Twining Plants: How Thick Should their Supports Be?

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Abstract

When twining plants grow they revolve around a support on which they exert a force in order to achieve vertical growth. In 1865 Charles Darwin realized that twining plants cannot grow on supports that are too wide. Here, mechanical aspects of this problem are investigated by modeling the stem close to the apex as a growing planar elastic rod with intrinsic curvature. The shape and forces as a function of the pole size are obtained and a critical radius limiting growth is identified.

Introduction

One of the most fascinating aspects of growth in plants is found in the movements and habits of climbing plants. Climbing plants are not self-supporting and they use their surrounding to achieve vertical growth. They are usually divided into plants that either ascend trees or supports by making use of tendrils or hooks, or plants that spiral around their support (see illustration 1). For instance tendril-bearers use tendrils to pull themselves upward. The spiral contraction in tendrils leads to an interesting mechanical problem where filamentary structure with regions of opposite handedness is created, the so-called *perversion of tendrils* [1]. Charles Darwin described this beautiful phenomenon at length in 1865 in his essay on "The Movements and Habits of Climbing Plants" [2].

The purpose of this note is to address another remarkable mechanical puzzle offered by climbing plants and first discussed by Darwin in his essay. The stem of a twining plant has a naturally spiral shape [3]. During the growing (climbing) process, the part of the stem near the apex is spontaneously revolving, following a continuous bowing movement directed successively to all points of the compass. This favors the shoot finding a support. When the apex reaches the support the stem goes on moving, nearing the support and finally achieving close continuous contact. In his study of twiners, Darwin ponders, "Most twining plants are adapted to ascend supports of moderate though of different thicknesses. Our English twiners, as far as I have seen, never twine round trees...". Darwin realized

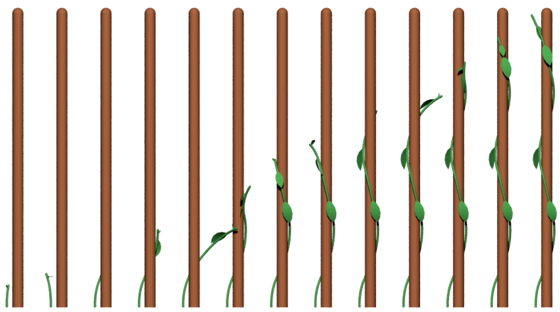

Illustration 1 : sketch of a twining plant obtained by computer graphics (from: Knut Arild Erstad www.ii.uib.no/~knute).

llustration 2 : a twiner may ascend a slim support and failed to wind around a thicker one.

that whereas a given twining plant can grow around certain pole sizes, it fails to climb on larger ones (see illustration 2). This naturally leads to the question: how thick a support can a given twining plant ascend?

To address this question the stem is idealized by an elastic growing rod on a rigid cylinder and the existence of configurations with realistic physical boundary conditions is established for varying pole sizes. The critical cylinder radius is found when such solutions cease to exist.

Material and methods

The goal here is to show that the critical climbing radius can be computed from basic mechanical principles and obtained as a function of the spiral size. Before lignification when the stem is still elastic [4], it is reasonable to model it as an inextensible unshearable elastic Kirchhoff rod with intrinsic curvature and torsion. A further simplification of the problem is obtained by assuming that the rod is confined in the plane. In this case the support is a rigid disc of radius R and the problem is to find equilibrium configurations of an elastic rod with intrinsic curvature $1/Rc$ in contact with the disc.

The Kirchhoff equations obtained as the static balance of linear and angular momenta in the plane read simply [5]

$$n' = 0,$$

$$m' + r' \times n = 0,$$

where n,m are, respectively, the force and moment applied to the rod centerline located at position $r(s)$ and are parameterized by the arc length s (the prime denotes the derivative with respect to s).

The filament is assumed to be strongly anchored at its base while its tip located at $s=L$ is simply supported. This implies that at $s=0$, $r(0)=(R,0)$, $r'(0)=(0,1)$ whereas at $s=L$, the rod lies on the disc and its curvature is equal to the intrinsic curvature, that is $\kappa=1/Rc$. The Kirchhoff equations together with the boundary conditions completely specify the problem for any given values of L,R,Rc and numerical continuation of equilibrium solutions of this boundary value problem can be performed.

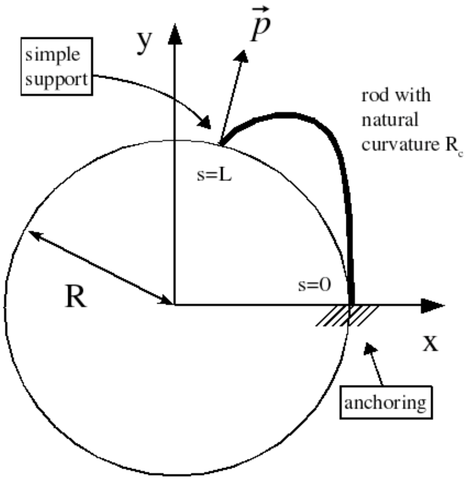

Illustration 3: model of an elastic filament around a rigid disc

Results and discussion

In order to gain insight on the possible configurations that satsify the boundary conditions, two series of equilibrium configurations for filaments with different intrinsic curvatures are shown in illustration 4 and 5. In both cases, the disc radius is kept constant while the length is increased. For the first series (illustration 4), a stable configuration develops and a continuous contact with the support is established. Conversely, in the second series (illustration 5), no continuous contact configurations exist, instead past a certain length the filament starts to wind outside the disc. Therefore, the limiting ratio *R/Rc* between climbing and non climbing states is found to be in the interval $3 < R/Rc < 3.5$. A more detailed bifurcation analysis reveals that the critical ratio is found to be $R/Rc \approx 3.31$.

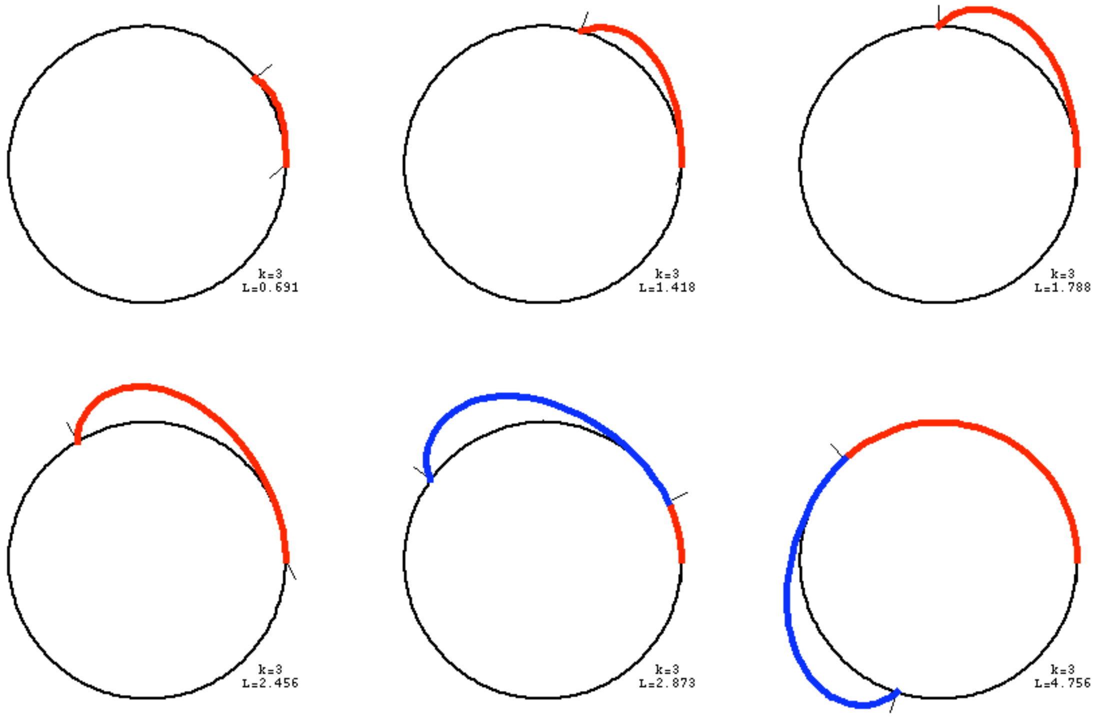

Illustration 5: Equilibrium configurations for a naturally curved filament around a disc with R/Rc=3. Configurations with continuous contact eventually develop.

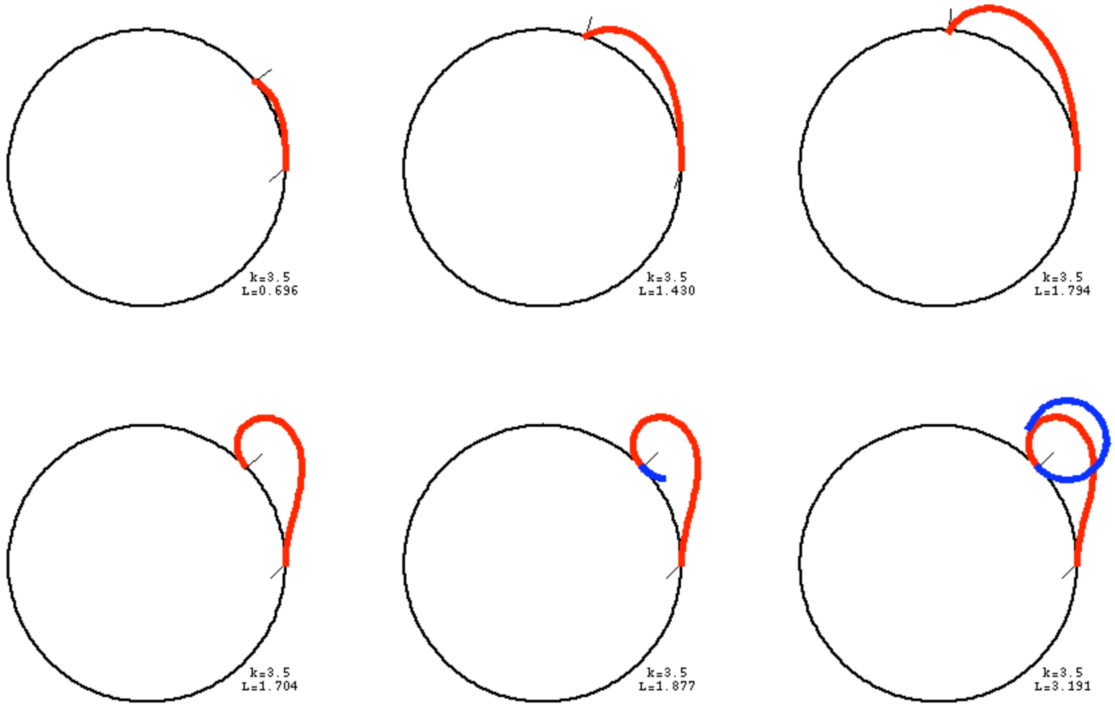

Illustration 4: Equilibrium configurations of a naturally curved filament with R/Rc=3.5. No configuration with continuous contact exist.

Conclusions

The mechanical action of twining was modelled as a growing elastic filament with an intrinsic curvature in contact with a rigid disc. The analysis of stable equilibrium configurations reveals that past a critical disc radius (about 3.3 times the plant intrinsic radius of curvature), the plant cannot successfully circle around the disc to continue its growth (see last figure in Illustration 5). Hence, an actual twiner with these mechanical properties could not grasp the pole and achieve vertical growth. The analysis presented here is only a starting point for realistic models of climbing plants. Nevertheless, it clearly underscores the interplay between growth, mechanics, and contact. Many other mechanical effects are in play in the growth of twiners and should be included in a realistic model. First growth is three-dimensional and the model can readily be generalized to a helix growing on a pole at the expense of tedious bifurcation analysis. Second, intrinsic curvature and torsion are in general neither constant nor uniform. A proper model of the evolution of such quantities with time requires a detailed analysis of growth and remodelling processes as a function of shapes, stresses and biochemical regulators. Finally, friction plays a subtle but important role in the overall ability of the plant to remain on the pole. Again, this effect can be included and will play a role on both the final shape and the critical radius.

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References

1. T. McMillen and A. Goriely, (2002): *Tendril perversion in intrinsically curved rods.* Journal of Nonlinear Science **12**, 241-281.
2. Ch. Darwin, (1865): *The Movements and Habits of Climbing Plants.* The Journal of the Linnean Society **9,** 1-118.
3. W. K. Silk and M. Hubbard, (1991): *Axial forces and normal distributed loads in twining stems of morning glory*. Journal of Biomechanics **24**, 599-606.
4. N. Rowe, S. Isnard and T. Speck, (2004): *Diversity of Mechanical Architectures in Climbing Plants: An Evolutionary Perspective*. Journal of Plant Growth Regulation **23**, 108-128.
5. A. E. H. Love, *A Treatise on the Mathematical Theory of Elasticity* (Cambridge University Press, Cambridge, 1892).