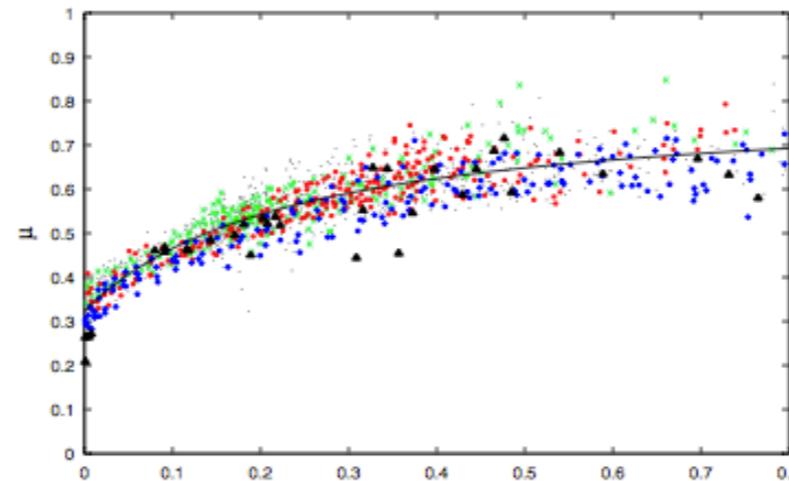
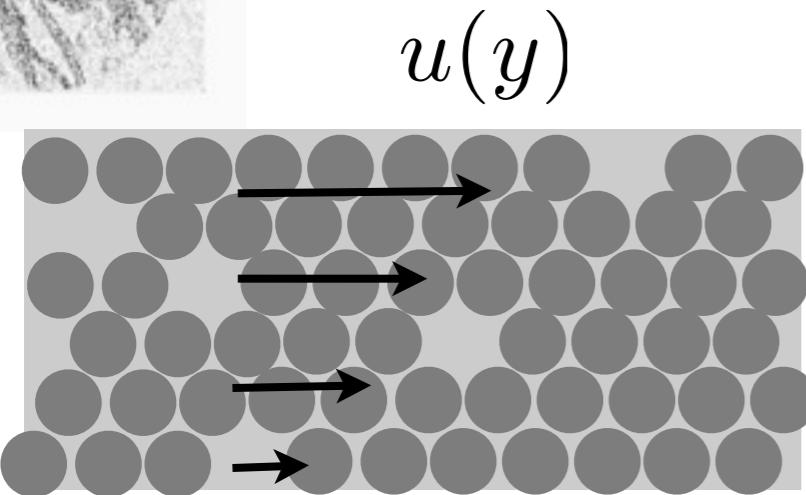
$$I = \frac{d\frac{\partial u}{\partial y}}{\sqrt{P/\rho}}$$

Pouliquen 99
Pouliquen Forterre JSM 06
Da Cruz 04-05
GDR Midi 04
Josserand Lagrée Lhuillier 04



The $\mu(I)$ -rheology

by grain dynamics



Lacaze Kerswell 09

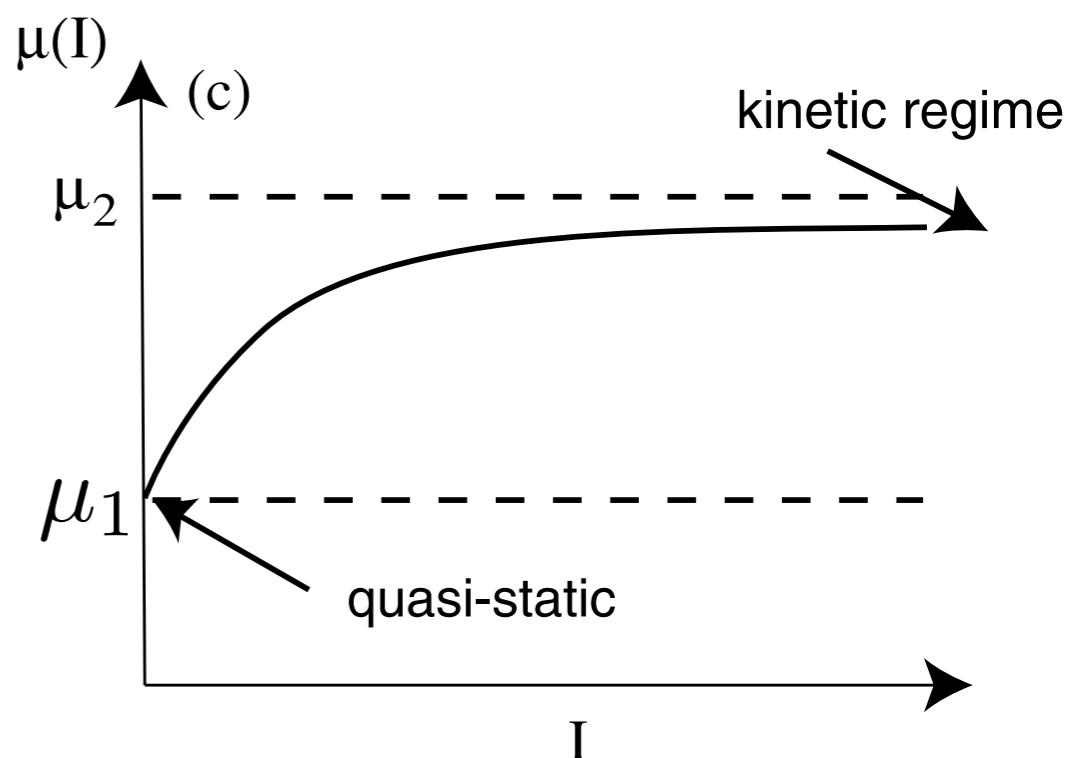
Coulomb friction law

$$\tau = \mu(I)P$$

$$\mu(I) = \mu_1 + \frac{\mu_2 - \mu_1}{I_0/I + 1}$$

$$\mu_1 \simeq 0.32 \quad (\mu_2 - \mu_1) \simeq 0.23 \quad I_0 \simeq 0.3$$

$$I = \frac{d \frac{\partial u}{\partial y}}{\sqrt{P/\rho}}$$

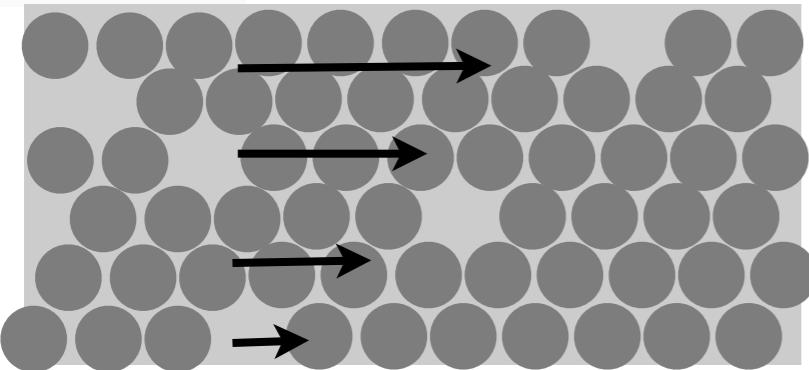




The $\mu(I)$ -rheology

Jop Forterre Pouliquen 2005

$$u(y)$$



Coulomb friction law

$$\tau = \mu(I)P$$

$$I = \frac{d\frac{\partial u}{\partial y}}{\sqrt{P/\rho}}$$

«Drucker-Prager»
plastic flow

«equivalent» viscosity

$$\eta \frac{\partial u}{\partial y} = \mu(I)p \rightarrow \eta = \frac{\mu(I)p}{\frac{\partial u}{\partial y}}$$



The $\mu(I)$ -rheology implementation in Navier Stokes?

Jop Forterre Pouliquen 2005

$$\mu(I) = \mu_1 + \frac{\mu_2 - \mu_1}{I_0/I + 1}$$

$$D_2 = \sqrt{D_{ij}D_{ij}} \quad D_{ij} = \frac{u_{i,j} + u_{j,i}}{2}$$

construction of a viscosity based on the D_2 invariant and redefinition of I

$$I = d\sqrt{2}D_2/\sqrt(|p|/\rho).$$

$$\eta = \left(\frac{\mu(I)}{\sqrt{2}D_2} p \right) \quad \text{«Drucker-Prager»}$$

$$\nabla \cdot \mathbf{u} = 0, \quad \rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \nabla \cdot (2\eta \mathbf{D}) + \rho g,$$

Boundary Conditions: no slip and $P=0$ at the interface



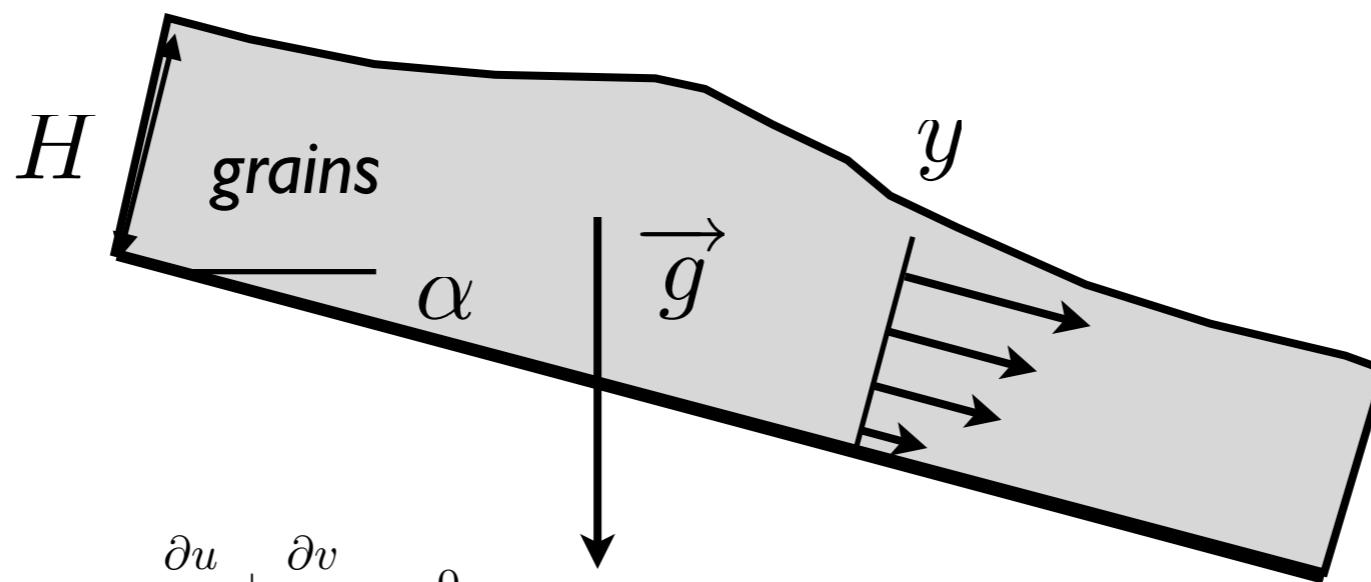
outline

- what is a granular fluid? some images
- the $\mu(l)$ friction law obtained from experiments and discrete simulation
- the viscosity associated to the $\mu(l)$ friction law
- the Saint Venant Savage Hutter Hyperbolic model
- implementing the $\mu(l)$ friction law in Navier Stokes
- Examples of flows: focusing on the granular column collapse (limits of Saint Venant Savage Hutter Hyperbolic model)

S T VENANT



● Couche Mince Saint Venant Shallow Water Savage Hutter



$$\varepsilon = \frac{H}{L}$$

$$\begin{cases} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \\ \rho \left(\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} \right) = -g\rho \sin \alpha - \frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \\ \rho \left(\frac{\partial v}{\partial t} + \frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} \right) = -g\rho \cos \alpha - \frac{\partial p}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} \end{cases}$$

$$p = -\rho g \cos \alpha (\eta(x, t) - y)$$

$$\tau_{xy} = \rho g H \sin \alpha \bar{\tau}_{xy}$$

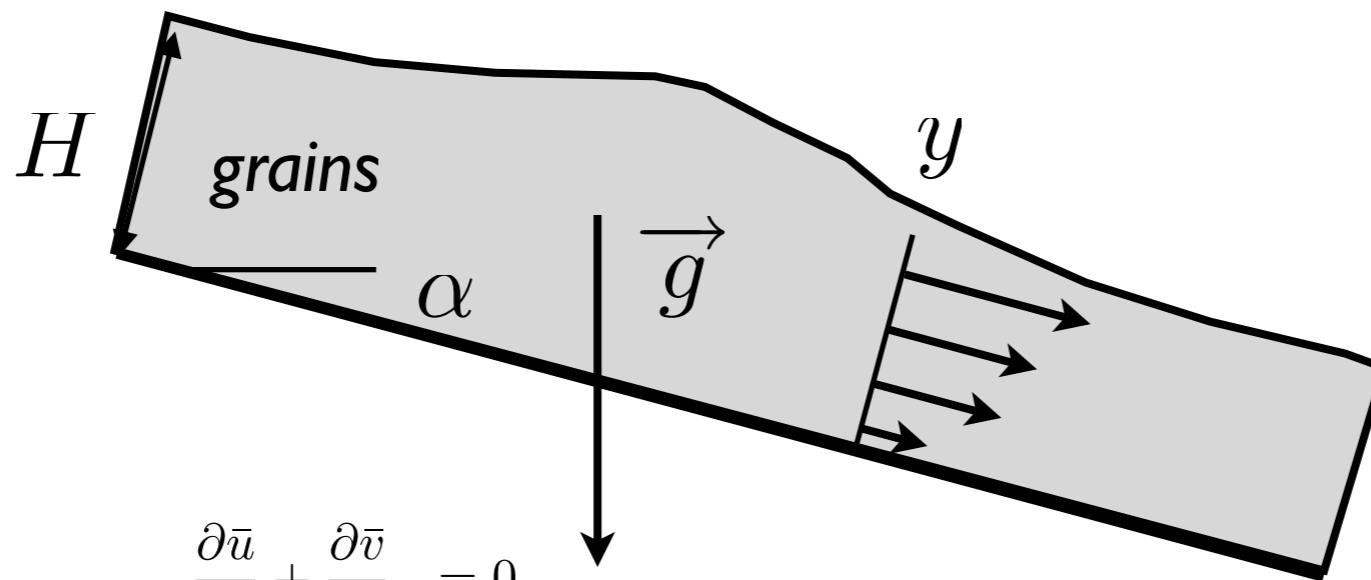
$$\rho U_0^2 / L \longleftrightarrow -\frac{\partial p}{\partial x} = -\rho g \frac{\partial \eta}{\partial x}$$

$$\rho U_0^2 / (H/\varepsilon) = \rho g \varepsilon.$$

$$U_0 = \sqrt{gH}.$$



Saint-Venant Savage Hutter



$$\varepsilon = \dot{H}/L$$

$$\left\{ \begin{array}{l} \frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \\ \varepsilon \left(\frac{\partial \bar{u}}{\partial \bar{t}} + \frac{\partial \bar{u}^2}{\partial \bar{x}} + \frac{\partial \bar{u}\bar{v}}{\partial \bar{y}} \right) = -\sin \alpha - \varepsilon \frac{\partial \bar{p}}{\partial \bar{x}} + \sin \alpha \varepsilon \frac{\partial \bar{\tau}_{xx}}{\partial \bar{x}} + \sin \alpha \frac{\partial \bar{\tau}_{xy}}{\partial \bar{y}} \\ \varepsilon^2 \left(\frac{\partial \bar{v}}{\partial \bar{t}} + \frac{\partial \bar{u}\bar{v}}{\partial \bar{x}} + \frac{\partial \bar{v}^2}{\partial \bar{y}} \right) = -\cos \alpha - \frac{\partial \bar{p}}{\partial \bar{y}} + \sin \alpha \varepsilon \frac{\partial \bar{\tau}_{yy}}{\partial \bar{x}} + \sin \alpha \frac{\partial \bar{\tau}_{yx}}{\partial \bar{y}} \end{array} \right.$$

$$p = -\rho g \cos \alpha (\eta(x, t) - y)$$

$$\tau_{xy} = \rho g H \sin \alpha \bar{\tau}_{xy}$$

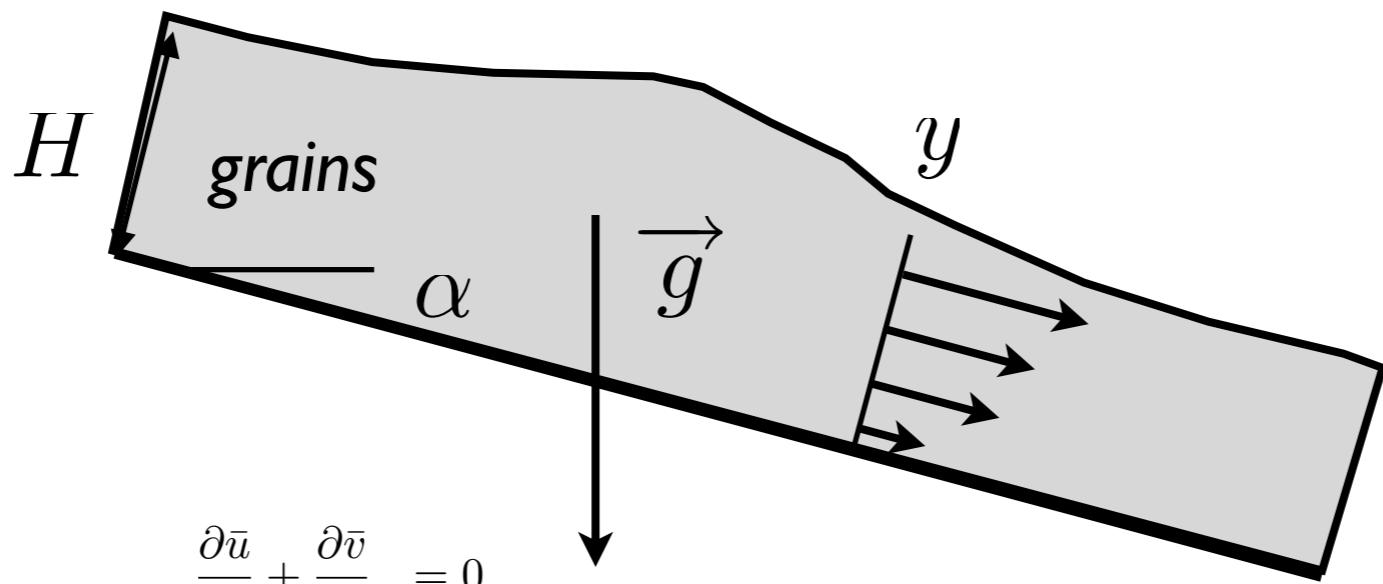
$$\rho U_0^2/L \longleftrightarrow -\frac{\partial p}{\partial x} = -\rho g \frac{\partial \eta}{\partial x}$$

$$\rho U_0^2/(H/\varepsilon) = \rho g \varepsilon.$$

$$U_0 = \sqrt{gH}.$$



Saint-Venant Savage Hutter



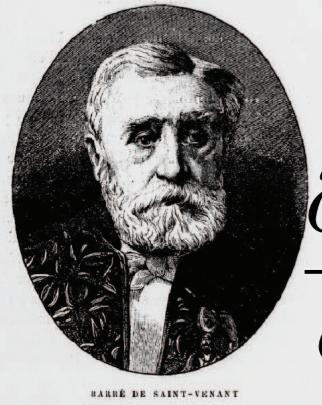
$$\left\{ \begin{array}{l} \frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \\ \varepsilon \left(\frac{\partial \bar{u}}{\partial \bar{t}} + \frac{\partial \bar{u}^2}{\partial \bar{x}} + \frac{\partial \bar{u}\bar{v}}{\partial \bar{y}} \right) = -\sin \alpha - \varepsilon \frac{\partial \bar{p}}{\partial \bar{x}} + \sin \alpha \frac{\partial \bar{\tau}_{xy}}{\partial \bar{y}} \\ 0 = -\cos \alpha - \frac{\partial \bar{p}}{\partial \bar{y}} \end{array} \right.$$

$$\int \Big| dy$$

$\mu(l)$ at the wall

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x} \int_f^\eta u(x, y, t) dy = 0$$

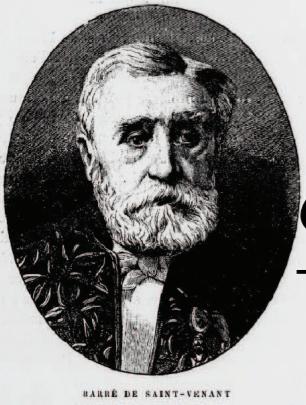
$$\frac{\partial}{\partial t} \int_f^\eta u(x, y, t) dy + \frac{\partial}{\partial x} \left(\int_f^\eta u(x, y, t)^2 dy + \cos \alpha \frac{g}{2} (h^2) \right) = -gh \sin(\alpha) - \cos \alpha g h \frac{d}{dx} f - \mu(I(0)) gh \cos(\alpha).$$



Saint-Venant Savage Hutter

$$\frac{\partial h}{\partial t} + \frac{\partial(Q)}{\partial x} = 0 \quad \frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{5Q^2}{4h} + \frac{g}{2}(h^2) \right) = -gh\mu(I) \frac{Q}{|Q|}$$

Integral over the layer of grains



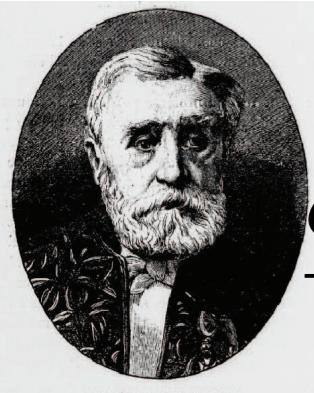
Saint-Venant Savage Hutter *Gerris*

$$\frac{\partial h}{\partial t} + \frac{\partial(Q)}{\partial x} = 0 \quad \frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{5Q^2}{4h} + \frac{g}{2}(h^2)\right) = -gh\mu(I)\frac{Q}{|Q|}$$

Gerris is a free finite volume code by Stéphane Popinet
one part of the code is a Shallow Water solver

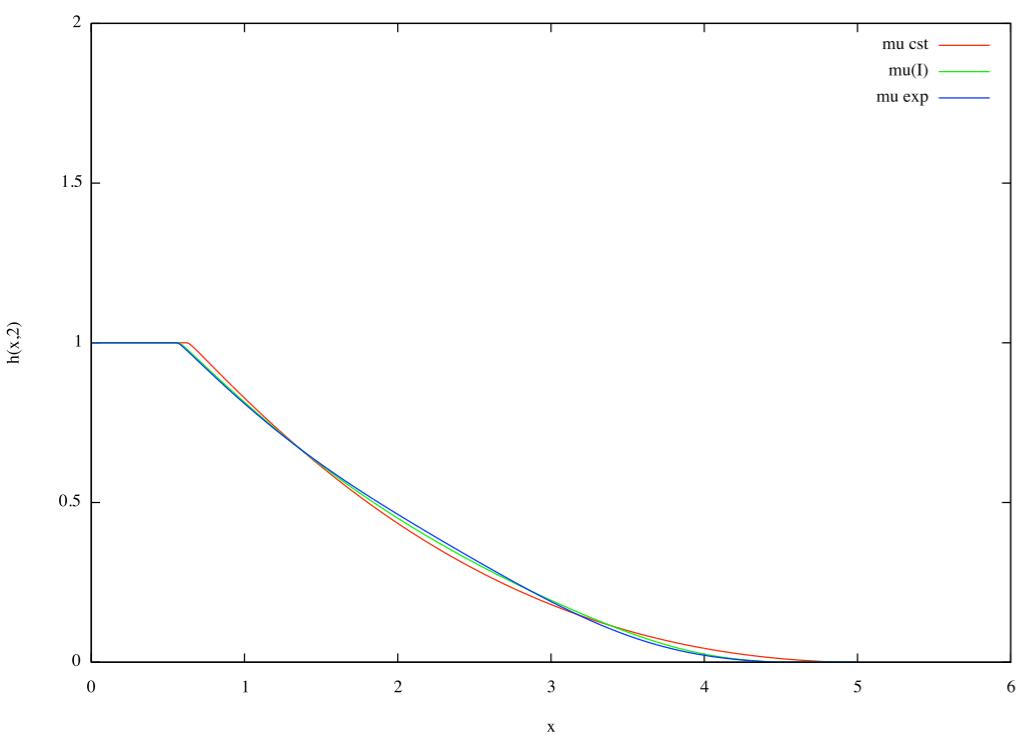
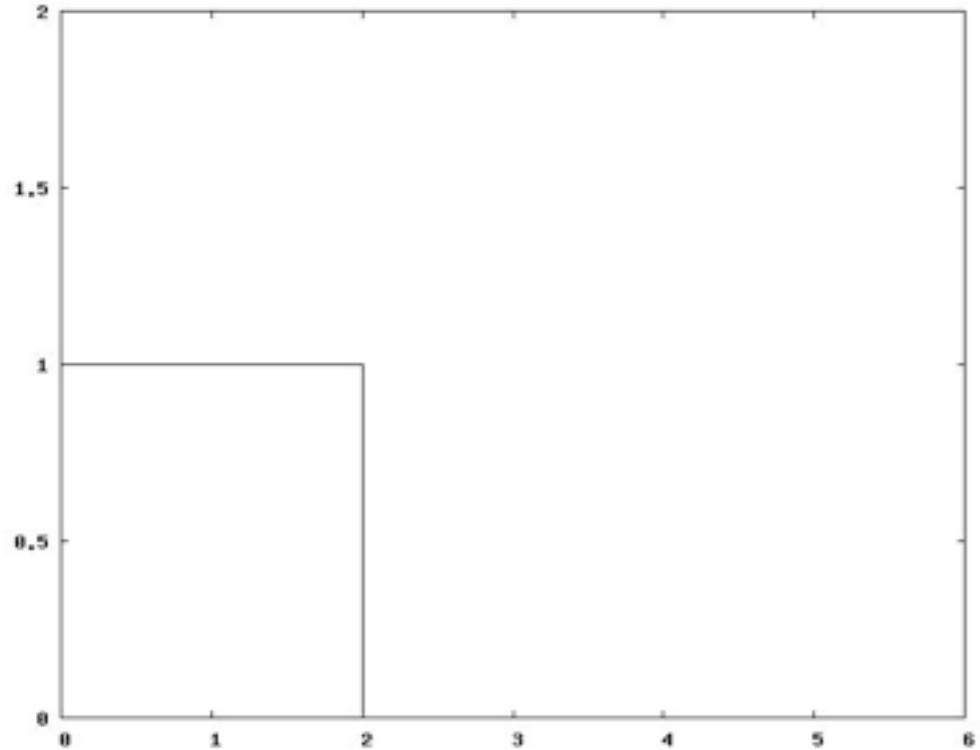
$$\frac{Q^* - Q^n}{\Delta t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{h} + \frac{g}{2}(h^2)\right) = 0 \quad \frac{Q^{n+1} - Q^*}{\Delta t} = -gh^*\mu(I^*)\frac{Q^{n+1}}{|Q^*|}$$

Audusse et al.



Saint-Venant Savage Hutter *Gerris*

$$\frac{\partial h}{\partial t} + \frac{\partial(Q)}{\partial x} = 0 \quad \frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{5Q^2}{4h} + \frac{g}{2}(h^2) \right) = -gh\mu(I) \frac{Q}{|Q|}$$



$$\mu(I) = \mu_s$$

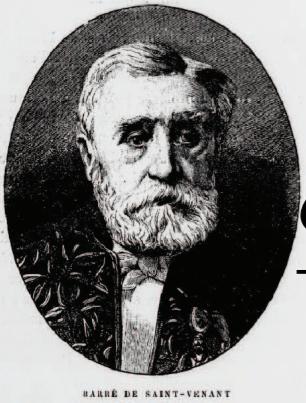
$$\mu(I) = \mu_s + \frac{\Delta\mu}{\frac{I_0}{I} + 1}$$

$$\mu(I) = \mu_s + \Delta\mu e^{-\beta/I}$$

$$\mu = 0.45$$

$$\mu = (0.4 + 0.26/(0.4/\ln + 1))$$

$$\mu = (0.4 + 0.26 * \exp(-0.136/\ln))$$

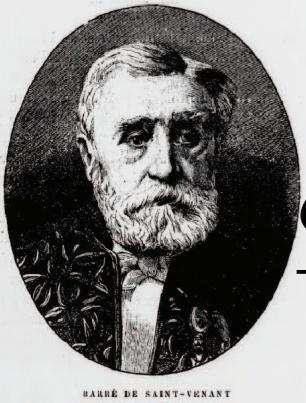


Saint-Venant Savage Hutter *Gerris*

$$\frac{\partial h}{\partial t} + \frac{\partial(Q)}{\partial x} = 0 \quad \frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{5Q^2}{4h} + \frac{g}{2}(h^2) \right) = -gh\mu(I) \frac{Q}{|Q|}$$

valid by hypothesis for small aspect ratio

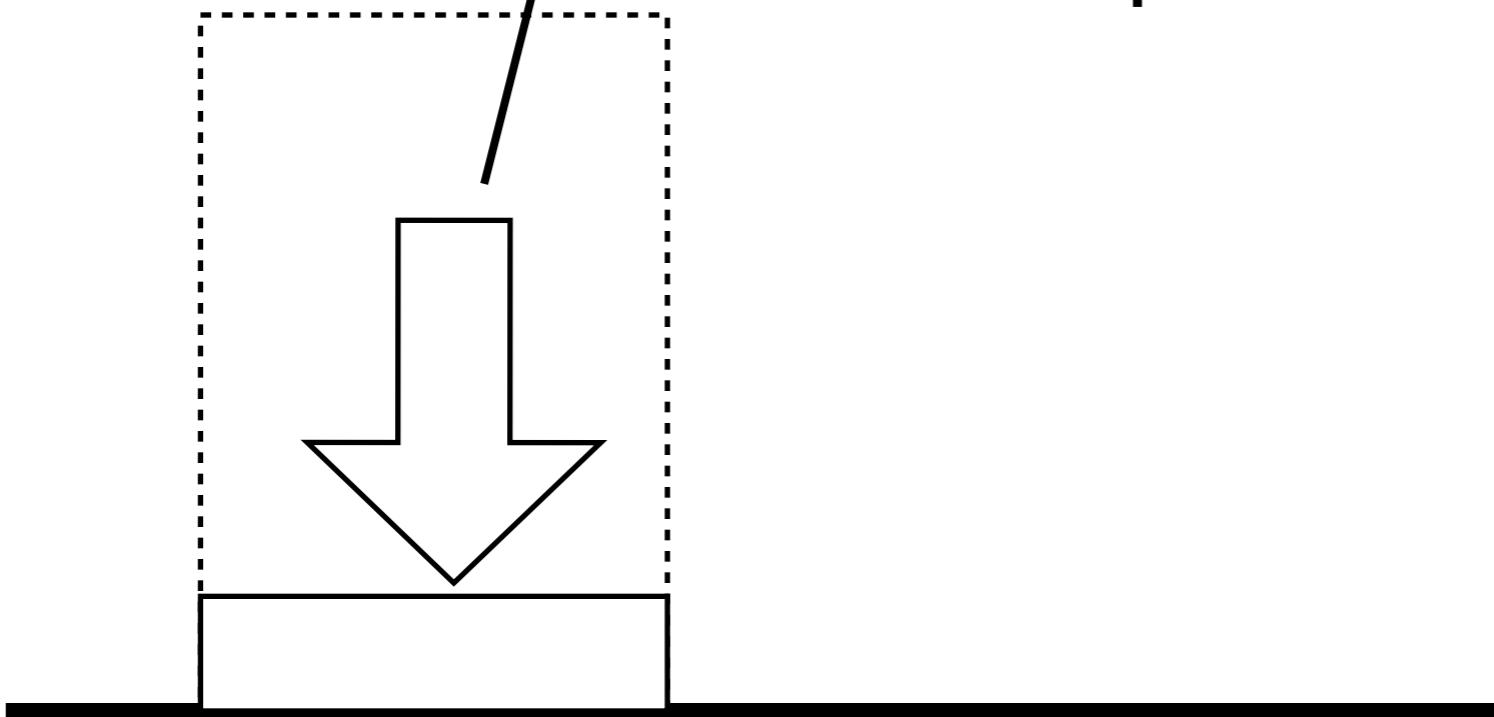




Saint-Venant Savage Hutter *Gerris*

$$\frac{\partial h}{\partial t} + \frac{\partial(Q)}{\partial x} = 0 \quad \frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{5Q^2}{4h} + \frac{g}{2}(h^2)\right) = -gh\mu(I)\frac{Q}{|Q|}$$

add a source term corresponding
to pluviation

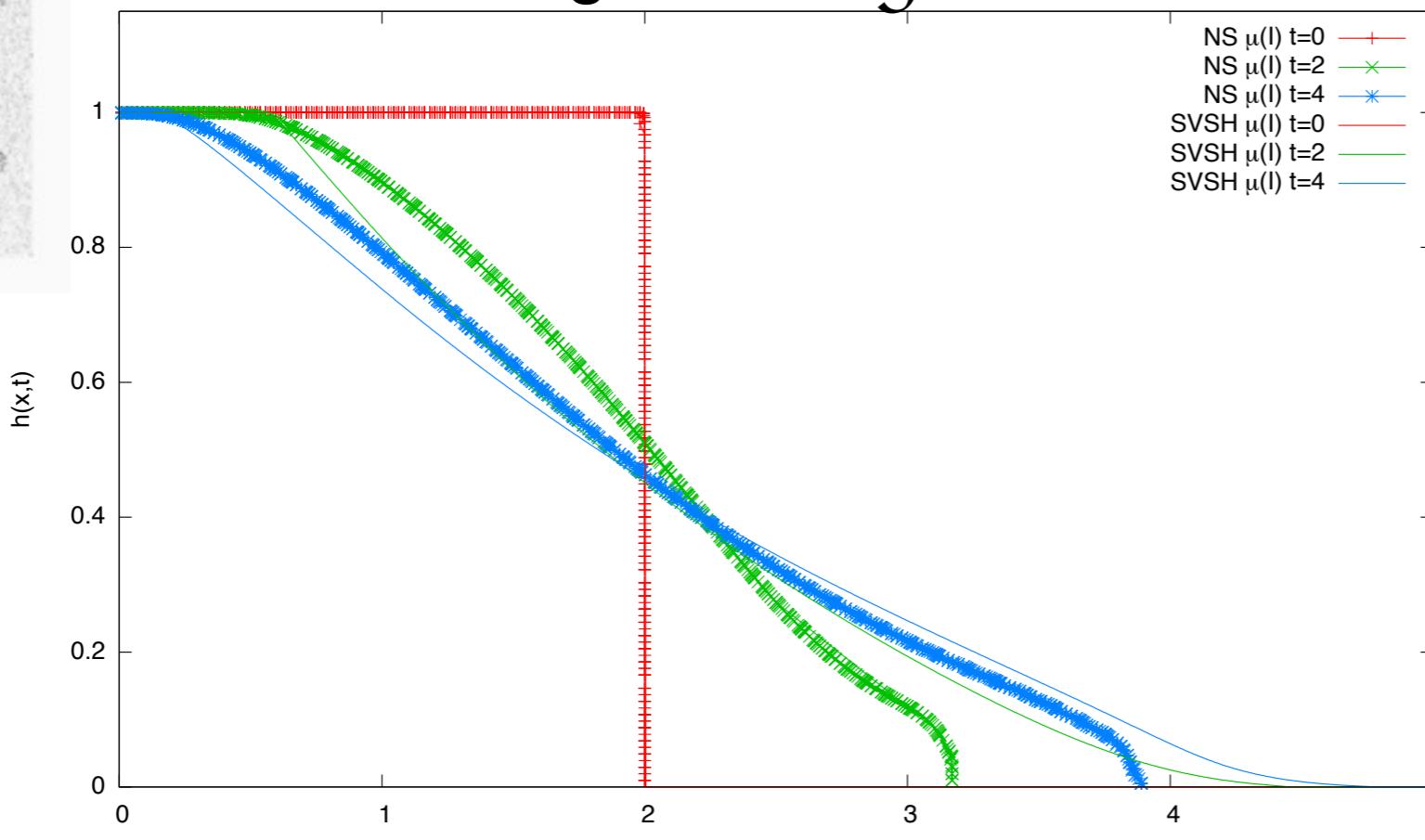




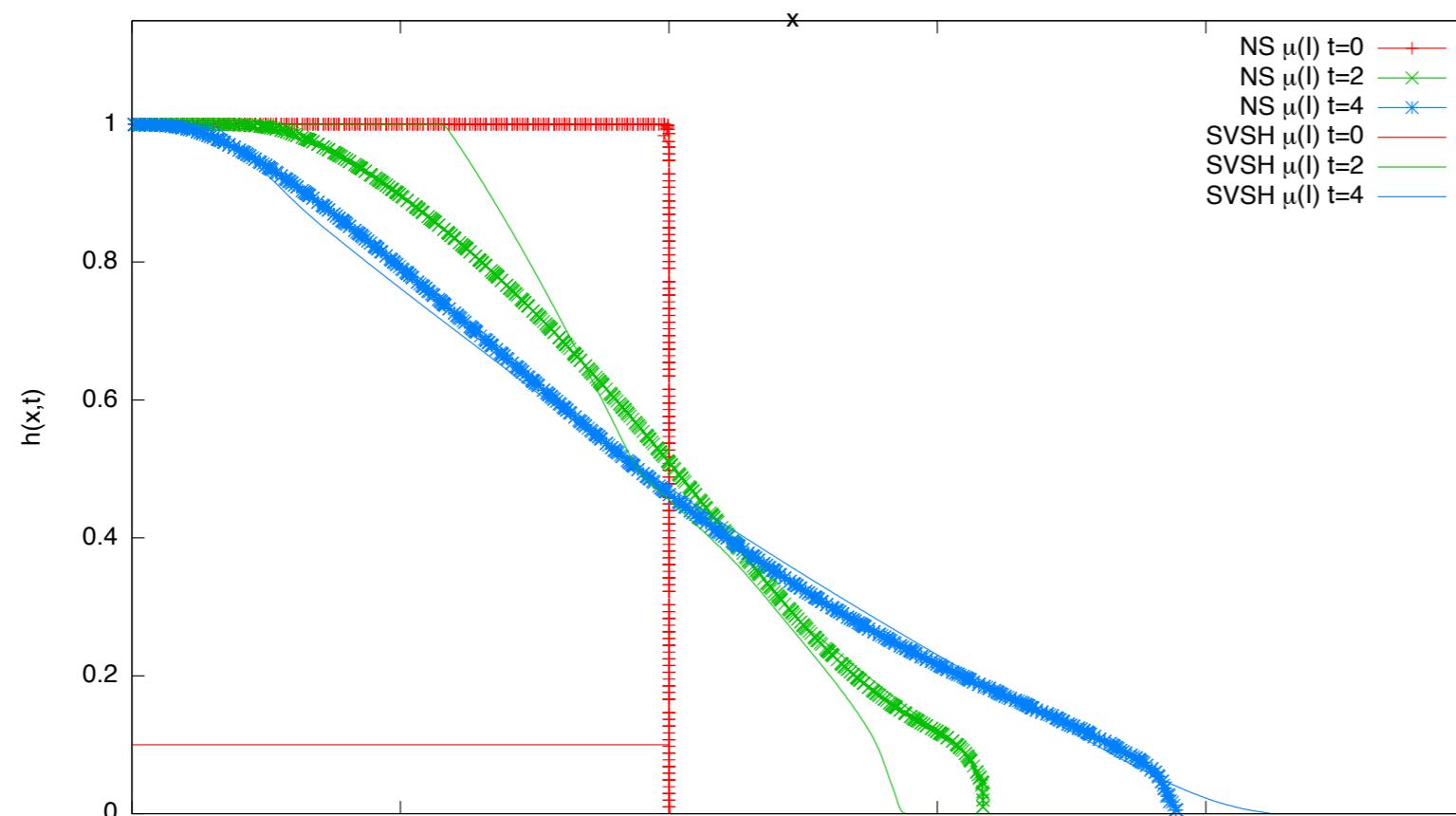
Saint-Venant Savage Hutter *Gerris*

better description of the run-out

no pluviation



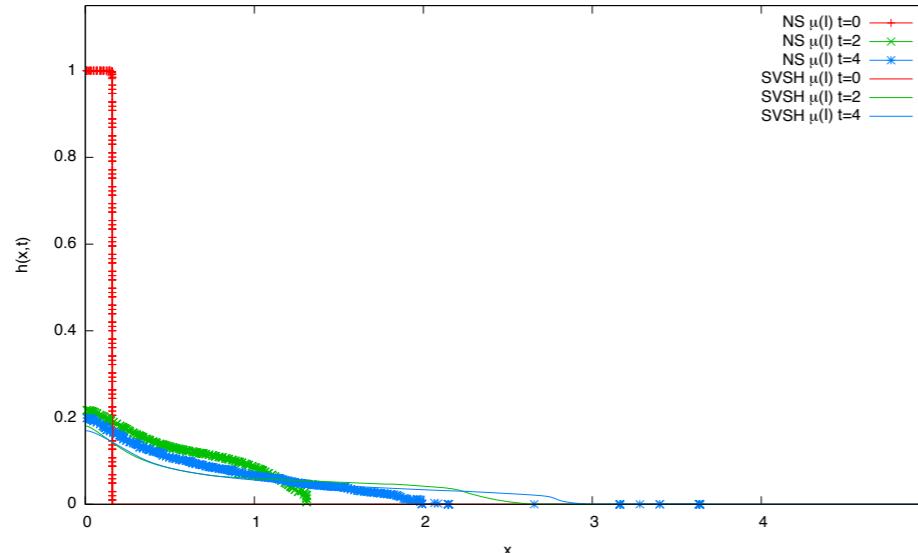
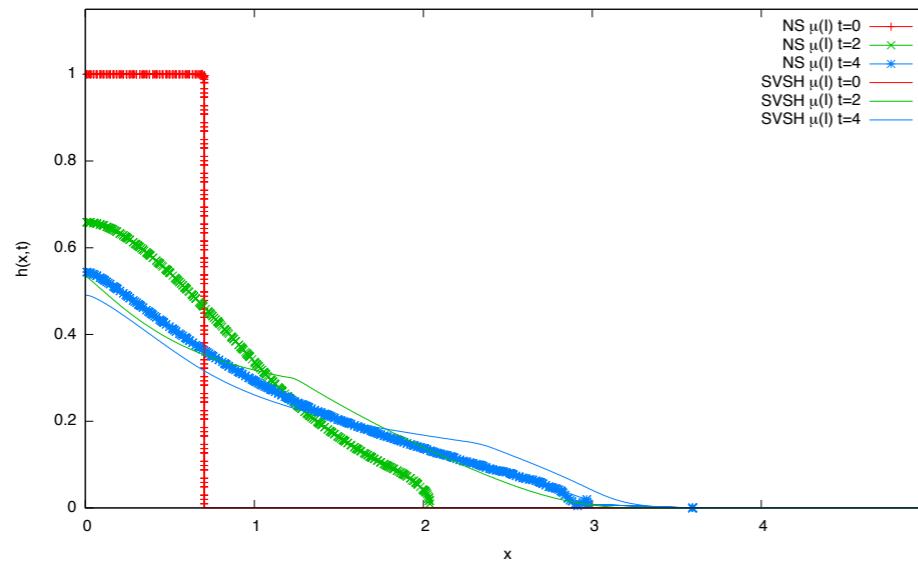
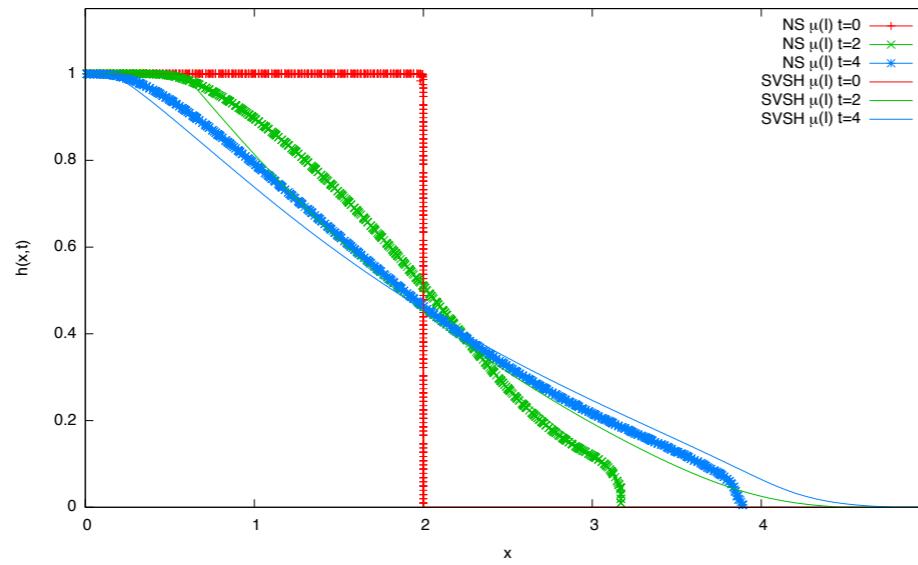
pluviation



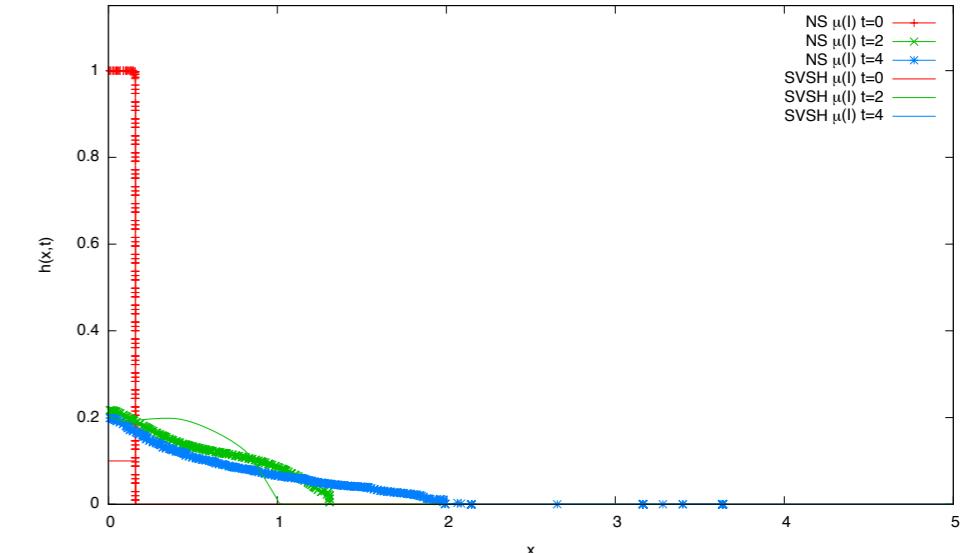
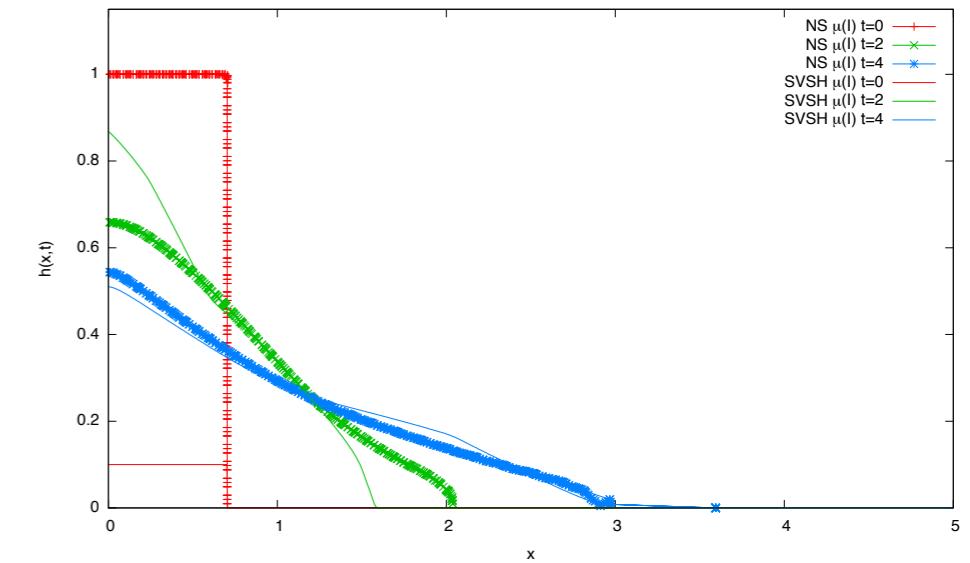
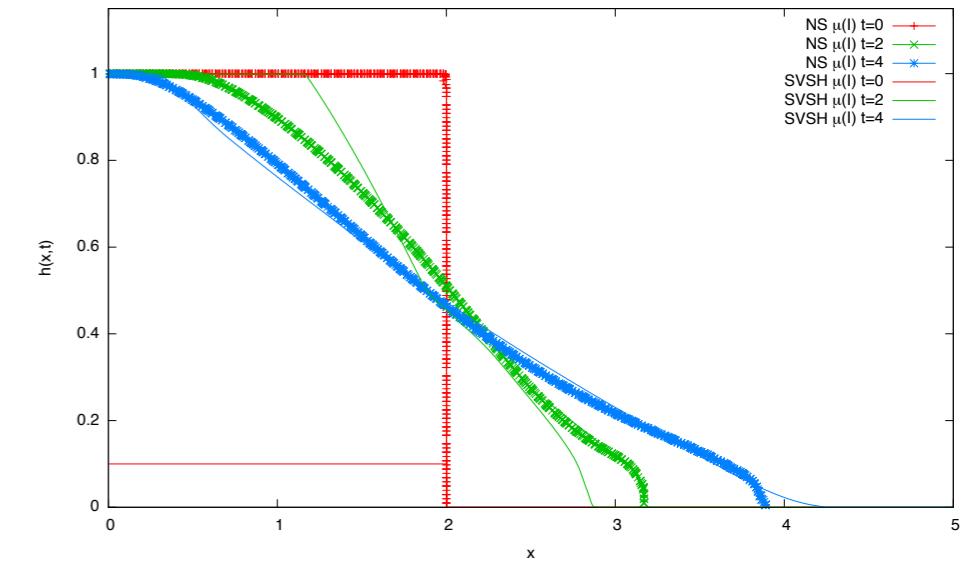


Saint-Venant Savage Hutter *Gerris*

no pluviation



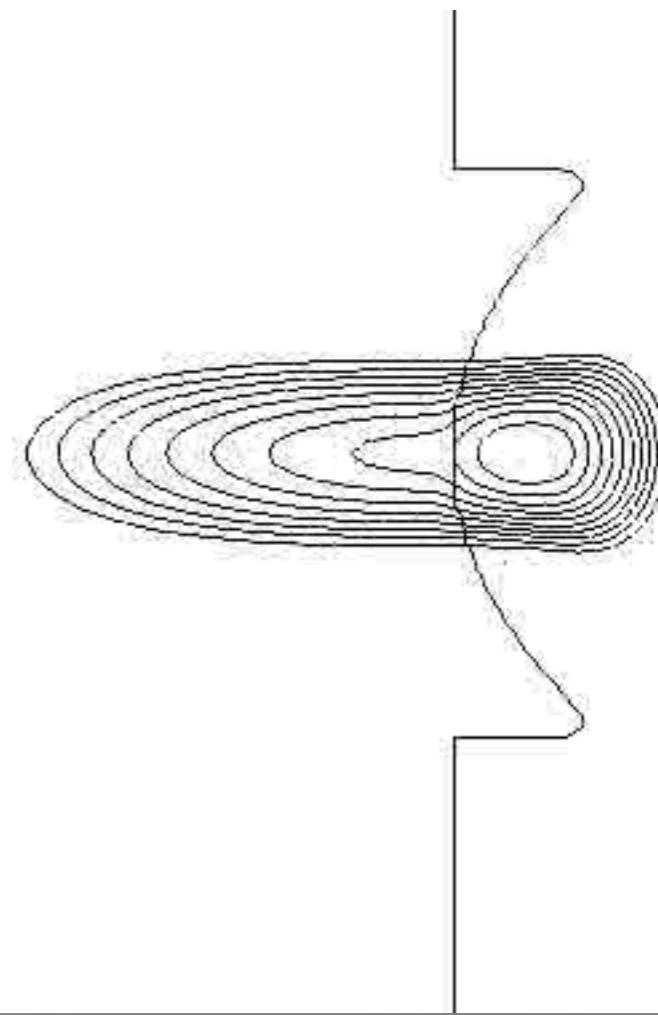
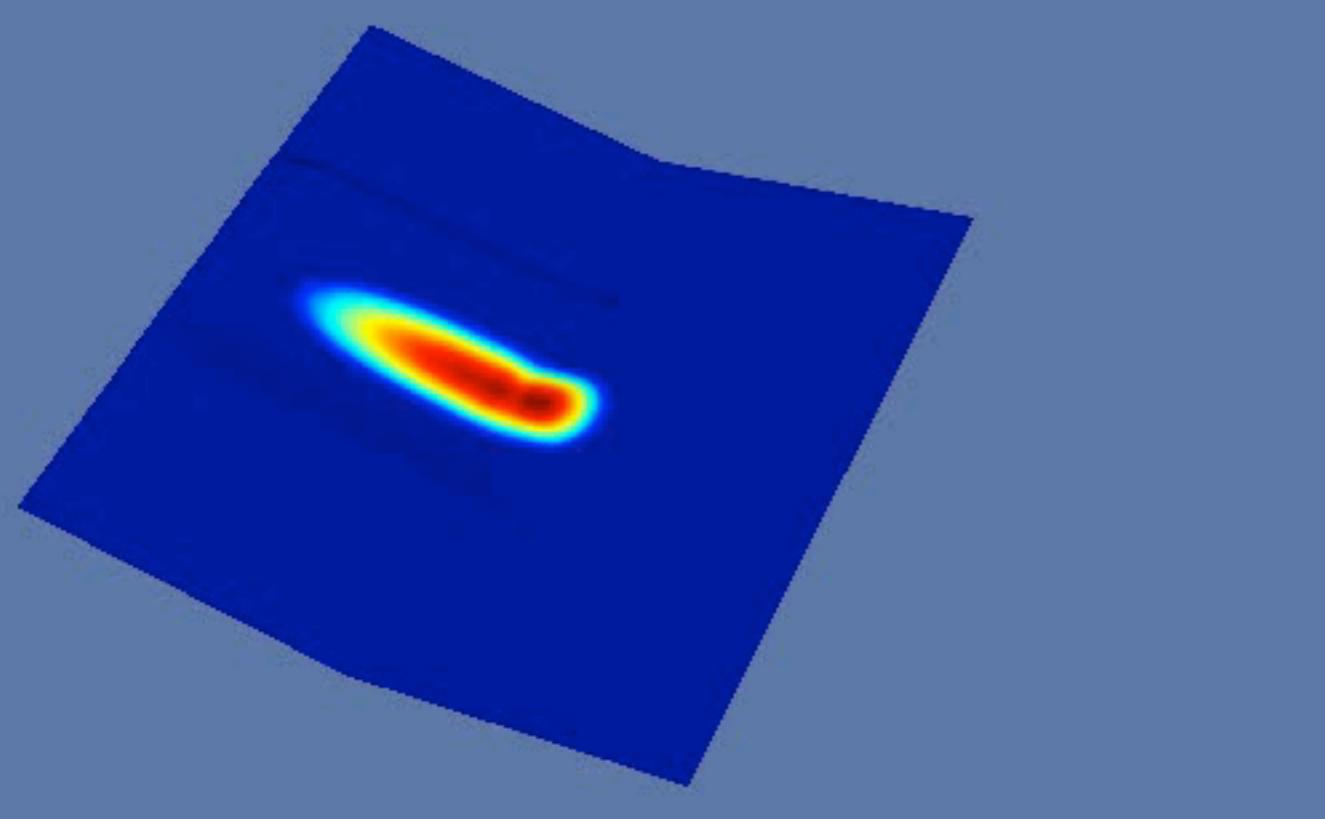
pluviation



better description of the run-out



Saint-Venant Savage Hutter *Gerris*



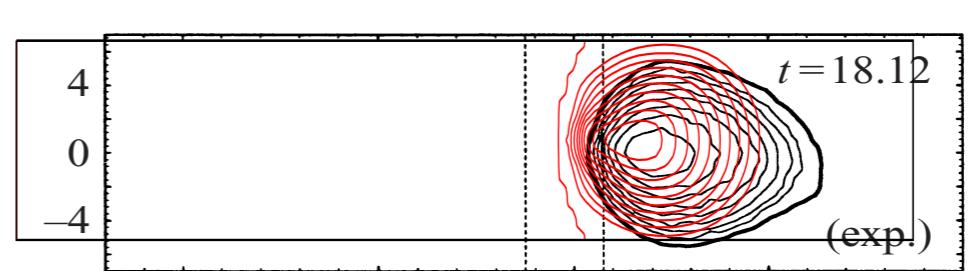
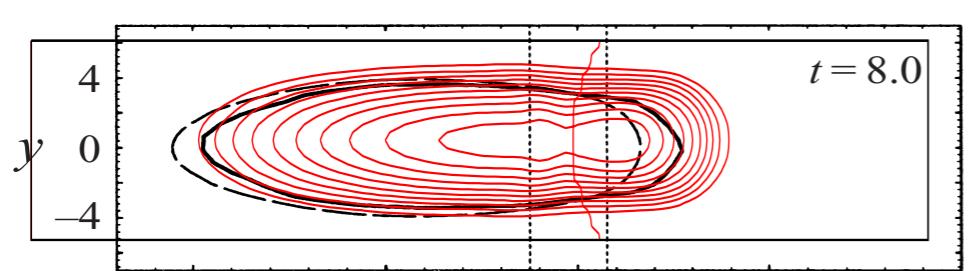
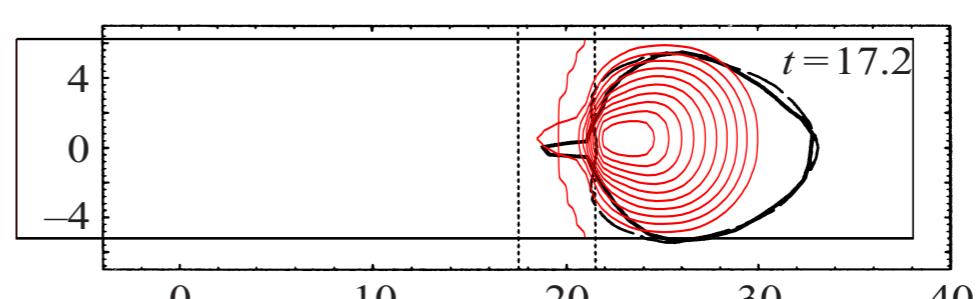
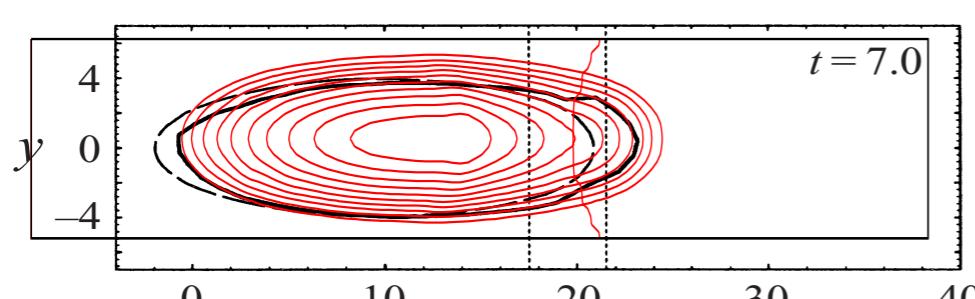
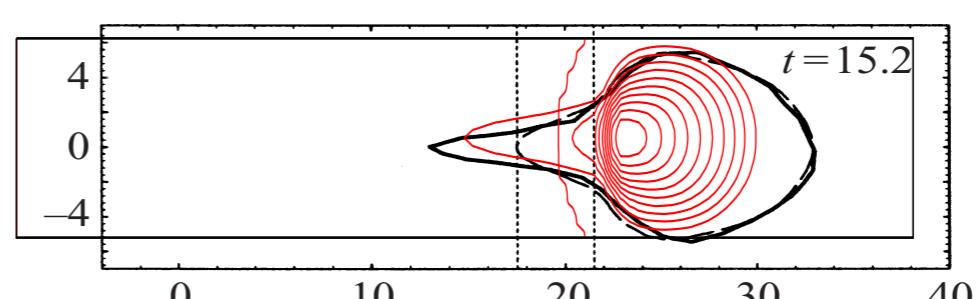
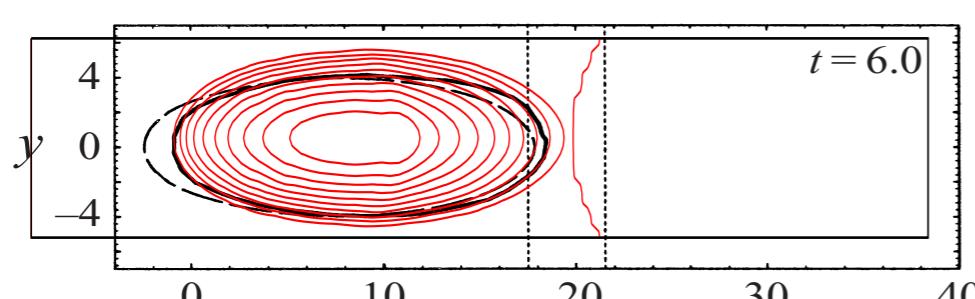
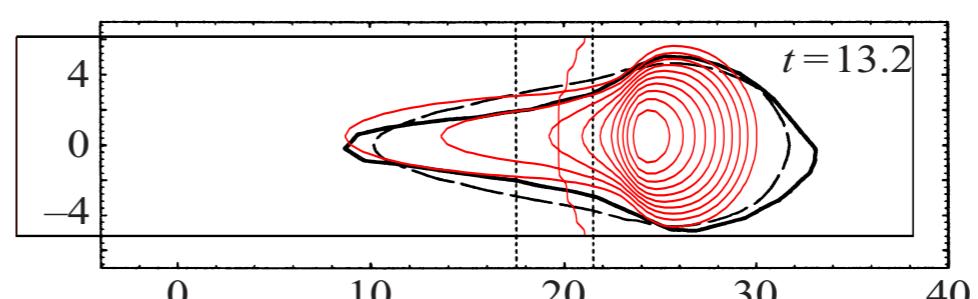
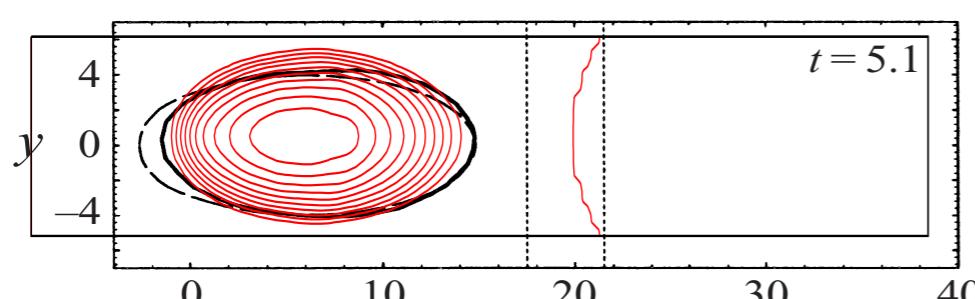
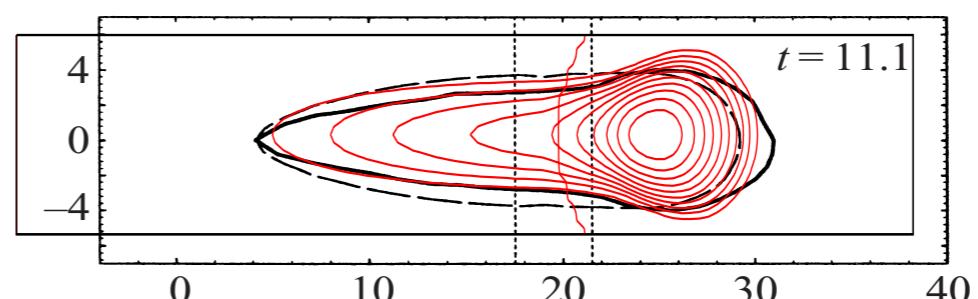
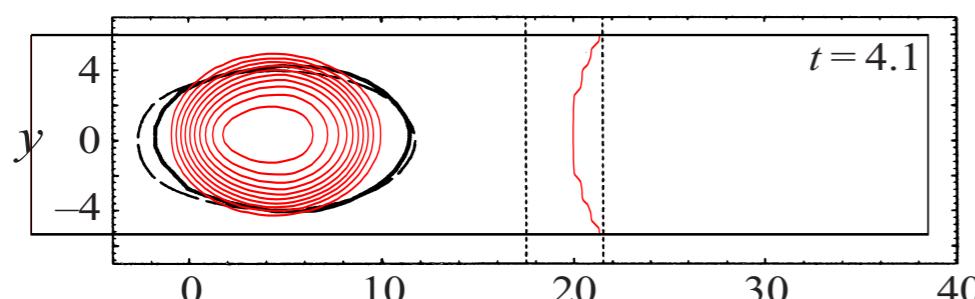
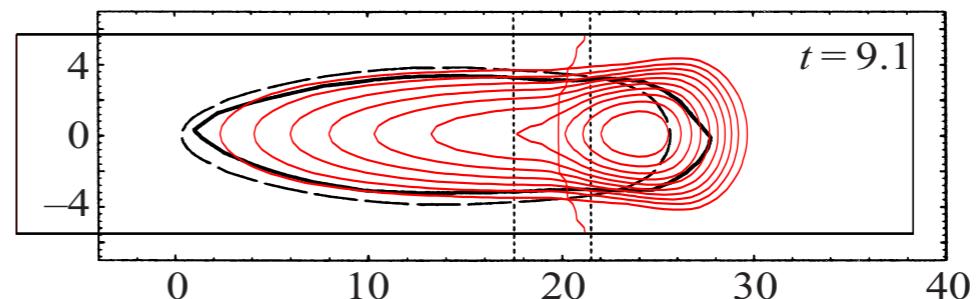
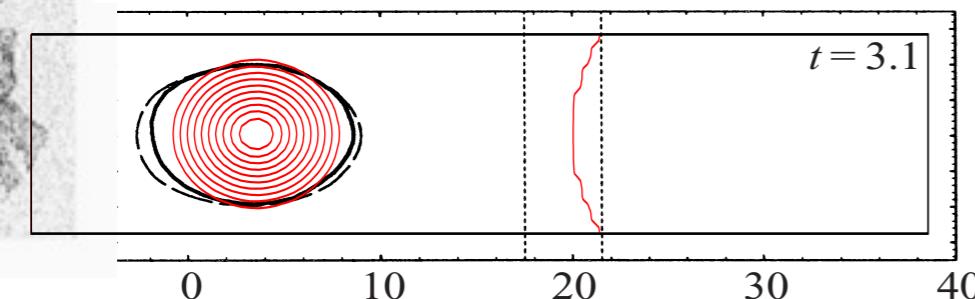


Saint-Venant Savage Hutter

Gerris

WIELAND, J. M. N. T. GRAY AND K. HUTTER 1999

Marina Pirulli, Marie-Odile Bristeau Anne Mangeney, Claudio Scavia 2006



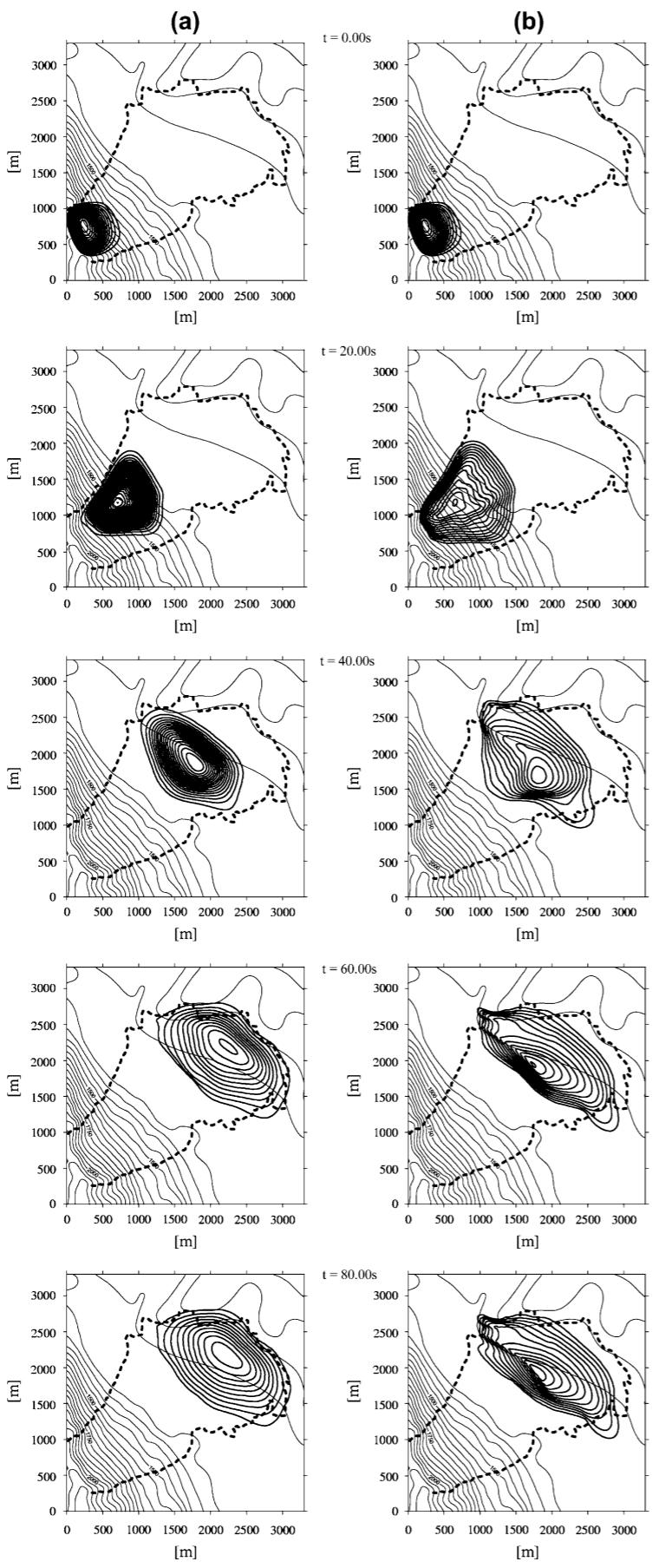


Fig. 24. Analysis of the Frank slide with RASH^{3D}. Plan of the simulated flow position at 20 s intervals (0 s–20 s–40 s–60 s–80 s) in condition of (a) anisotropy and (b) isotropy of normal stresses. The flow depth contours are at 3 m intervals. The sliding surface contours are at 50 m intervals. The dashed line indicated the extent of the real event.

$H \times L = 1000\text{m} \times 2500\text{m}$

150m initial 16 m for the train





E10001

LUCAS ET AL.: MARTIAN LANDSLIDES SCAR AND DYNAMICS

E10001

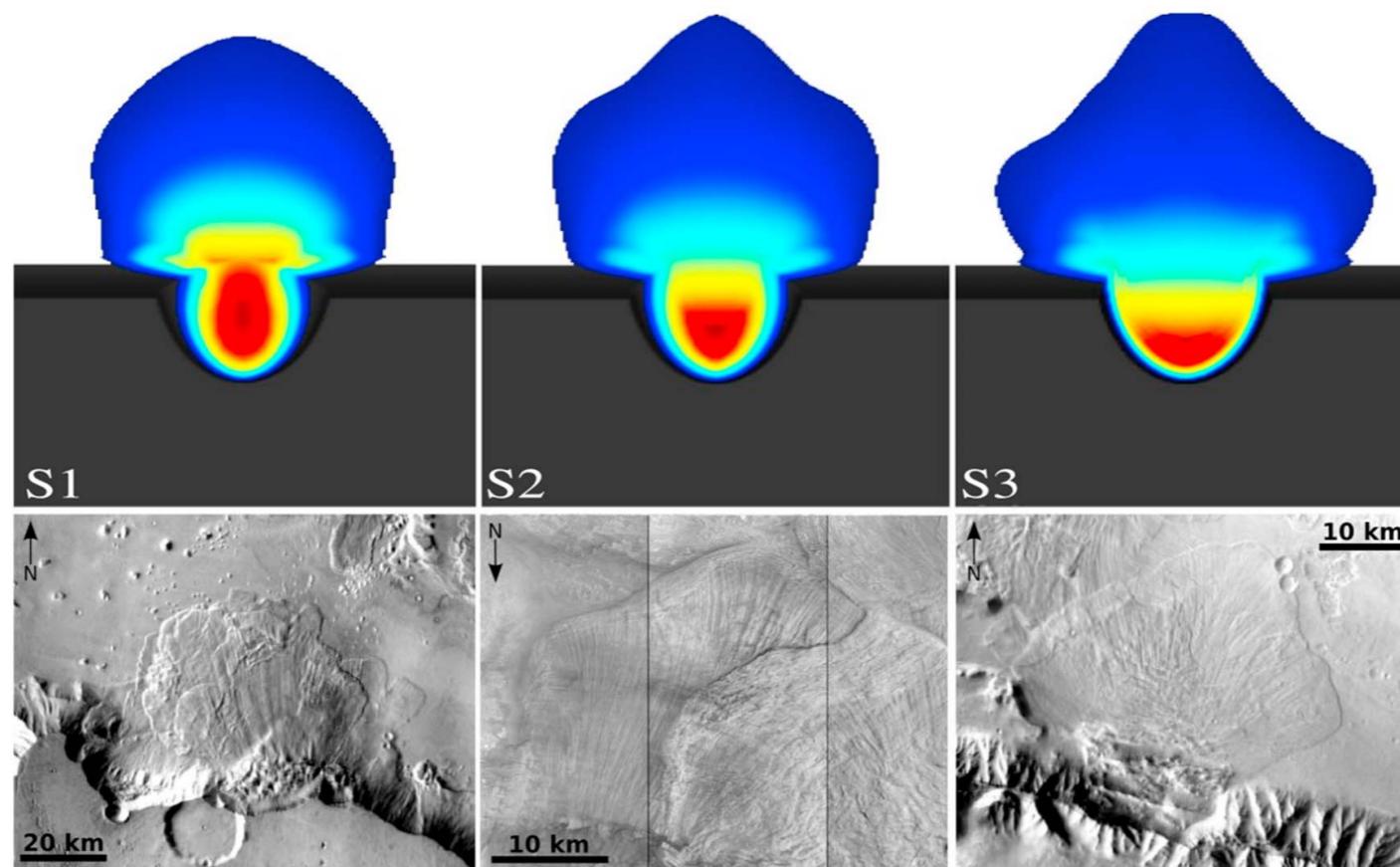
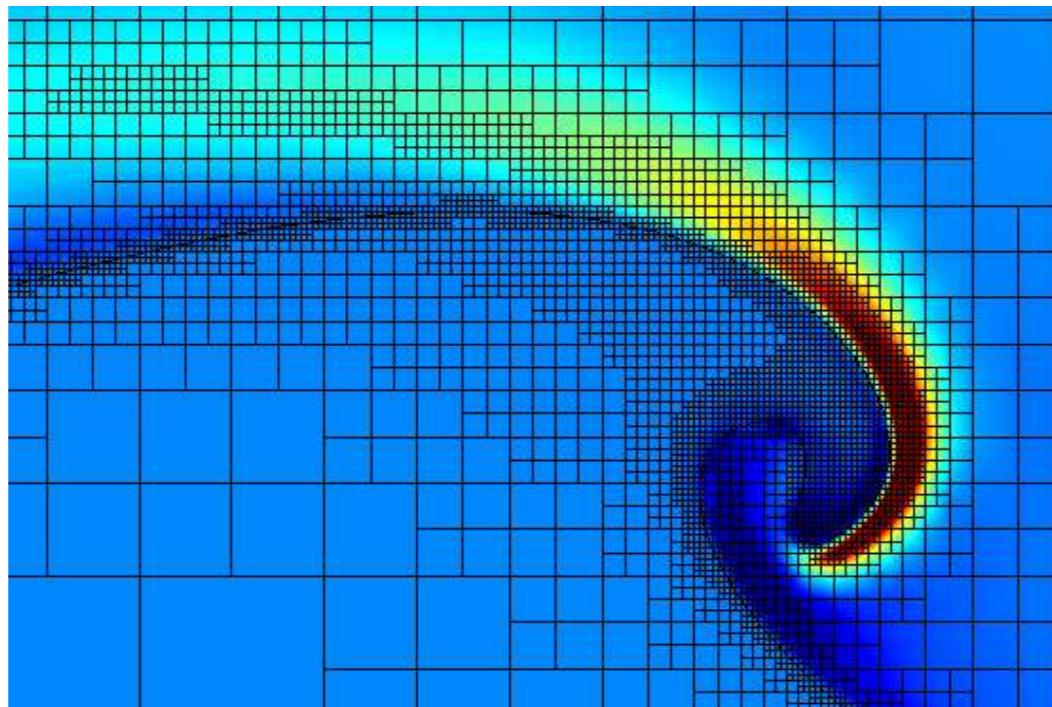
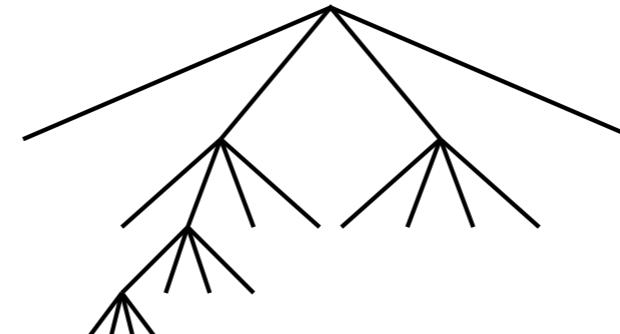
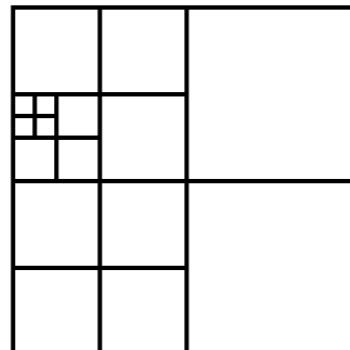


Figure 6. (top) Deposits of simulated landslides obtained for each scar geometry S_i using $\delta = 10^\circ$. (bottom) Martian landslides observed on THEMIS IR present similar deposit shapes, respectively, from left to right: Ganges Chasma, East Ius Chasma, and Coprates Chasma.



outline

- what is a granular fluid? some images
- the $\mu(I)$ friction law obtained from experiments and discrete simulation
- the viscosity associated to the $\mu(I)$ friction law
- the Saint Venant Savage Hutter Hyperbolic model
- implementing the $\mu(I)$ friction law in Navier Stokes
- Examples of flows: focusing on the granular column collapse (limits of Saint Venant Savage Hutter Hyperbolic model)



- *Gerris* is a finite volume code by Stéphane Popinet NIWA
one part of the code is a Navier Stokes solver
- automatic mesh adaptation
- Volume Of Fluid method for two phase flows
- free on sourceforge



rheology; defining a viscosity

$$\mu(I) = \mu_1 + \frac{\mu_2 - \mu_1}{I_0/I + 1}$$

$$\eta \frac{\partial u}{\partial y} = \mu(I) P$$

local equilibrium

$$\eta = \frac{\mu \left(\frac{d \frac{\partial u}{\partial y}}{\sqrt{P/\rho}} \right) P}{\frac{\partial u}{\partial y}}$$

construction of a viscosity



implementation in *Gerris* flow solver?

$$\mu(I) = \mu_1 + \frac{\mu_2 - \mu_1}{I_0/I + 1}$$

$$D_2 = \sqrt{D_{ij}D_{ij}} \quad D_{ij} = \frac{u_{i,j} + u_{j,i}}{2}$$

construction of a viscosity based on the D_2 invariant and redefinition of I

$$\eta = \min(\eta_{max}, \max\left(\frac{\mu(I)}{\sqrt{2}D_2} p, 0\right)) \quad I = d\sqrt{2}D_2/\sqrt(|p|/\rho).$$

- the «min» limits viscosity to a large value
- always flow, even slow

Boundary Conditions: no slip and $P=0$ at the interface



implementation in *Gerris* flow solver?

$$\mu(I) = \mu_1 + \frac{\mu_2 - \mu_1}{I_0/I + 1}$$

$$D_2 = \sqrt{D_{ij}D_{ij}} \quad D_{ij} = \frac{u_{i,j} + u_{j,i}}{2}$$

construction of a viscosity based on the D_2 invariant and redefinition of I

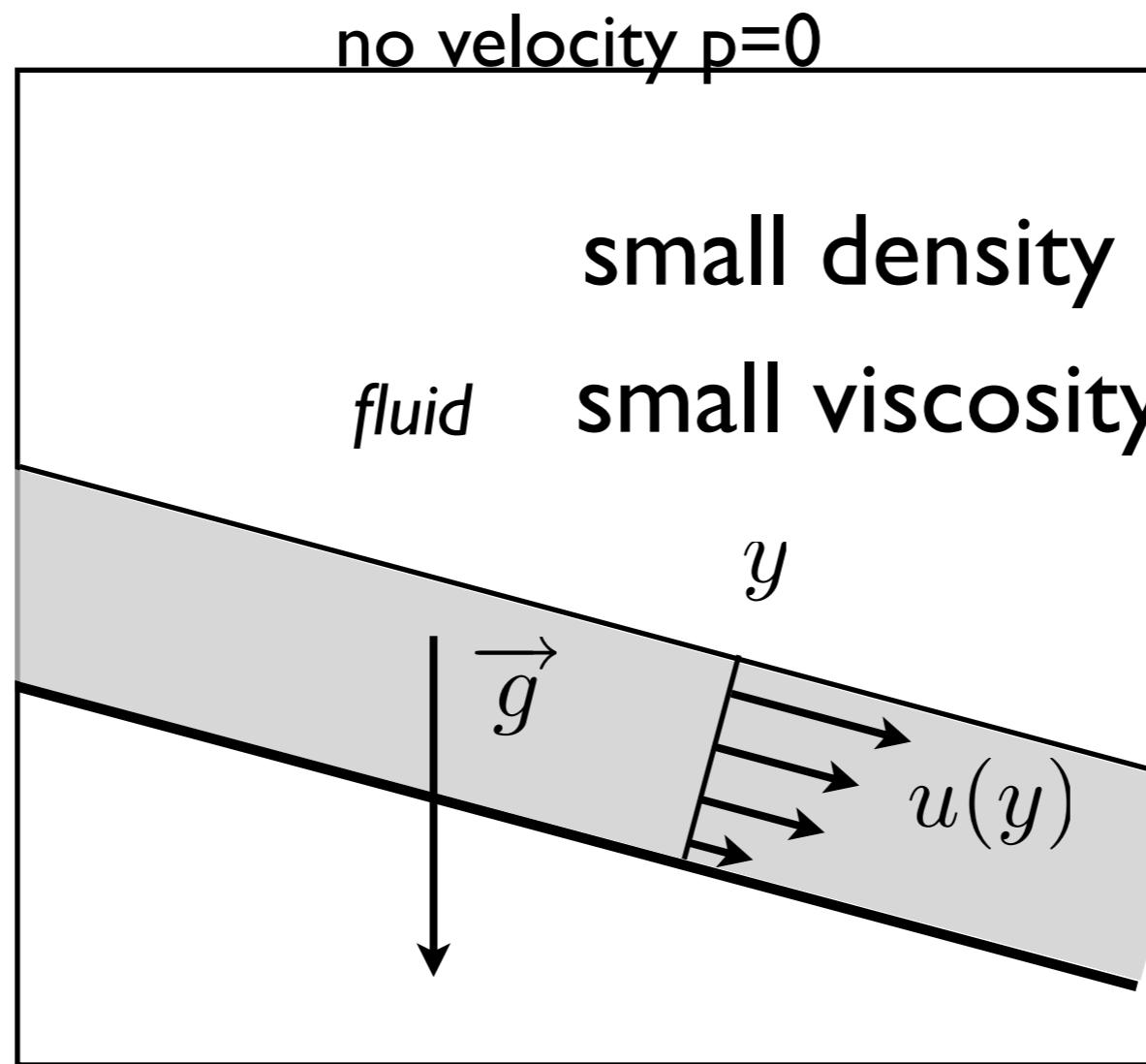
$$\eta = \min(\eta_{max}, \max\left(\frac{\mu(I)}{\sqrt{2}D_2}p, 0\right)) \quad I = d\sqrt{2}D_2/\sqrt(|p|/\rho).$$

$$\nabla \cdot \mathbf{u} = 0, \quad \rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \nabla \cdot (2\eta \mathbf{D}) + \rho g,$$

$$\frac{\partial c}{\partial t} + \nabla \cdot (c\mathbf{u}) = 0, \quad \rho = c\rho_1 + (1-c)\rho_2, \quad \eta = c\eta_1 + (1-c)\eta_2$$

The granular fluid is covered by a passive light fluid (it allows for a zero pressure boundary condition at the surface, bypassing an up to now difficulty which was to impose this condition on a unknown moving boundary).

Boundary Conditions: no slip and $P=0$ at the top



$$\frac{\partial c}{\partial t} + \nabla \cdot (c\mathbf{u}) = 0, \quad \rho = c\rho_1 + (1 - c)\rho_2, \quad \eta = c\eta_1 + (1 - c)\eta_2$$

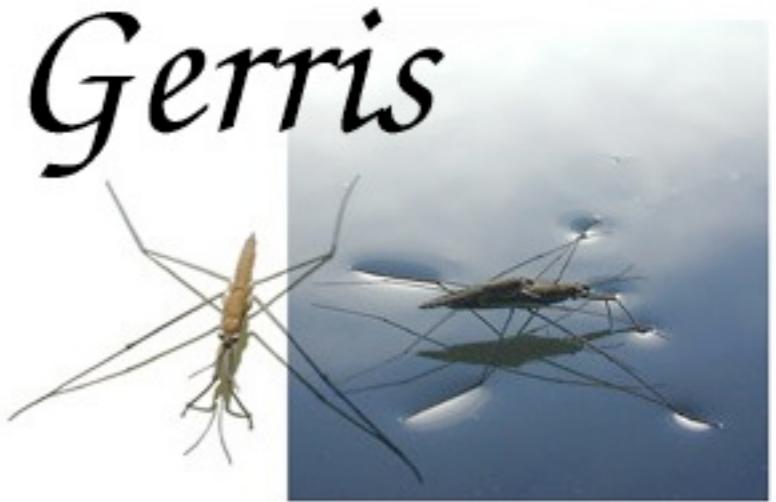
The granular fluid is covered by a passive light fluid (it allows for a zero pressure boundary condition at the surface, bypassing an up to now difficulty which was to impose this condition on a unknown moving boundary).

Boundary Conditions: no slip and $P=0$ at the top



Projection Method

$$\begin{aligned}\rho_{n+\frac{1}{2}} \left(\frac{\mathbf{u}_* - \mathbf{u}_n}{\Delta t} + \mathbf{u}_{n+\frac{1}{2}} \cdot \nabla \mathbf{u}_{n+\frac{1}{2}} \right) &= \nabla \cdot (\eta_{n+\frac{1}{2}} \mathbf{D}_*) - \nabla p_{n-\frac{1}{2}}, \\ \mathbf{u}_{n+1} &= \mathbf{u}_* - \frac{\Delta t}{\rho_{n+\frac{1}{2}}} (\nabla p_{n+\frac{1}{2}} - \nabla p_{n-\frac{1}{2}}), \\ \nabla \cdot \mathbf{u}_{n+1} &= 0.\end{aligned}$$



multigrid solver for Laplacien of pressure

$$\nabla \cdot \left(\frac{\Delta t}{\rho_{n+\frac{1}{2}}} \nabla p_{n+\frac{1}{2}} \right) = \nabla \cdot \left(\mathbf{u}_* + \frac{\Delta t}{\rho_{n+\frac{1}{2}}} \nabla p_{n-\frac{1}{2}} \right)$$

implicit for \mathbf{u}^*

$$\frac{\rho_{n+\frac{1}{2}}}{\Delta t} \mathbf{u}_* - \frac{1}{2} \nabla \cdot (\eta_{n+\frac{1}{2}} \nabla \mathbf{u}_*) = \rho_{n+\frac{1}{2}} \left[\frac{\mathbf{u}_n}{\Delta t} - \mathbf{u}_{n+\frac{1}{2}} \cdot \nabla \mathbf{u}_{n+\frac{1}{2}} \right] - \nabla p_{n-\frac{1}{2}} + \frac{1}{2} \nabla \mathbf{u}_n^T \nabla \eta_{n+\frac{1}{2}}.$$

VOF reconstruction

$$\frac{c_{n+\frac{1}{2}} - c_{n-\frac{1}{2}}}{\Delta t} + \nabla \cdot (c_n \mathbf{u}_n) = 0$$

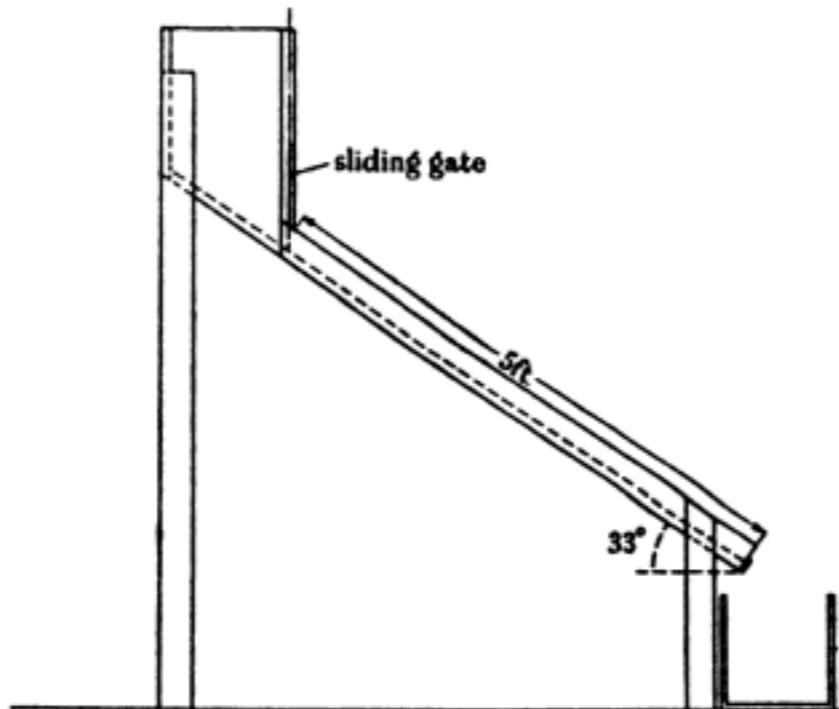


outline

- what is a granular fluid? some images
- the $\mu(l)$ friction law obtained from experiments and discrete simulation
- the viscosity associated to the $\mu(l)$ friction law
- the Saint Venant Savage Hutter Hyperbolic model
- implementing the $\mu(l)$ friction law in Navier Stokes
- **Examples of flows: focusing on the granular column collapse (limits of Saint Venant Savage Hutter Hyperbolic model)**



Test of the code: «Bagnold» avalanche



kind of Nußelt solution

$$T \propto \sigma(\lambda D)^2 (dU/dy)^2$$

$$U = \frac{2}{3} \times 0.165 (g \sin \beta)^{1/2} \frac{y'^{3/2}}{D},$$

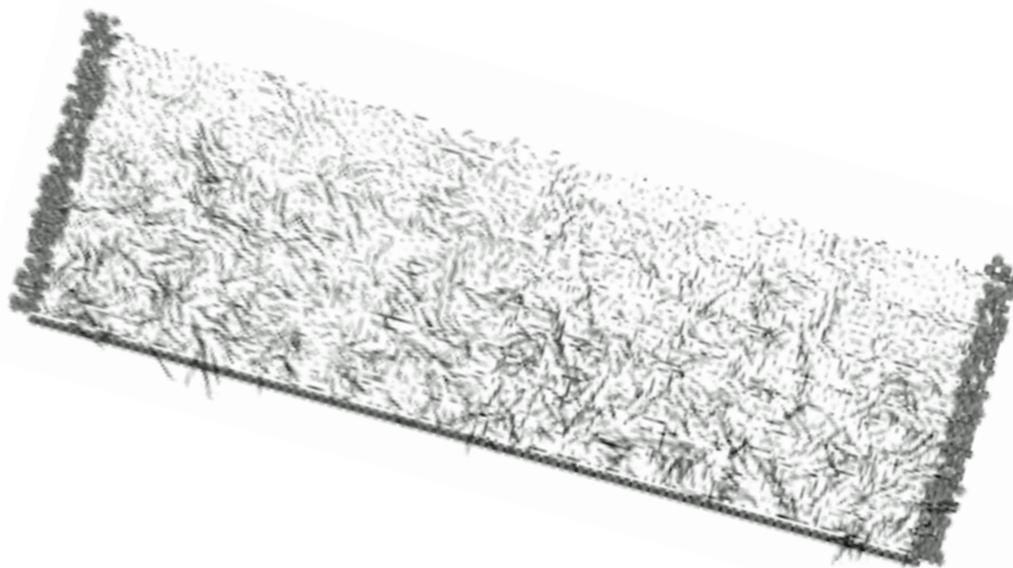
TABLE 1.

flow height Y (cm)	measured speed (cm/sec)	speed, from (9) (cm/sec)	ratio
0.5	17.2	26.4	1.53
0.65	27.5	38.8	1.41
0.75	30.0	48.0	1.6
0.9	39.0	63.0	1.61

Bagnold 1954



Test of the code: «Bagnold» avalanche

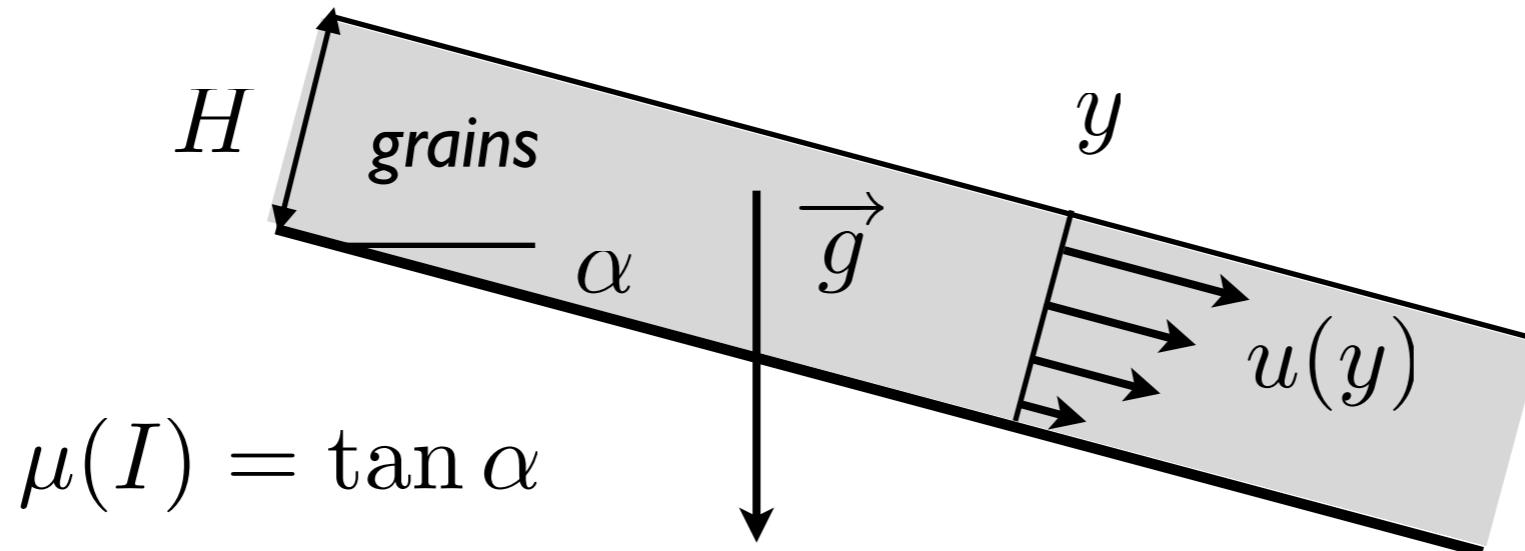


kind of Nußelt solution

Contact Dynamic
simulation Lydie Staron



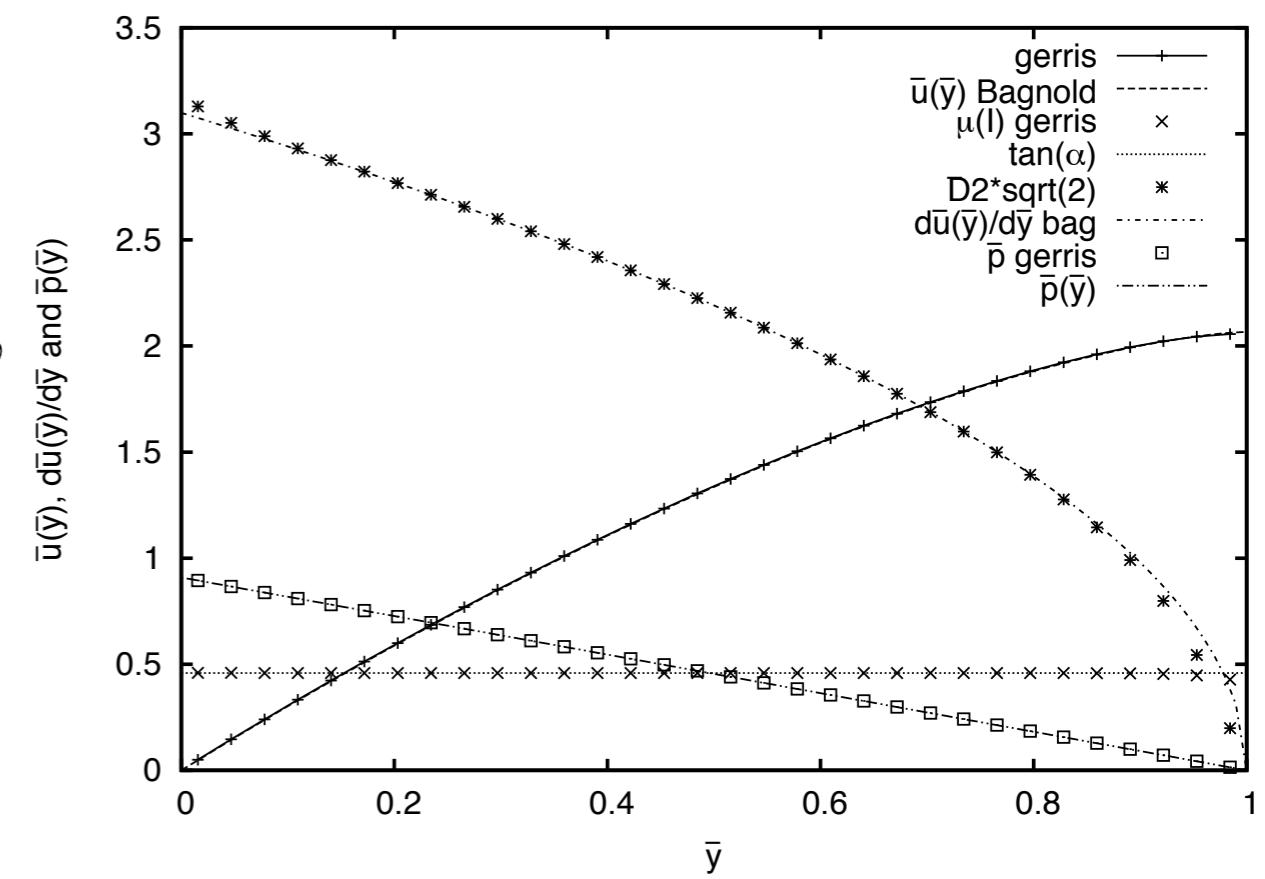
Test of the code: «Bagnold» avalanche



$$\mu(I) = \tan \alpha$$

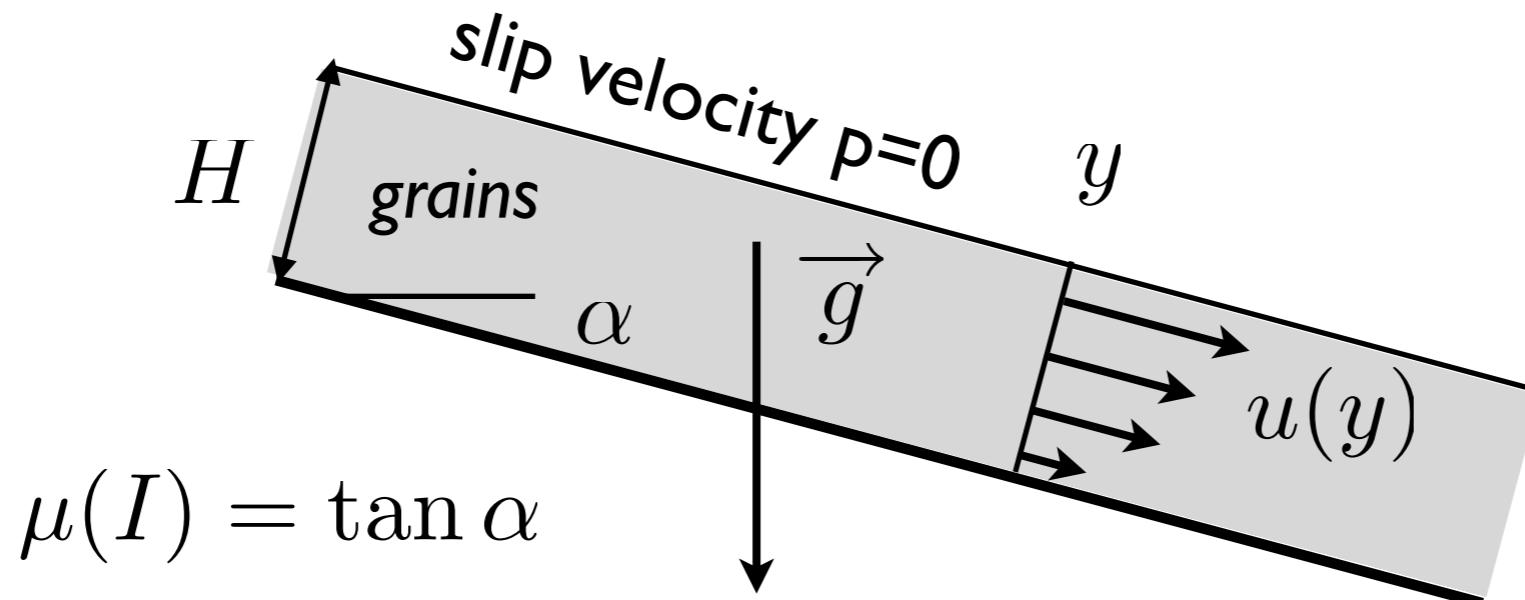
$$u = \frac{2}{3} I_\alpha \sqrt{gd \cos \alpha \frac{H^3}{d^3}} \left(1 - \left(1 - \frac{y}{H} \right)^{3/2} \right),$$

$$v = 0, \quad p = \rho g H \left(1 - \frac{y}{H} \right) \cos \alpha.$$





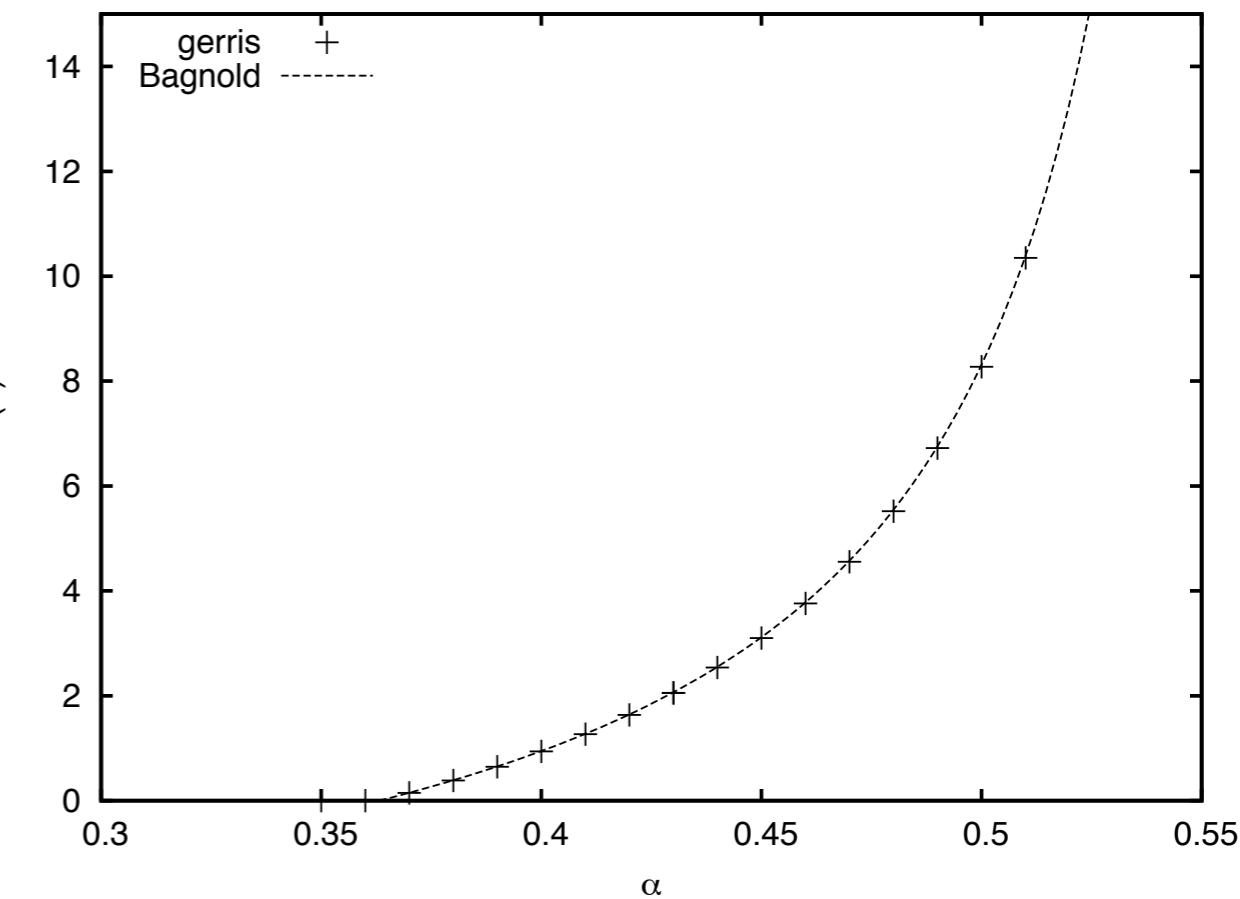
Test of the code: «Bagnold» avalanche



$$\mu(I) = \tan \alpha$$

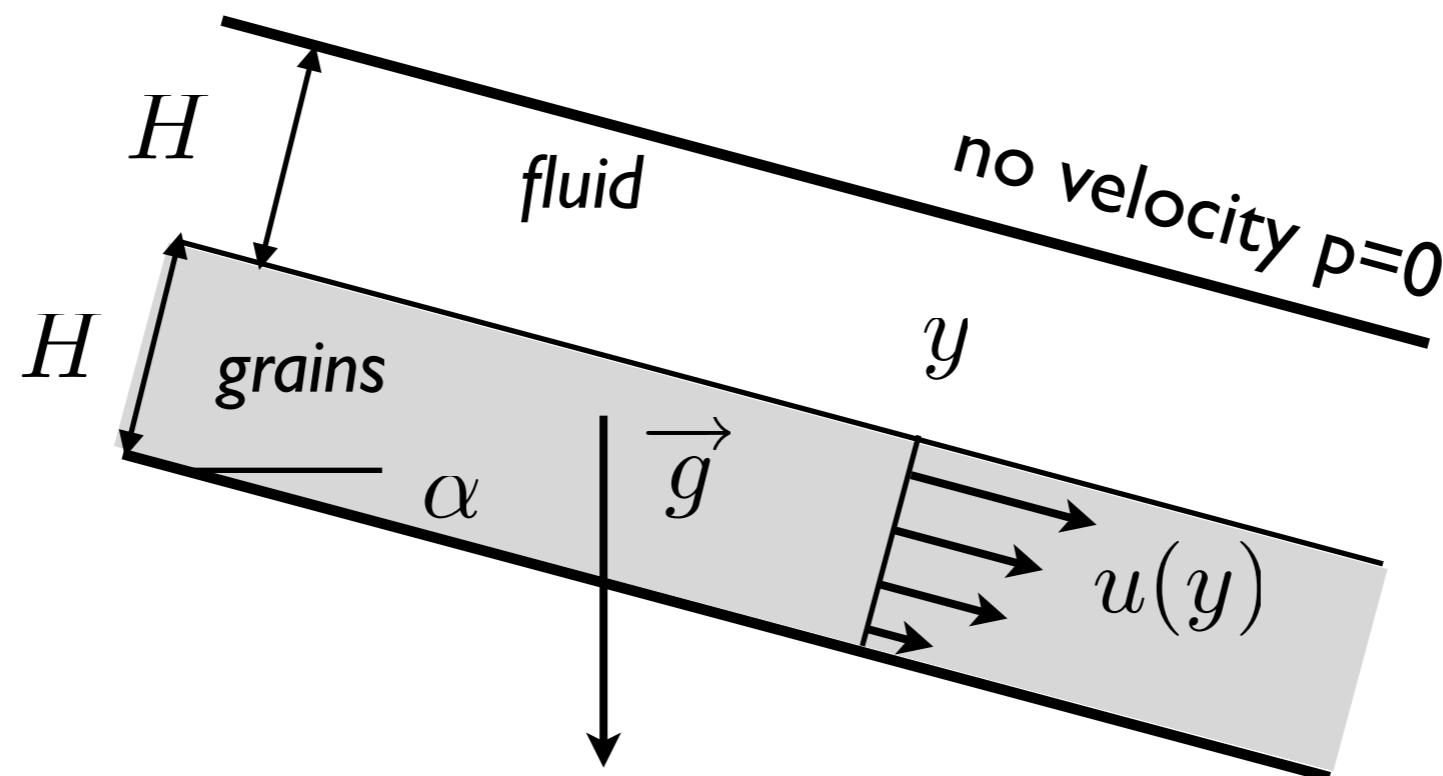
$$u = \frac{2}{3} I_\alpha \sqrt{gd \cos \alpha \frac{H^3}{d^3}} \left(1 - \left(1 - \frac{y}{H} \right)^{3/2} \right), \quad \bar{u}(1)$$

$$v = 0, \quad p = \rho g H \left(1 - \frac{y}{H} \right) \cos \alpha.$$





Test of the code: «Bagnold» avalanche



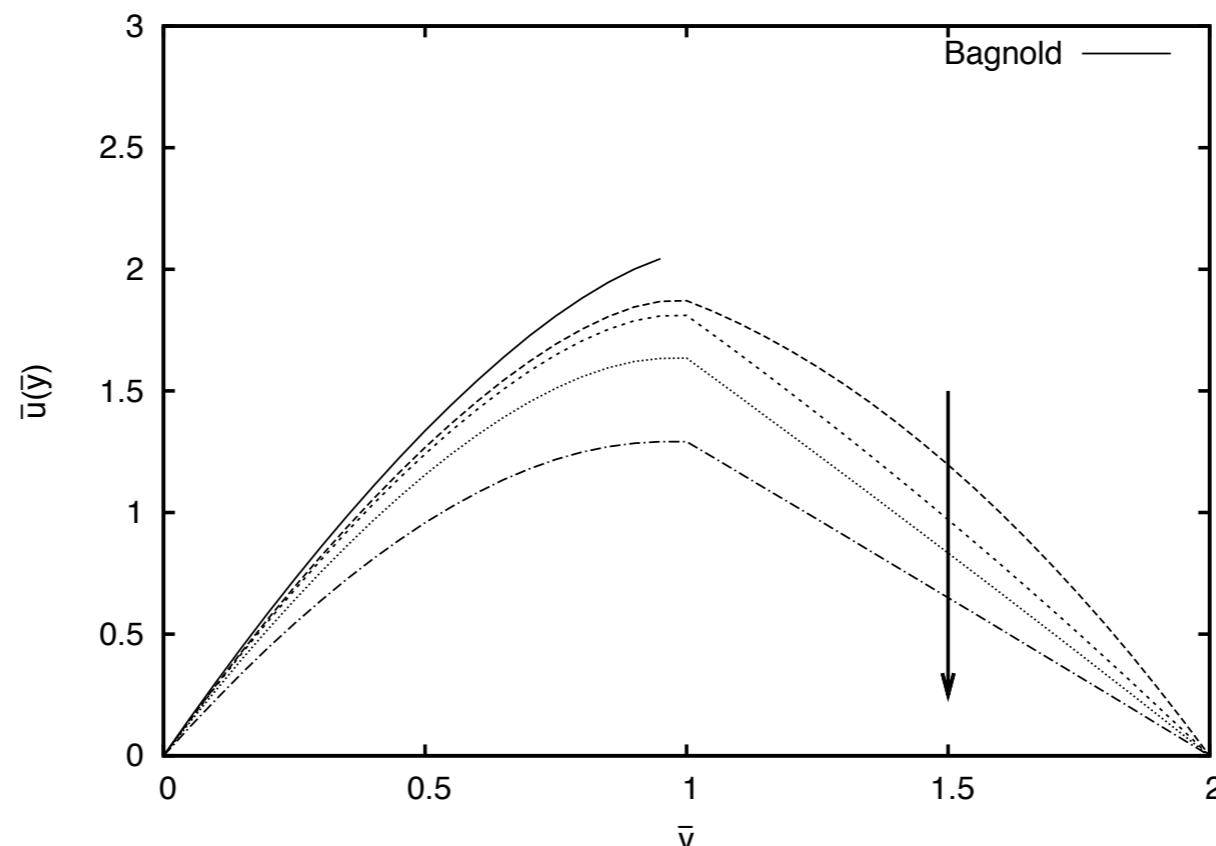
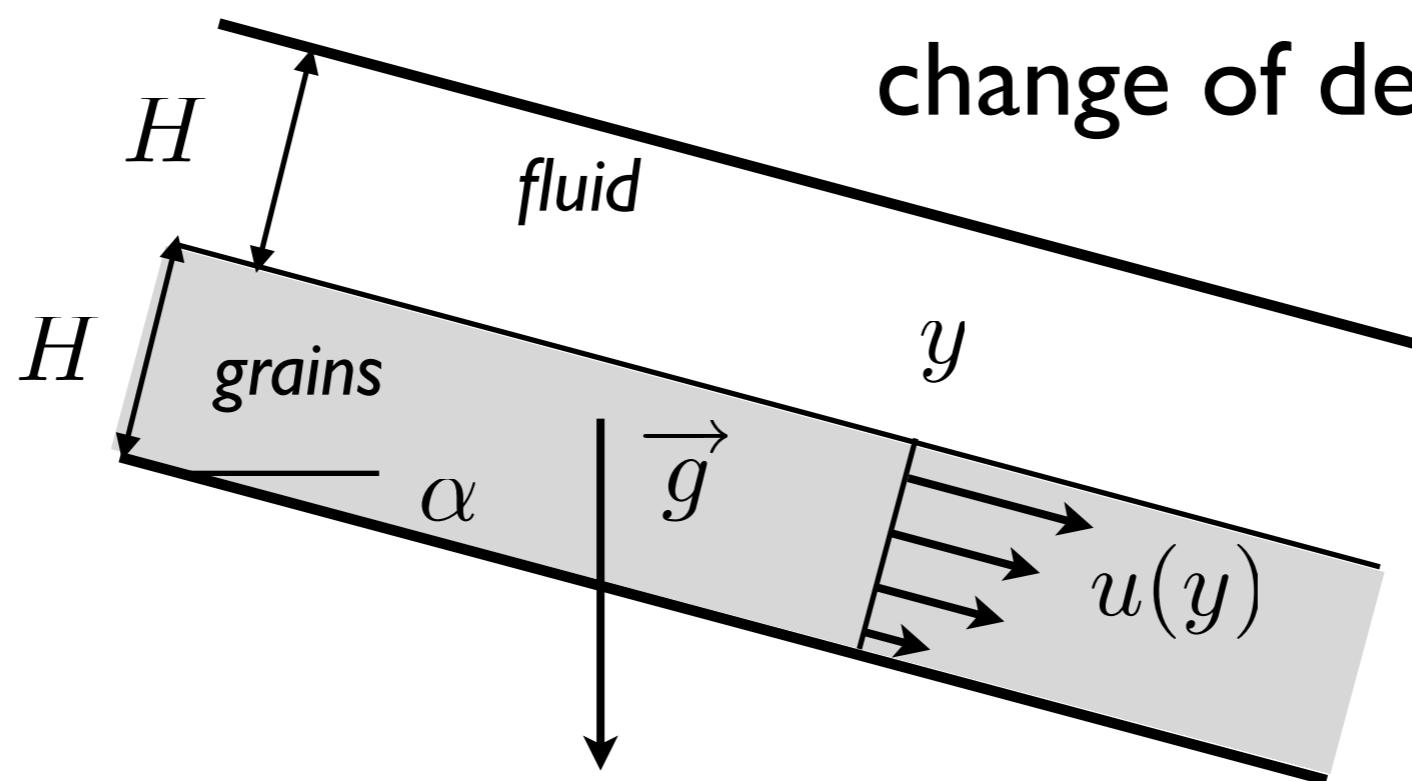
$$u(y) = \frac{(2H - y)(\rho_f gy \sin \alpha - 2\tau_0)}{2\mu_f}$$

$$d\frac{\partial u}{\partial y} = \max \left[\sqrt{p_0/\rho + gH \left(1 - \frac{y}{H}\right) \cos \alpha} \times \mu^{-1} \left(\frac{\tau_0 + \rho g H \left(1 - \frac{y}{H}\right) \sin \alpha}{p_0 + \rho g H \left(1 - \frac{y}{H}\right) \cos \alpha} \right), 0 \right].$$

$$u(H^-) - u(H^+) = 0$$

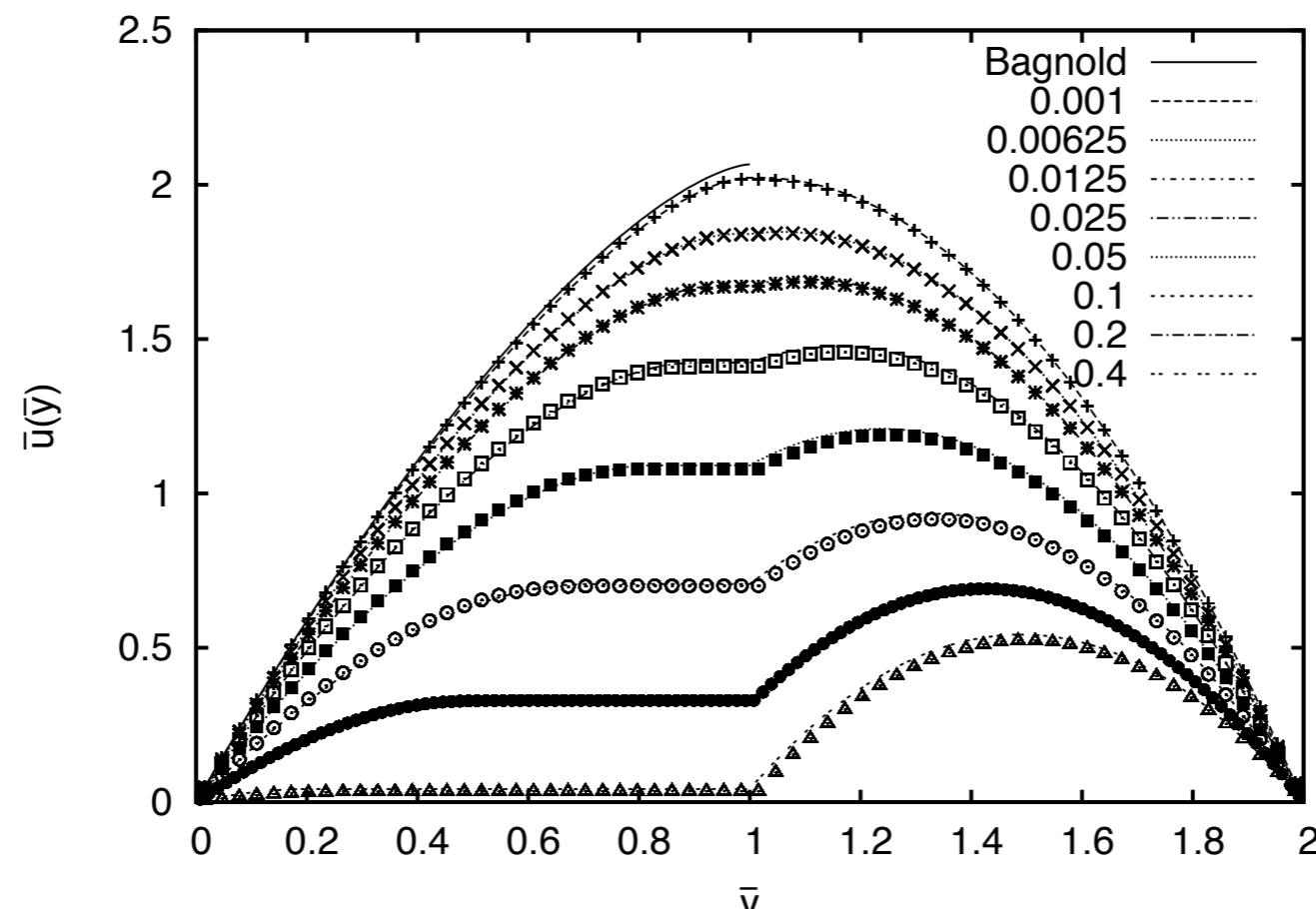
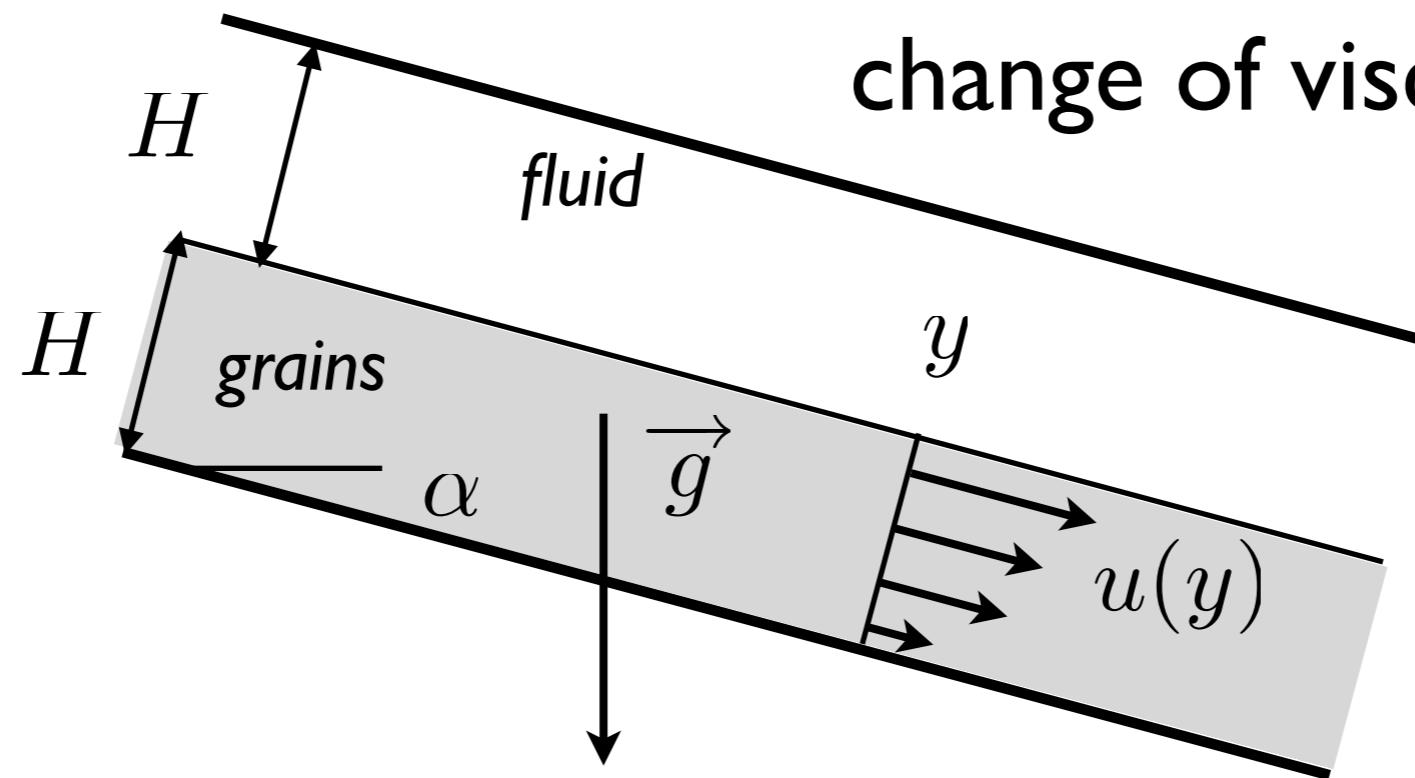


Test of the code: «Bagnold» avalanche change of density of the fluid





Test of the code: «Bagnold» avalanche change of viscosity of the fluid



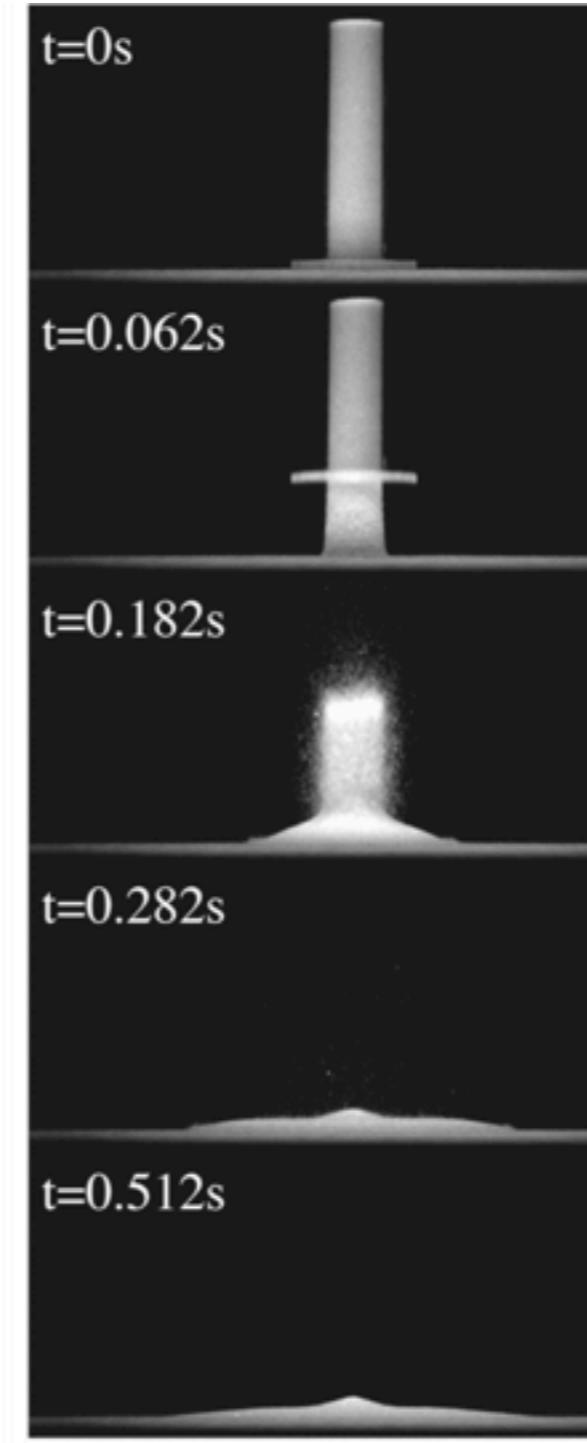
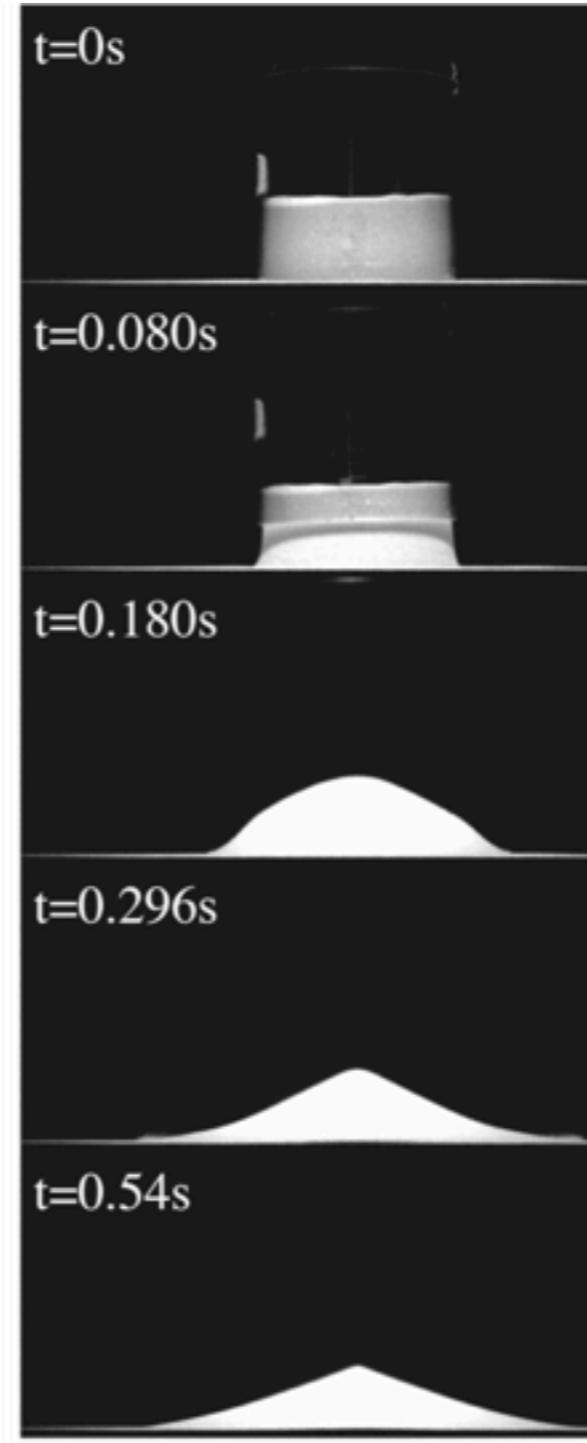
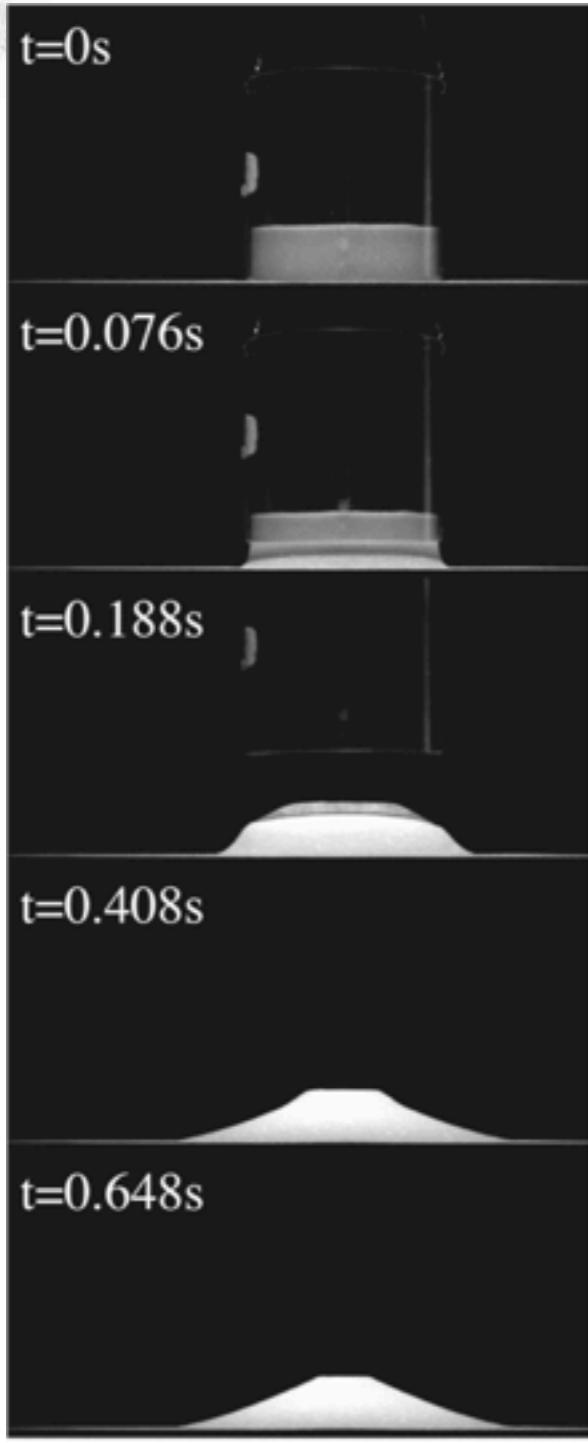


The sand pit problem: quickly remove the bucket of sand



Granular Column Collapse

E. Lajeunesse A. Mangeney-Castelnau and J. P. Vilotte PoF 204



(a)

(b)

(c)

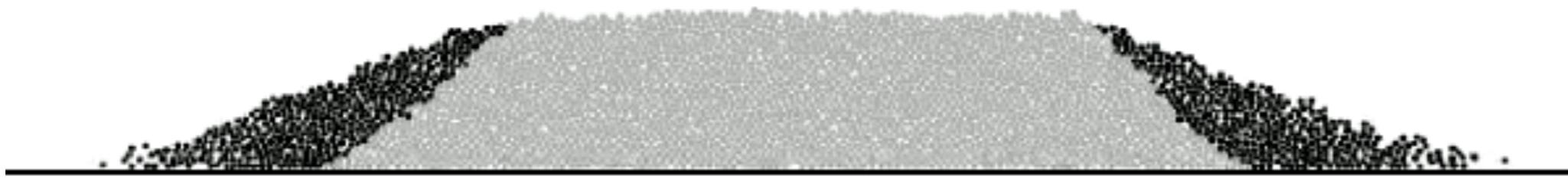


The sand pit problem: quickly remove the bucket of sand



Collapse of columns

a=0.37



Contact Dynamic
simulation Lydie Staron





Collapse of columns

a=0.90



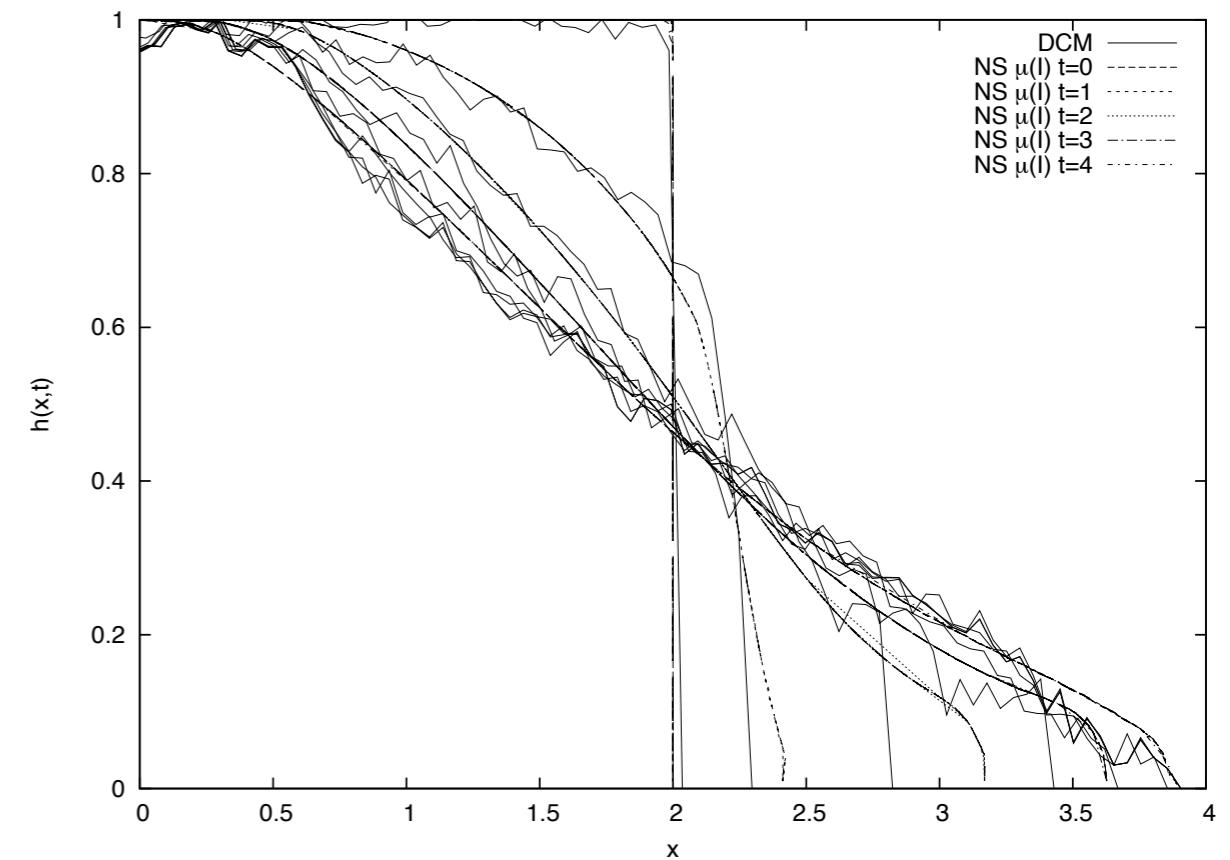
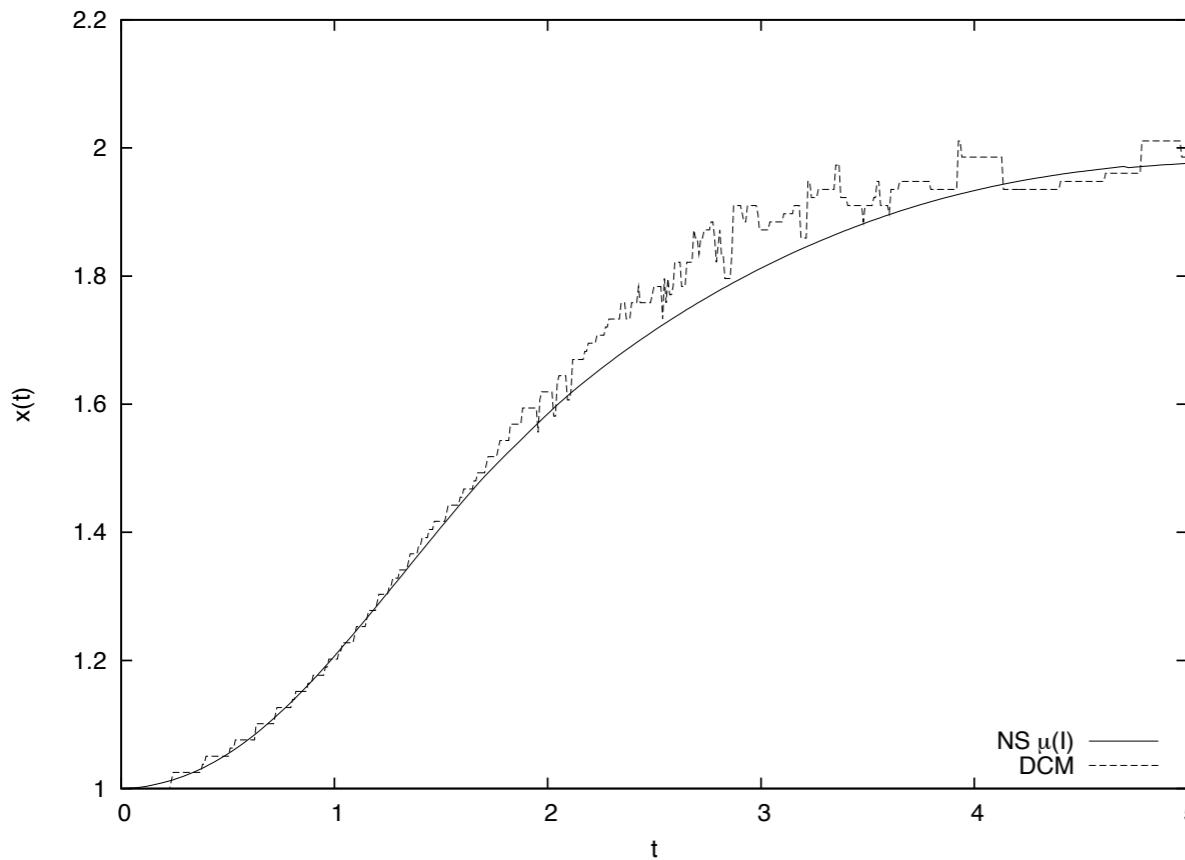
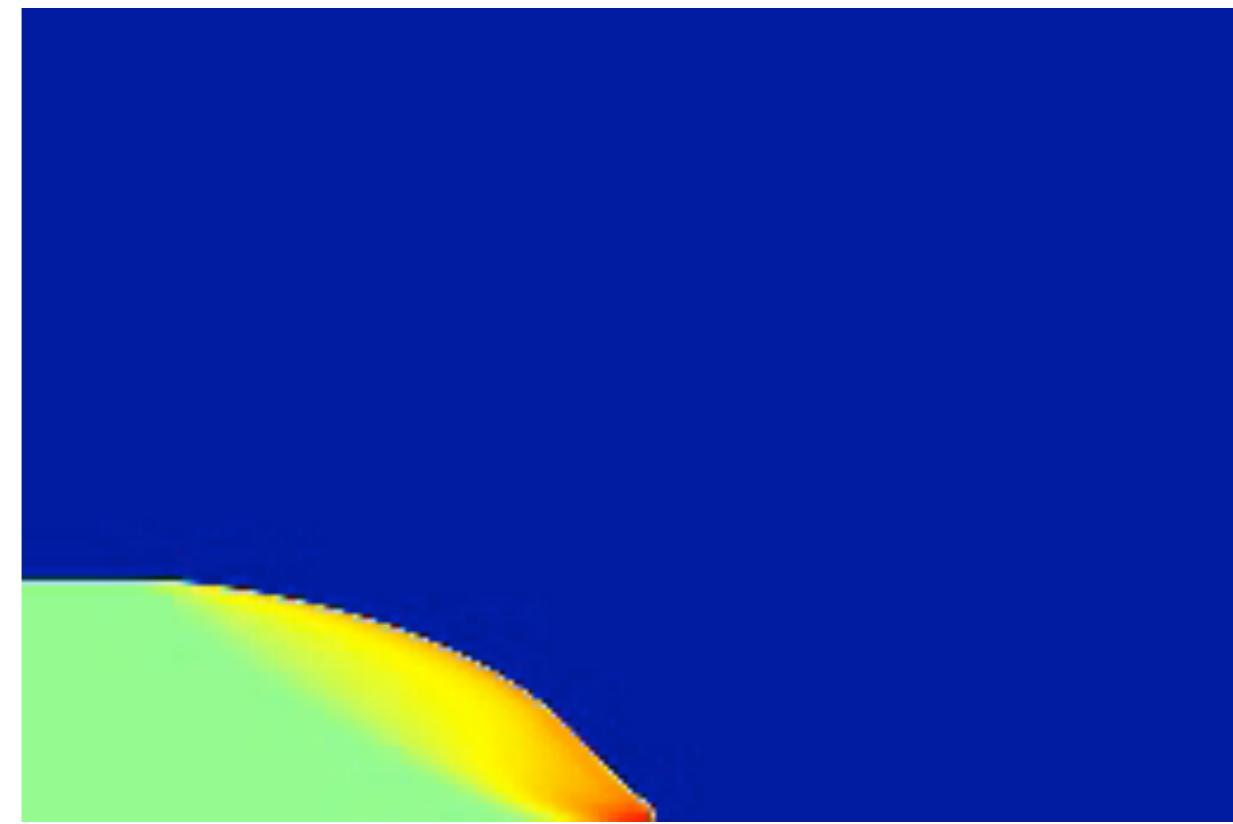
Contact Dynamic
simulation Lydie Staron





Collapse of columns simulation *Gerris* $\mu(l)$

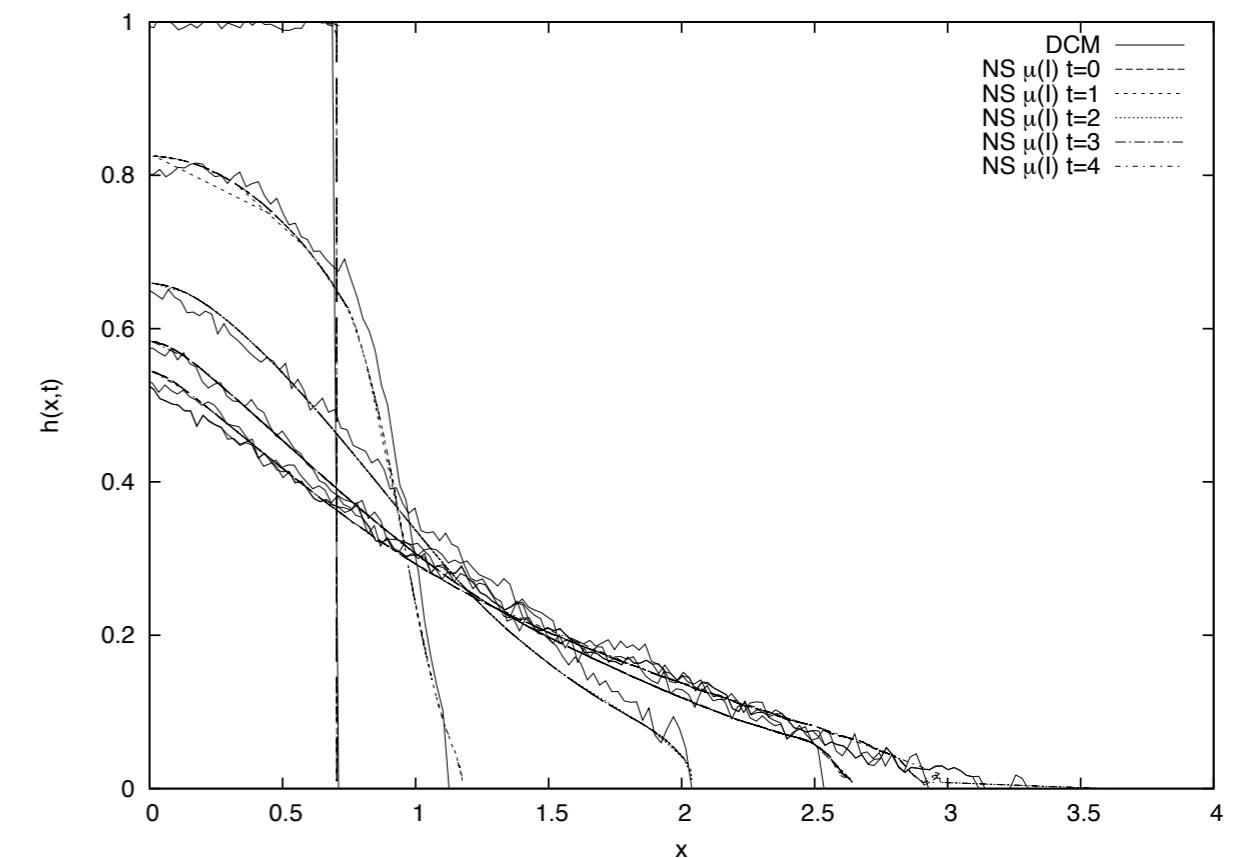
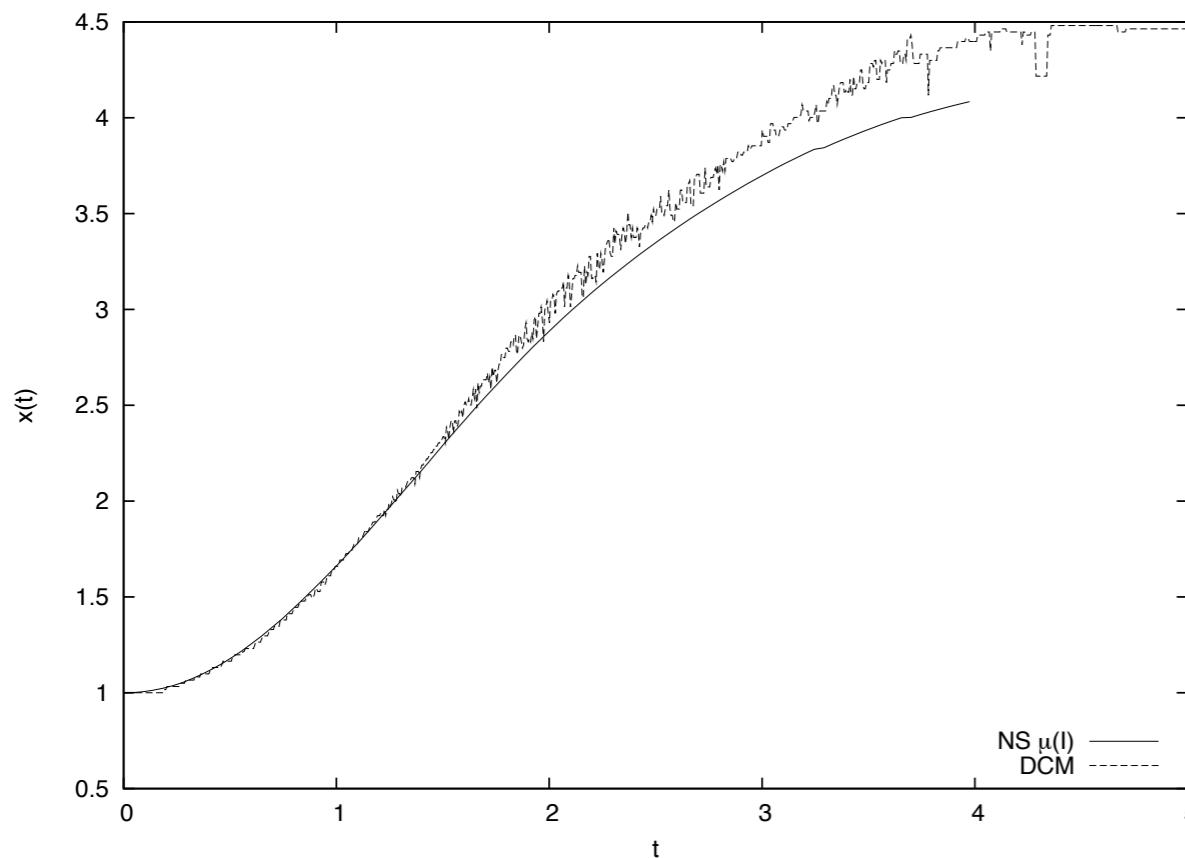
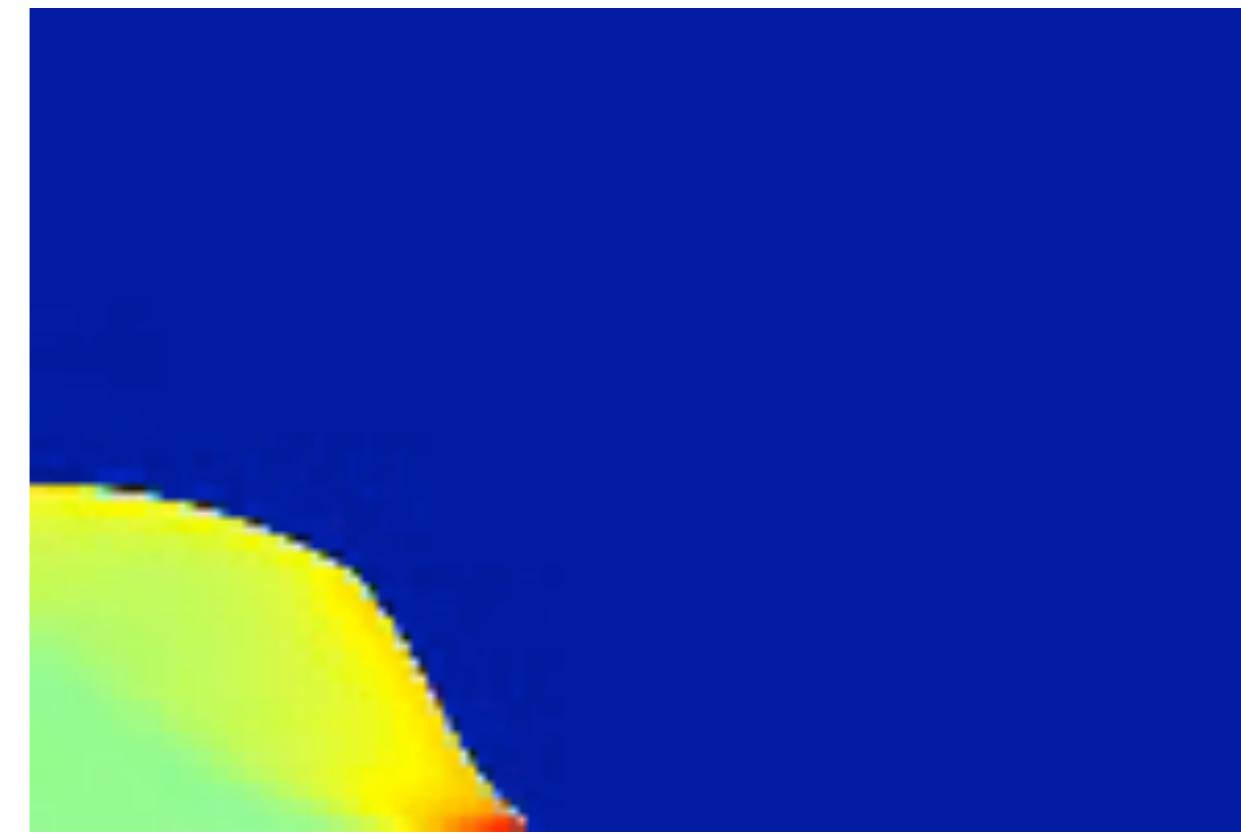
Collapse of columns of aspect ratio 0.5
comparison of Discrete Simulation Contact
Method and Navier Stokes gerris, shape at time
0, 1, 2, 3, 4 and position of the front of the
avalanche as function of time (time measured
with $\sqrt{H_0/g}$ and space with aH_0)





Collapse of columns simulation *Gerris* $\mu(l)$

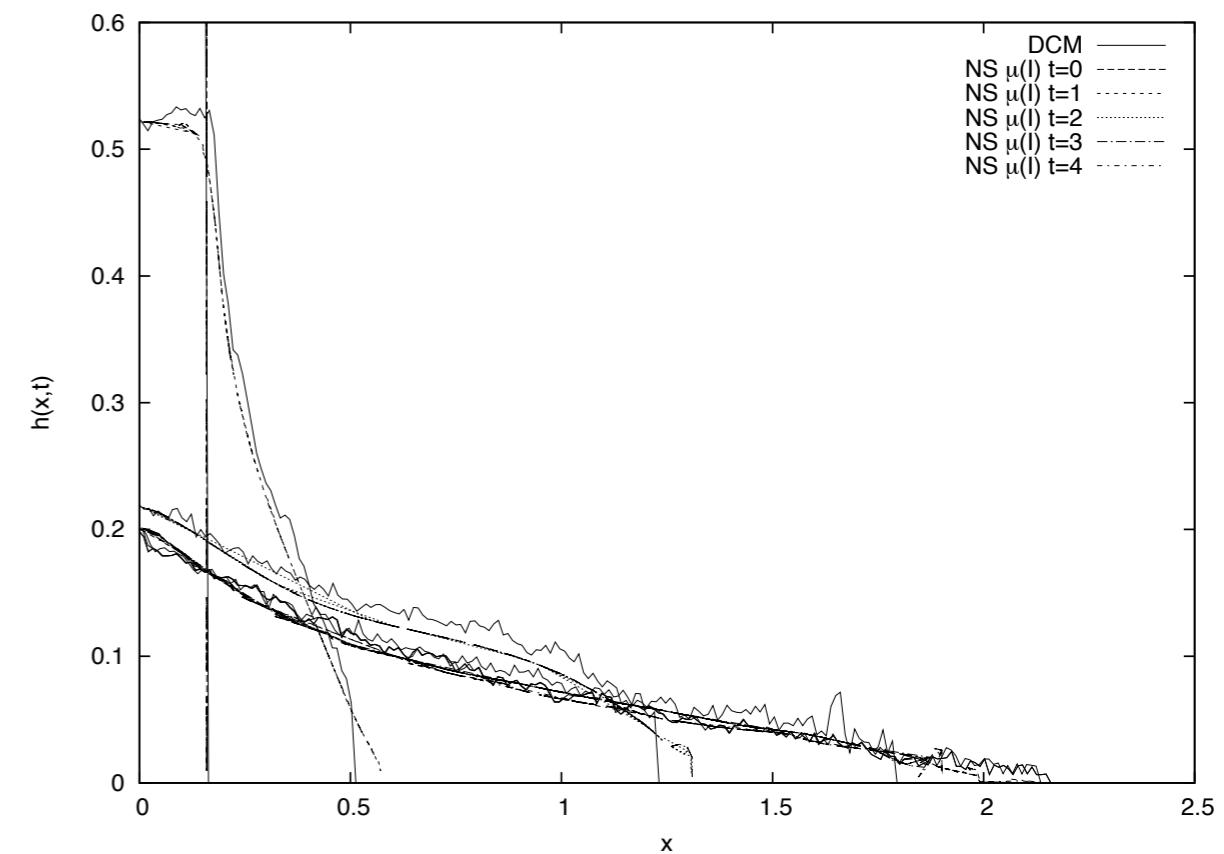
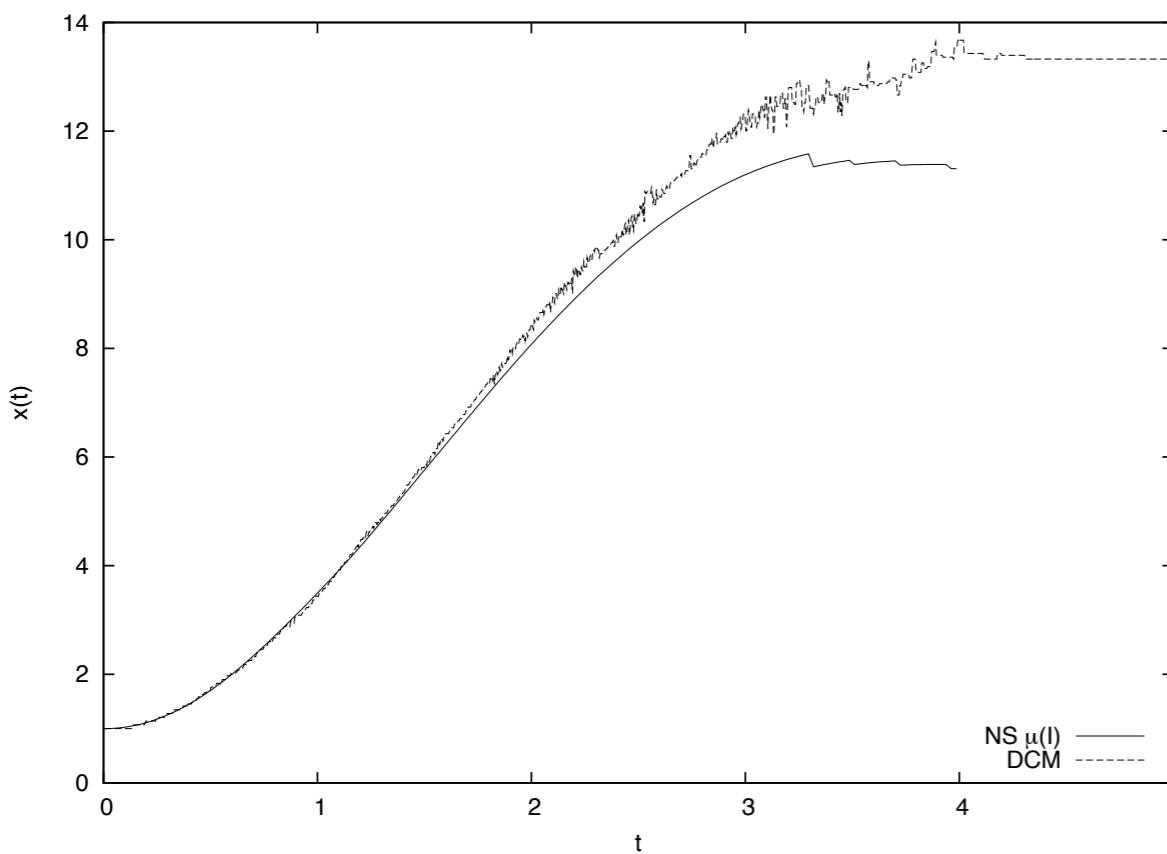
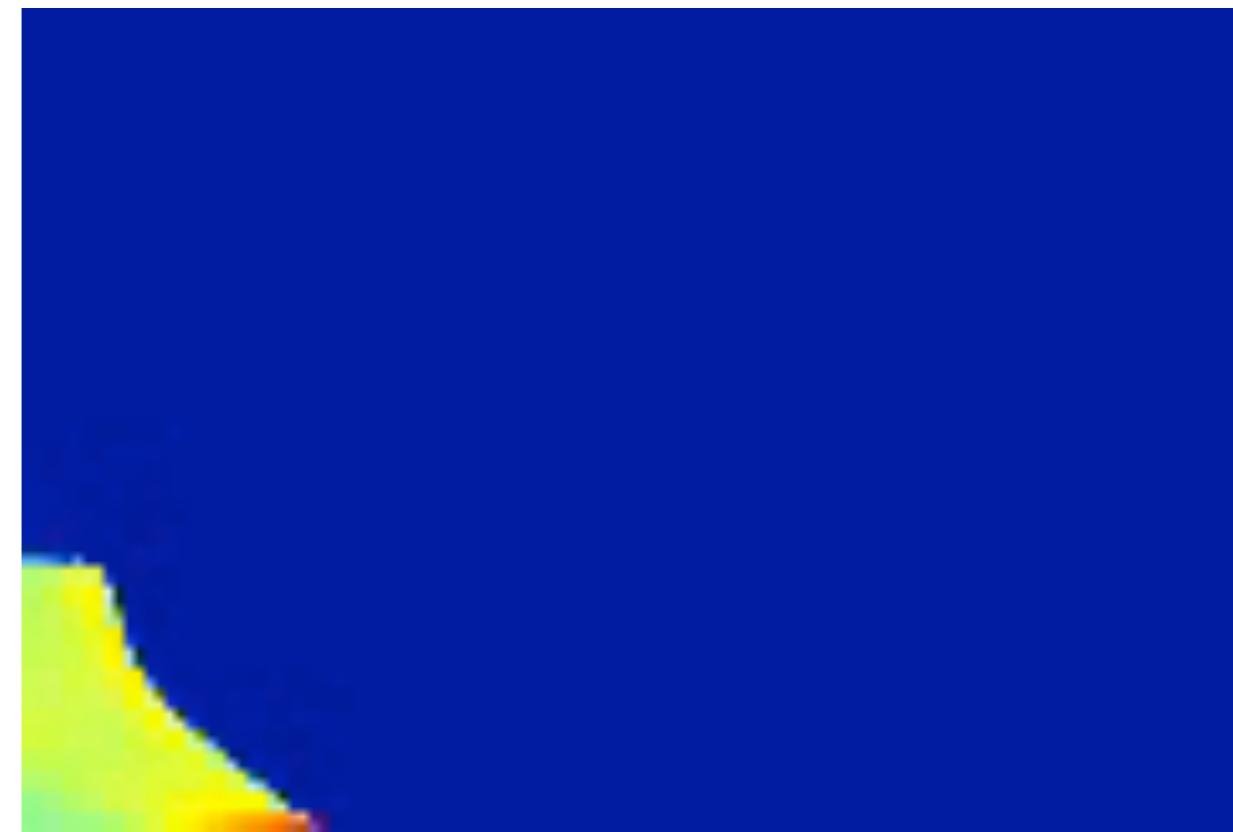
Collapse of columns of aspect ratio 1.42
comparison of Discrete Simulation Contact
Method and Navier Stokes gerris, shape at time
0, 1, 2, 3, 4 and position of the front of the
avalanche as function of time (time measured
with $\sqrt{H_0/g}$ and space with aH_0)





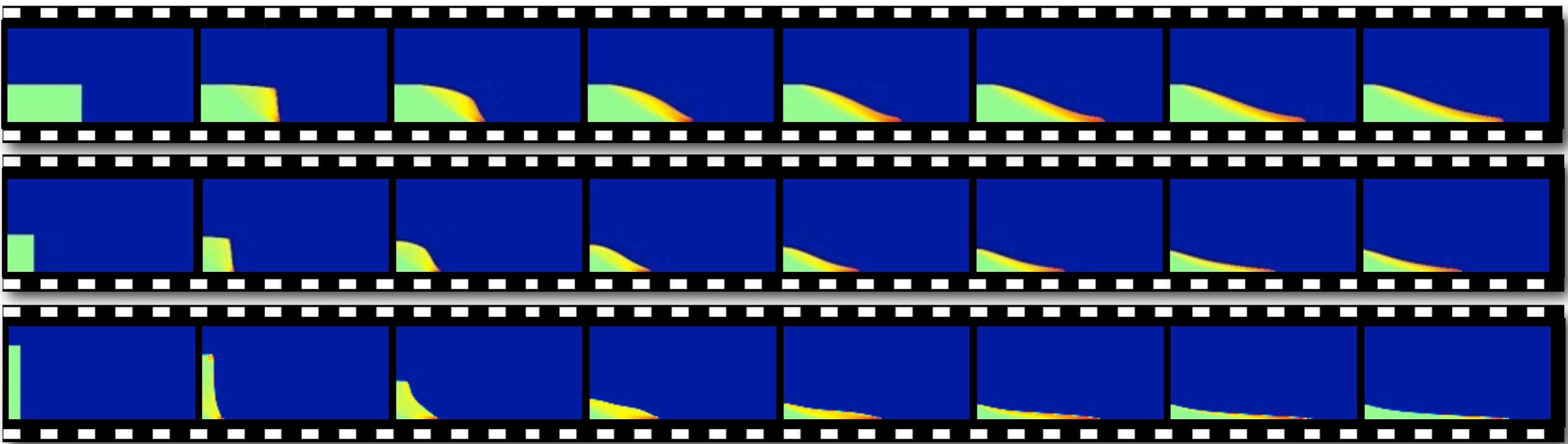
Collapse of columns simulation *Gerris* $\mu(l)$

Collapse of columns of aspect ratio 6.26
comparison of Discrete Simulation Contact
Method and Navier Stokes gerris, shape at time
0, 1, 2, 3, 4 and position of the front of the
avalanche as function of time (time measured
with $\sqrt{H_0/g}$ and space with aH_0)





Collapse of columns simulation *Gerris* $\mu(l)$



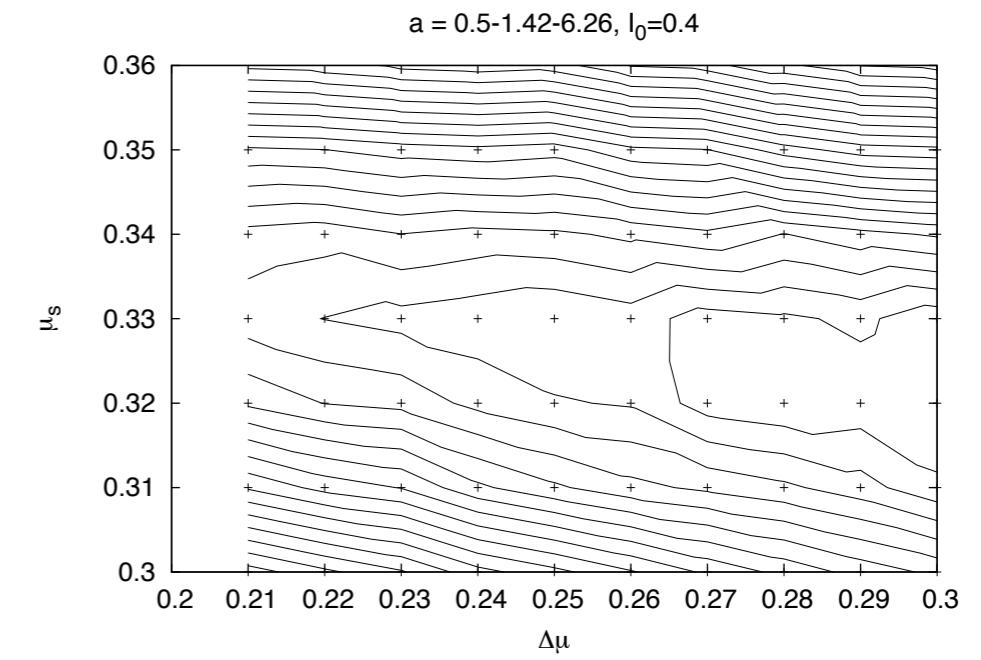
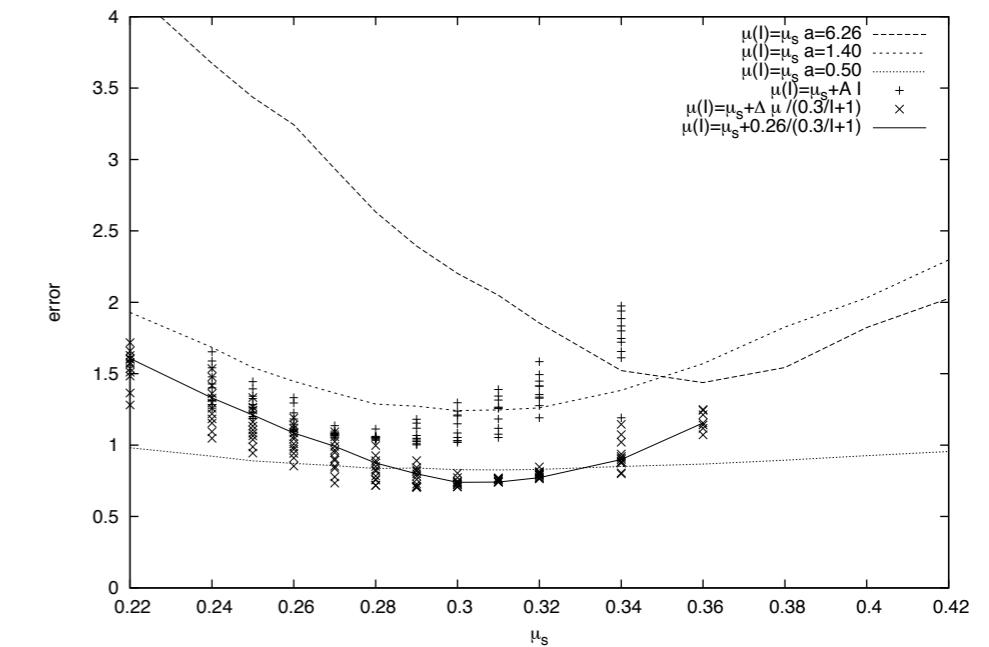
Snapshots of collapse of three columns of aspect ratio 0.5 1.42 and 6.26 (top to bottom)

Collapse of columns simulation *Gerris* $\mu(I)$



optimisation

$$\mu(I) = \mu_s + \frac{\Delta\mu}{I_0 + 1}$$

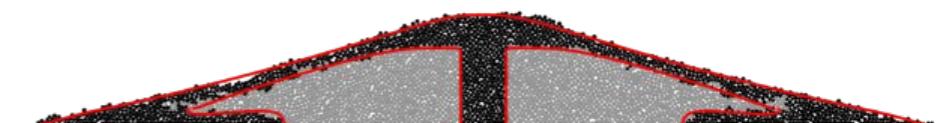
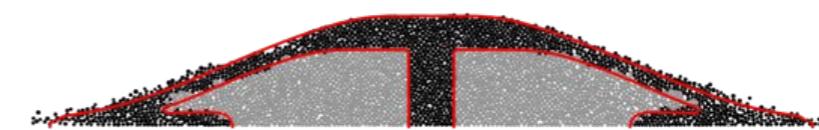
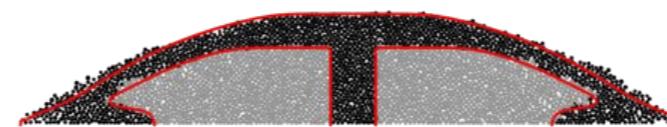
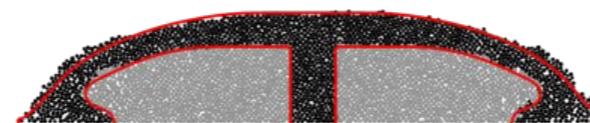
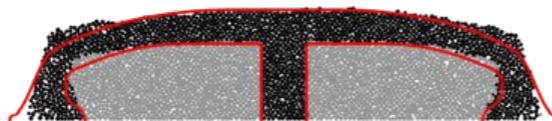
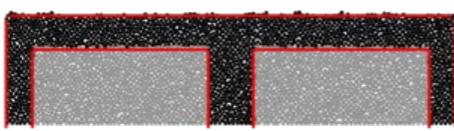


final values

$$\mu_s = 0.32 \quad \Delta\mu = 0.28 \quad I_0 = 0.4$$



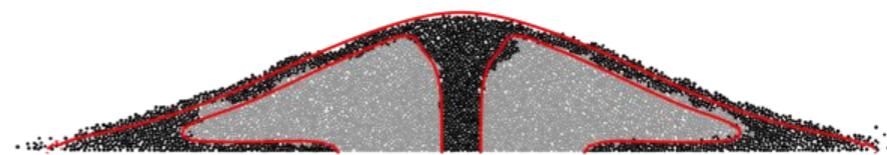
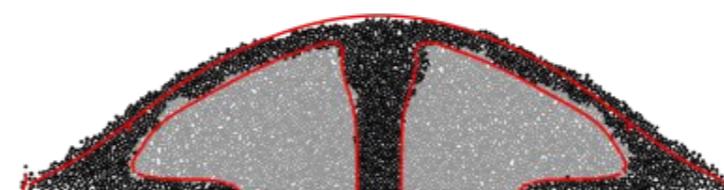
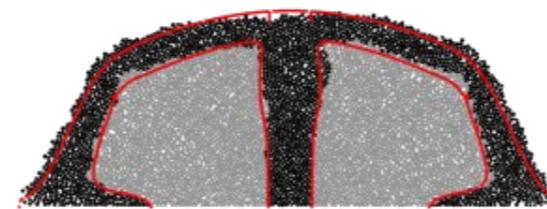
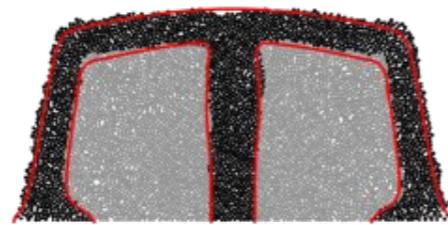
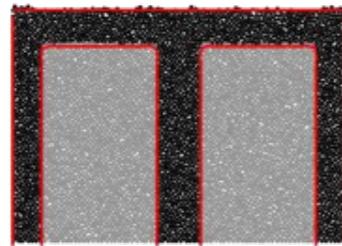
Collapse of columns simulation *Gerris* $\mu(l)$



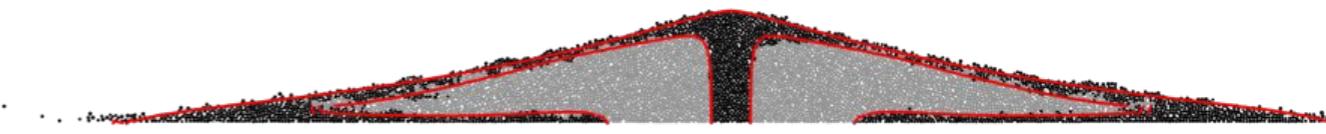
$a = 0.5$ DCM vs *Gerris* $\mu(l)$



Collapse of columns simulation *Gerris* $\mu(l)$

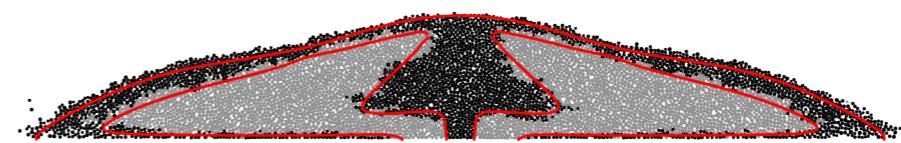
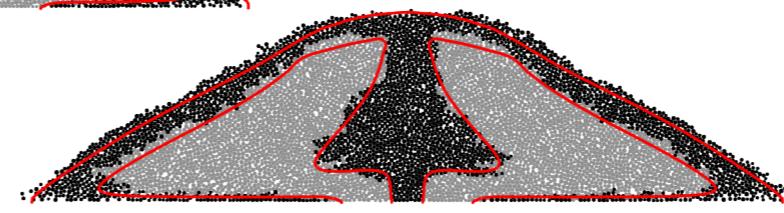
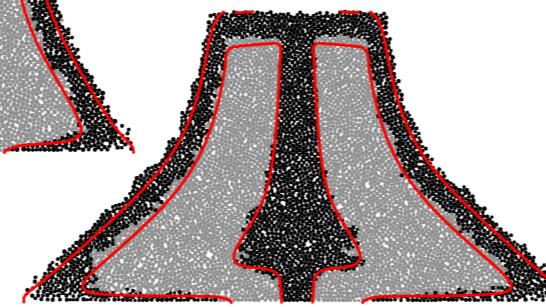
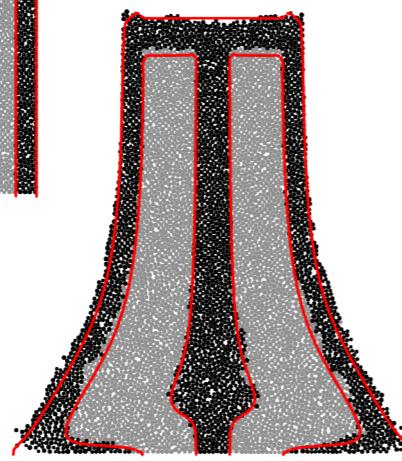
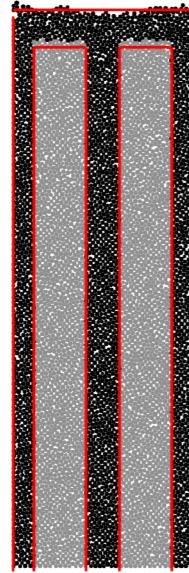


$a = 1.42$ DCM vs *Gerris* $\mu(l)$

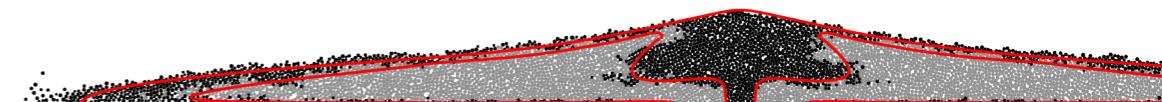




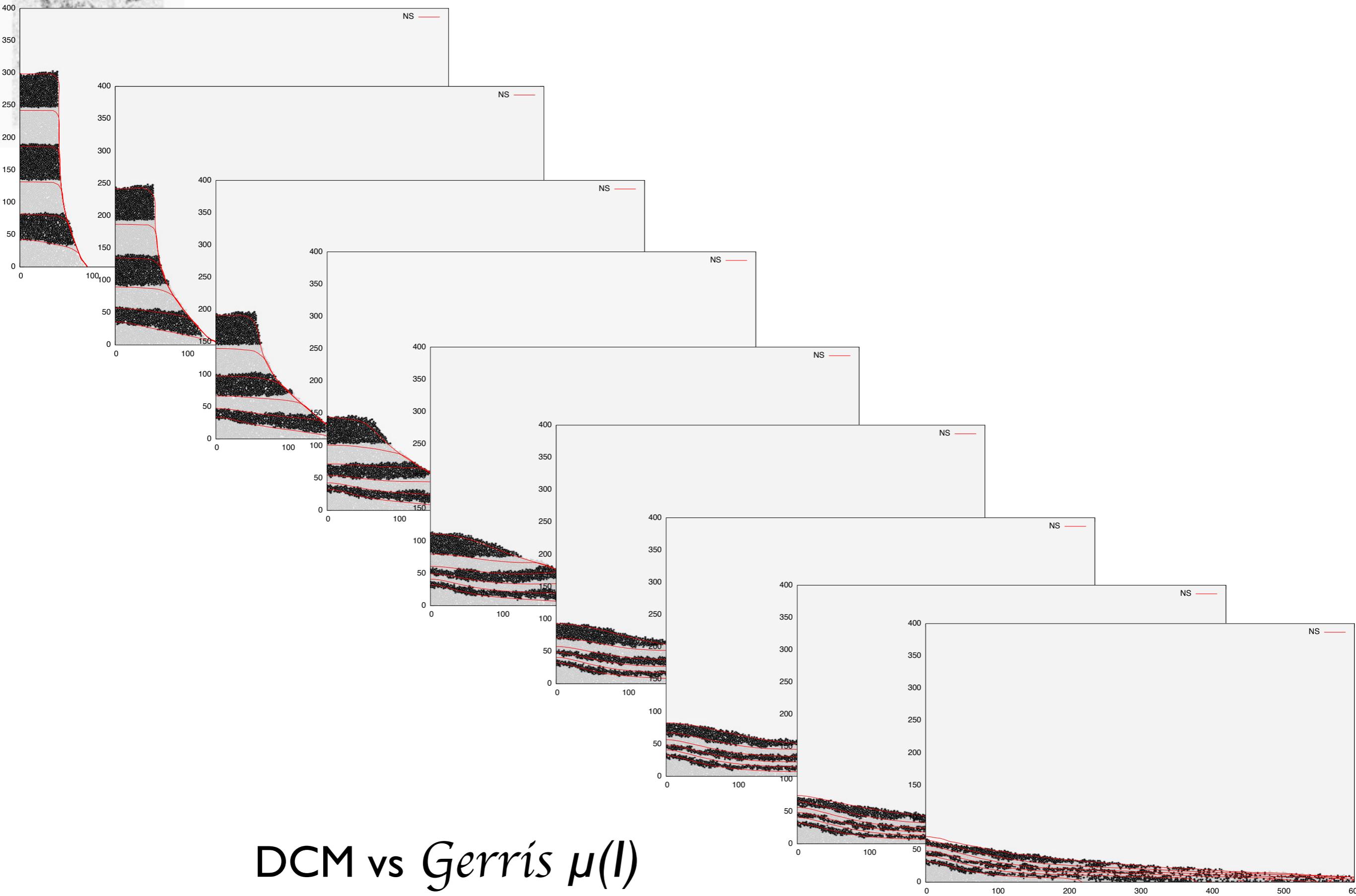
Collapse of columns simulation *Gerris* $\mu(l)$



$a = 6.6$ DCM vs *Gerris* $\mu(l)$



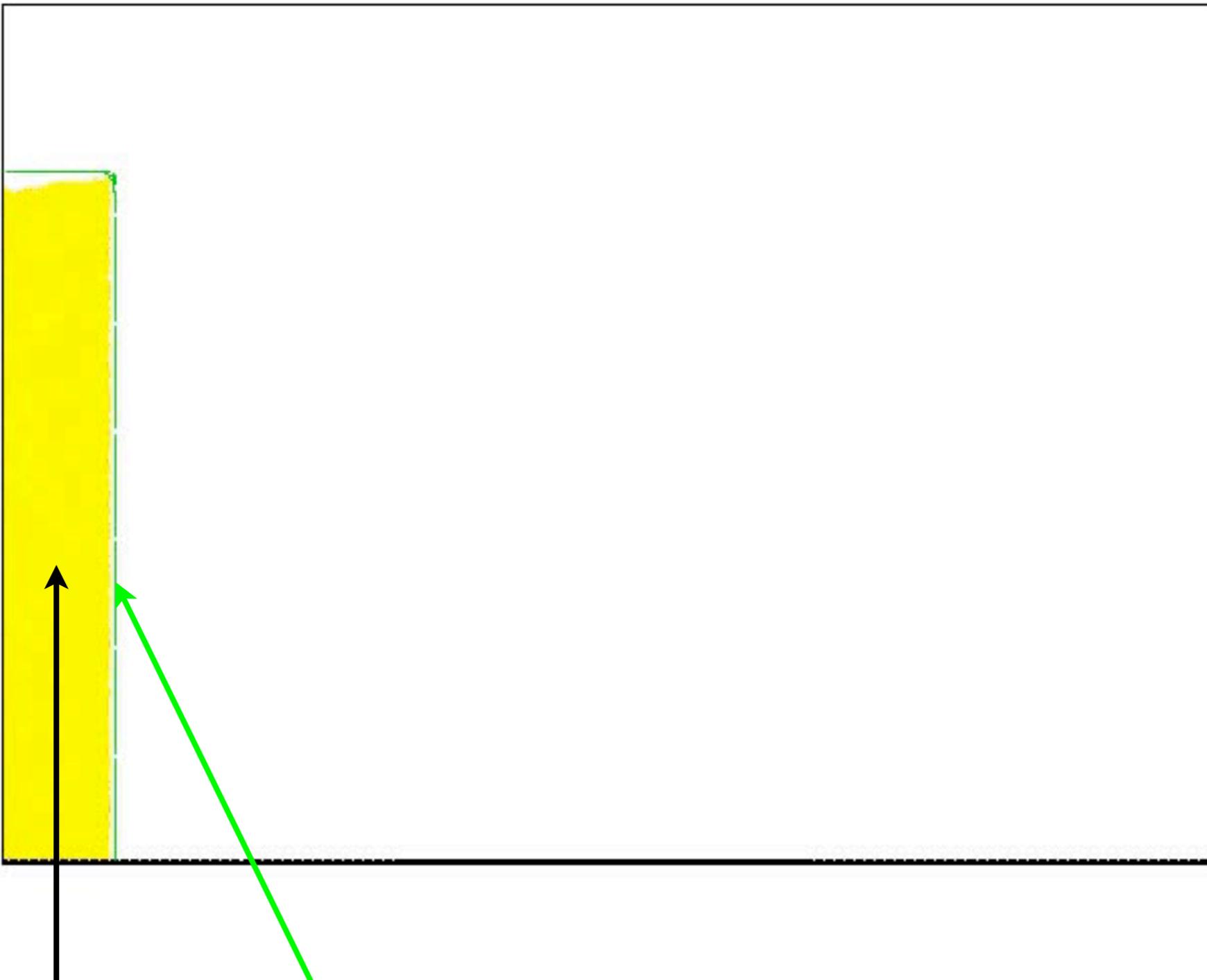
Collapse of columns simulation *Gerris* $\mu(l)$





Collapse of columns simulation *Gerris* $\mu(l)$

NS/CD t=0.0190

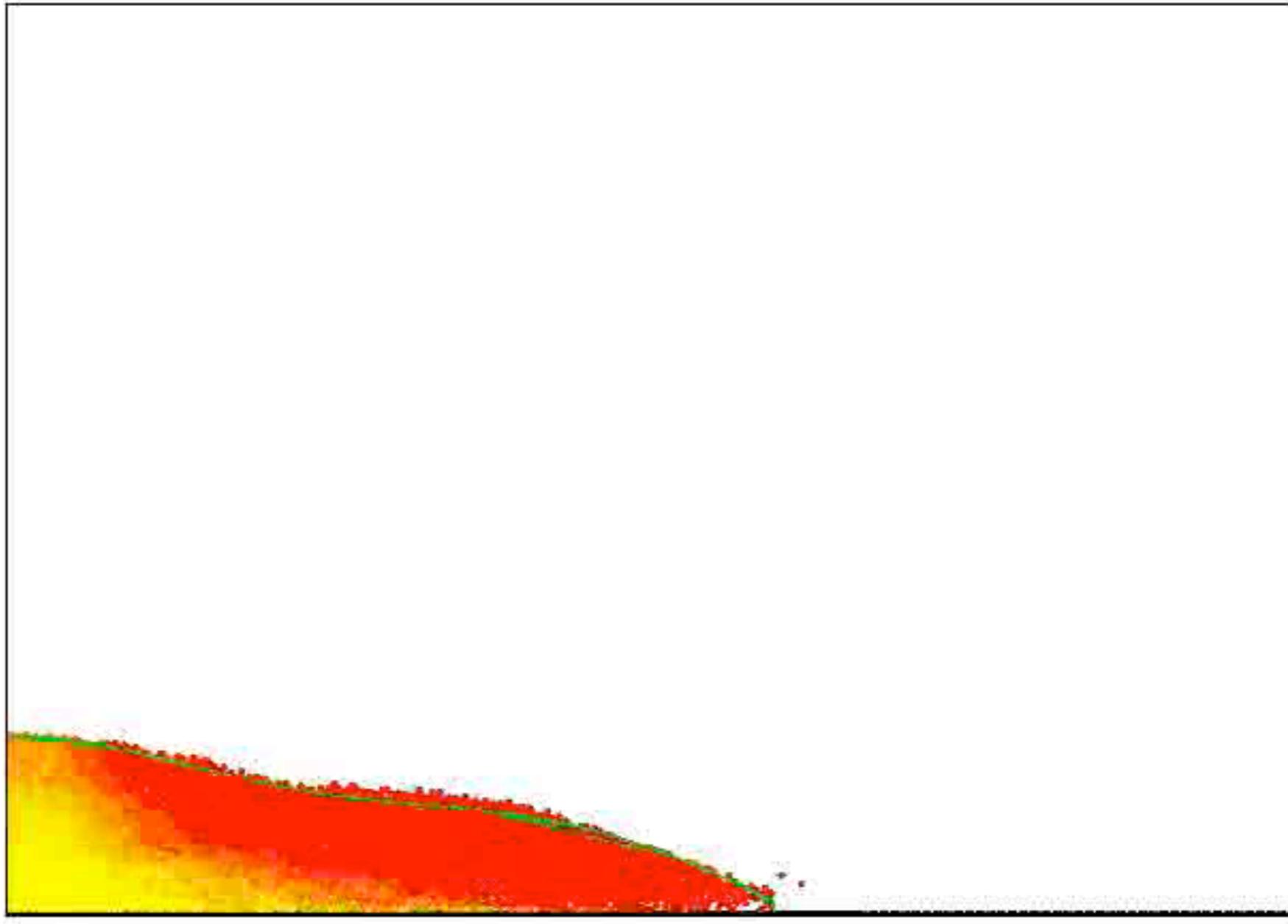


DCM vs *Gerris* $\mu(l)$



Collapse of columns simulation *Gerris* $\mu(l)$

NS/CD t=1.6720

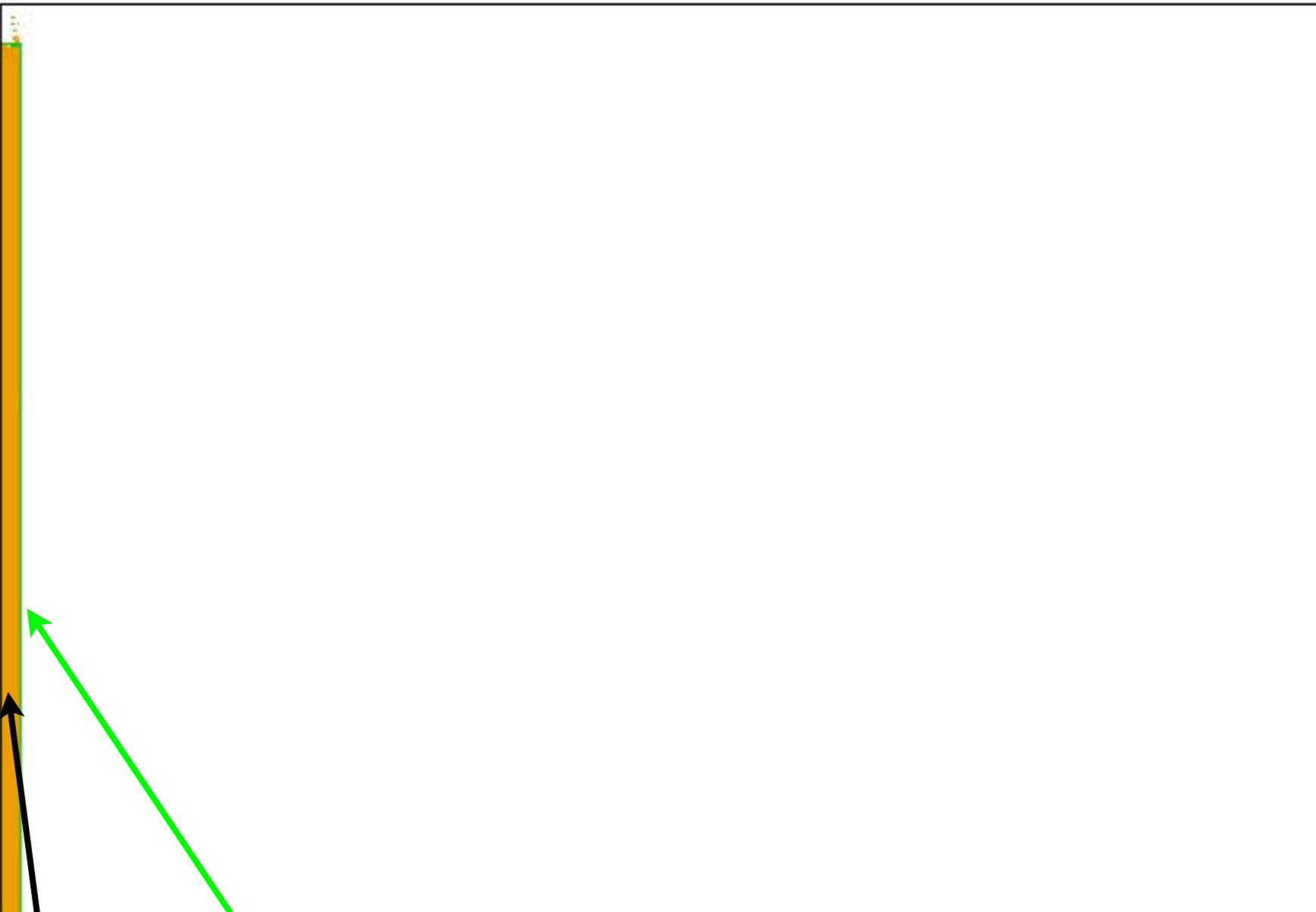


DCM vs *Gerris* $\mu(l)$



Collapse of columns simulation *Gerris* $\mu(l)$

NS/CD $t=0.0075$

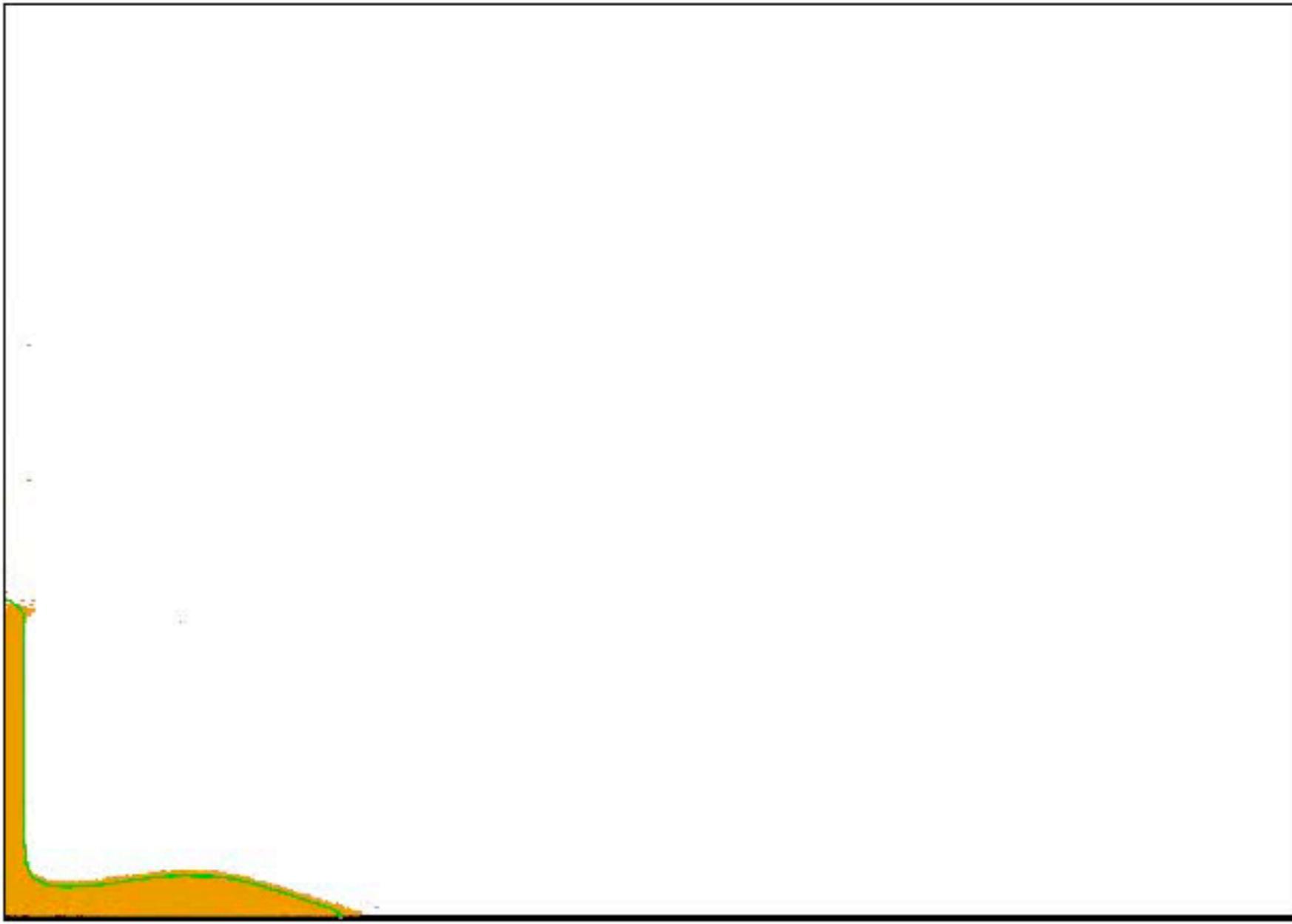


DCM vs *Gerris* $\mu(l)$



Collapse of columns simulation *Gerris* $\mu(l)$

NS/CD t=1.1400



DCM vs *Gerris* $\mu(l)$



Collapse of columns simulation *Gerris* $\mu(l)$

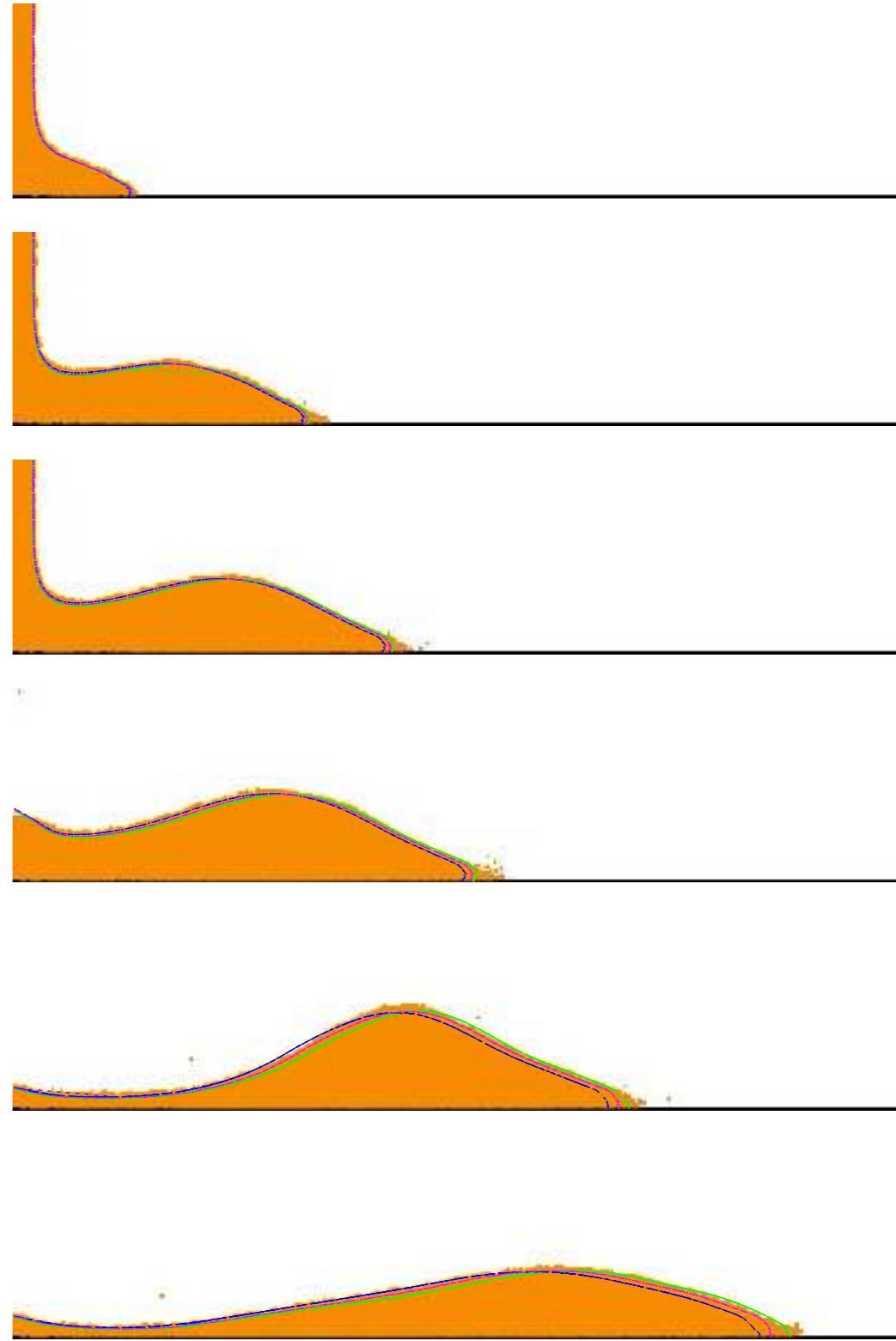
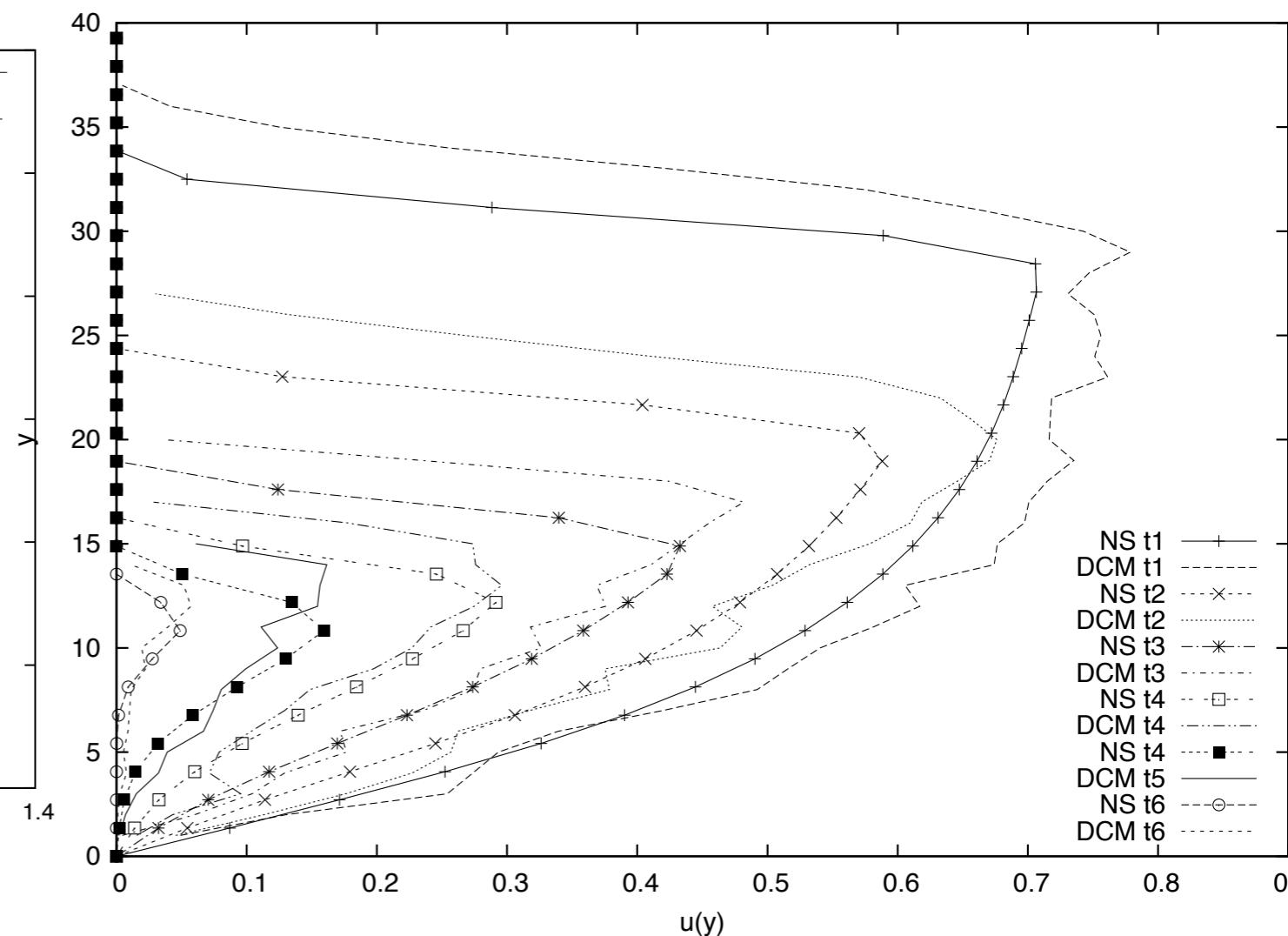
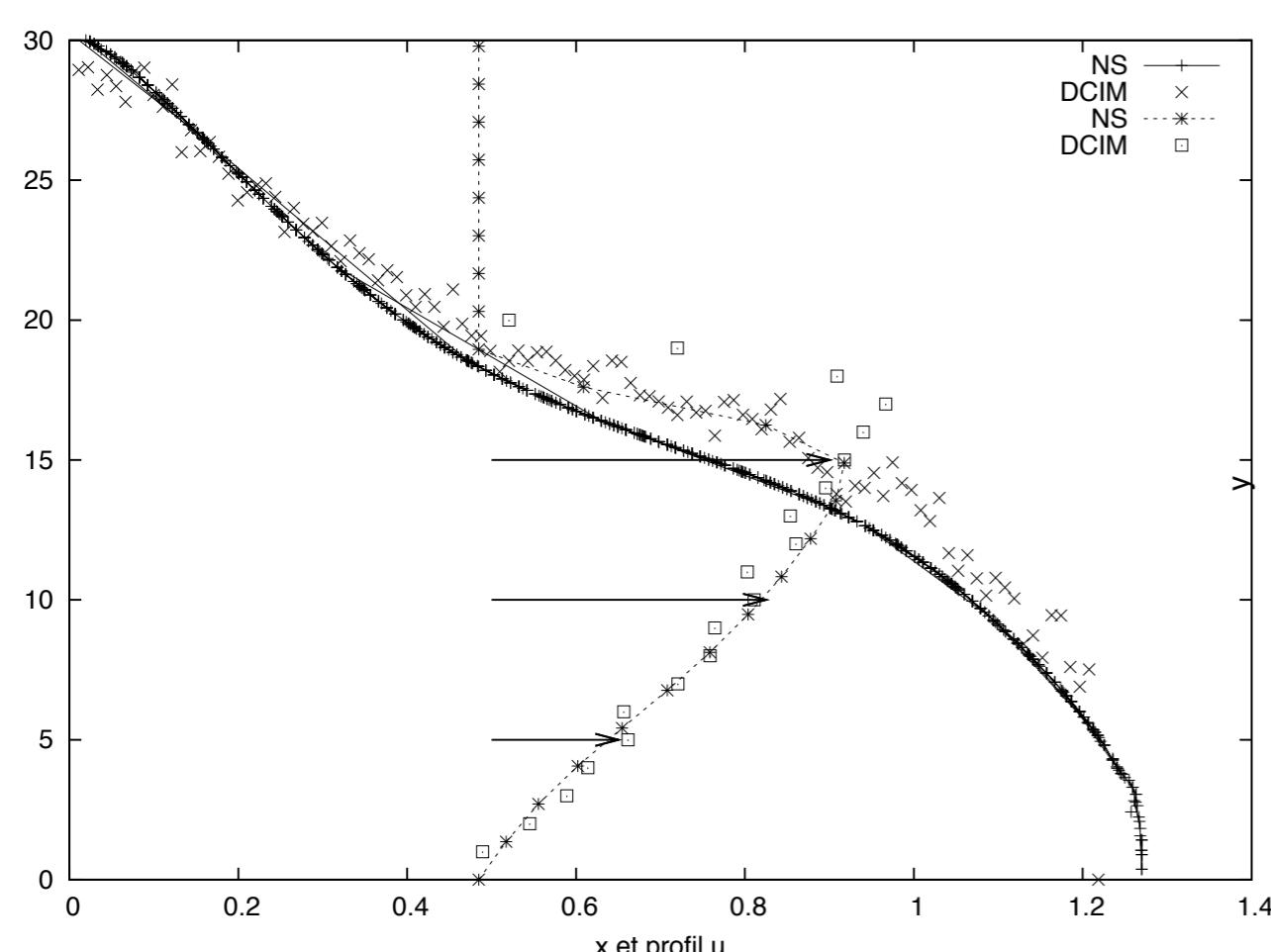


Figure 10: Strip representing a series of snapshots ($t = 0.5, 1.0, 1.2, 1.4, 1.7$, and 2.0) of a column collapse with aspect ratio $a = 68$. The most advanced curve (in green) corresponds to $\mu_s = 0.3$ $\Delta mu = 0.26$ and $I_0 = 0.30$. the less advanced (in blue) $\mu_s = 0.32$ $\Delta mu = 0.28$ and $I_0 = 0.30$ fits better the end of the heap. The curve in between (in cyan) corresponds to $\mu_s = 0.32$ $\Delta mu = 0.28$ and $I_0 = 0.40$ and fits better the top of the surge.

DCM vs *Gerris* $\mu(l)$



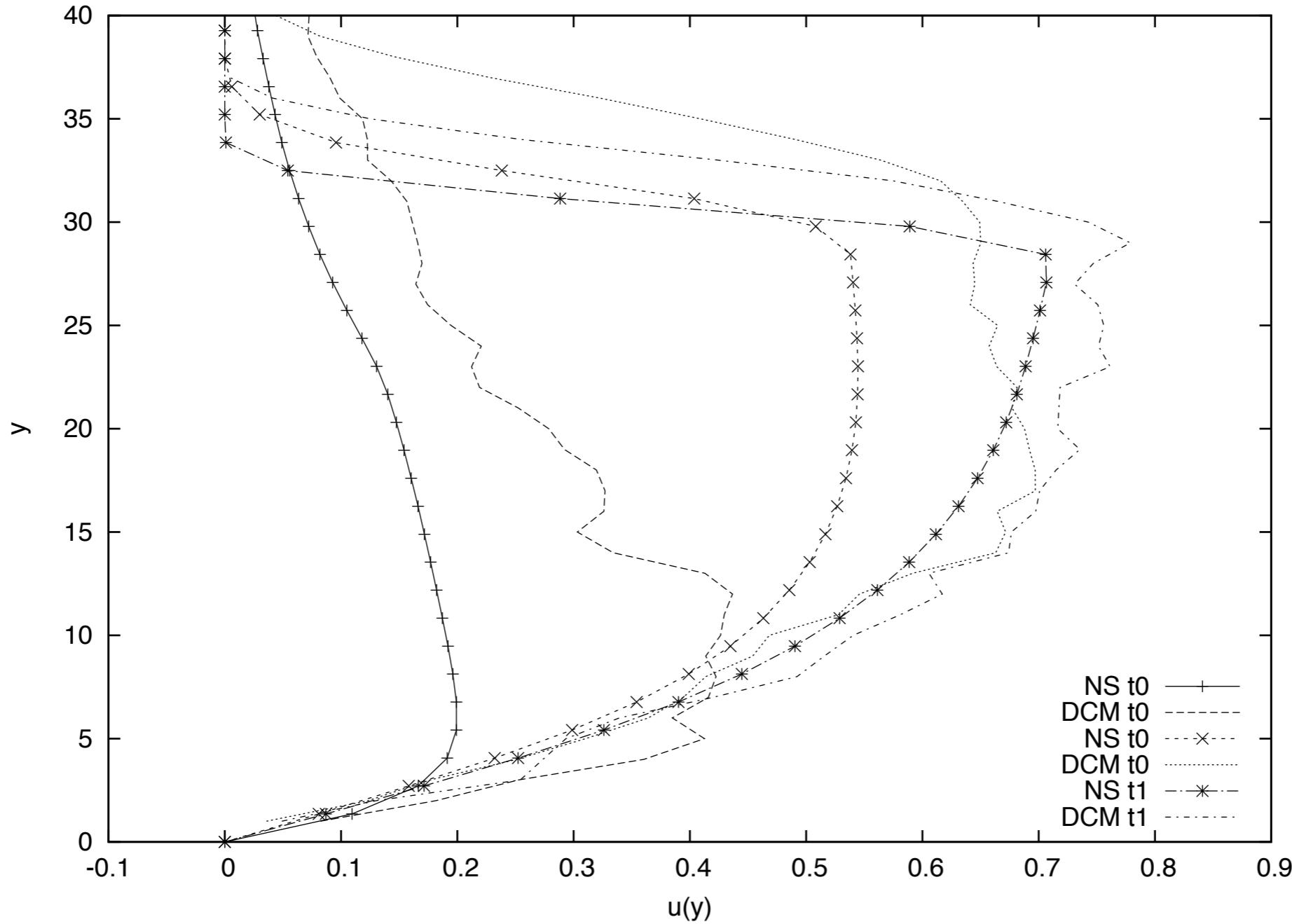
● comparaison de profiles de vitesse



DCM vs *Gerris* $\mu(l)$



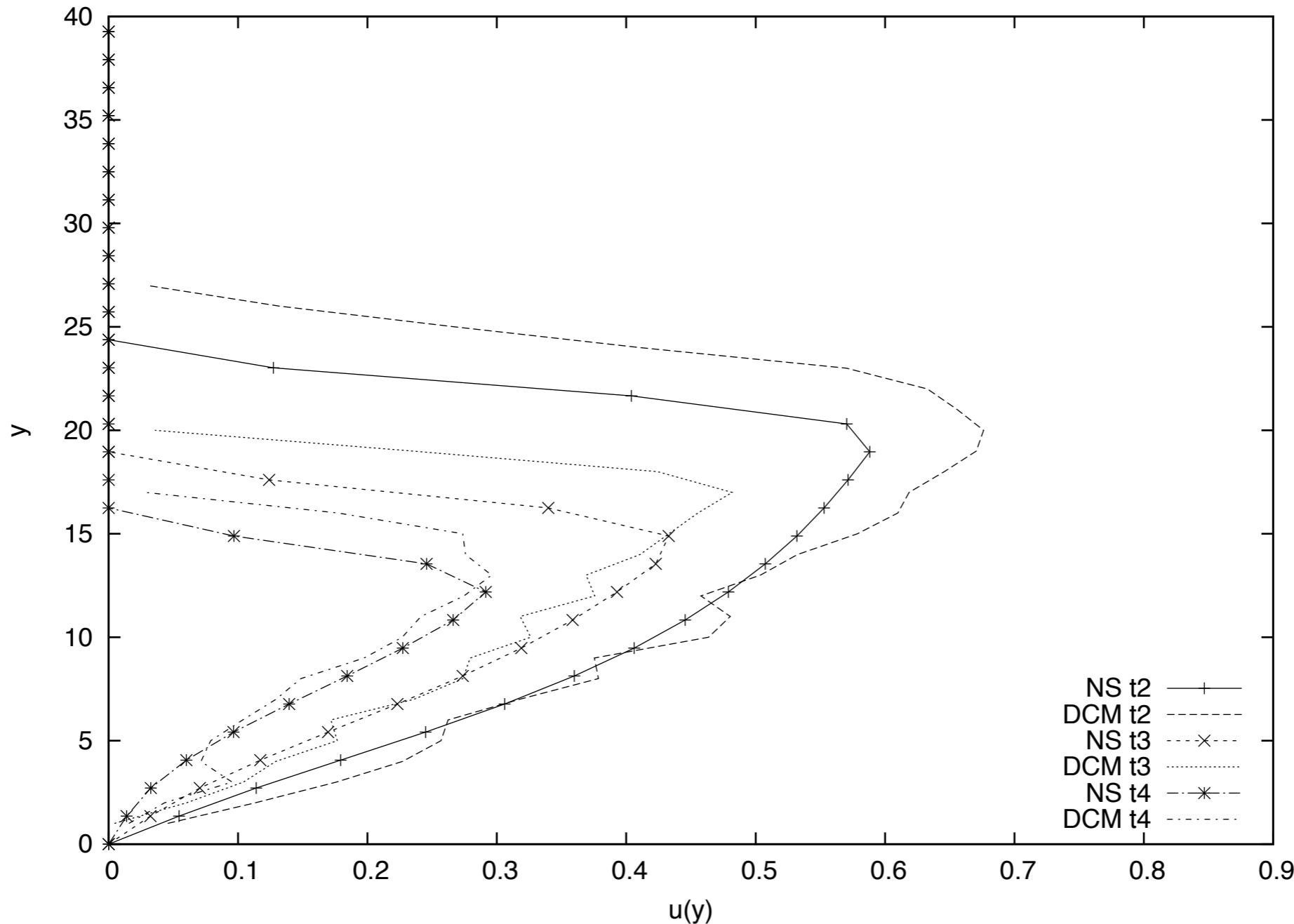
- comparaison of velocity profiles



DCM vs *Gerris* $\mu(l)$



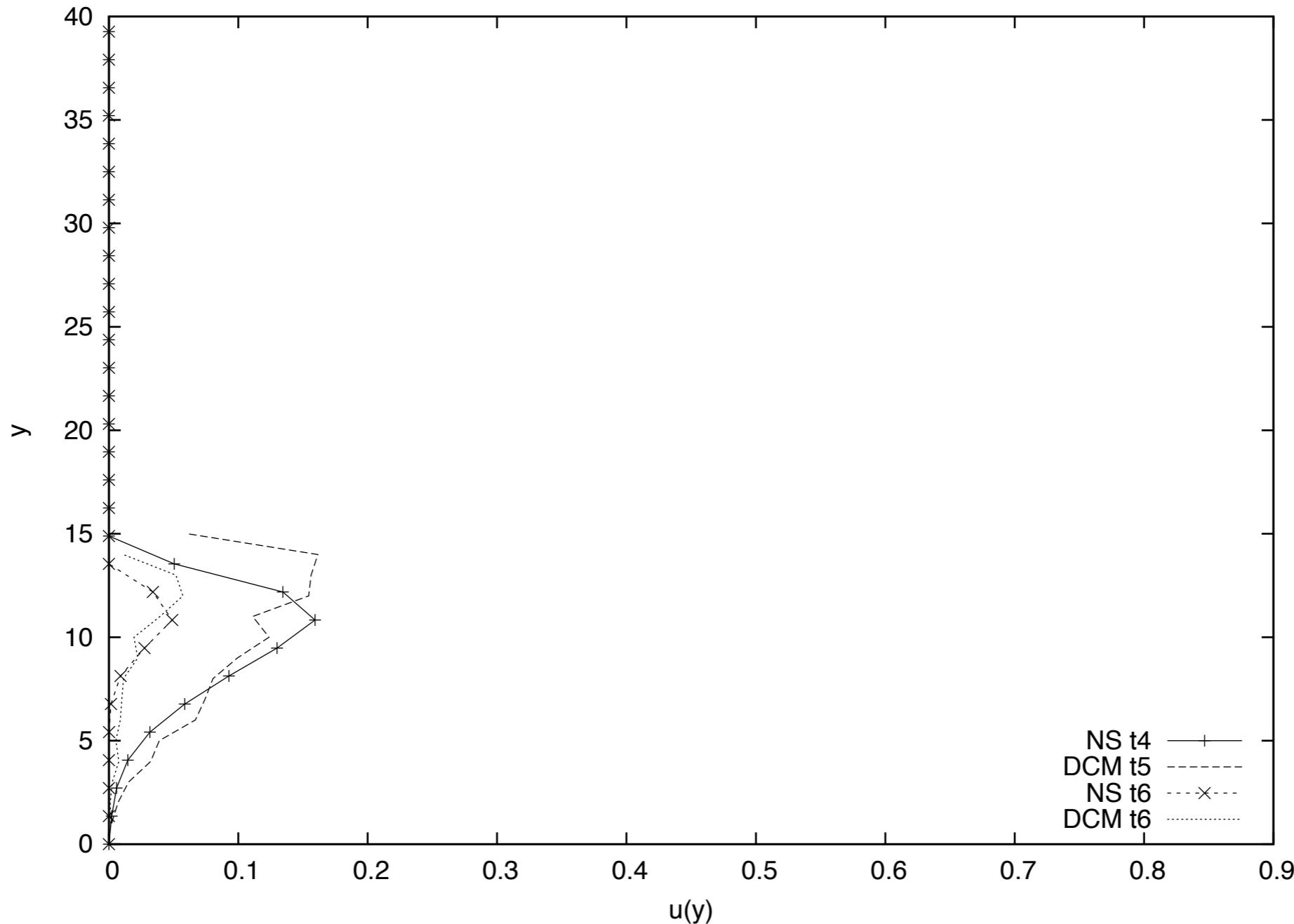
- comparaison of velocity profiles



DCM vs *Gerris* $\mu(l)$



- comparaison of velocity profiles

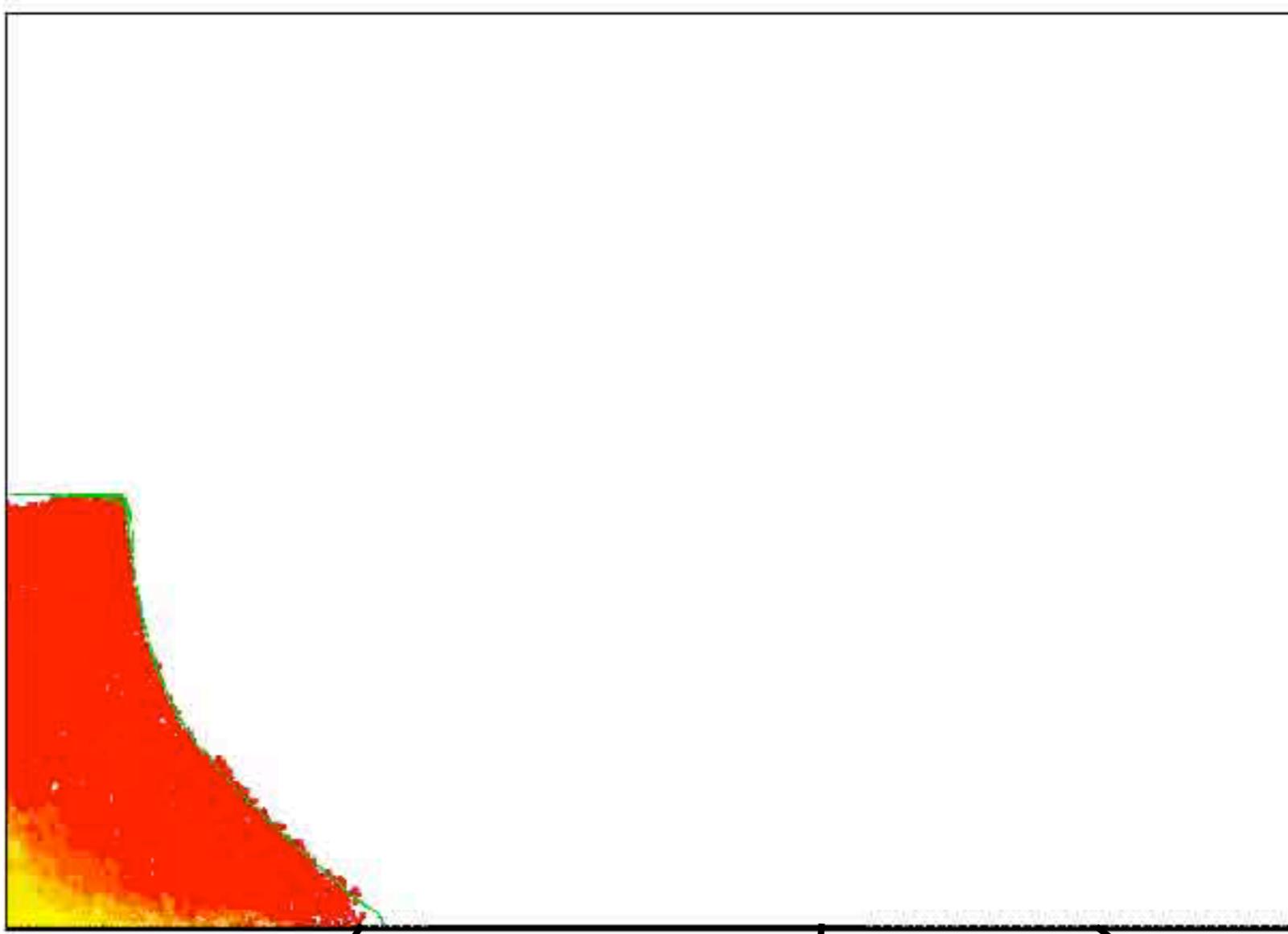


DCM vs *Gerris* $\mu(l)$

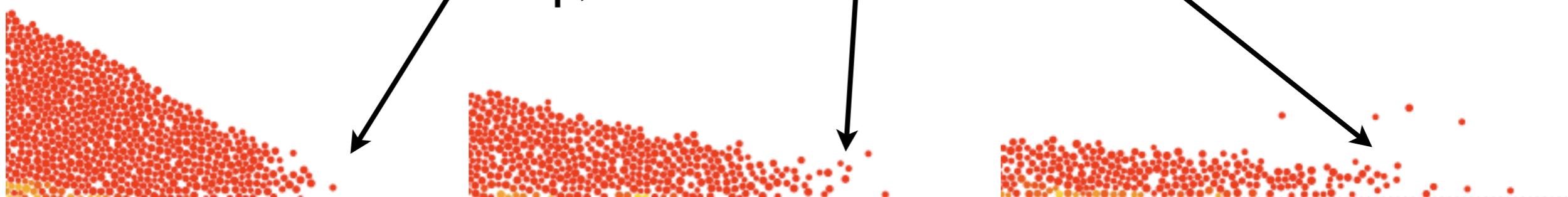


Collapse of columns simulation *Gerris* $\mu(l)$

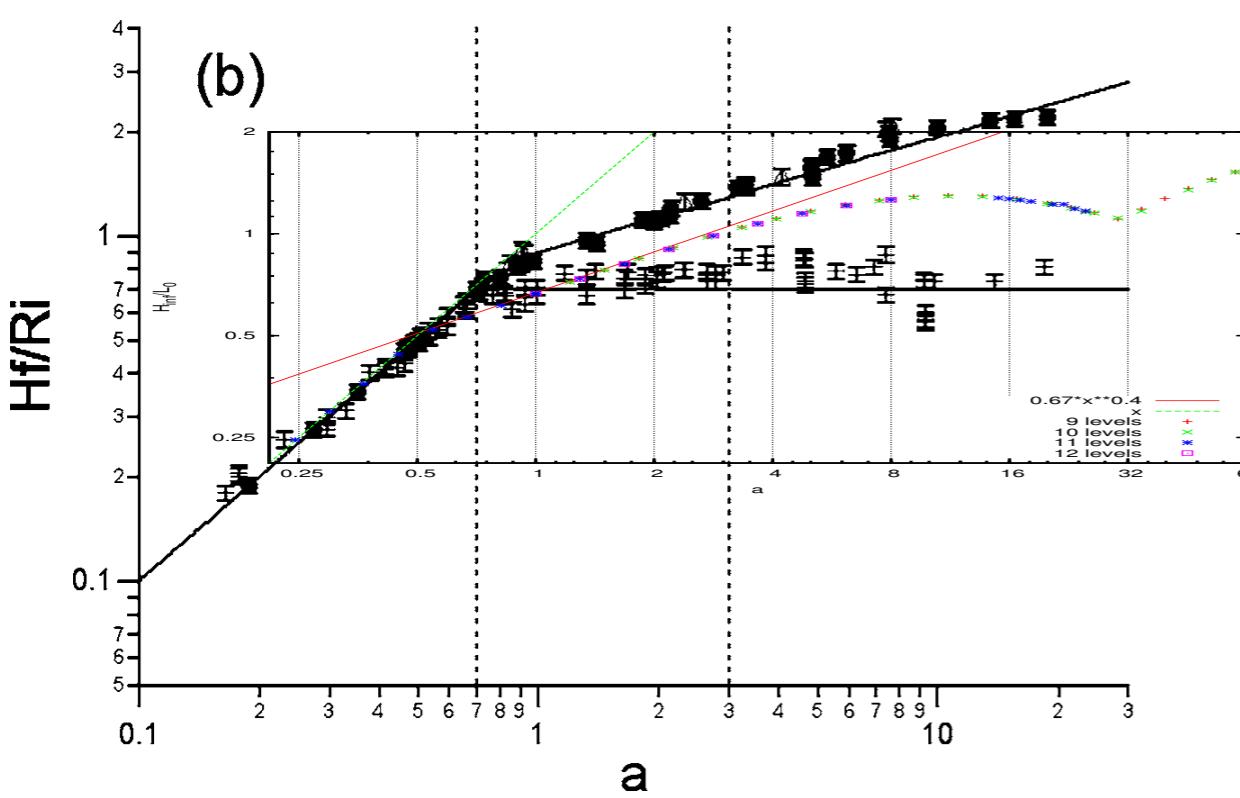
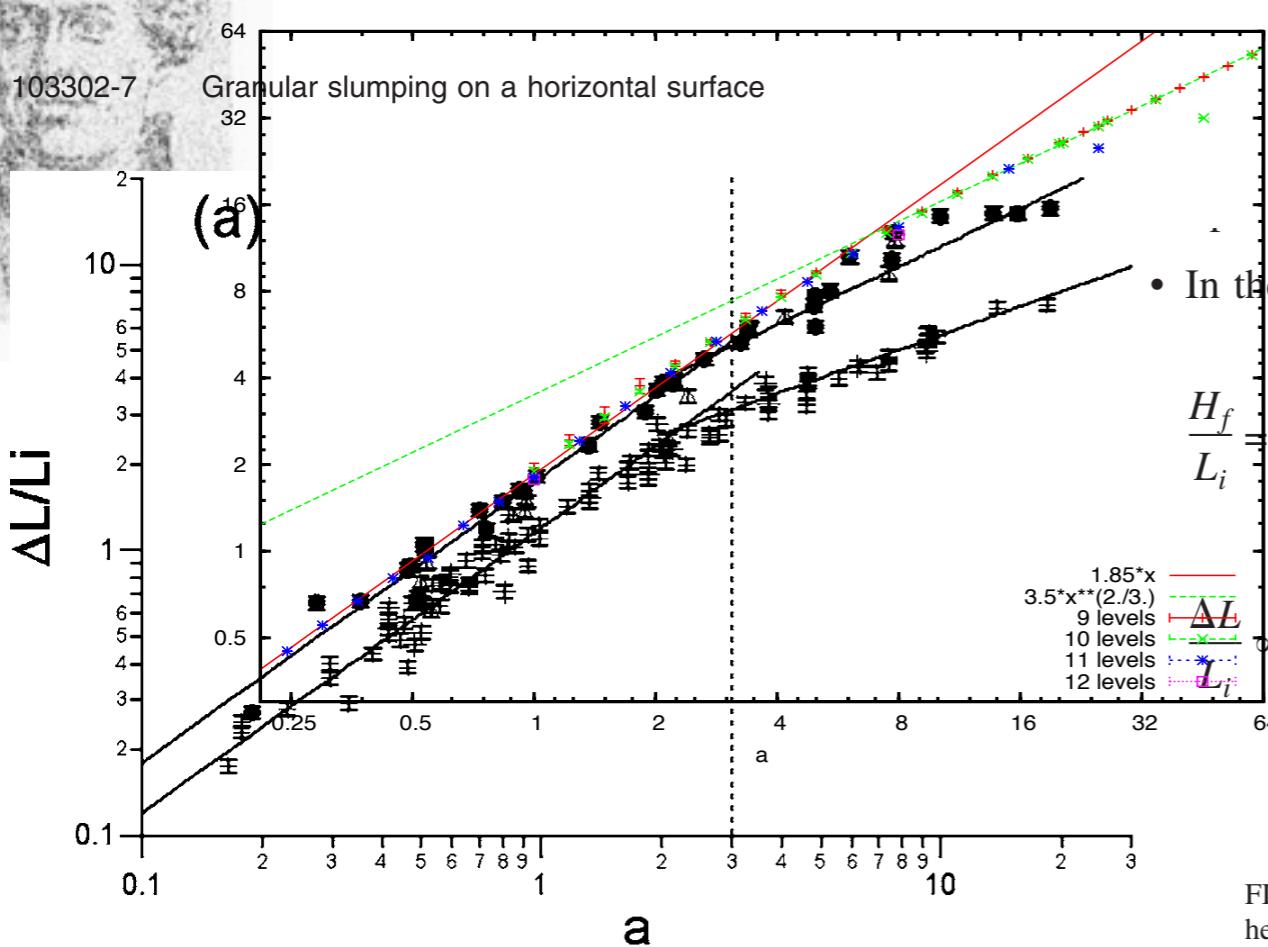
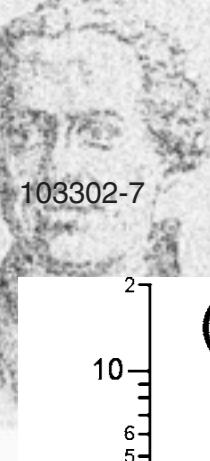
NS/CD t=0.9318



at the tip, $a=6.6$ t=1.33 2 2.66

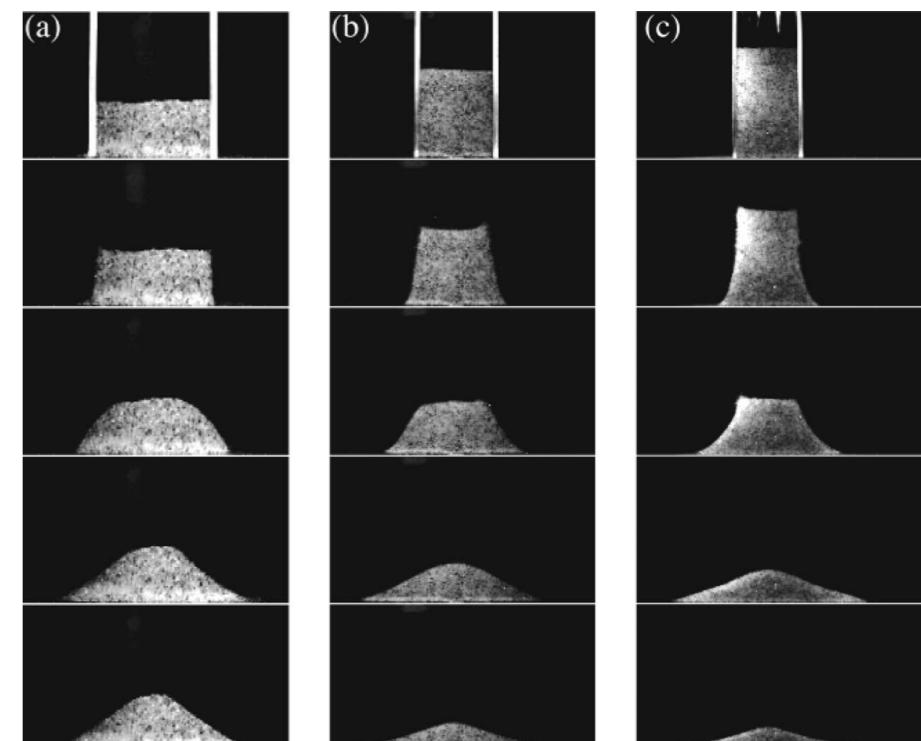


DCM vs *Gerris* $\mu(l)$



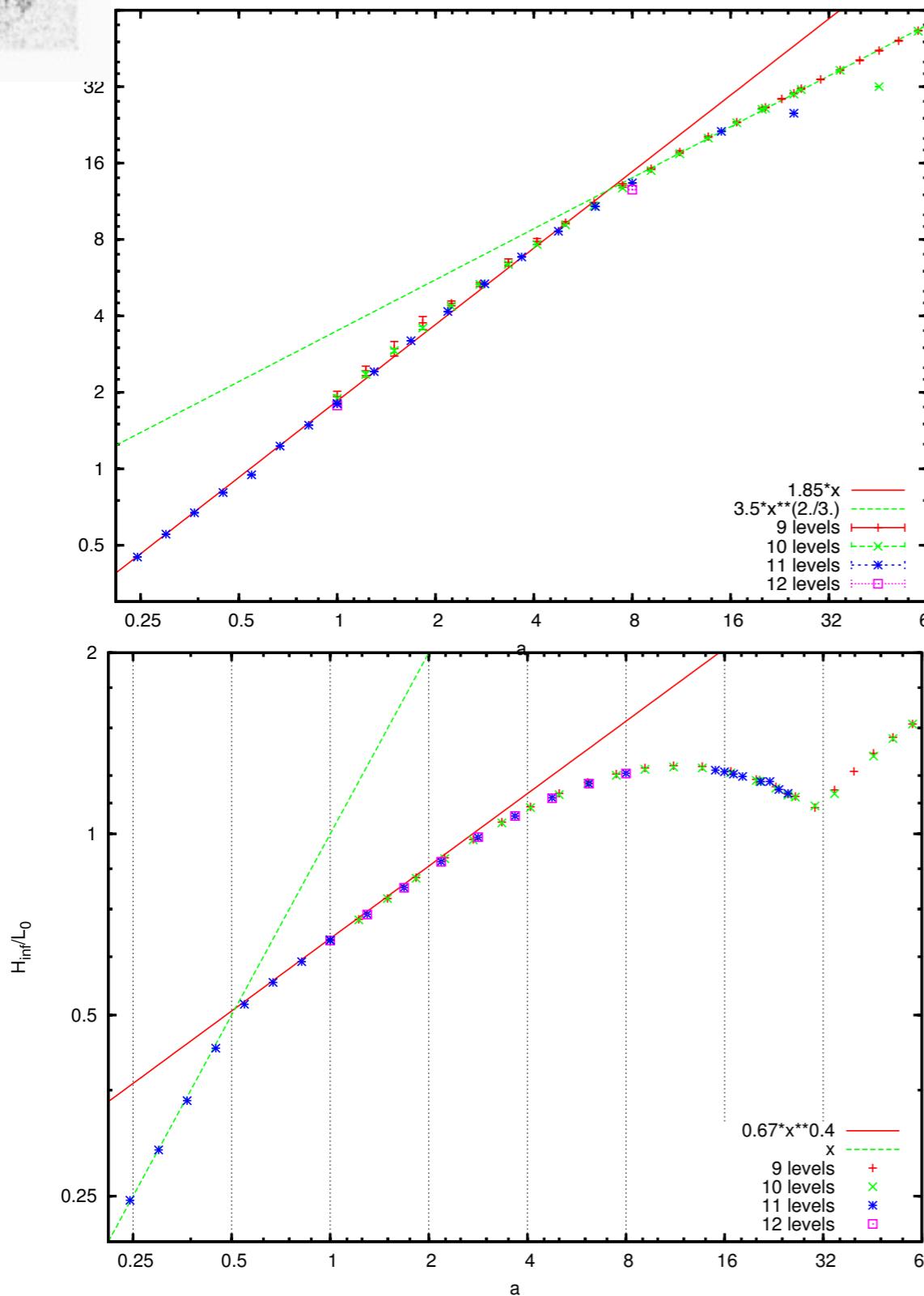
Phys. Fluids 17, 103302 (2005)

FIG. 6. Scaled runout $\Delta L/L_i$ (a) and scaled deposit height H_f/L_i (b) as functions of a . Circles and triangles correspond to experiments performed in the 2D channel working respectively with glass beads of diameter $d = 1.15$ mm or $d = 3$ mm. Crosses correspond to the data set of axisymmetric collapses from Lajeunesse *et al.* (Ref. 10).





Collapse of columns simulation *Gerris* $\mu(I)$



Normalised final deposit extent as a function of aspect ratio a .

Well-defined power law dependencies with exponents of 1 and $2/3$ respectively.

We recover the experimental scaling [Lajeunesse et al. 04] and [Staron et al. 05]. Differences between the values of the prefactors are due to the difficulties to obtain the run out length: friction in the Navier Stokes code tends to underestimate it, whereas direct simulation shows that the tip is very gaseous, it can no longer be explained by a continuum mechanic description.



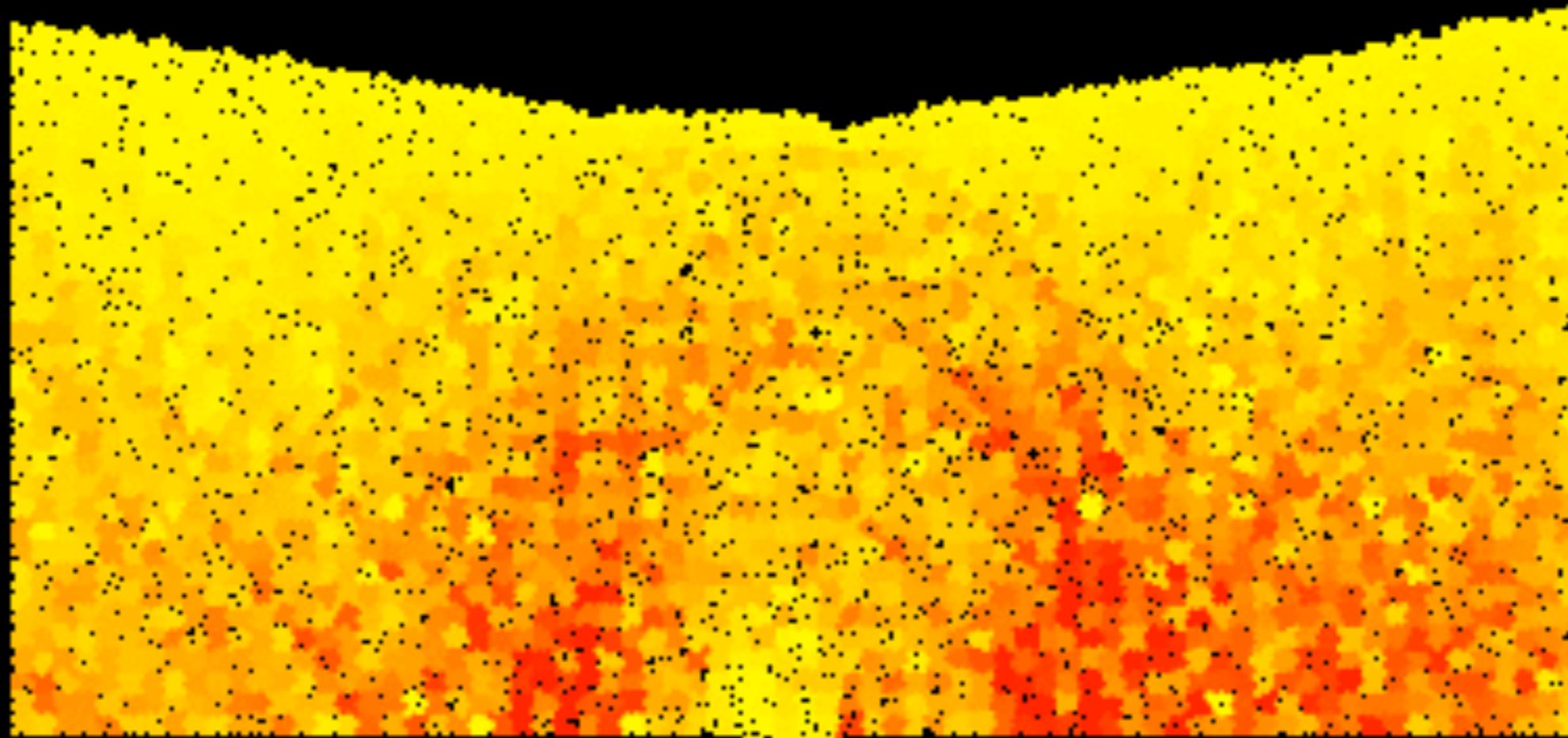
perspectives

- Other examples under investigation



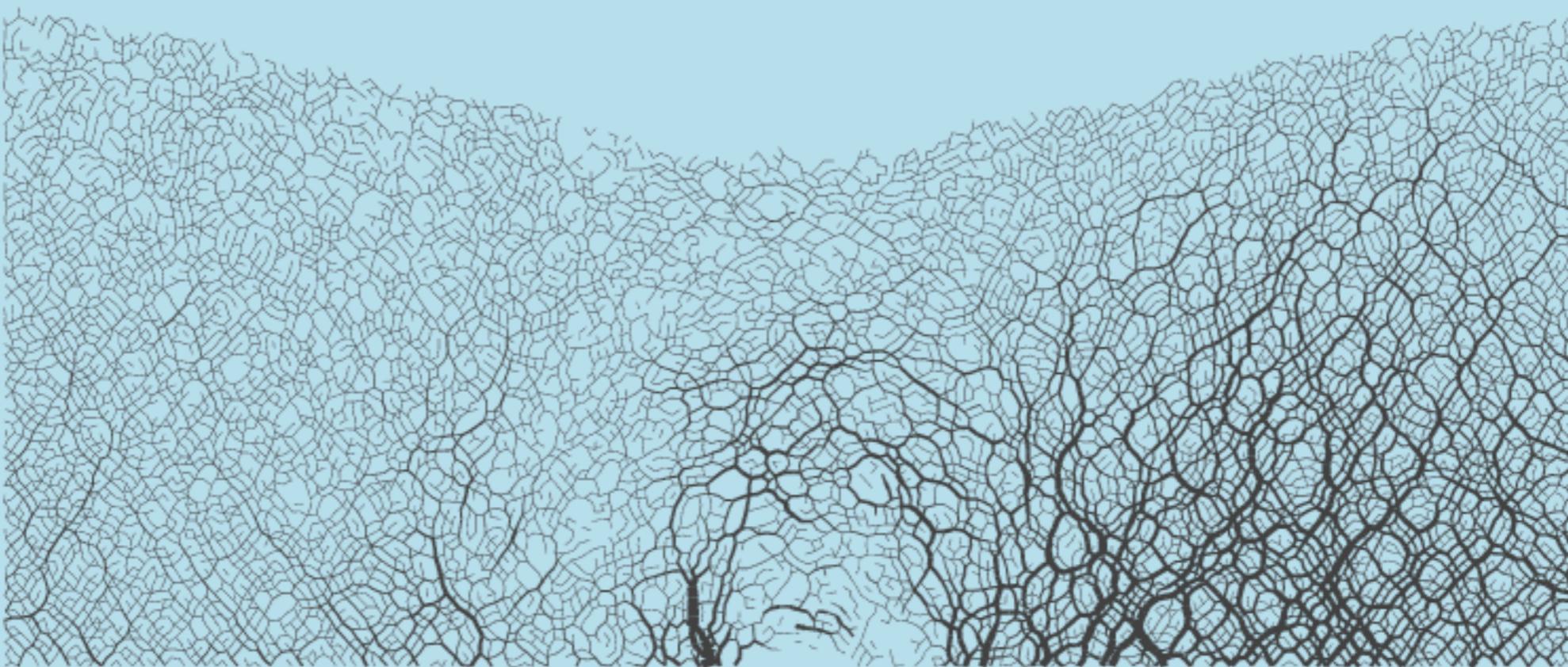
- Silo, hopper

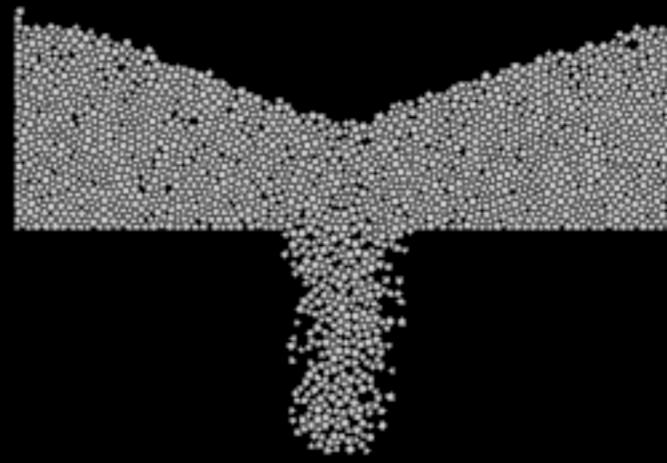






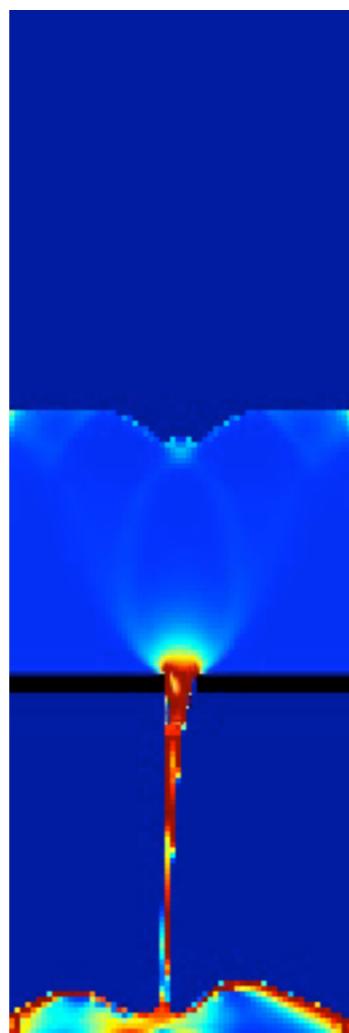
- Vidange de Silo



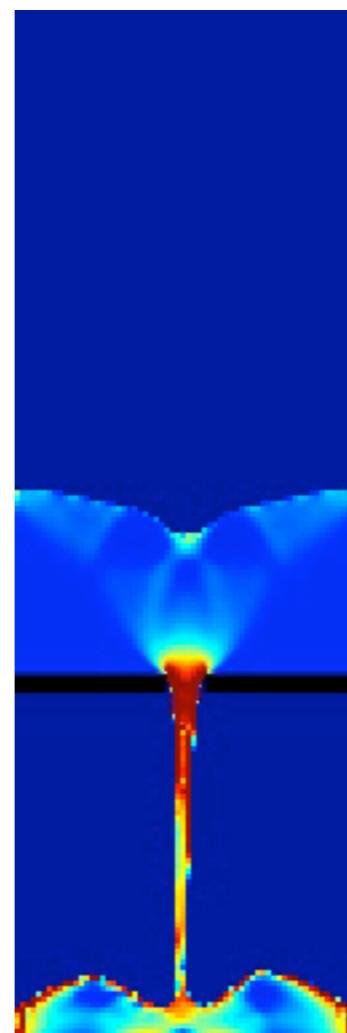




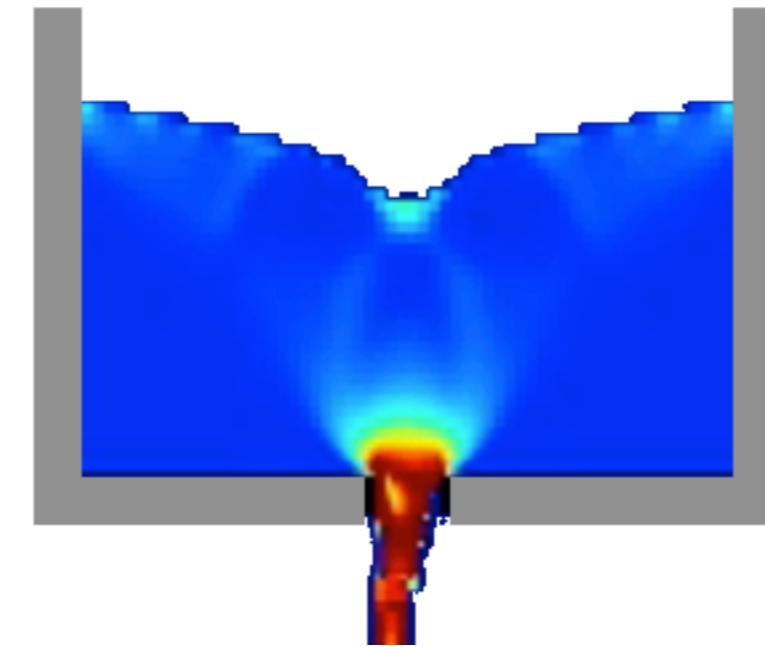
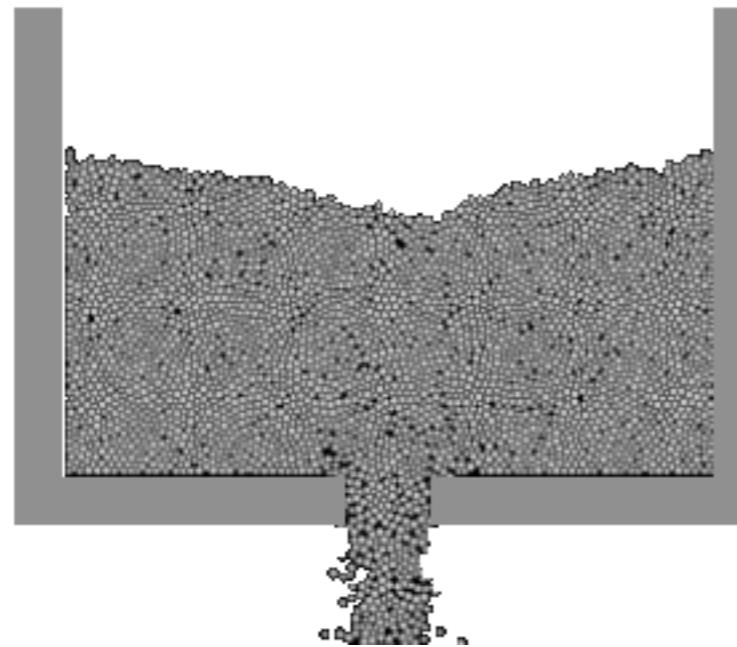
- Vidange de Silo



12 grains

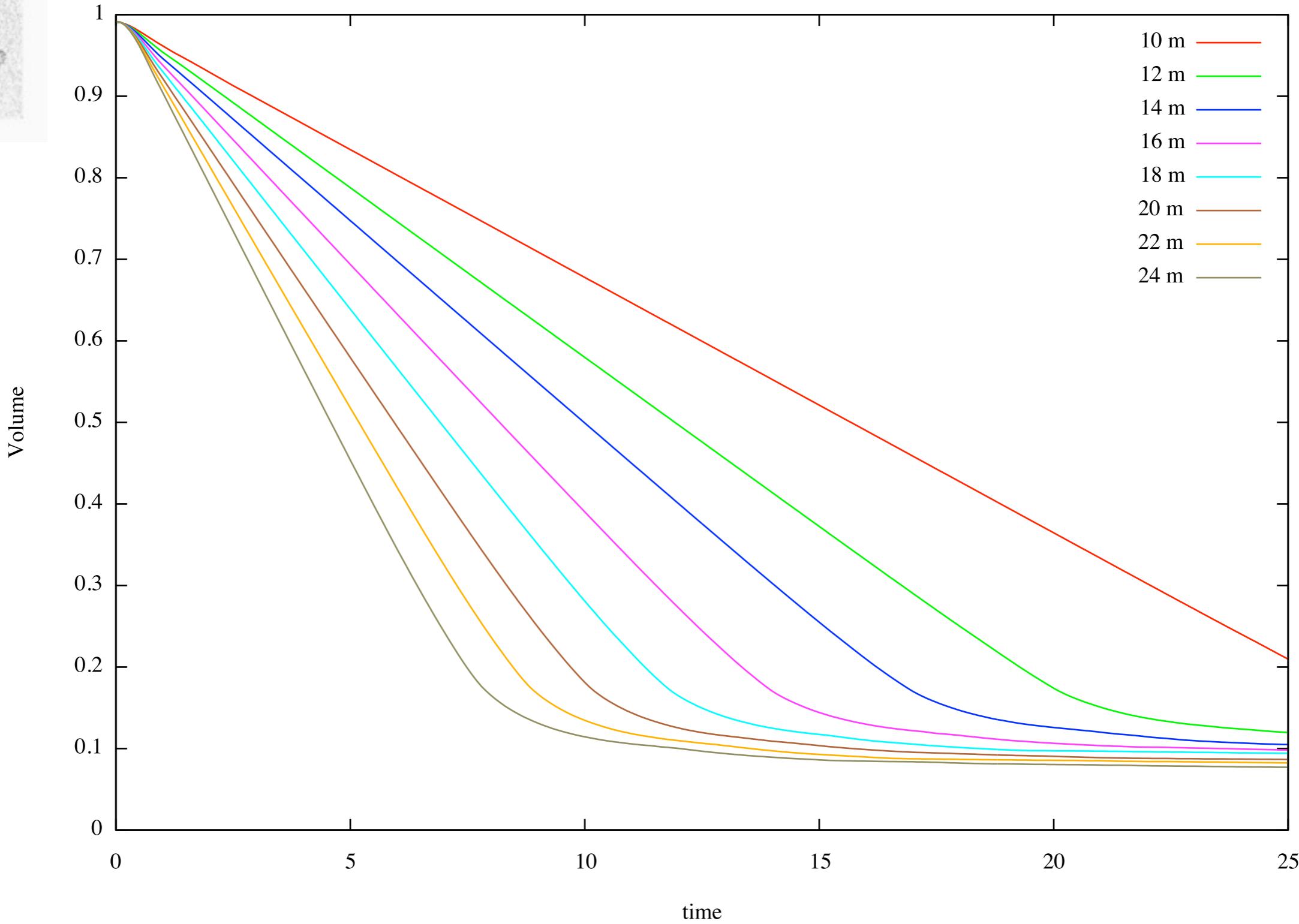


14 grains





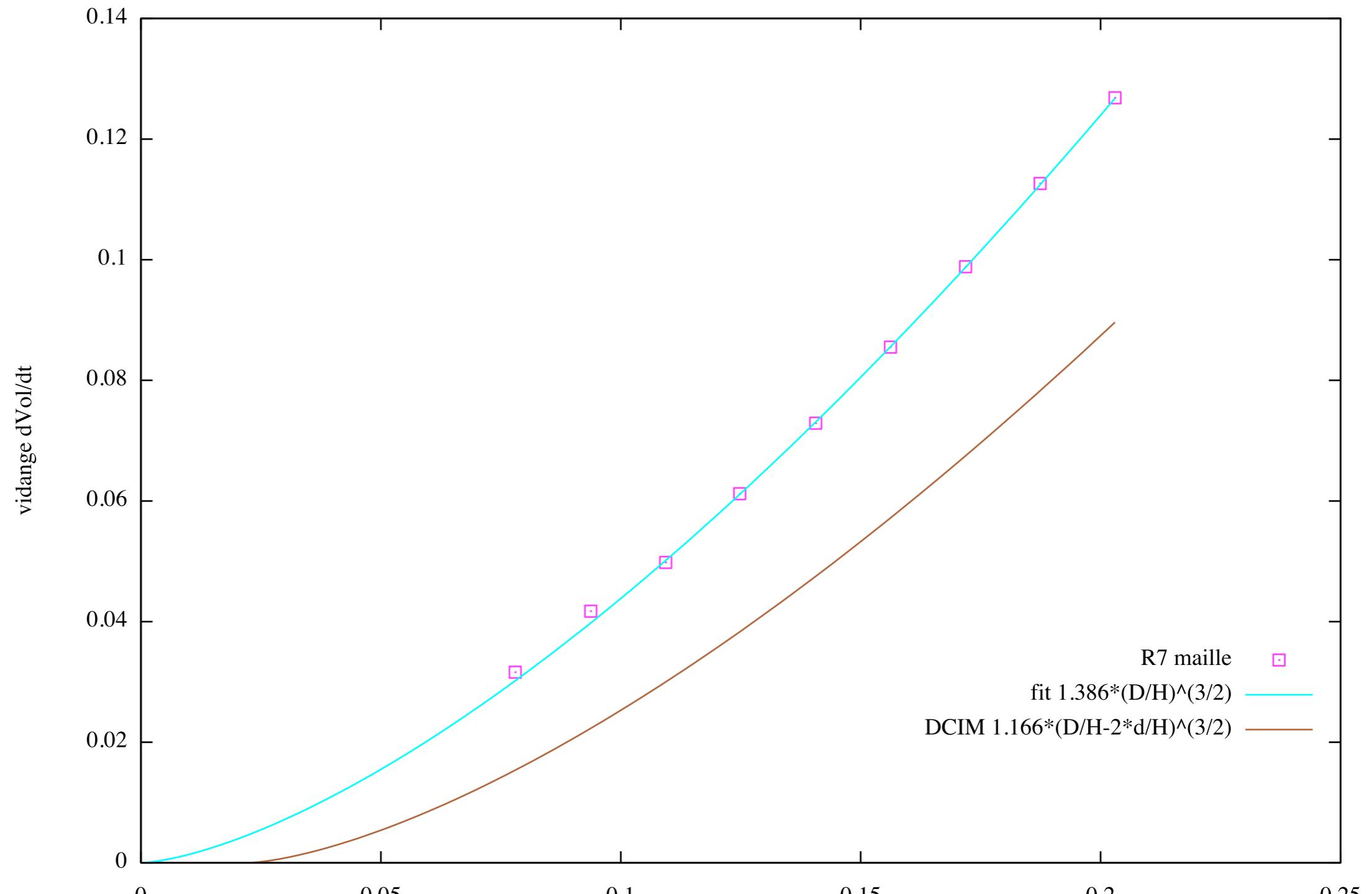
● Vidange de Silo





● Vidange de Silo

$$\frac{d\bar{V}}{dt} = 1.166 \left(\frac{D}{L} - k \frac{d}{L} \right)^{3/2}$$



loi de Beverloo Hagen



conclusion

- $\mu(l)$ obtained from experimental flows of dry granular flows [Jop et al. 06], implemented it in *Gerris*
- test case: analytical solution of steady avalanche (Bagnold solution)
- collapse of granular columns (shape as function of time compared to Discrete Simulations).
- The experimental trends of the scaling of the run out are reobtained
- Saint Venant Savage Hutter to be compared with.
- complete spectra: discrete grains/ Saint Venant/ Navier Stokes and plasticity

This opens the door to systematic studies of granular flows using this continuum approach.



références:

- P. Jop, Y. Forterre, O. Pouliquen, (2006) "A rheology for dense granular flows", *Nature* 441, pp. 727-730 (2006)
- P.-Y. Lagrée, L. Staron and S. Popinet (2011) "The granular column collapse as a continuum: validity of a two-dimensional Navier–Stokes model with a $\mu(I)$ -rheology", *JFM* pp 1-31 , doi:10.1017/jfm.2011.335
- E. Lajeunesse, A. Mangeney-Castelnau, and J.-P. Vilotte, (2004) "Spreading of a granular mass on an horizontal plane», *Phys. Fluids*, 16(7), 2371-2381.
- L. Staron & E. J. Hinch (2005) "Study of the collapse of granular columns using two-dimensional discrete-grain simulation", *J. Fluid Mech.* (2005), vol. 545, pp. 1–27.