ANR Grain de Sable 11/12/08

Pierre-Yves Lagrée

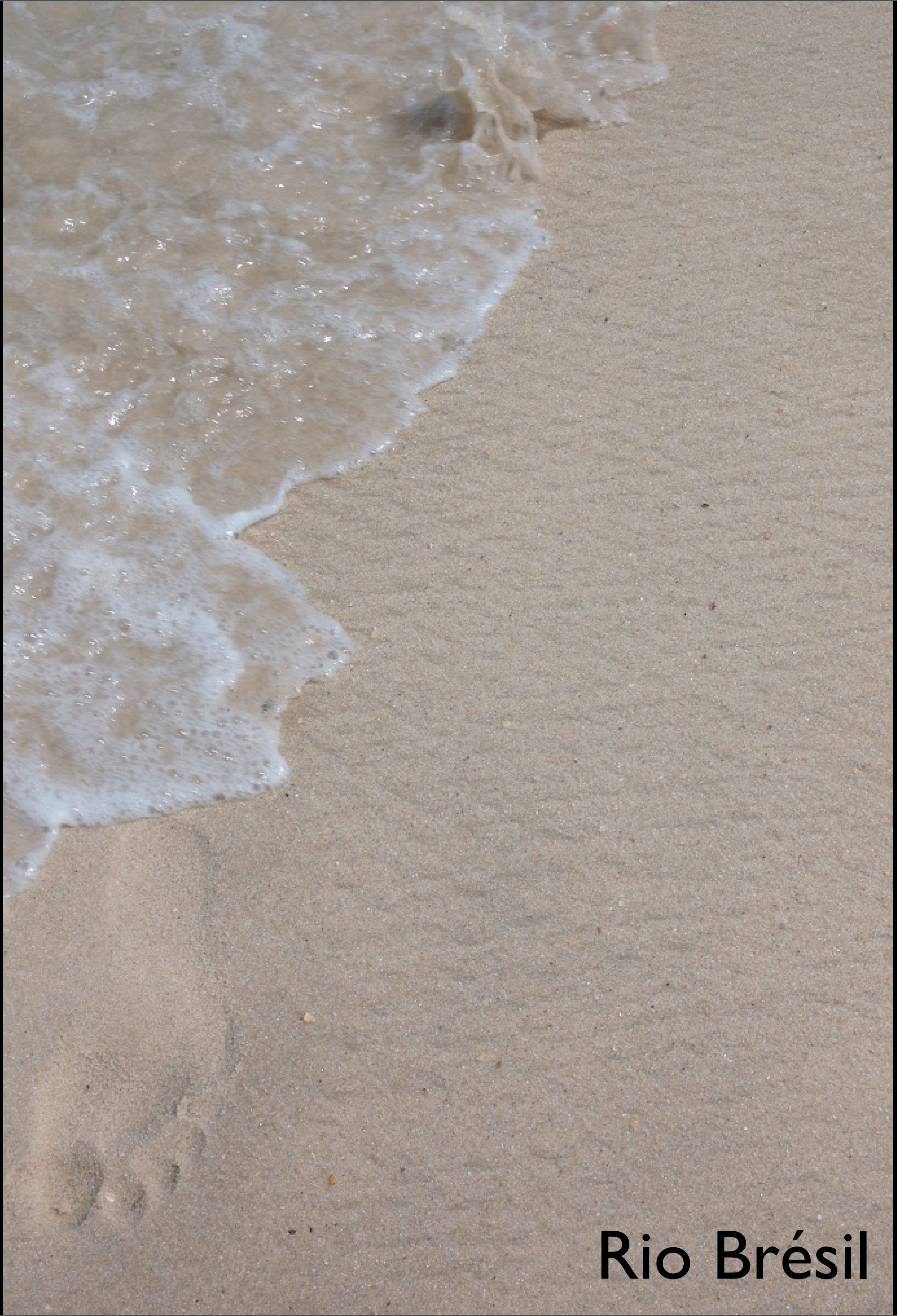
“Chevrons & Rides dans les écoulements à fond érodable”

Institut Jean Le Rond d’Alembert *ex LMM*

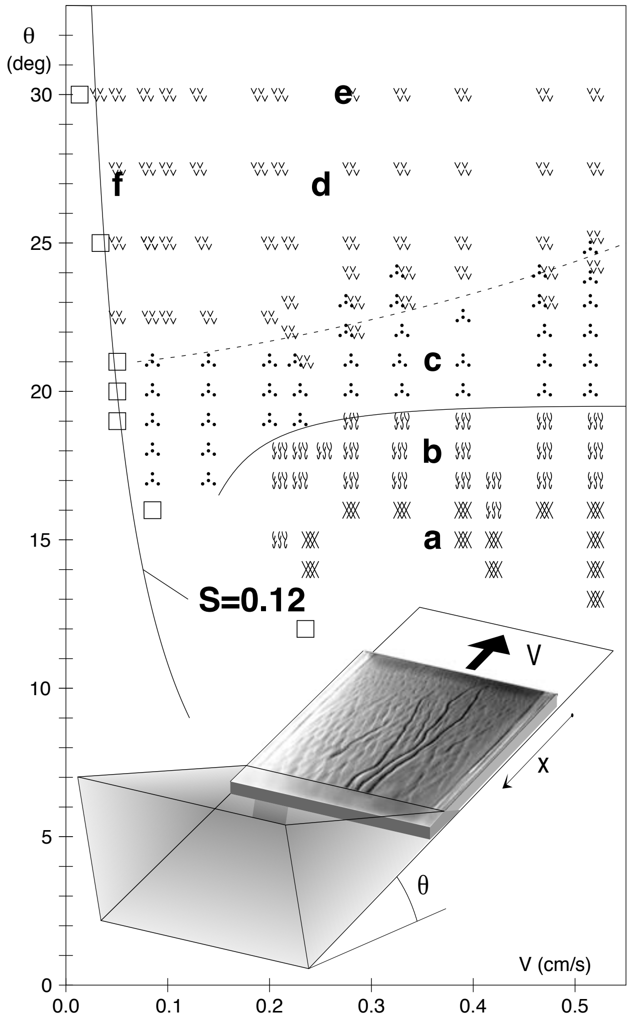
Olivier Devauchelle, Christophe Josserand
(Daniel Lhuillier, Lydie Staron): FCIH IJLRA

Kouamé Kouakou, Kanh-Dang Nguyen Thu-Lam

Luce Malverti, Eric Lajeunesse, François Métivier: IPGP


Rio Brésil

Goleta beach, Santa Barbara USA O. Devauchelle



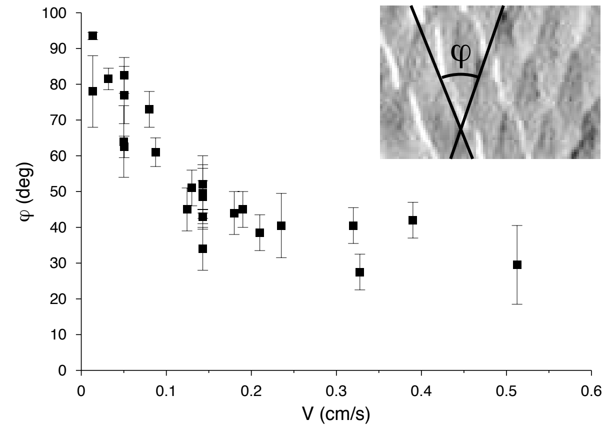


FIG. 3: Chevron alignment angle as a function of velocity. Error bars indicate measurement variations

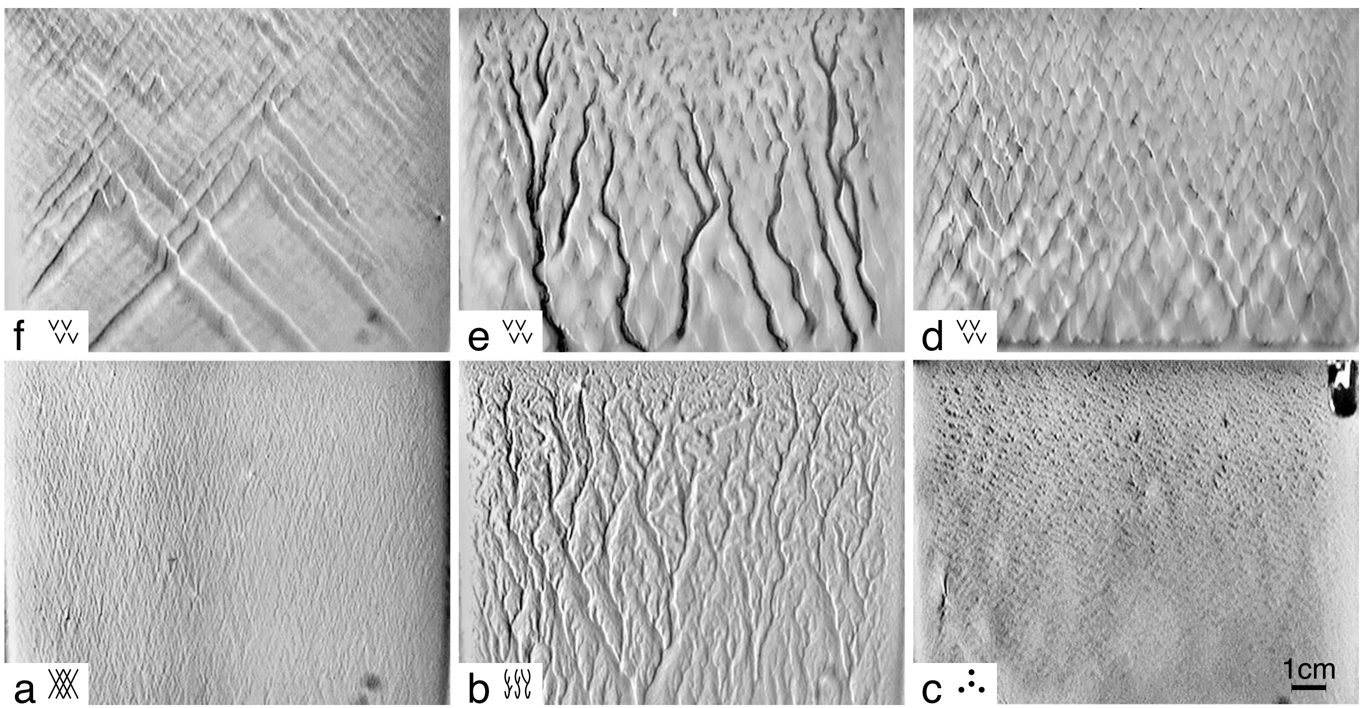


FIG. 2: Patterns observed in the erosion experiment: **a** crossed hatched pattern, **b** disordered branched pattern, **c** orange skin, **d** chevron structure, **e** chevrons with oblique channels, **f** localized pulses at chevron onset. The layer appears darker where it has been eroded because the bottom plate is black. A light source to the left creates additional shading.

Daerr, A., Lee, P., Lanuza, J. & Clement, E. 2003 Erosion patterns in a sediment layer. Physical Review E 67

RHOMBOID RIPPLE MARK.

A. O. WOODFORD.

Bucher (p. 153, 1919) has proposed the term "*rhomboid (current-) ripple*" for "*small rhomboidal, scale-like tongues of sand*, arranged in a reticular pattern" produced experimentally by Engels (1905) as the first effect of transportation by a water current in gentle, uniform flow. But violent currents in water also impress rhomboidal patterns on sand, and hence, in this paper, the term *rhomboid ripple mark* will be used in a descriptive sense, to include all sharply rhomboid patterns developed on the surface of a mobile sediment. An example is given in Fig. 1. Braided rills which are not sharply and regularly rhomboid in pattern, are not included. Neither are the numerous V-shaped grooves which spread from the snouts of partly buried sand crabs (Hippidae, *Emerita analoga* in California), and which may in combination suggest an irregularly rhombic pattern.

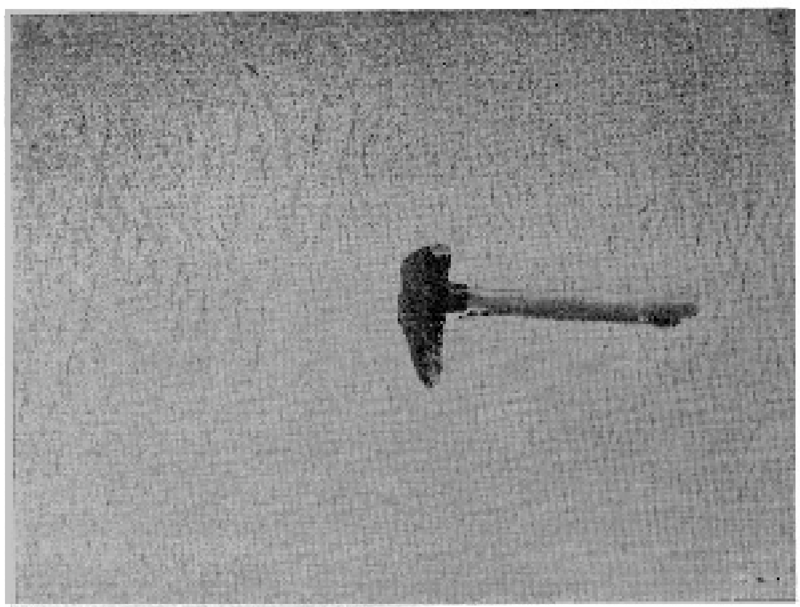


Fig. 1. Rhomboid ripple mark, Laguna Beach, Calif., March 29, 1933. The hammer gives the scale.

Several authors (Kindle: p. 34 and pl. 19b, 1917; Johnson: pp. 515-517, 1919; Kindle and Bucher: pp. 655, 656, 1932) describe and figure rhomboid ripple marks from modern beaches. In 1917 Kindle ascribed the imbricated pattern to, "The action of very small waves lapping and crossing each other from opposite sides of a miniature spit," but in 1932 Kindle and Bucher were inclined to explain the pattern in the light of the Engels' experiment mentioned above. Johnson calls the structures "backwash marks," and says (p. 517, 1919): "The thin sheet of water returning down the beach slope appeared to be split into diverging minor currents by every patch of more compact sand or particle of coarser material which impeded its progress, and the crossing of these minor currents resulted in the criss-cross pattern in the sand."

INTERFERENCE PATTERN UNDER RAPID FLOW.

The rhomboid pattern formed on sand looks very much like an interference effect. Therefore, before describing the

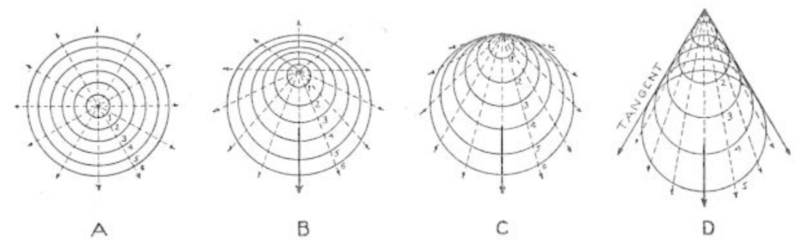
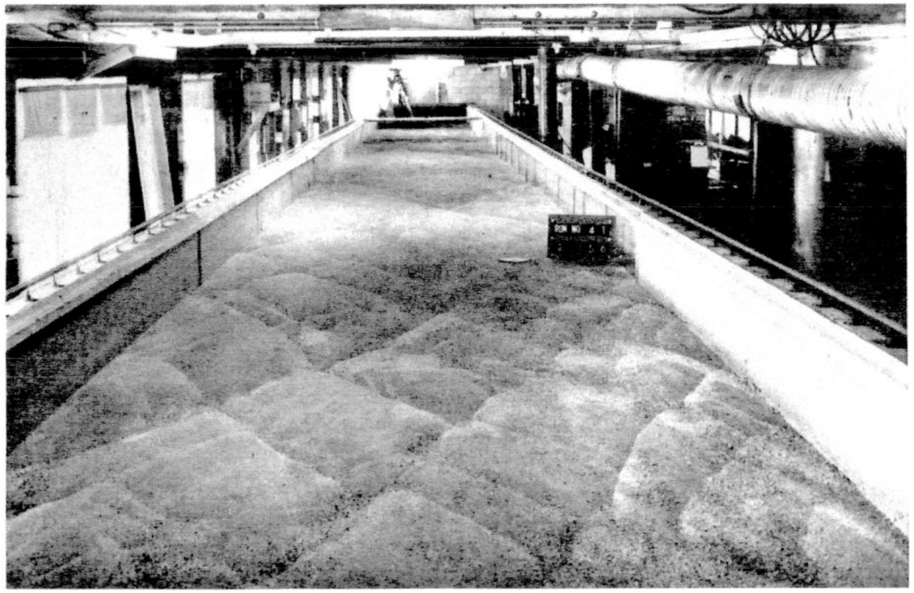


Fig. 2. Schematic sketches showing wave impulses spreading from a point, affected by various rates of flow. See text for explanation. After Rehbock.

observed pattern in detail, there will be presented some generalities concerning the waves which may form in water currents.

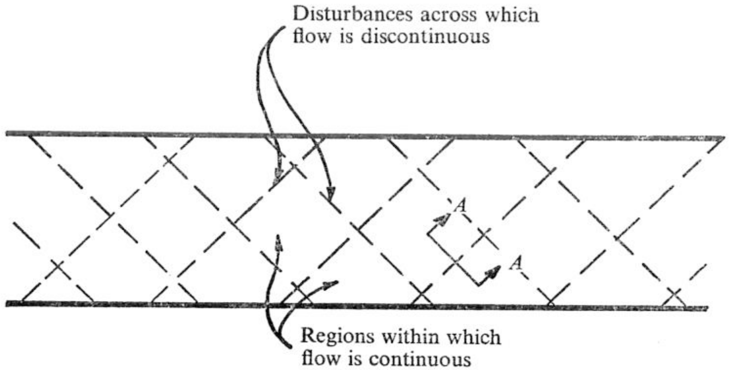
First of all, the distinction must be made between *tranquil flow* and *rapid flow* (Rehbock: 1930; Bakhmeteff: 1932). In tranquil flow, the average velocity of the water is less than the wave velocity for the given depth; in rapid flow it is greater. The effect on waves is shown in Fig. 2, after Rehbock. If a pebble is tossed into quiet water, concentric waves are produced (A). If the water is in tranquil flow, the ripples are distorted (B). If a certain critical velocity is equaled or exceeded, the waves cannot be propagated upstream, but only down (C and D). In D there is suggested a cause for the

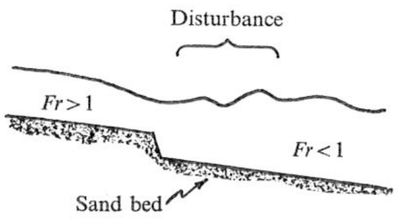
Chang Simons JFM 70



(c)

FIGURE 2. Diagonal bed patterns in a laboratory flume with large width to depth ratios and with the flow nearly critical. (a) Froude number = 0·92, width to depth ratio = 24. (b) Froude number = 0·83, width to depth ratio = 28·5. (c) Froude number = 1·12, width to depth ratio = 18.



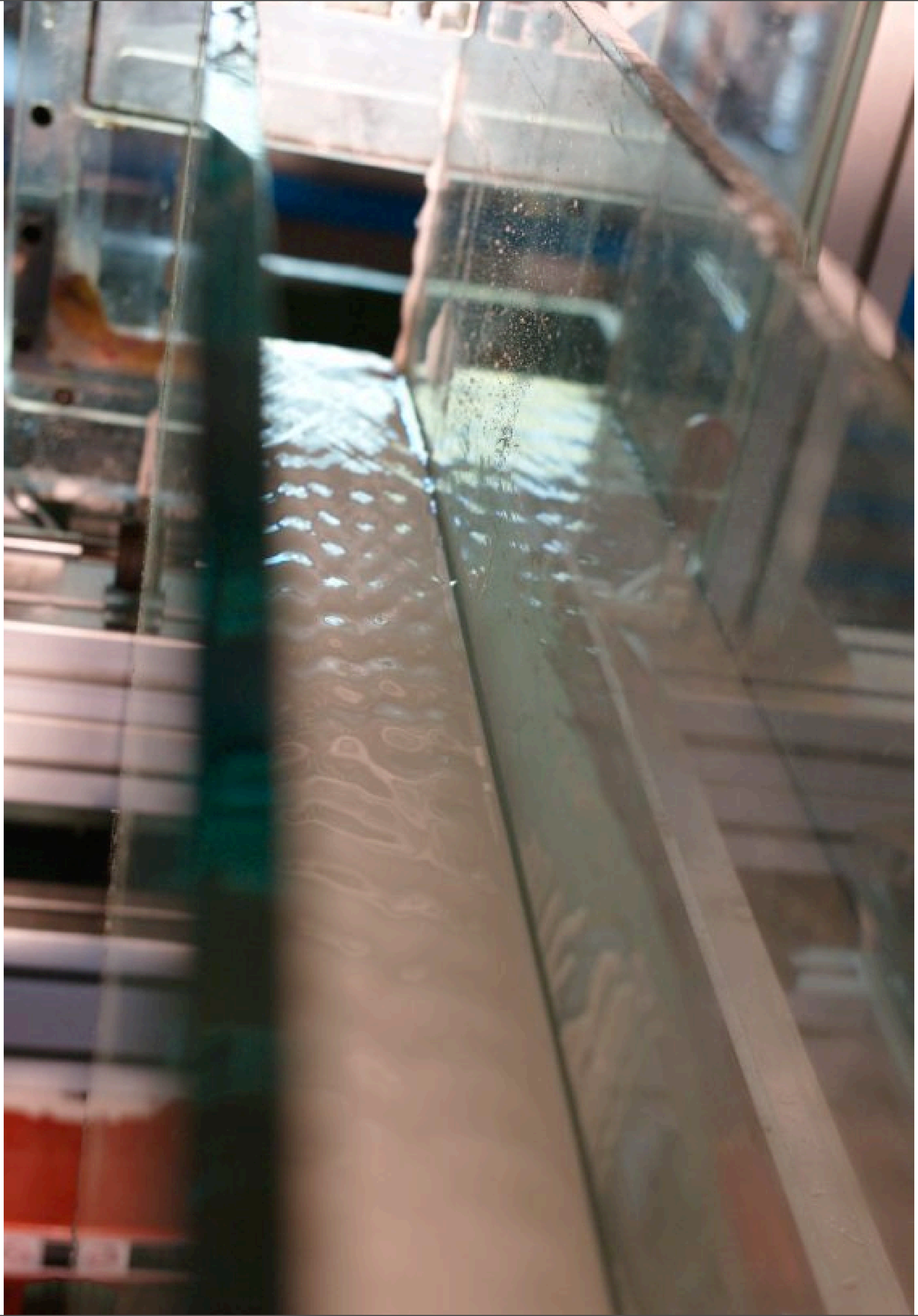


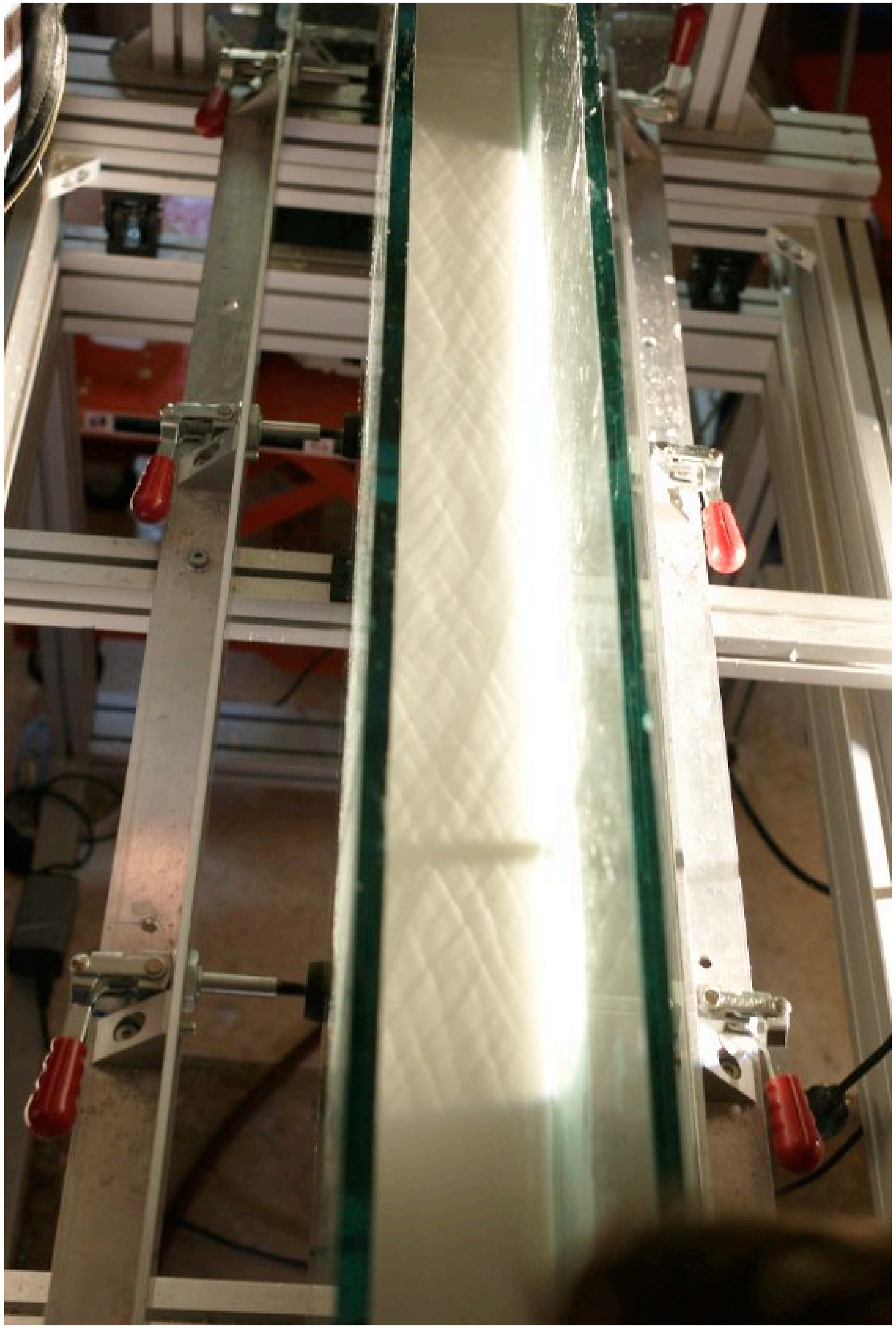
Section *A–A*

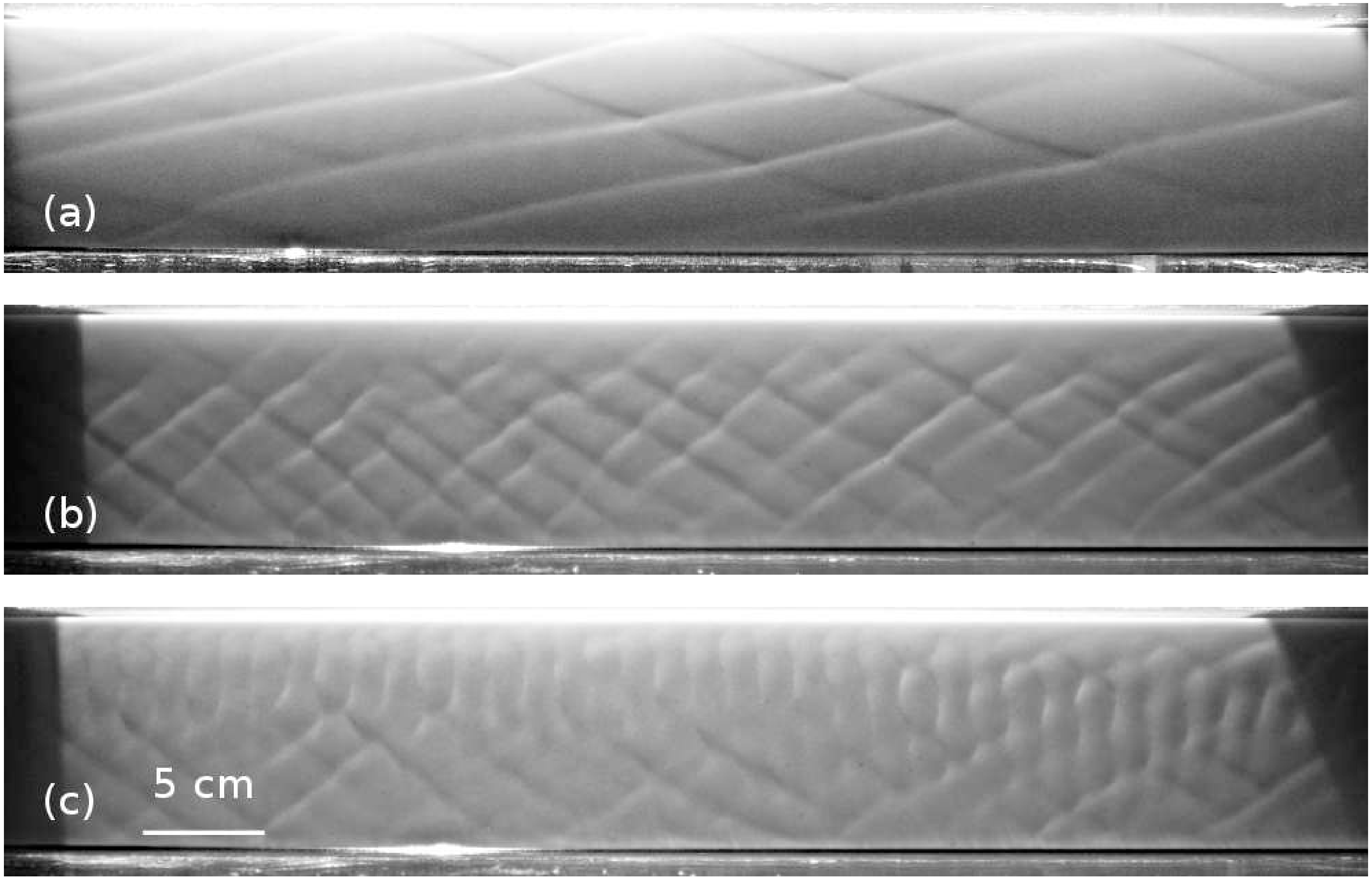
Schematic drawing showing diagonal lines in shallow channel flow with Froude number near unity.

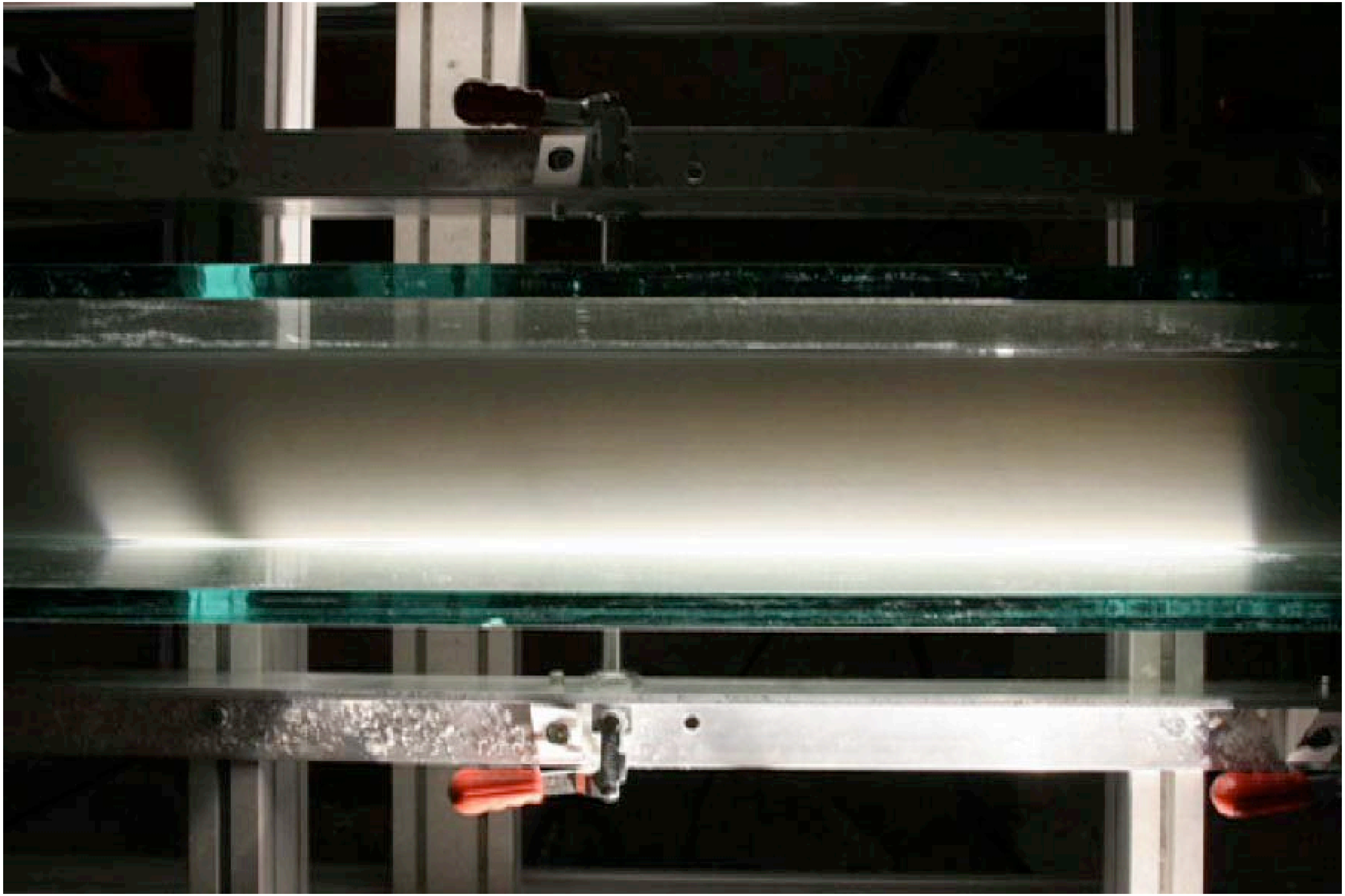
$$N = \begin{vmatrix} U & W & 0 & 0 & g & 0 & 0 & 0 \\ 0 & 0 & U & W & 0 & g & 0 & 0 \\ h & 0 & 0 & h & U & W & 0 & 0 \\ 0 & \dfrac{Wq_1}{U^2} & 0 & -\dfrac{q_1}{U} & 0 & 0 & -1 & -\dfrac{W}{U} \\ dx & dz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & dx & dz & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & dx & dz & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & dx & dz \end{vmatrix}.$$

IPGP Saint-Maur



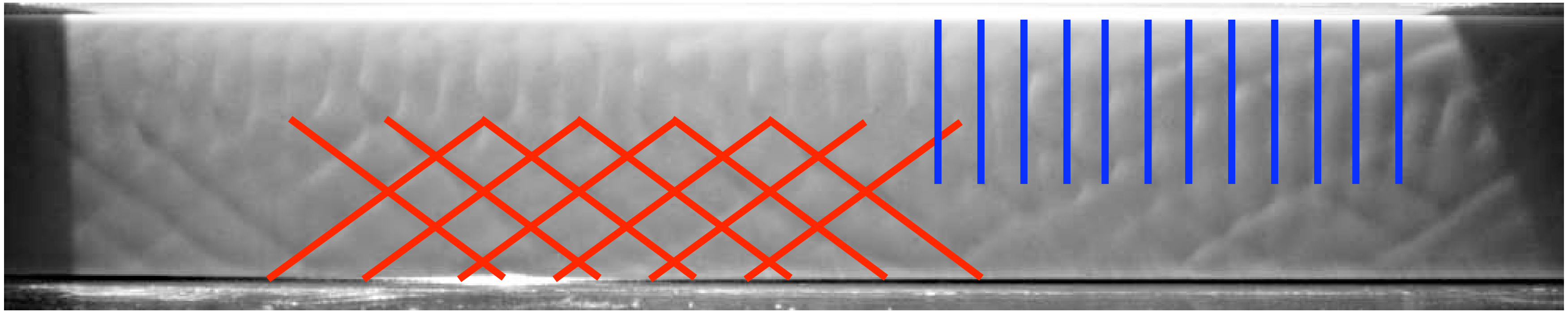




(a)
(b)
(c)
5 cm



approche Saint-Venant

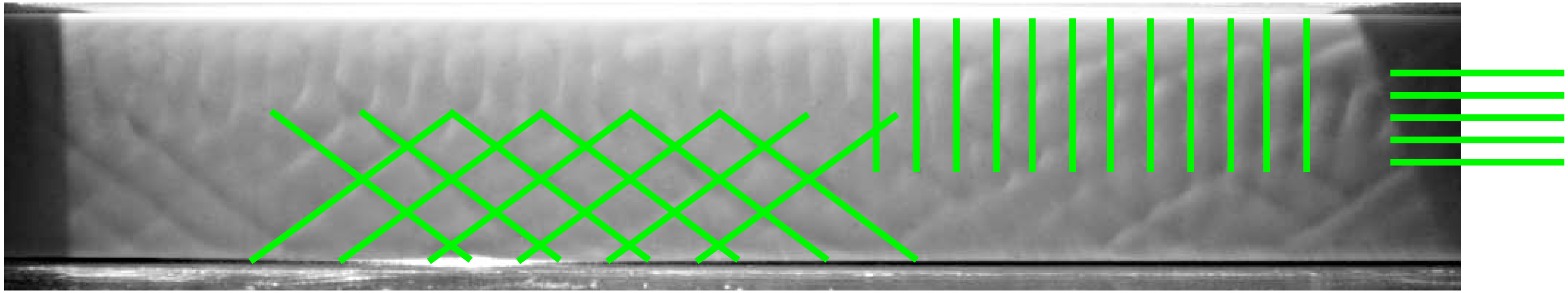
approche asymptotique



approche Saint-Venant

approche asymptotique

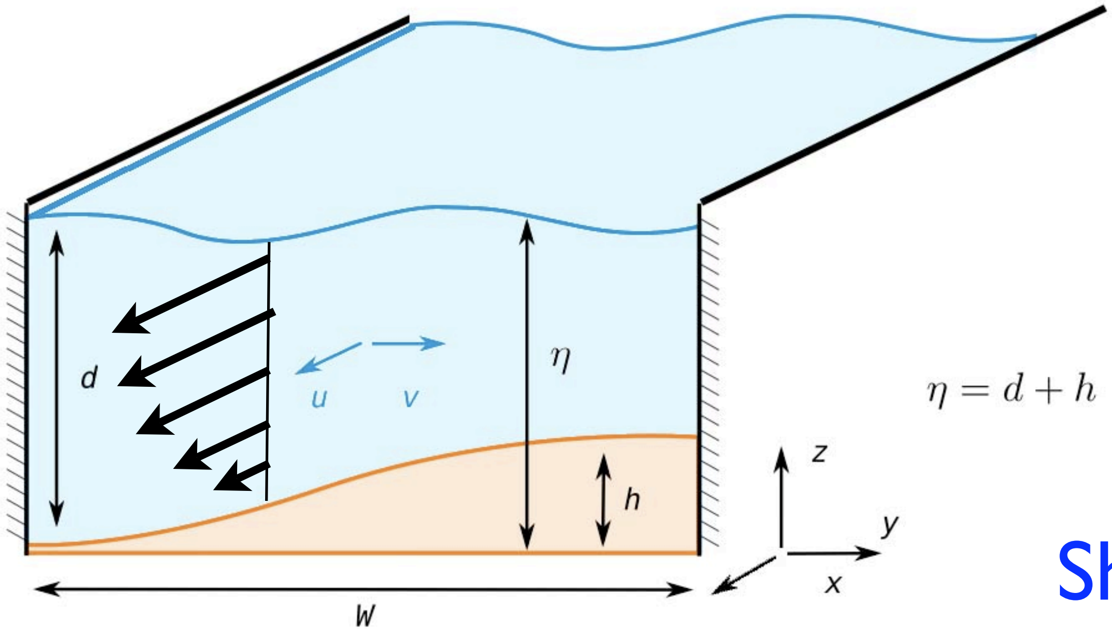
approche stabilité linéaire 3D complète



approche Saint-Venant

Flow Model

$$\int_{z=h}^{z=\eta} dz \quad \text{(Navier Stokes)}$$



+Poiseuille profile

+ hydrostatic balance

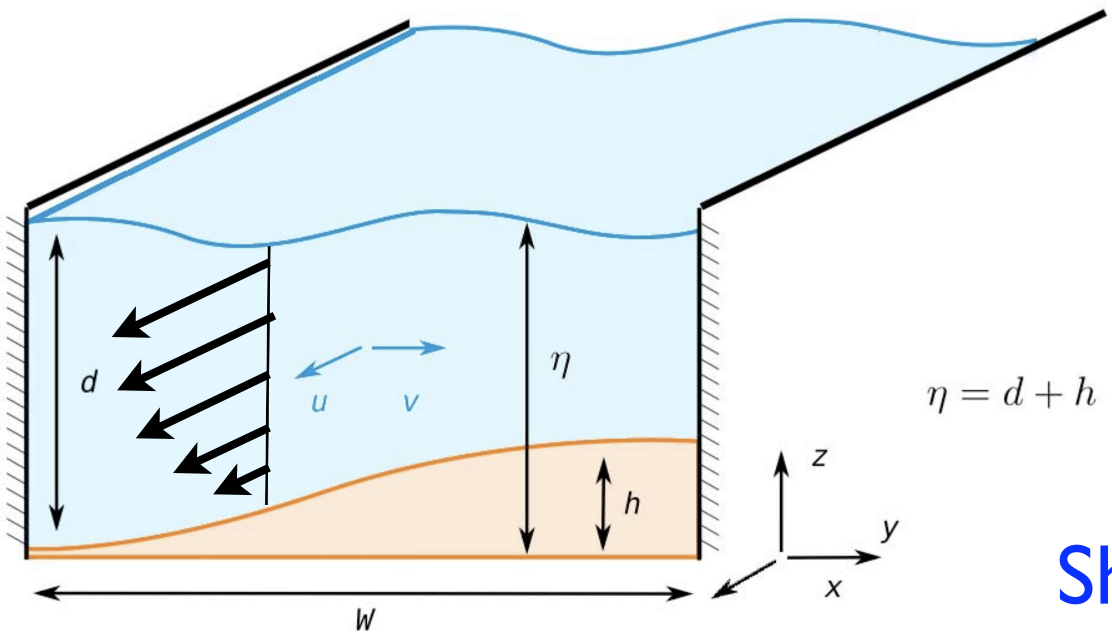
Shallow water - Saint Venant

approche Saint-Venant

Flow Model

$$\frac{6}{5}(\overrightarrow{u}\cdot\overrightarrow{\nabla})\overrightarrow{u} = -g(\overrightarrow{\nabla}\eta + sin(\theta)\overrightarrow{e}_x) - \frac{3\nu\overrightarrow{u}}{(\eta-h)^2}$$

$$\overrightarrow{\nabla}\cdot(\overrightarrow{u}(\eta-h)) = 0$$



+Poiseuille profile

+ hydrostatic balance

Shallow water - Saint Venant

approche Saint-Venant

Flow Model

laminar

$$\frac{6}{5}F^2 u_l \partial_l u_i = S\delta_{i,1} - \partial_i(d+h) - S\frac{u_i}{d^2}$$

$$\partial_l(du_l) = 0$$

$$F = \frac{U}{\sqrt{gh}}$$

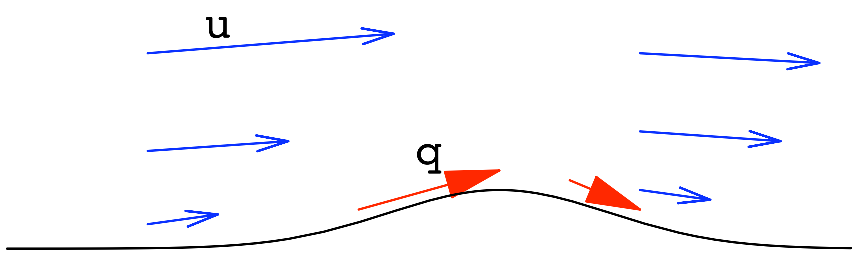
$$\mathrm{Re} = \frac{3F^2}{S}.$$

$$\tau_i = \frac{u_i}{d}$$

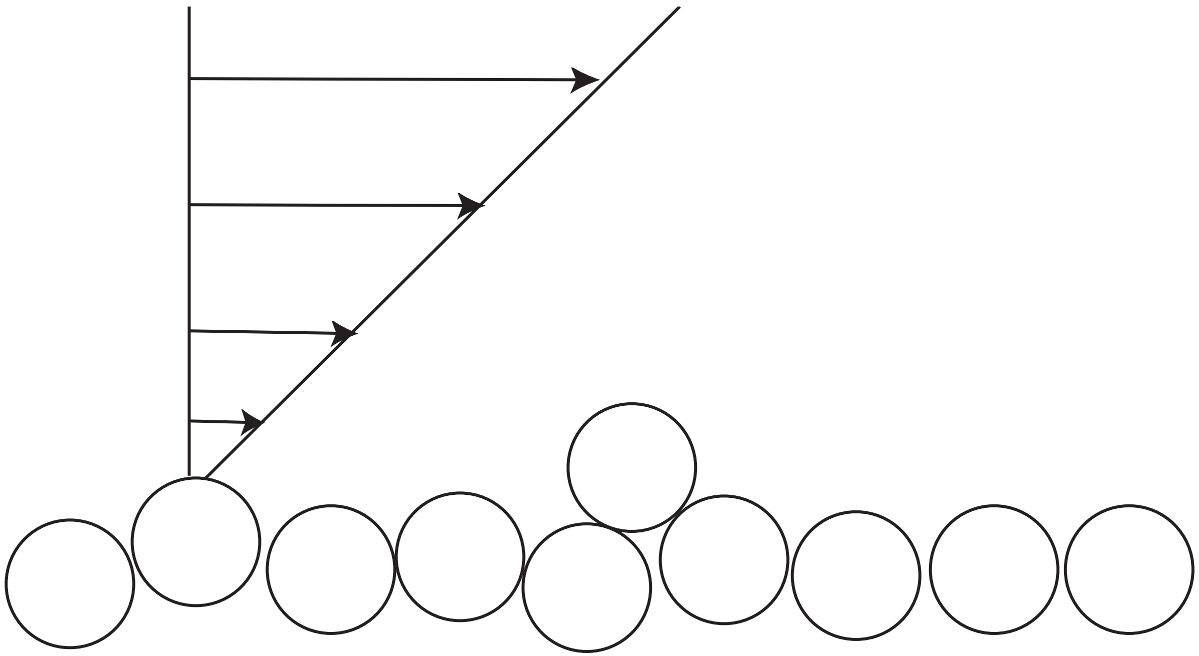
Erosion Model

link between the flow of water and the flow of grains

Problem :
What is the relationship between q and the flow?
hint: the larger u the larger the erosion, the larger q
q seems to be proportional to the skin friction



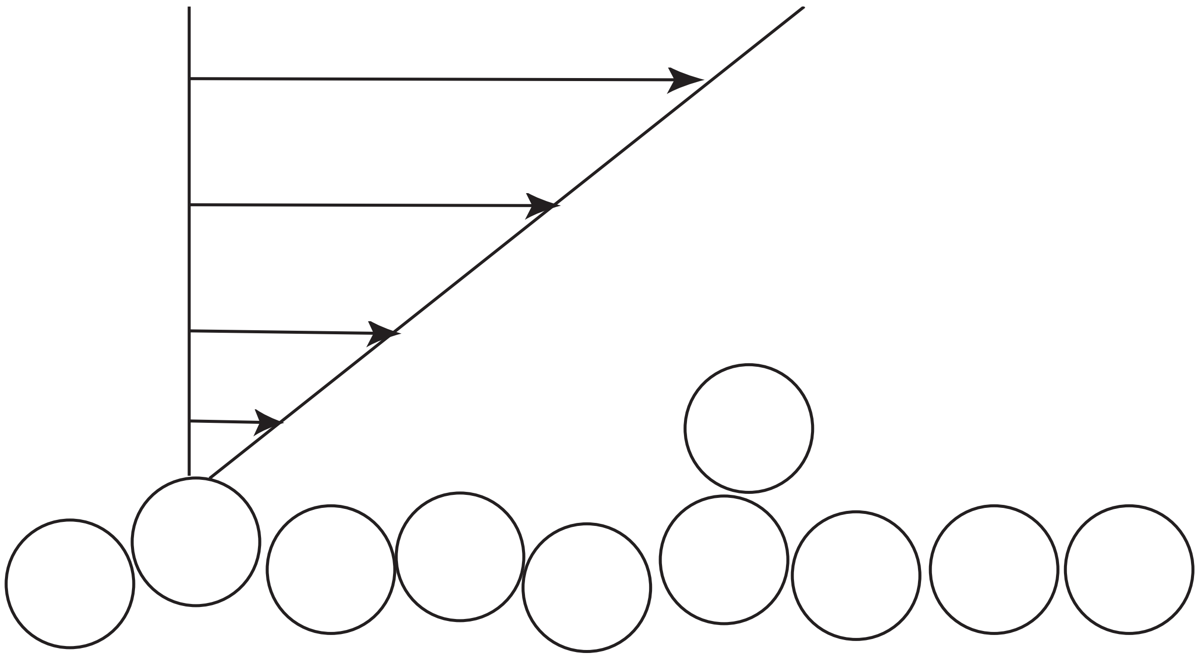
Erosion Model



Stress larger than a threshold $\tau > \tau_s$

Shields number $$\frac{\tau}{(\rho_p - \rho) g D}$$

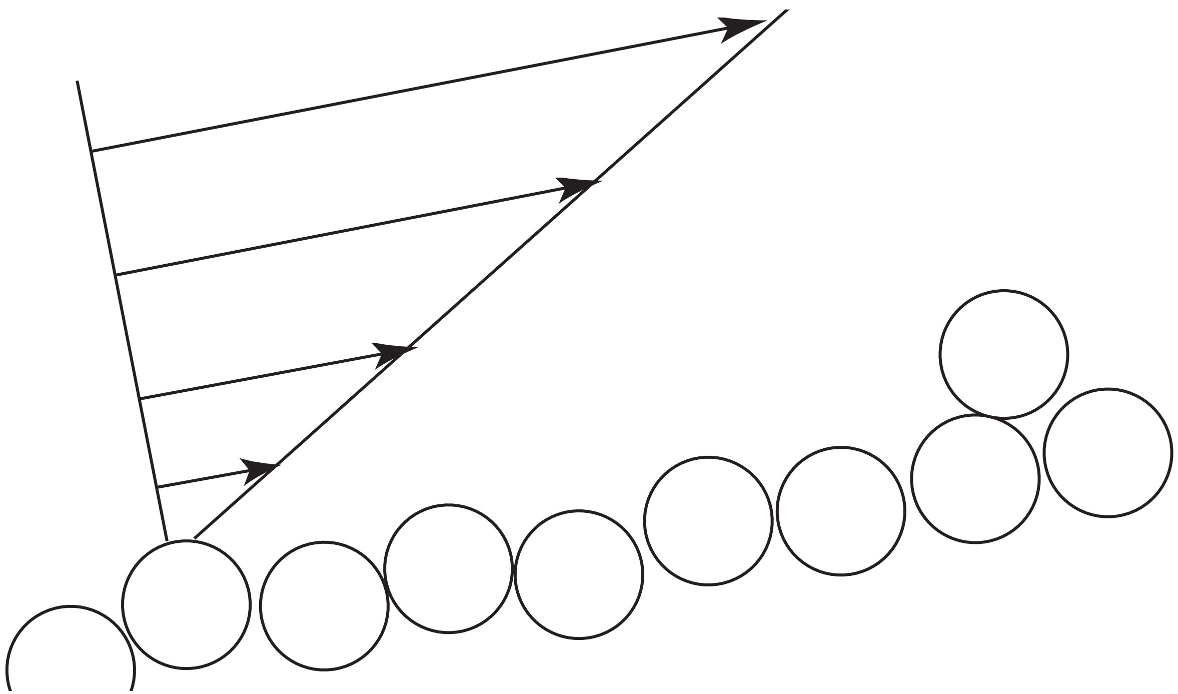
Erosion Model



Stress larger than a threshold $\tau > \tau_s$

Shields number $\dfrac{\tau}{(\rho_p - \rho) g D}$

Erosion Model



Les lois d'entraînement de M. Scipion Gras
sur les torrents des Alpes (Annales des ponts et Chaussées, 1857, 2^e semestre) résumées par du Boys 1879 :

"*un caillou posé au fond d'un courant liquide, peut être déplacé par l'impulsion des filets qui le rencontrent : le mouvement aura lieu si la vitesse est supérieure à une certaine limite qu'il (S. Gras) nomme vitesse d'entraînement. Cette vitesse limite dépend de la densité, du volume et de la forme du caillou ; elle dépend aussi de la densité du liquide et de la profondeur du courant.*"

Erosion Model

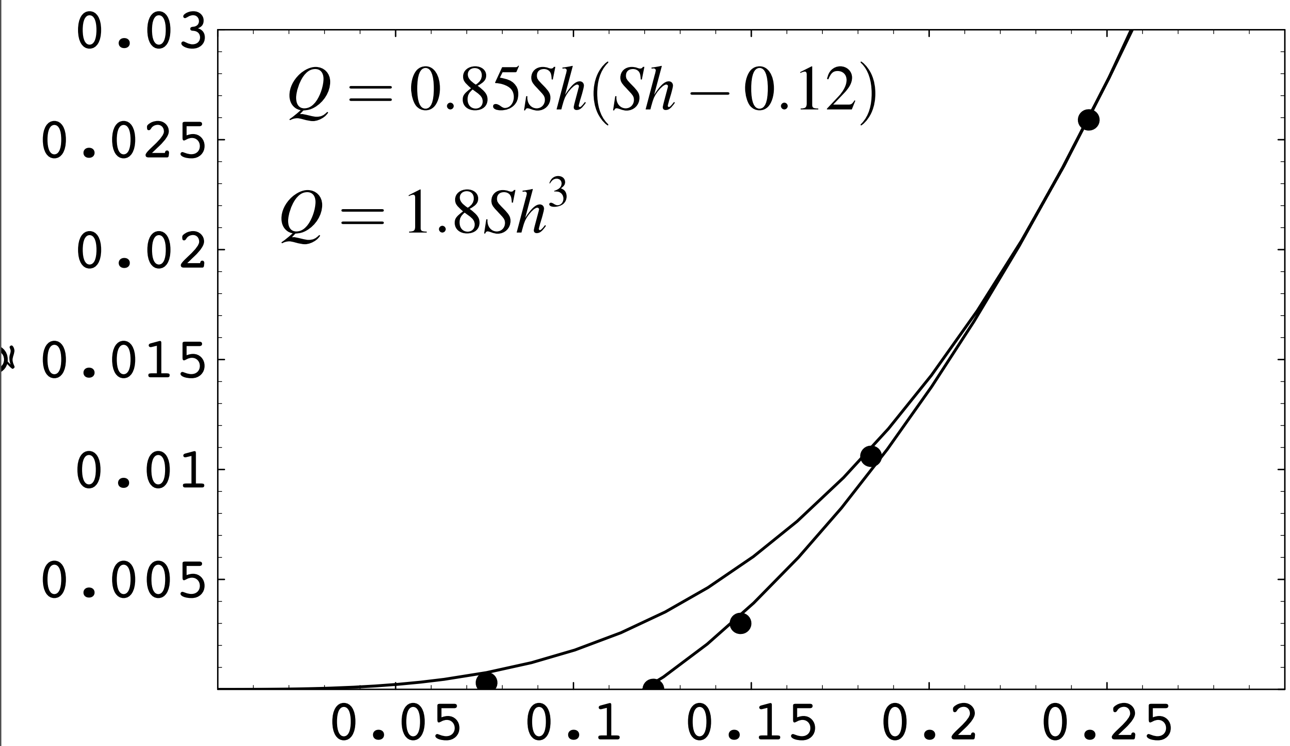
In the literature one founds Charru /Izumi & Parker / Yang / Blondeau Du Boys

$$q_s = E\varpi(\tau^a(\tau - \tau_s)^b)$$

if $x > 0$ then $\varpi(x) = x$ else $\varpi(x) = 0$.

or with a slope correction for the threshold value:

a, E coefficients, $a = 0, b = 3$ or $a = b = 1$ or $a = 1/2, b = 1$ or ...



$$\theta = \frac{\eta\gamma}{(\rho_s - \rho_f)gd}$$

Charru

Sh

Erosion Model

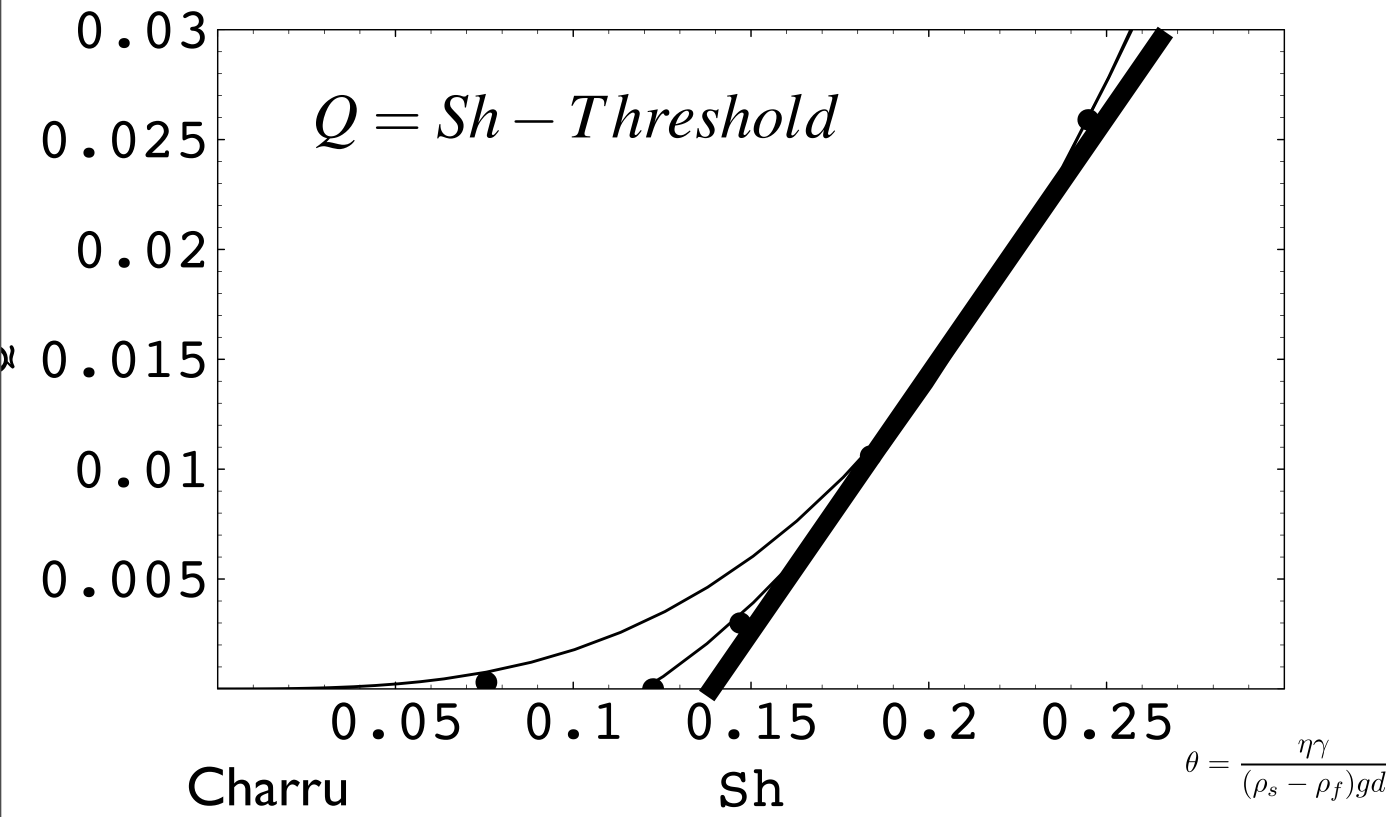
In the literature one founds Charru /Izumi & Parker / Yang / Blondeau Du Boys

$$q_s = E\varpi(\tau^a(\tau - \tau_s)^b)$$

if $x > 0$ then $\varpi(x) = x$ else $\varpi(x) = 0$.

or with a slope correction for the threshold value:

a, E coefficients, $a = 0, b = 3$ or $a = b = 1$ or $a = 1/2, b = 1$ or ...



Erosion Model

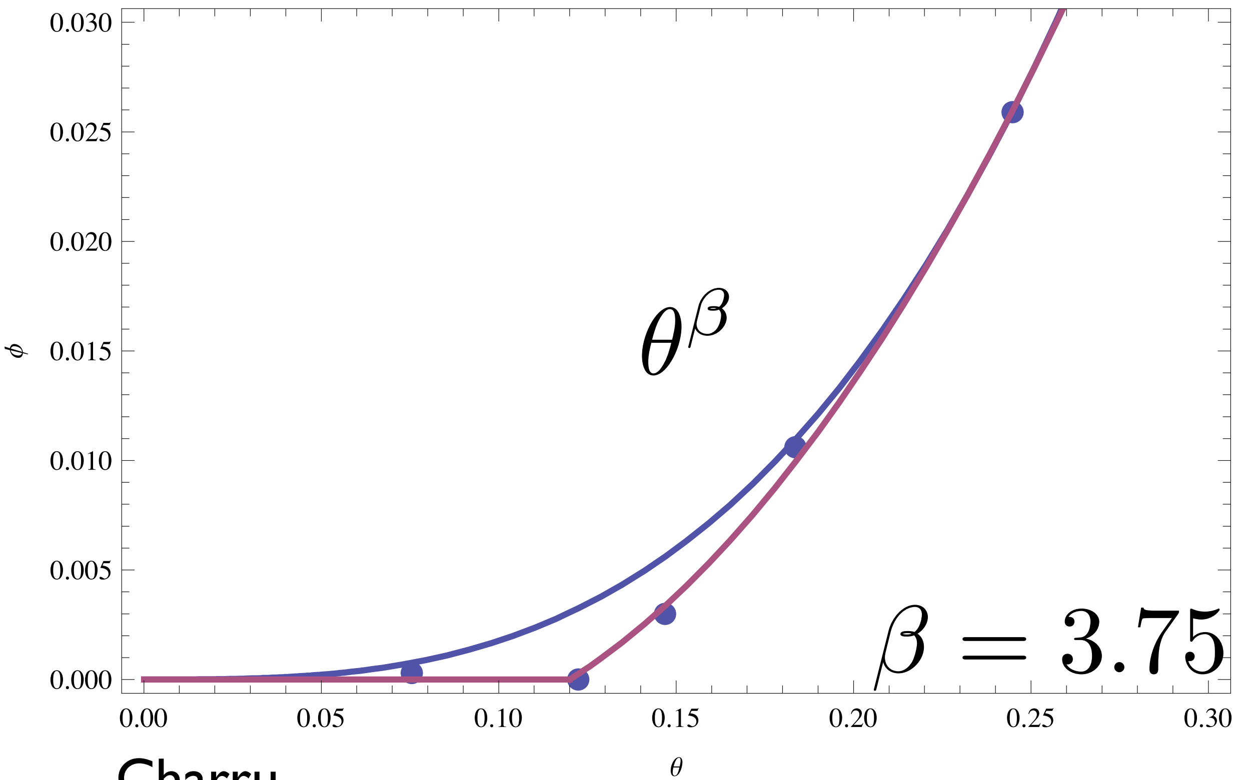
In the literature one founds Charru /Izumi & Parker / Yang / Blondeau Du Boys

$$q_s = E\varpi(\tau^a(\tau - \tau_s)^b)$$

if $x > 0$ then $\varpi(x) = x$ else $\varpi(x) = 0$.

or with a slope correction for the threshold value:

a, E coefficients, $a = 0, b = 3$ or $a = b = 1$ or $a = 1/2, b = 1$ or ...



Charru

$$\vec{q} = \phi\theta^{\beta}\left(\frac{\vec{u}}{\|\vec{u}\|} - \gamma\vec{\nabla}h\right)$$

Navier Stokes

coupled system

Saint Venant

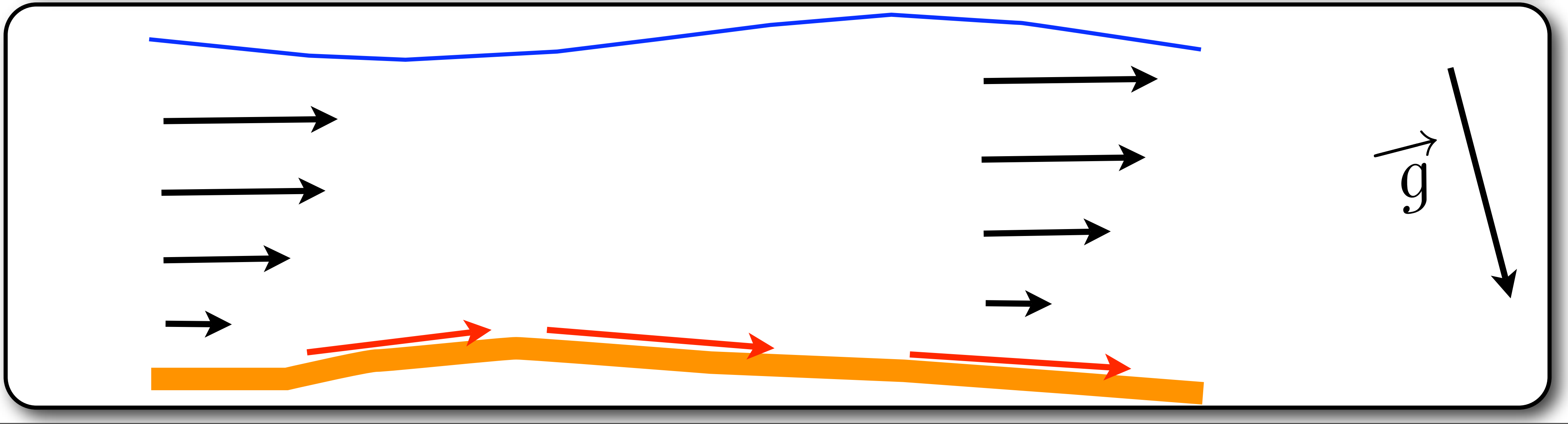
$$\frac{6}{5}(\overrightarrow{u}\cdot\overrightarrow{\nabla})\overrightarrow{u} = -g(\overrightarrow{\nabla}\eta + sin(\theta)\overrightarrow{e}_x) - \frac{3\nu\overrightarrow{u}}{(\eta-h)^2}$$

Mass conservation of fluid

$$\overrightarrow{\nabla}\cdot(\overrightarrow{u}(\eta-h)) = 0$$

$$\vec{q} = \phi\theta^{\beta}\left(\frac{\vec{u}}{\|\vec{u}\|} - \gamma\vec{\nabla}h\right)$$

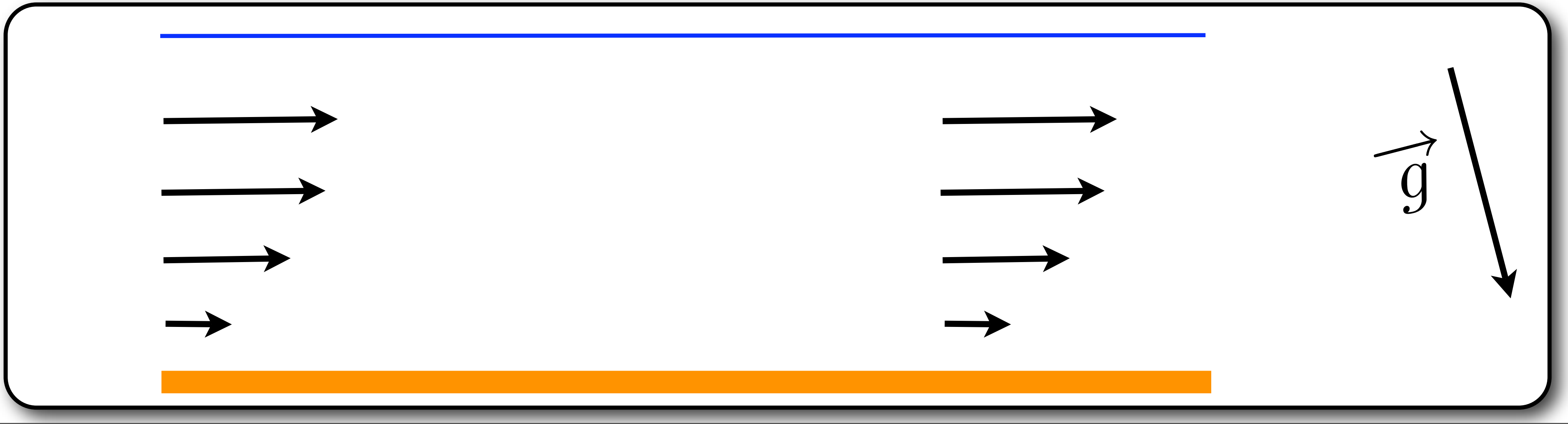
$$\frac{\partial h}{\partial t} = -\overrightarrow{\nabla}\cdot\overrightarrow{q}$$



Linear Stability

Basic flow $u_0 = 1,\ d_0 = 1$

perturbations $\propto \exp(i(k_l x_l - \omega t))$



Linear Stability

Basic flow

$$u_0 = 1,\ d_0 = 1$$

perturbations

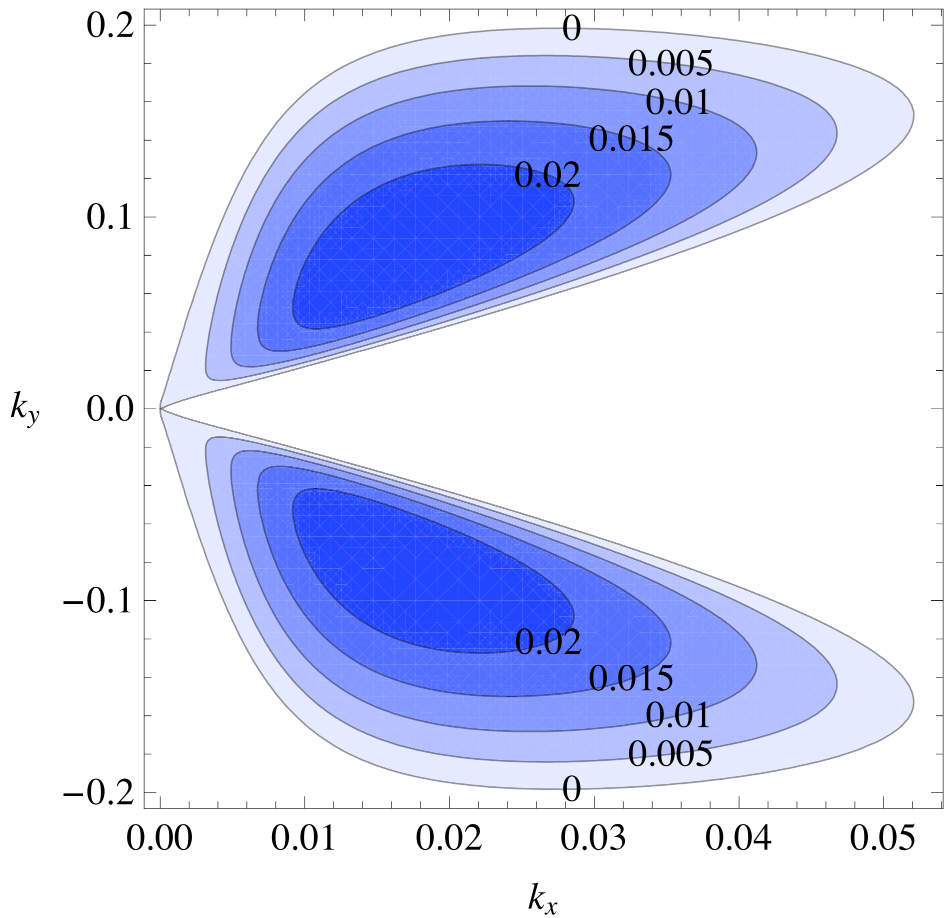
$$\propto \exp(i(k_l x_l - \omega t))$$

dispersion relation

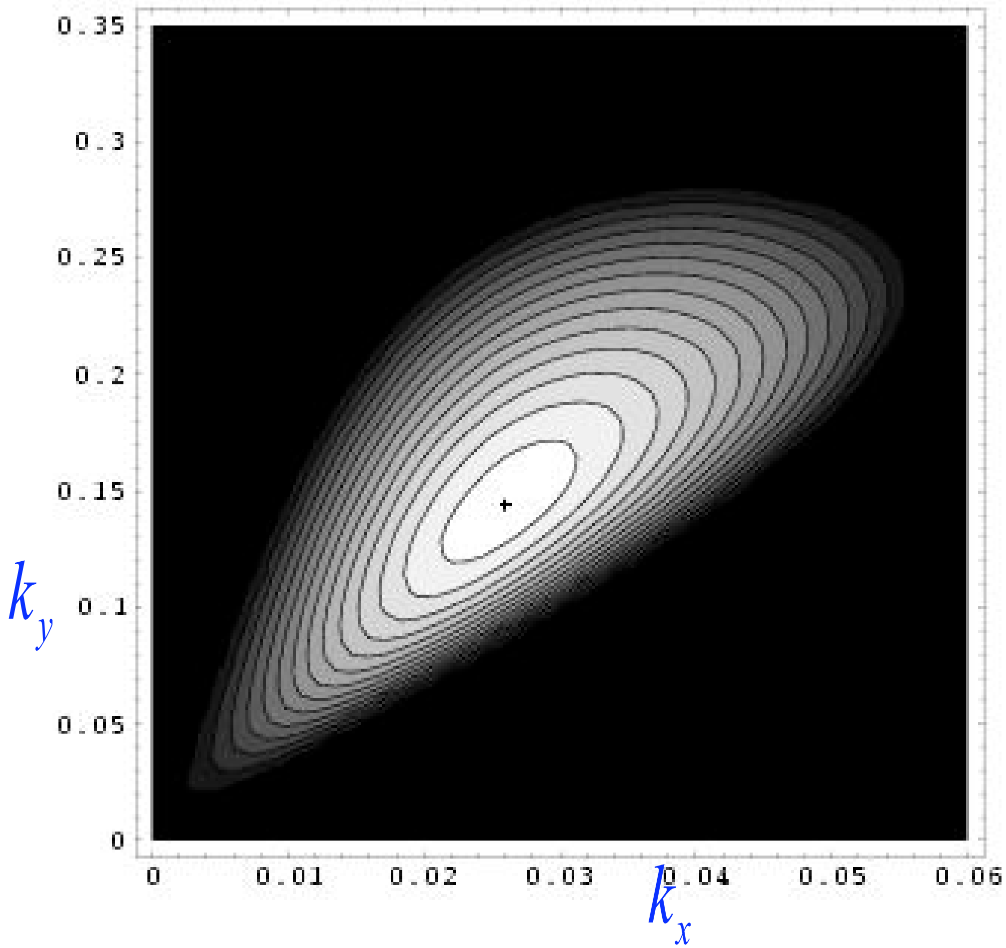
$$\begin{aligned}\omega = \big(&- 36 i F^4 k_x^3 \big(k_x^2 + k_y^2\big)\gamma + 30 i F^2 k_x \big(k_x^4 \gamma + 2 k_x^2 k_y^2 \gamma + k_y^4 \gamma \\ &+ 2 i k_x^3 (\beta + S(2+\beta)\gamma) + i k_x k_y^2 (1 + \beta + S(4+\beta)\gamma)\big) \\ &+ 25 S \big(k_x^4 \gamma + 2 k_x^2 k_y^2 \gamma + k_y^4 \gamma - i k_x k_y^2 (-3+\beta)(1+S\gamma) \\ &+ i k_x^3 (2\beta + S(3+2\beta)\gamma)\big)\big) / \\ &\big(\big(6F^2 k_x - 5 i S\big)\big(\big(-5 + 6F^2\big) k_x^2 - 5 k_y^2 - 15 i k_x S\big)(1 + S\gamma)\end{aligned}$$

$$\beta = \frac{\theta_0 \phi'(\theta_0)}{\phi(\theta_0)}$$

Linear Stability



Linear Stability



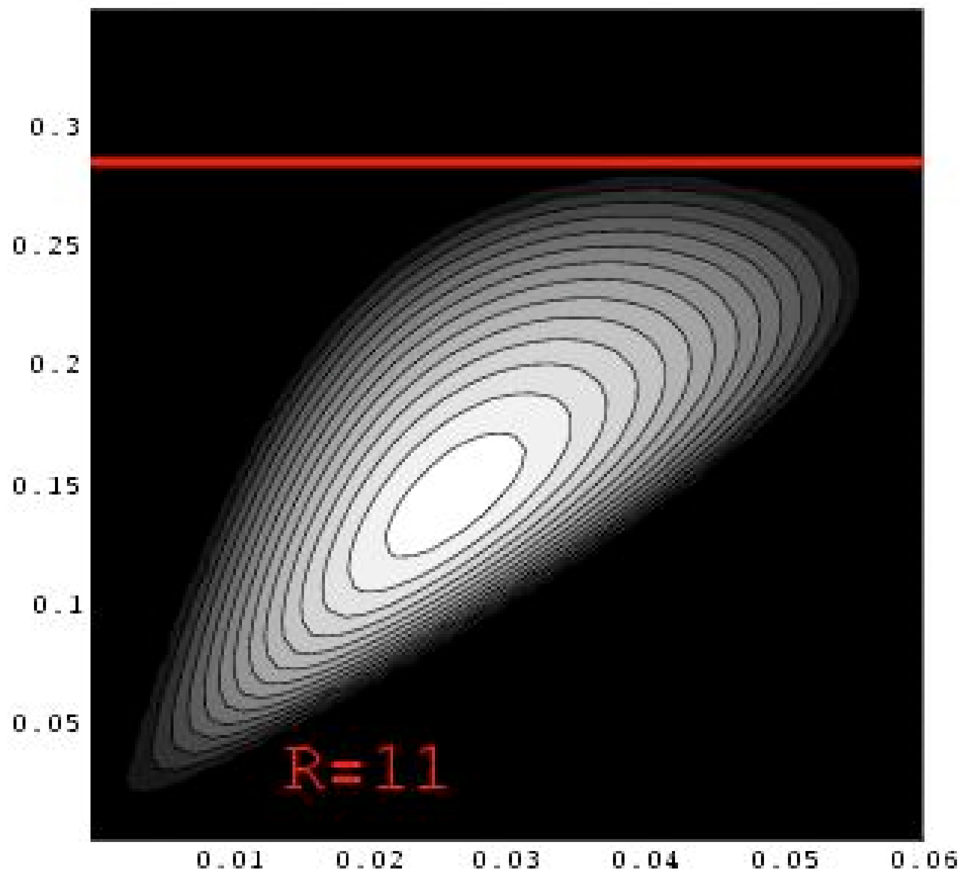
2 D Instability :
inclindes bancs

No 1D instability (k_y=0):

$$F = 1,5 \quad \varphi = 3°$$

$$\beta = 3,75 \quad \gamma = 1$$

Linear Stability

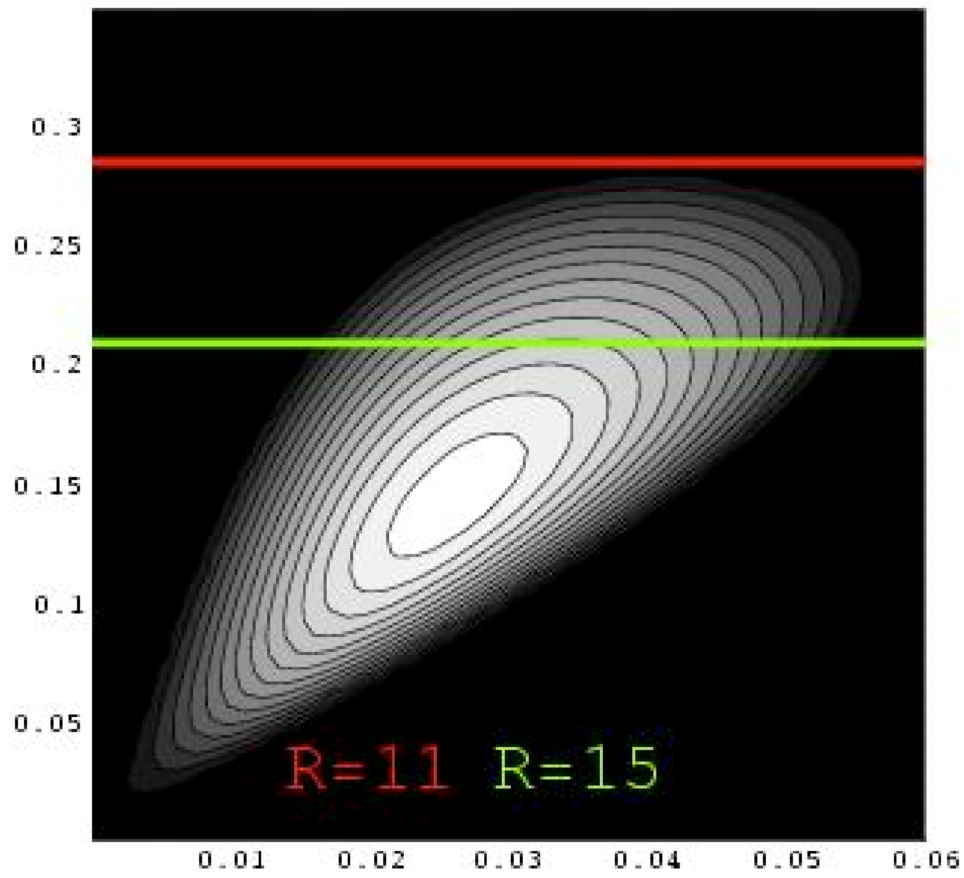


width of the river R promotes the modes

$F = 1,5 \quad \varphi = 3°$

$\beta = 3,75 \quad \gamma = 1$

Linear Stability

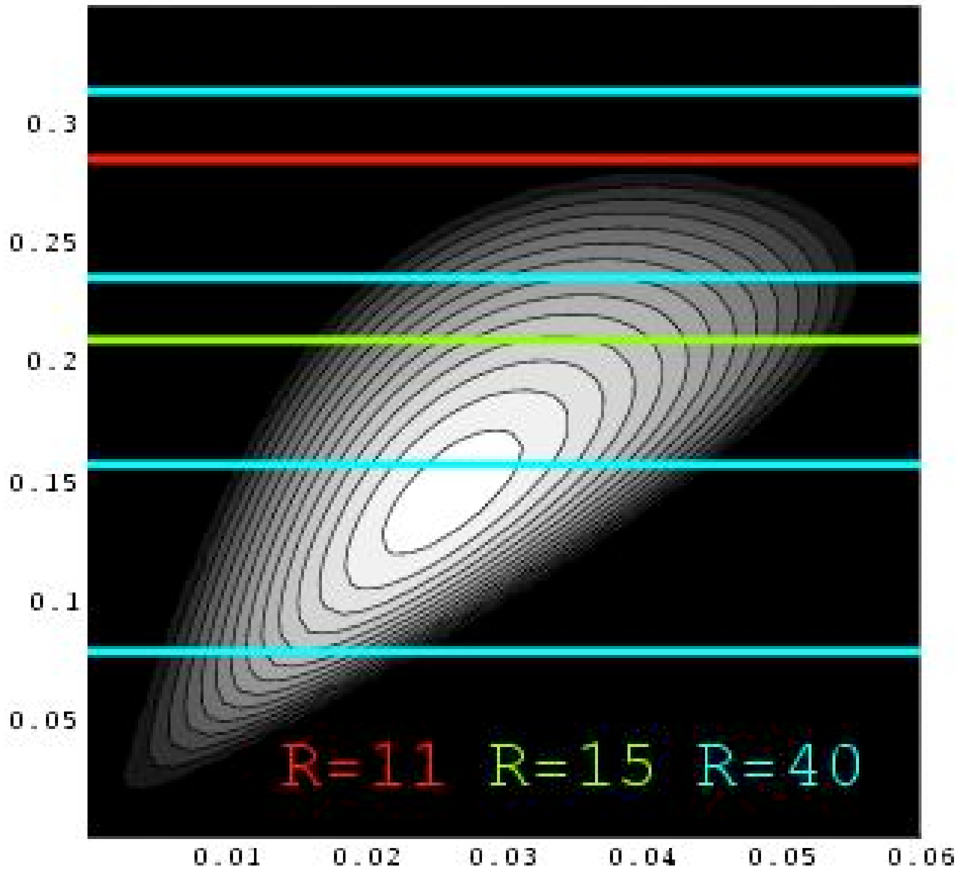


width of the river R promotes the modes

$F = 1{,}5 \quad \varphi = 3^\circ$

$\beta = 3{,}75 \quad \gamma = 1$

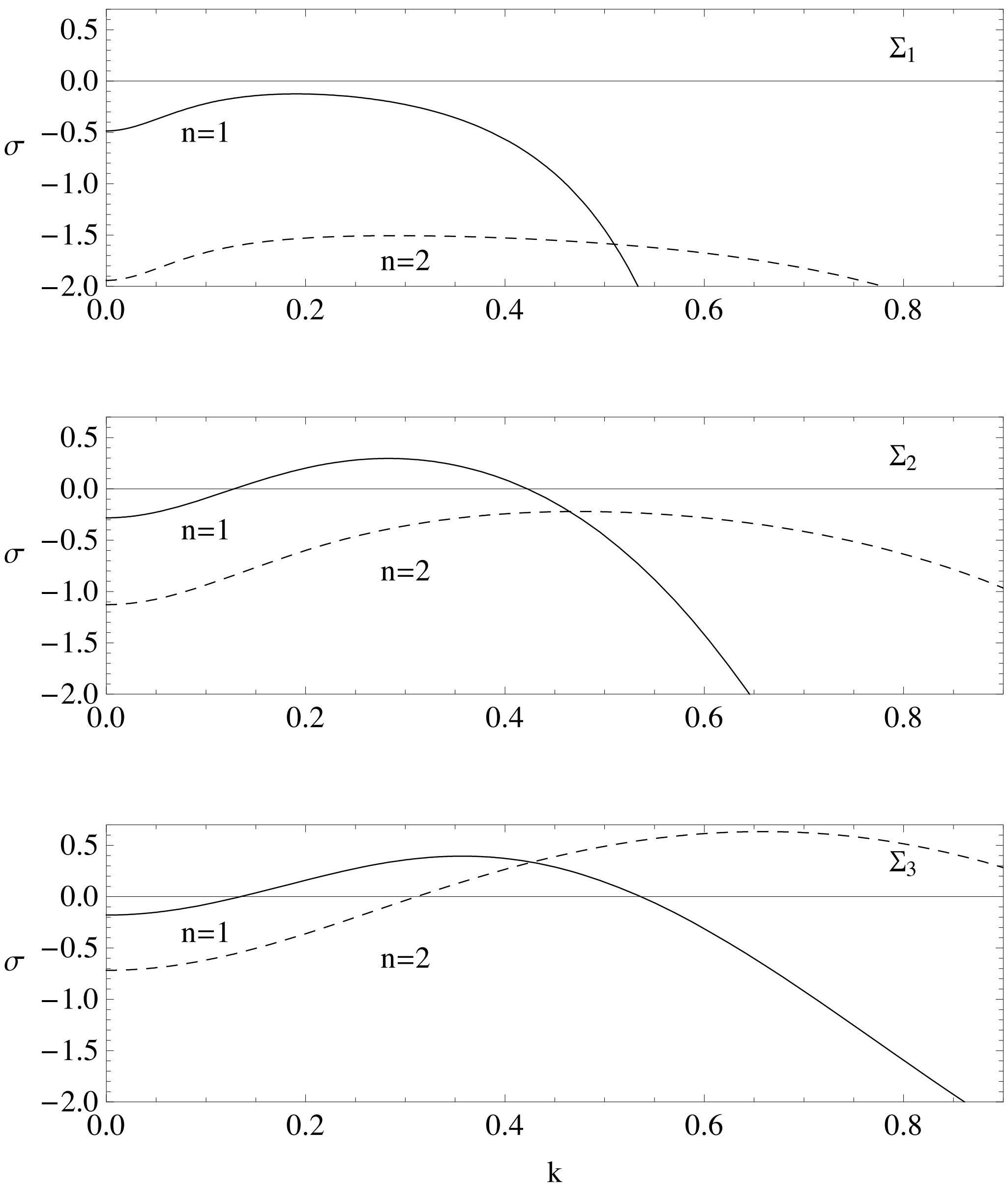
Linear Stability

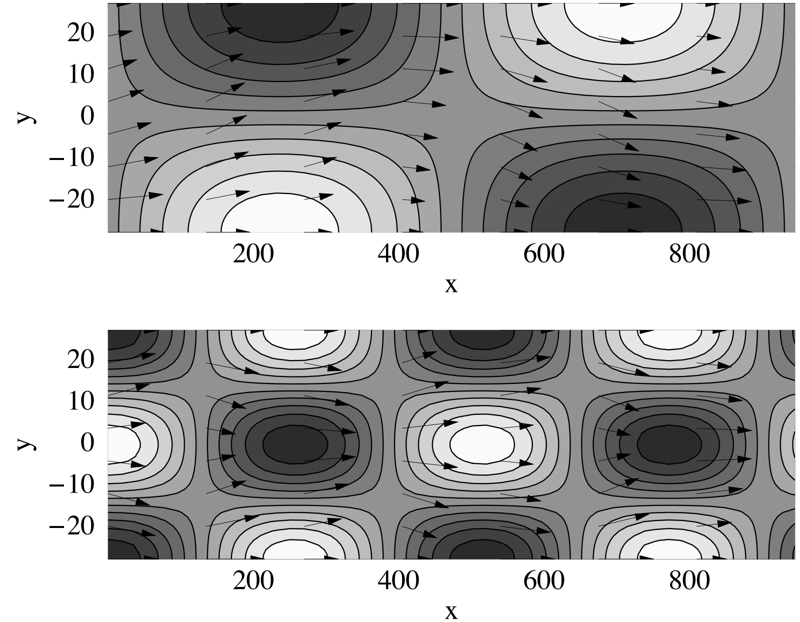


width of the river R promotes the modes

$$F = 1{,}5 \quad \varphi = 3^\circ$$

$$\beta = 3{,}75 \quad \gamma = 1$$


Σ1
n=1
n=2
Σ2
n=1
n=2
Σ3
n=1
n=2
σ
k


y
x

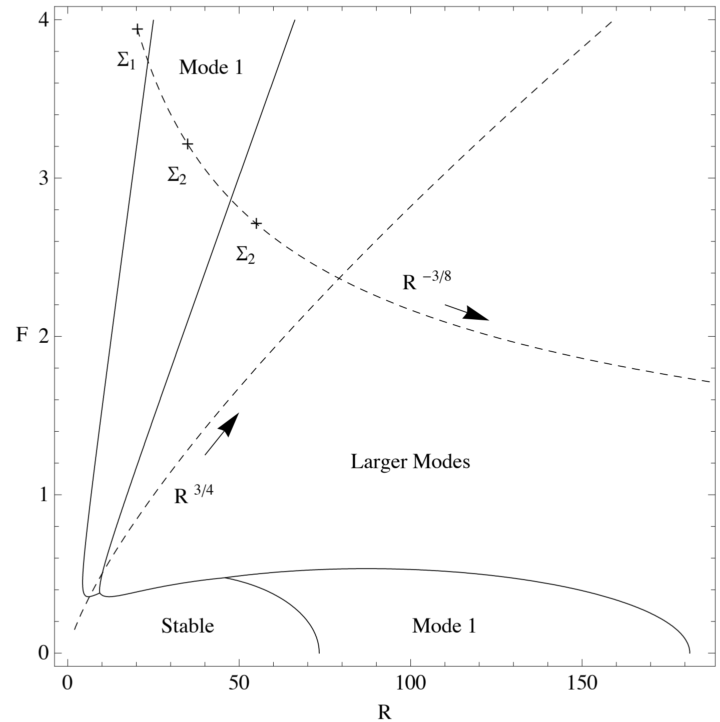
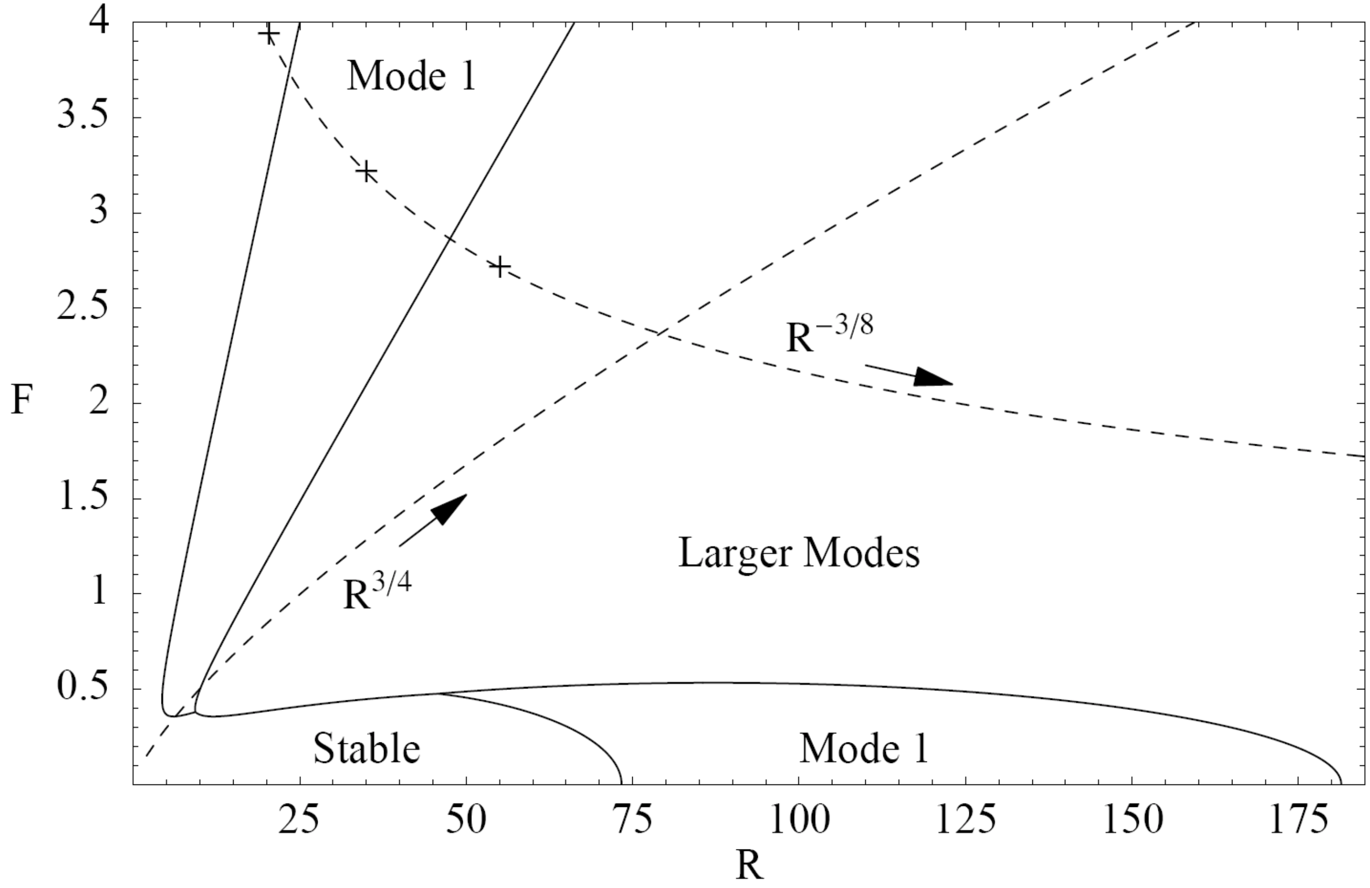

Σ1
Mode 1
Σ2
Σ2
R −3/8
R 3/4
Larger Modes
Stable
Mode 1
F
R

Diagramme de stabilité

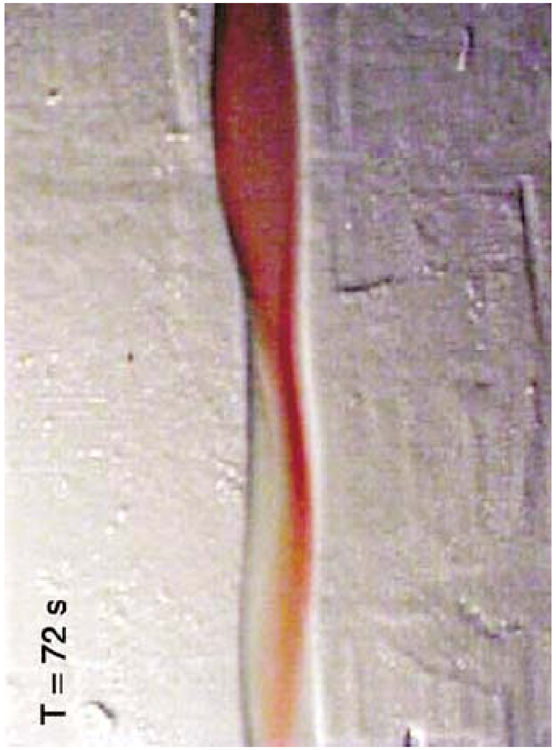


• Bancs instables à Froude nul !

• A pente fixée, l'élargissement d'une rivière modifie F et R

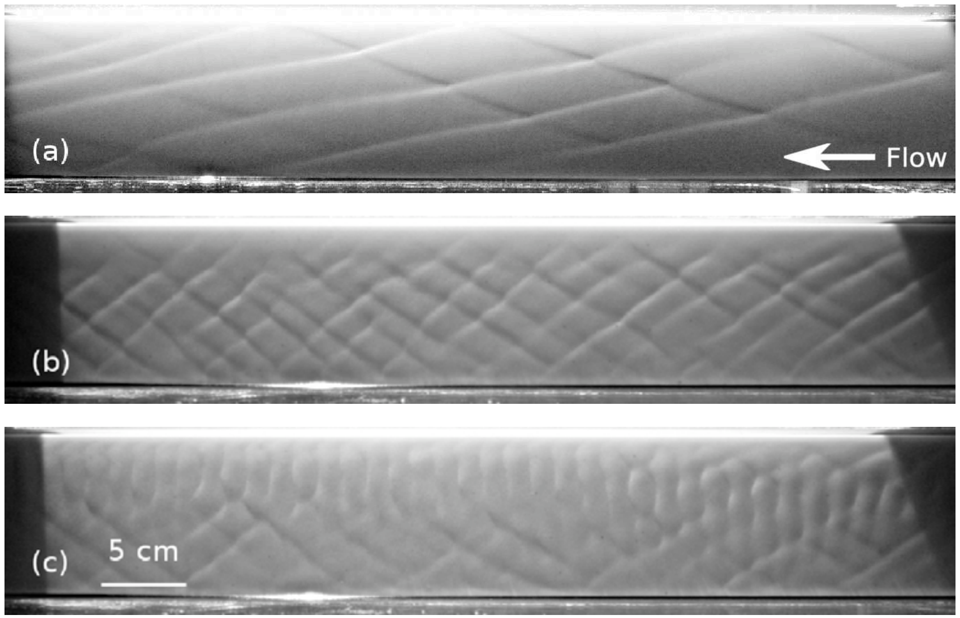
Evolution de micro-rivières

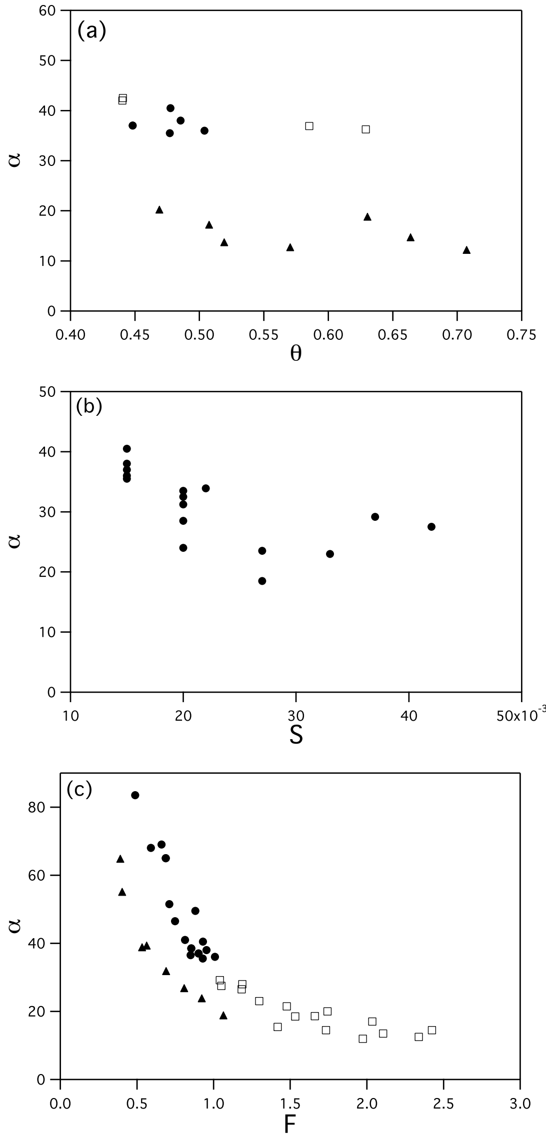
F. Métivier, P. Meunier -
Journal of Hydrology 271 (2003) 22–38



Rapport d'aspect petit : pas d'instabilité
Rapport d'aspect augmente : apparition du mode 1
Rapport d'aspect grand : instabilité

comparaisons mesures théorie



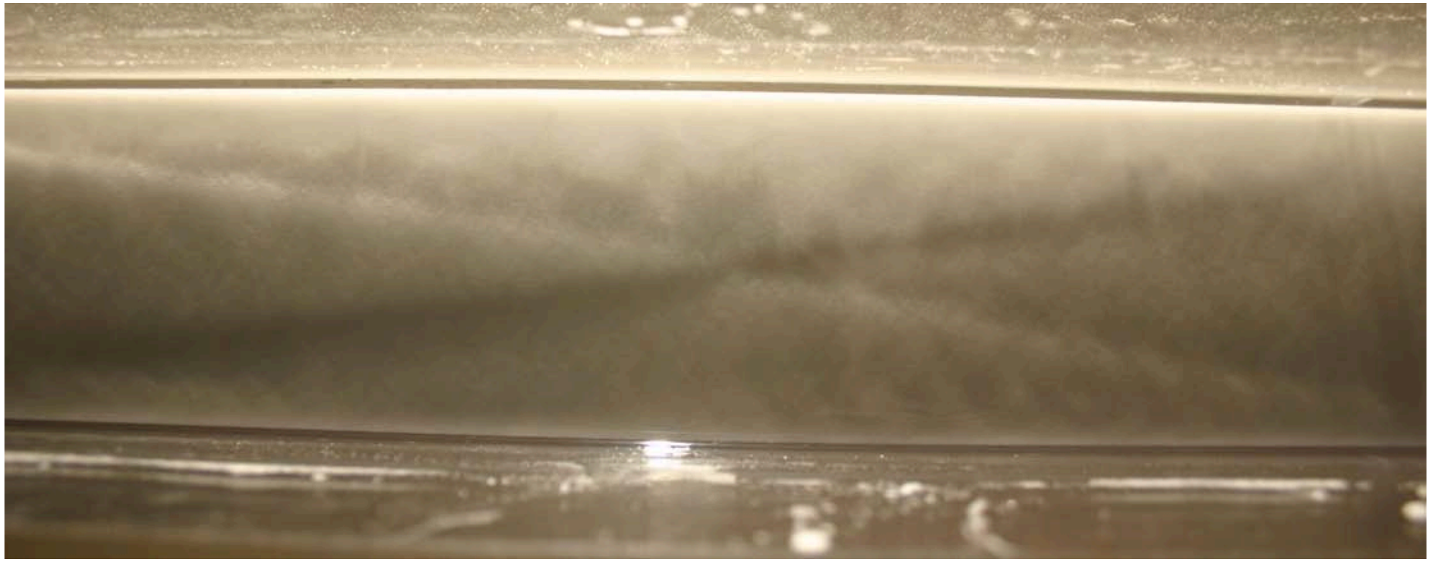


(a) Large rhomboid pattern (Fr = 1.76, S = 0.03, Bo = 1.31 and Sh = 0.616).

(b) Small rhomboid pattern (Fr = 0.95, S = 0.015, Bo = 3.25 and Sh = 0.485).

(c) Rhomboid pattern mixed with ripples (Fr = 1.01, S = 0.015, Bo = 3.50 and Sh = 0.504)

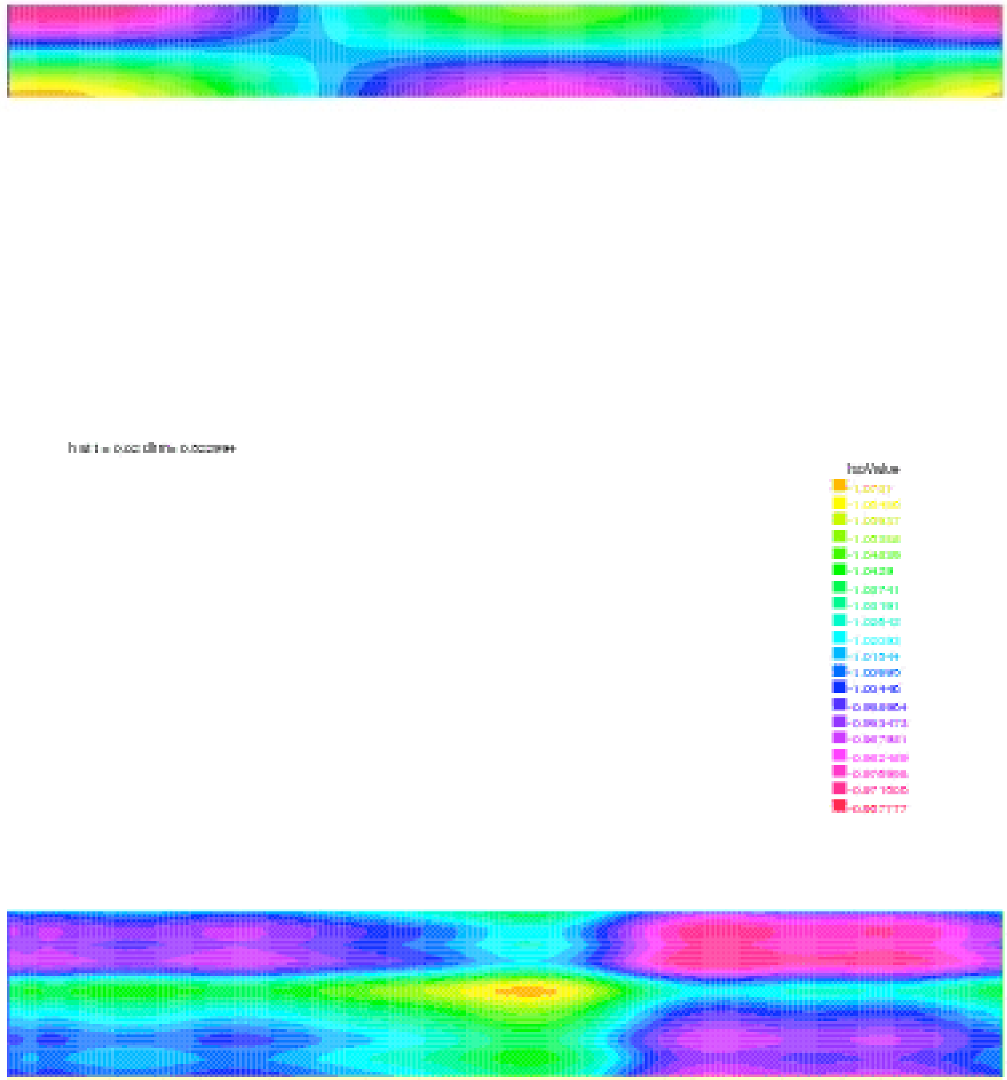
non-linear evolution of mode 1



$F \approx 1$

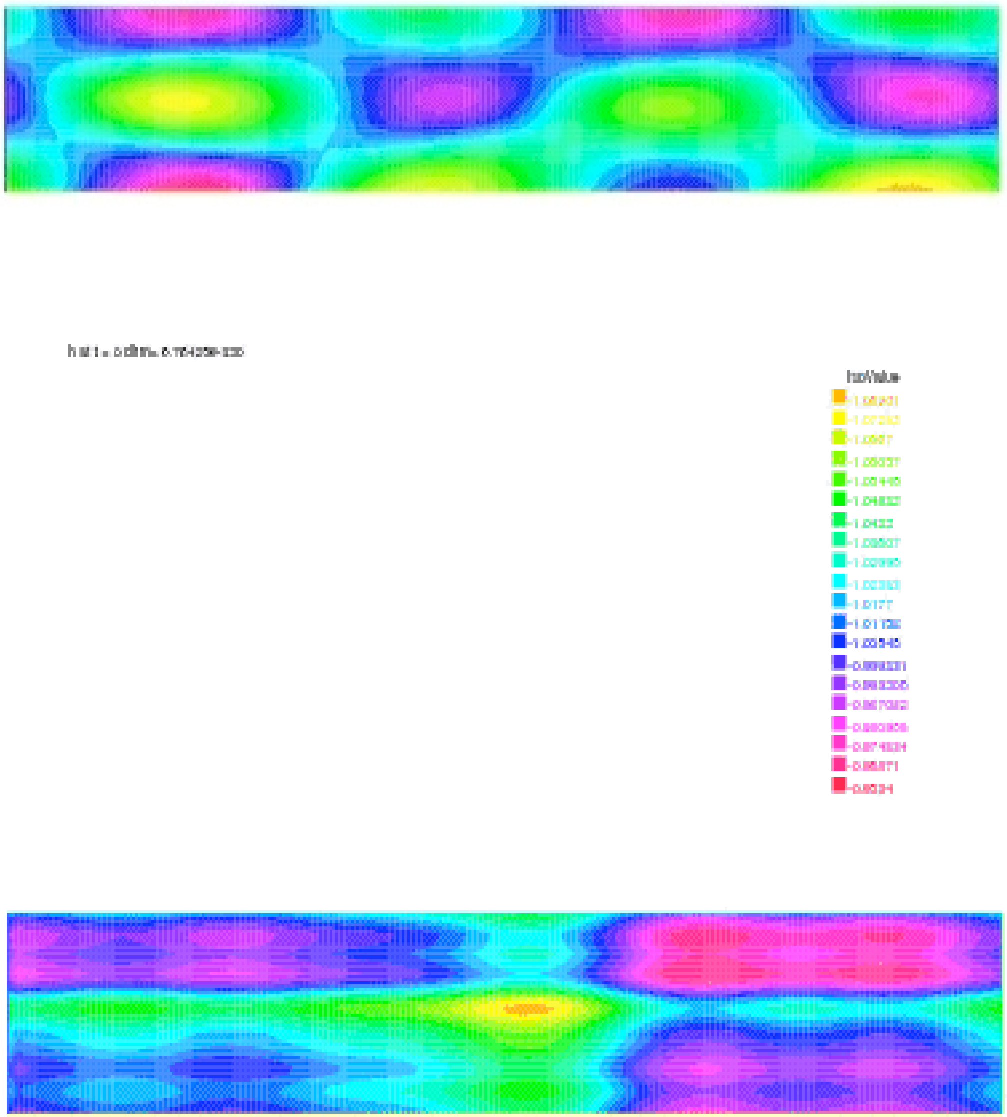

carte d'assurance maladie
vitale
EMISE LE 25/08/2006
2 81 11 72 181 252 24
DEVAUCHELLE
OLIVIER

évolution en temps d'un fond initialement bruité



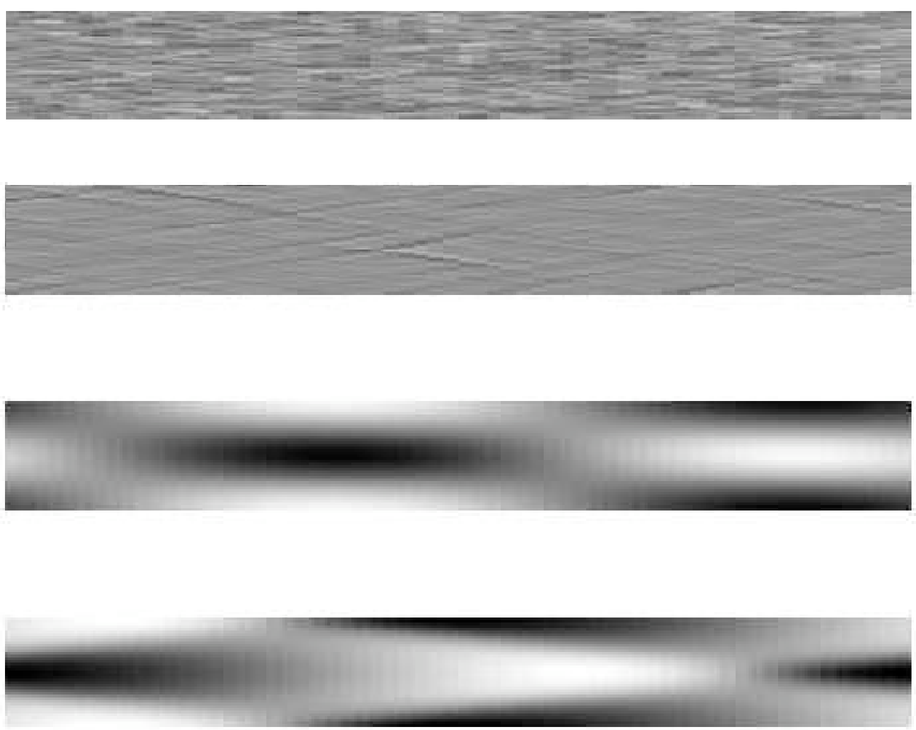
éléments finis périodicité en x

évolution en temps d'un fond initialement bruité



éléments finis périodicité en x

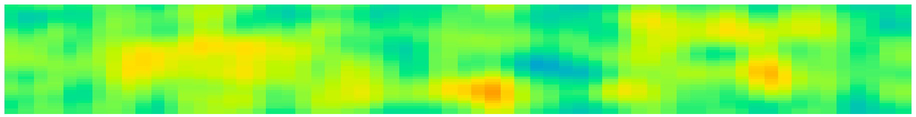
évolution en temps d'un fond initialement bruité
rides inclinées et motif en diamant

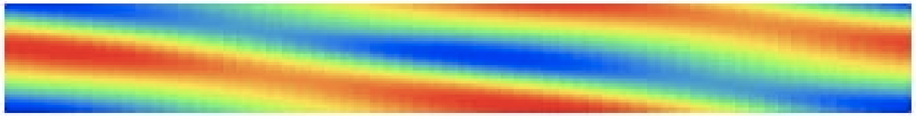


Fourier/ non linearité (θ^{β}), périodicité en x et y

évolution en temps d'un fond initialement bruité

rides inclinées et motif en diamant

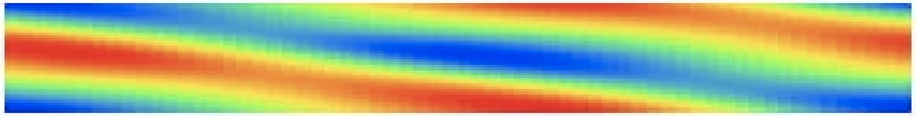
initialement 

ensuite 

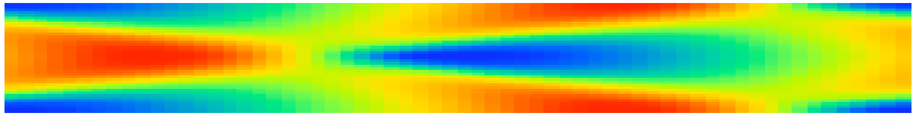
Fourier/ non linearité (θ^{β}), périodicité en x et y

évolution en temps d'un fond initialement bruité
rides inclinées et motif en diamant

initialement



ensuite



Fourier/ non linearité (θ^{β}), périodicité en x et y

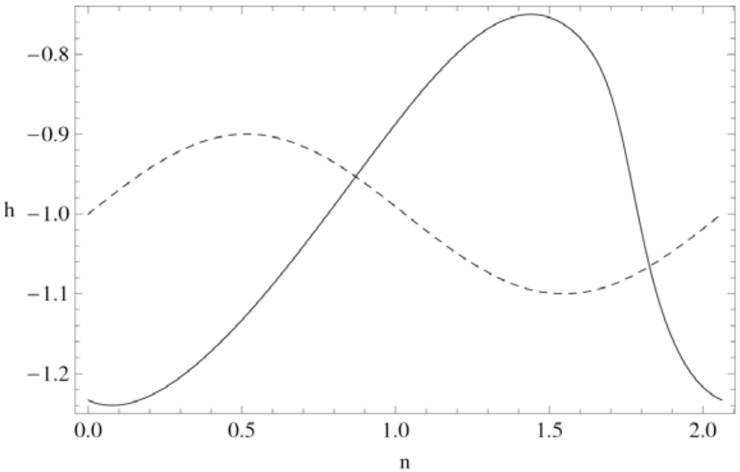
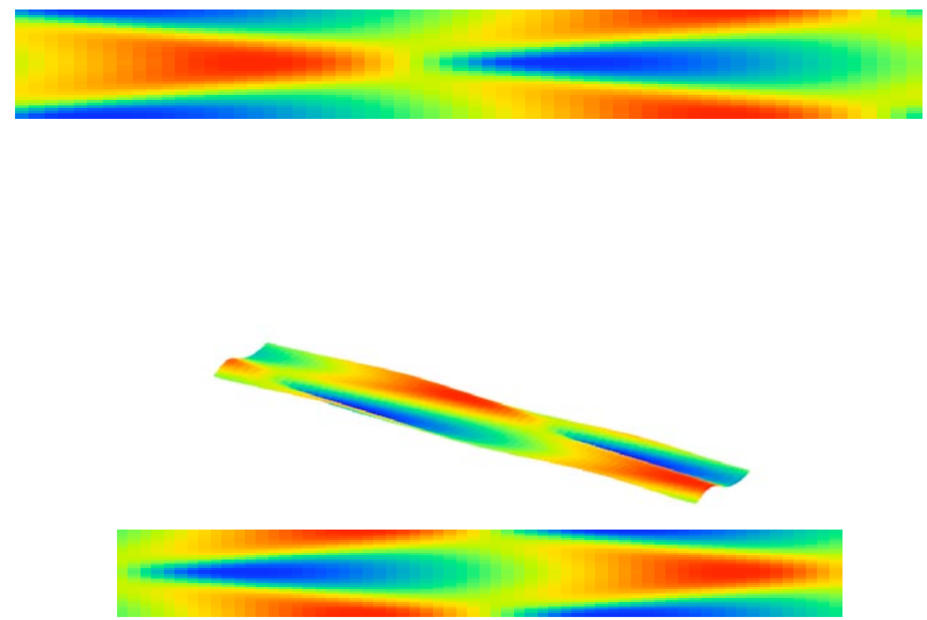


Figure 3. Evolution of an isolated erosion wave (numerical simulation). Non linear terms in the erosion equations lead to a steep front formation. *n* denotes the propagation direction.



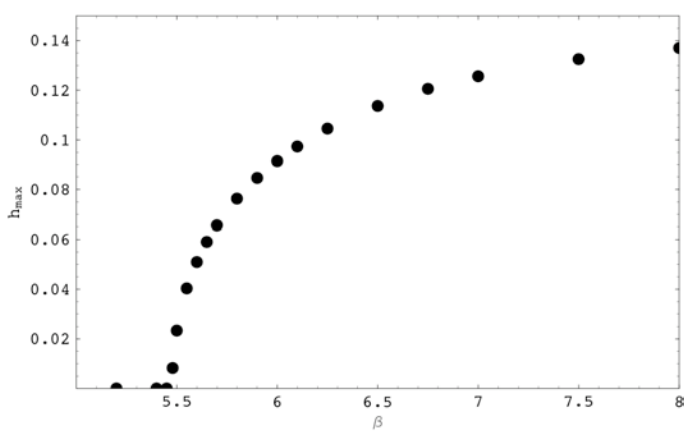
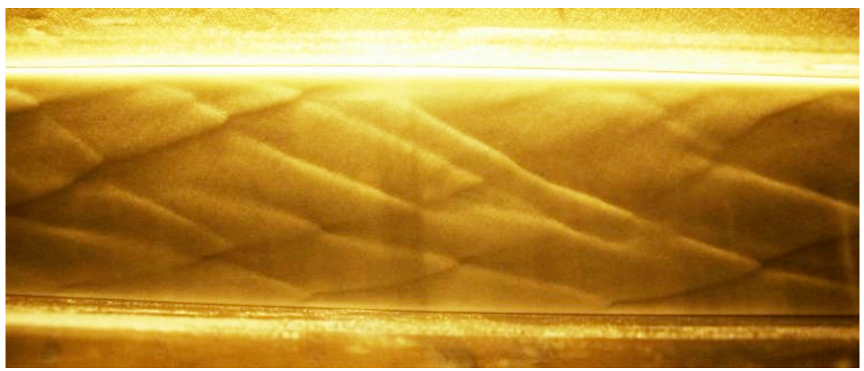
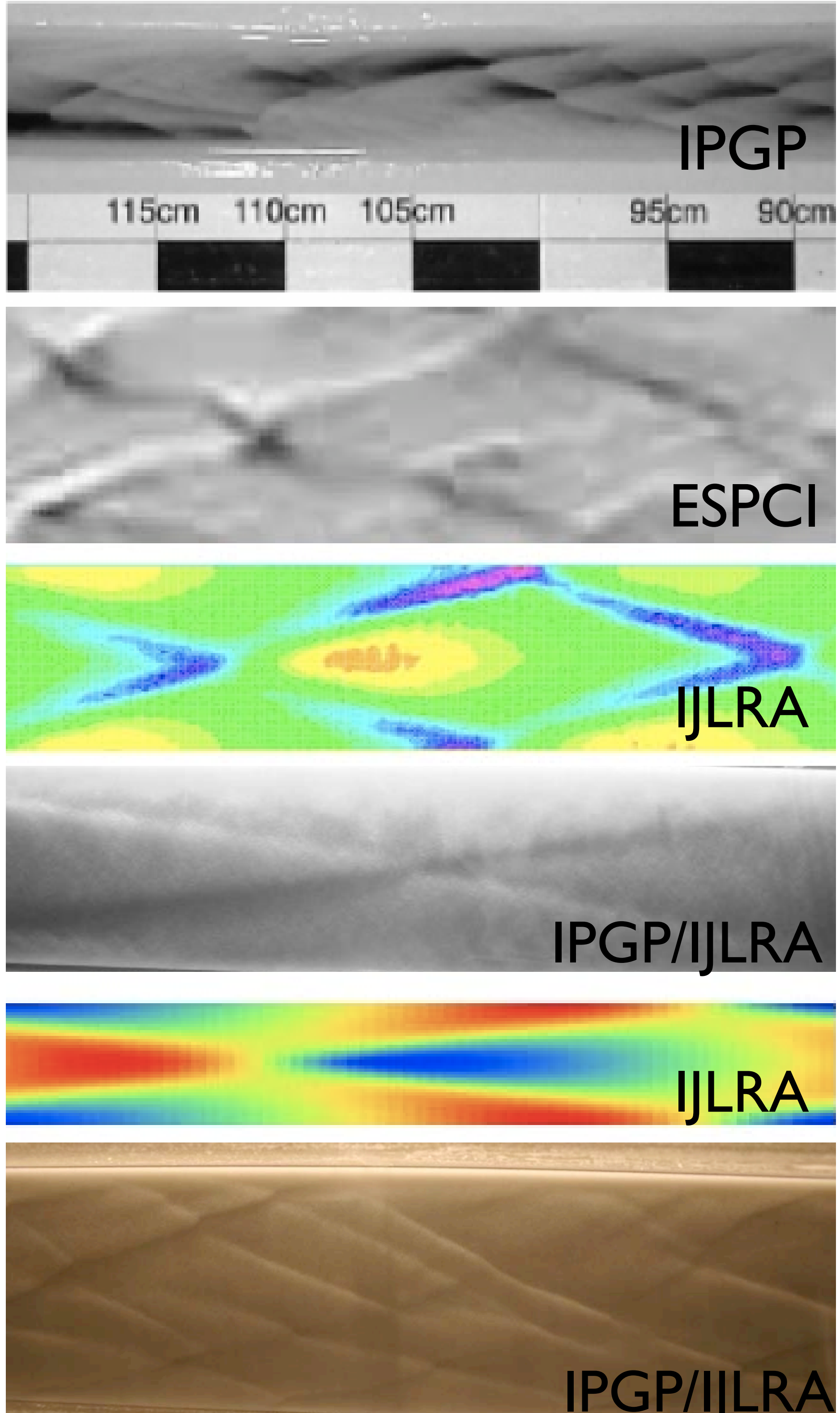


Figure 4. Saturation amplitude h_{max} of the erosion wave vs. the erosion law parameter β.



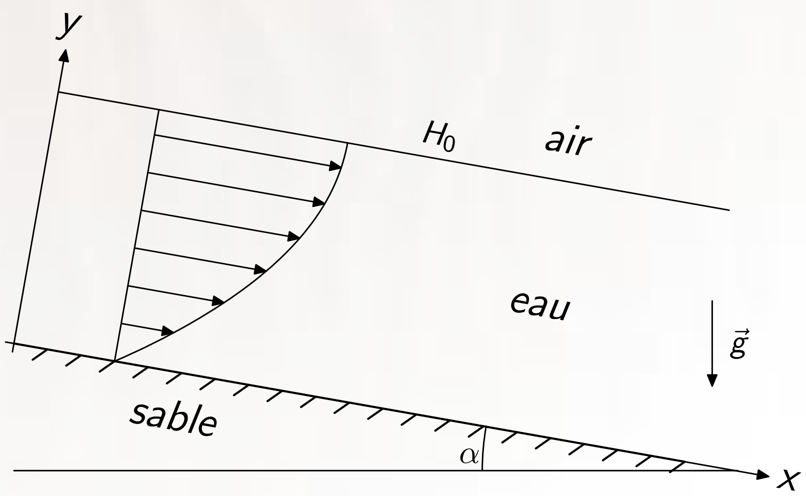

IPGP
115cm 110cm 105cm 95cm 90cm
ESPCI
IJLRA
IPGP/IJLRA
IJLRA
IPGP/IJLRA

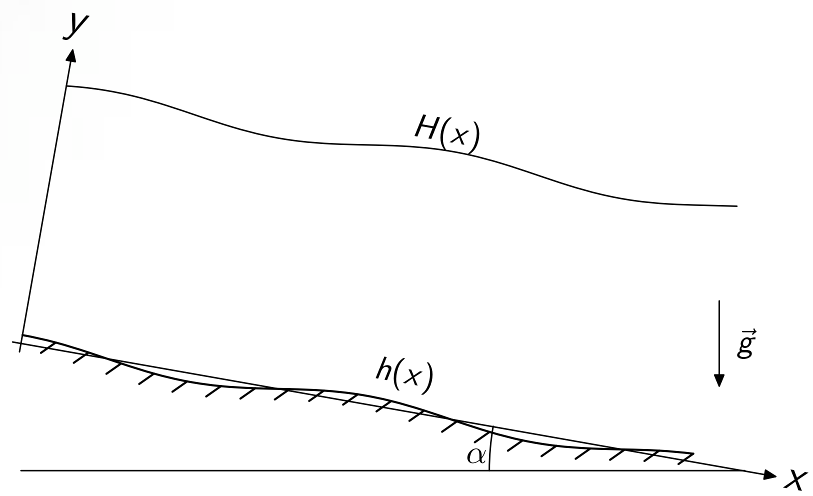
Saint Venant:
- ne permet pas de comprendre les rides
- accord qualitatif pour les chevrons

bien tenir compte des effets visqueux

> Orr Sommerfeld Stationnaire

- Écoulement quasi-**stationnaire**





Écoulement de base :
$u_0(y)$,
$v_0 = 0$.

Perturbation :
$h(x,t) = \varepsilon H_0 e^{ikx - i\omega t}$.

- Conservation de la matière :

$$\vec{\nabla} \cdot \vec{u} = 0$$

- Conservation de la quantité de mouvement (Navier-Stokes) :

$$(\vec{u} \cdot \vec{\nabla})\vec{u} = -\frac{1}{\rho}\vec{\nabla} p + \mu \vec{\nabla}^2 \vec{u} + \vec{g}$$

- Non-glissement au fond :

$$\vec{u} = \vec{0} \qquad \text{en} \quad y = h(x)$$

- Continuité de la contrainte tangentielle à la surface :

$$\sum_k \tau_{ik} n_k = 0 \qquad \text{en} \quad y = H(x)$$

Trois paramètres

- $k = 2\pi/\lambda$ (longueur d'onde de la perturbation)
- $\mathrm{Re} = UH/\nu$
- $\mathrm{S} = \tan\alpha$ ou $\mathrm{Fr} = U/\sqrt{gH}$

Équations linéarisées

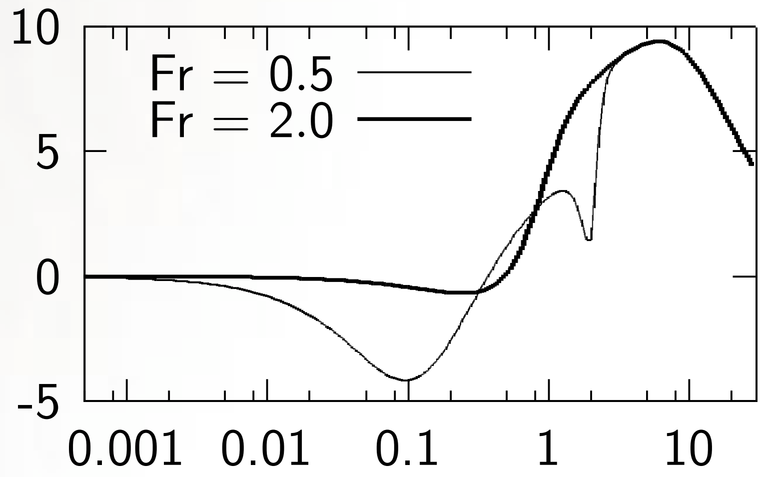
- Pour $u = u_0(y) + \varepsilon\psi'(y)e^{ikx}$ et $v = -\varepsilon ik\psi(y)e^{ikx}$,

$$\psi'''' - 2k^2\psi'' + k^4\psi = ik\mathrm{Re}\left[u_0(\psi'' - k^2\psi) - u_0''\psi\right];$$

- conditions aux limites en $y = 0$ et en $y = 1$.

Résolution numérique (méthode du tir linéaire)

$\Im m\, \psi''(0)$
en fonction de k :
($\mathrm{Re} = 30$)



Résolution analytique

- $k < 1$: développement en série de k^n
- $k \gg 1$: méthode de perturbation singulière (raccords asymptotiques)
- $k = \mathcal{O}(1)$, $\mathrm{Re} \to \infty$: ?

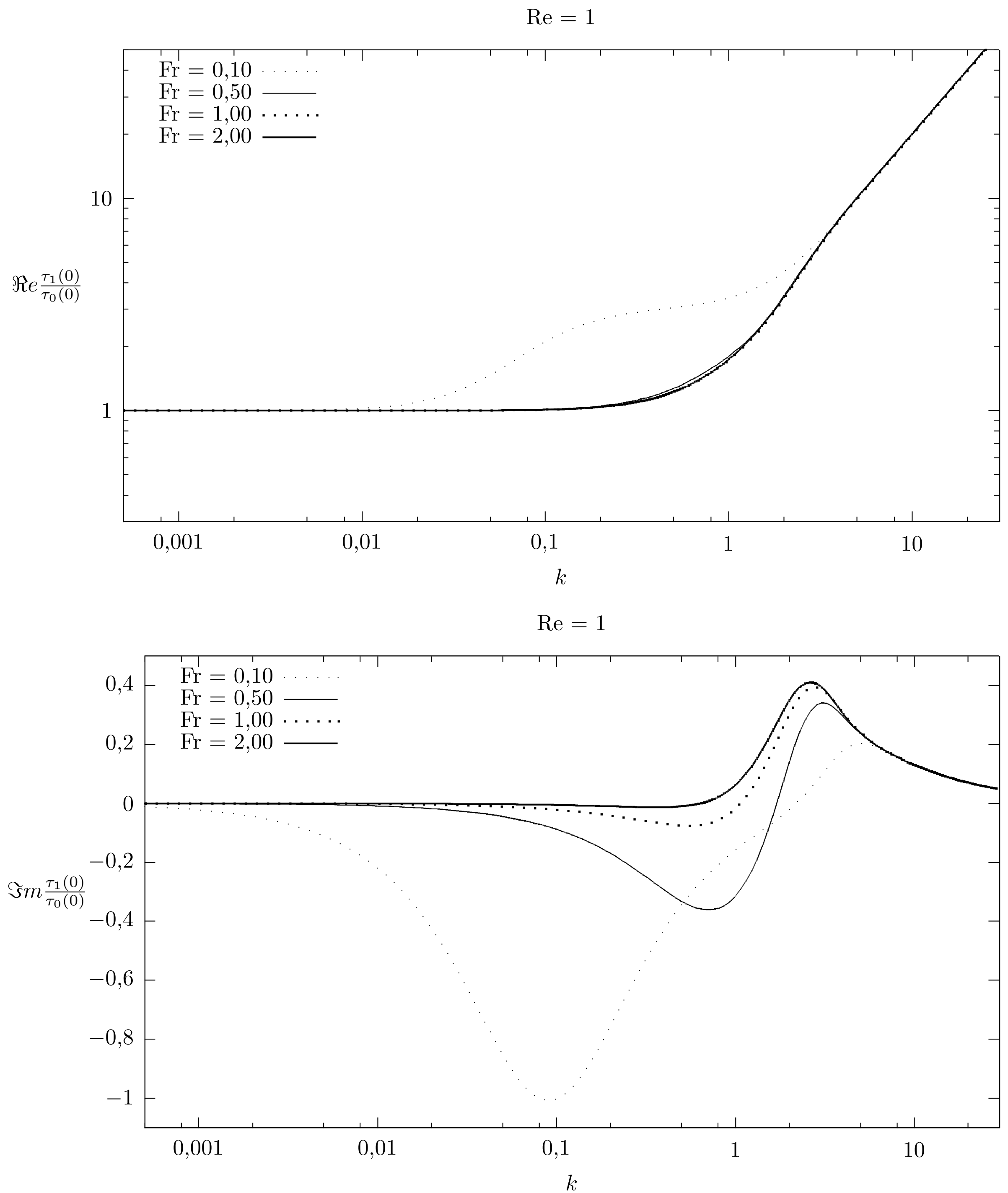


Fig. 2.3 – *Parties réelles (en haut) et parties imaginaires (en bas) de la perturbation du cisaillement au fond renormalisée, pour* $Re = 1$ *et différentes valeurs de Fr.*

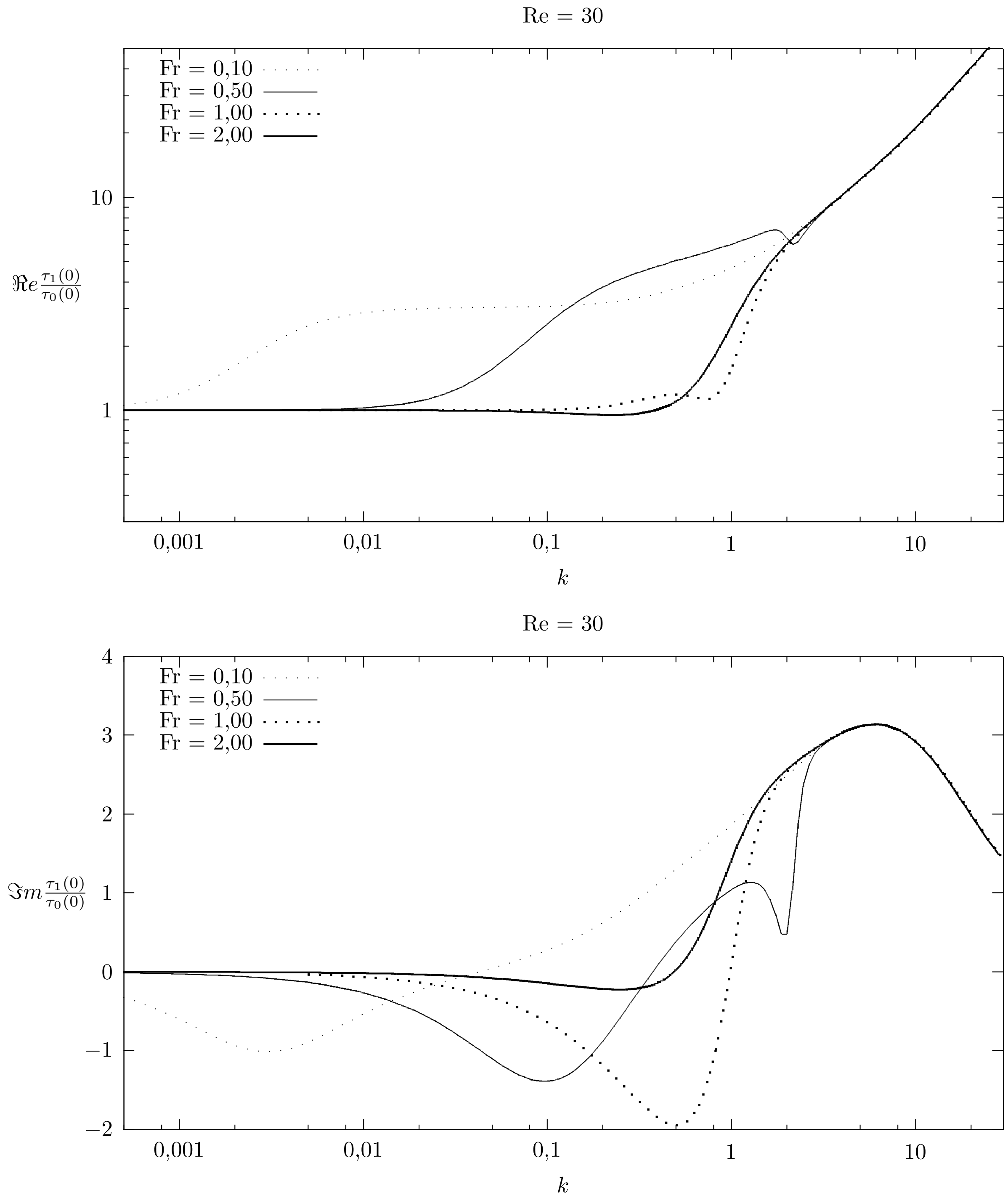


FIG. 2.4 – *Parties réelles (en haut) et parties imaginaires (en bas) de la perturbation du cisaillement au fond renormalisée, pour* $Re = 30$ *et différentes valeurs de Fr.*

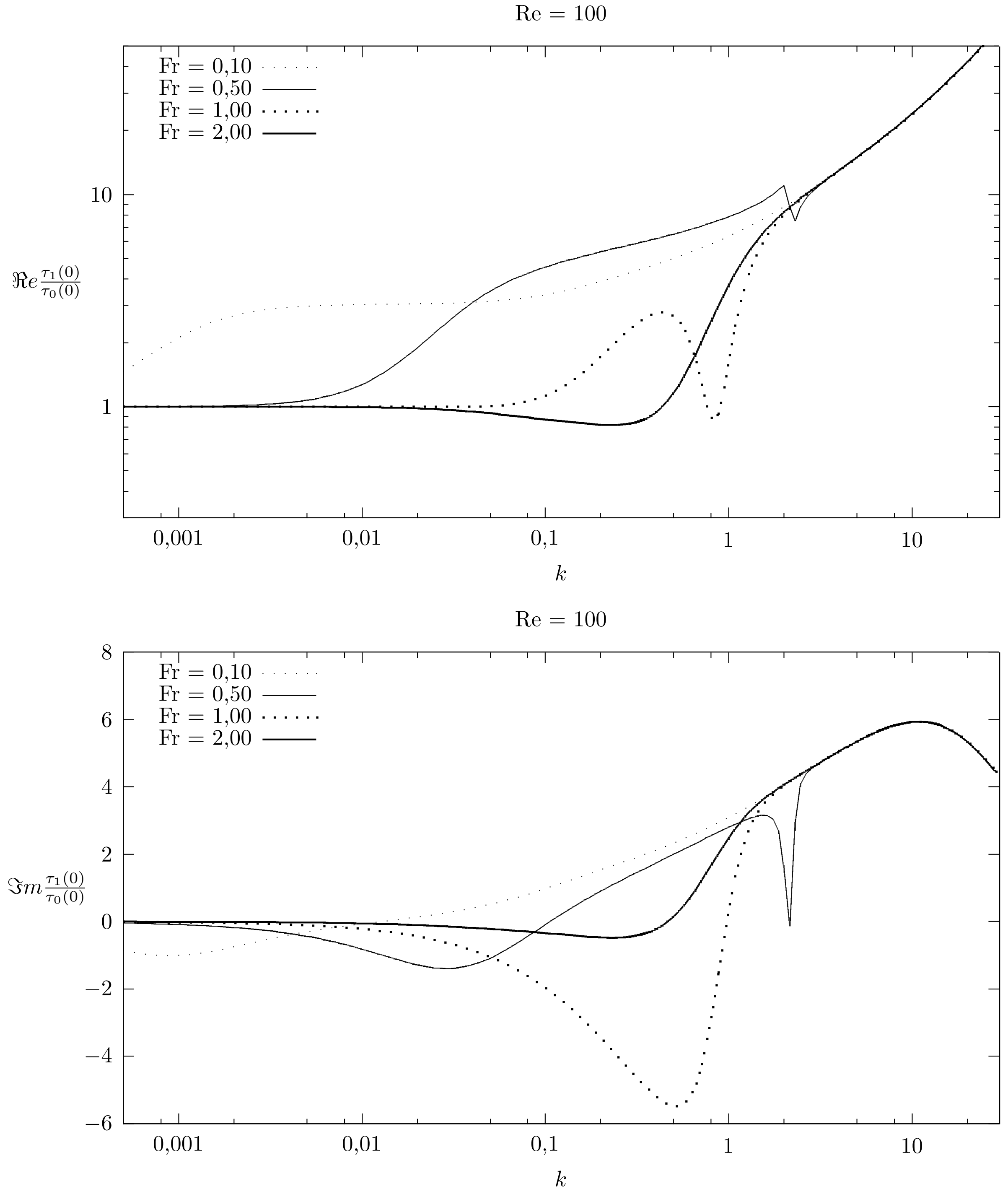


FIG. 2.5 – *Parties réelles (en haut) et parties imaginaires (en bas) de la perturbation du cisaillement au fond renormalisée, pour $Re = 100$ et différentes valeurs de Fr.*

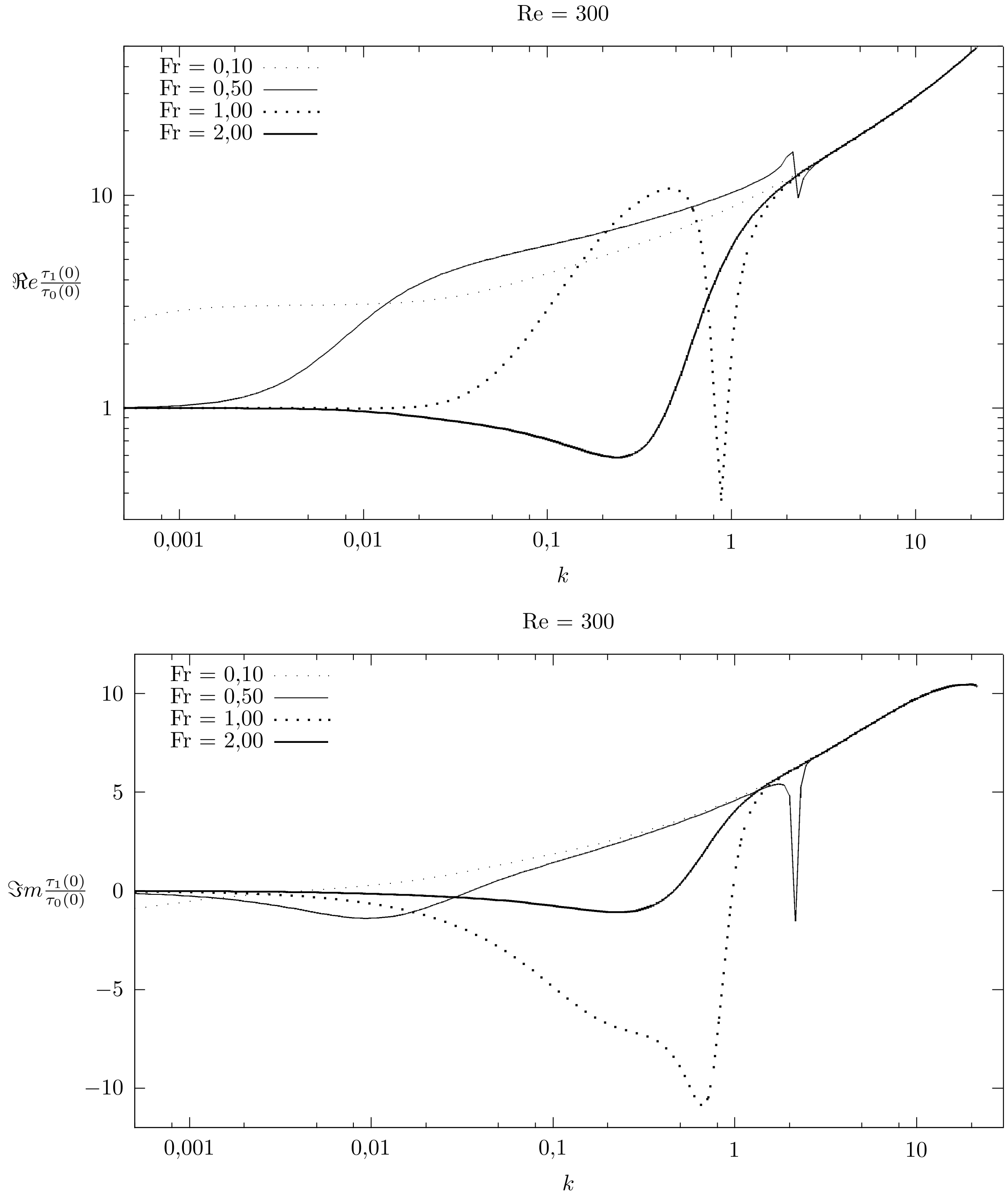
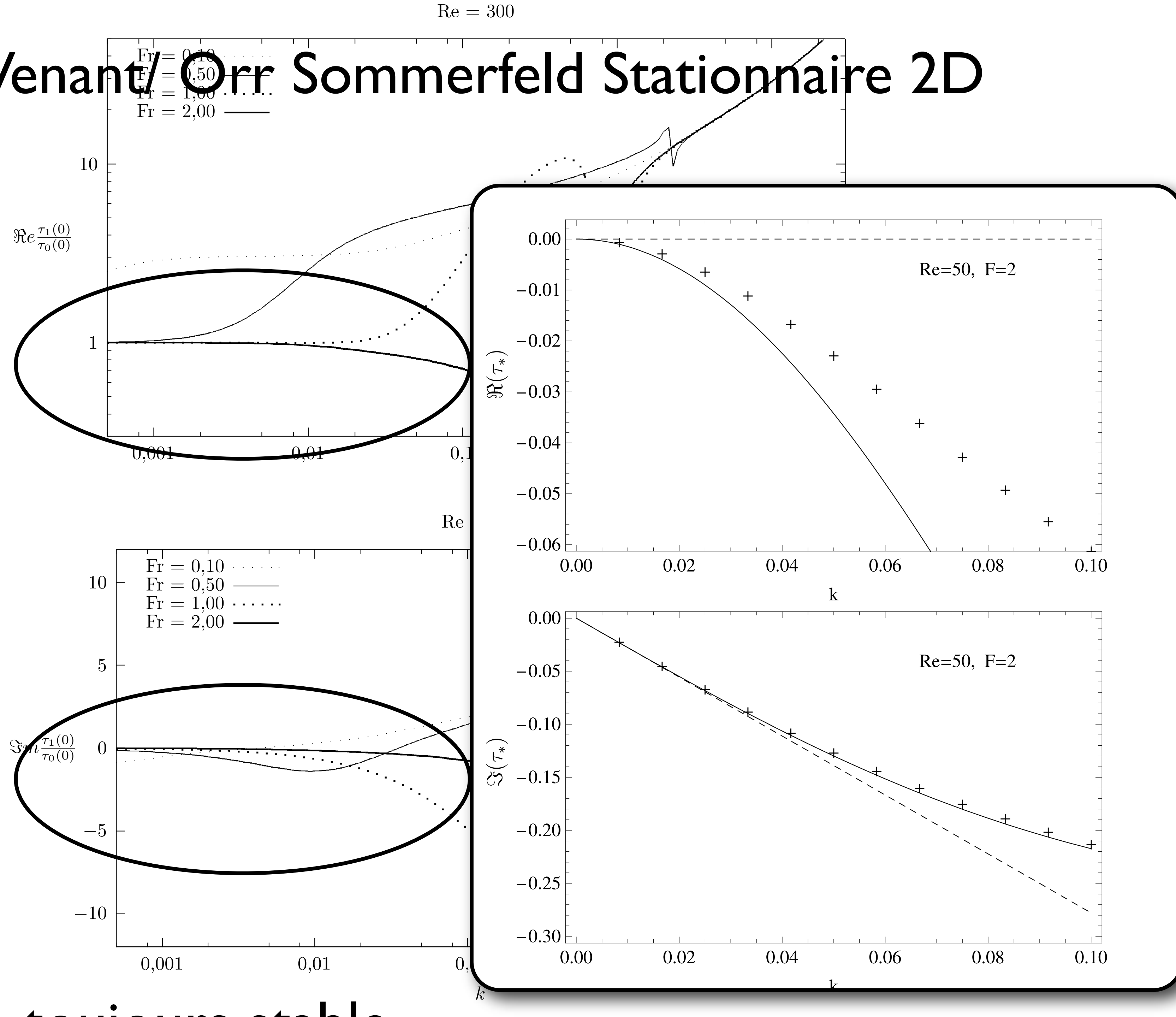


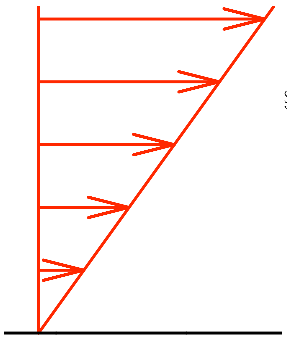
FIG. 2.6 – *Parties réelles (en haut) et parties imaginaires (en bas) de la perturbation du cisaillement au fond renormalisée, pour $Re = 300$ et différentes valeurs de Fr.*

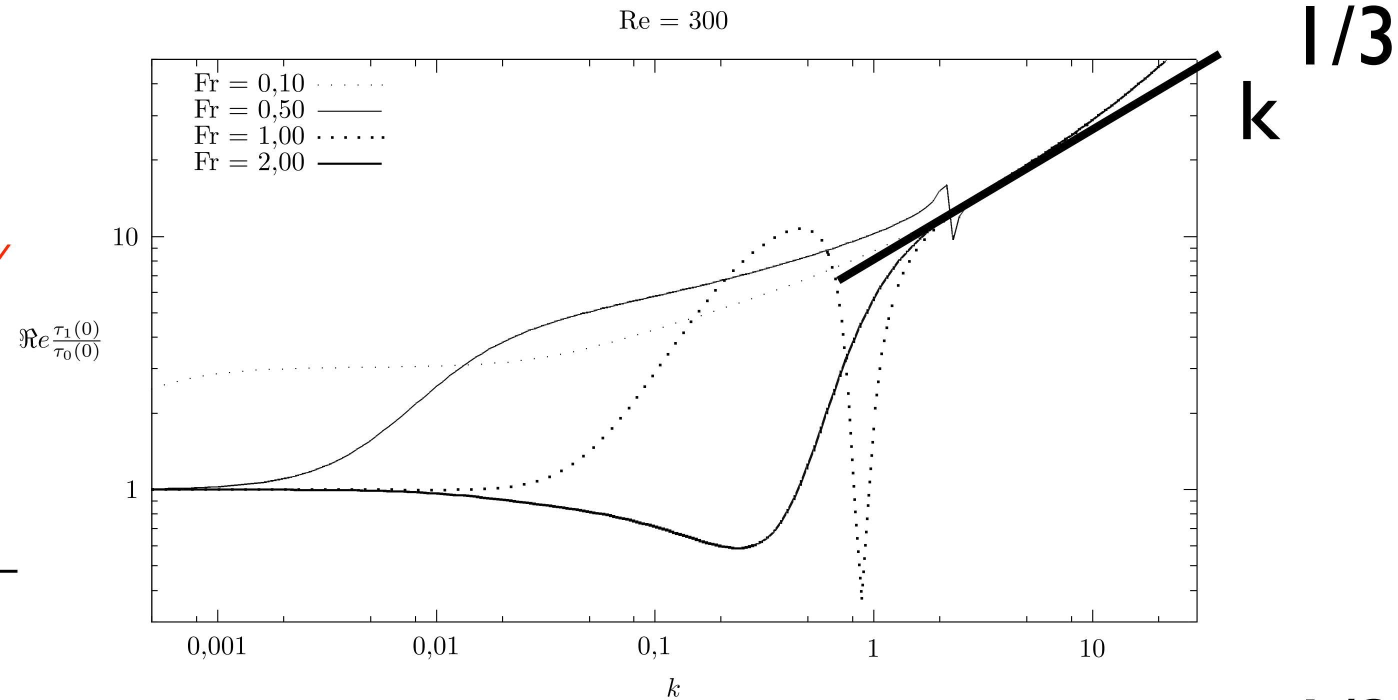
Saint Venant/ Orr Sommerfeld Stationnaire 2D

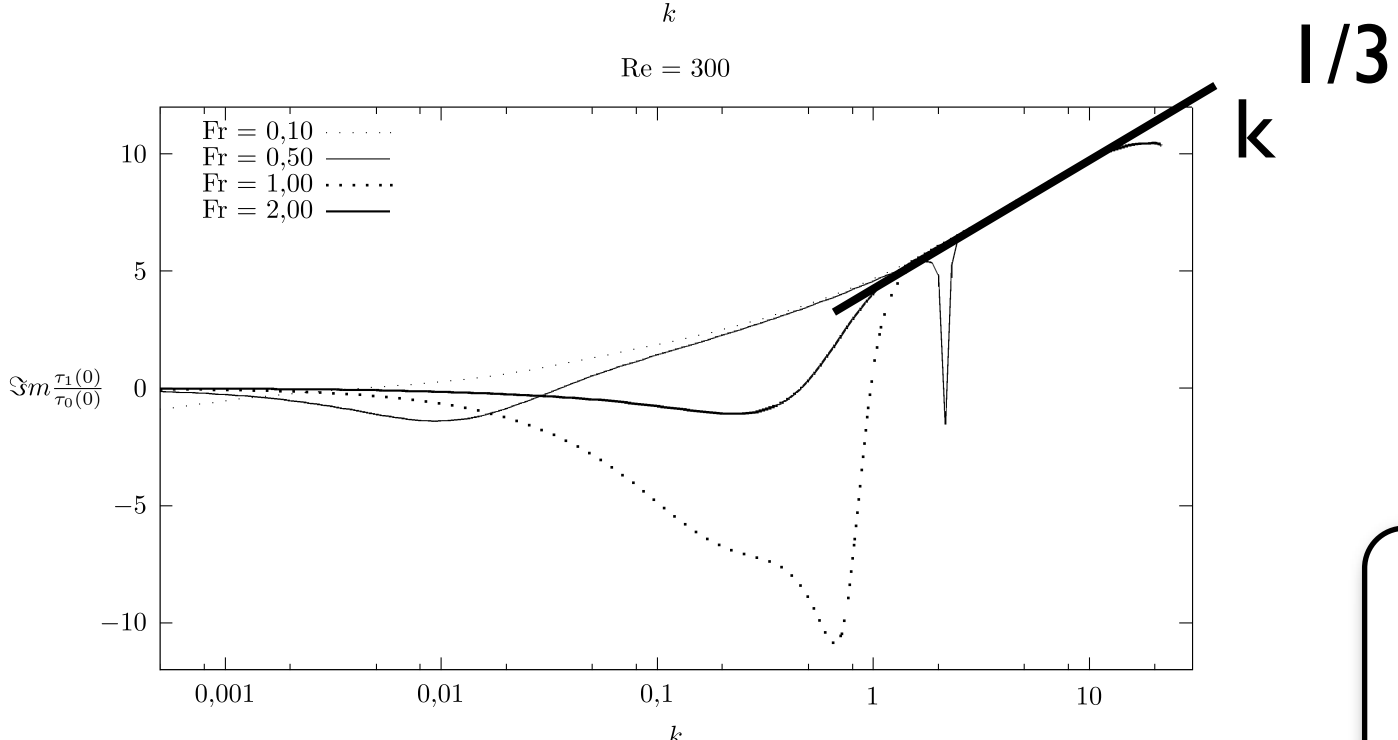


c'est bien toujours stable

FIG. 2.6 – *Parties réelles (en haut) et parties imaginaires (en bas) de la perturbation du cisaillement au fond renormalisée, pour $Re = 300$ et différentes valeurs de Fr.*



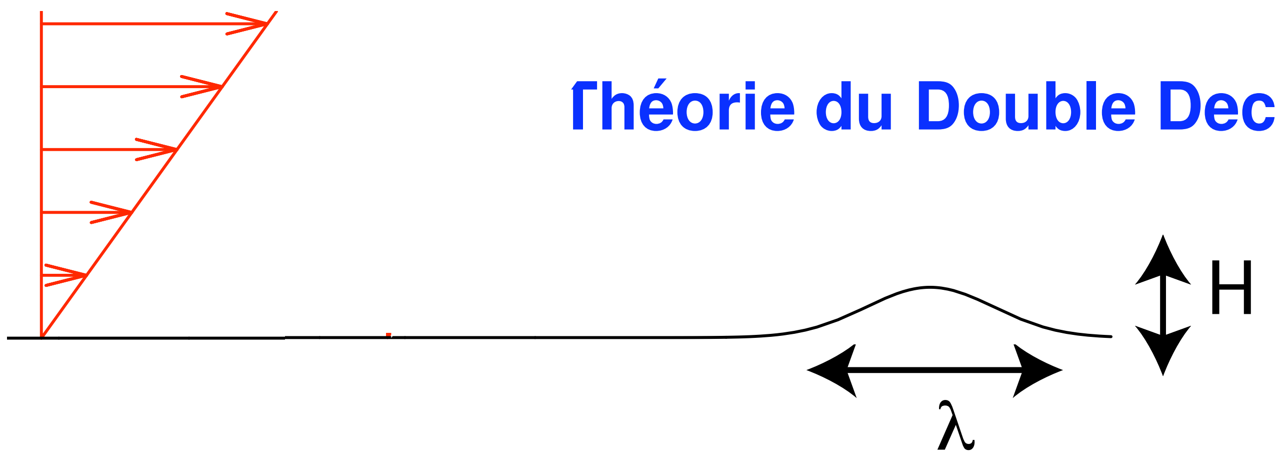




$k^{1/3}$

FIG. 2.6 – *Parties réelles (en haut) et parties imaginaires (en bas) de la perturbation du cisaillement au fond renormalisée, pour Re = 300 et différentes valeurs de Fr.*

Théorie du Double Deck



Pour une bosse de longueur d'ordre λ et de hauteur d'ordre $H << \delta$:

$$\tau = \mu U_0'(\bar{U}_S'(1 + (\frac{U_0'}{\nu\lambda})^{1/3} H \tilde{c})), \text{avec } \tilde{c} = FT^{-1}[FT[\tilde{f}]3Ai(0)(-(i2\pi\tilde{k})\bar{U}_S')^{1/3}]$$

la fonction du temps $\bar{U}_S'$ est un nombre d'ordre 1.

$$(\frac{U_0'}{\nu\lambda})^{1/3} H \leq 1$$

$k^{1/3}$

dans la littérature :

$$q_s = E\varpi(\tau^a(\tau - \tau_s)^b)$$

si$(\tau - \tau_s) > 0$ alors $\varpi(\tau - \tau_s) = (\tau - \tau_s)$ sinon $\varpi((\tau - \tau_s)) = 0$.

avec une correction de pente pour le seuil :

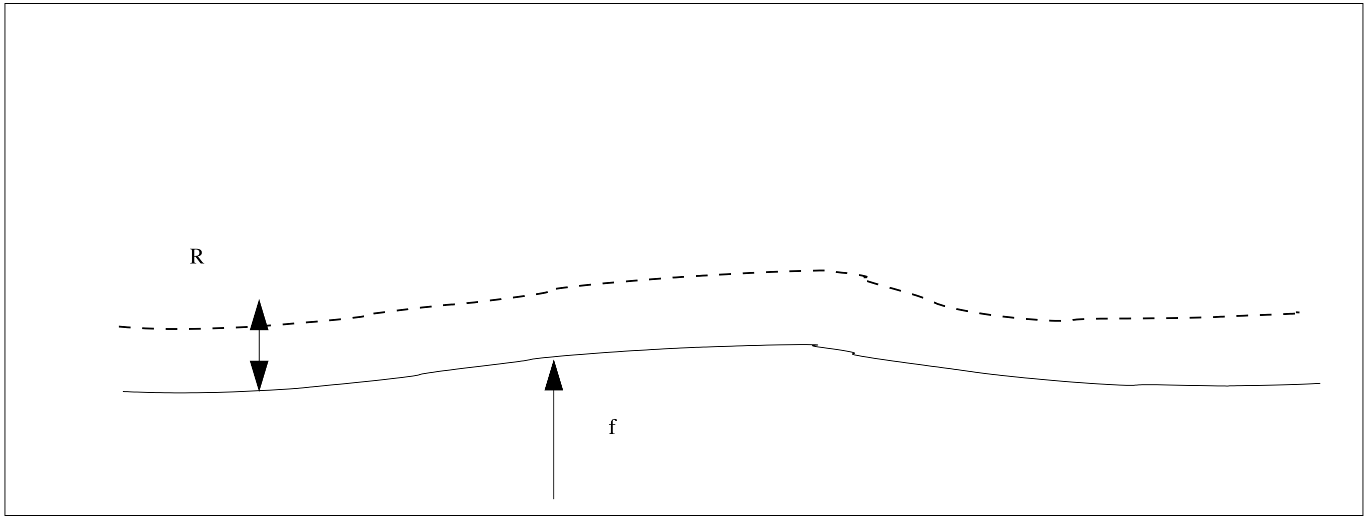
$$\tau_s + \Lambda\frac{\partial f}{\partial x},$$

a, E coefficients, $a = 0, b = 3$ ou $a = b = 1$ ou $a = 1/2, b = 1$ ou ...

écrire l'équation de conservation de la masse

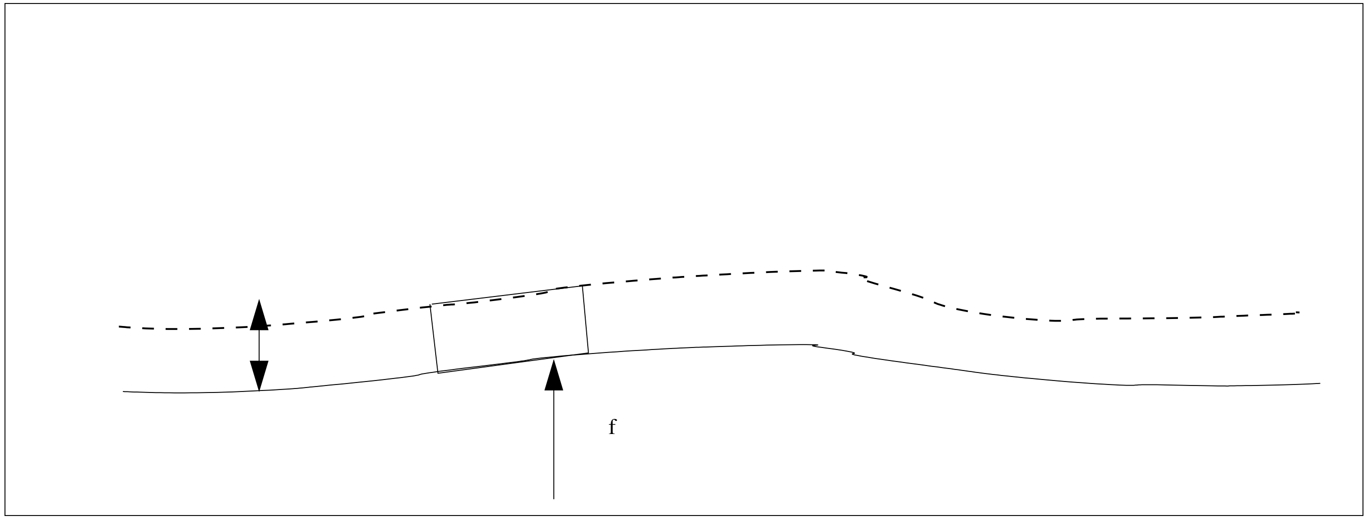
ce qui rentre - ce qui sort

Kroy/ Hermann/ Sauermann 02, Lagrée 03, Valance Langlois 05, Charru Hinch 06, Charru 06



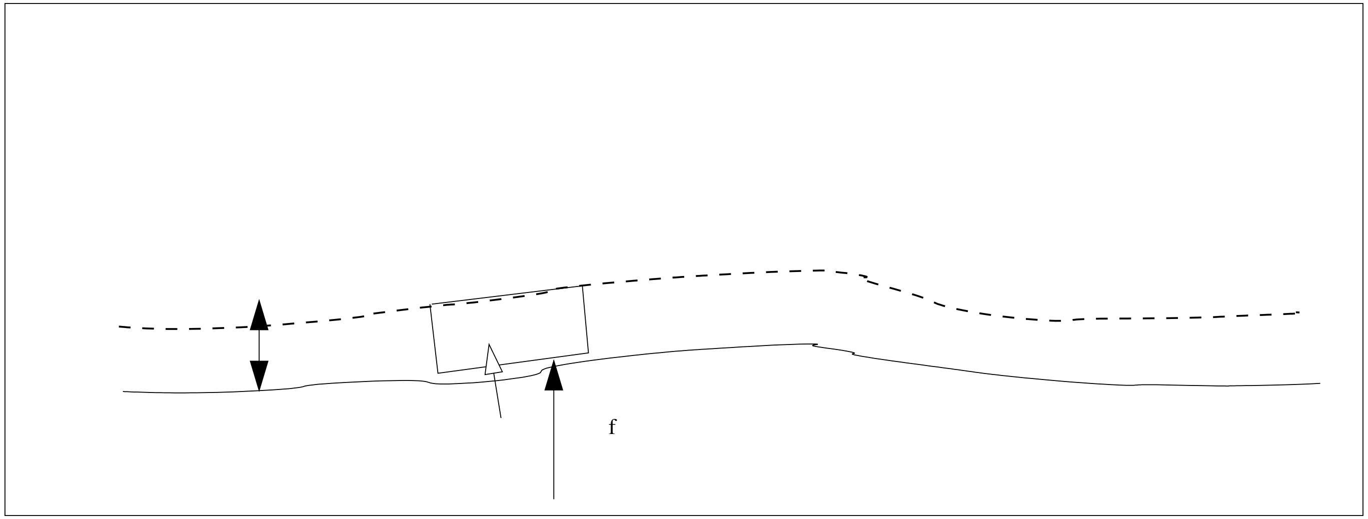
$$\frac{\partial R}{\partial t} = \ldots$$

$$\frac{\partial f}{\partial t} = \ldots$$



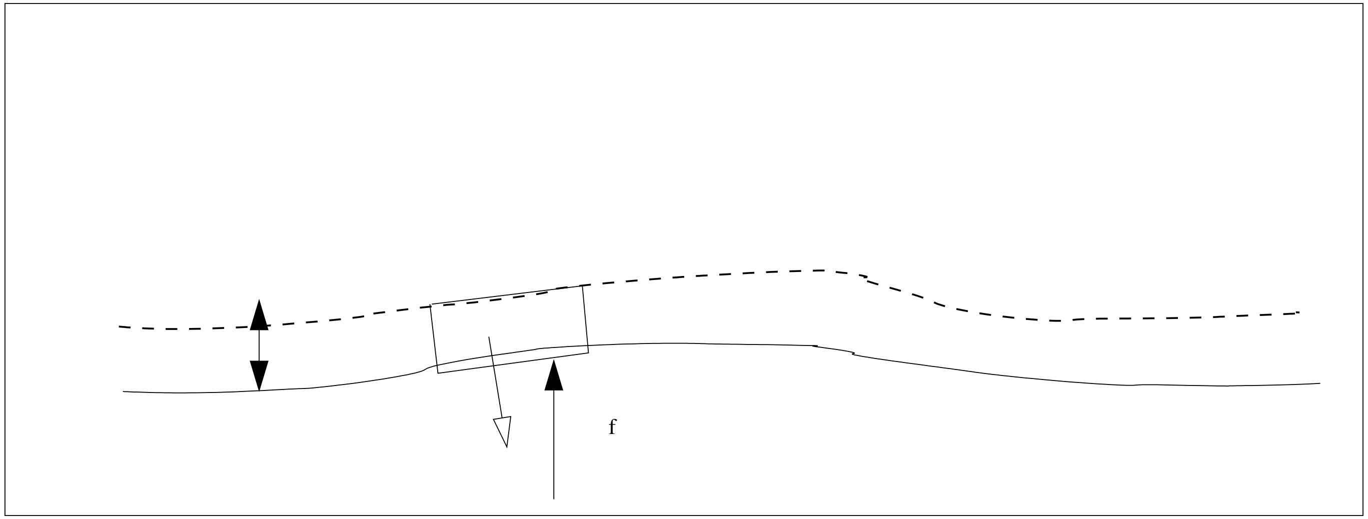
$$\frac{\partial R}{\partial t} = \ldots$$

$$\frac{\partial f}{\partial t} = \ldots$$



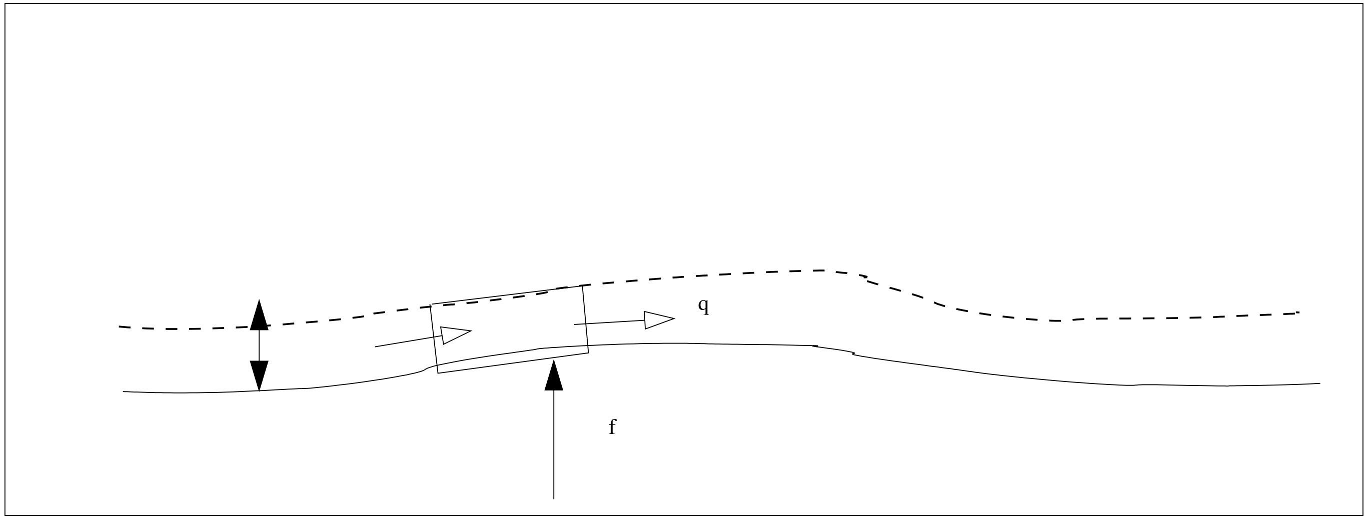
$$\frac{\partial R}{\partial t} = ... + \Gamma$$

$$\frac{\partial f}{\partial t} = -\Gamma$$



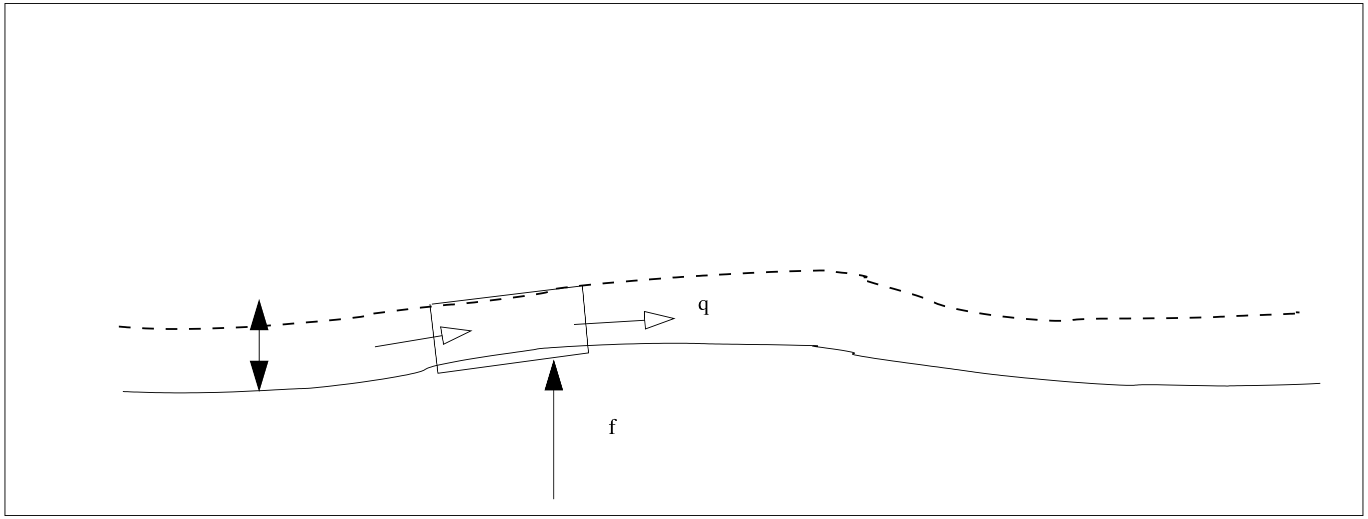
$$\frac{\partial R}{\partial t} = ... + \Gamma$$

$$\frac{\partial f}{\partial t} = -\Gamma$$



$$\frac{\partial R}{\partial t} = -\frac{\partial q}{\partial x} + \Gamma$$

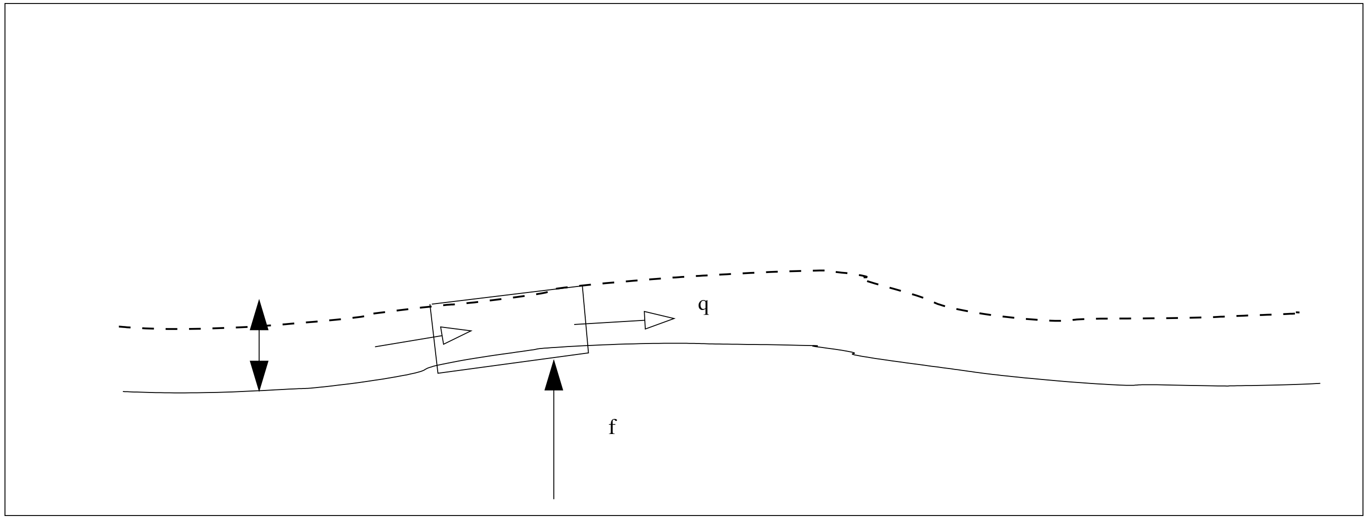
$$\frac{\partial f}{\partial t} = -\Gamma$$



$$\frac{\partial R}{\partial t} = -\frac{\partial q}{\partial x} + \Gamma \qquad \frac{\partial f}{\partial t} = -\Gamma$$

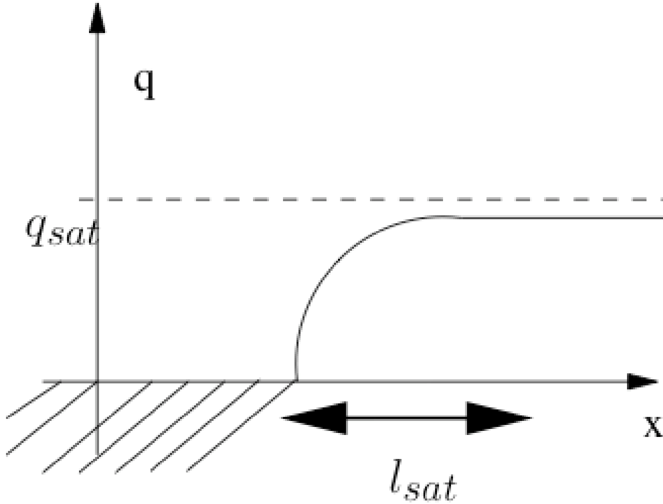
$\Gamma =$ (érosion)-(déposition)

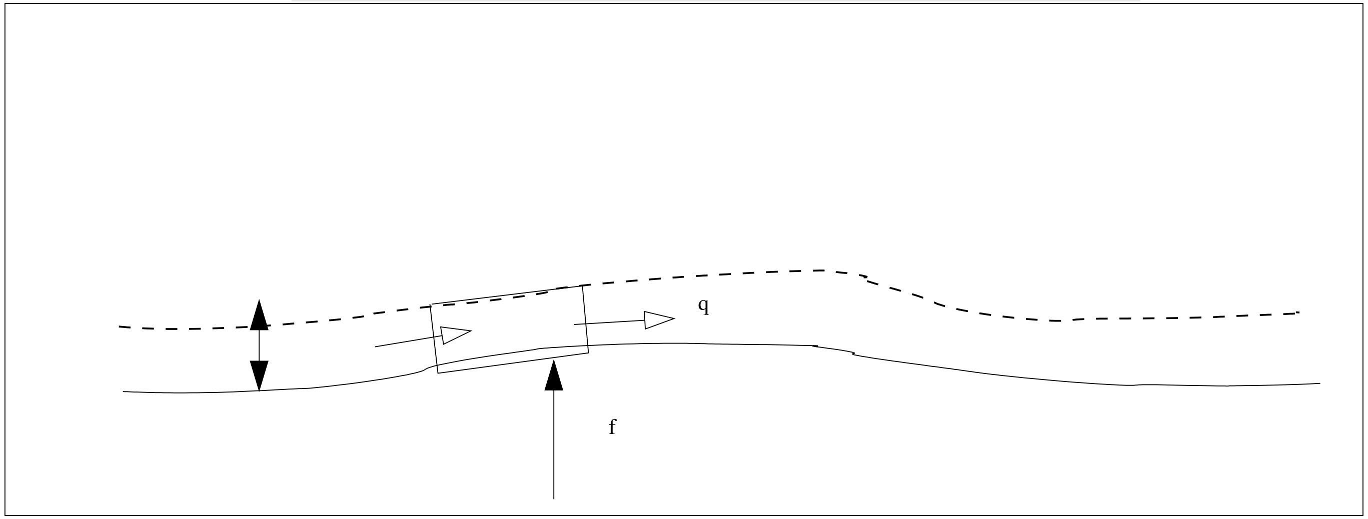
-(déposition) $\propto -R$ érosion $\propto (\tau - \tau_s)$ et $q \propto R\,\tau$



$$l_s \frac{\partial q}{\partial x} + q = q_s \qquad \frac{\partial f}{\partial t} = -\frac{\partial q}{\partial x}$$

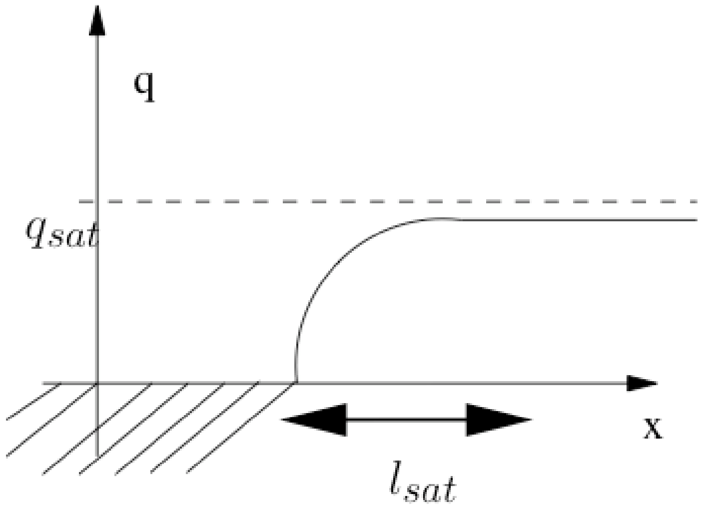
$$q_s = E\varpi(\tau - \tau_s)$$

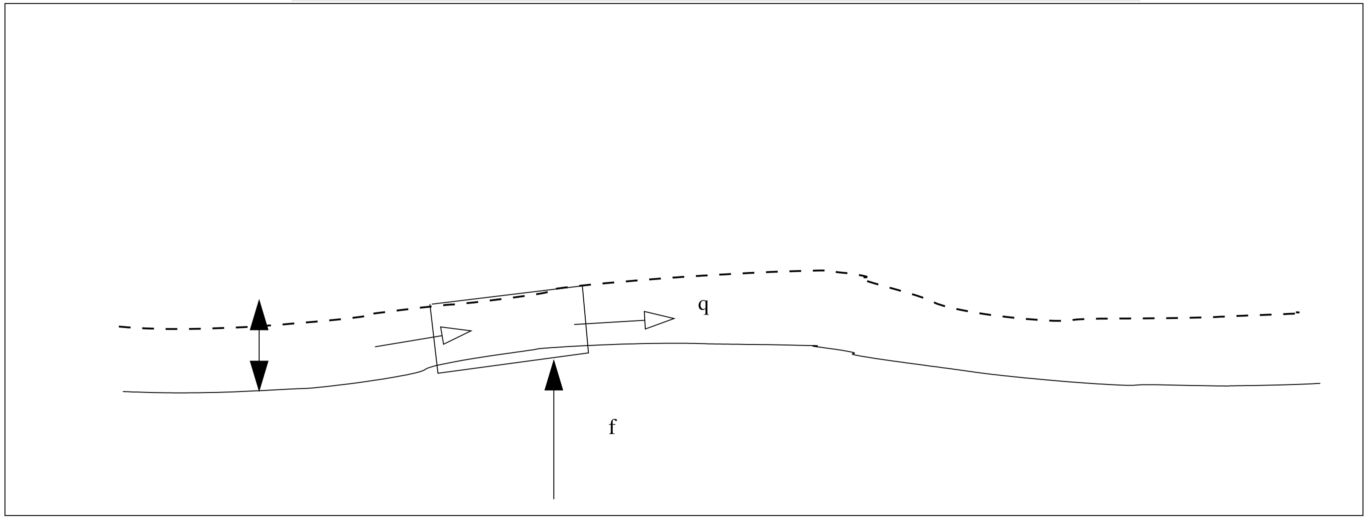




Sauerman, Kroy, Hermann 01, Andreotti Claudin Douady 02,

$$l_{sat}\frac{\partial q}{\partial x} + q = q_{sat}$$



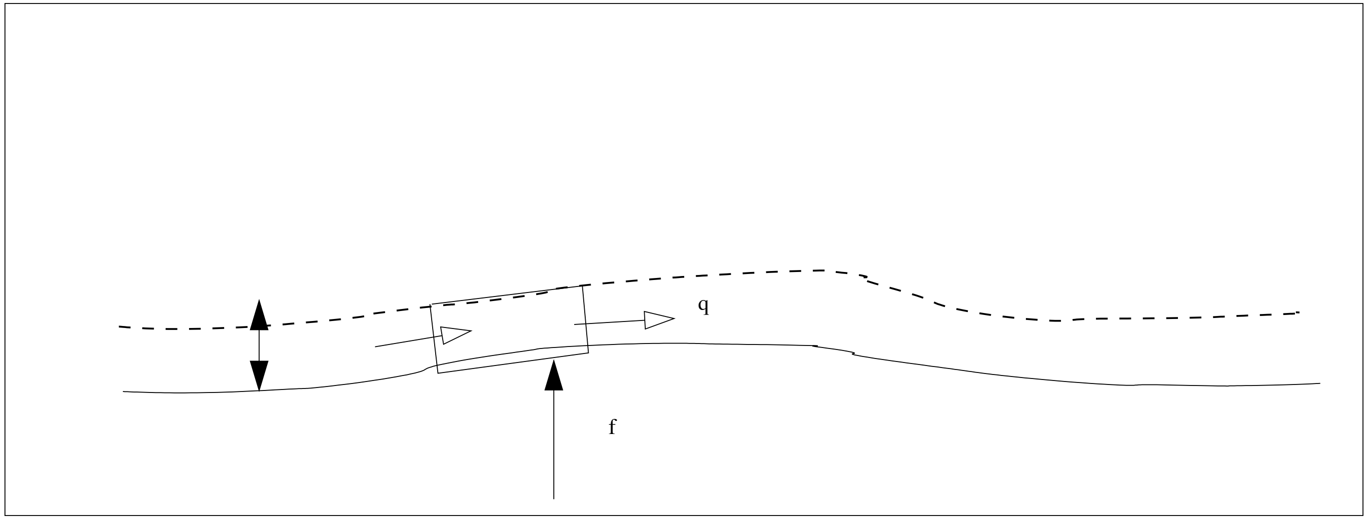


Du Boy (1879) :

"*une fois une certaine quantité de matières en mouvement sur le fond du lit, la vitesse des filets liquides devient trop faible pour entraîner davantage : le cours d'eau est alors saturé. Un cours d'eau non saturé tend à le devenir en entraînant une partie des matériaux qui composent son lit, et en choisissant de préférence les plus petits.*"

Charru 06

$$\frac{\partial n}{\partial t} = \dot{n}_e - \dot{n}_d - \frac{\partial q_x}{\partial x} - \frac{\partial q_y}{\partial y},$$



$$\dot{n}_d = c_d \frac{U_s}{d_s} n,$$

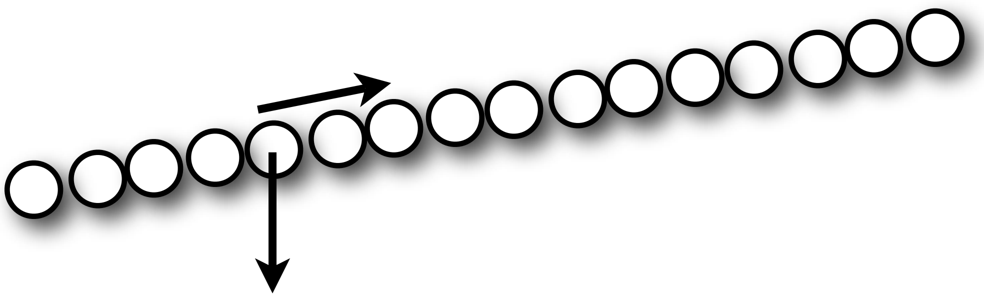
$$C \partial_t h = -\frac{\pi d_s^3}{6} \left(\partial_x q_x + \partial_y q_y \right),$$

$$\dot{n}_e = \frac{18 c_e U_s}{d_s^3} (c_g \theta - \theta_t),$$

$$q_x = n c_u d_s \frac{\partial u_x}{\partial z}, \quad q_y = n c_u d_s \frac{\partial u_y}{\partial z},$$

$$Sh = \frac{\rho \|\boldsymbol{\tau}_h\|}{(\rho_s - \rho)\|\boldsymbol{g}\| d_s},$$

$$f_i^{\nu} = \tau_{ik} n_k^b \epsilon.$$



$$f_i^g = (\rho_s - \rho) g_i \epsilon c_g d_s.$$

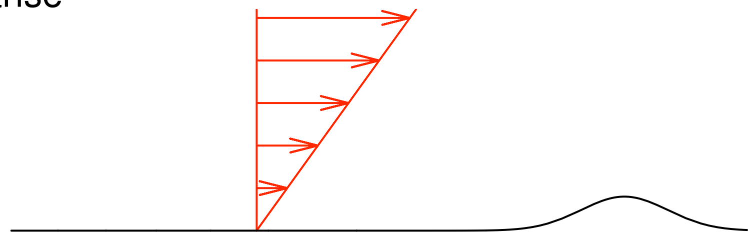
$$f^n = f_k n_k^b,$$

$$\boldsymbol{f}^t = \boldsymbol{f} - f^n \boldsymbol{n}^b.$$

$$\theta = -\frac{\|\boldsymbol{f}^t\|}{f^n}.$$

Asymptotic solution of the flow over a bump; double deck theory

Viscous effects are important near the wall
Perturbation of a shear flow
Non linear resolution (with flow separation) possible
But first we linearise



$$\tau = \mu U_0'(\bar{U}_S'(1 + (\frac{U_0'}{\nu\lambda})^{1/3} H\tilde{c})), \text{with } \tilde{c} = FT^{-1}[FT[\tilde{f}]3Ai(0)(-(i2\pi\tilde{k})\bar{U}_S')^{1/3}]$$

Completely erodible soil, Linear Stability

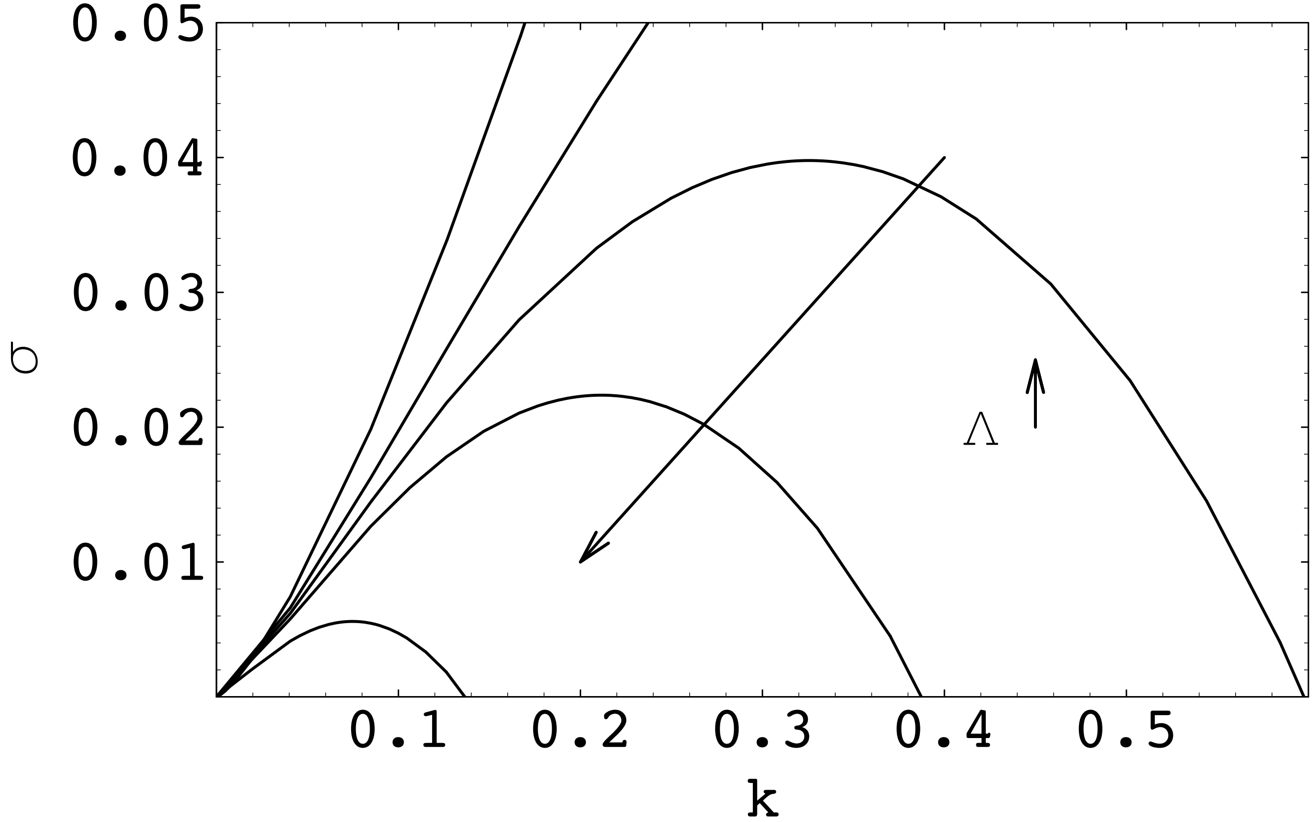
Solution of

$$\tau = TF^{-1}[(3Ai(0))(-ik)^{1/3}TF[f]]$$

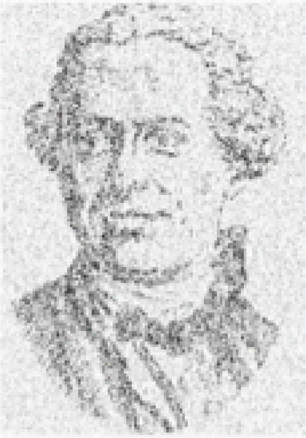
$$l_s \frac{\partial q}{\partial x} + q = \varpi(\tau - \tau_s - \Lambda \frac{\partial f}{\partial x})$$

$$\frac{\partial f}{\partial t} = -\frac{\partial q}{\partial x}$$

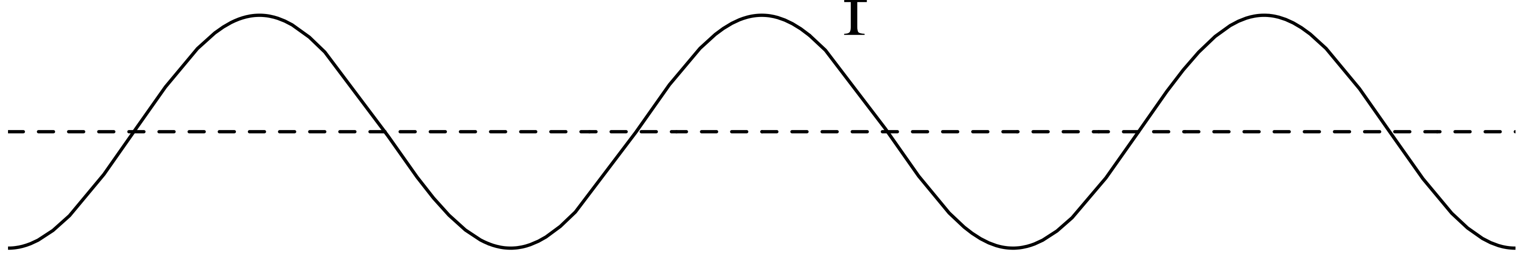
Completely erodible soil, Linear Stability



Λ increases, $l_s = 0$



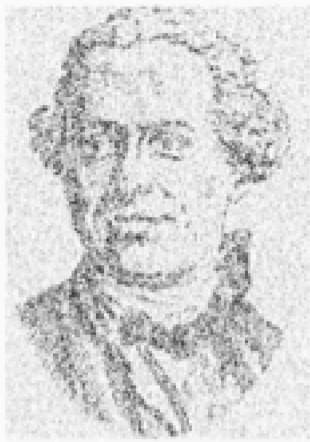
fluid → Fourier

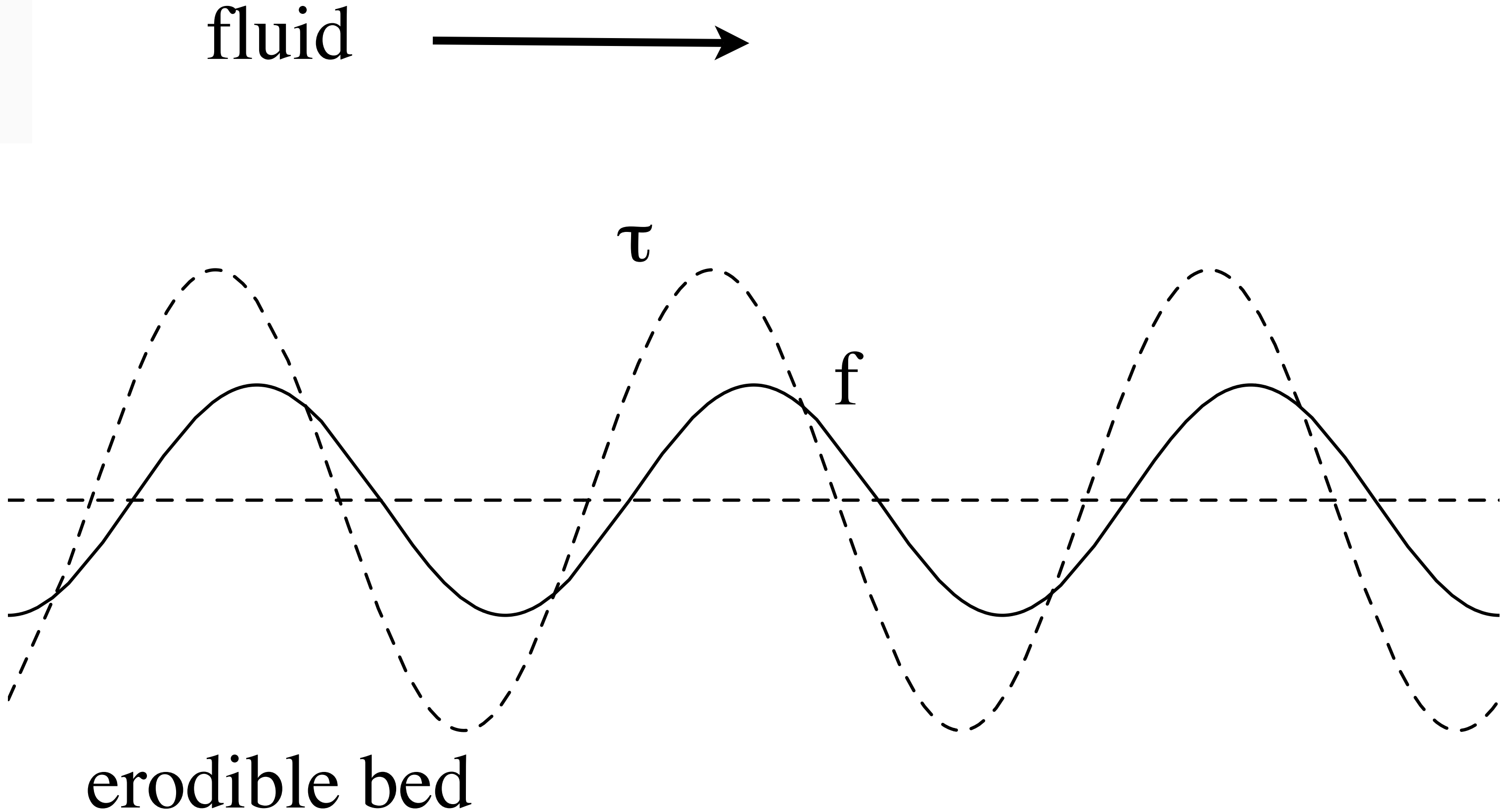


erodible bed

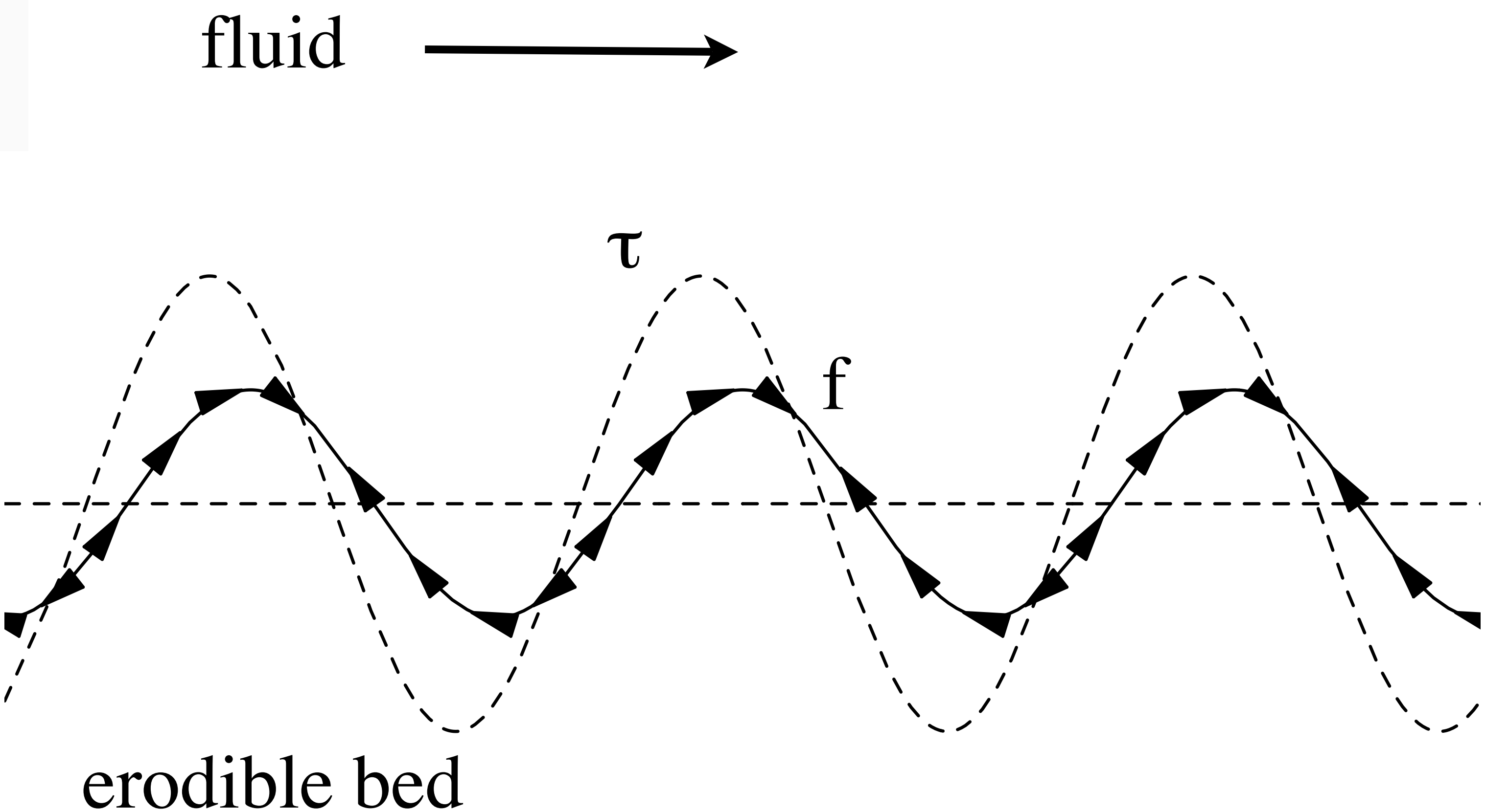
$$e^{\sigma t - ikx}$$

$$Re(\sigma(k))$$





le fond   le frottement



le fond   le frottement

le flux

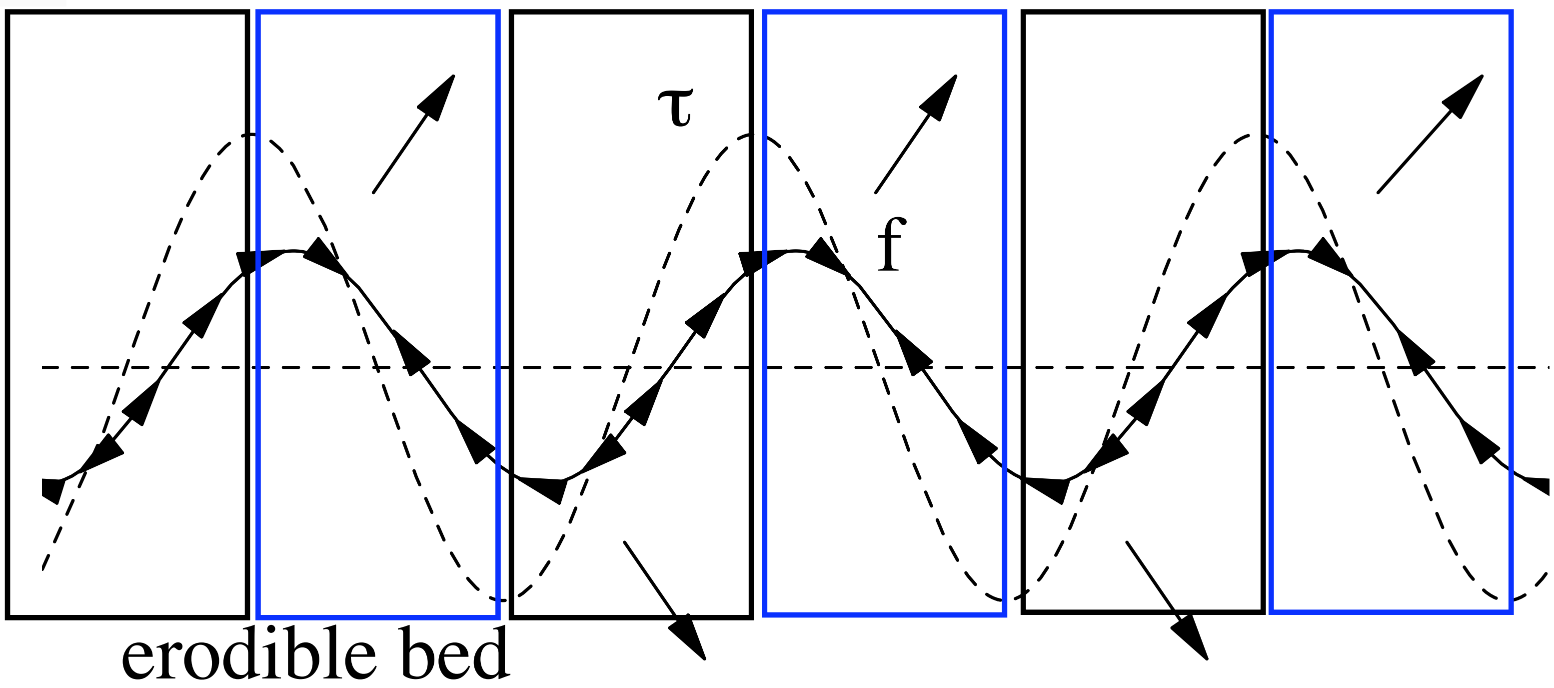
région où le frottement croît

le fond diminue

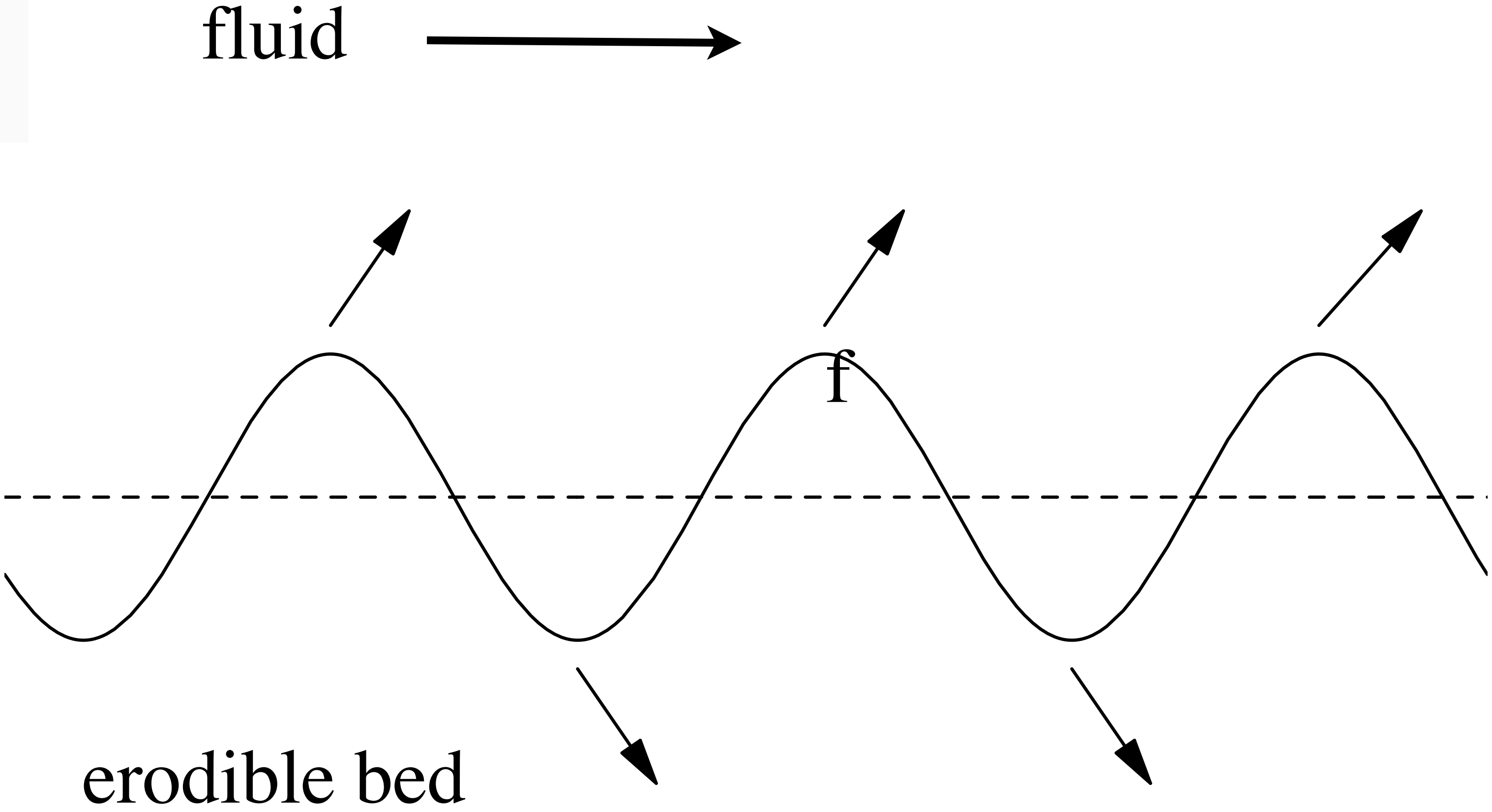
région où le frottement décroît

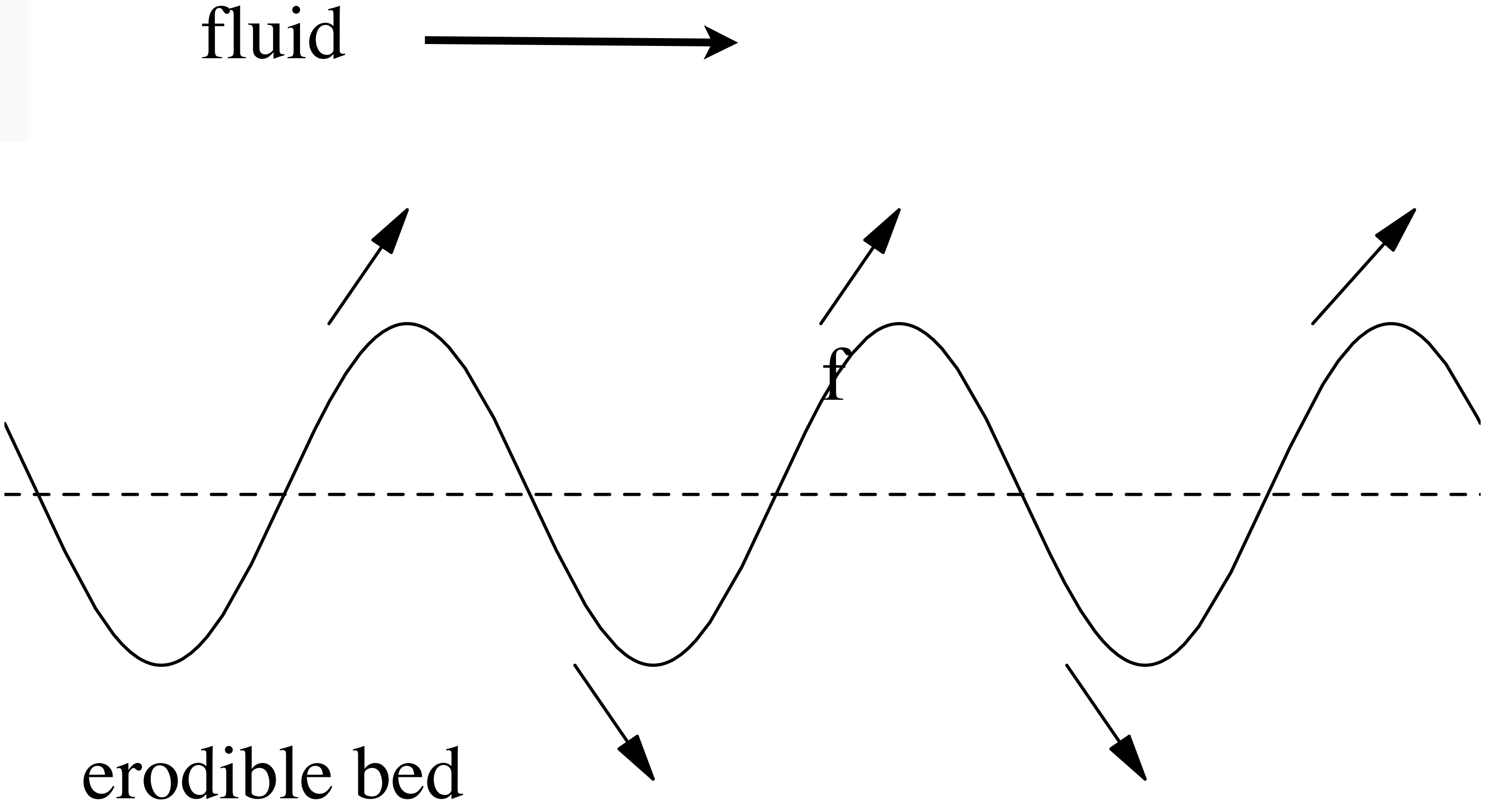
fluid

le fond augmente

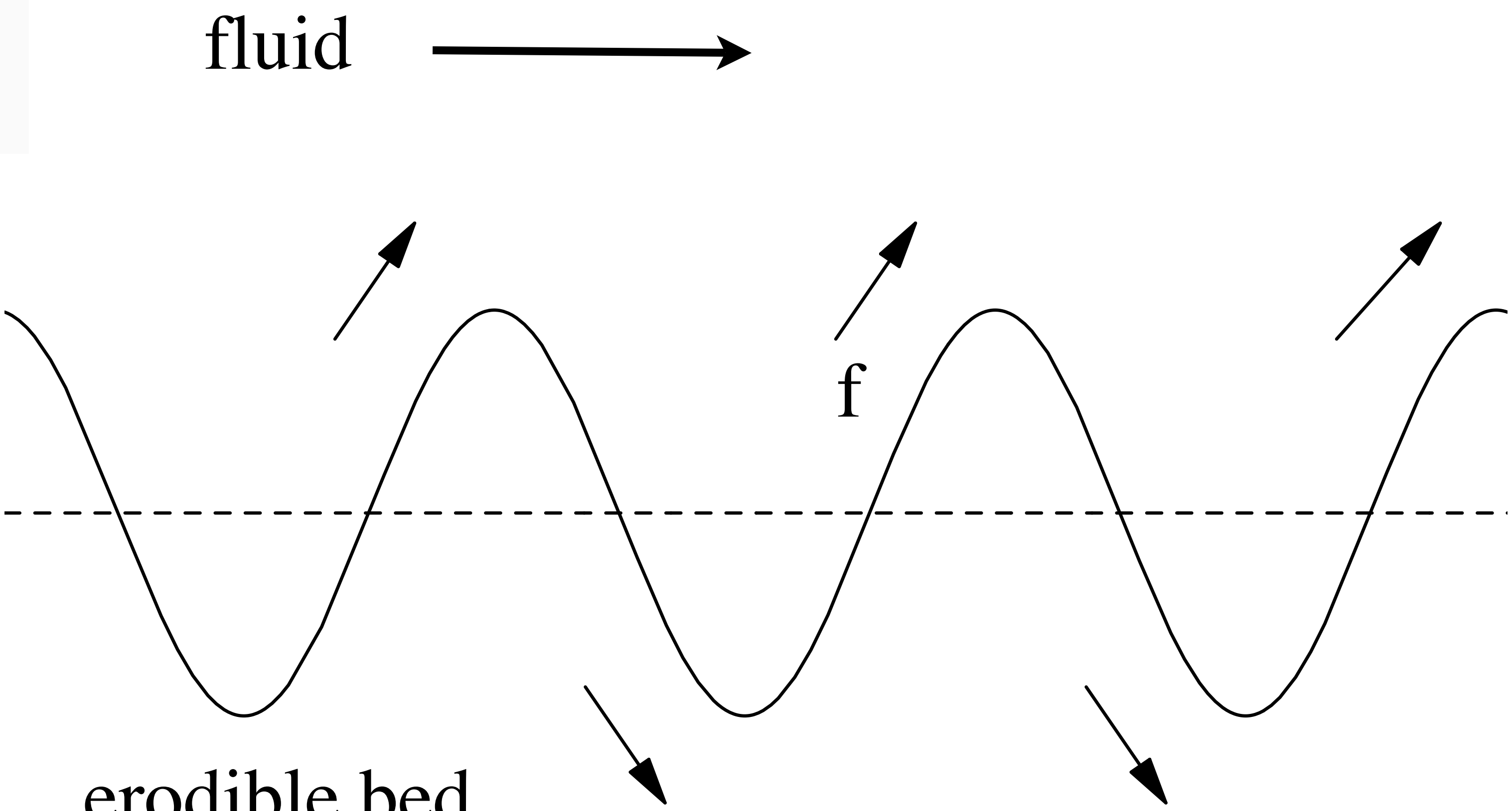


le flux est positif après le sommet, on creuse dans les creux

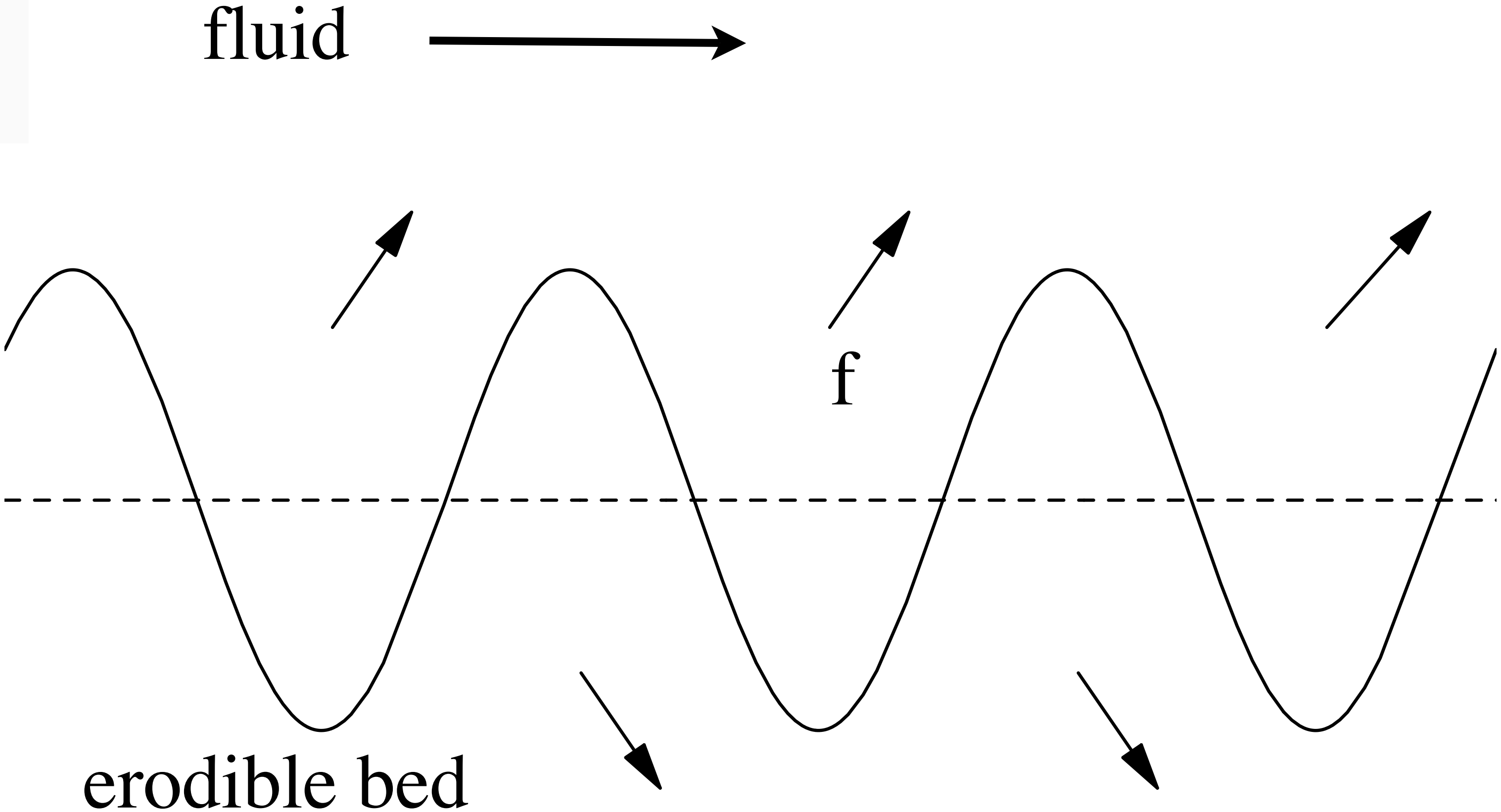

fluid
f
erodible bed



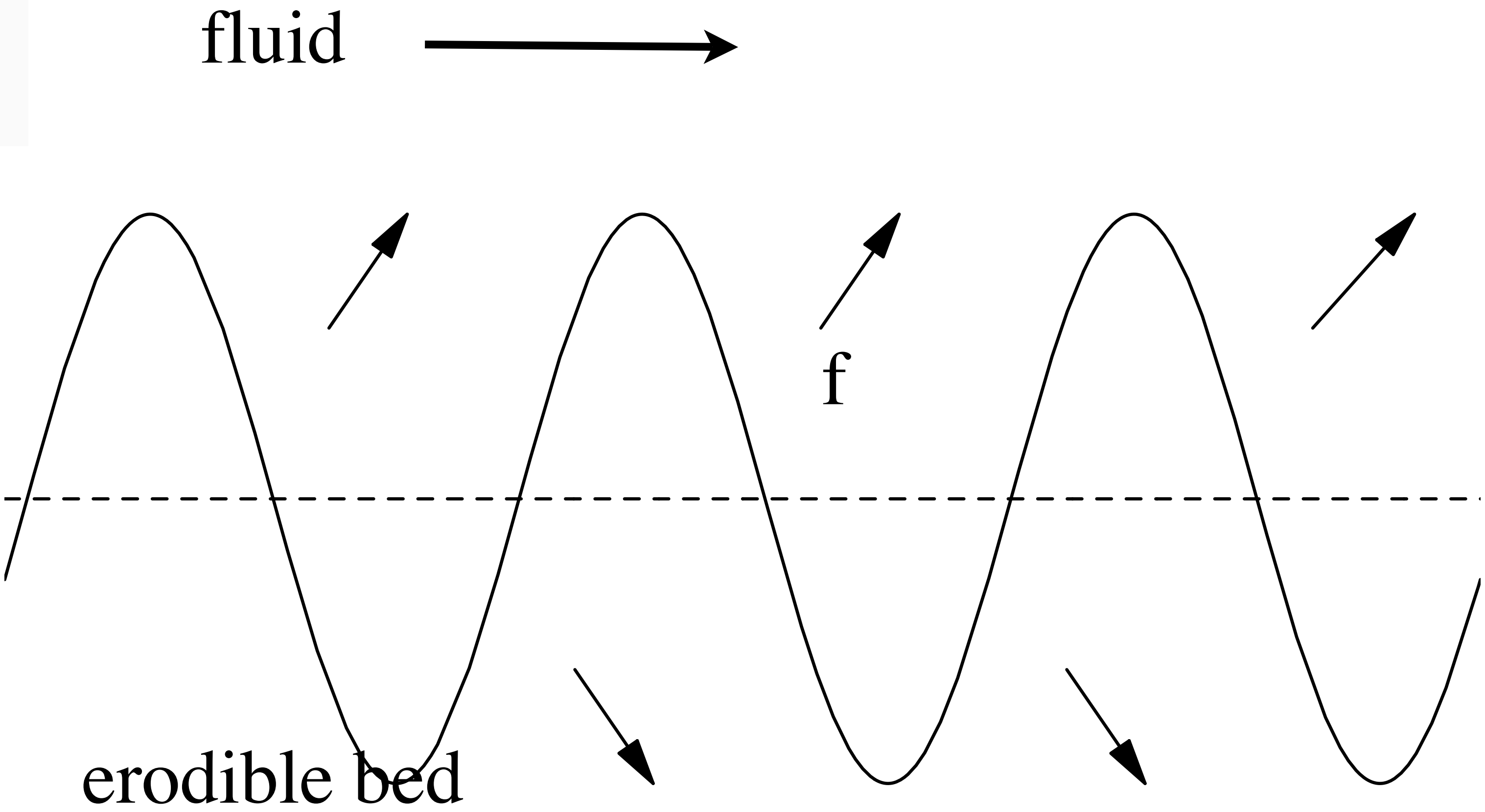
on a bien une instabilité



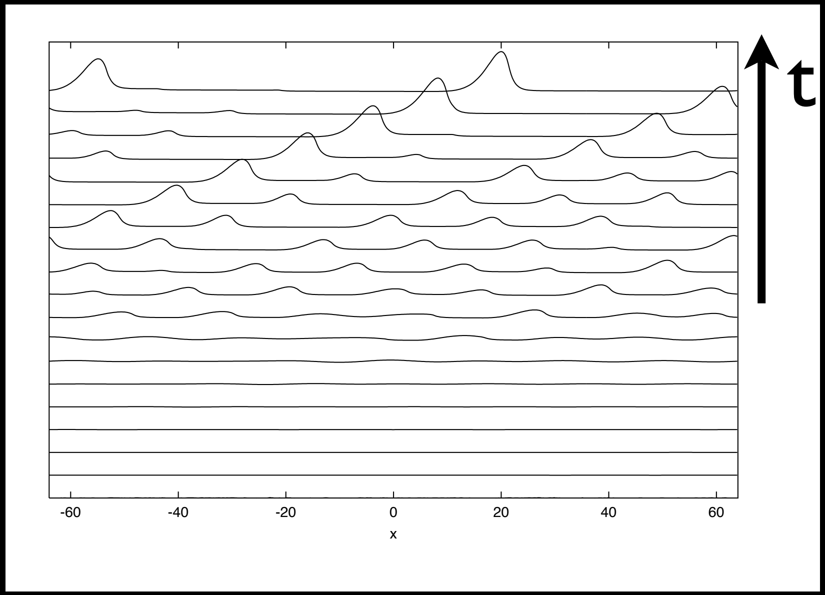
on a bien une instabilité

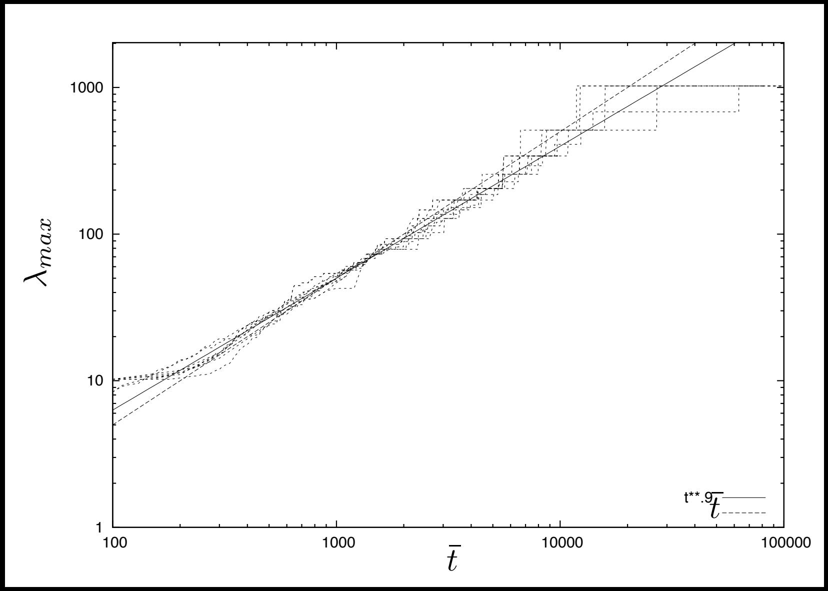


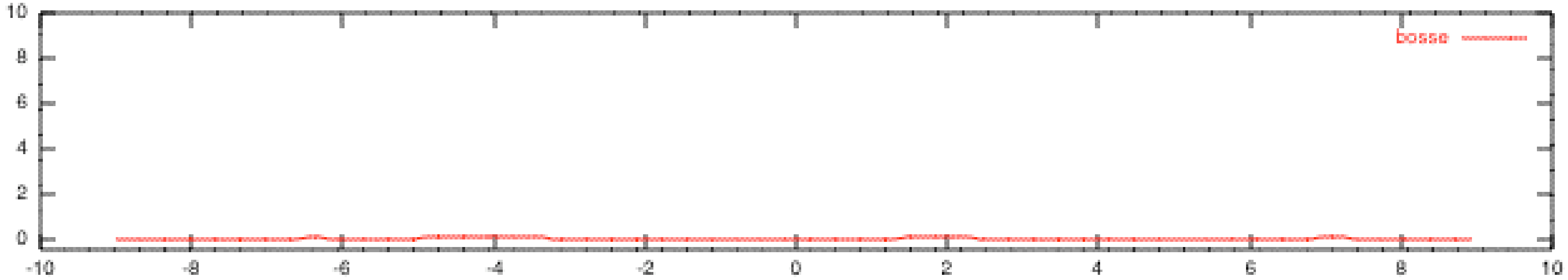
on a bien une instabilité



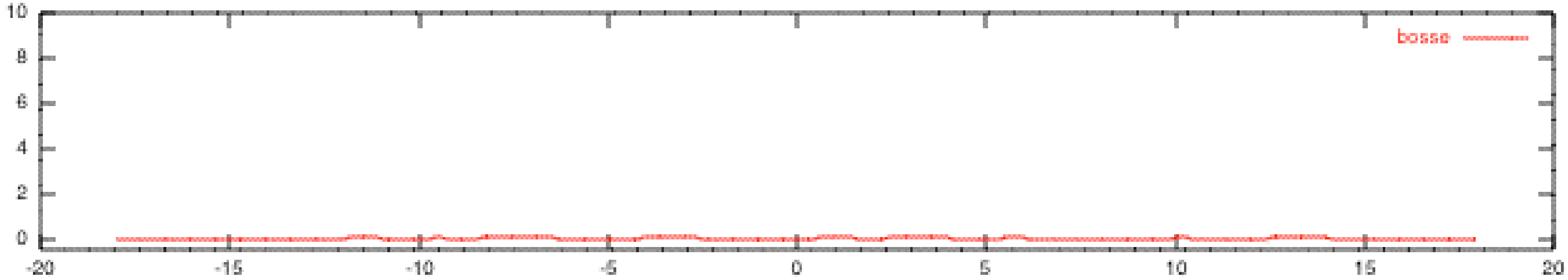
on a bien une instabilité


t
x
-60
-40
-20
0
20
40
60


λ_{max}
$\bar{t}$
1000
100
10
1
100
1000
10000
100000

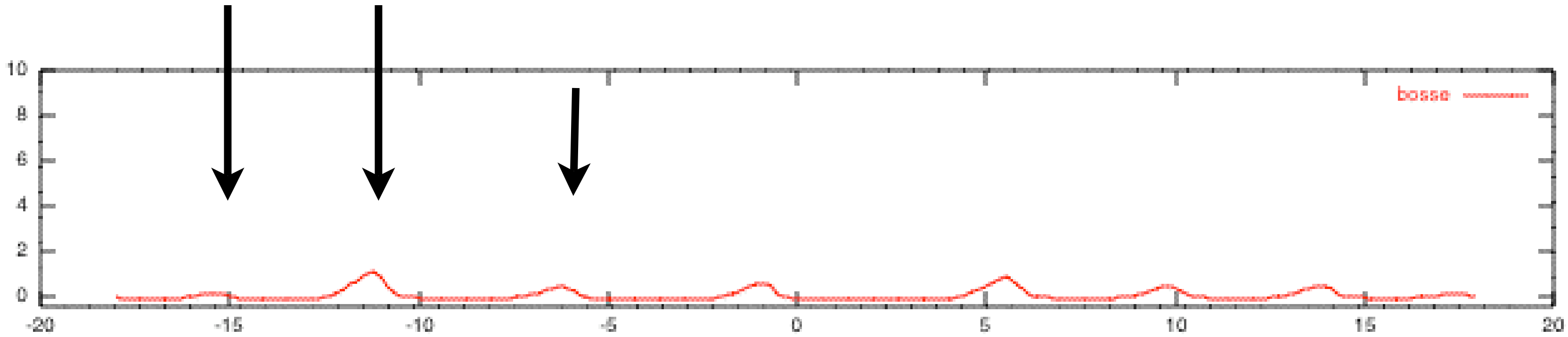


bosse
10
8
6
4
2
0
-10
-8
-6
-4
-2
0
2
4
6
8
10

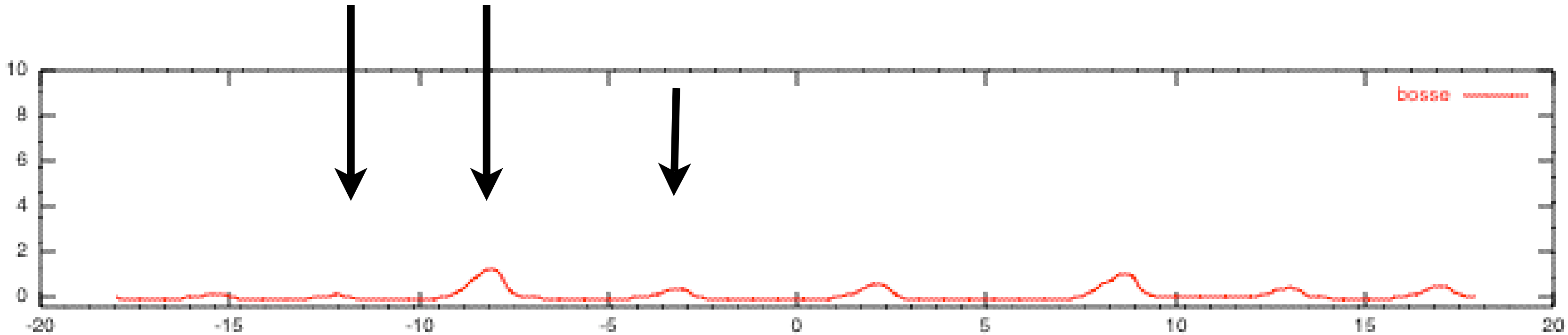


bosse
10
8
6
4
2
0
-20
-15
-10
-5
0
5
10
15
20

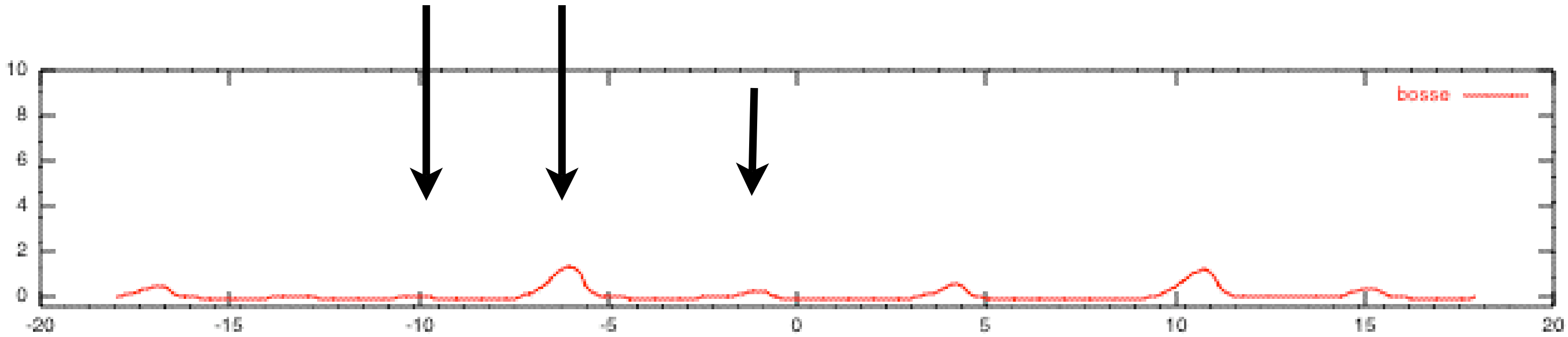
coarsening



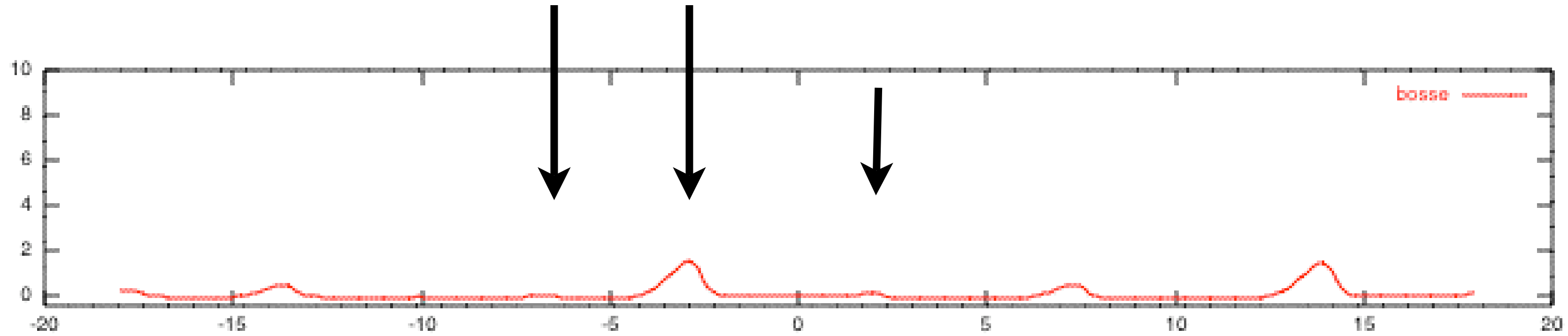
coarsening



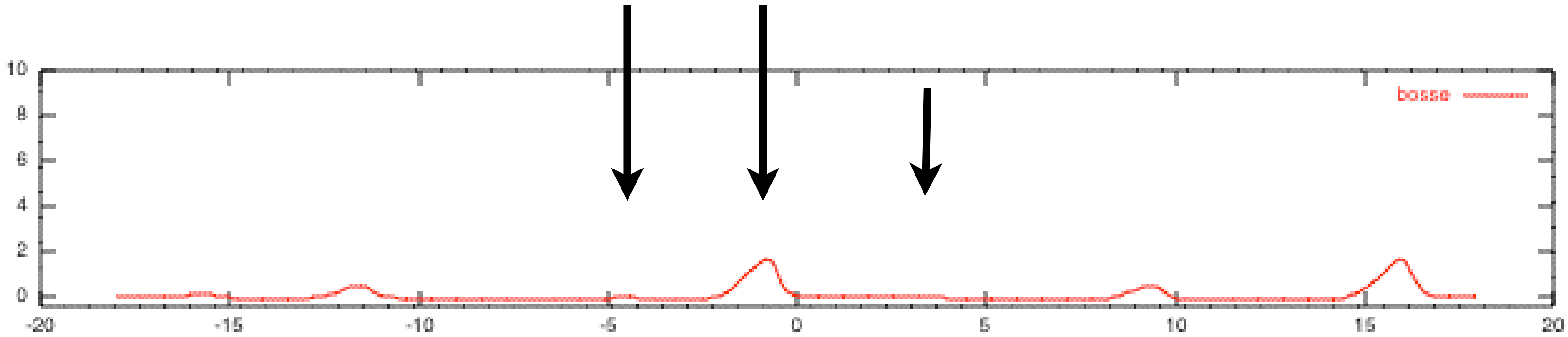
coarsening



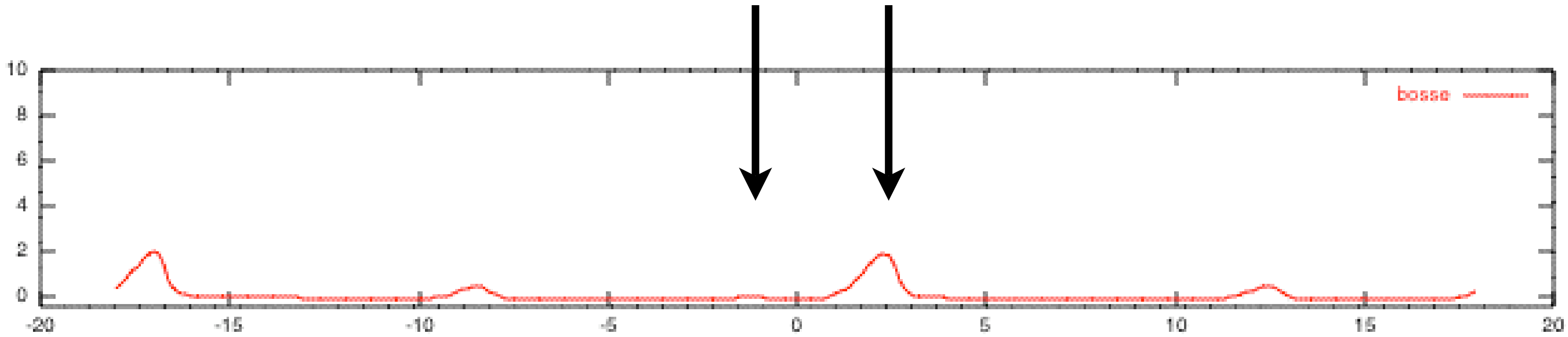
coarsening



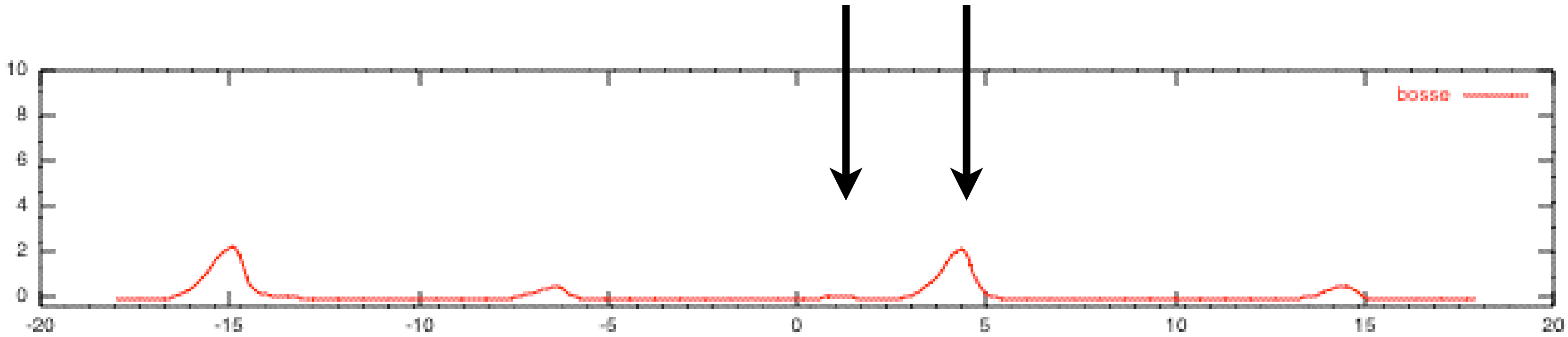
coarsening



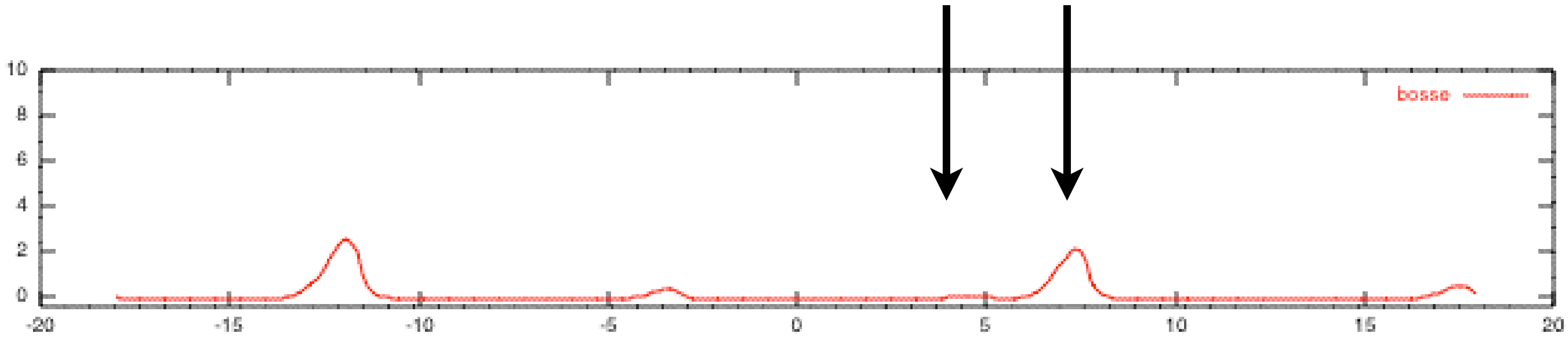
coarsening



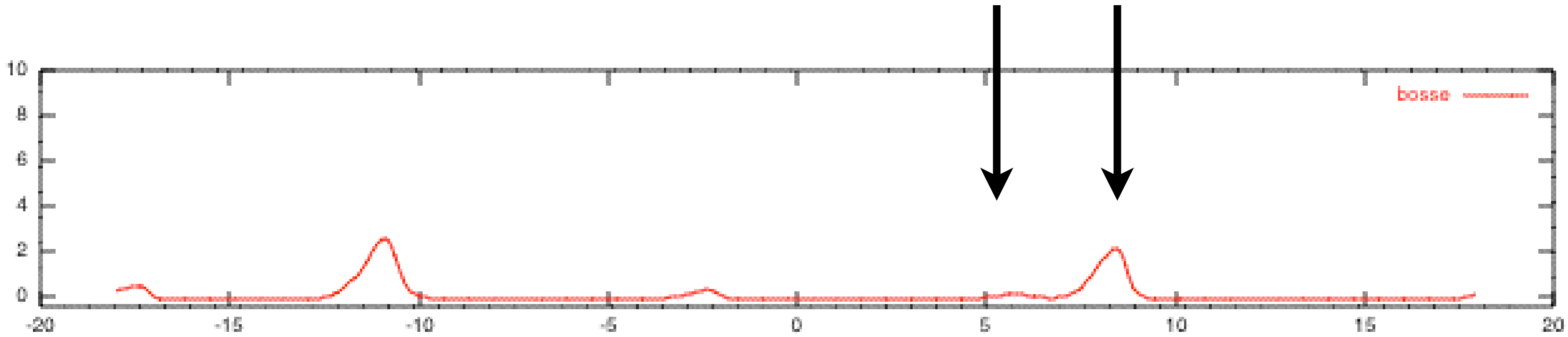
coarsening



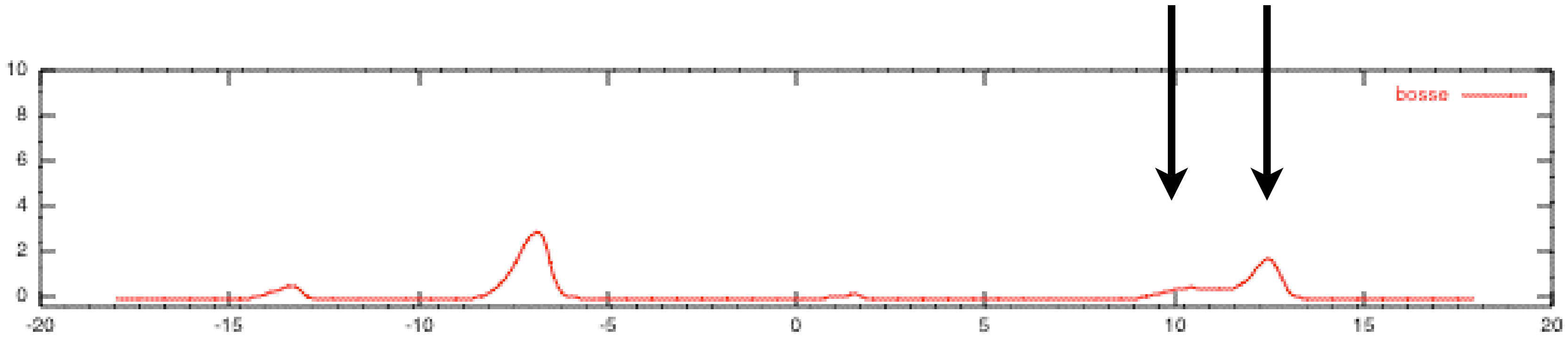
coarsening



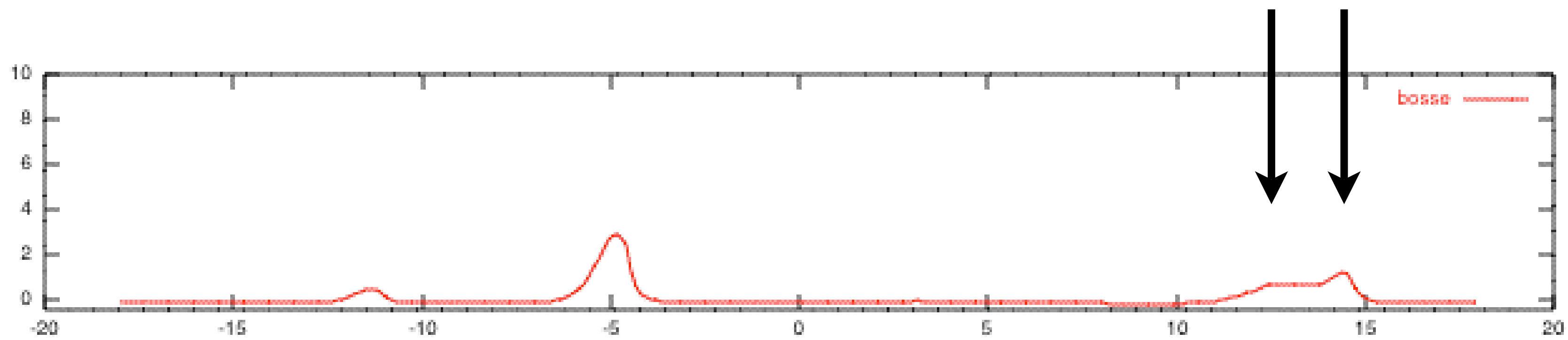
coarsening



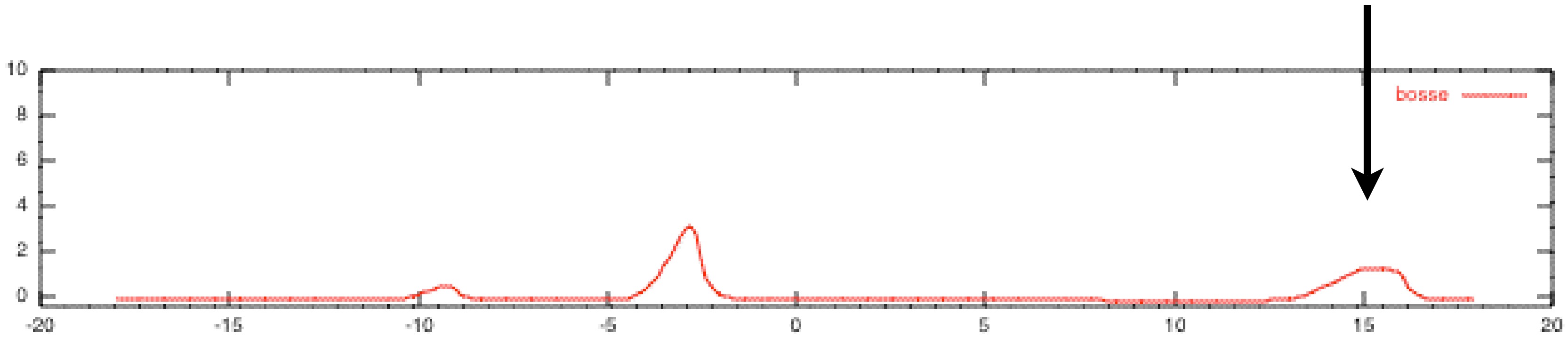
coarsening



coarsening

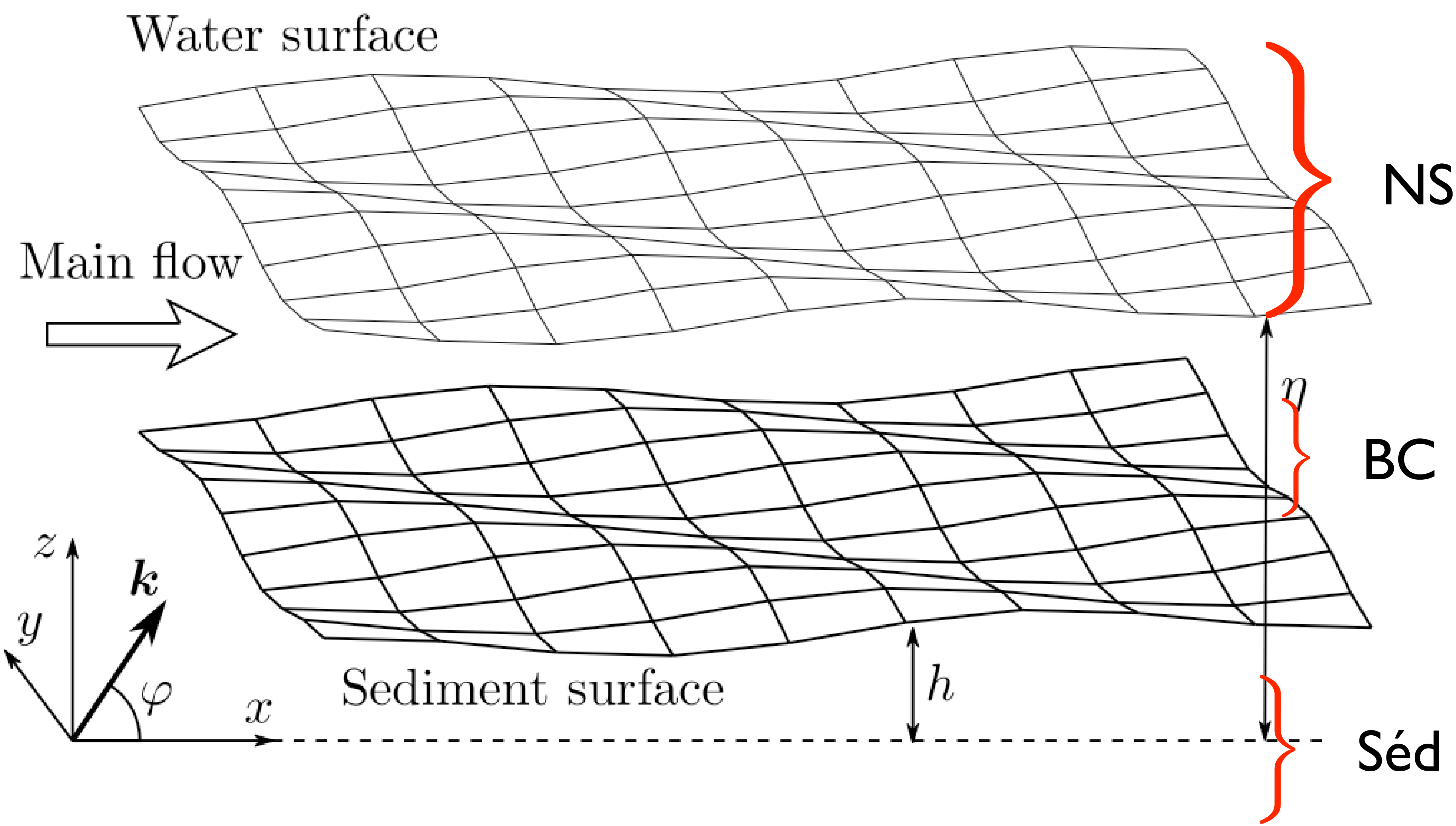


coarsening



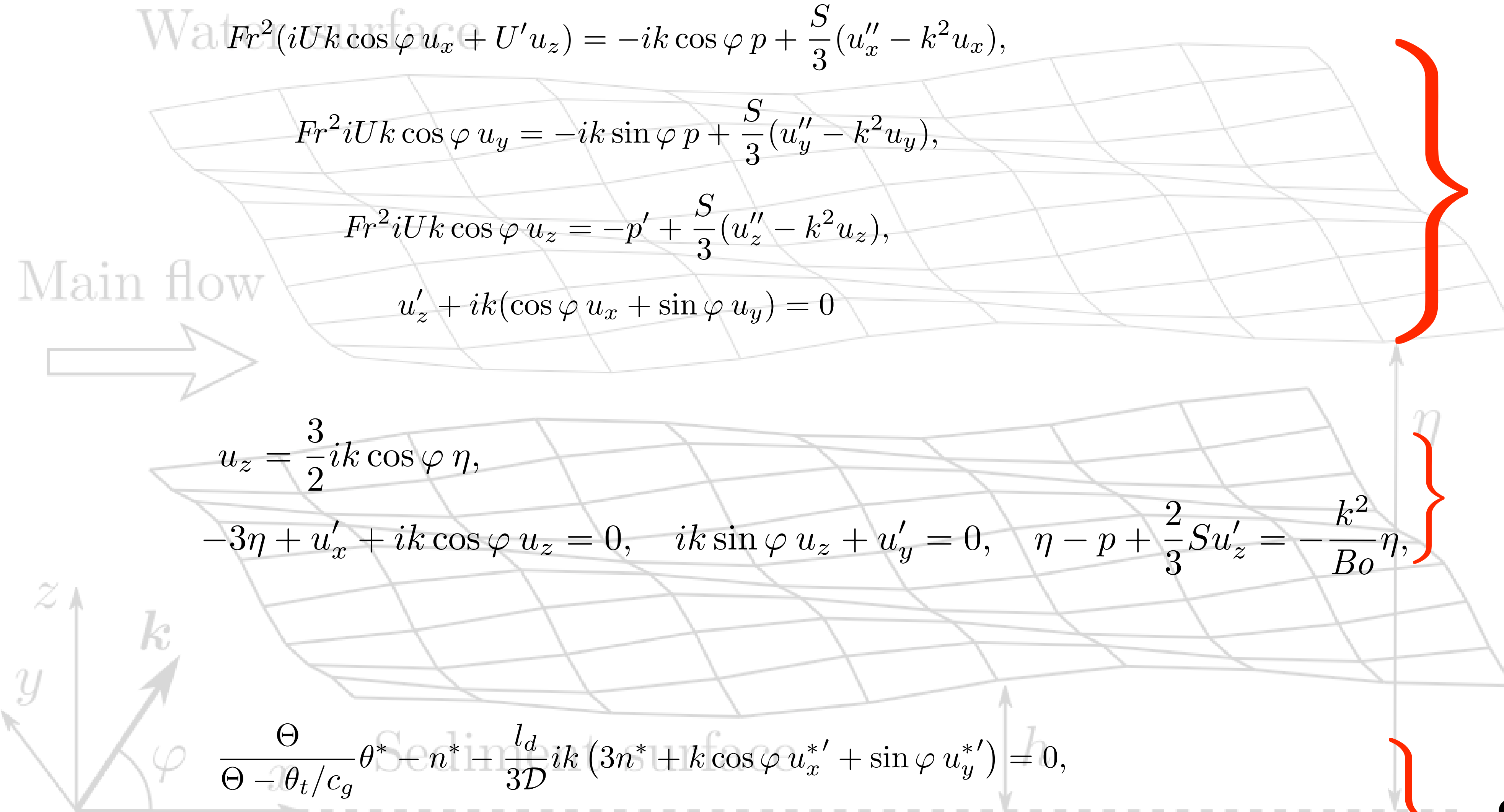
passage 3D

approche stabilité linéaire 3D complète



passage 3D

approche stabilité linéaire 3D complète



NS

$$Fr^2(iUk\cos\varphi\, u_x + U' u_z) = -ik\cos\varphi\, p + \frac{S}{3}(u_x'' - k^2 u_x),$$

$$Fr^2 iUk\cos\varphi\, u_y = -ik\sin\varphi\, p + \frac{S}{3}(u_y'' - k^2 u_y),$$

$$Fr^2 iUk\cos\varphi\, u_z = -p' + \frac{S}{3}(u_z'' - k^2 u_z),$$

$$u_z' + ik(\cos\varphi\, u_x + \sin\varphi\, u_y) = 0$$

BC

$$u_z = \frac{3}{2} ik\cos\varphi\, \eta,$$

$$-3\eta + u_x' + ik\cos\varphi\, u_z = 0, \quad ik\sin\varphi\, u_z + u_y' = 0, \quad \eta - p + \frac{2}{3}Su_z' = -\frac{k^2}{Bo}\eta,$$

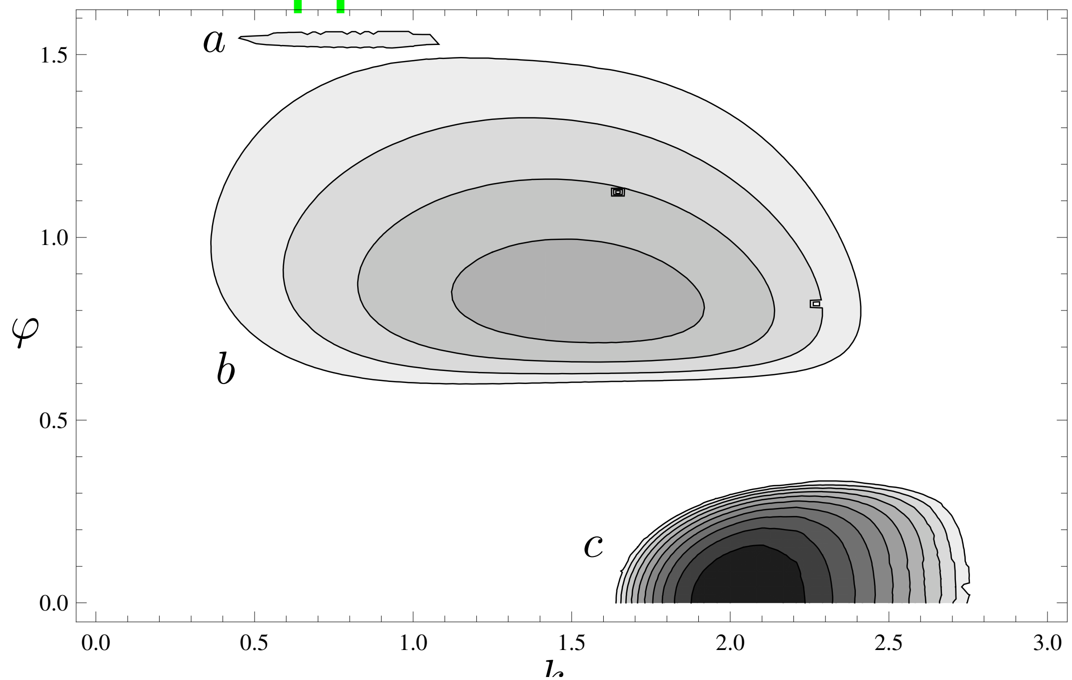
Séd

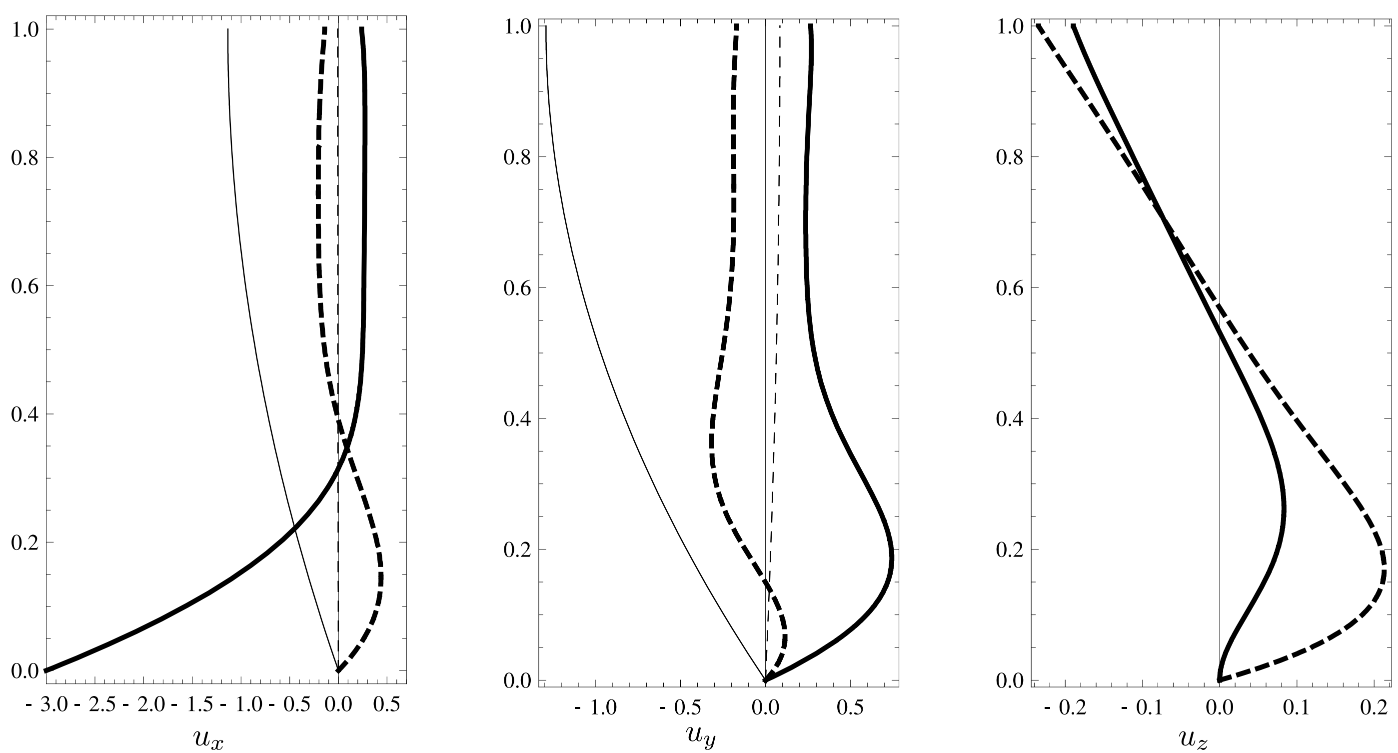
$$\frac{\Theta}{\Theta - \theta_t/c_g}\theta^* - n^* - \frac{l_d}{3\mathcal{D}} ik\left(3n^* + k\cos\varphi\, u_x^{*\prime} + \sin\varphi\, u_y^{*\prime}\right) = 0,$$

$$\theta^* = \frac{1}{3}\Big(2\tilde{\theta}^2\left(u_z^{*\prime} - 3ih^*k\cos\varphi\right) - 3ih^*k\cos\varphi\,\left(1 + S^2\right) + \frac{Sh}{c_g}\left(u_x^{*\prime} + 2Su_z^{*\prime} - 3h^*(1 + 3ik\cos\varphi\, S)\right)\Big).$$

passage 3D

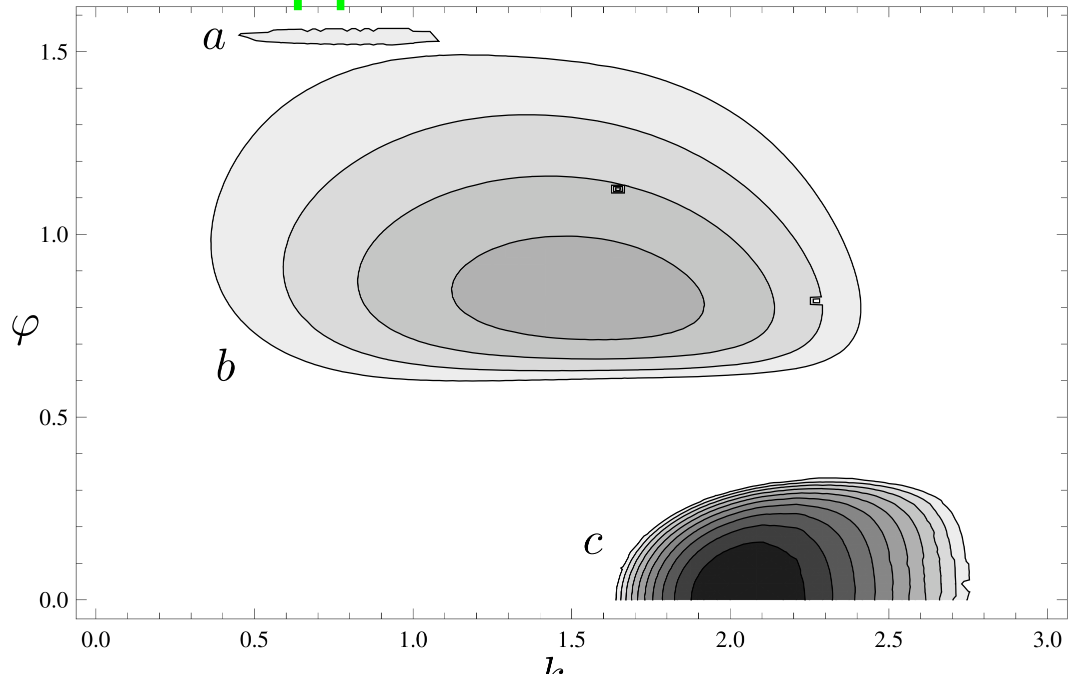
approche stabilité linéaire 3D complète



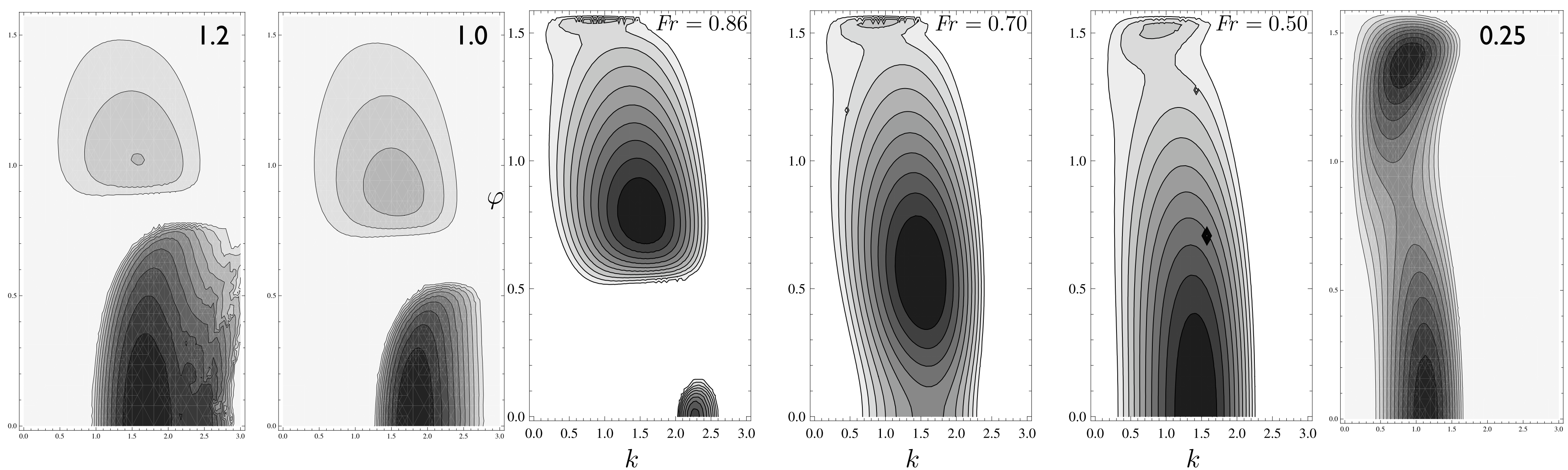


passage 3D

approche stabilité linéaire 3D complète

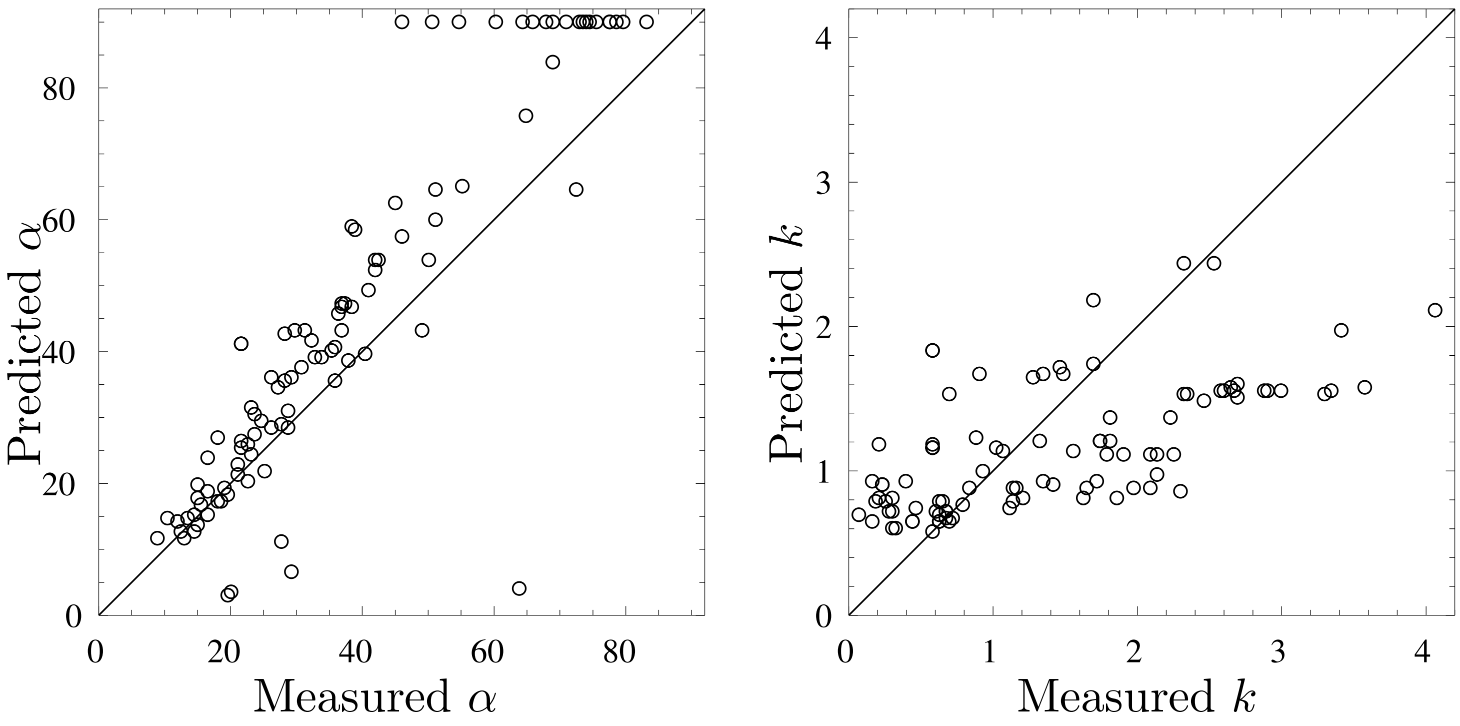


$\bar{Fr} = 0.90,\ S = 0.015,\ Bo = 2.77$ and $\bar{Sh} = 0.448$



passage 3D

approche stabilité linéaire 3D complète



conclusion

modèle complet de l'écoulement
- lois avec longueur de saturation
- barres: Saint Venant
- rides: perturbation d'un écoulement cisaillé
- barres+rides+stries: OSS

À FAIRE

- Théorie non linéaire: chevrons
- quelques raccords asymptotiques
- autres écoulements

Publications

O. Devauchelle, C. Josserand, P.-Y. Lagrée and S. Zaleski (2008):
"Mobile Bank Conditions for Laminar Micro-Rivers"
C. R. Geoscience (2008), doi:10.1016/j.crte.2008.07.010

F. Bouchut, E.D. Fernández-Nieto, A. Mangeney & P.-Y. Lagrée (2008):
"Erosion in avalanches".
Acta Mecanica, 10.1007/s00707-007-0534-9

O. Devauchelle, C. Josserand, P.-Y. Lagrée, and S. Zaleski (2007):
"Morphodynamic modeling of erodible laminar channels"
Phys. Rev. E 76, 056318

P.-Y. Lagrée (2007):
"Interactive Boundary Layer in a Hele Shaw cell".
Z. Angew. Math. Mech. 87, No. 7, pp. 486-498

P.-Y. Lagrée & D. Lhuillier (2006):
"On steady avalanches of dense granular media" Uzbek J. of Phys. Vol 8, N 4-5, pp. 201-207

P.-Y. Lagrée & D. Lhuillier (2006):
"The Couette flow of dense and fluid-saturated granular media"
European Journal of Mechanics B/ Fluids. 25 pp. 960-970

K.K.J. Kouakou & P.-Y. Lagrée (2006):
"Evolution of a model dune in a shear flow".
European Journal of Mechanics B/ Fluids Vol 25 (2006) pp 348-359.

C. Josserand, P.-Y. Lagrée, D. Lhuillier (2006):
" Granular pressure and the thickness of a layer jamming on a rough incline"
Europhys. Lett., 73 (3), pp. 363–369 (2006)

K.K.J. Kouakou & P.-Y. Lagrée (2005):
"Stability of an erodible bed in various shear flow".
European Physical Journal B - Condensed Matter, Volume 47, Issue 1, Sep 2005, Pages 115 - 125

C. Josserand, P.-Y. Lagrée, D. Lhuillier (2004):
"Stationary shear flows of dense granular materials : a tentative continuum modelling",
Eur. Phys. J. E. vol 14, pp. 127-135.

P.-Y. Lagrée, K.K.J. Kouakou & E. Danho (2003):
"Effet dispersif de la loi d'Exner menant à l'équation de Benjamin-Ono: formation de rides sur un sol meuble",
C. R. Acad. Sci. Paris, vol 331/3 pp 231 - 235

P.-Y. Lagrée (2003):
"A Triple Deck model of ripple formation and evolution",
Physics of Fluids, Vol 15 n 8, pp. 2355-2368.

Lagrée P.-Y. (2000):
" Erosion and sedimentation of a bump in fluvial flow",
C. R. Acad. Sci. Paris, t328, Série II b, p869-874, 2000

-O. Devauchelle, L. Malverti, É. La Jeunesse, C. Josserand, P.-Y. Lagrée, & F. Métivier
"Rhomboid Beach Pattern: a Benchmark for Shallow water Geomorphology"
Subm

- P.-Y. Lagrée & D. Lhuillier:
"Viscous sediment transport".
Subm

- O. Devauchelle, L. Malverti, É. La Jeunesse, P.-Y. Lagrée, C. Josserand & K.-D. Nguyen Thu-Lam
Stability of bedforms in laminar flows with free-surface: from bars to ripples
Subm