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Comment on 'Elliptic integral solutions of spatial elastica of a thin straight rod bent under concentrated terminal forces'

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In a recent paper published in Meccanica [1] it is claimed that solutions of the equilibrium equations for the spatial elastica are given in explicit form. I would like to point out that such solutions are not new and already published elsewhere.

In the title, in the abstract and in the conclusion of [1] it is claimed that, analytical, closed-form solutions of the nonlinear equations for the elastica in space are given for the first time. In this sense equations (6), (9), and (14) are the main result of the paper.

I would like to point out that closed-form solutions of the spatial elastica are known, and that equations (6), (9), and (14) are (a sub-case of) equations (21), (27), and (20) of a published paper [2]. It is rather confusing to notice that this paper [2] is cited in [1]!

More precisely, the scheme which is followed in [1] (i.e. passing to cylindrical coordinates, translation of the reference frame, etc.) is thoroughly explained in [2] (Sect. 2, pp. 10991–10993). In fact, the scheme was introduced (as far as I know) in the book by Landau and Lifshitz [3] (see Problem 5 of Sect. 19: *The equations of equilibrium of rods*).

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Moreover I would like to insist on the fact that [1] only deals with a *sub-case* of what is exposed in [2]. Paper [1] only considers *untwisted rods* (no torsional moment in the spatial elastica): when [2] considers $T = (C/A)(\omega - \omega_0)$ (see just after equation 10), paper [1] restricts to $T = 0$. Paper [1] further restricts to cases where the applied force F (at the end of the rod) is perpendicular to the tangent of the rod at $s = 0$ (equation $z'(0) = 0$ in [1]). Specifically the constant C in [1] (or a in [2], see equation (9)) is taken to be zero: $C = 0$.

In other words, in [1] the spatial elastica has to be anchored perpendicularly to the applied force, and there has to be no twisting moment. These are strong restrictions with regard to the general case treated in [2].

I chose [2] as the most illustrative example of my point but there are several other papers where closed-form solutions of the twisted spatial elastica are given, e.g. [4, 5].

References

1. Mingari Scarpello G, Ritelli D (2006) Elliptic integral solutions of spatial elastica of a thin straight rod bent under concentrated terminal forces. Meccanica 41:519–527
2. Tobias I, Coleman BD, Olson WK (1994) The dependence of dna tertiary structure on end conditions: Theory and implications for topological transitions. J Chem Phys 101(12):10990–10996

3. Landau LD, Lifshitz EM (1986) Theory of elasticity. Course of theoretical physics, vol 7, 3rd edn. Pergamon, Oxford
4. Nizette M, Goriely A (1999) Toward a classification of Euler–Kirchhoff filaments. J Math Phys 40:2830–2866
5. Coleman BD, Swigon D (2000) Theory of supercoiled elastic rings with self-contact and its application to DNA plasmids. J Elast 60:173–221