Vibrations planes de poutres: singularité de la limite inextensible

S. Neukirch, J. Frelat, C. Maurini

Institut d'Alembert , CNRS & UPMC Univ. Paris 6

A. Goriely

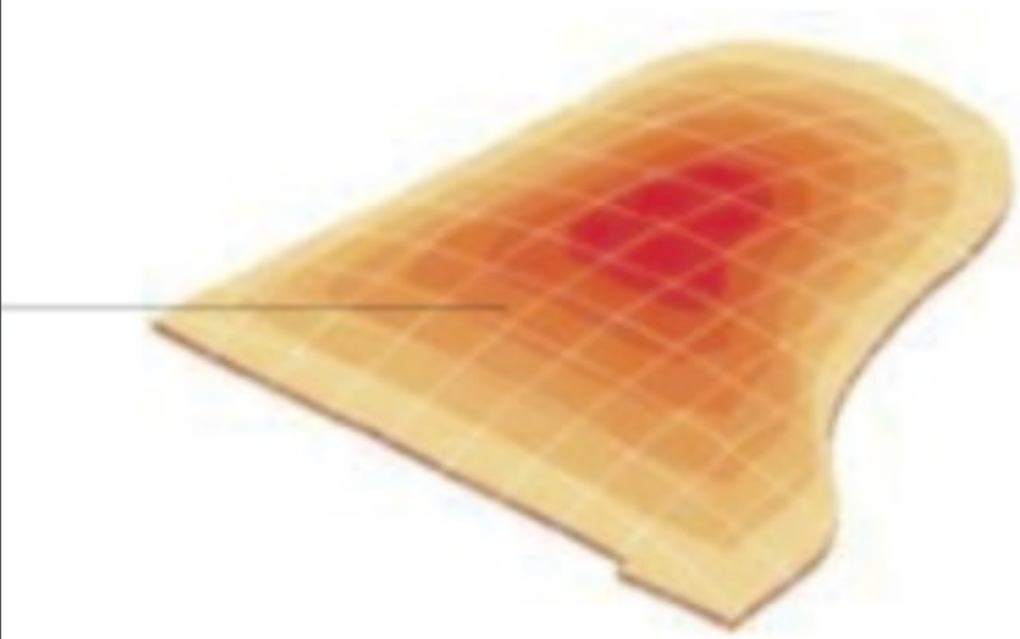
Centre for Applied Math. (OCCAM), Oxford, U.K.

Piano soundboard

http://www.steinway.com/

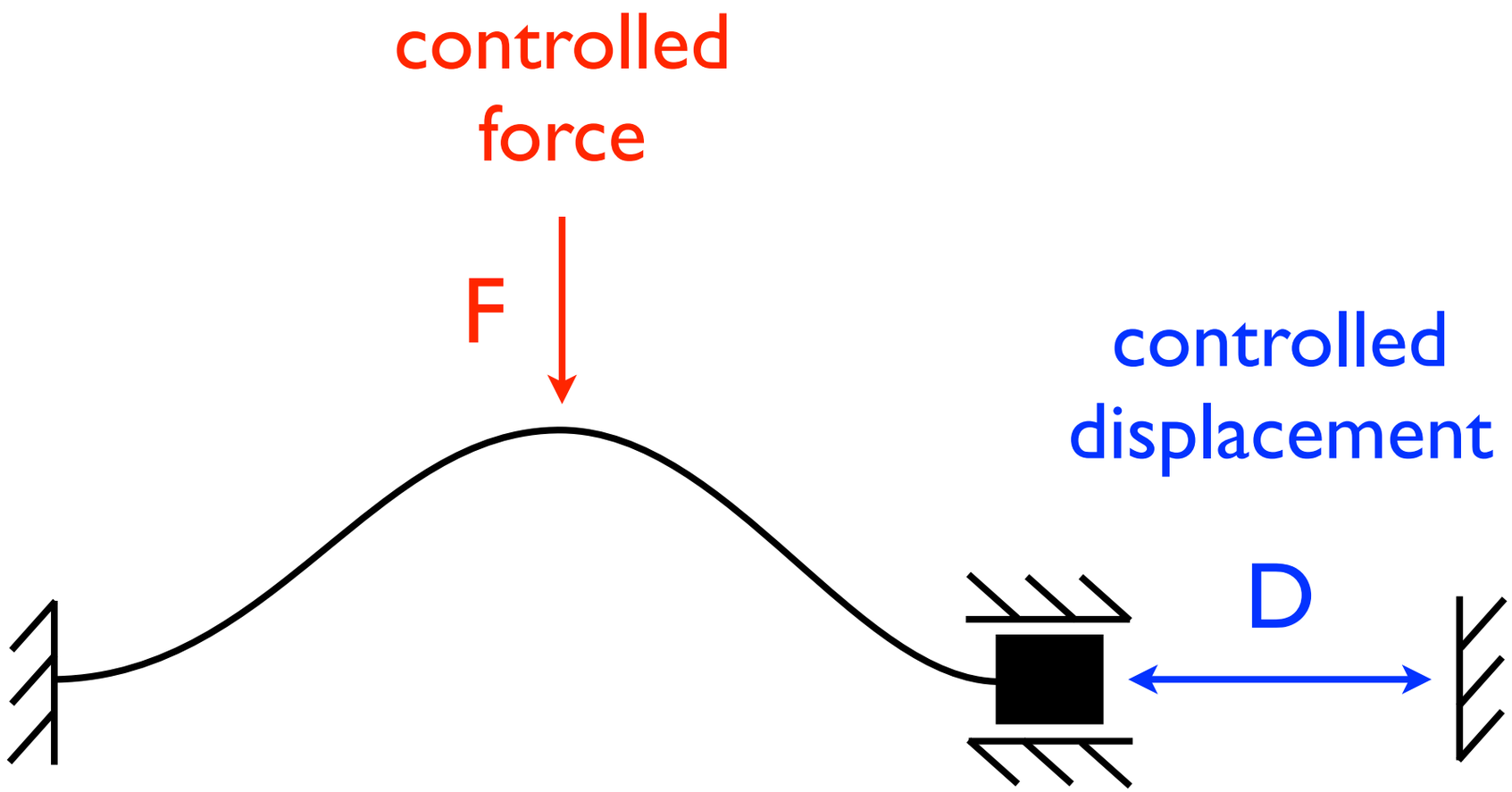
Piano soundboard

acoustic radiation from the soundboard (not the strings)



Model: pre-stressed beam

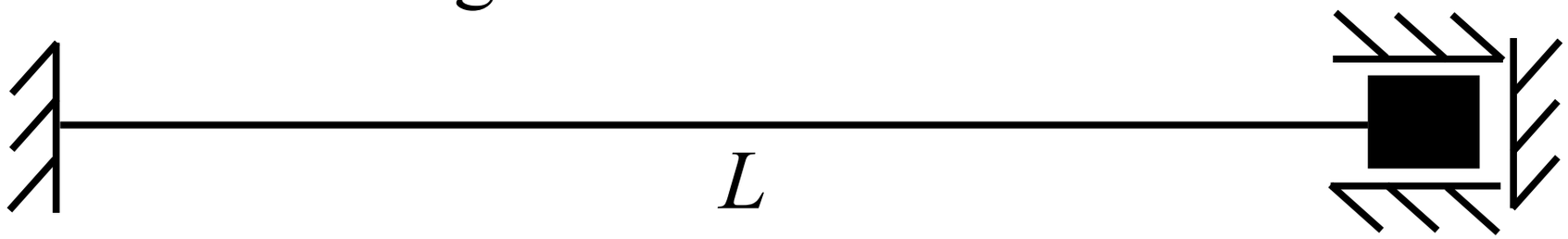
vibrations	slender elastic beam	in the plane

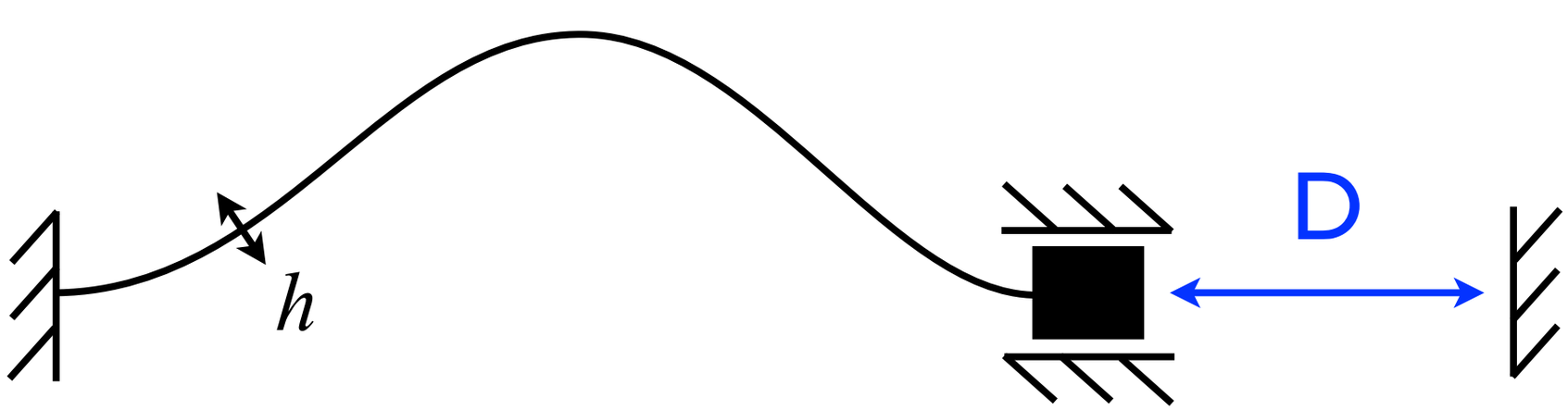


Influence of F, D
on the frequencies ?

Elastic beam in the plane

L : length in unstressed state





h : section thickness w : section width	$I = \dfrac{1}{12}\, h^3\, w$	$A = h\, w$

Model : do we need extensibility ?

$$E_{\text{strain}} = \underbrace{\frac{1}{2}\int_0^L EI\,\kappa^2(s)ds}_{\text{curvature}} + \underbrace{\frac{1}{2}\int_0^L EA\,e^2(s)ds}_{\text{extension}}$$

h : section thickness

w : section width

$h^3\,w$ $\qquad$ $h\,w$

$$\epsilon = \frac{I}{AL^2} = \frac{1}{12}\left(\frac{h}{L}\right)^2 \ll 1$$

$\epsilon = 0$ inextensible

$\epsilon > 0$ extension

Marigo Classification

charge croissante

1. Poutres flexibles Petits Déplacements
2. Poutres flexibles Grands Déplacements
3. Fils inextensibles
4. Fils extensibles

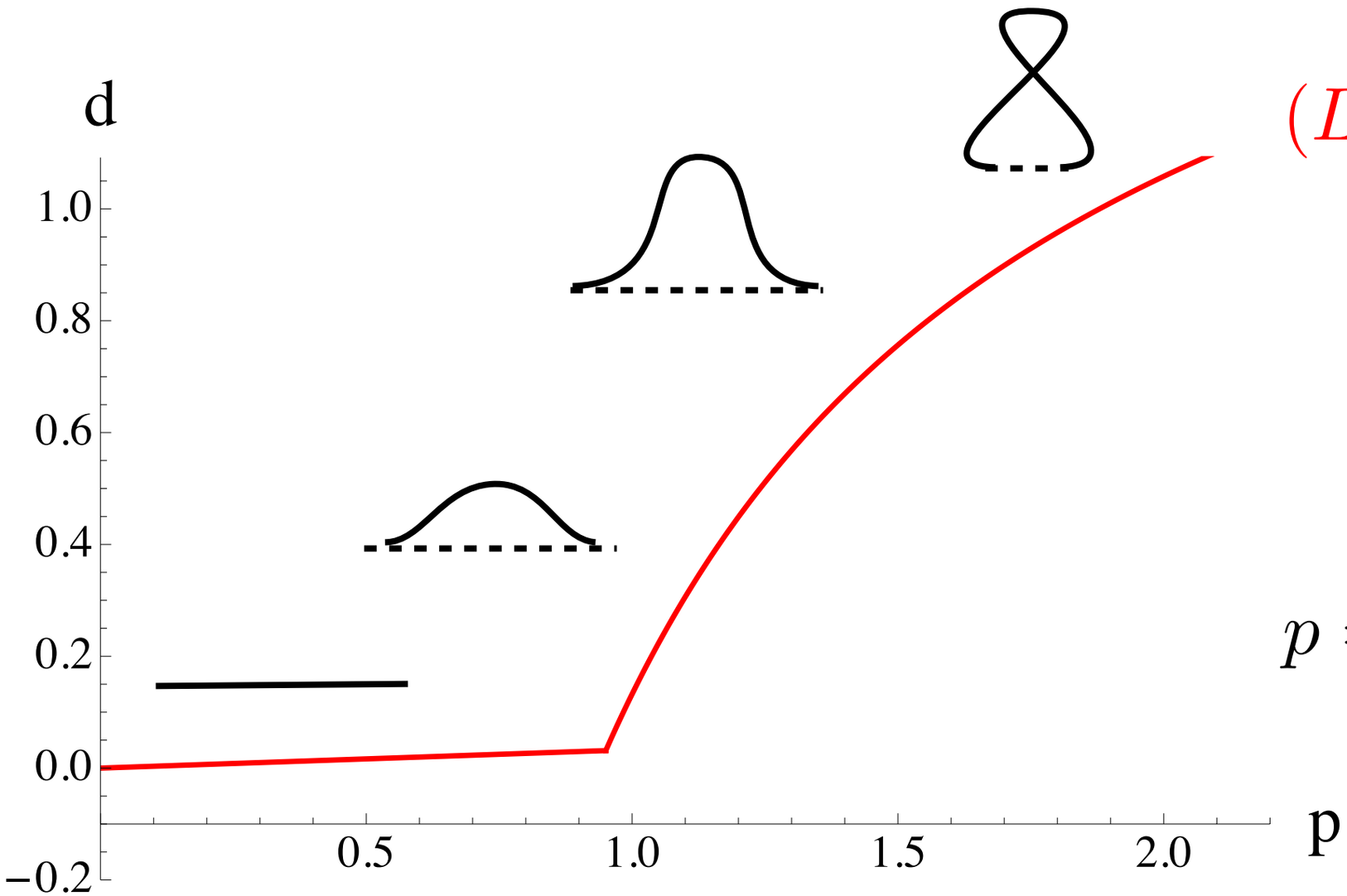
JJM & Ghidouche & Sedkaoui, CRAS (1998)
JJM & Madani, CRAS (1998)
JJM & Meunier, Journal of Elasticity (2006)
JJM & Madani, Journal of Elasticity (2004)

Equilibrium (numerical study)

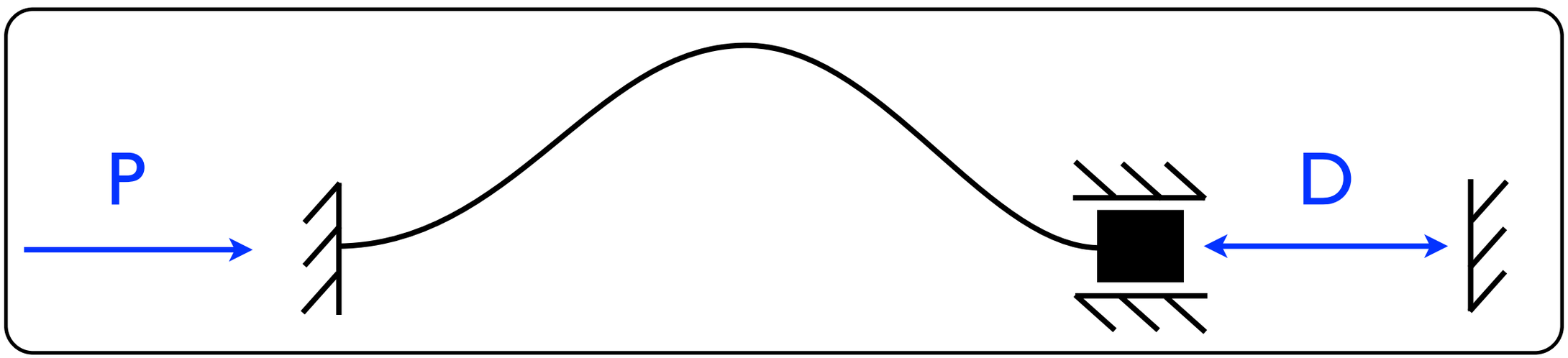
$$d = \frac{D}{L}$$

$$\epsilon = \frac{1}{1200}$$

$$(L = 10h)$$

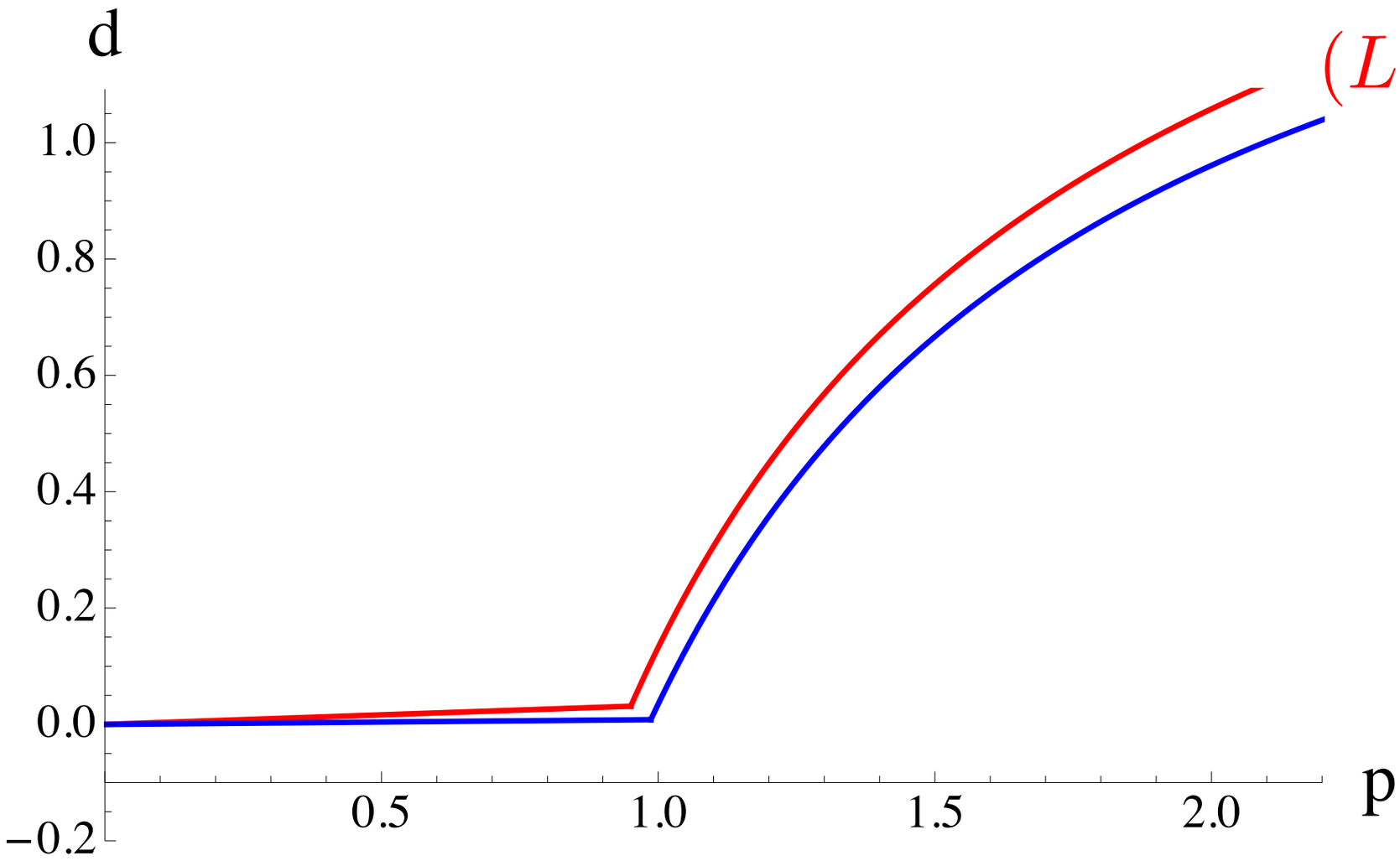


$$p = \frac{PL^2}{4\pi^2 EI}$$



Equilibrium (numerical study)

$$d = \frac{D}{L}$$

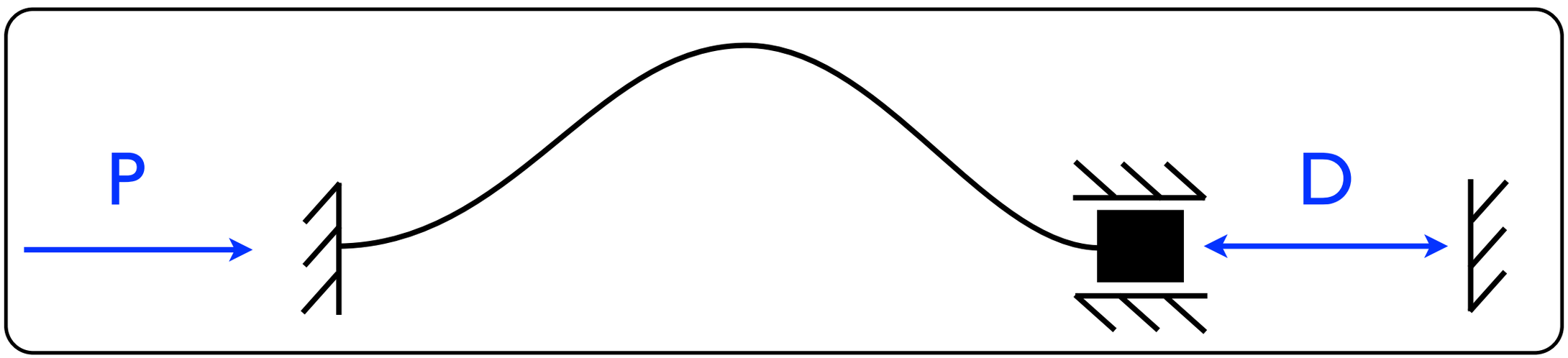


$$\epsilon = \frac{1}{1200}$$

$$(L = 10h)$$

$$\epsilon = \frac{1}{4800}$$

$$(L = 20h)$$



Equilibrium (numerical study)

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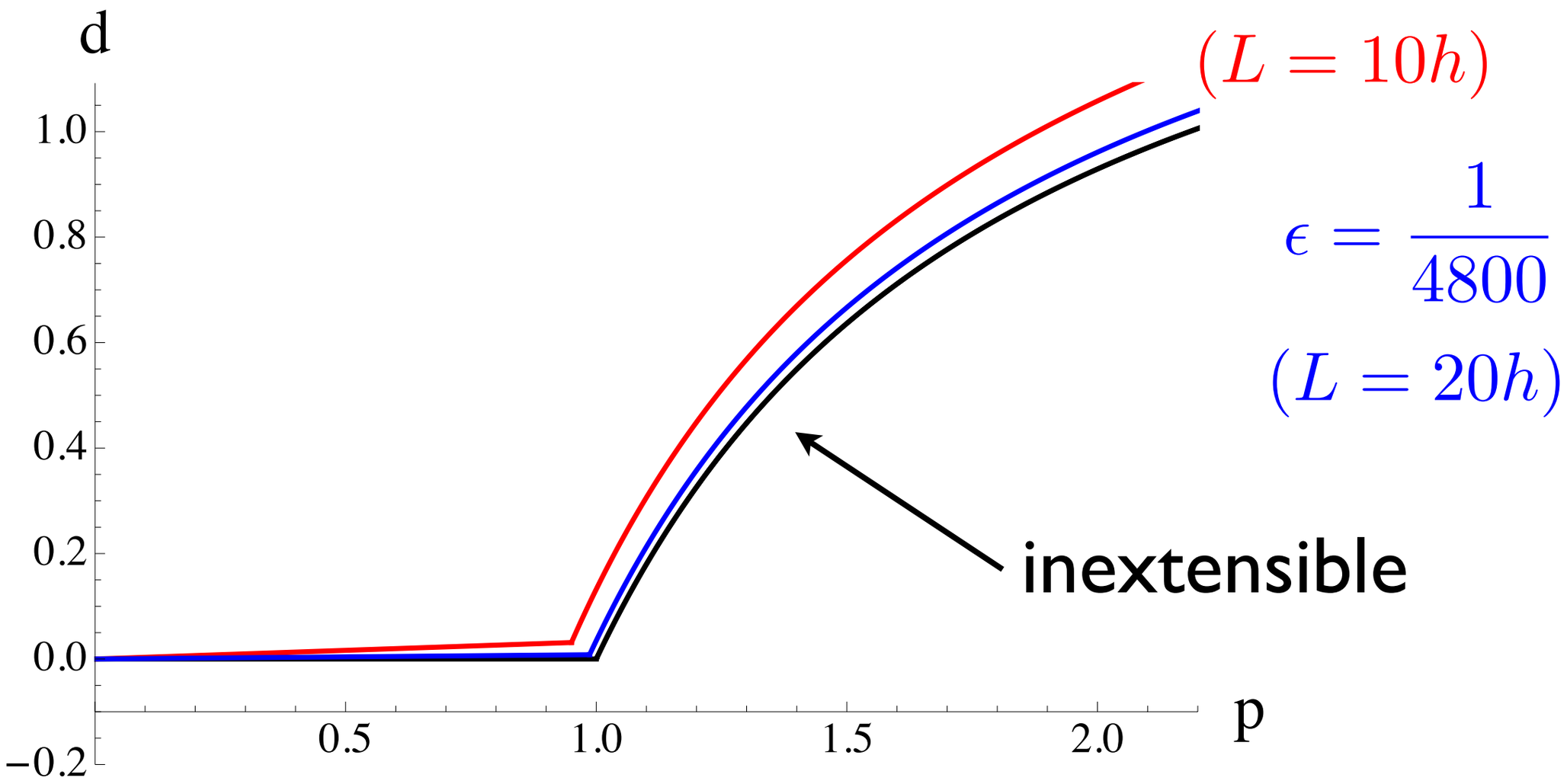
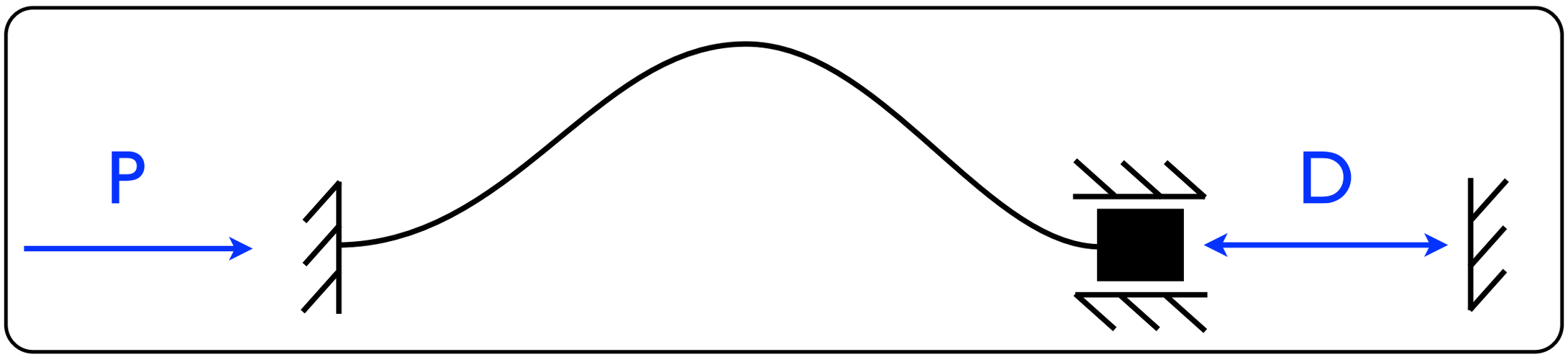
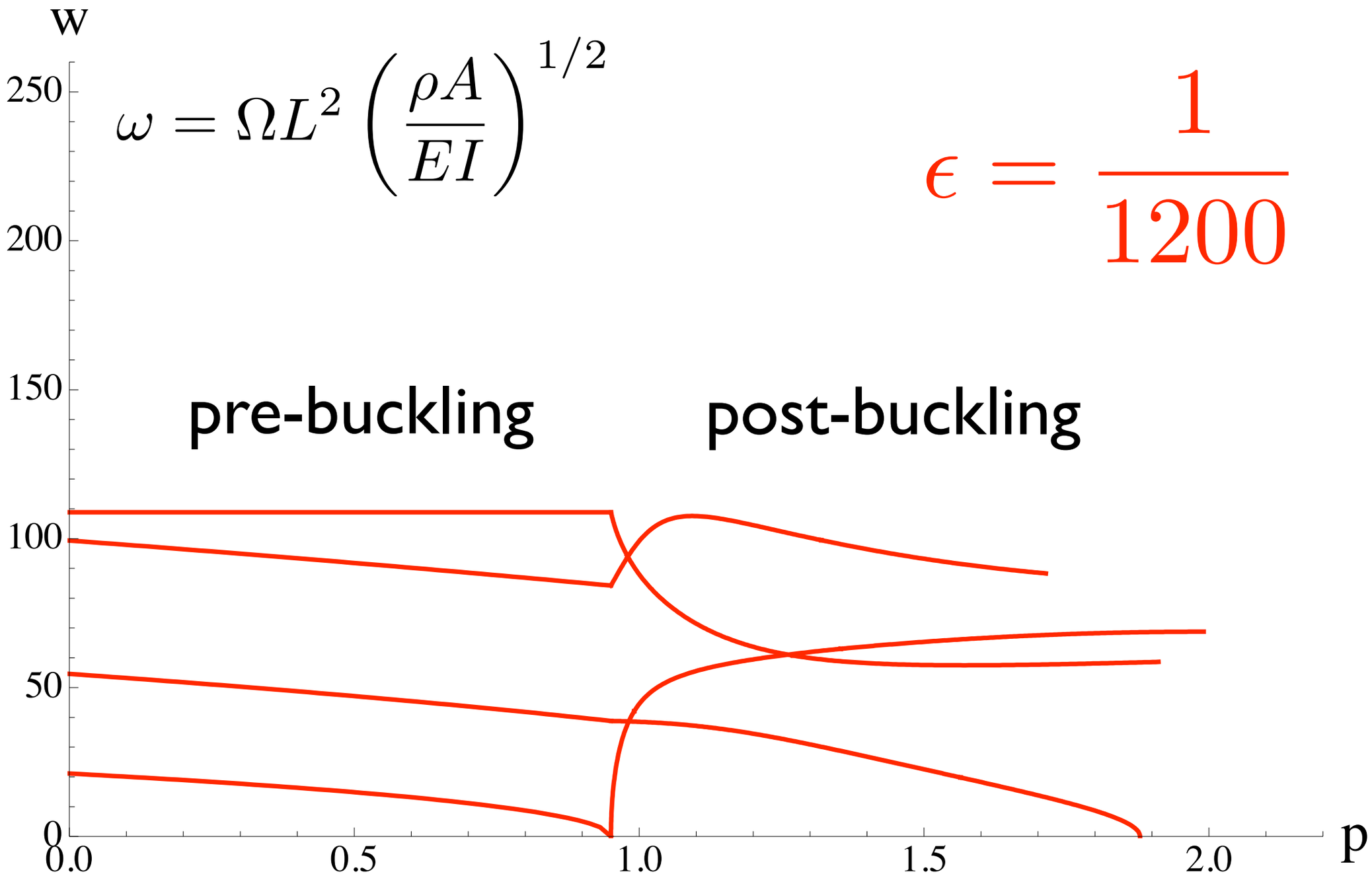




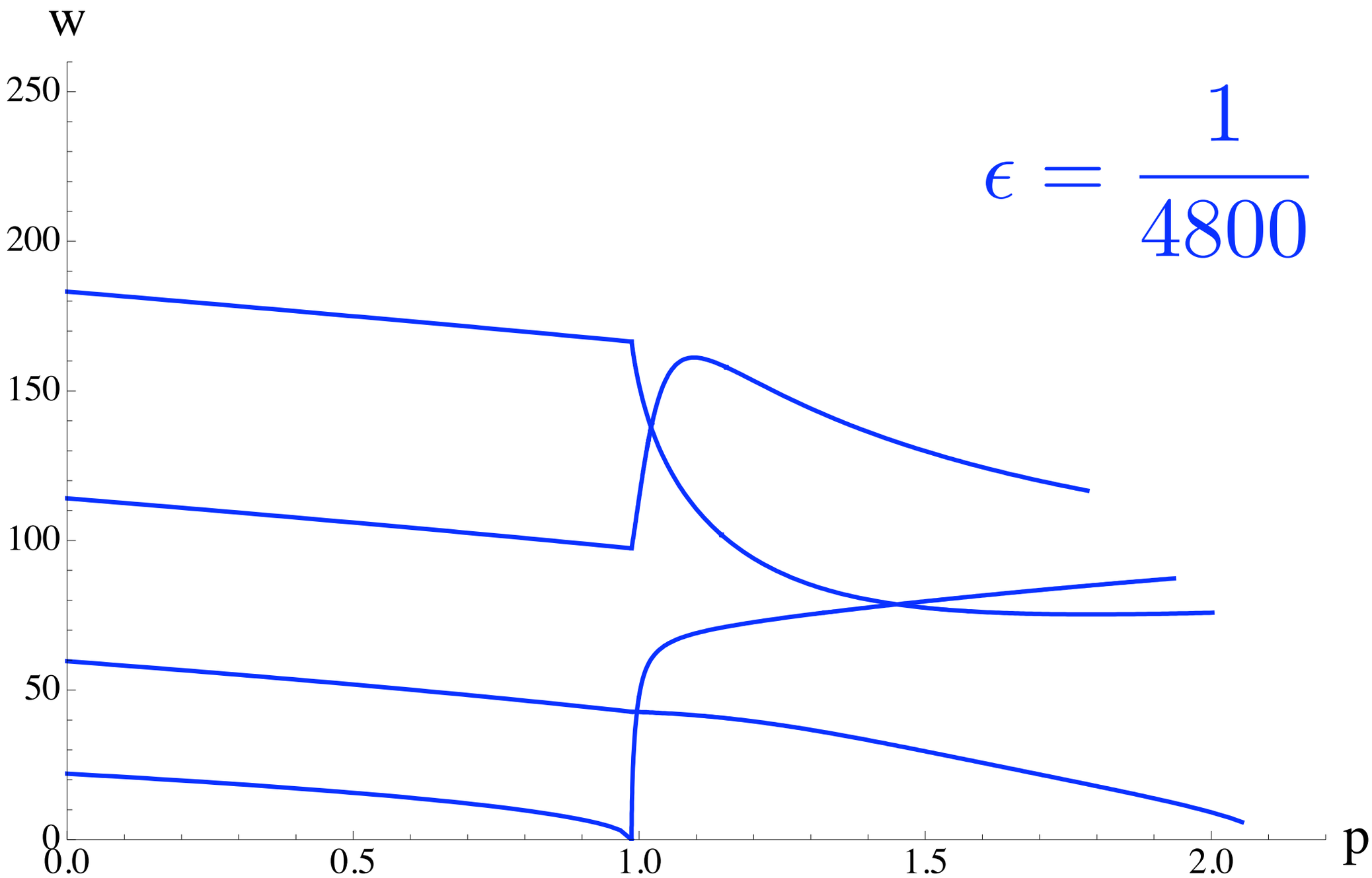
Vibrations (extensible case)

$$\omega = \Omega L^2 \left(\frac{\rho A}{EI}\right)^{1/2}$$

$$\epsilon = \frac{1}{1200}$$

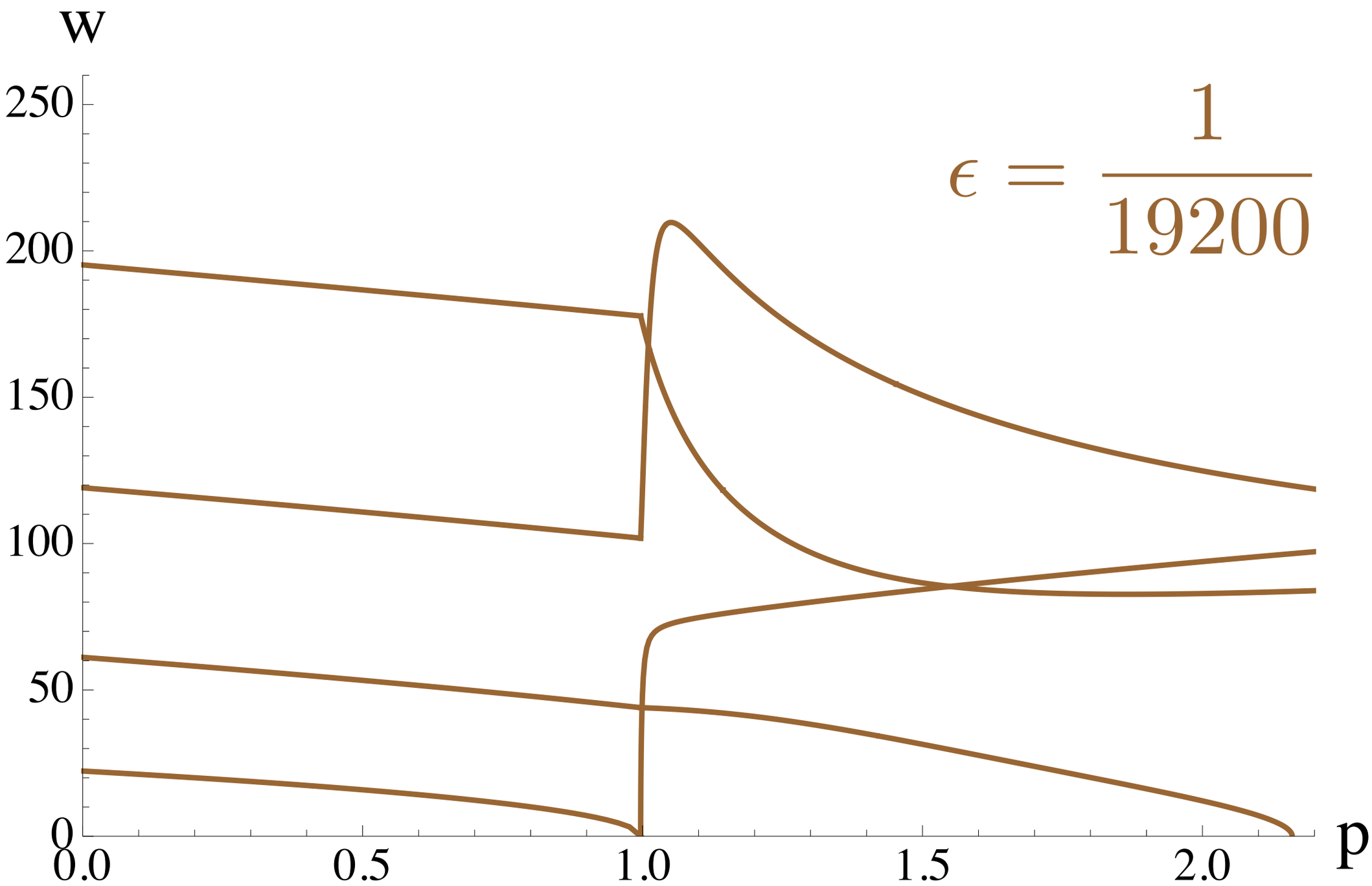


Vibrations (extensible case)



$$\epsilon = \frac{1}{4800}$$

Vibrations (extensible case)

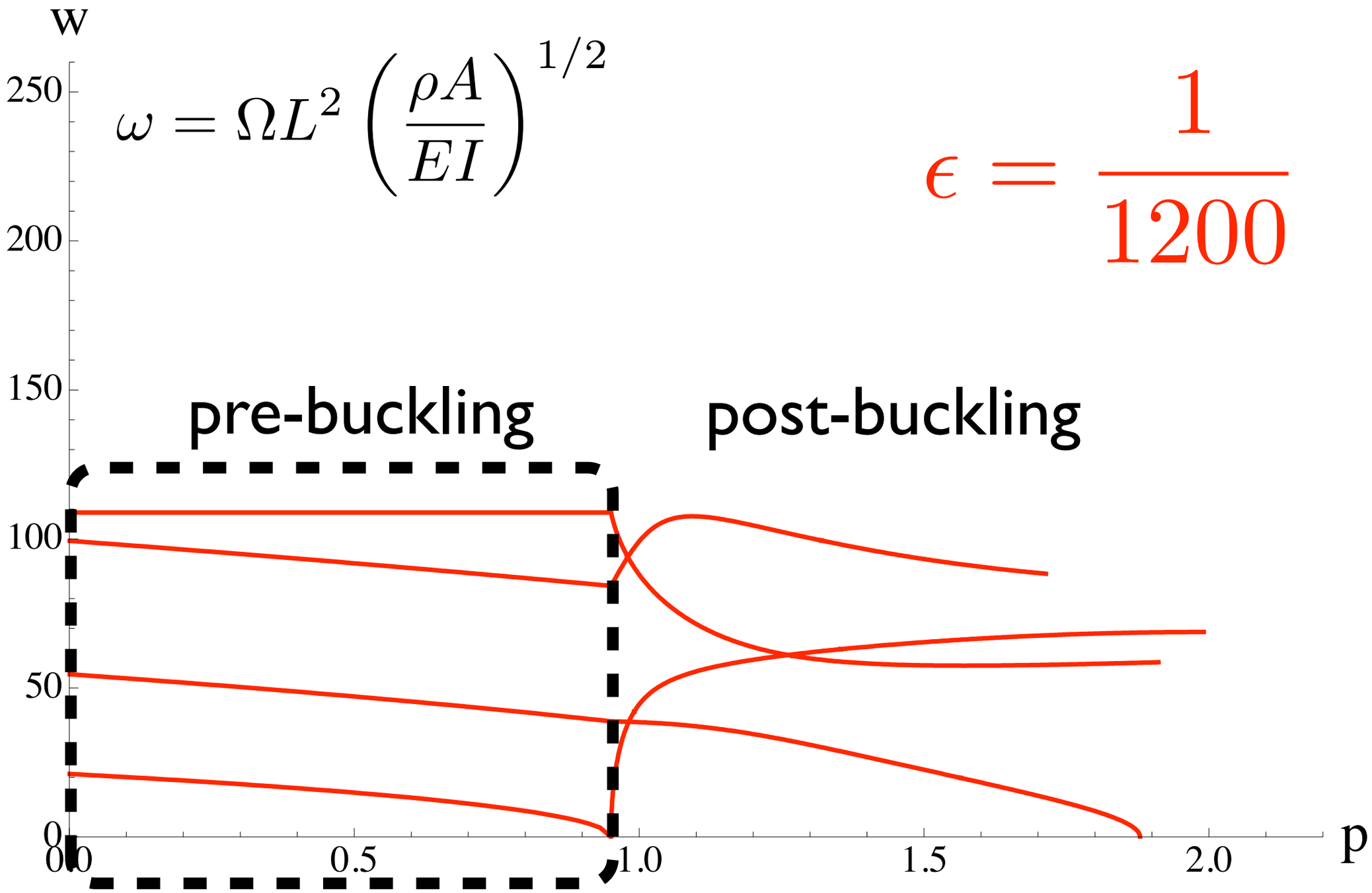


$$\epsilon = \frac{1}{19200}$$

Vibrations (extensible case)

$$\omega = \Omega L^2 \left(\frac{\rho A}{EI} \right)^{1/2}$$

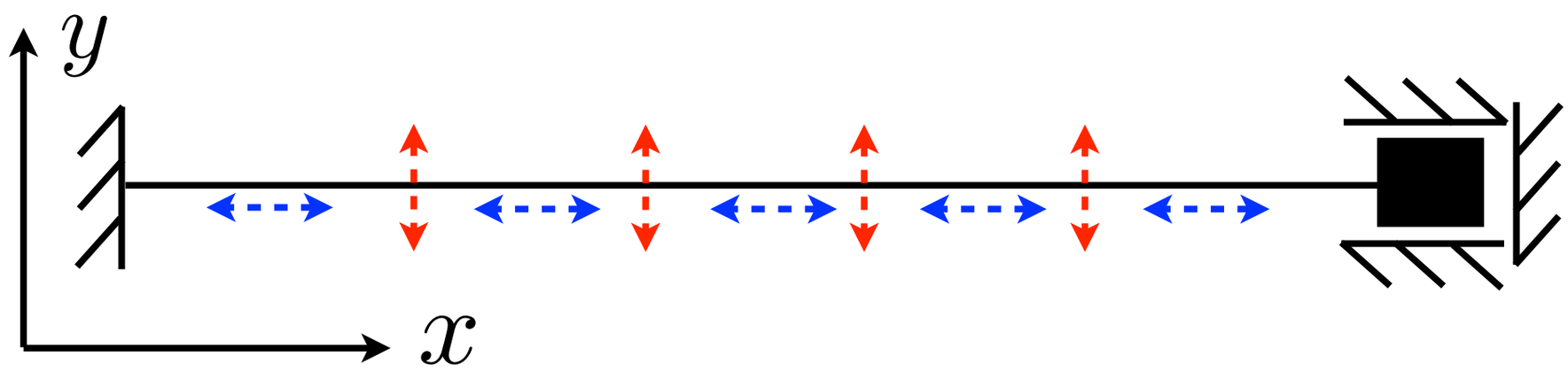
$$\epsilon = \frac{1}{1200}$$



Vibrations

(extensible case)

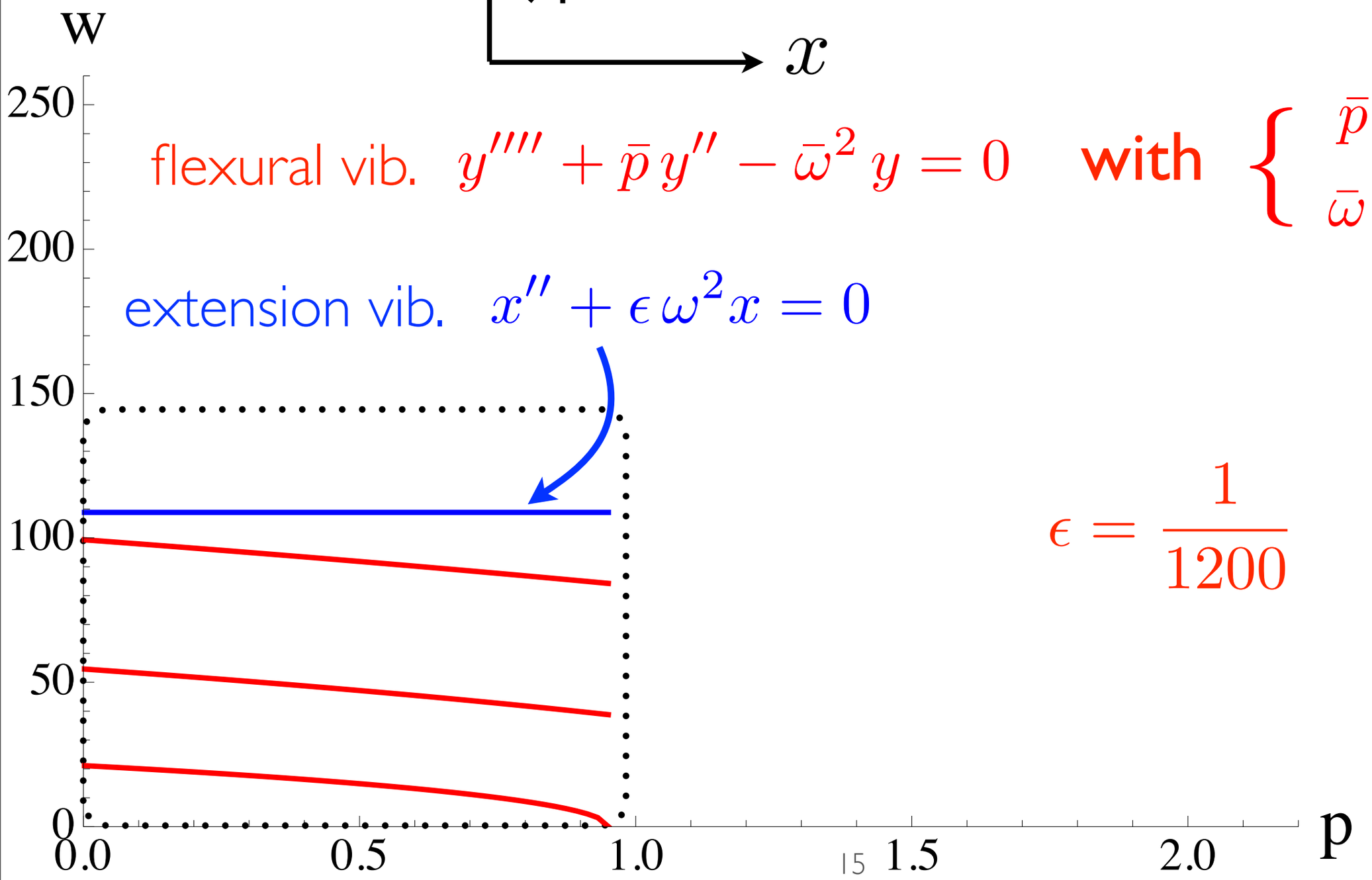
pre-buckling



flexural vib. $y'''' + \bar{p}\, y'' - \bar{\omega}^2\, y = 0$ **with** $\begin{cases} \bar{p} = p(1-\epsilon p) \\ \bar{\omega} = \omega(1-\epsilon p) \end{cases}$

extension vib. $x'' + \epsilon\, \omega^2 x = 0$

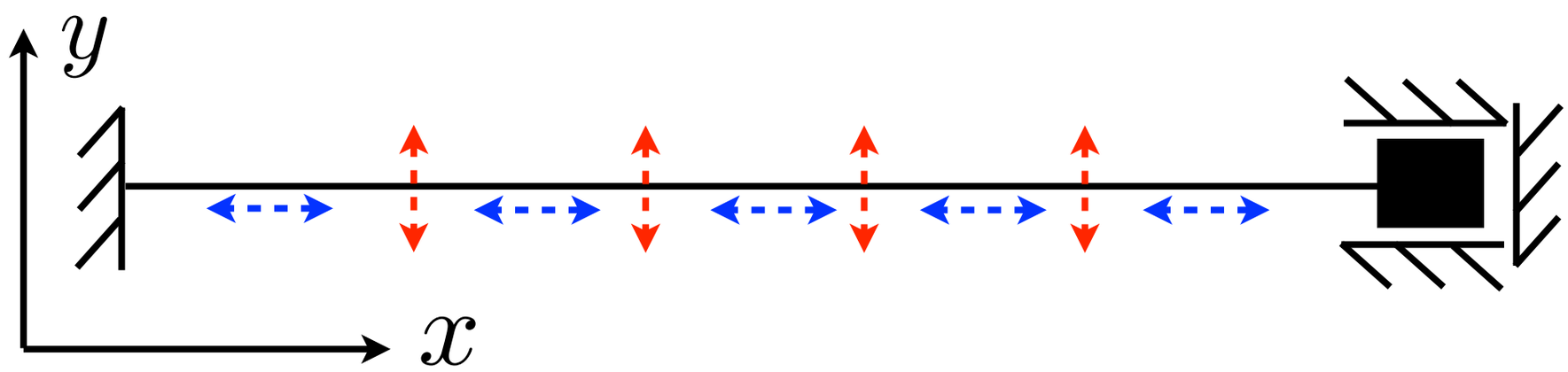
$$\epsilon = \frac{1}{1200}$$



Vibrations

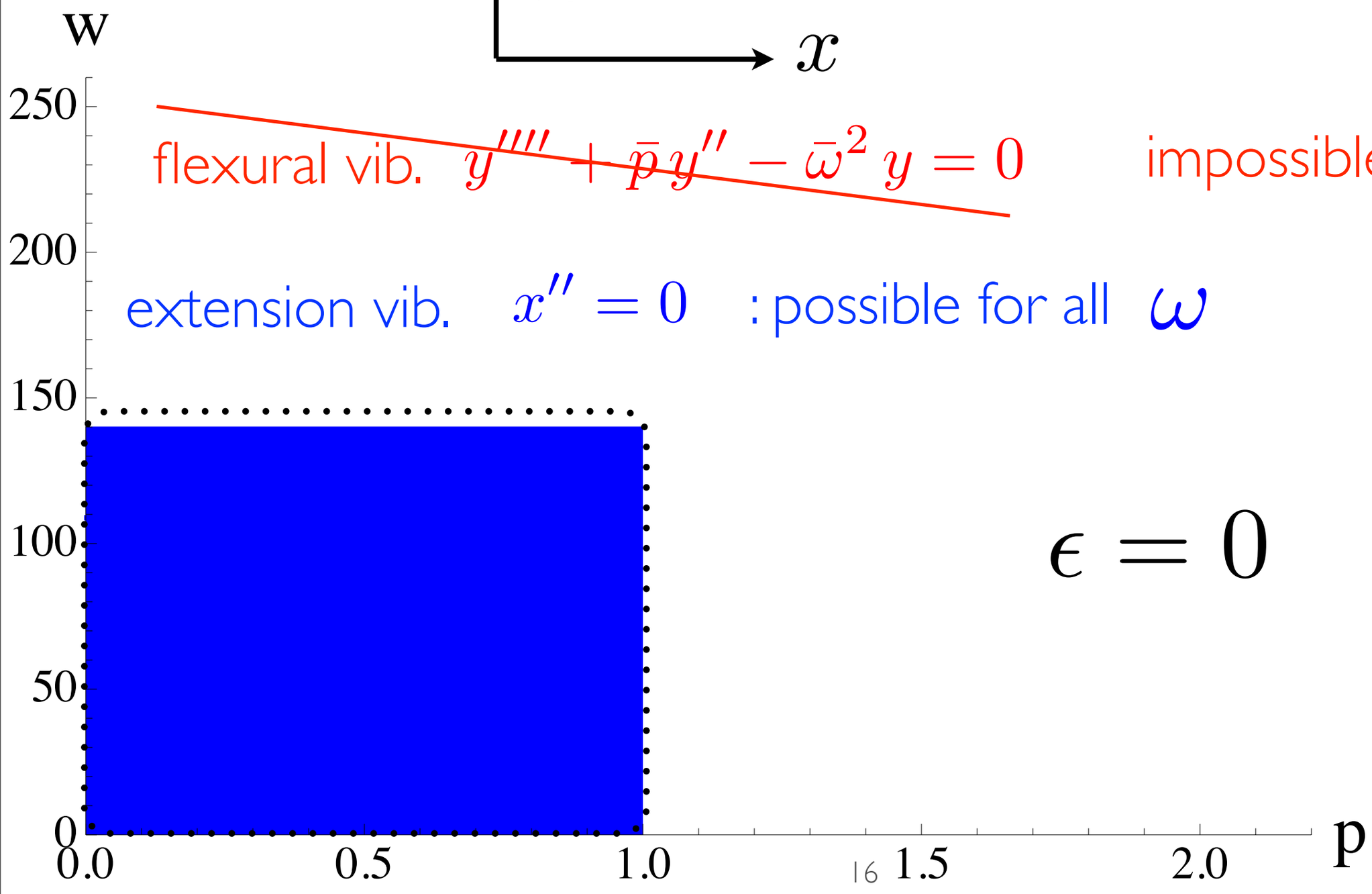
pre-buckling

(inextensible)



flexural vib. $\bar{y}'''' + \bar{p}\,\bar{y}'' - \bar{\omega}^2\,\bar{y} = 0$ impossible

extension vib. $x'' = 0$: possible for all ω

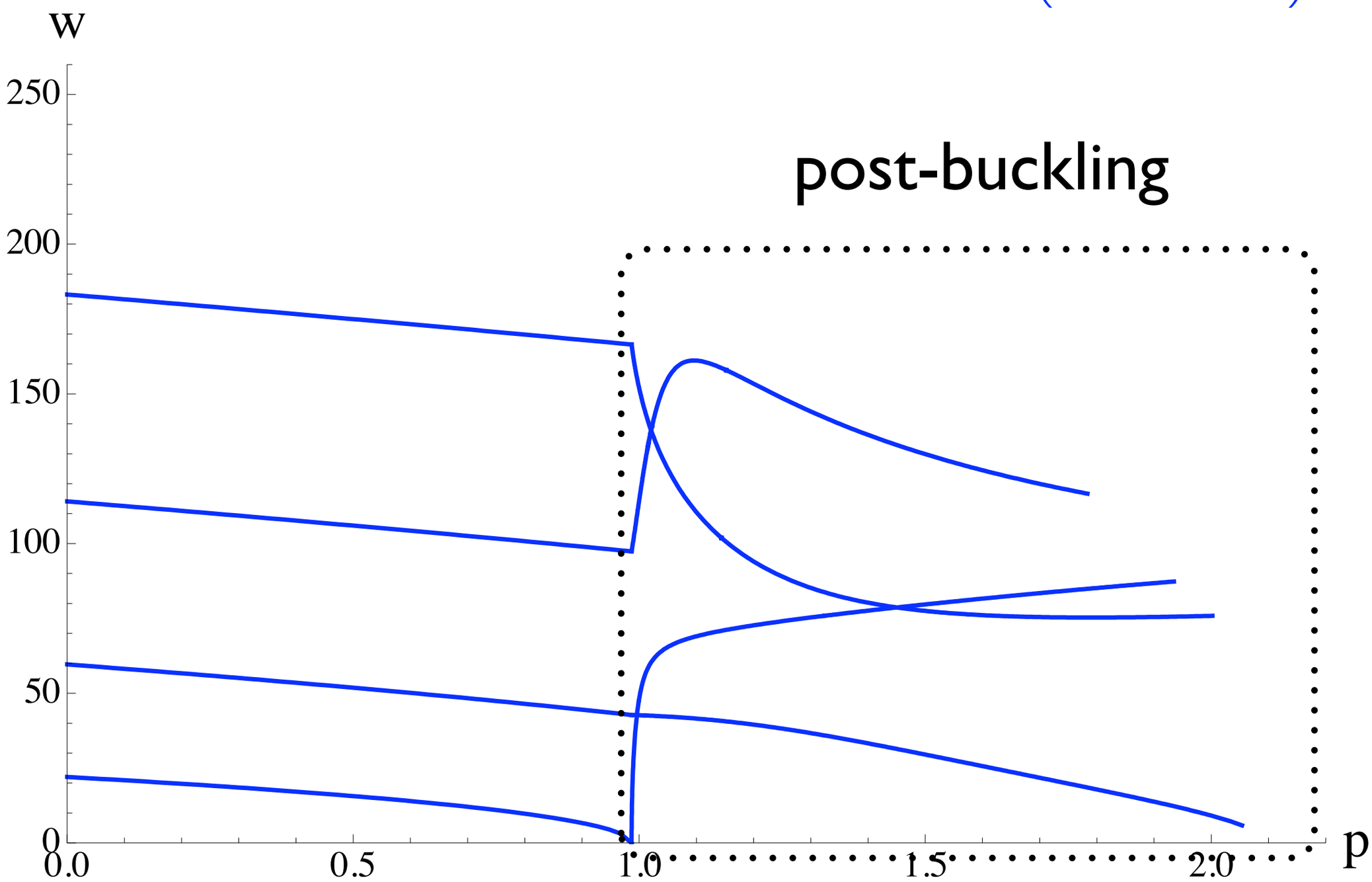


$\epsilon = 0$

Vibrations

$$\epsilon = \frac{1}{4800}$$

(extensible)



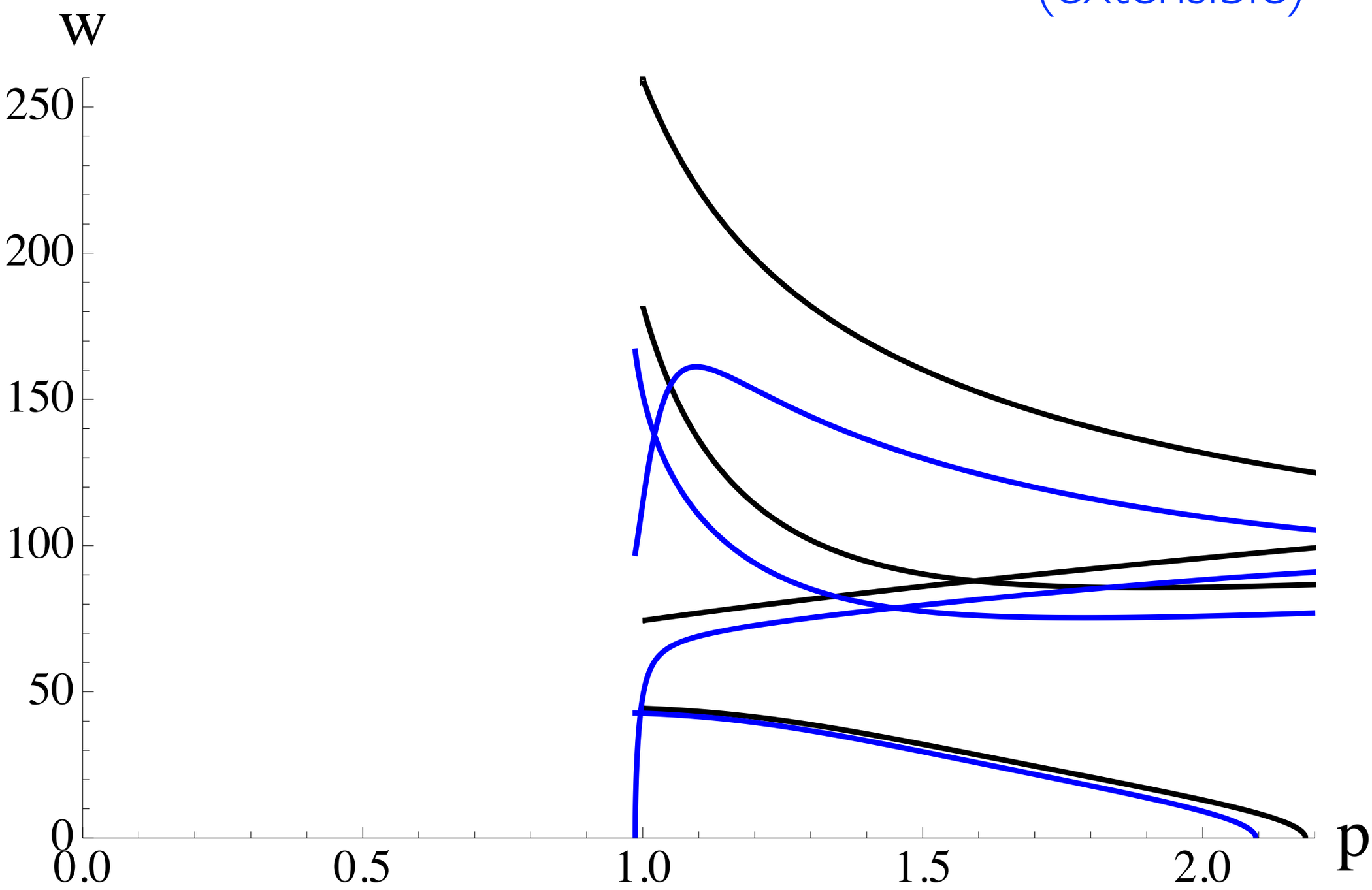
Vibrations

$$\epsilon = 0$$

(inextensible)

$$\epsilon = \frac{1}{4800}$$

(extensible)



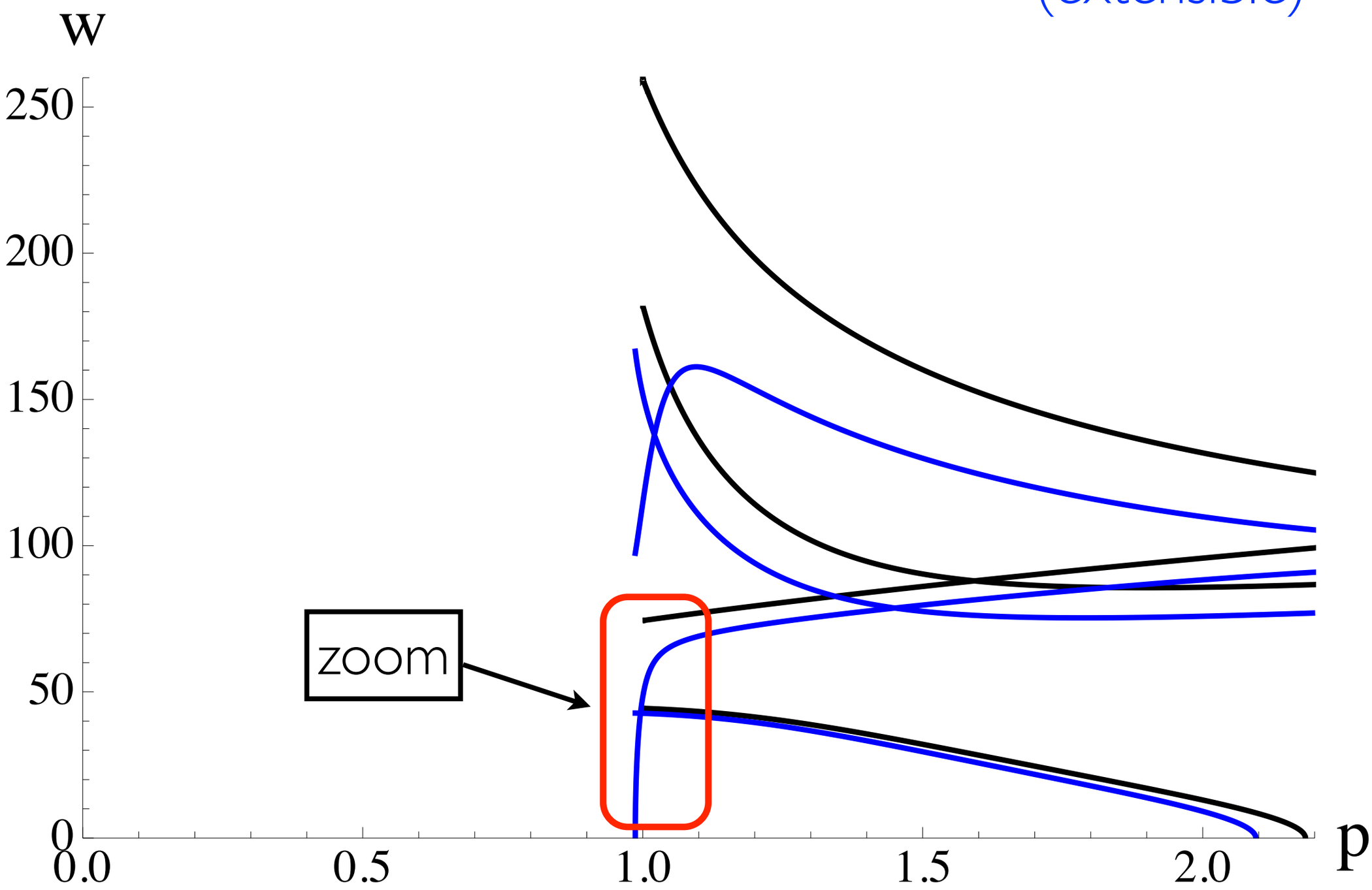
Vibrations

$$\epsilon = 0$$

(inextensible)

$$\epsilon = \frac{1}{4800}$$

(extensible)



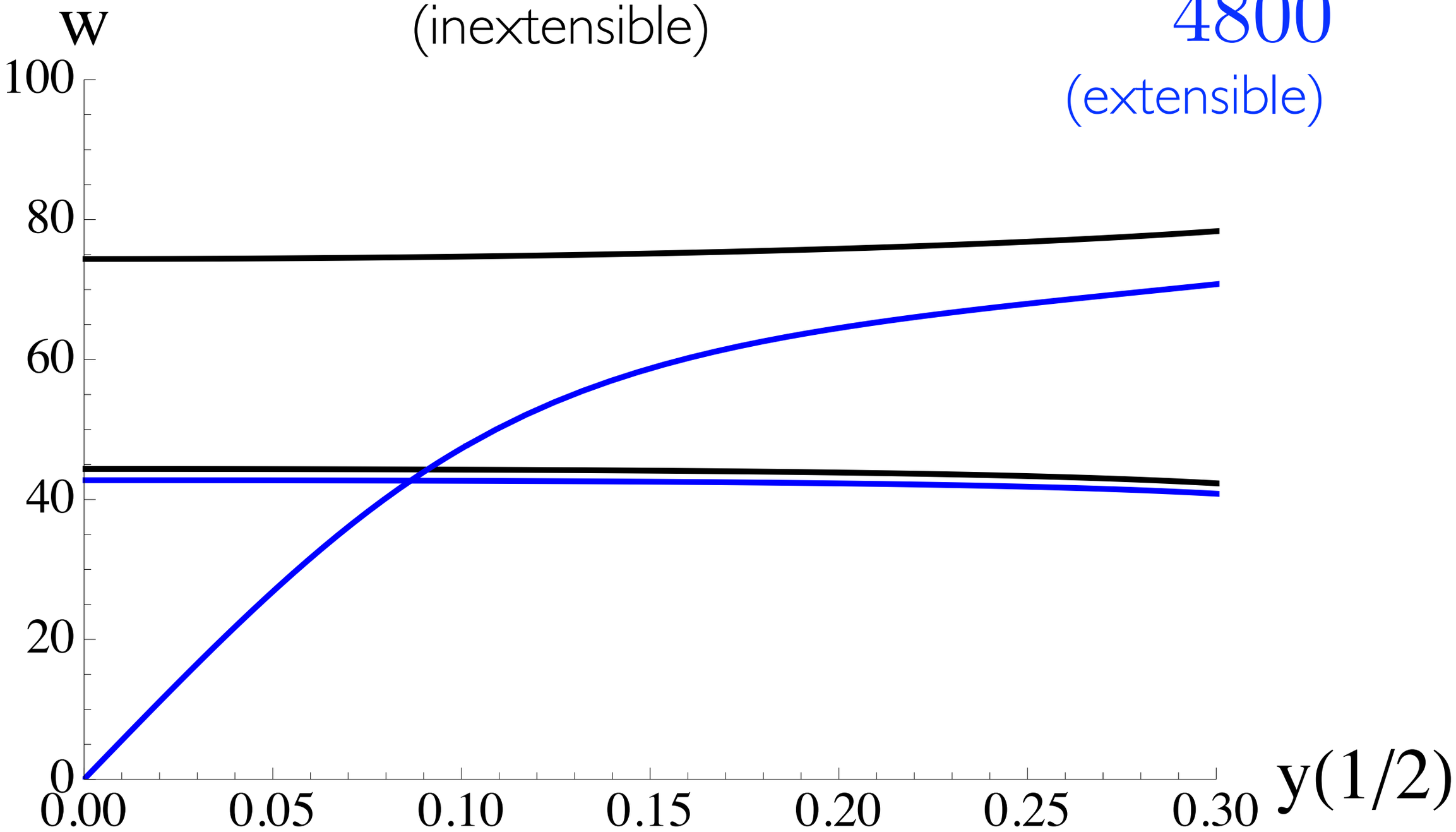
Vibrations

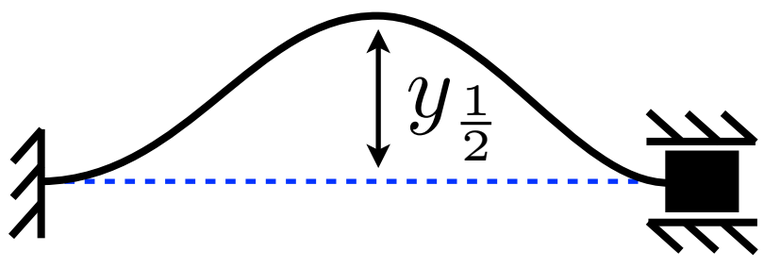
$\epsilon = 0$

(inextensible)

$\epsilon = \dfrac{1}{4800}$

(extensible)

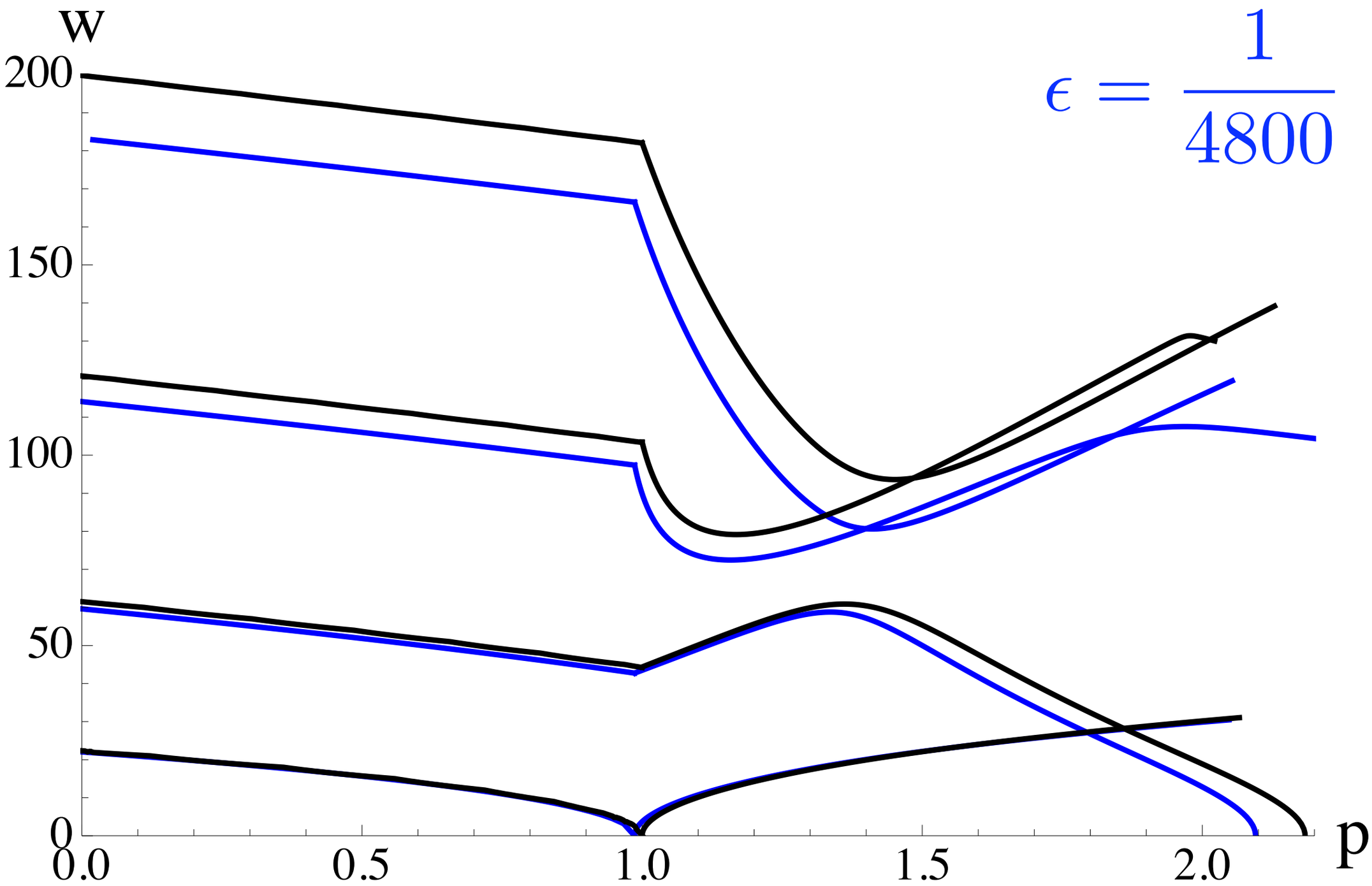




Vibrations : dead load

$$\epsilon = 0$$

$$\epsilon = \frac{1}{4800}$$



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