Meccanica (2007) 42: 611–613
DOI 10.1007/s11012-007-9084-4

Reply of the Authors to Prof. Neukirch comments

Giovanni Mingari Scarpello · Daniele Ritelli

Received: 30 April 2007 / Published online: 11 September 2007

The side comments of Professor Neukirch come after several (unfruitful) personal communications: now he keeps out from his own papers, focusing his claims on paper [3]. About our paper [2] he claims that "such solutions are not new and already published elsewhere", so that our treatment would be a "sub-case" of the aforementioned [3]: we will see what this could mean.

First of all a remarkable detail. Professor Neukirch makes understand we omitted to declare the cylindrical coordinates to elastica to go back to Landau and Lifchitz: but he could have avoided such a flaw simply reading our bibliographic entry [1] (Landau-Lifchitz) explicitly recalled at pages 521–522 [2], where we declare:

> Starting from the Wantzel model (see [34,35]), we develop a new solution for 3D elastica of a given loaded rod, in a clear, purely mechanical and self consisting way.

Now let us pass to the "sub-case". A result is really a sub-case of a more general one, whenever, specializing some of the parameters inside the last, one will obtain as a particular case *all* the things of the former. E.g. the motion equations of a charge under electrostatic field of restricted Relativity: putting there $v \ll c$, they will collapse into the classic ones which everybody could have obtained without Relativity.

Would a sub-case be a crime? could one bring a charge against Cauchy or De Saint Venant for being their elasticity a sub-case of the relevant work of Euler? or Lagrange? Certainly not: nevertheless we are going to prove that such a case doesn't apply, namely: [2] is not a sub-case of [3].

For the reader's better intelligence, the sequence we use to follow in elastica modeling problems will be recalled:

- define carefully all the input data of the unstressed rod, and grasp the nature of constraints (clamping, simple support, hinge, and so on) based on the movements they allow or prevent;
- for each of the rod terminal ends translate what above in suitable boundary conditions of displacement or rotation;
- after integration has been carried out, evaluate accordingly both integration constants trying to define their possible ranges on the basis of the first statement;
- discuss the axial curve equation, specially its roots and their *reality*, its shape, symmetry, periodicity and the overall behavior, showing how much the detected features comply with the first statement;

G. Mingari Scarpello (✉)
Università Milano Bicocca, via Negroli, 6, 20136 Milan, Italy
e-mail: giovannimingari@yahoo.it

D. Ritelli
Dipartimento di Matematica per le Scienze Economiche e Sociali, viale Filopanti, 5, 40126 Bologna, Italy
e-mail: daniele.ritelli@unibo.it

- provide some example of computation and/or plot of it.

We will restrict to the first paragraphs (I and II) of [3] where this procedure is *not* completely applied, for highlighting that:

No calculated equilibrium configuration is shown

As a matter of fact the provided figures concern the DNA nicked segments, out of our discussion.

The constraints definition is missing

The constraints are not defined at all on the body filament which is assumed to undergo elastic flow under forces passing only through the terminal points.

Reality of the second and third roots of the cubic equation is not ensured

In order to perform integration, a third degree equation is considered [3], equations (13) and (14), p. 10992:

$$\begin{aligned} P_3(u) := -\Big[& u^3 + (\lambda^2 - 2a)u^2 \\ & + (a^2 - 2a\lambda^2 + T\lambda - 1)u \\ & - \left(\frac{1}{2}T - a\lambda\right)^2\Big] \\ = {} & (u-u_1)(u-u_2)(u_3-u) = 0. \end{aligned}$$

The boundary conditions being undefined, as a consequence λ and a integration constants have been not evaluated. Consequently no test can be done on the discriminant of the $P_3(u) = 0$. Depending such a discriminant on a, λ, T in not easy way, a long analysis should be performed in order to define the ranges to the above quantities so that formulæ (14), (15), (16) can be deemed valid: in fact they all require that each of the roots u_1, u_2, u_3 is real. This point should have been more carefully considered by the authors and/or their referees. For instance, if one takes $T = 20$, $\lambda = 0.1$ and $a = -0.1$, $P_3(u)$ has only one real negative root $u_1 = -4.64201$ and two complex conjugate roots $u_{2,3} = 2.21601 \pm 4.08348i$. Of course there are infinite triples (T, λ, a) of this kind and this omission, fully ignored in [3] and by Professor Neukirch, invalidates the later argument!

Final formulæ for ρ and z through the arclength s should be better analyzed

Comparing minutely the formulæ for ρ and z, namely (9), (14) of [2] and (27), (20) of [3], we see quite different expressions for $z(s)$ and it seems difficult to succeed in drawing a "sub-case" with elliptic integrals of III and I kind from a "main case" like (20) holding that of II kind. The same could be seen on $\rho(s)$ and the Jacobi elliptic functions. In any case this cannot be checked thoroughly, until someone else (we will not) decides the filament constraints nature, sets the relevant boundary conditions, computes the integration constants λ and a, checks reality and positiveness of $P_3(u) = 0$ roots, and finally operates for inquiring what above and other similar things.

Torque

The stress/strain analysis of 1-D loaded elastic bodies makes primary reference to the effects of bending: second order effects, like shear or torsion are usually ignored. The latter is meaningful in sizing, or checking critical frequencies of, rotating shafts of alternative engines or steam turbines, as any elasticist knows. Taking into account the exact curvature of a simple cantilever, we met in a next issue paper Abelian integrals also for simple continuous purely bending loads: then no sense would be including torsion in our treatment: to overcharge a model of details which nobody can solve is … illusive. Conversely we didn't know how much strain of DNA filaments is really ruled by a torque.

$C = 0$, $C \neq 0$

It is true that our spatial elastica has to be anchored perpendicularly to the applied force as a consequence of the particular boundary condition $z'(0) = 0$. A charge has been brought by Professor Neukirch against our paper for this. Such a choice has been done for a better decoupling among equations (4). Of course it would be possible to release such a restriction: then the elliptic differential (7) would be changed in another one, without appreciable improvement in the model, but of a more long discussion.

Conclusions

Being the model vaguely explained and formulated, there are scarce elements for calculating through [3] any filament strained configuration in space with such cryptic boundary conditions. Nothing can then be said about the effective meaning of formulæ in lack of any previous analysis warranting reality to the second and third roots of the cubic equation. Which is also necessary for ensuring reality and boundedness to the modulus m of the relevant elliptic integrals.

The reader should be aware that our final formulæ have been—as usually—validated successfully on sample cases otherwise computed through high precision Runge-Kutta methods. This provides a posteriori confirmation not only of their exactness (always beyond any doubt), but also of not being they a sub-case of [3], whose troubles have been expounded.

The idea that [2] could be a sub-case of [3] has then no grounds, because [3] describes very little the elastic equilibrium which [2] treats effectively.

References

1. Landau LD, Lifchitz EM (1967) Théorie de l'élasticité. MIR, Moscow
2. Mingari Scarpello G, Ritelli D (2006) Elliptic integral solutions of spatial elastica of a thin straight rod bent under concentrated terminal forces. Meccanica 41(5):519–527
3. Tobias I, Coleman BD, Olson WK (1994) The dependence of DNA tertiary structure on end conditions: theory and implications for topological transitions. J Chem Phys 101(12):10990–10996