Some contact problems for elastic rods

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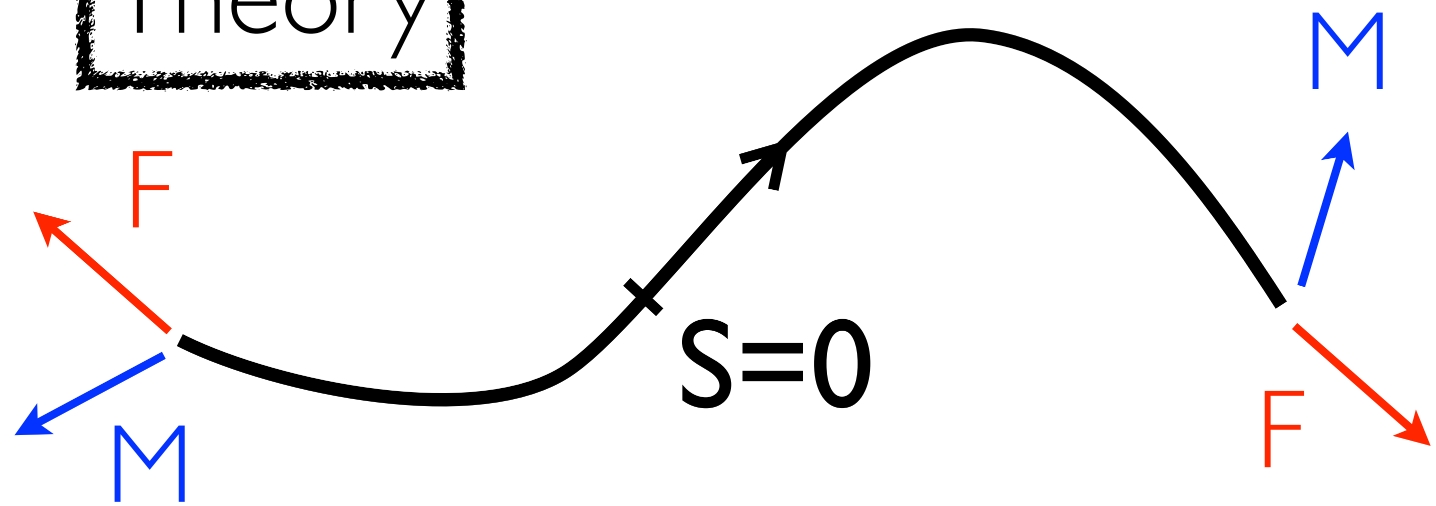
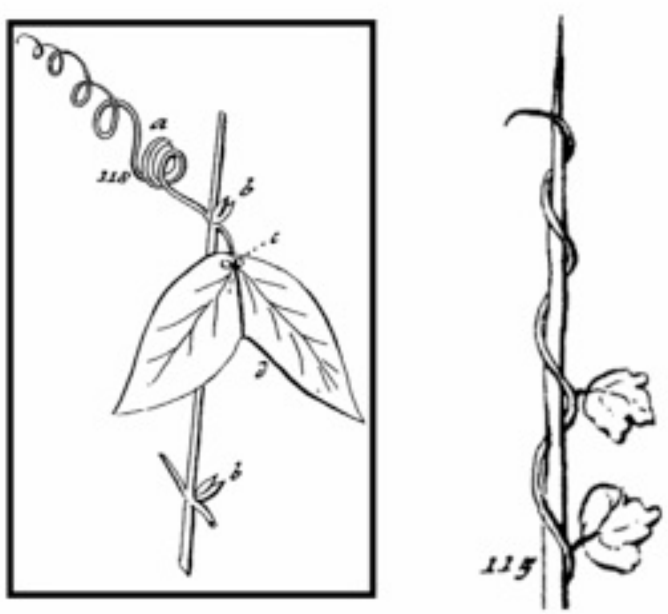
Elastic filaments

Theory

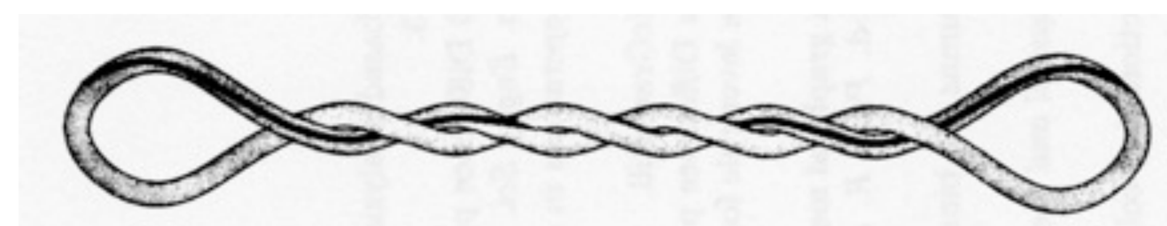


Applications

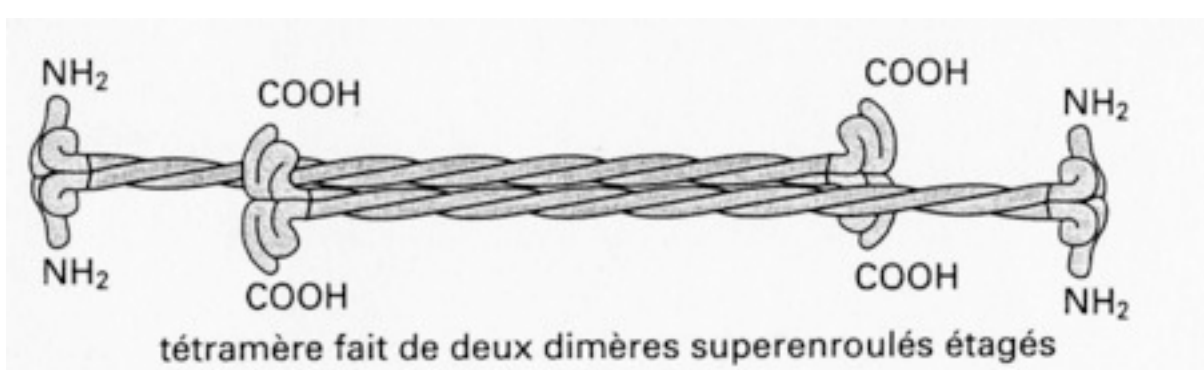
climbing plants



DNA supercoiling

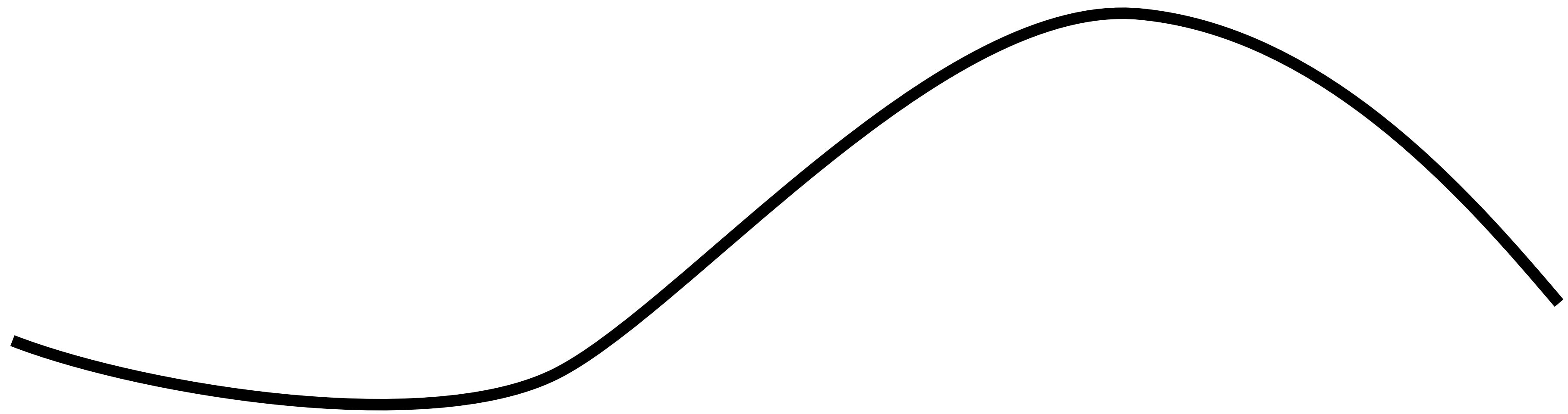


fibrous proteins



cables

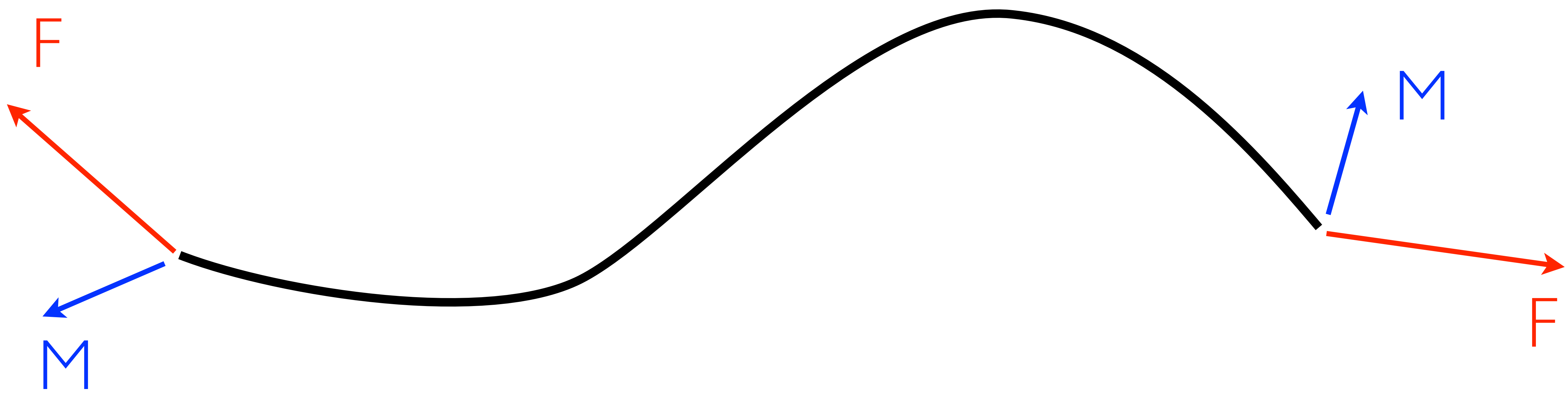
Kirchhoff equations



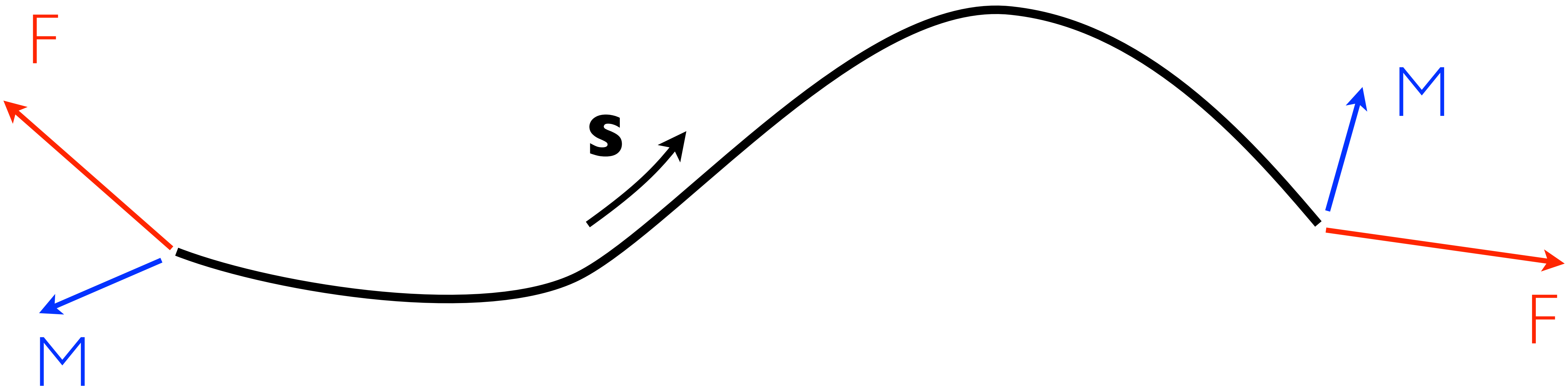
apply to :

- slender bodies
- not too bent

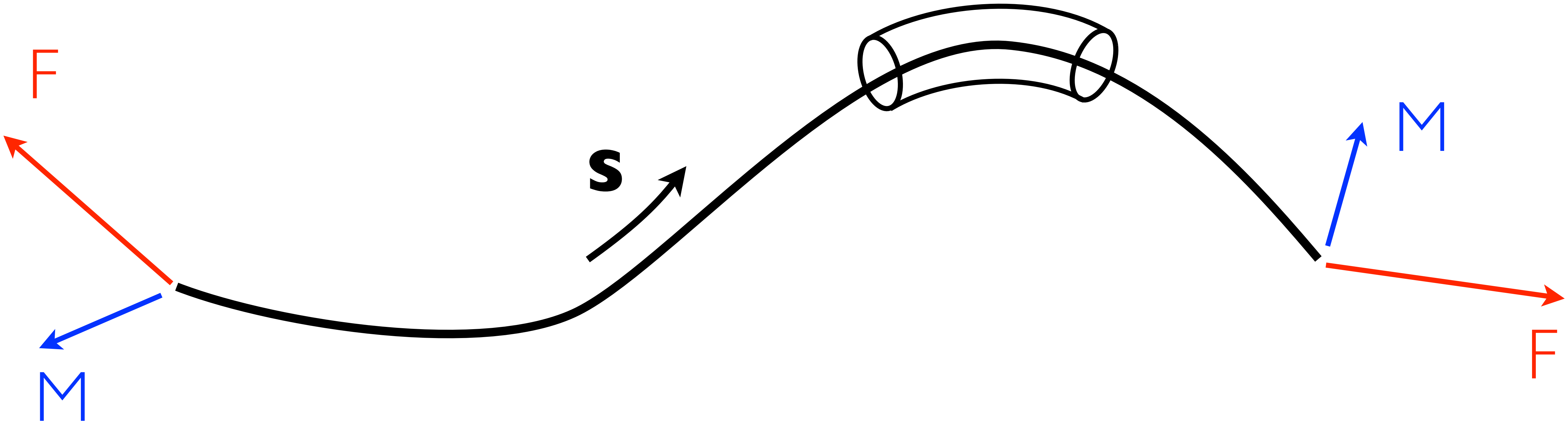
Kirchhoff equations



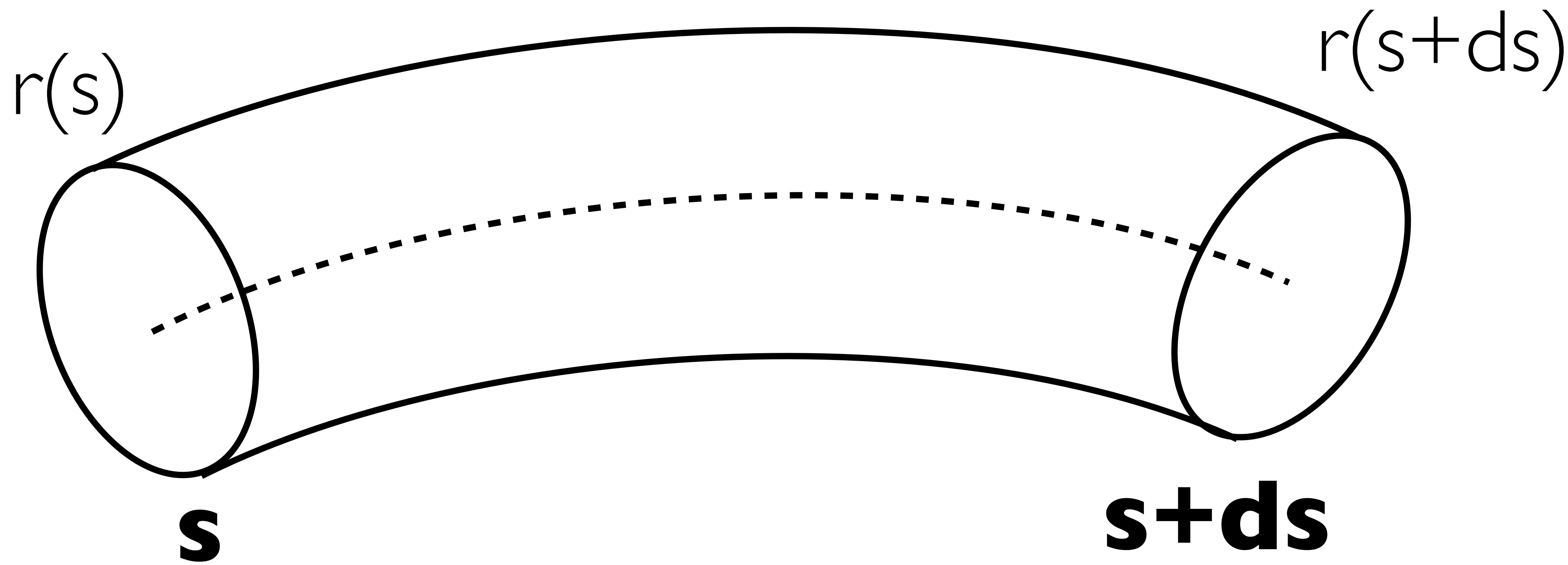
Kirchhoff equations



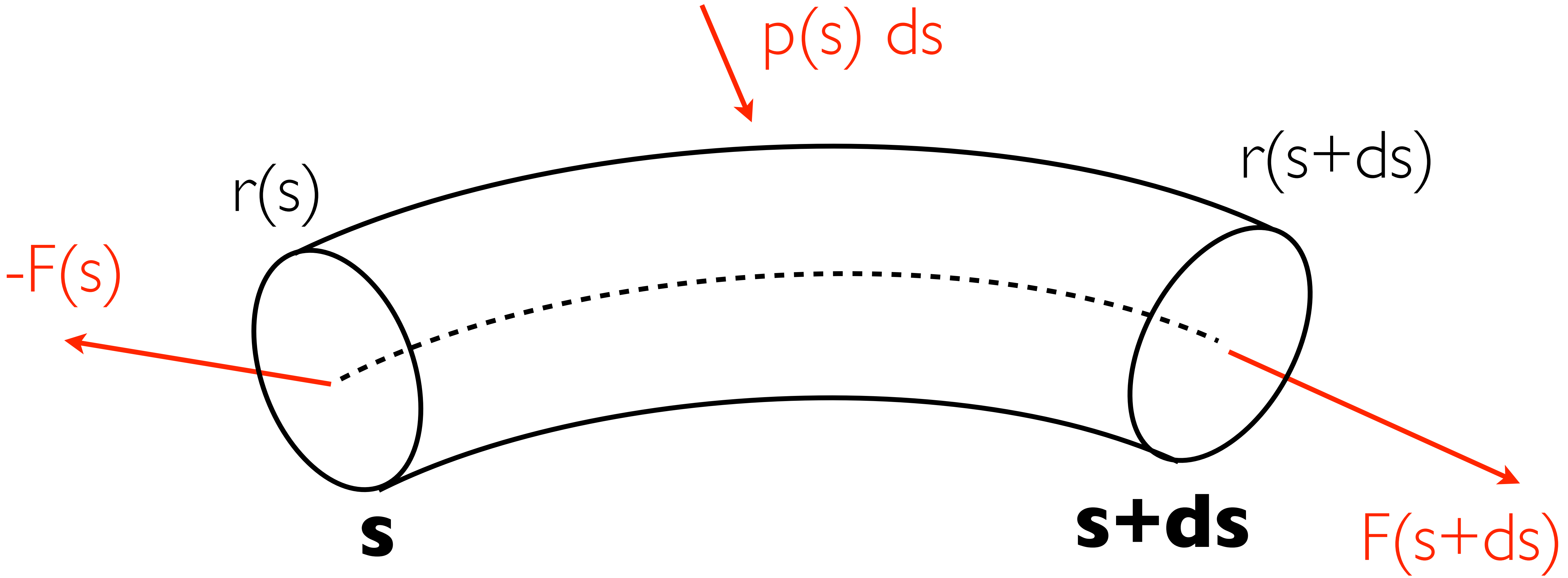
Kirchhoff equations



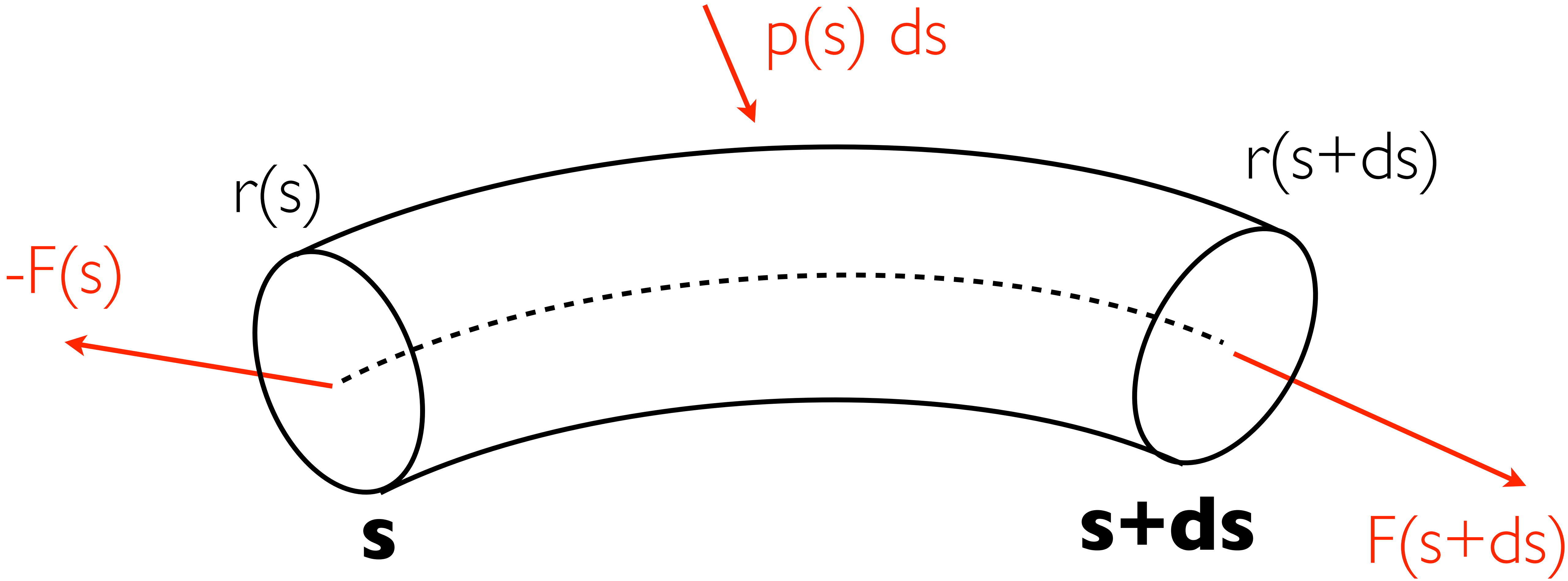
Kirchhoff equations



Kirchhoff equations



Kirchhoff equations

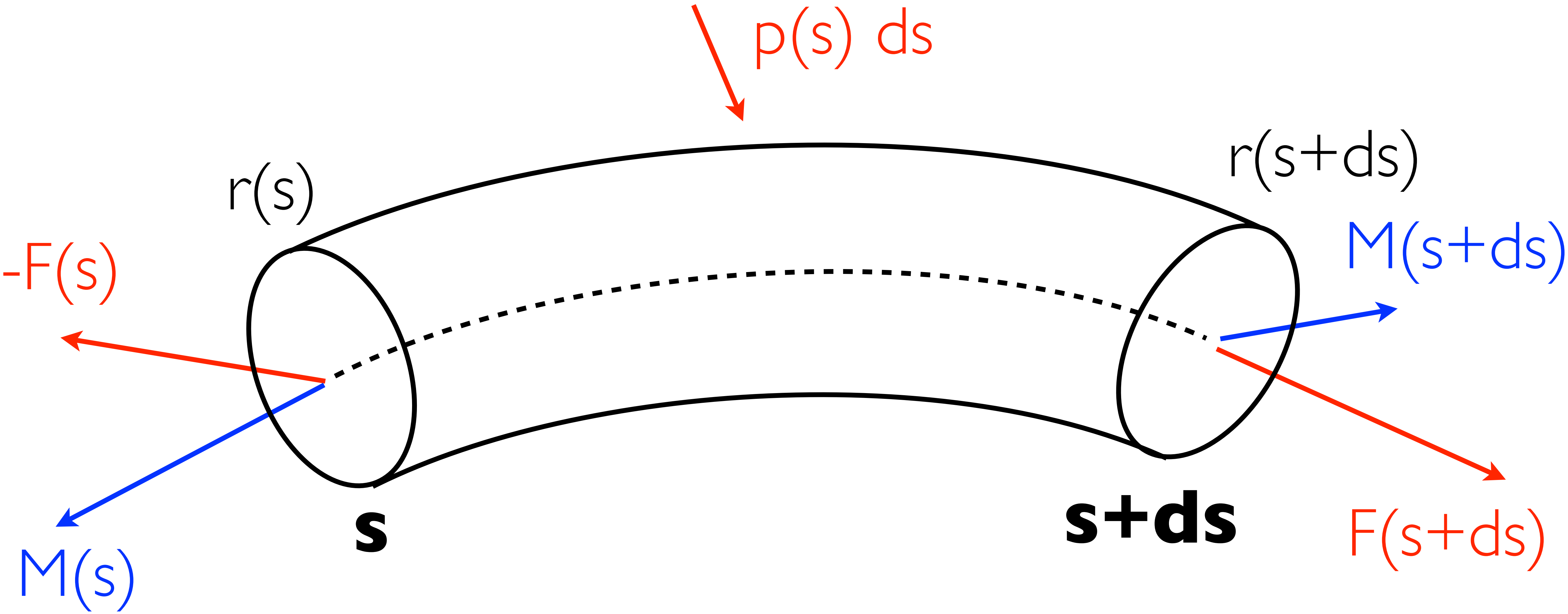


$$F(s+ds) - F(s) + p(s)\, ds = 0$$

Equilibrium

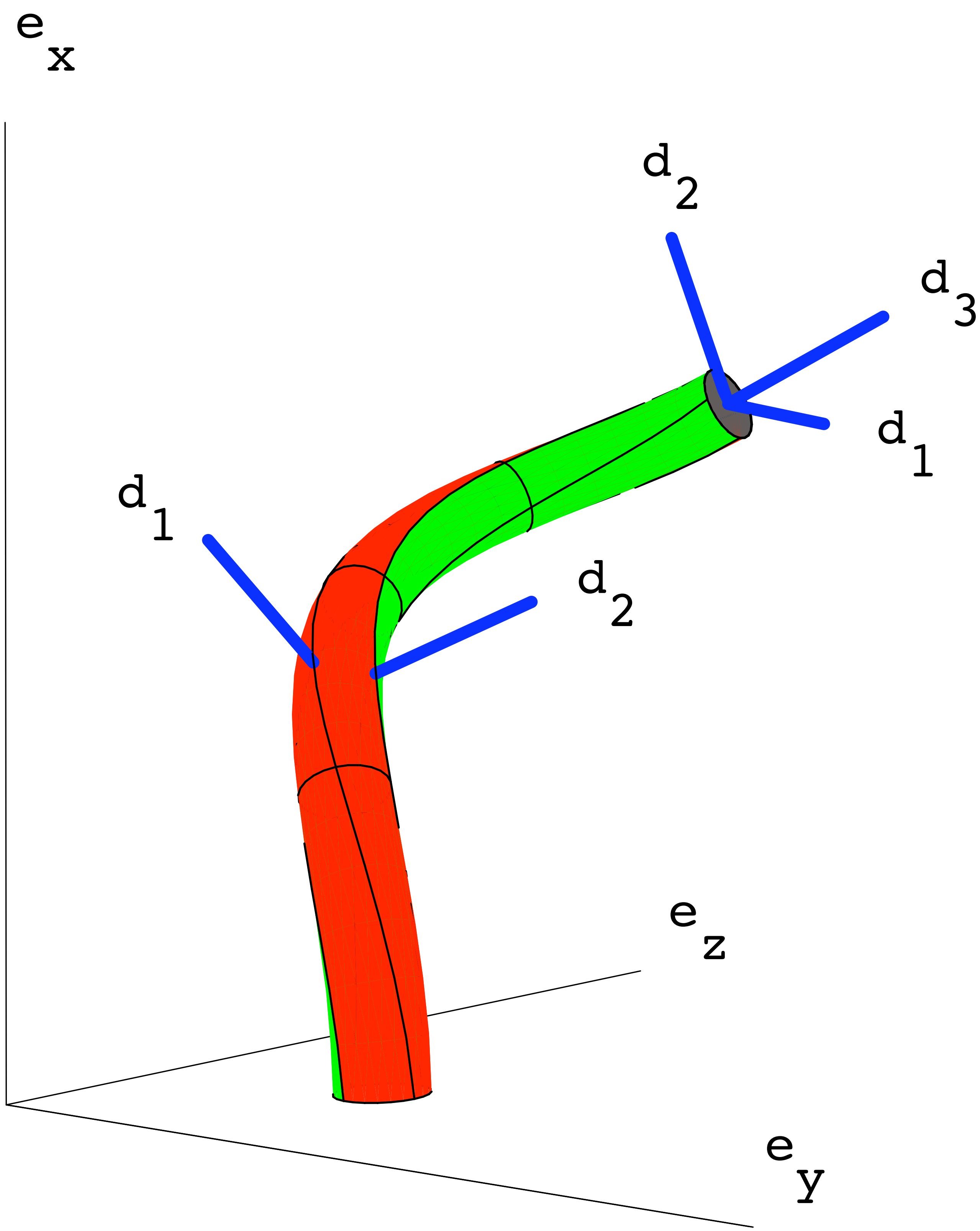
$$F'(s) + p(s) = 0$$

Kirchhoff equations



Equilibrium $M' + r' \times F = 0$

Kirchhoff equations



Cosserat frame

$$d_1' = u \times d_1$$
$$d_2' = u \times d_2$$
$$d_3' = u \times d_3$$

$$u = \{\kappa_1, \kappa_2, \tau\}_{d_i}$$

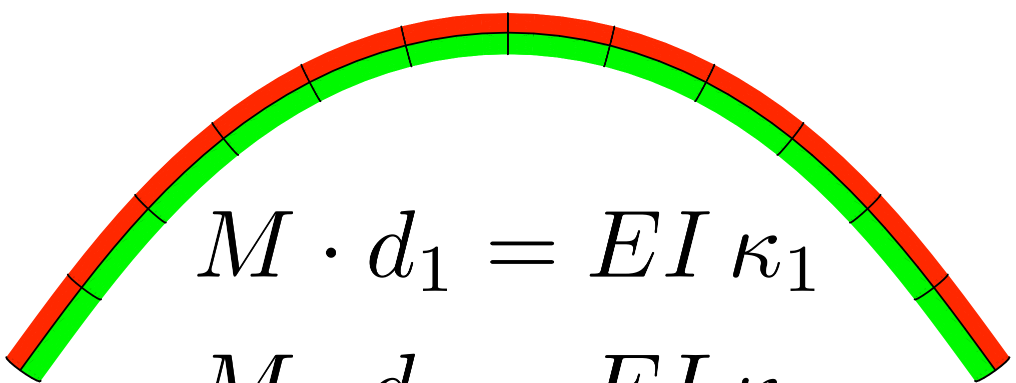
curvatures

twist

Kirchhoff equations

constitutive relations

curvature



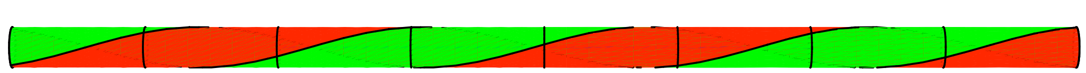
$$M \cdot d_1 = EI\,\kappa_1$$

$$M \cdot d_2 = EI\,\kappa_2$$

E Young's modulus

I second moment of area

twist



$$M \cdot d_3 = GJ\,\tau$$

G shear modulus

J polar moment of area

Kirchhoff equations

21 ODEs with variable : s

ordinary differential equations

$$\frac{d}{ds}\vec{F}=\vec{p}$$

$$\frac{d}{ds}\vec{M}=\vec{F}\wedge\vec{d}_3$$

$$\frac{d}{ds}\vec{r}=\vec{d}_3$$

$$\frac{d}{ds}\vec{d}_i=\vec{u}\wedge\vec{d}_i$$

$$m_i=K_i u_i$$

linear elasticity

21 unknowns

$\vec{F}(s)$

$\vec{M}(s)$

$\vec{r}(s)$

$\vec{d}_3(s)$

$\vec{d}_2(s)$

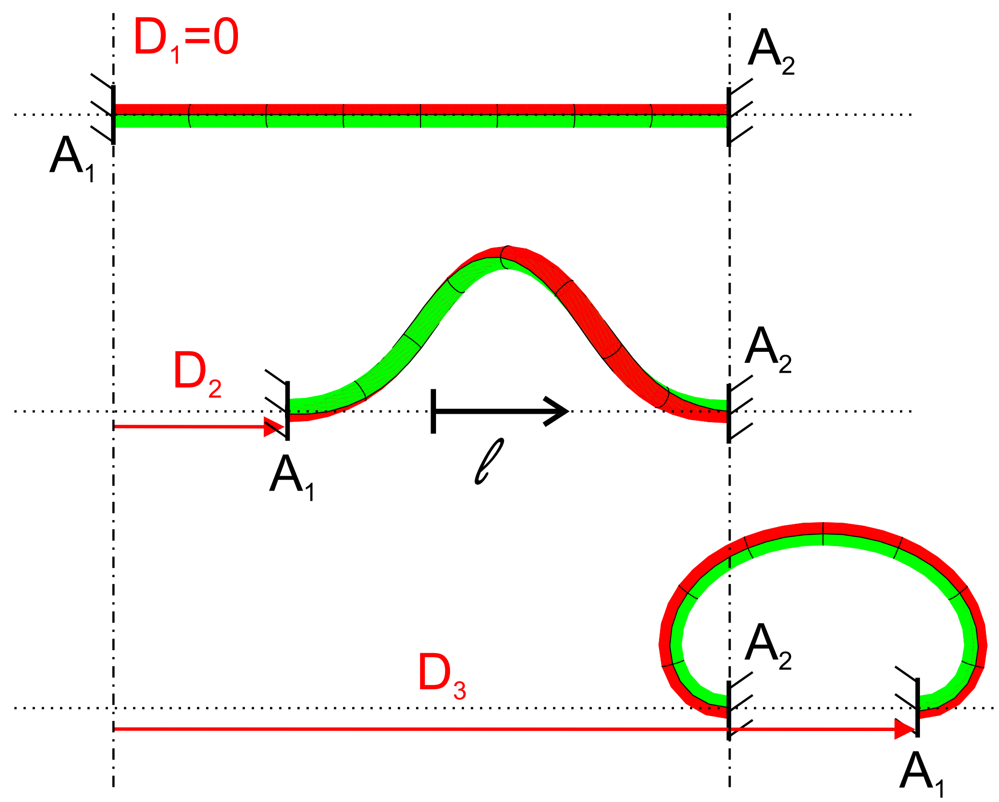
$\vec{d}_3(s)$

$\vec{u}(s)$

$i=1,2,3$

<u>boundary conditions</u>

- how the rod is held
- few solutions are admissibles



$$\vec{d}_3(A_1)=\vec{d}_3(A_2)$$

$$\vec{r}(A_2)-\vec{r}(A_1)=k\,\vec{d}_3(A_2)$$

$$(D=L-k)$$

Find admissible equilibrium solutions : shooting method

initial conditions

$r(0)=(0,0,0)$

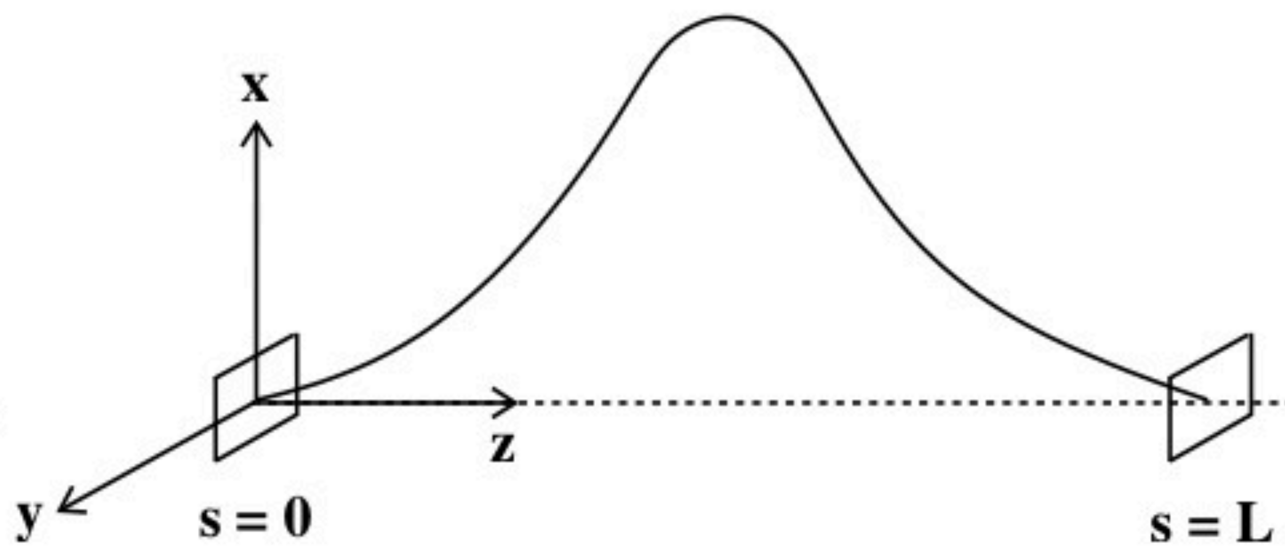
$d_1(0)=(1,0,0)$

$d_2(0)=(0,1,0)$

$d_3(0)=(0,0,1)$

parameters

$\vec{F}(0), \vec{M}(0)$



end conditions

$$\left.\begin{aligned} x(L)&=0\\ y(L)&=0\\ d_3 x(L)&=0\\ d_3 y(L)&=0 \end{aligned}\right\}\phi$$

---

solution of ODEs

$$\phi(\overbrace{\vec{F}(0), \vec{M}(0)}^{u})=0$$

$$\Leftrightarrow \phi(u)=0$$

$$\phi \in \mathbb{R}^L$$

$$u \in \mathbb{R}^P$$

this defines a *P-L* solution manifold

1D solution manifold : path following predictor-corrector scheme

1D solution manifold

$$\begin{cases} \phi_1(u_1, u_2, u_3) = 0 \\ \phi_2(u_1, u_2, u_3) = 0 \end{cases}$$

At each point :

1-(predictor)

we take a guess : Z_i

2-(corrector)

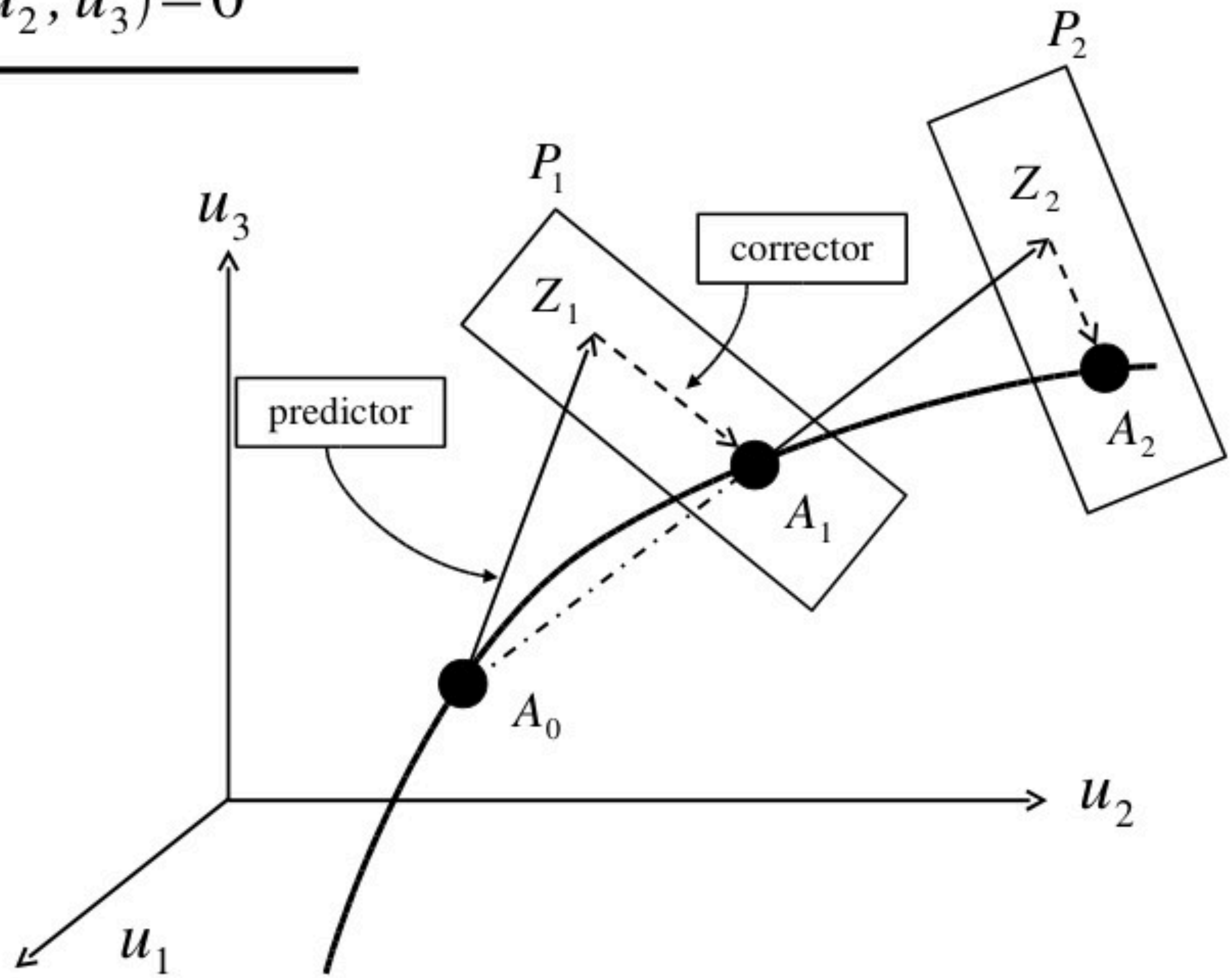
we define a projection :

$P_i(u_1, u_2, u_3) = 0$

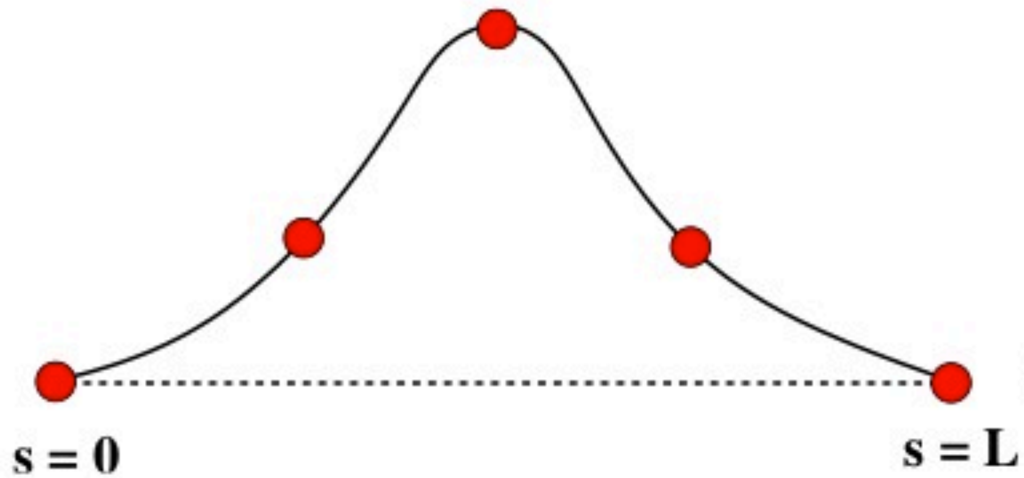
and we solve :

$$\begin{cases} \phi_1(u_1, u_2, u_3) = 0 \\ \phi_2(u_1, u_2, u_3) = 0 \\ P_i(u_1, u_2, u_3) = 0 \end{cases}$$

to obtain A_i



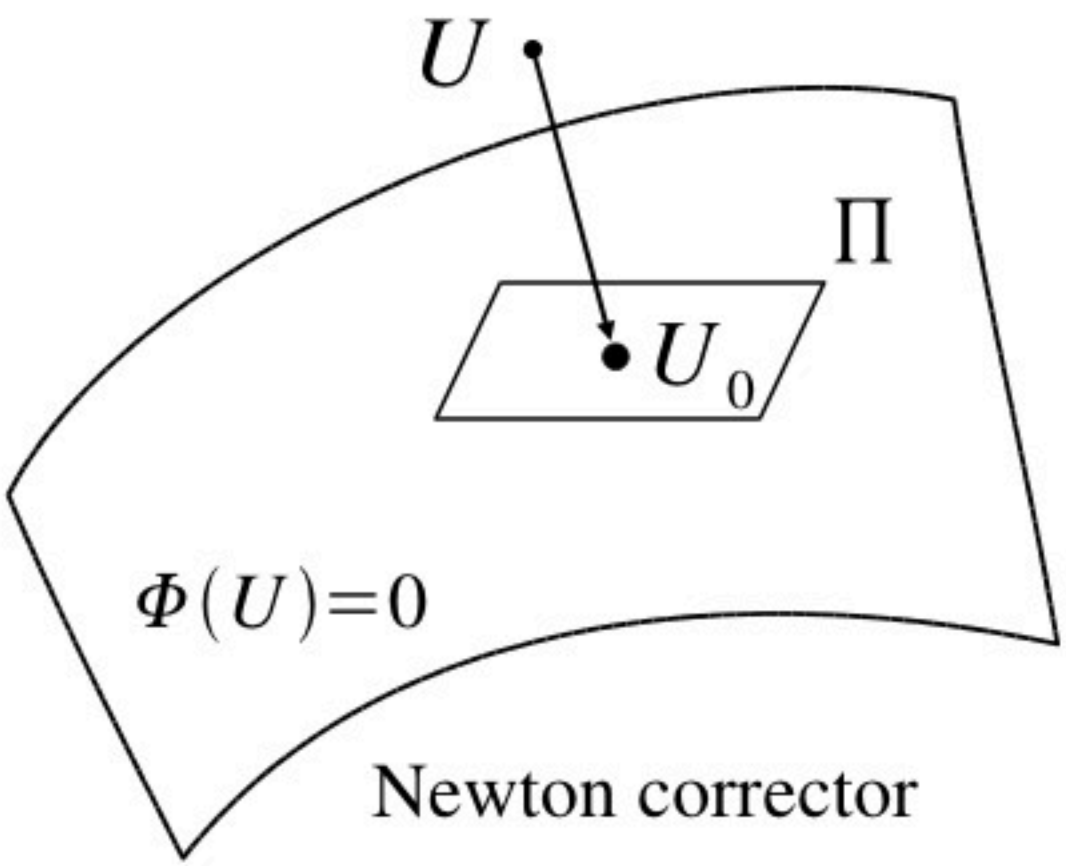
Find admissible equilibrium solutions : discretization methods



discretization over N intervals

$\Phi(U) = 0$ → boundary conditions, matching conditions

system of nonlinear algebraic equations

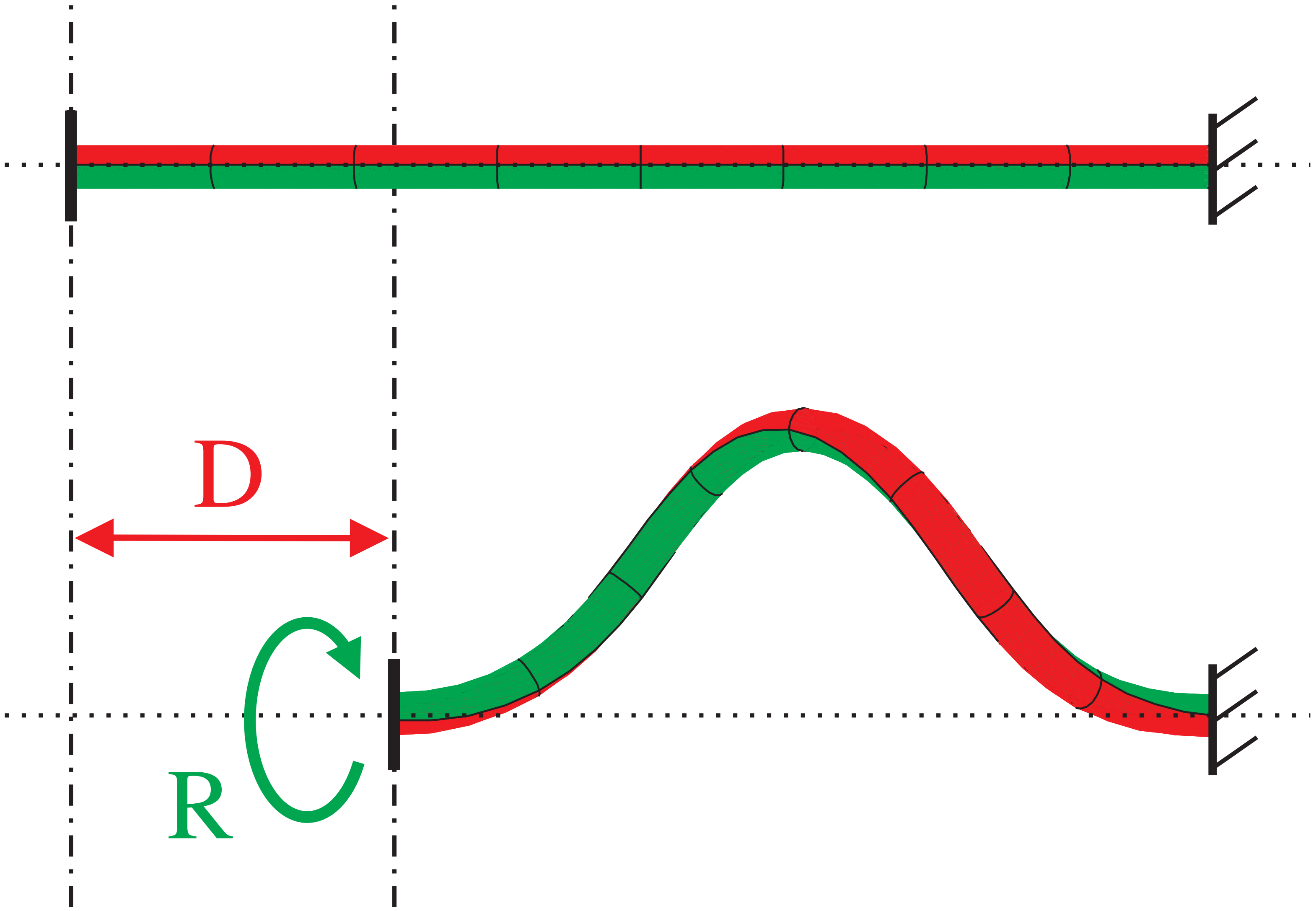


$$\underbrace{\Phi(U_0)}_{=0} = \Phi(U) + \frac{D\Phi}{DU}(U_0 - U) + \ldots$$

1-we take a point U
2-compute Jacobian
3-kernel is tangent plane Π
4-we project orthogonaly : $U \rightarrow U_0$

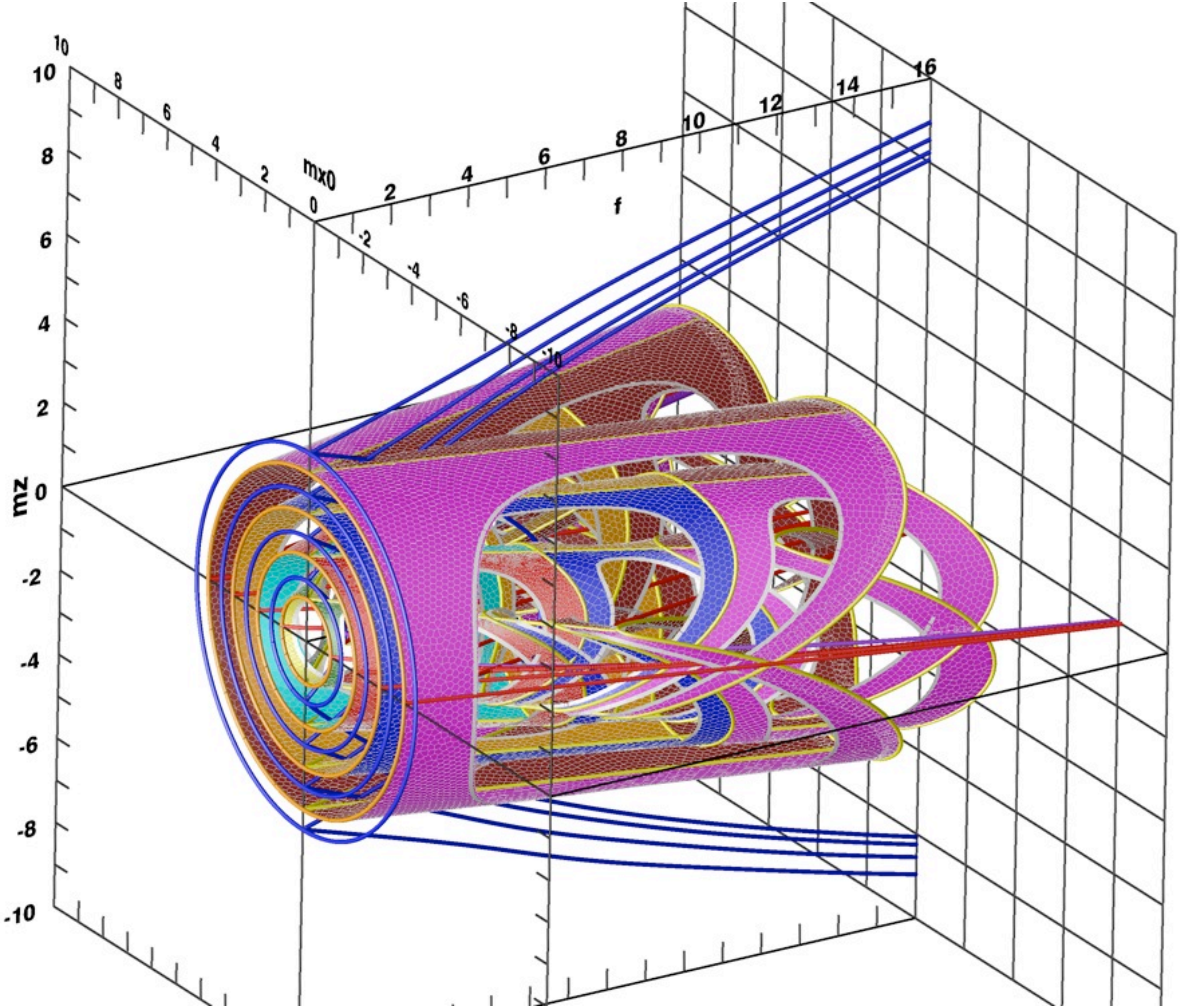
1st `experiment'

Clamped rod loaded in tension and torque



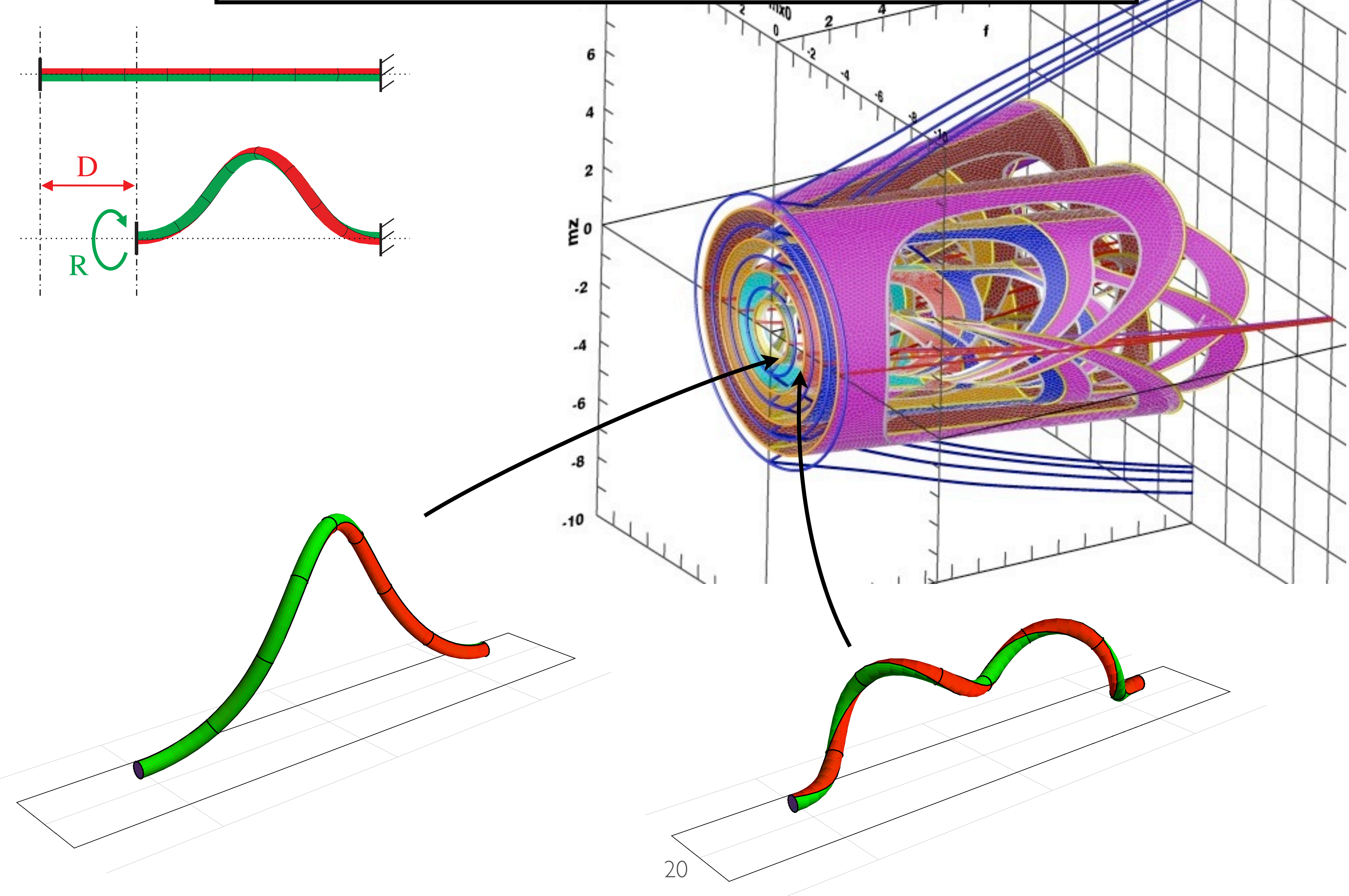
D : raccourcissement

R : rotation

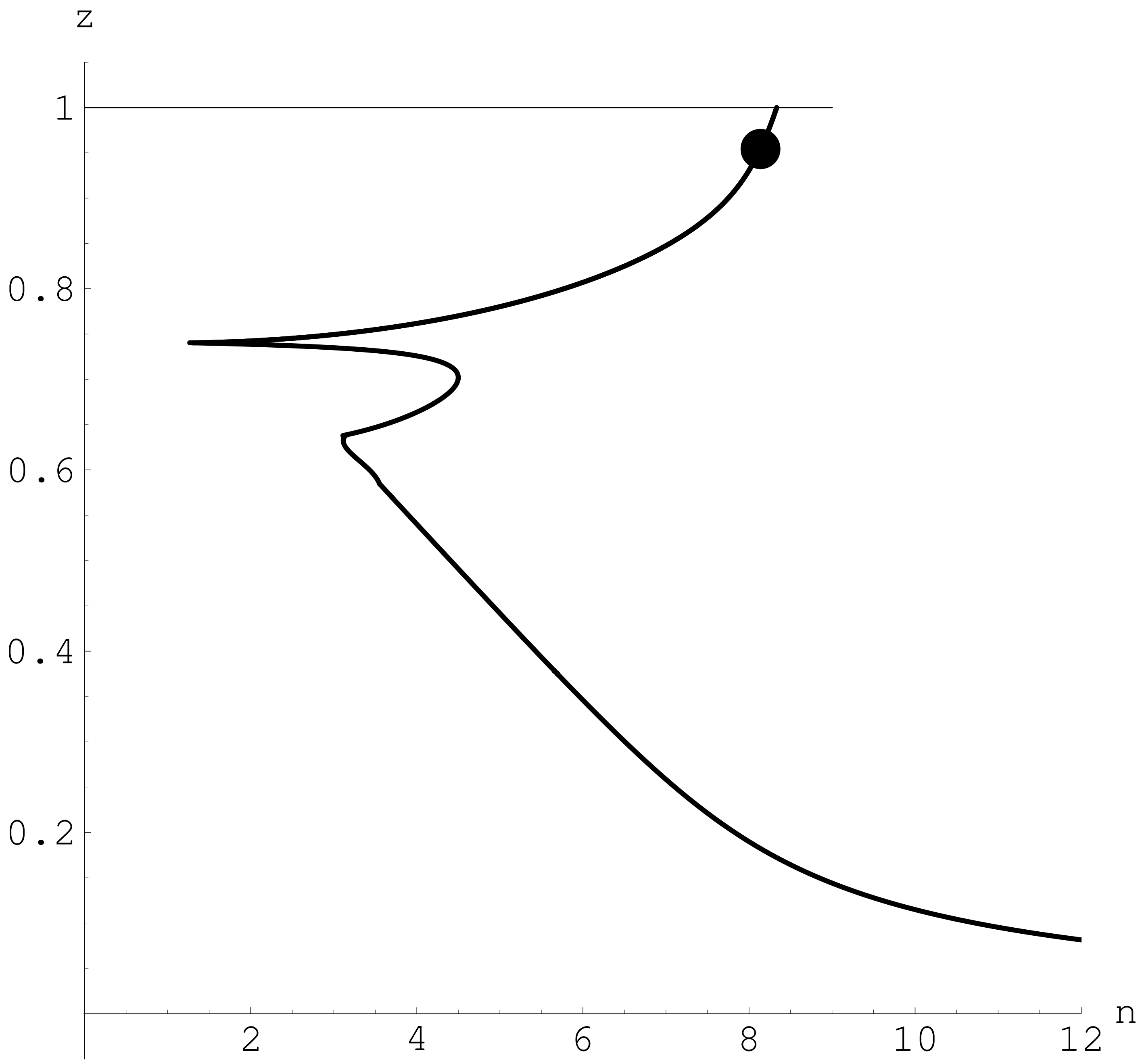


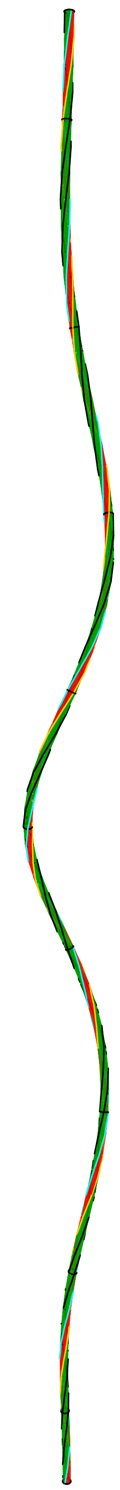
mz
mx0
f
10
8
6
4
2
0
-2
-4
-6
-8
-10
12
14
16

Typical setup: the clamped rod



Results : how a twisted rod coils





$$\frac{L}{2R} = 170$$

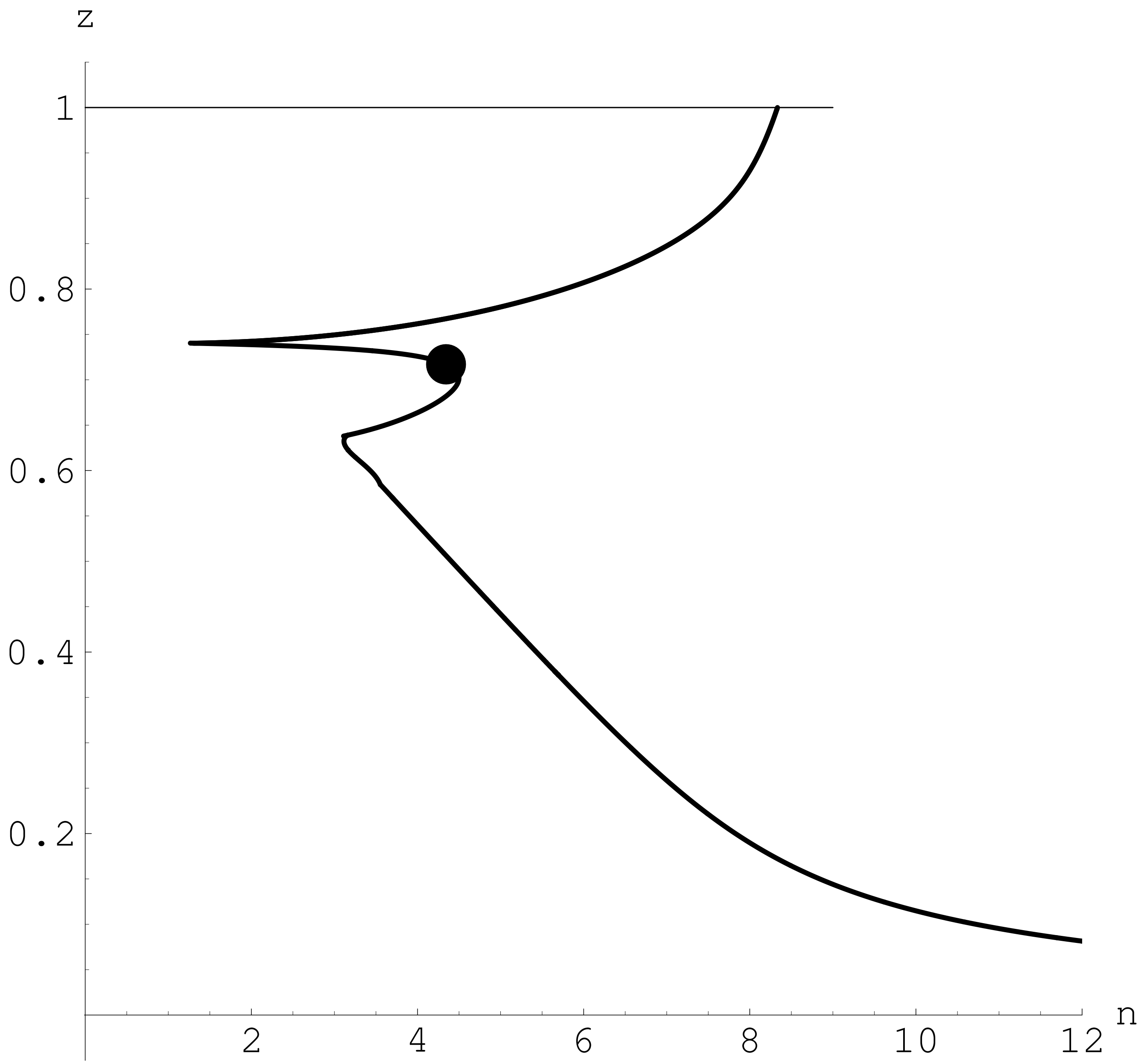
$$t = \frac{TL^2}{4\pi^2 K_0} = 6$$

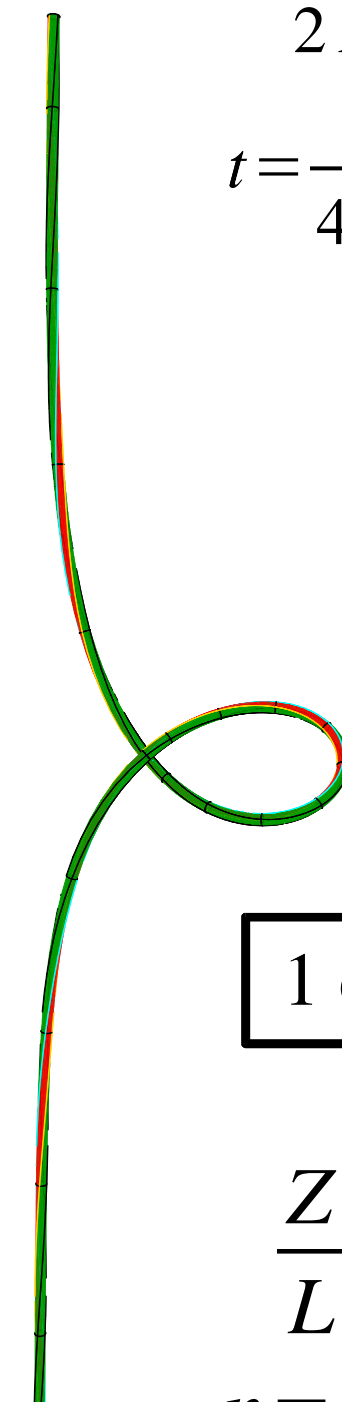
0 contact(s)

$$\frac{Z}{L} = 0.95$$

$$n = 8.1 \, turns$$

Results : how a twisted rod coils





$$\frac{L}{2R} = 170$$

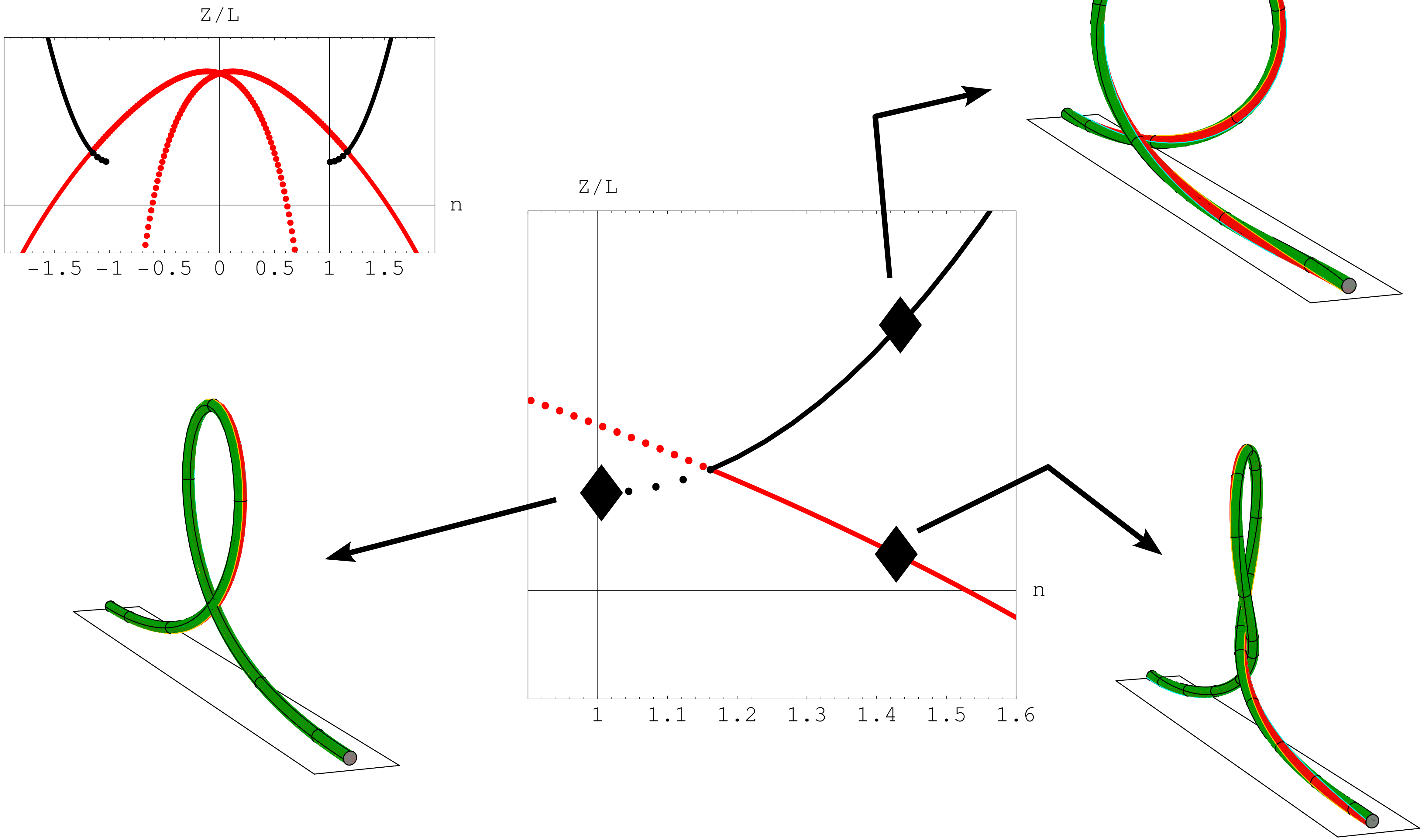
$$t = \frac{TL^2}{4\pi^2 K_0} = 6$$

1 contact(s)

$$\frac{Z}{L} = 0.72$$

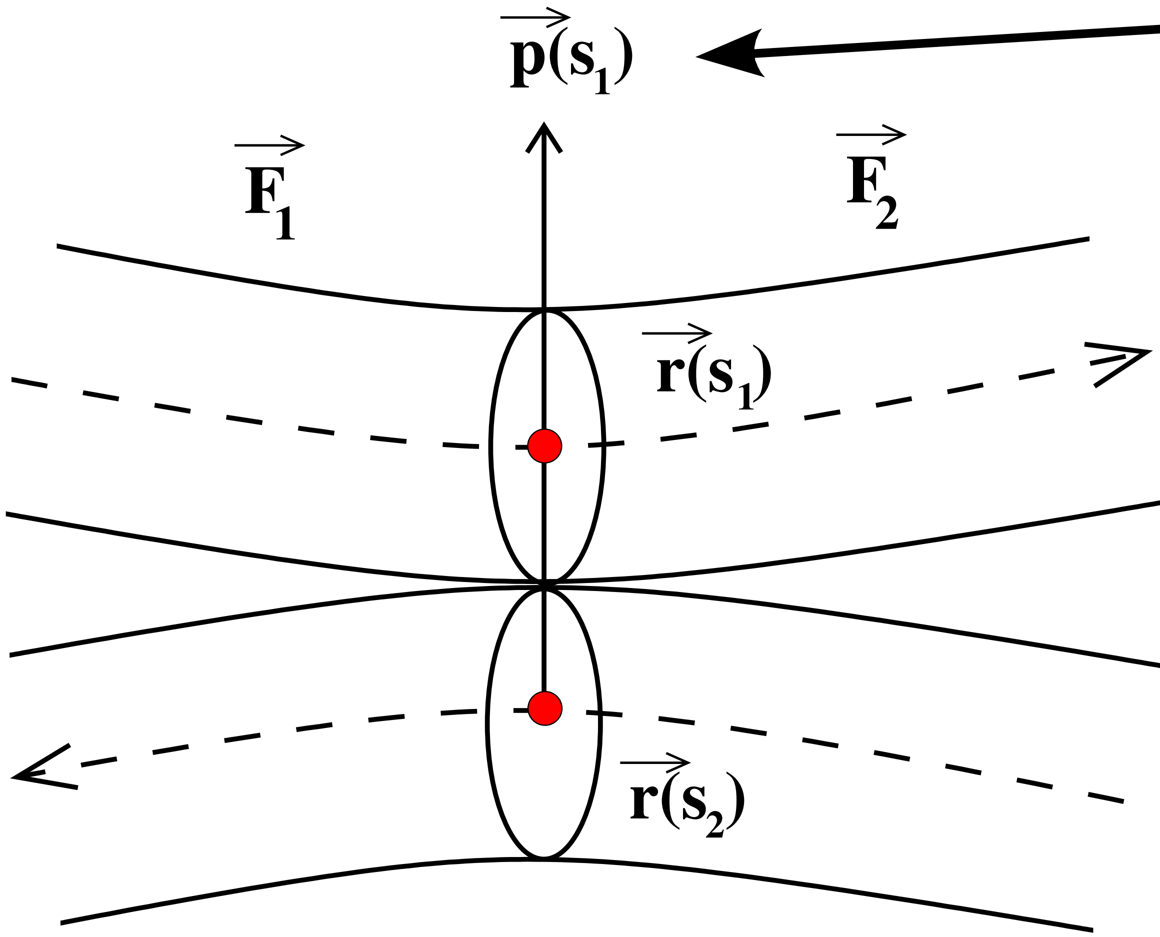
$n = 4.3$ turns

Bifurcation : 0 contact -> 1 contact



Hard-wall contact, no friction

force from strand at s_2 acting on strand at s_1



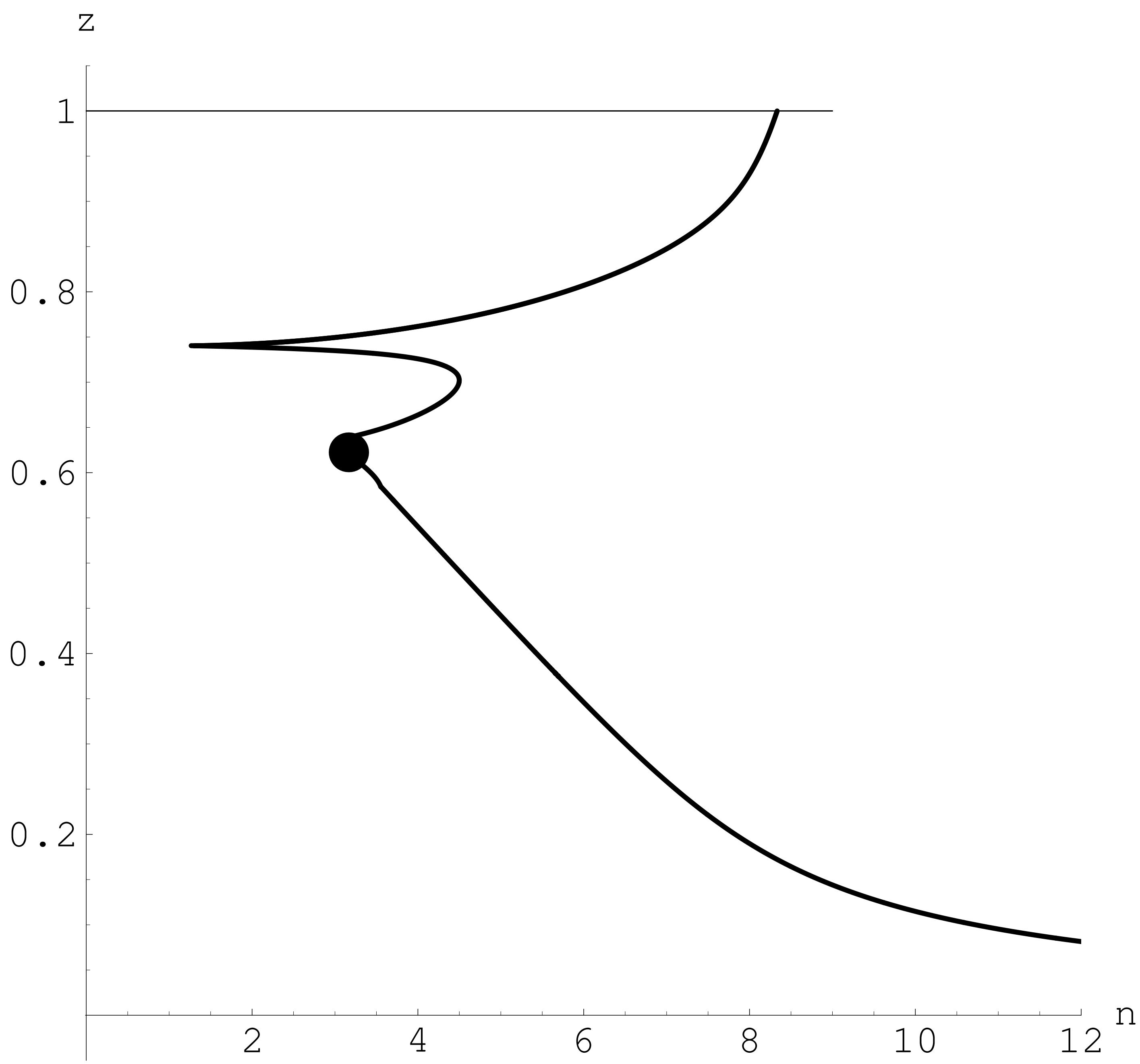
$$\vec{F}_1 = \vec{p} + \vec{F}_2$$

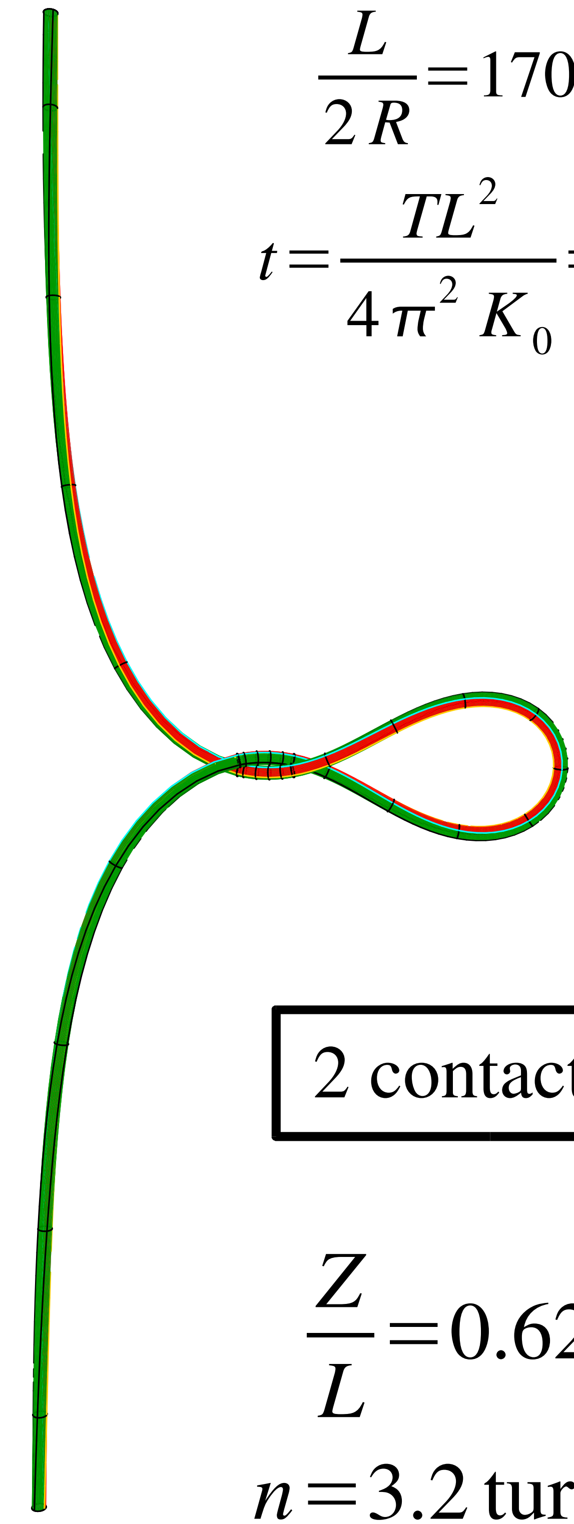
$$\vec{p} = p\,\frac{\vec{r}(s_1) - \vec{r}(s_2)}{|\vec{r}(s_1) - \vec{r}(s_2)|}$$

touching conditions :

$$\begin{cases} |\vec{r}(s_1) - \vec{r}(s_2)| = \text{thickness} \\ (\vec{r}(s_1) - \vec{r}(s_2)) \perp \vec{d}_3(s_1) \\ (\vec{r}(s_1) - \vec{r}(s_2)) \perp \vec{d}_3(s_2) \end{cases}$$

Results : how a twisted rod coils





$$\frac{L}{2R} = 170$$

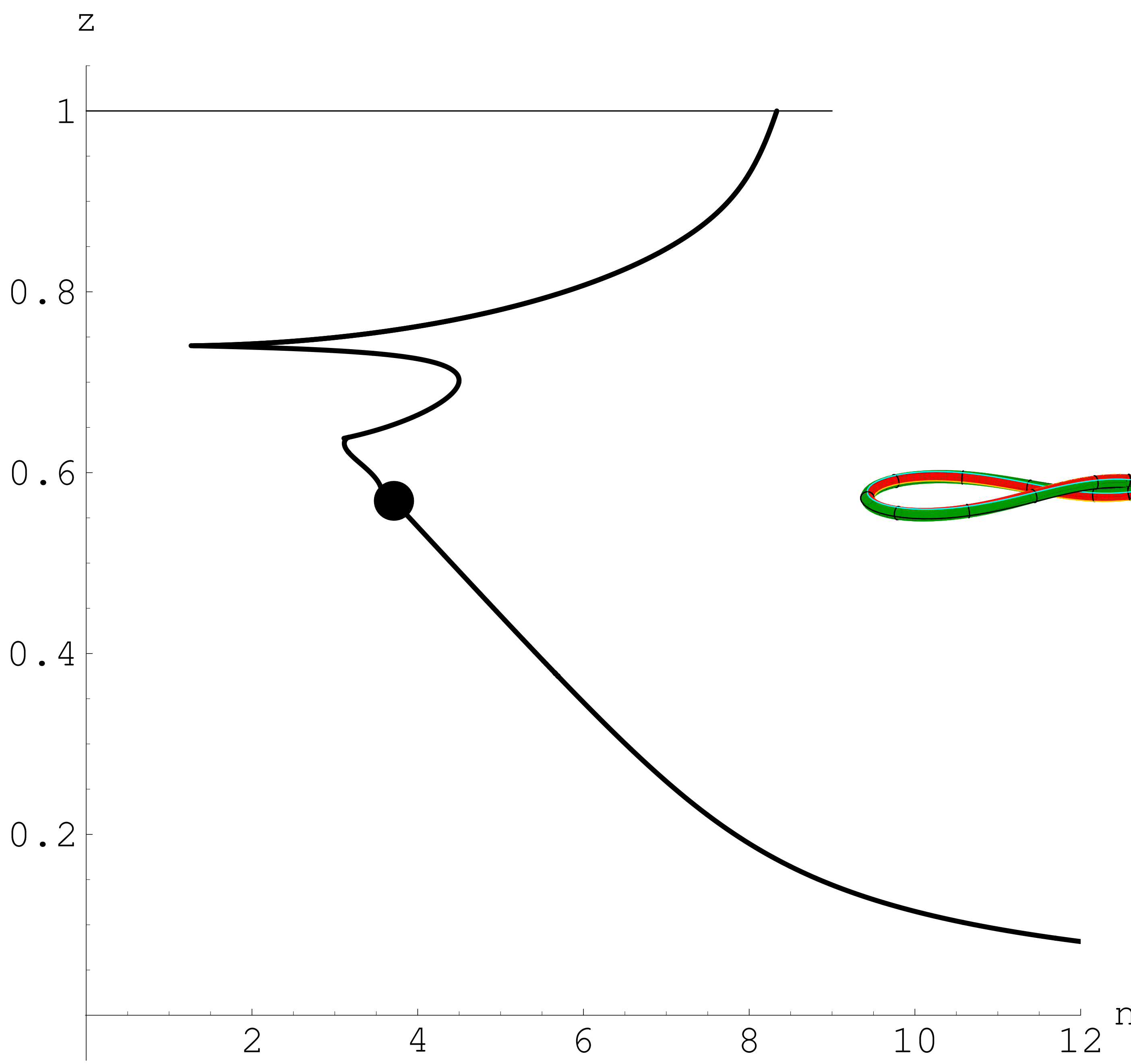
$$t = \frac{TL^2}{4\pi^2 K_0} = 6$$

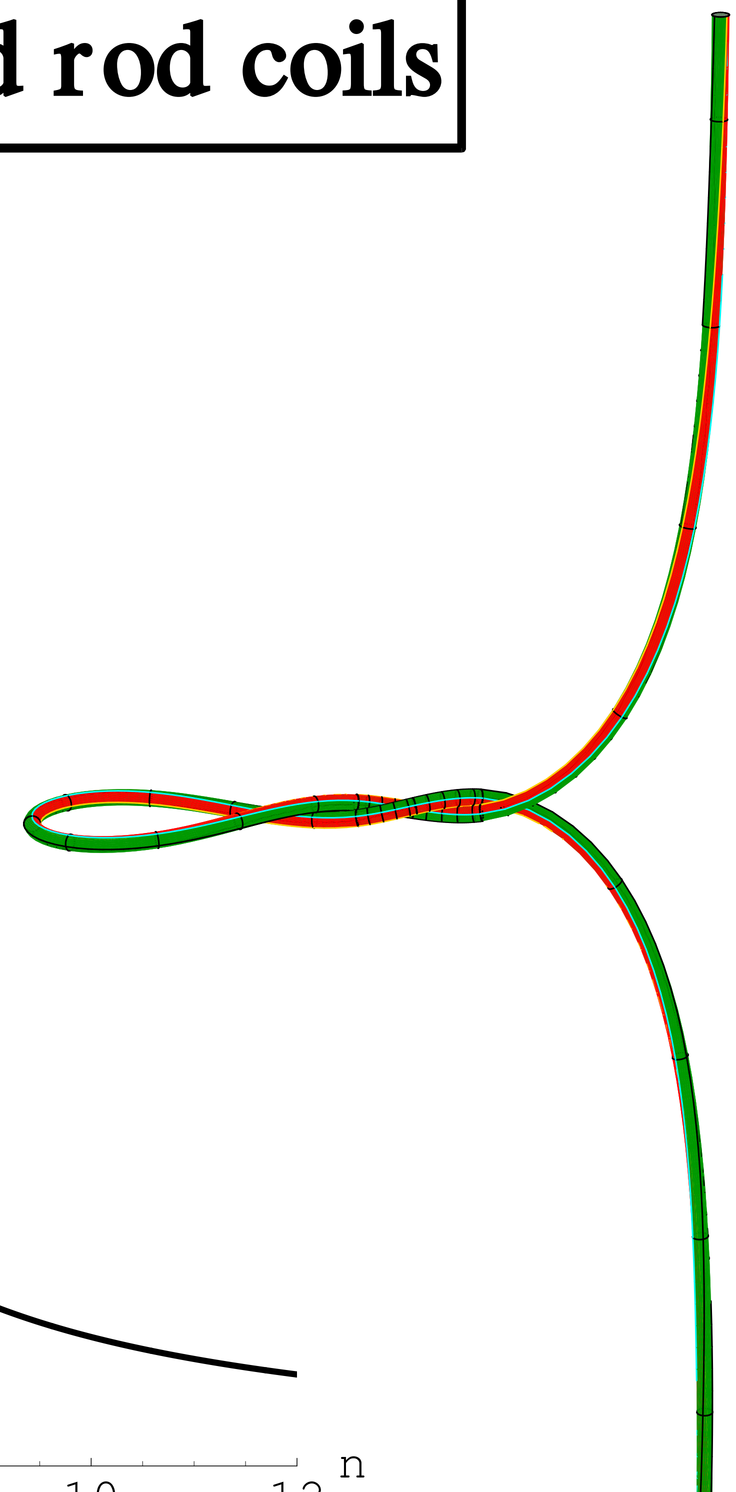
2 contact(s)

$$\frac{Z}{L} = 0.62$$

$n = 3.2$ turns

Results : how a twisted rod coils





$$\frac{L}{2R} = 170$$

$$t = \frac{TL^2}{4\pi^2 K_0} = 6$$

3 contact(s)

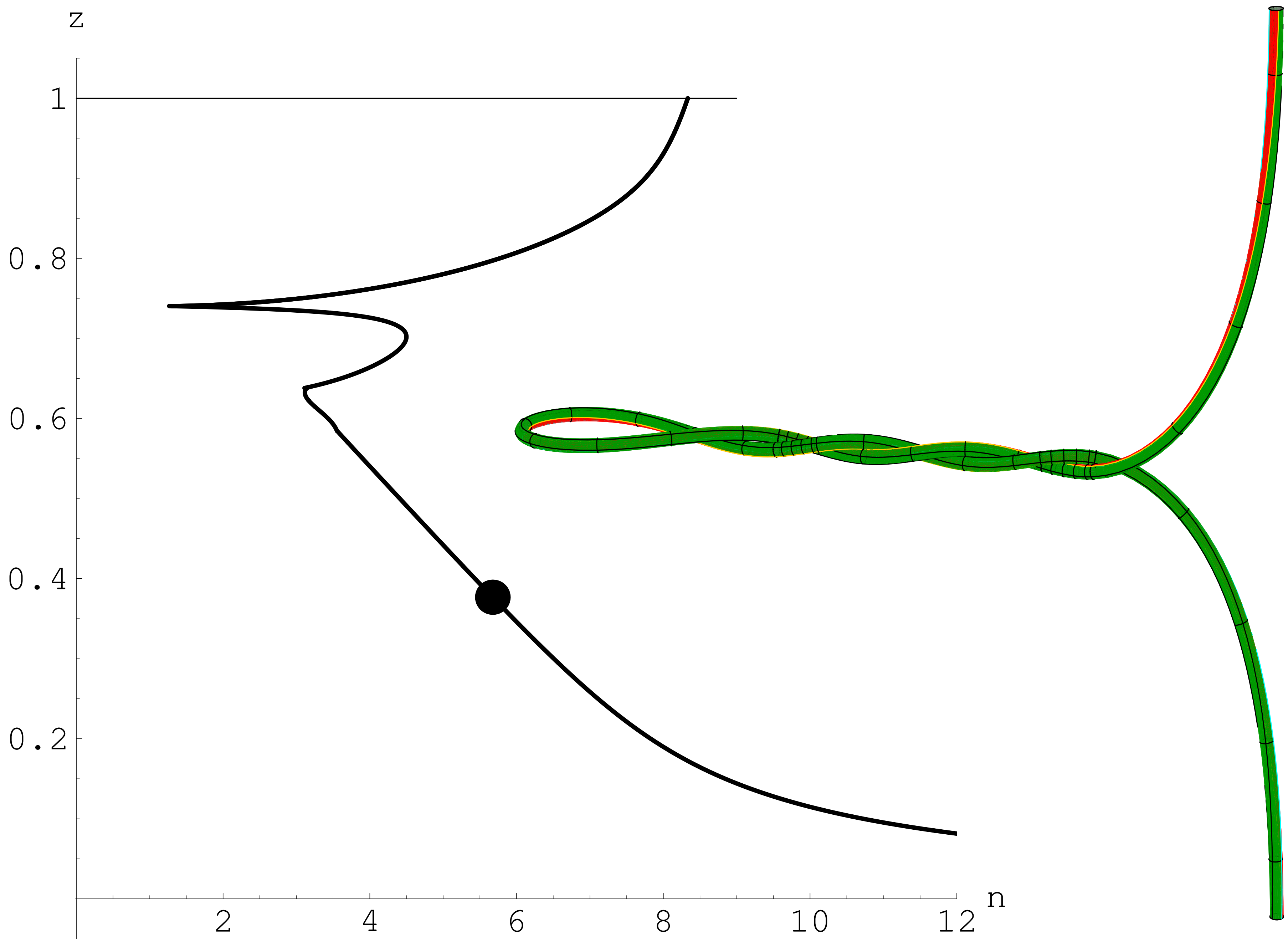
$$\frac{Z}{L} = 0.57$$

$n = 3.7$ turns

Results : how a twisted rod coils

$$\frac{L}{2R} = 170$$

$$t = \frac{TL^2}{4\pi^2 K_0} = 6$$

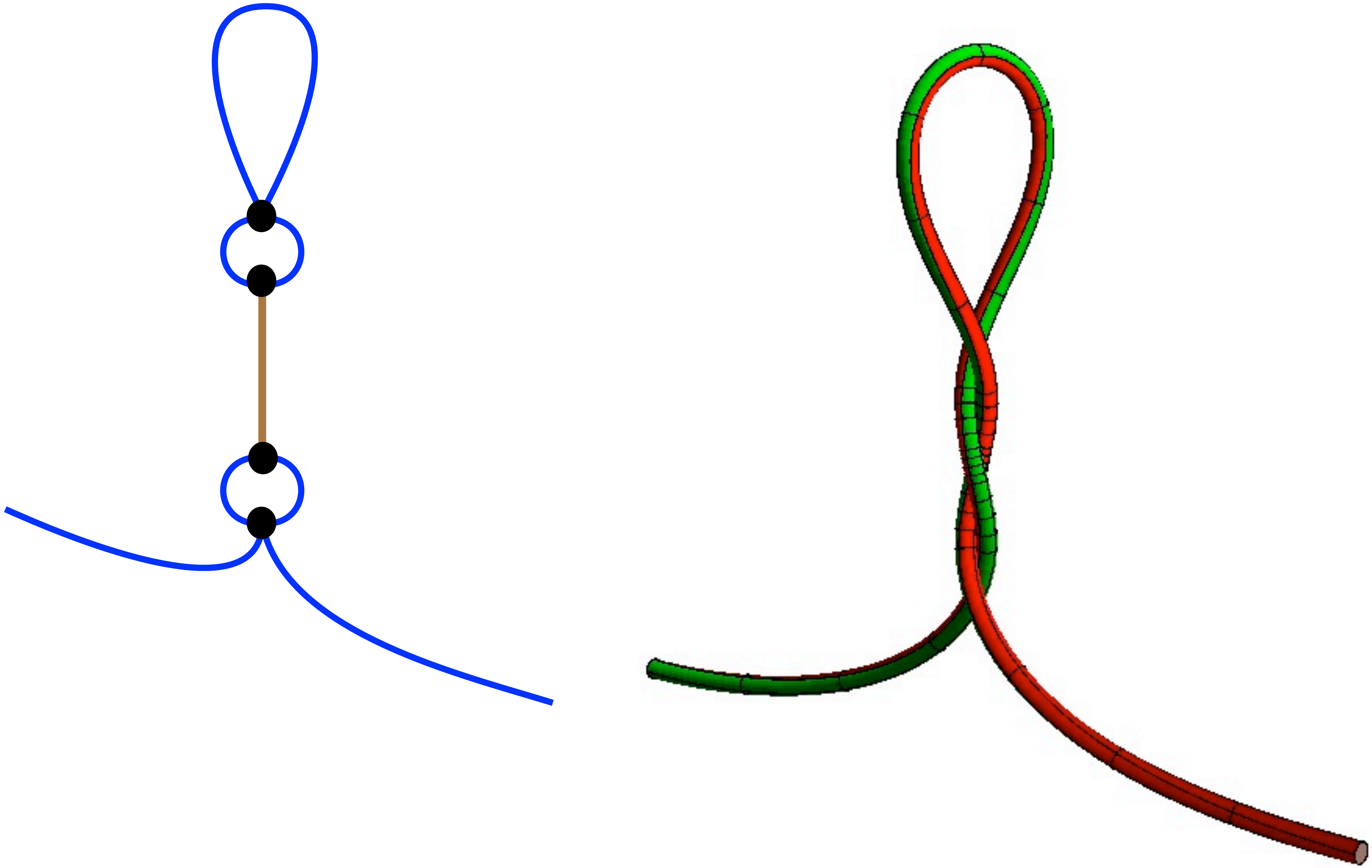


1L1 contact(s)

$$\frac{Z}{L} = 0.38$$

$n = 5.7$ turns

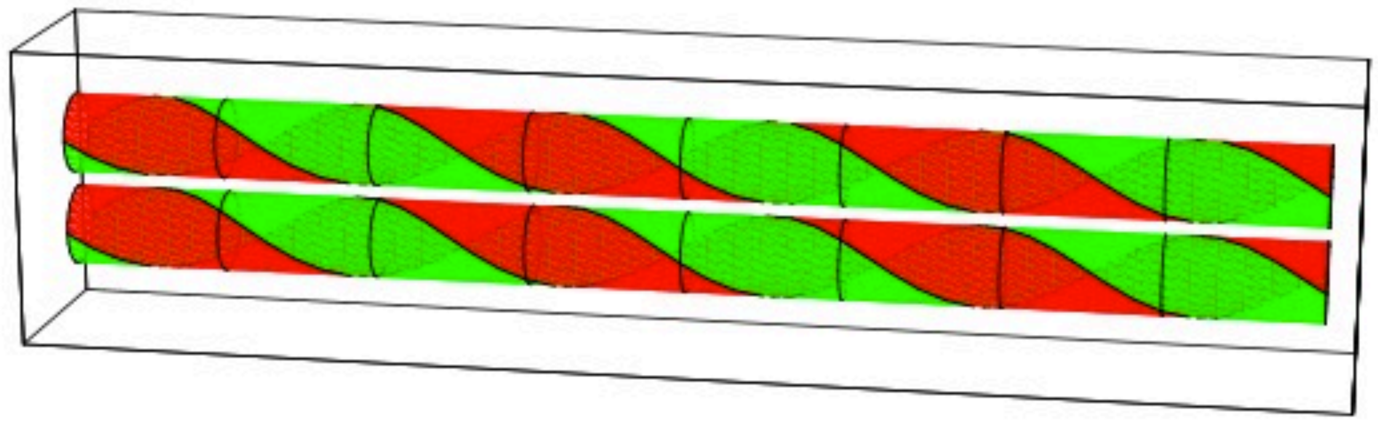
Self-contact topology



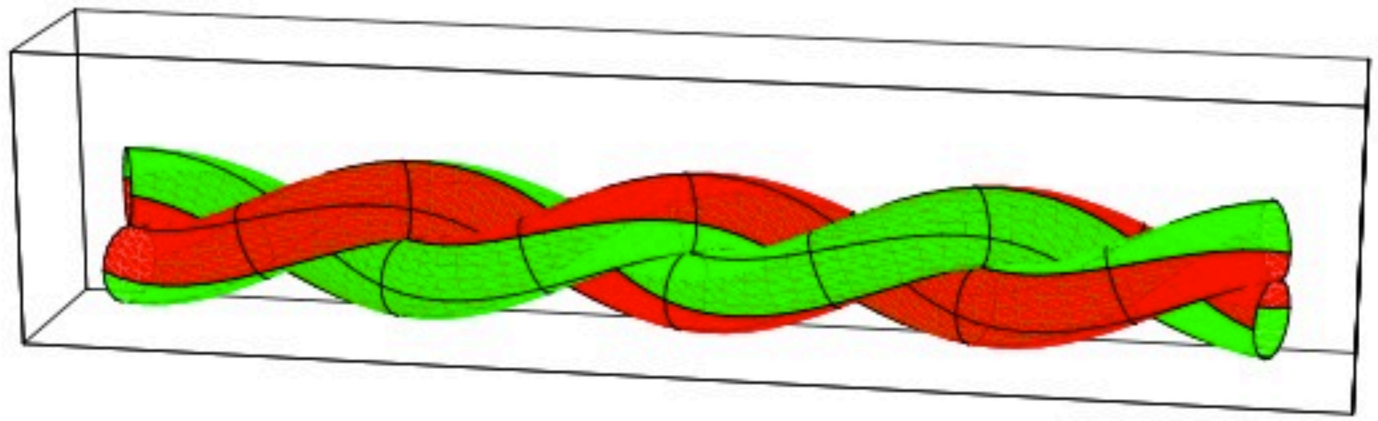
Filaments coiled in helical structures

Previous work : Fraser & Stump (1998) , Coleman & Swigon (2000)

Lk = 6
Tw = 6
Wr = 0



Lk = 6
Tw = 2.35
Wr = 3.65

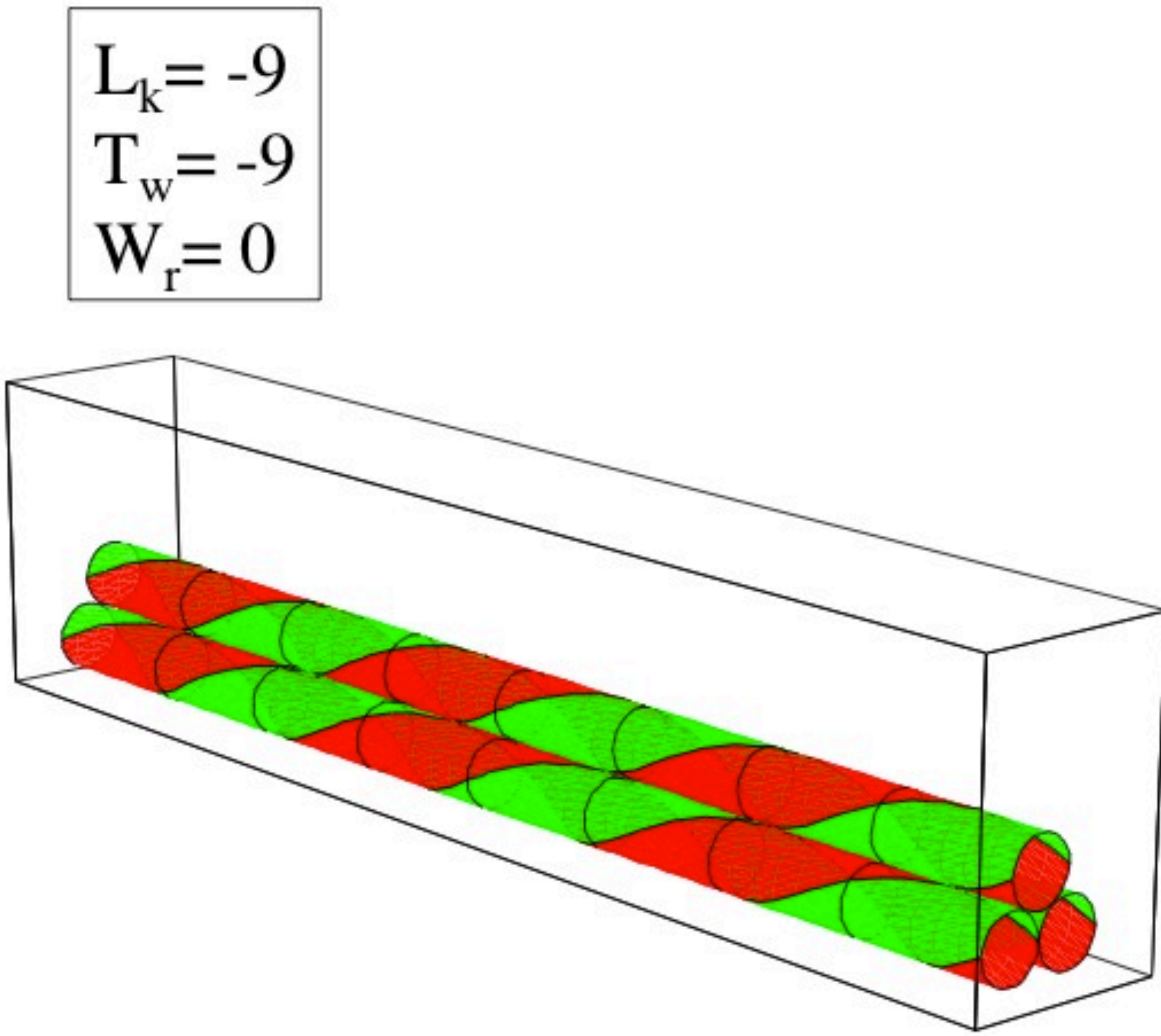


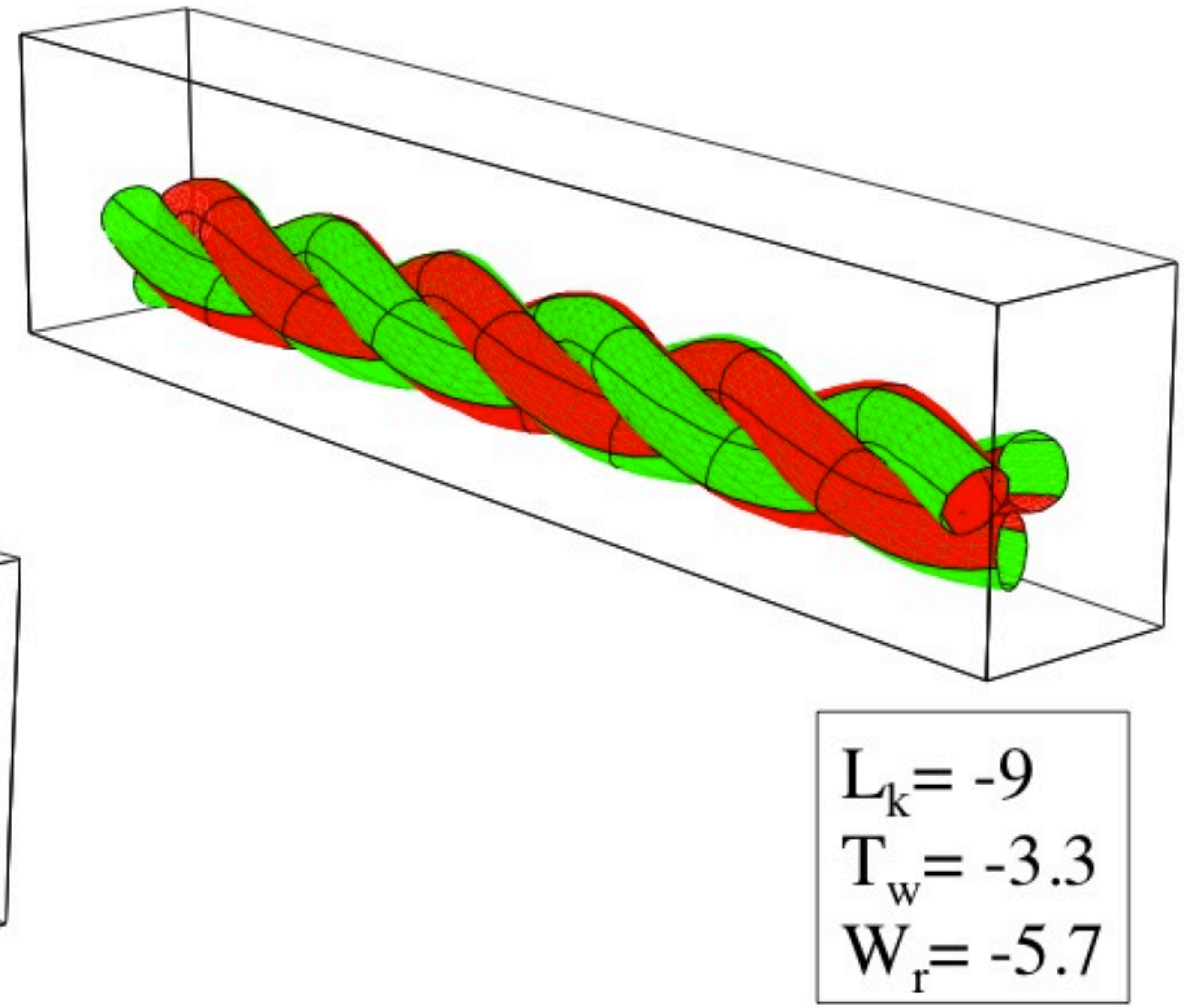
$$2\ K_0\, n \sin^3\theta \cos\theta + \epsilon\, n\, K_3\, R\tau \cos 2\theta + R^2\, F \sin\theta - \epsilon\, R\, M \cos\theta = 0$$

$$p R^3 = \frac{\sin^2\theta}{\cos 2\theta}\left(K_0 \sin^2\theta + \frac{R^2 F}{n}\cos\theta - \epsilon\frac{R M}{n}\sin\theta\right)$$

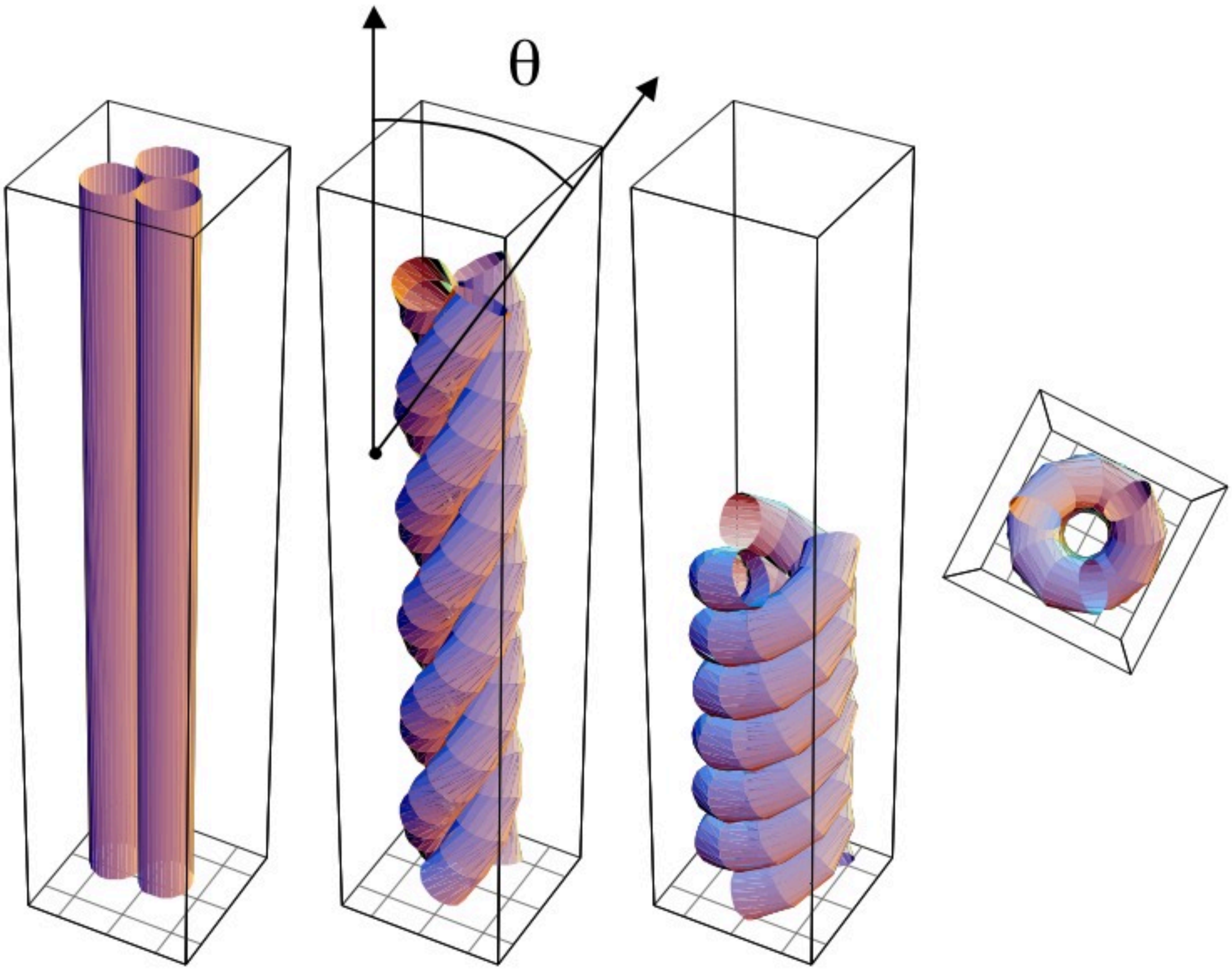
ε=+-1 : handedness
n : nb of strands
F, M : external stress

Equilibrium of plies





Equilibrium of plies



Equilibrium of plies

Minimize $$V = \frac{1}{2}\int_0^L \kappa^2(s)ds + \frac{1}{2}\int_0^L \tau^2(s)ds$$

subject to the constraint:
rod has to lie on the cylinder

Equations for:

the coiling angle $\theta(s)$
the contact pressure $p(s)$

Equilibrium of plies

First way to obtain equilibrium equations

Variational approach:

$$V[\theta, \theta', \phi, \phi'] = \int_0^L W(\theta(s), \theta'(s), \phi(s), \phi'(s))\; ds$$

$$\left.\begin{aligned} \frac{\partial W}{\partial \theta} &= \frac{d}{ds}\frac{\partial W}{\partial \theta'} \\ \frac{\partial W}{\partial \phi} &= \frac{d}{ds}\frac{\partial W}{\partial \phi'} \end{aligned}\right\}$$

Euler-Lagrange equations

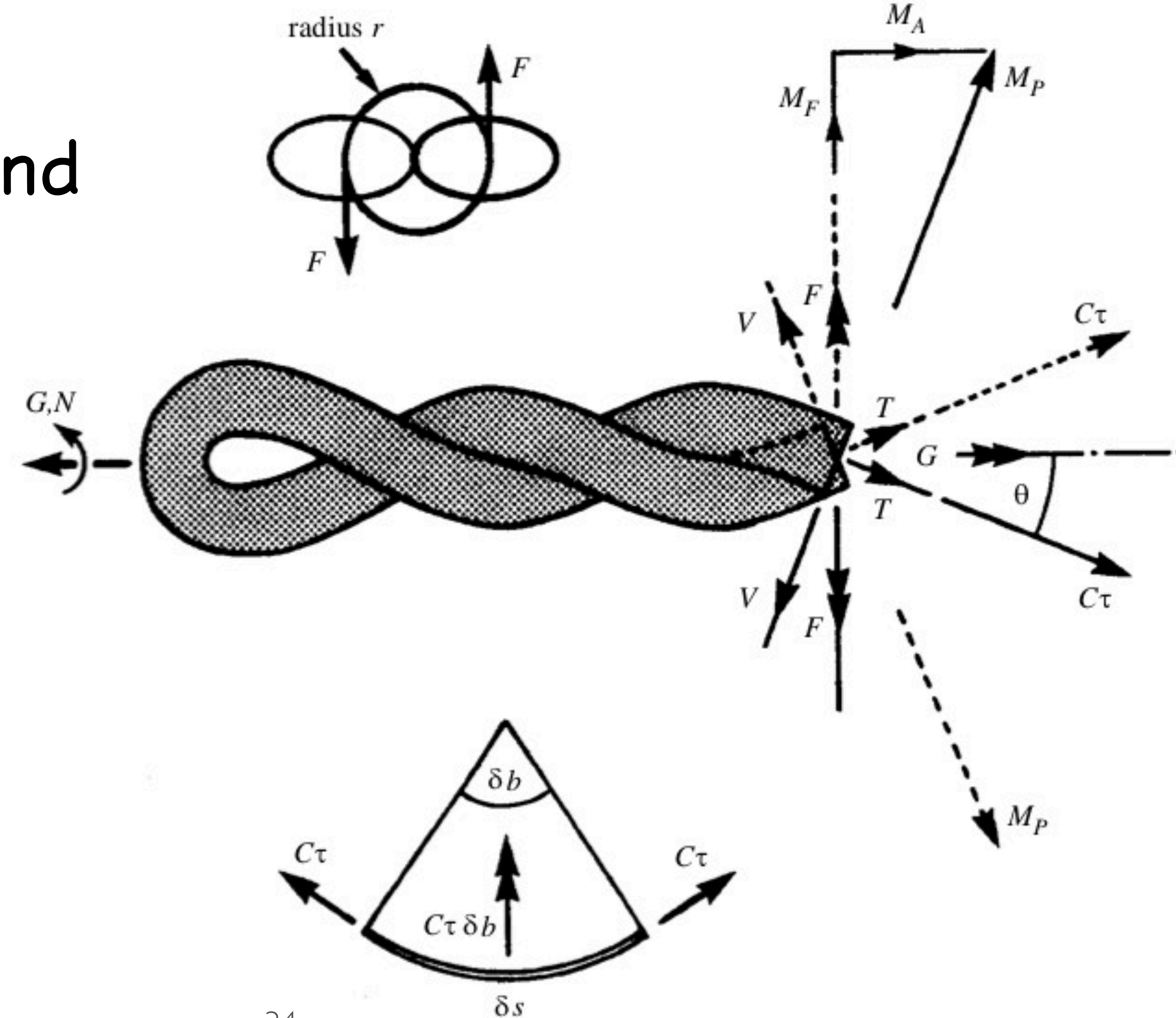
Equilibrium of plies

Second way to obtain equilibrium equations

Forces balance

(1) for each strand

(2) for the ply



Equilibrium of plies

Third way to obtain equilibrium equations

Ansatz (semi-inverse problem)

We impose:

$$r(s) = \begin{pmatrix} +R\sin\psi(s) \\ -R\cos\psi(s) \\ s\,\cos\theta \end{pmatrix}$$

$$\psi(s) = \frac{\sin\theta}{R}\, s$$

→ Kirchhoff equations →

We obtain:

$$\varphi(\theta, R, F_0, M_0) = 0$$

$\vec{F}(s)$ force

$\vec{M}(s)$ moment

contact conditions

$$\Delta(\theta, x) := 2 + x^2 \cos^2\theta - 2\cos\left(x\sin\theta - \frac{2\pi}{n}\right) = \frac{4}{\rho^2}$$

$$x\cos^2\theta + \sin\theta\sin\left(x\sin\theta - \frac{2\pi}{n}\right) = 0,$$

where

$$x = \frac{\varepsilon(s_1 - s_2)}{R}, \qquad \rho = \frac{R}{r}.$$

$$\gamma = \frac{C}{B}, \qquad f = \frac{r^2 F_0}{B}, \qquad m = \frac{r M_0}{B}$$

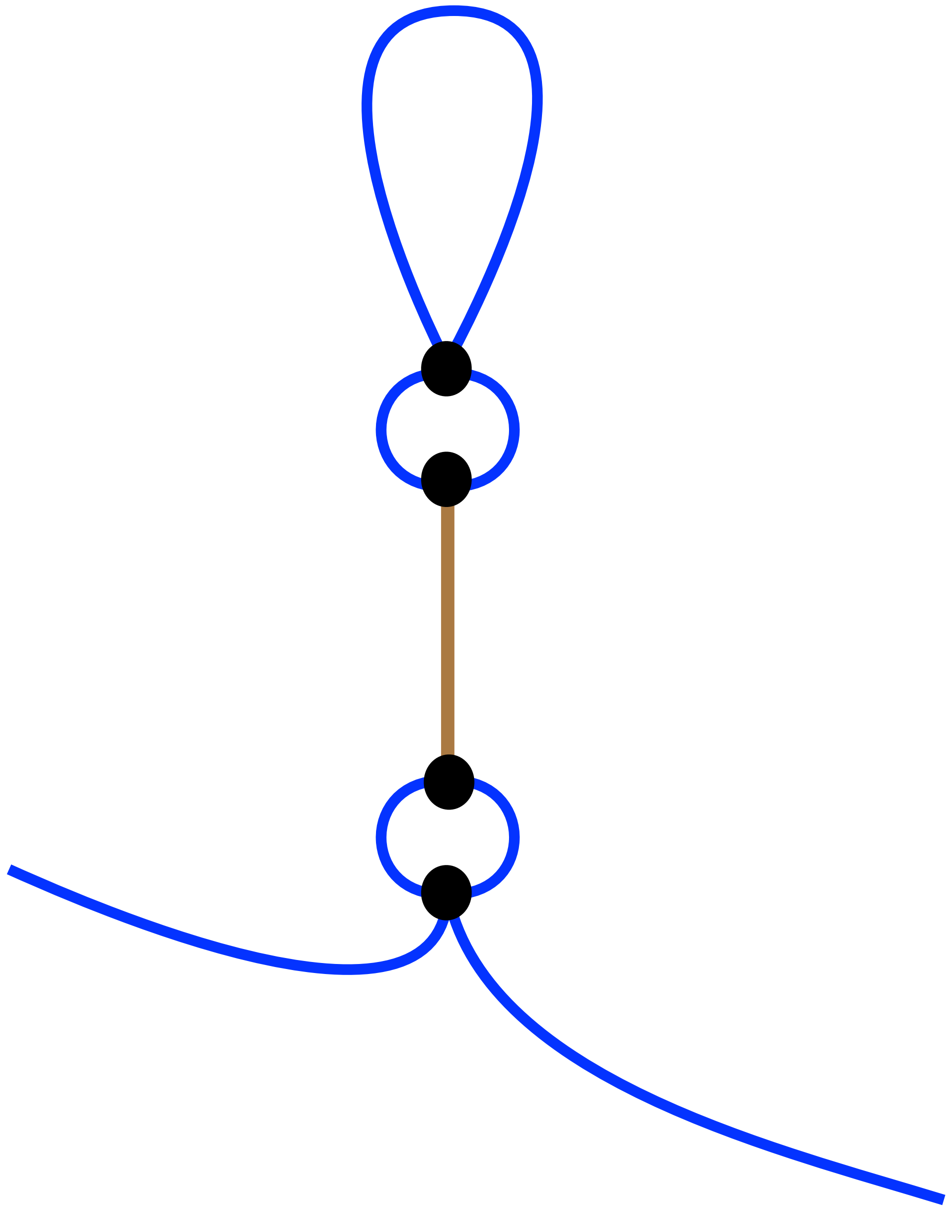
external loads

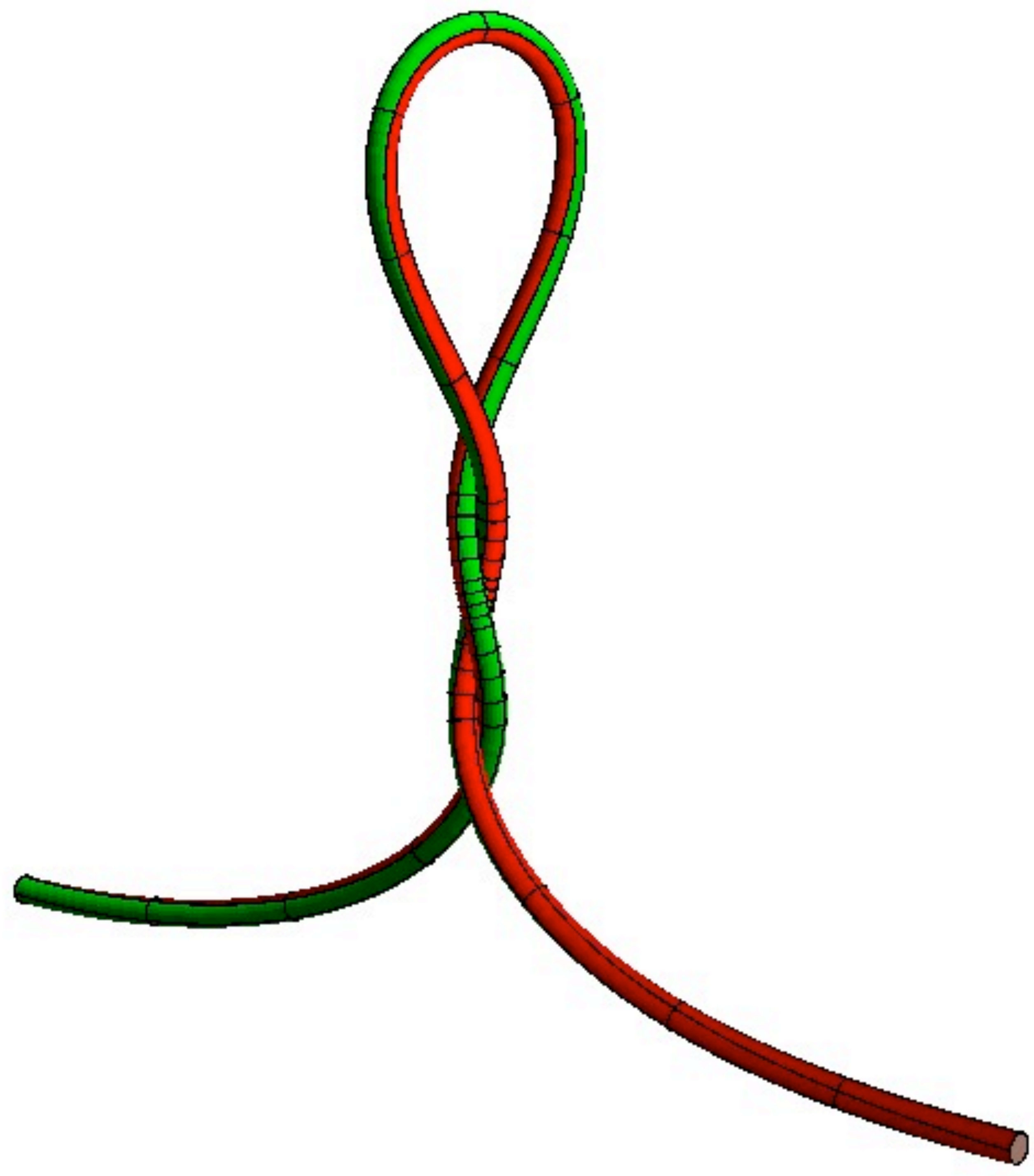
equilibrium

$$2n\sin^3\theta\cos\theta + \varepsilon n\rho\gamma r u_3 \cos 2\theta + \rho^2 f \sin\theta - \varepsilon\rho m\cos\theta = 0.$$

$$\frac{pr^3}{B} = \frac{\sin^2\theta}{n\rho^3\cos 2\theta}(n\sin^2\theta + \rho^2 f\cos\theta - \varepsilon\rho m\sin\theta).$$

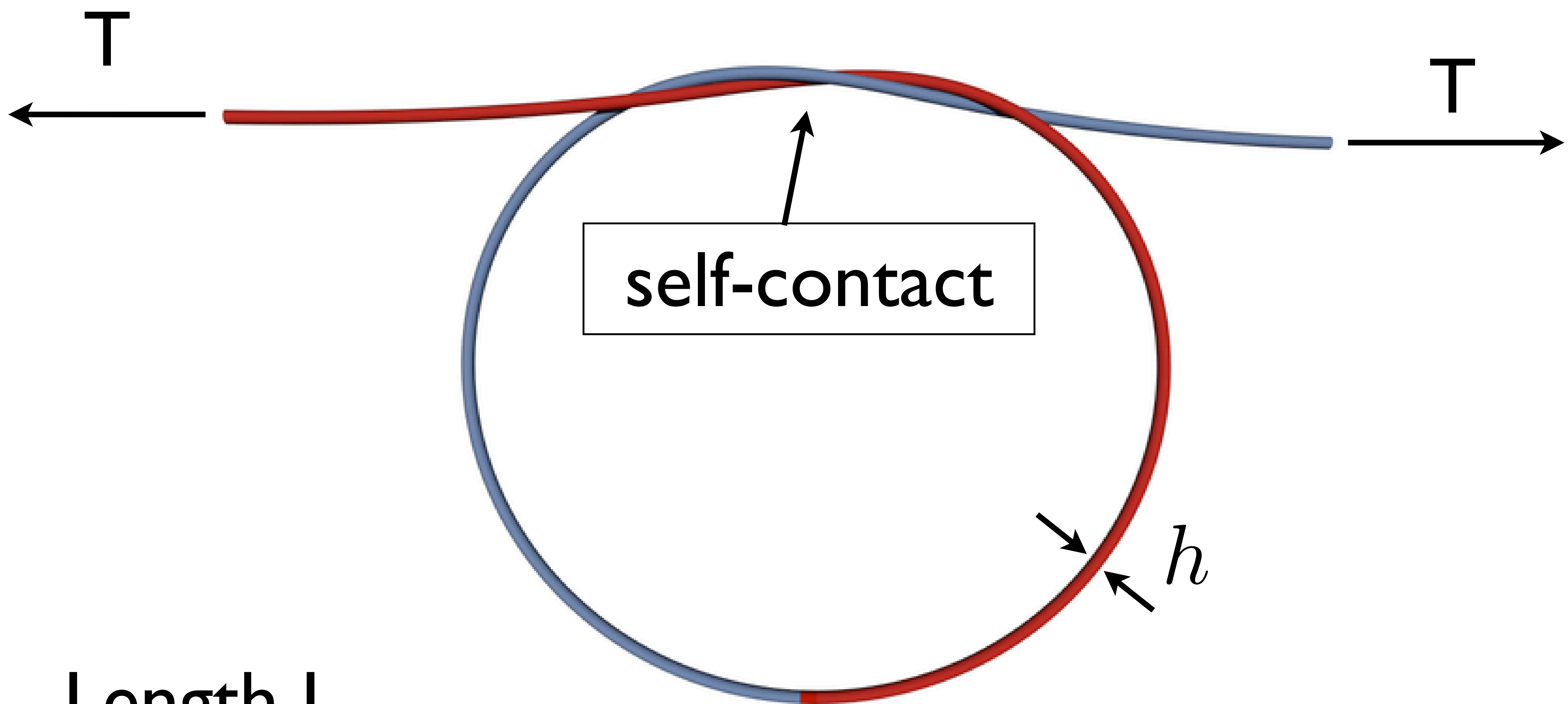
Self-contact topology





2nd `experiment'

Elastic knots



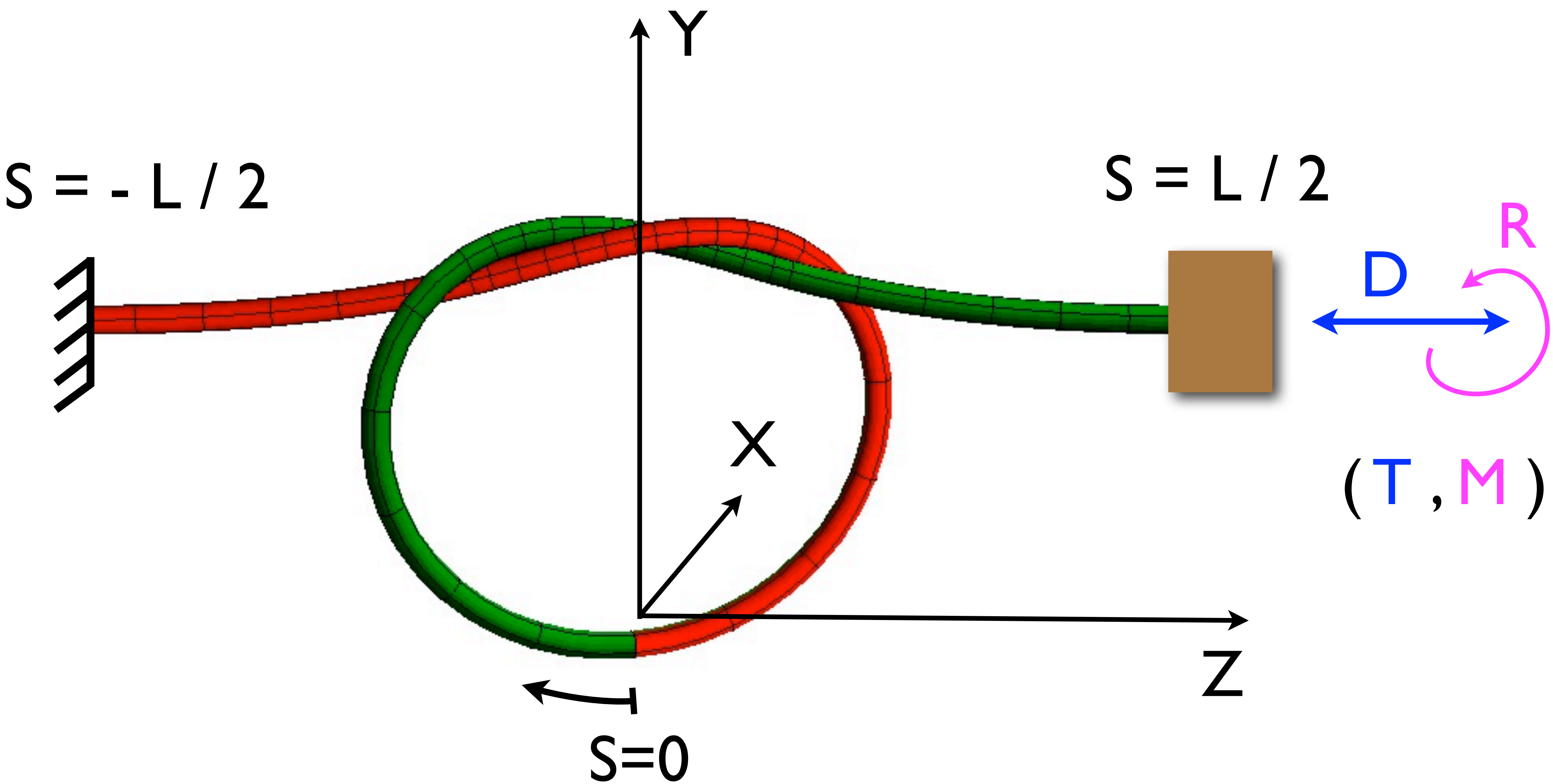
- Length L
- Circular cross-section: radius h
- Bending rigidity : E I
- Twist rigidity : G J

$$I = \frac{\pi h^4}{4}$$

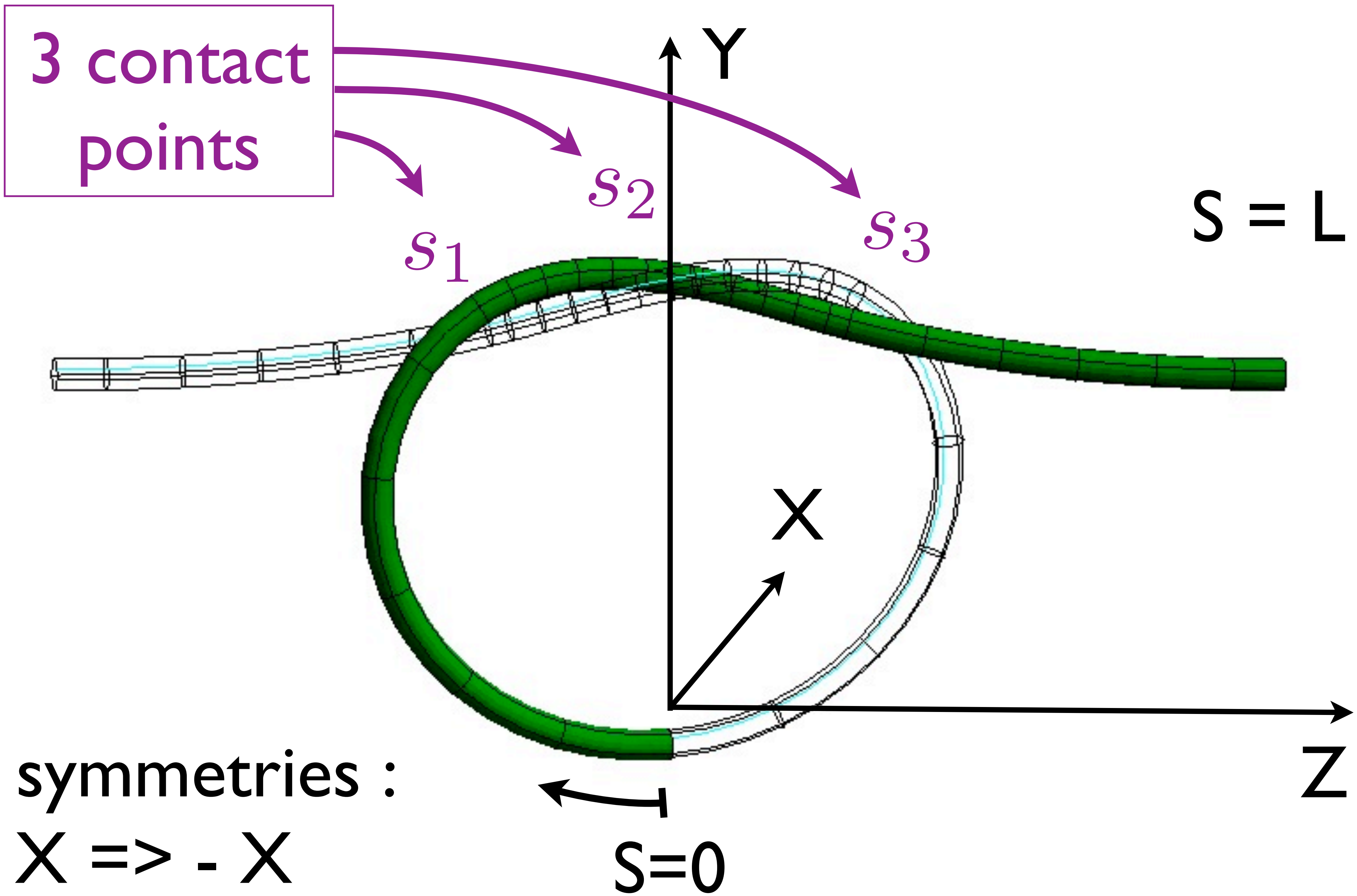
$$J = \frac{\pi h^4}{2}$$

E : Young's modulus G : shear modulus

Boundary value problem



Boundary value problem



S = L / 2

symmetries :

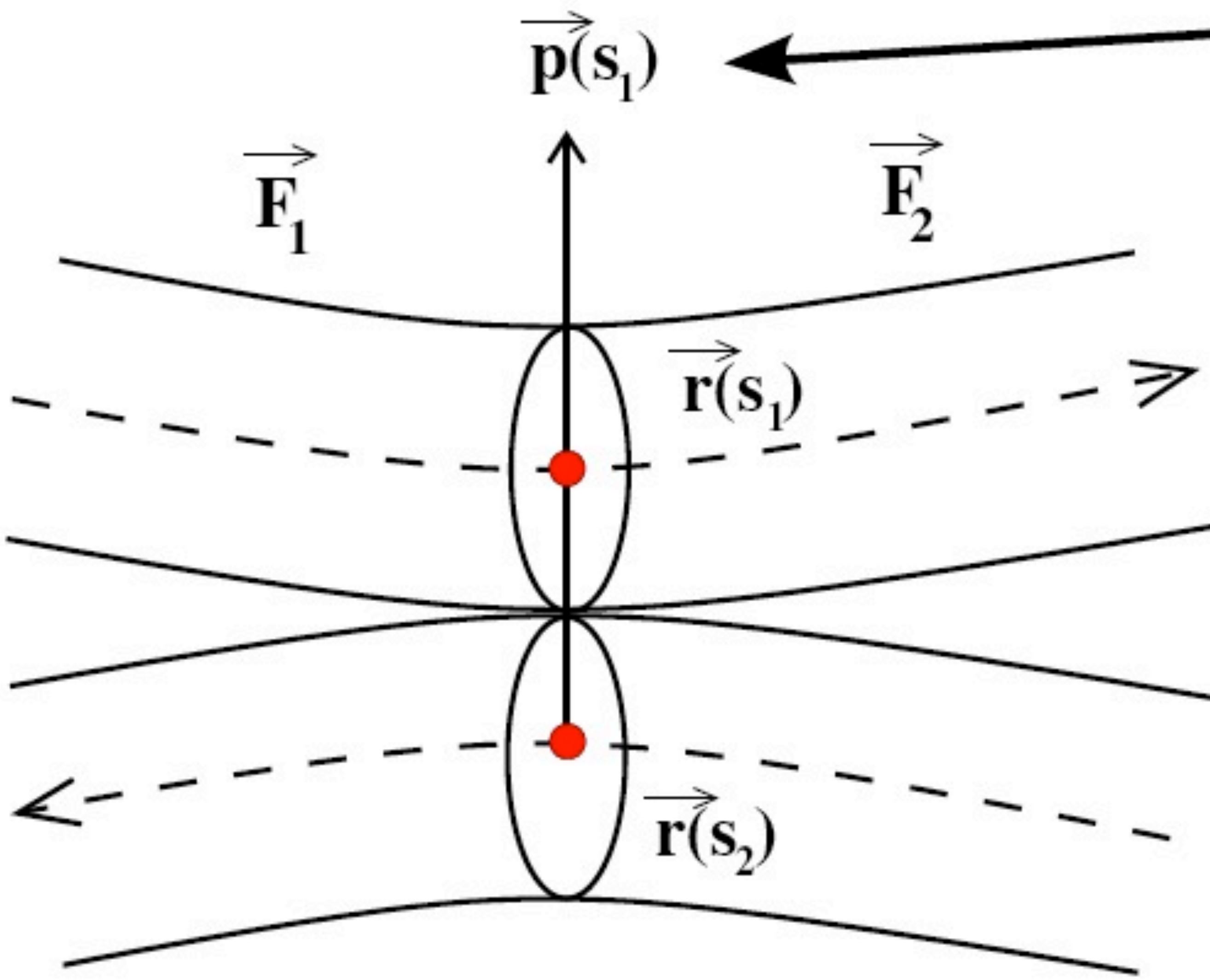
X => - X

Y => Y

Z => - Z

- Shooting method (Mathematica)
- Gauss colocation (AUTO)

Hard-wall contact, no friction



force from strand at s_2 acting on strand at s_1

$$\vec{F}_1 = \vec{p} + \vec{F}_2$$

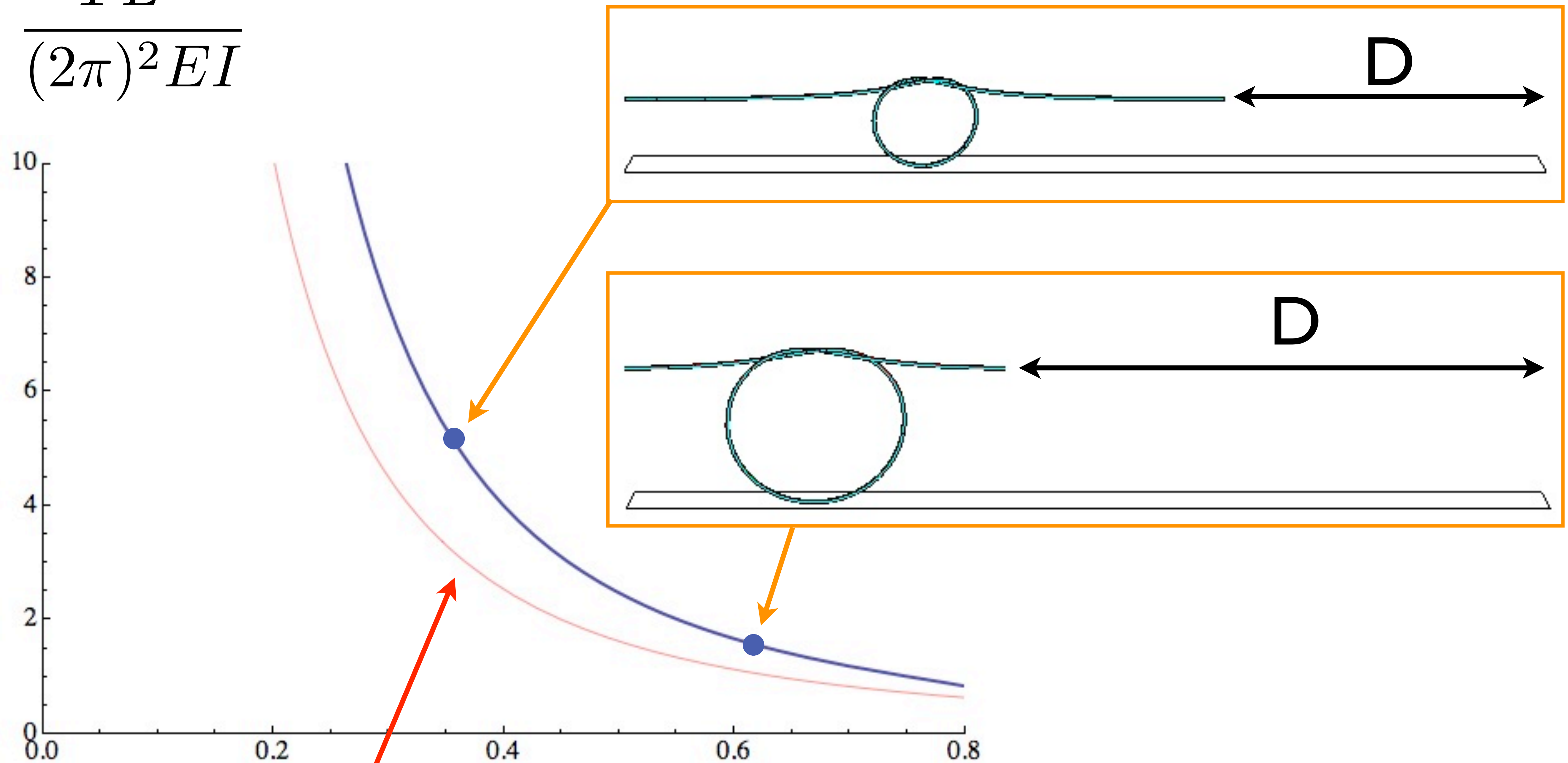
$$\vec{p} = p \frac{\vec{r}(s_1) - \vec{r}(s_2)}{|\vec{r}(s_1) - \vec{r}(s_2)|}$$

touching conditions :

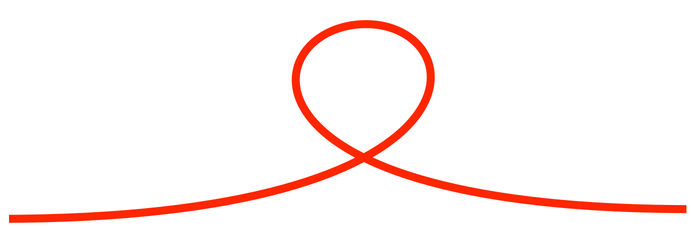
$$\begin{cases} |\vec{r}(s_1) - \vec{r}(s_2)| = \text{thickness} \\ (\vec{r}(s_1) - \vec{r}(s_2)) \perp \vec{d}_3(s_1) \\ (\vec{r}(s_1) - \vec{r}(s_2)) \perp \vec{d}_3(s_2) \end{cases}$$

Numerical Path Following : Results

$$t = \frac{TL^2}{(2\pi)^2 EI}$$



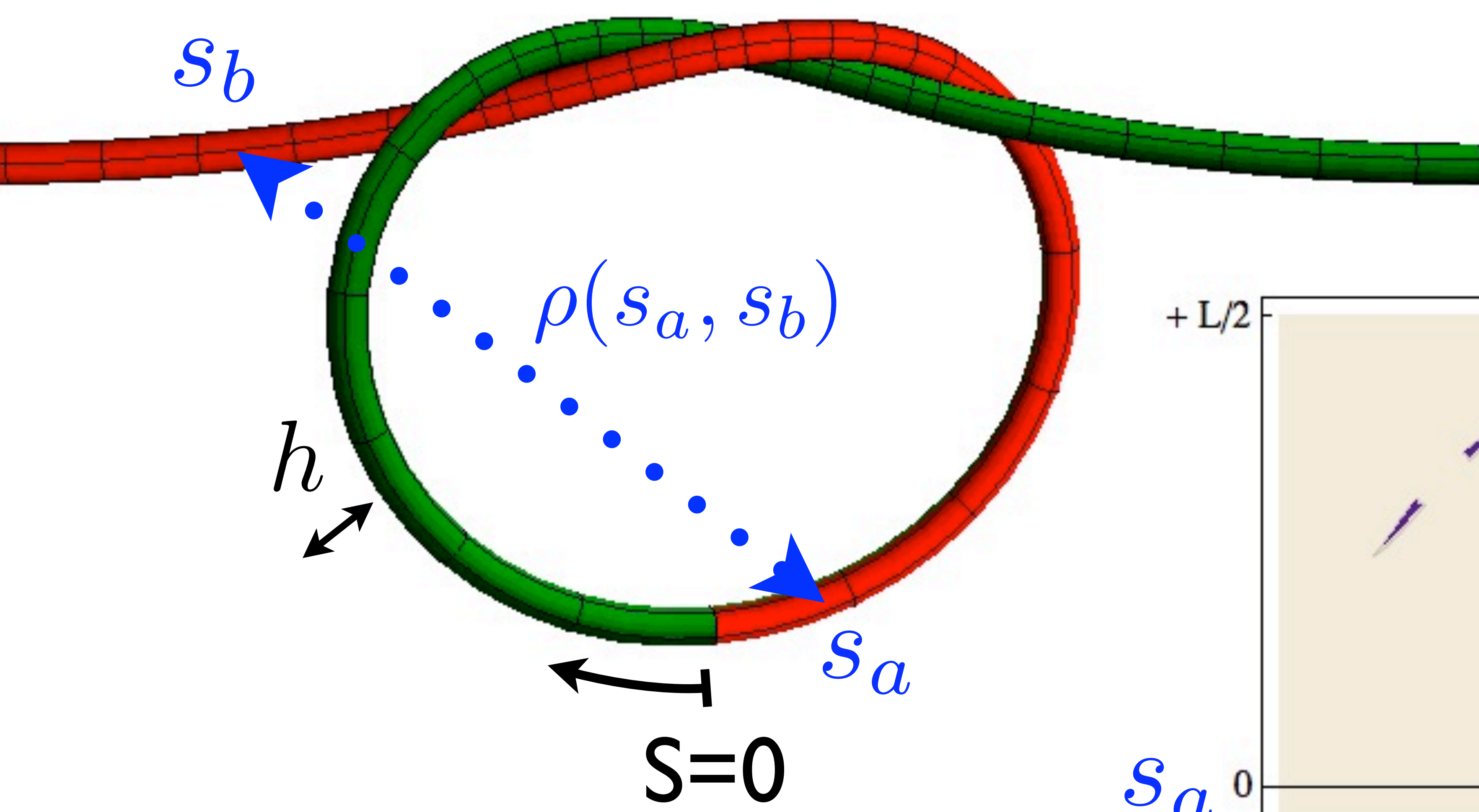
$$d = \frac{D}{L}$$

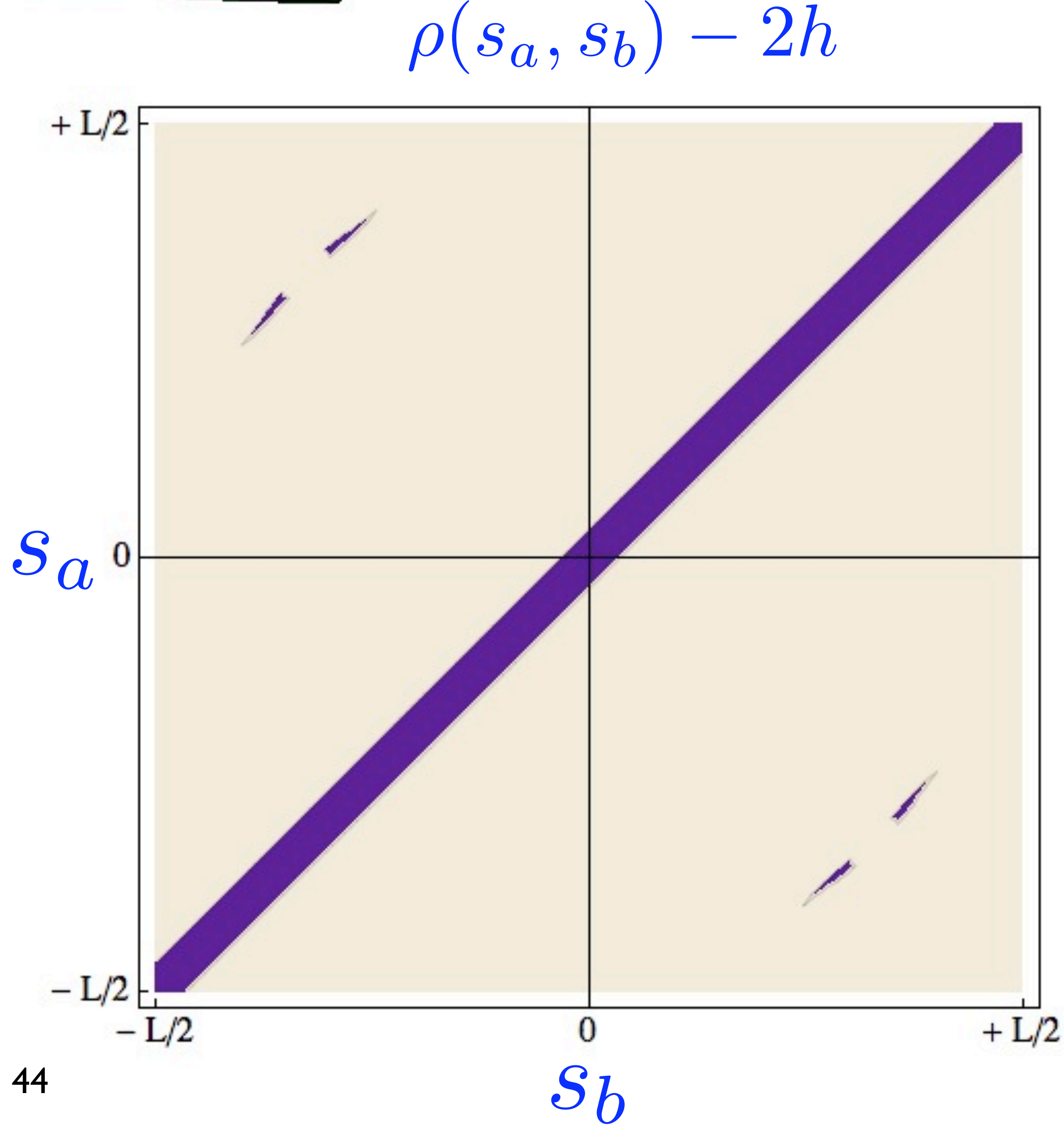


Planar Elastica

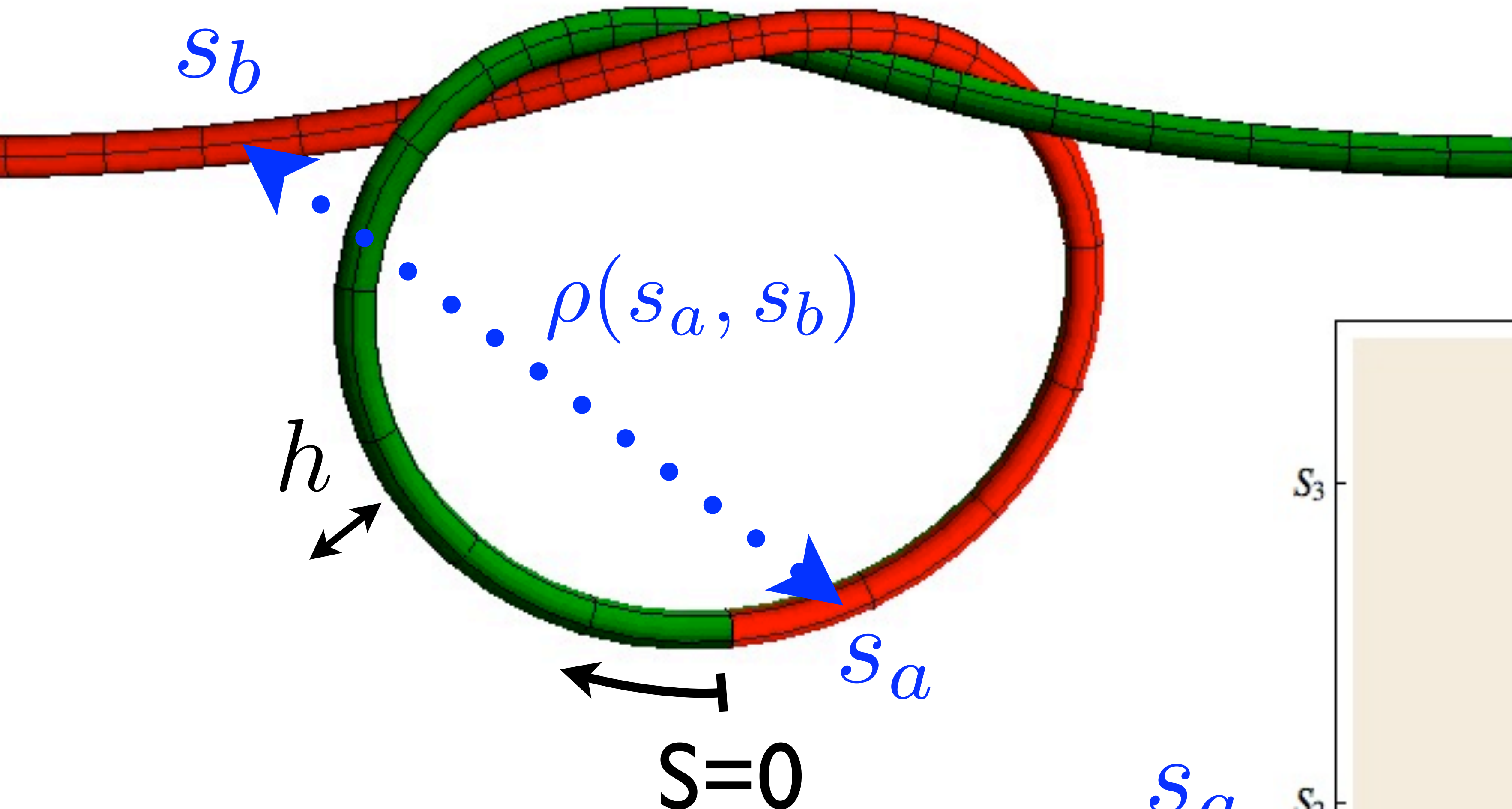
$$t = \left(\frac{2/\pi}{d}\right)^2$$

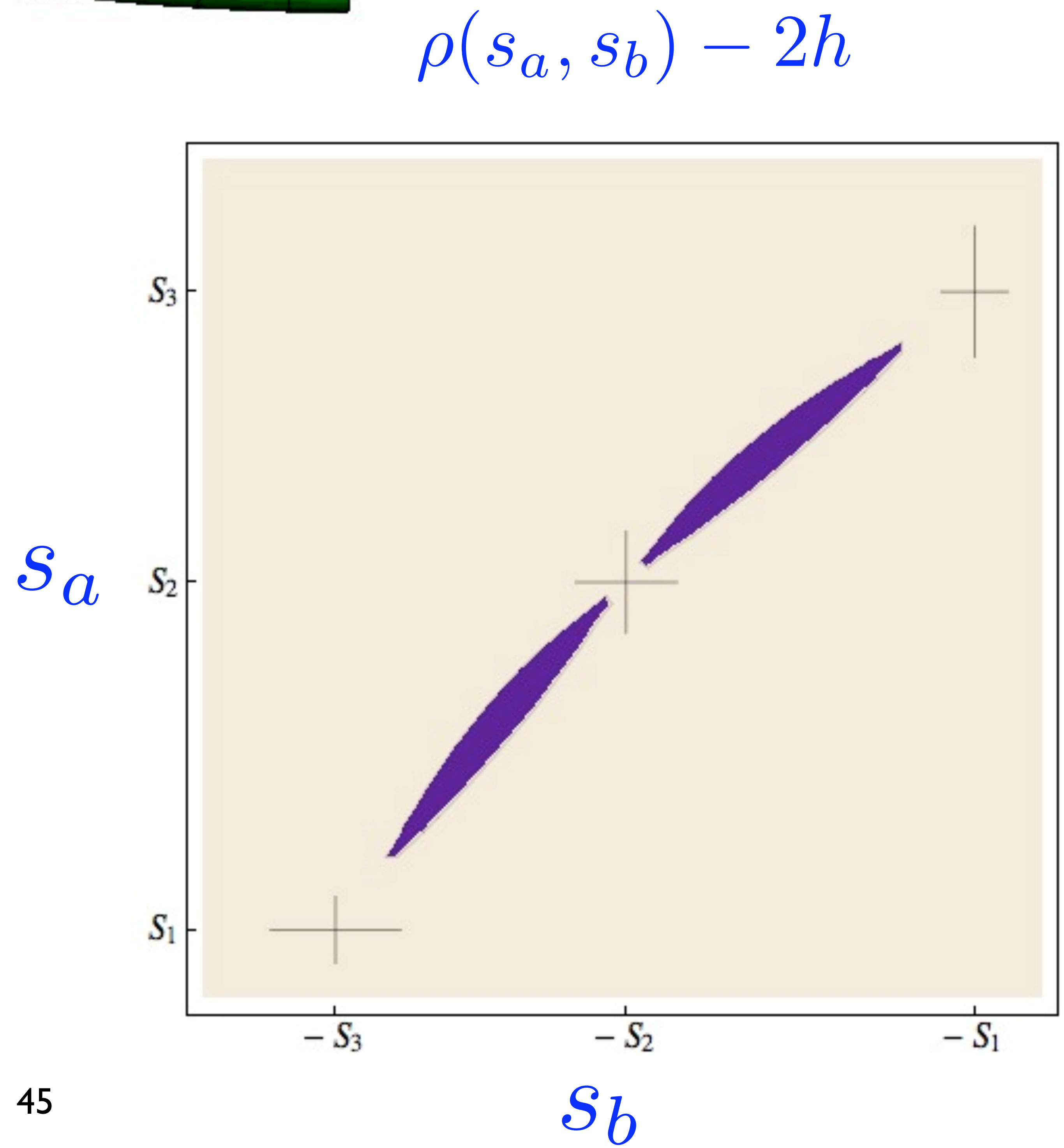
Distance of self-approach





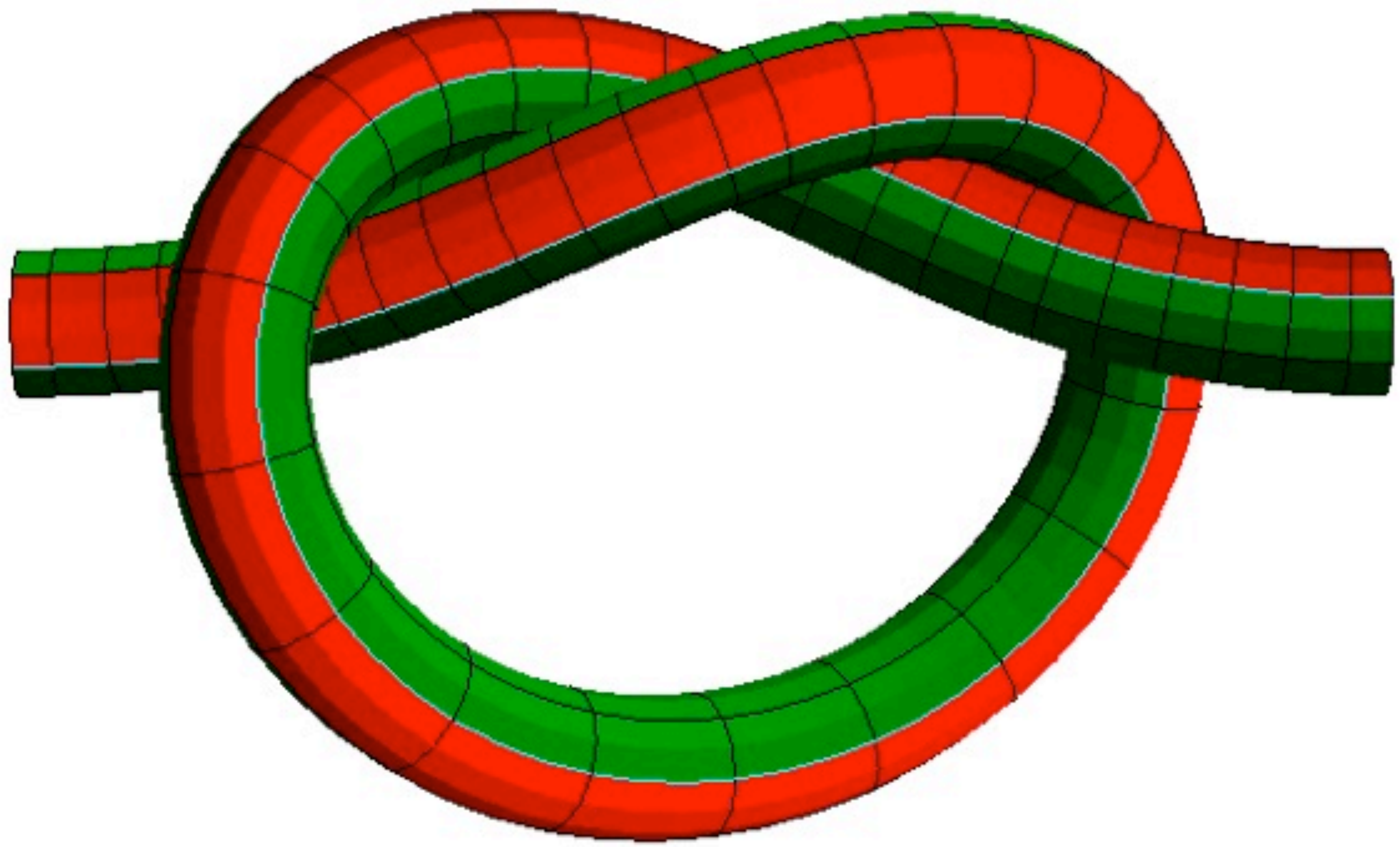
Distance of self-approach



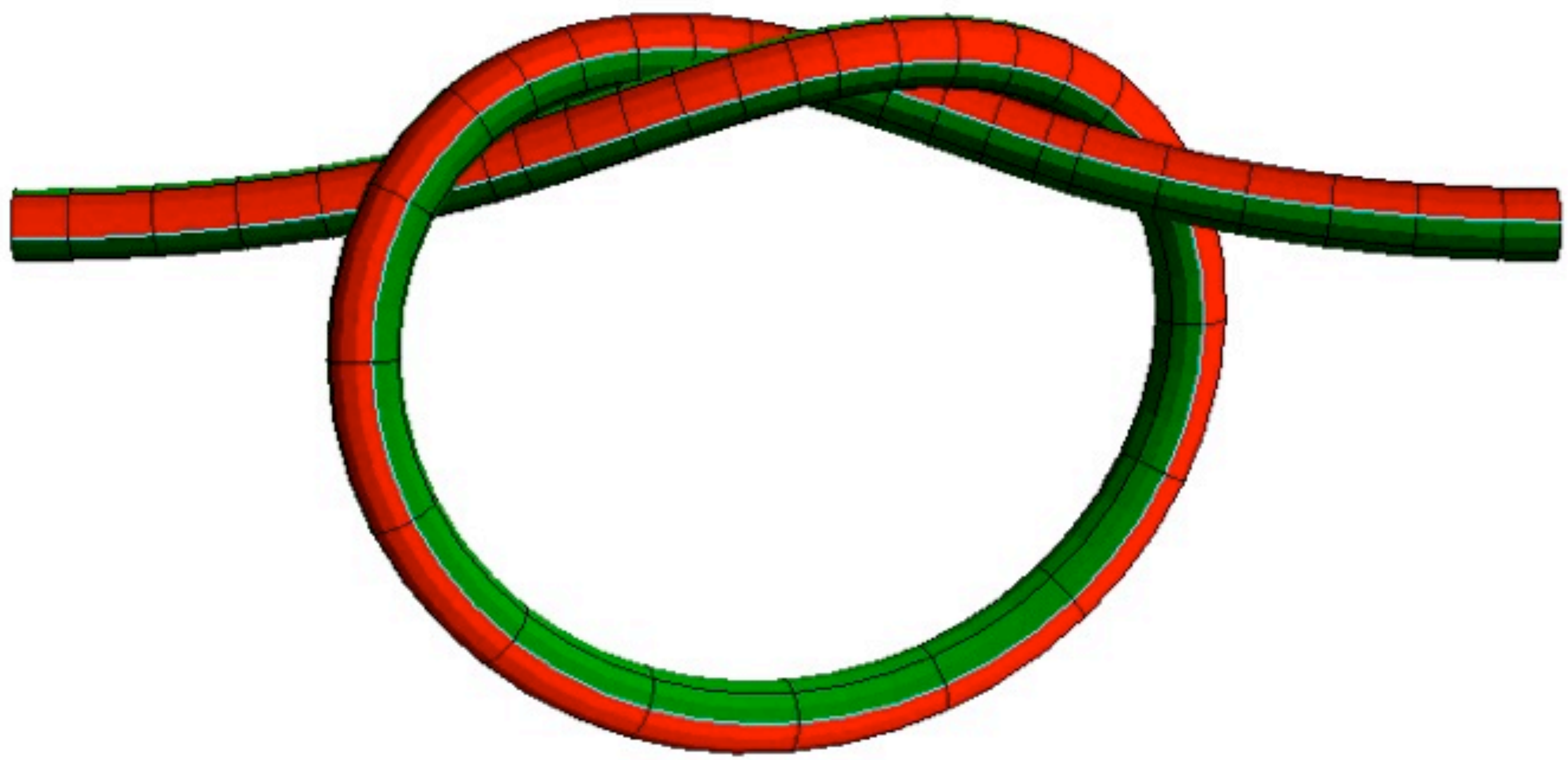


$\rho(s_a, s_b) < 2h$

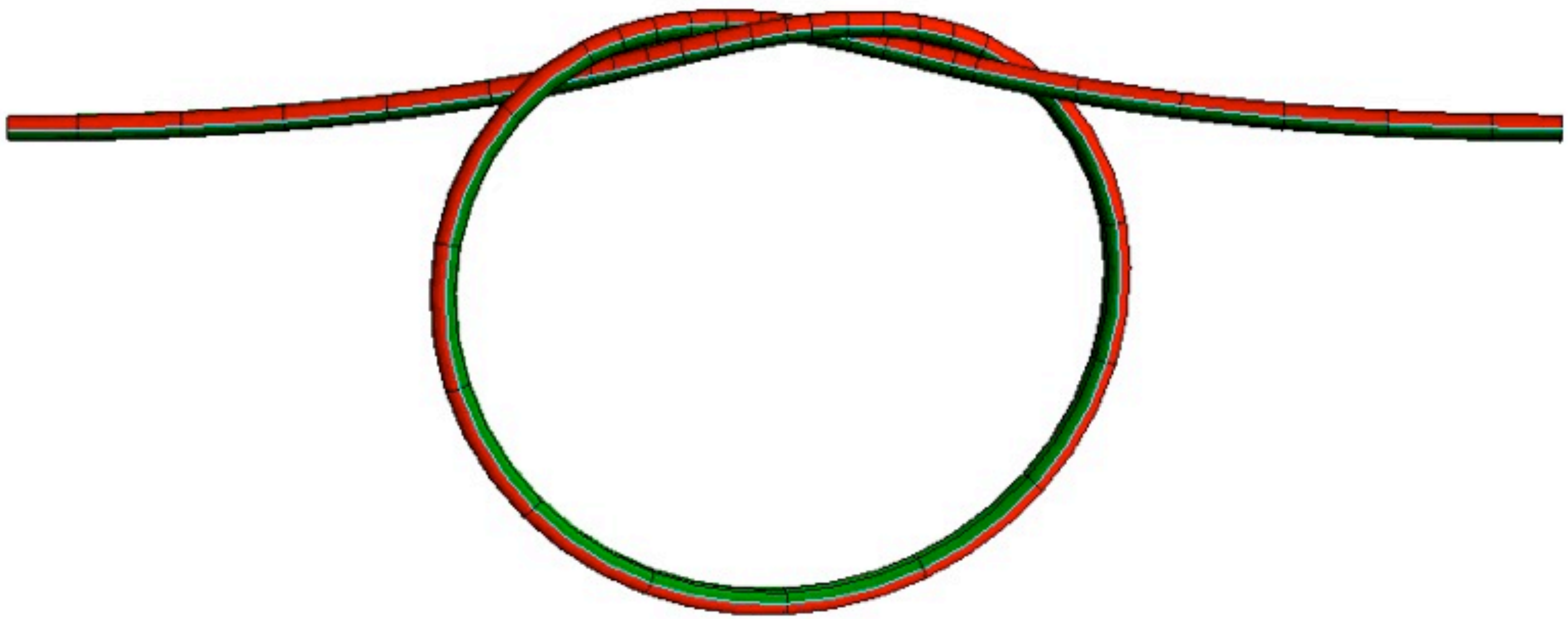
Making the rod thinner



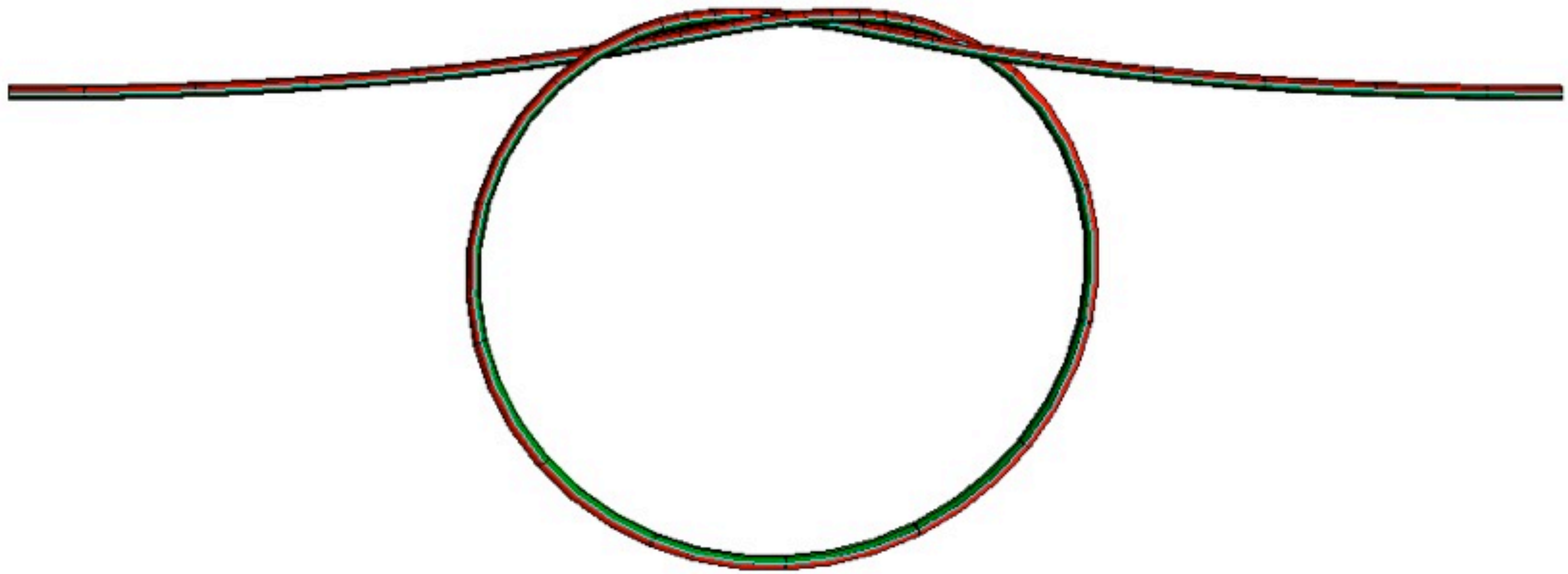
Making the rod thinner



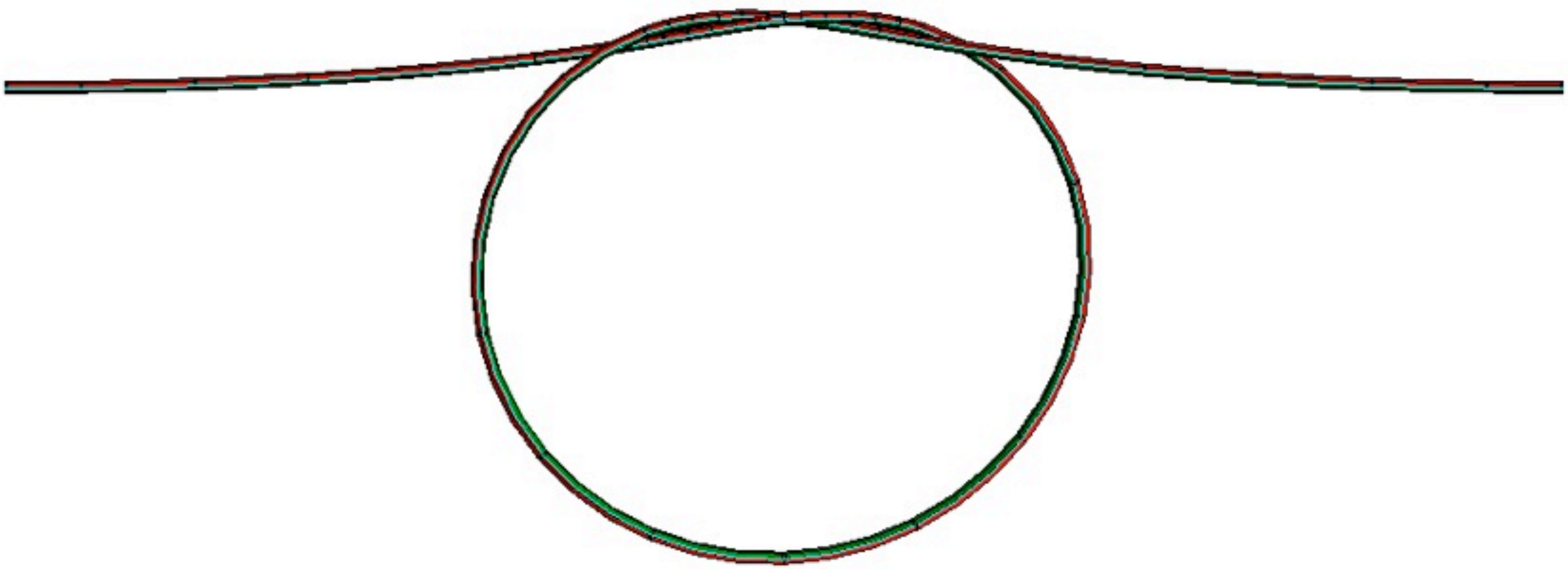
Making the rod thinner



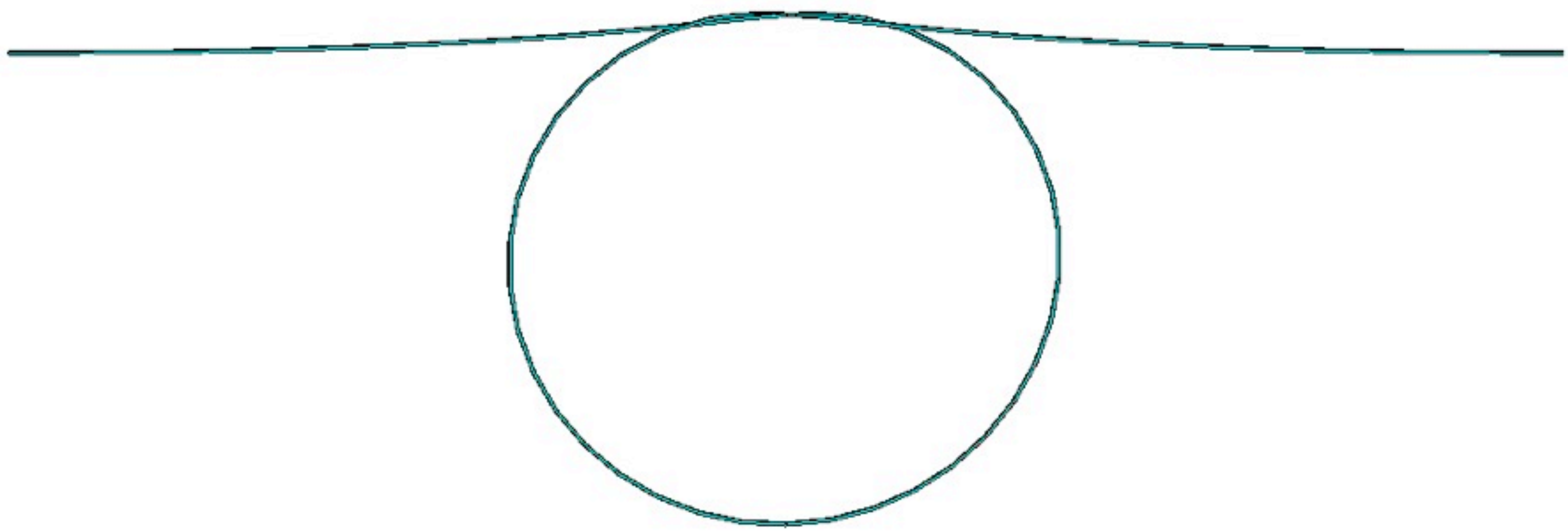
Making the rod thinner



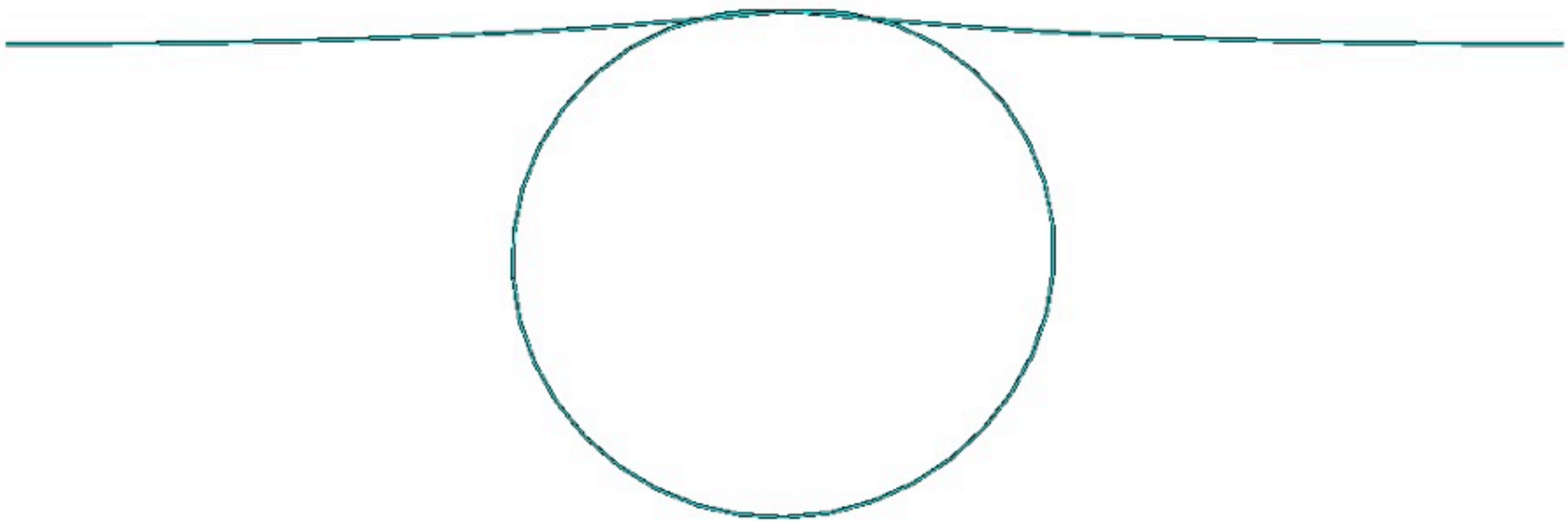
Making the rod thinner



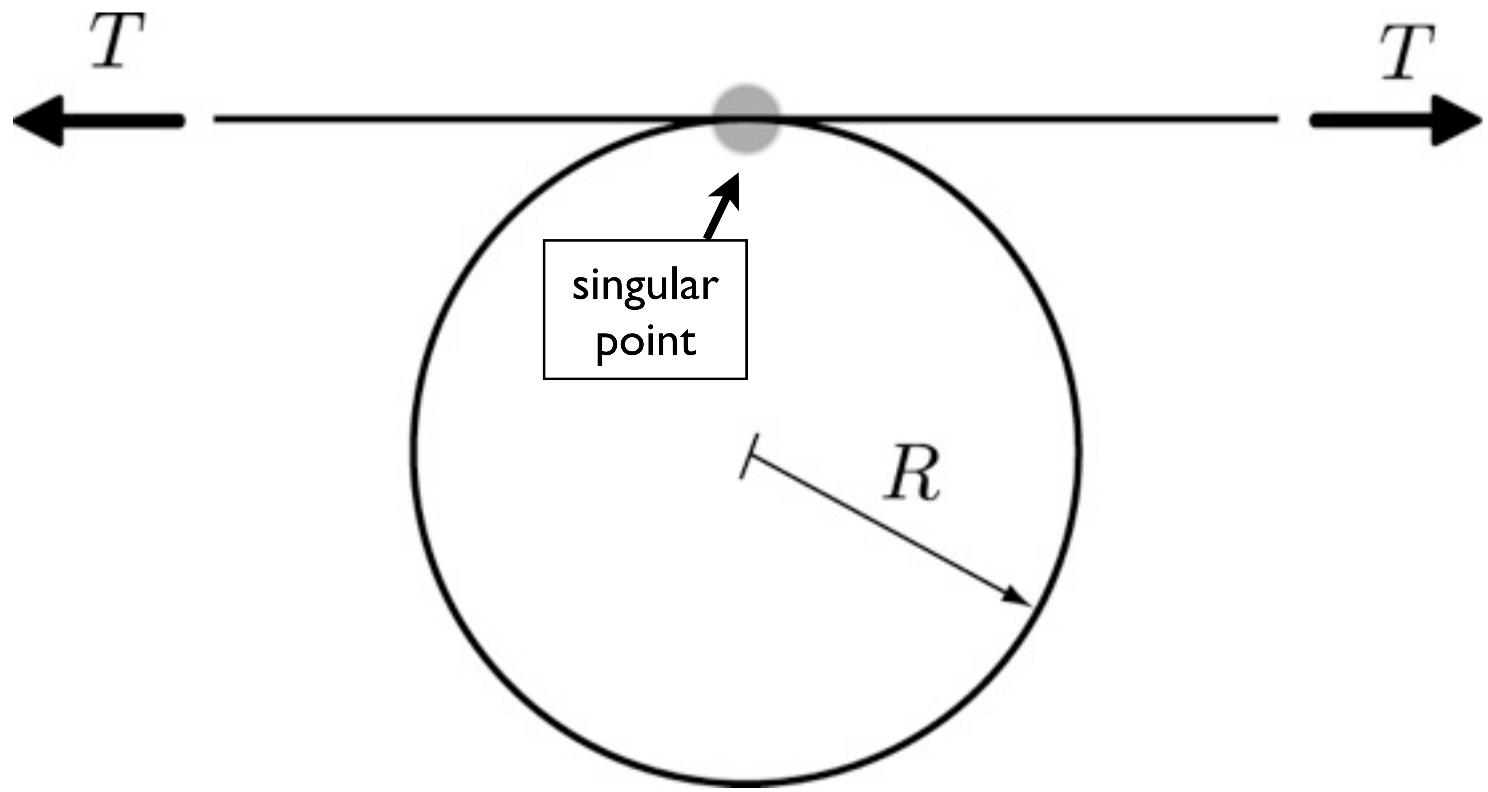
Making the rod thinner



Making the rod thinner

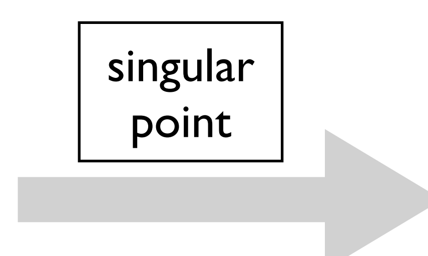


Zero thickness limit



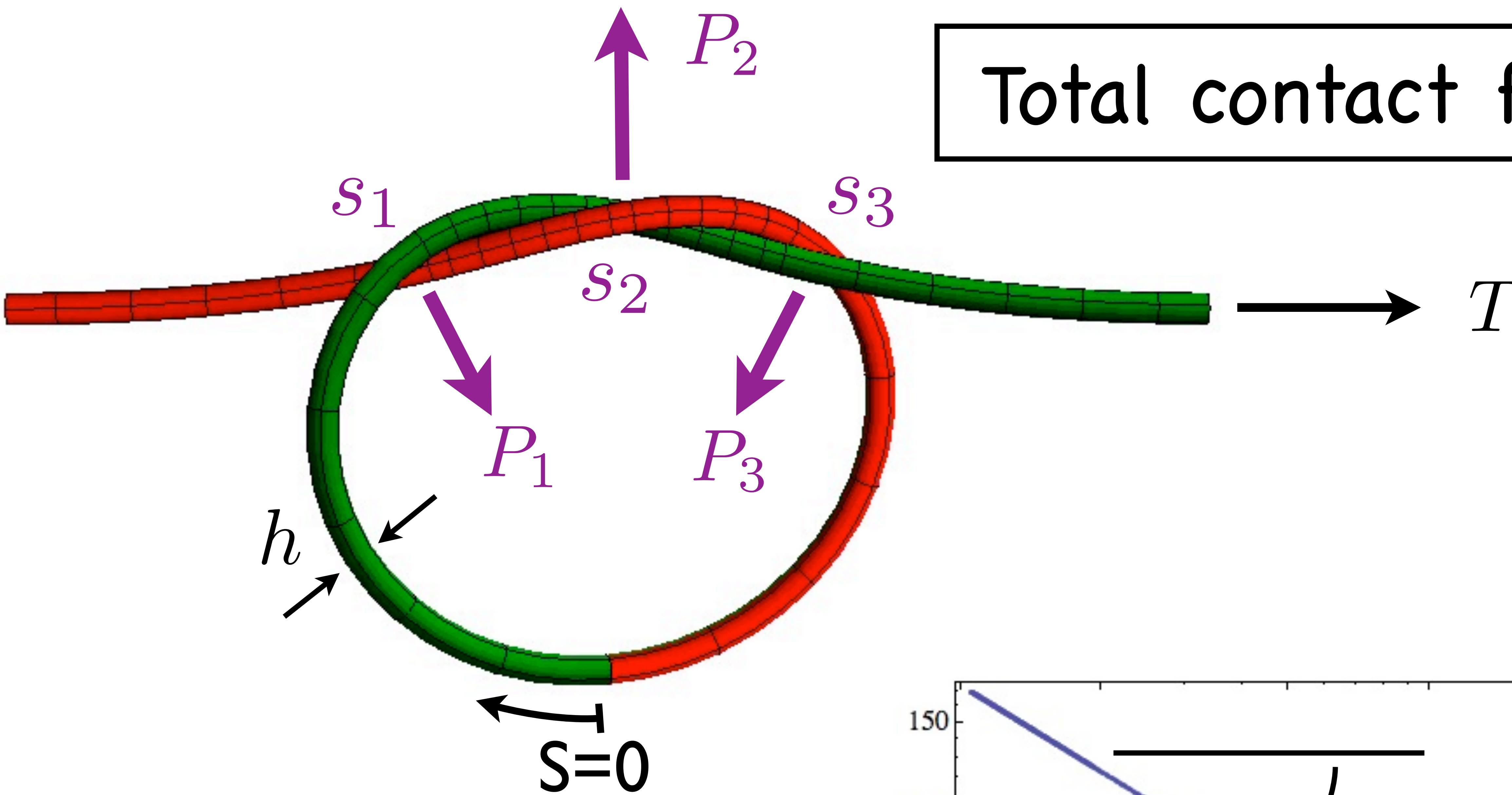
equilibrium : $T = \dfrac{EI}{2R^2}$ Arai et al (1999)

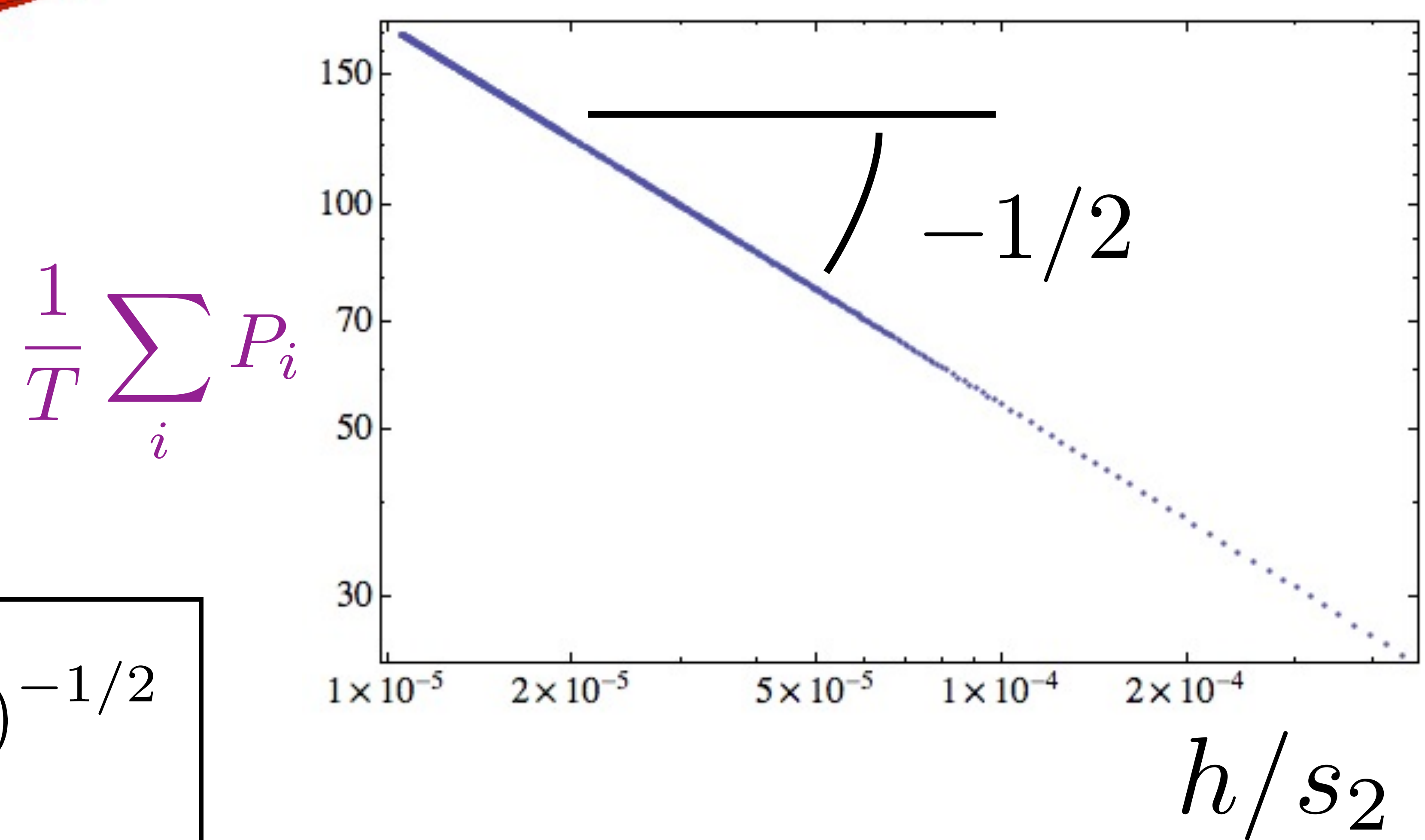
tensile force T



bending moment $\dfrac{EI}{R}$

Total contact force





$$\frac{1}{T}\sum_i P_i \simeq 0.55\,(h/s_2)^{-1/2}$$

Kirchhoff Equations

$$\left\{ \begin{array}{rcl} \vec{F}' & = & -\vec{p} \\ \vec{M}' & = & \vec{F} \times \vec{t} \\ \vec{t}' & = & \dfrac{1}{EI}\,\vec{M} \times \vec{t} \\ \vec{R}' & = & \vec{t} \end{array} \right.$$

forces equil.

moments equil.

kinematics

tangent def.

$$' \equiv \frac{d}{ds}$$

$\vec{p}(s)$ ext. pressure

$\vec{M}(s)$ internal moment

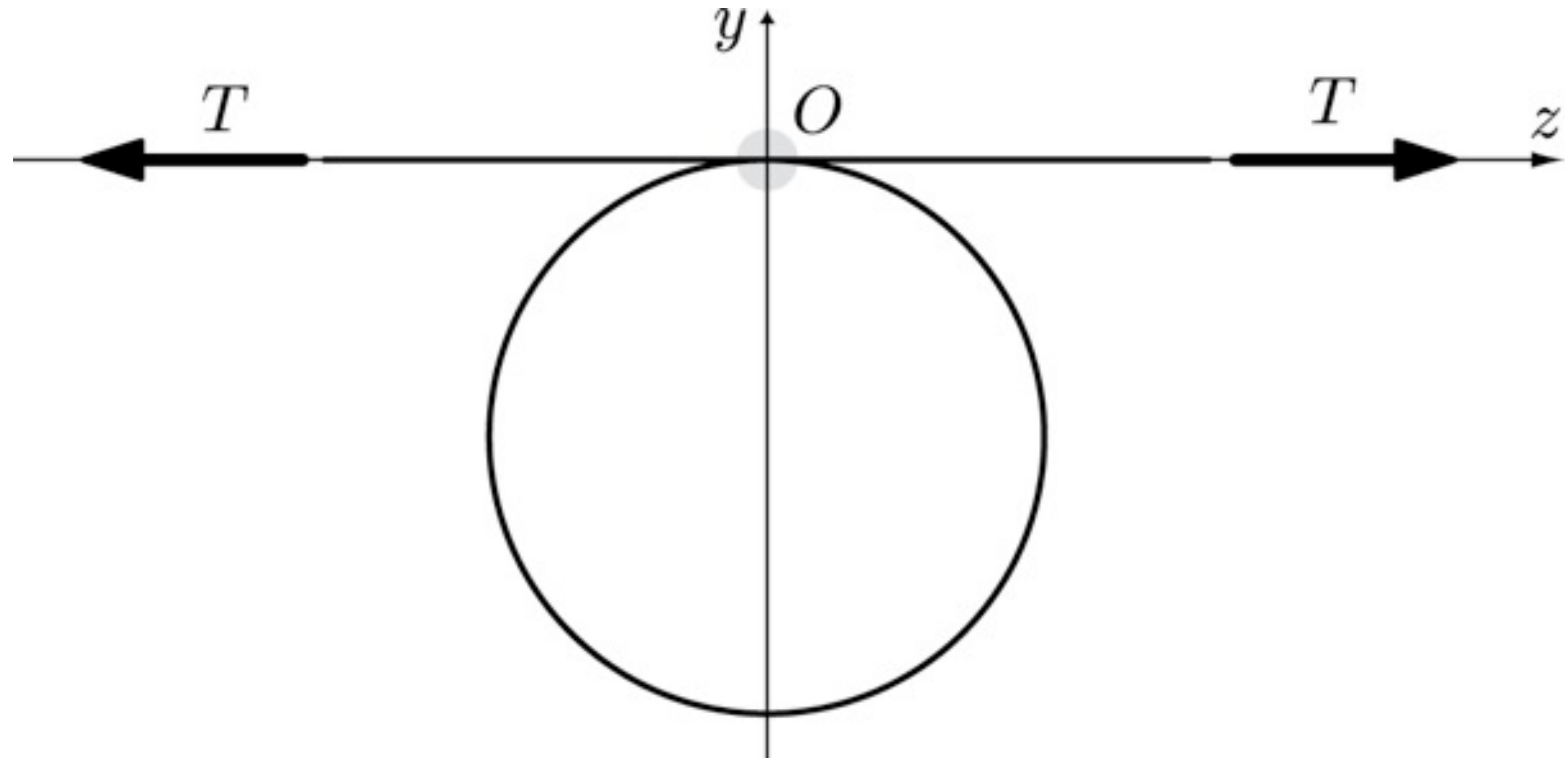
$\vec{F}(s)$ internal force

$\vec{R}(s)$ position

$\vec{t}(s)$ tangent

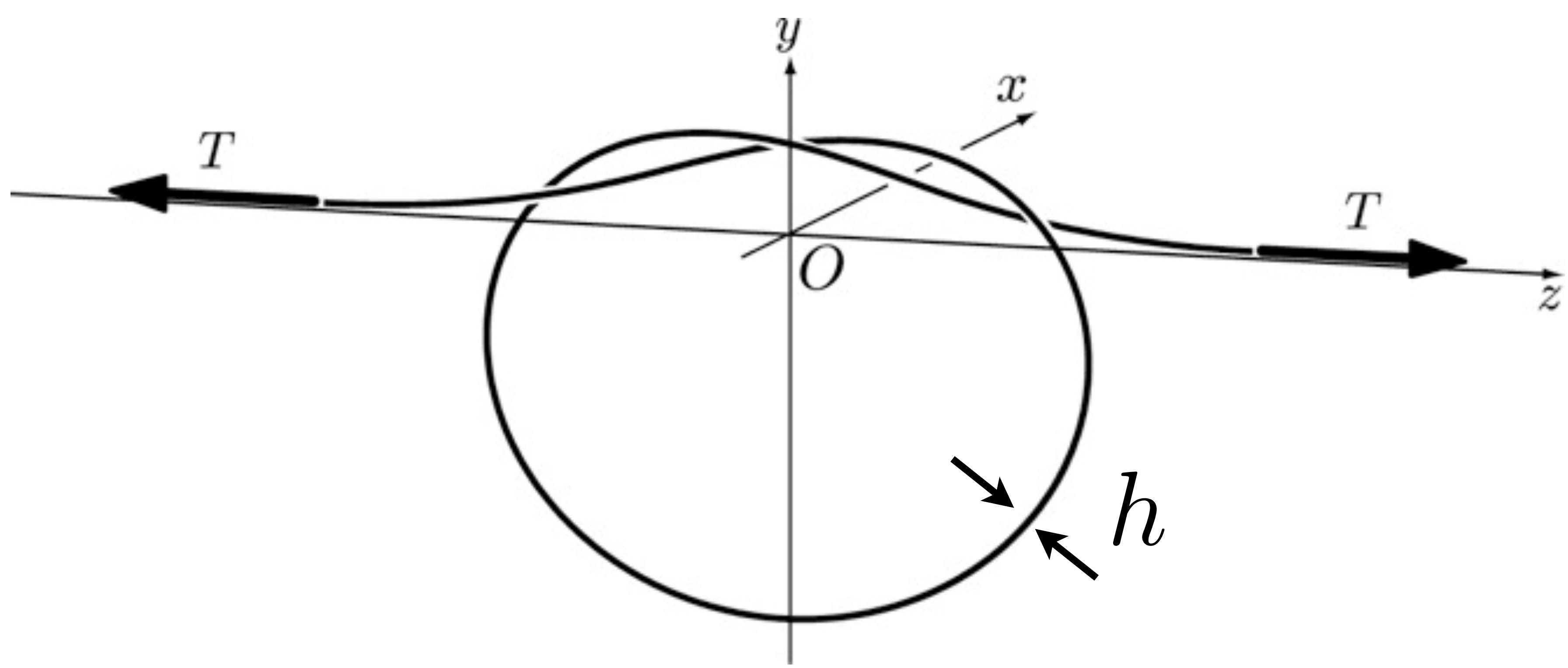
Perturbative problem

$\epsilon = 0$
$(h = 0)$

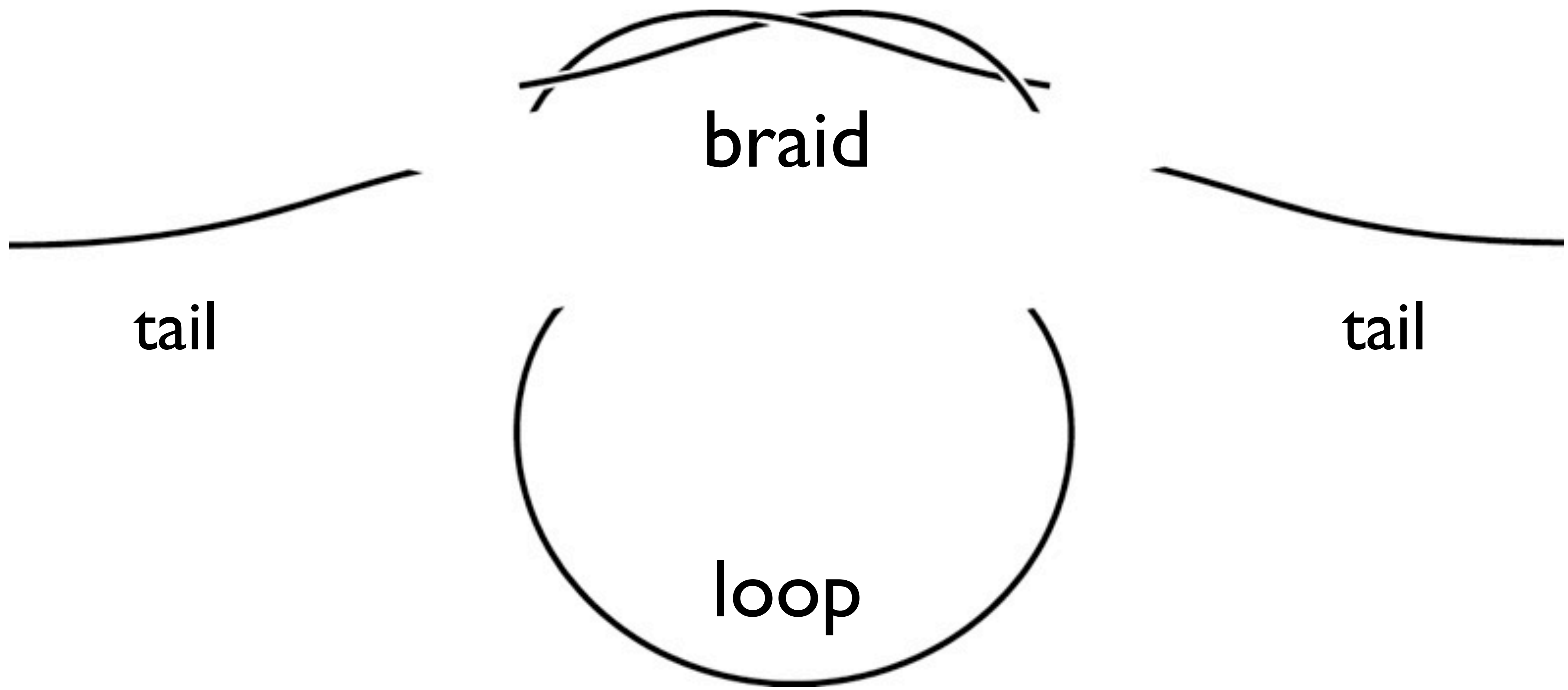


small parameter

$$\epsilon = \left(\frac{2h^2T}{EI}\right)^{1/4} \ll 1$$

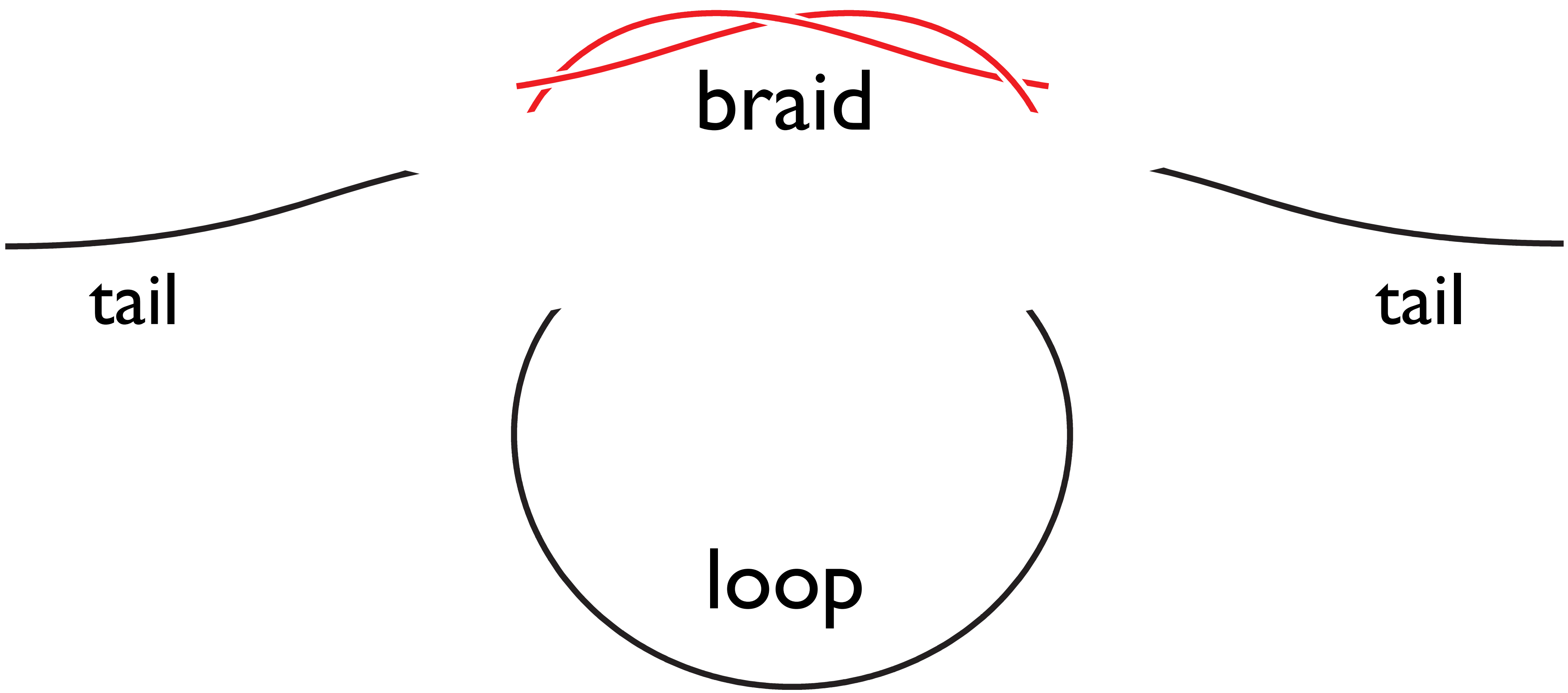


Matched asymptotic expansions

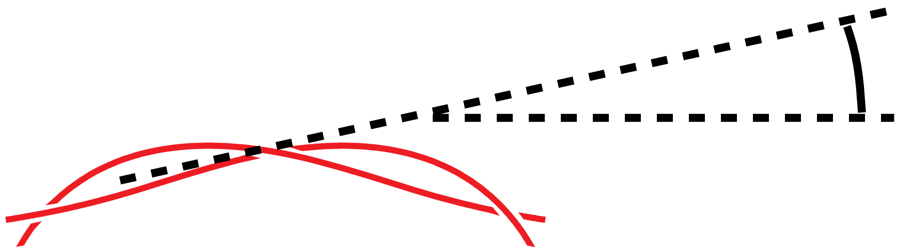


small parameter : $\epsilon = \left(\frac{2h^2T}{EI}\right)^{1/4} \ll 1$

Braid : self-contact zone

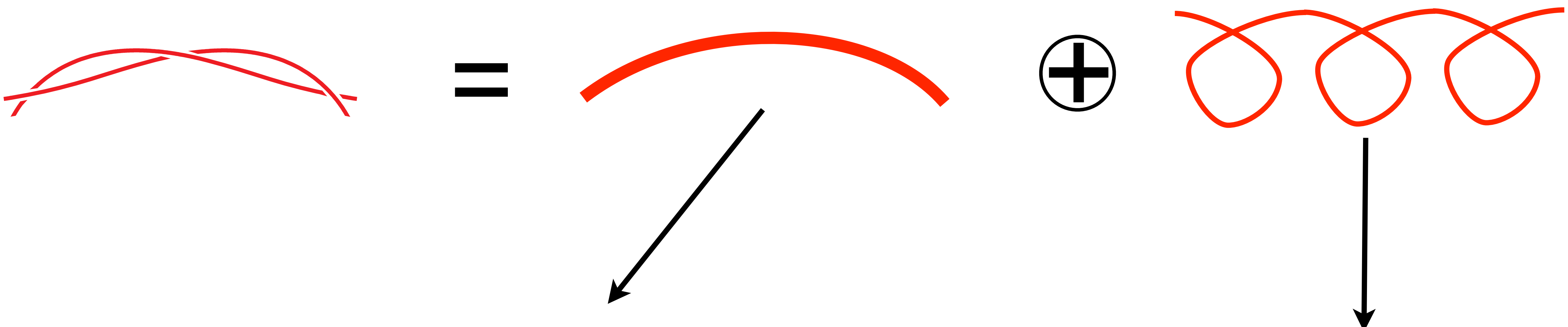


Braid : linear superposition



small deflections => linear problem

x^A , x^B = $x^B + x^A$ ⊕ $x^B - x^A$

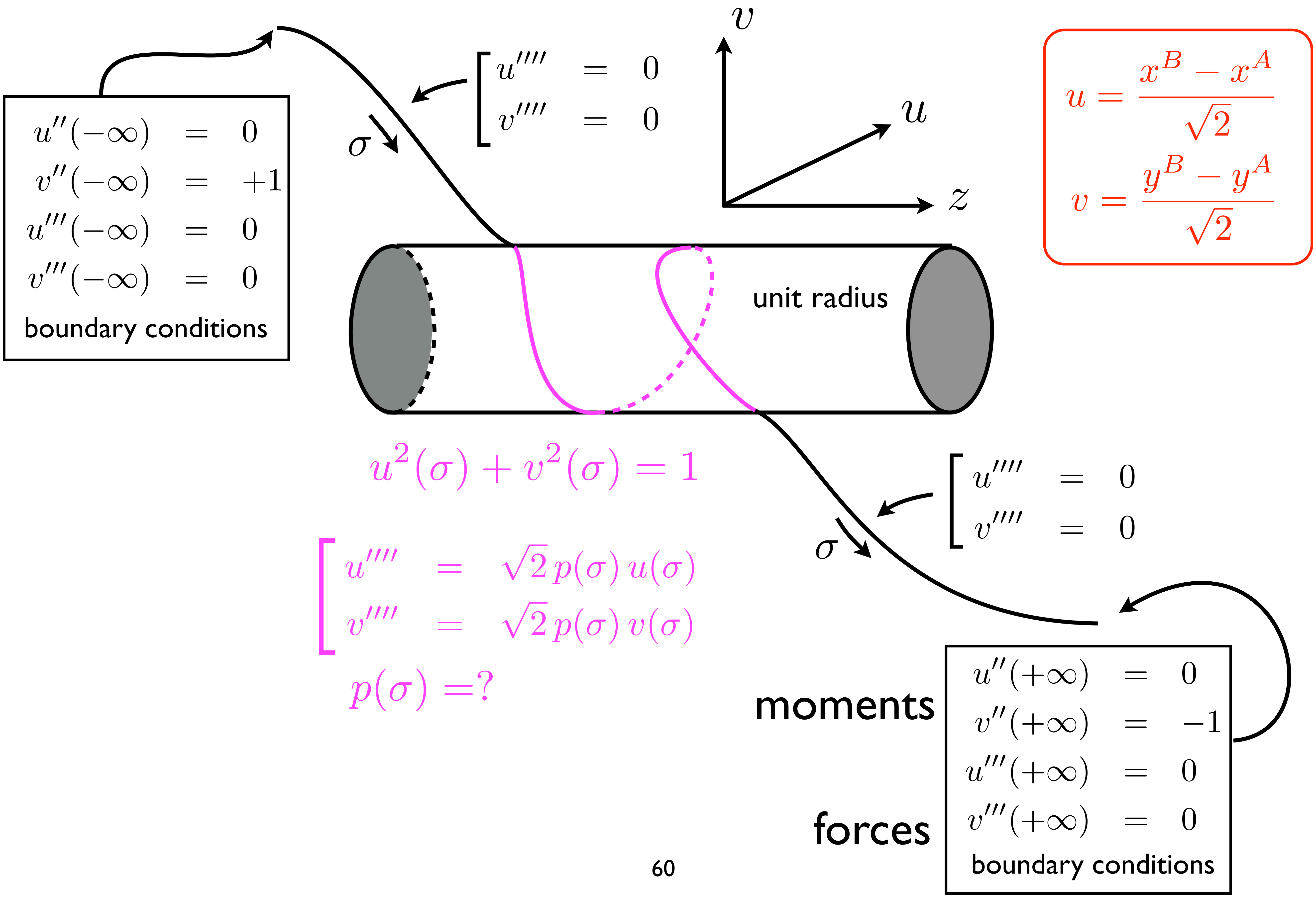


twice more rigid

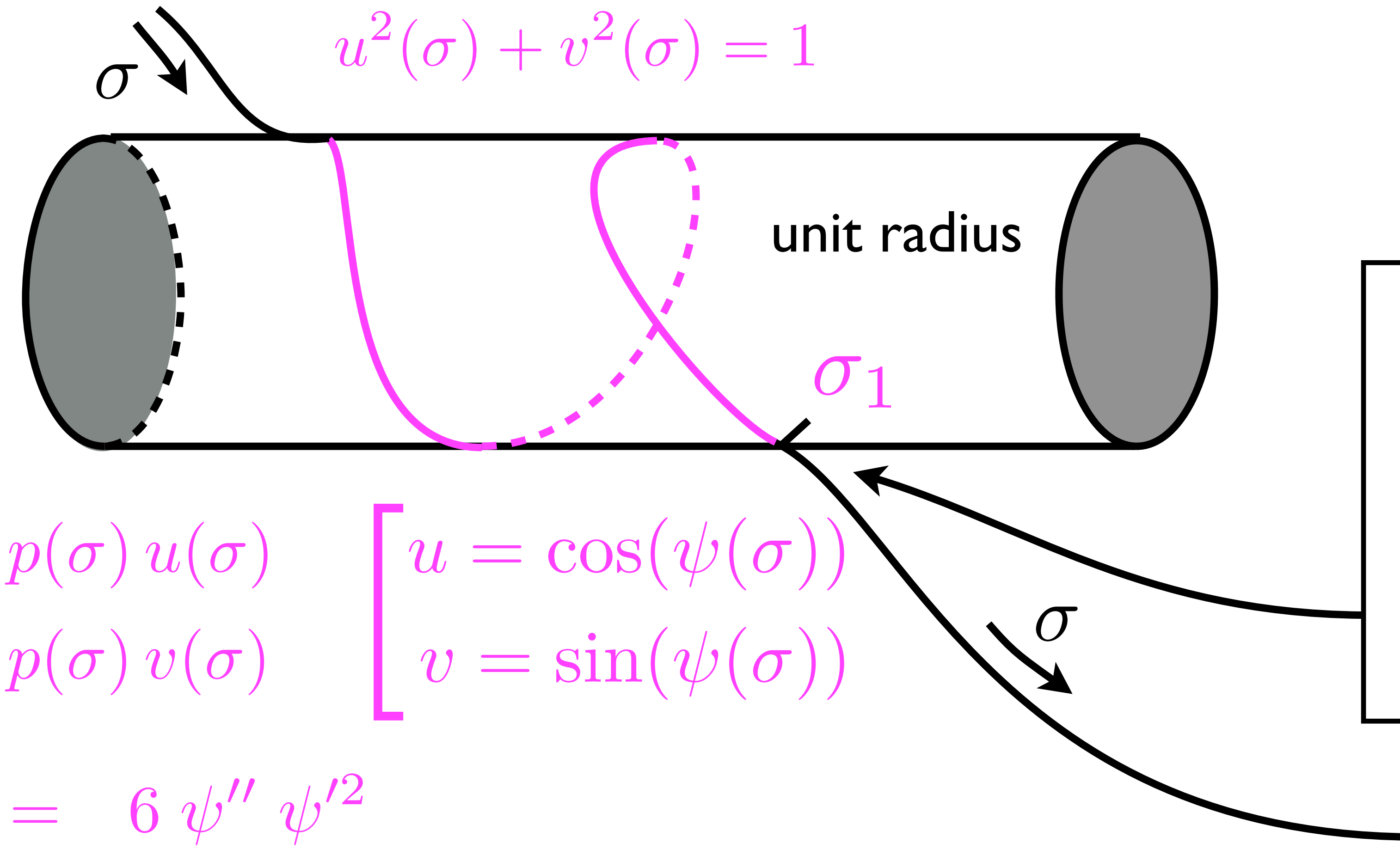
curvature: $\frac{1}{2}\frac{1}{R}$

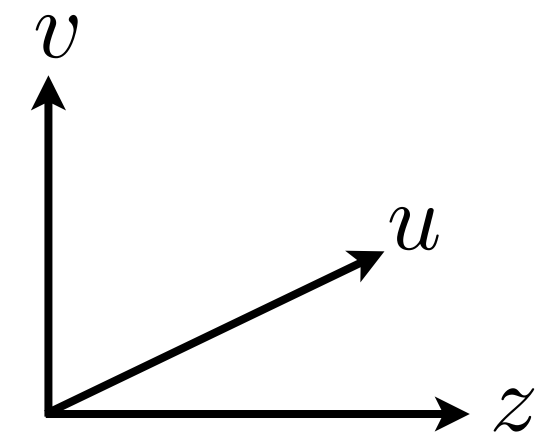
self-contact
=>
contact with obstacle

Braid : boundary value problem (BVP)



Braid : first kind of solutions





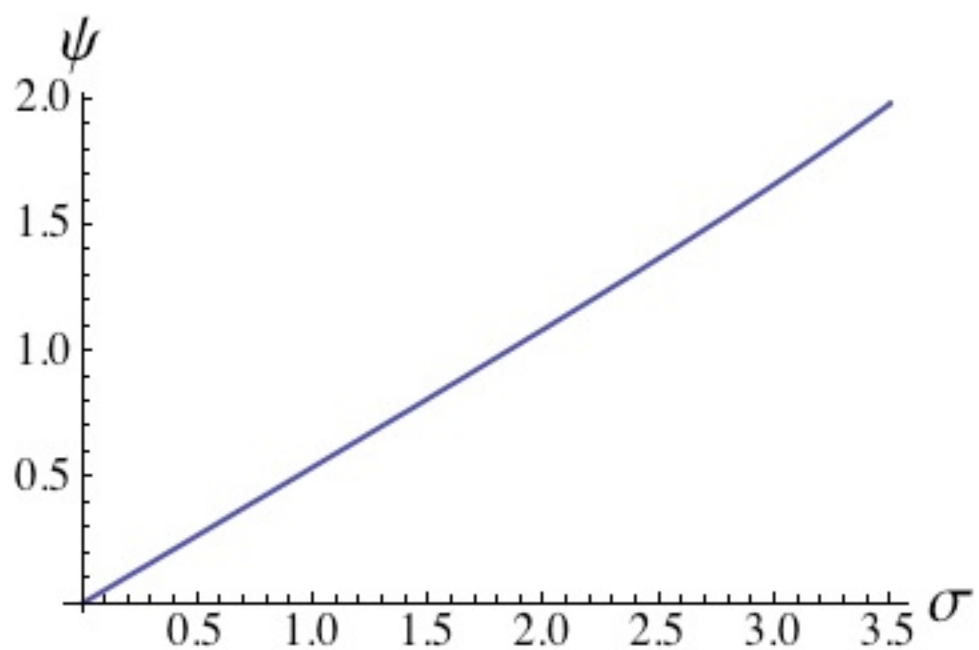
boundary conditions

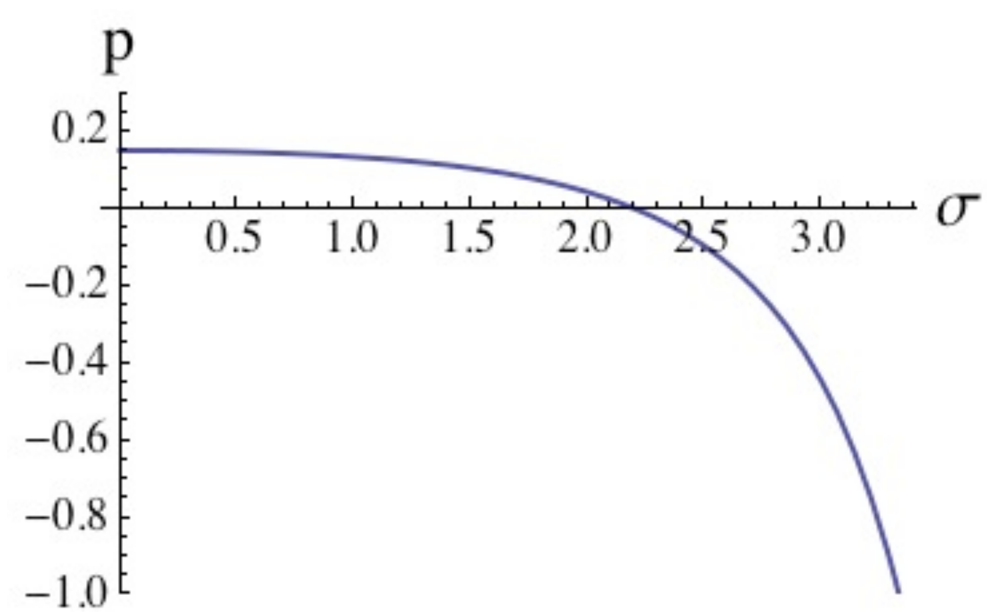
$$\begin{aligned} u''(\sigma_1) &= 0 \\ v''(\sigma_1) &= -1 \\ u'''(\sigma_1) &= 0 \\ v'''(\sigma_1) &= 0 \end{aligned}$$

$$\left[\begin{aligned} u'''' &= \sqrt{2}\, p(\sigma)\, u(\sigma) \\ v'''' &= \sqrt{2}\, p(\sigma)\, v(\sigma) \end{aligned} \right. \qquad \left[\begin{aligned} u &= \cos(\psi(\sigma)) \\ v &= \sin(\psi(\sigma)) \end{aligned} \right.$$

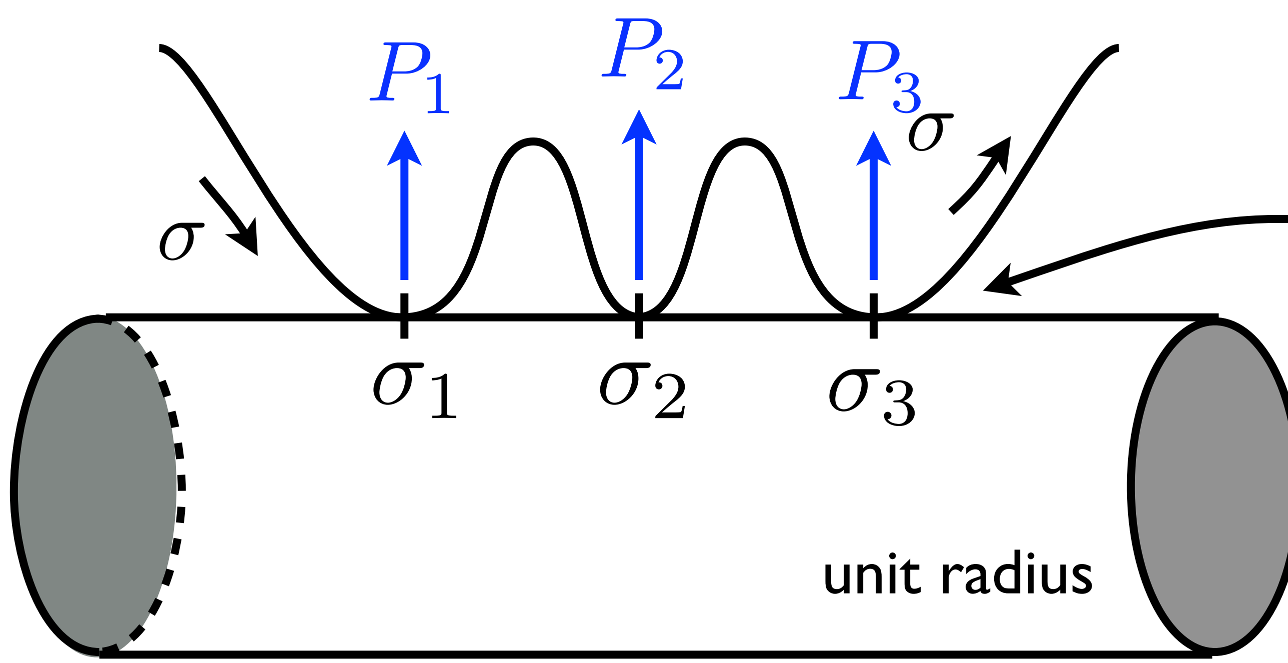
$$\left\{ \begin{aligned} \psi'''' &= 6\, \psi''\, \psi'^2 \\ p(\sigma) &= \left(\psi'^4 - 3\psi''^2 - 4\psi'\psi'''\right) / \sqrt{2} \end{aligned} \right.$$

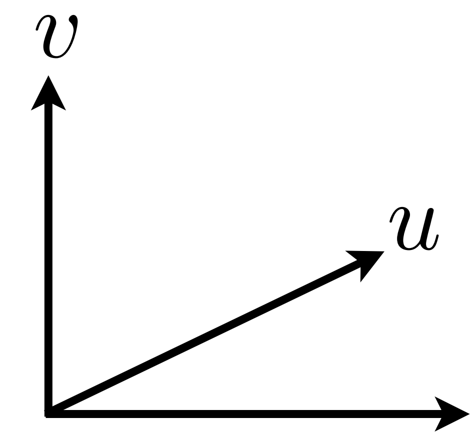
$$\begin{aligned} \psi(0) &= 0 \\ \psi'(0) &= 0.54 \\ \psi''(0) &= 0 \\ \psi'''(0) &= 0.004 \\ \sigma_1 &= 3.50 \end{aligned}$$





Braid : second kind of solutions





boundary conditions

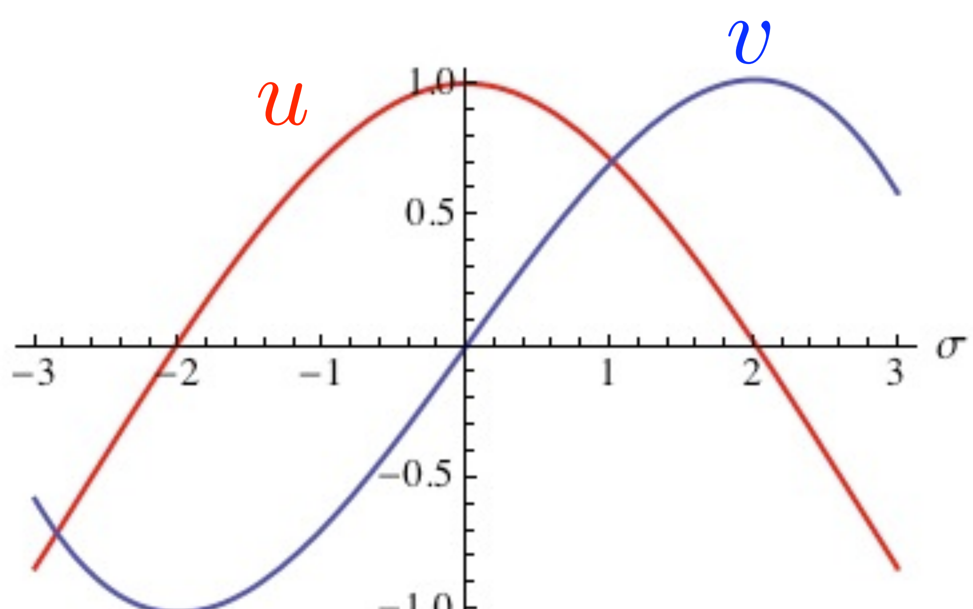
$$
\begin{aligned}
u''(+\sigma_3) &= 0 \\
v''(+\sigma_3) &= -1 \\
u'''(+\sigma_3) &= 0 \\
v'''(+\sigma_3) &= 0
\end{aligned}
$$

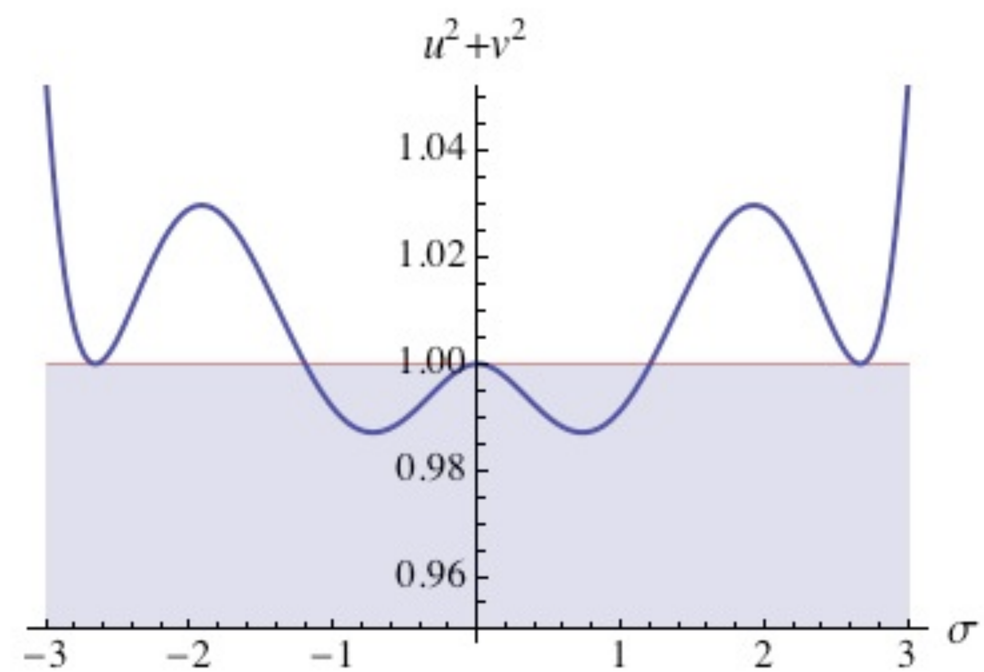
$$
\left\{
\begin{aligned}
u'''' &= \sqrt{2}\, p(\sigma)\, u(\sigma) \\
v'''' &= \sqrt{2}\, p(\sigma)\, v(\sigma)
\end{aligned}
\right.
$$

avec $p(\sigma) = P_1\,\delta(\sigma - \sigma_1) + P_2\,\delta(\sigma - \sigma_2) + P_3\,\delta(\sigma - \sigma_3)$

$$
\begin{aligned}
u(0) &= 1 \\
u'(0) &= 0 \\
u''(0) &= -0.66 \\
u'''(0) &= 0.25
\end{aligned}
$$

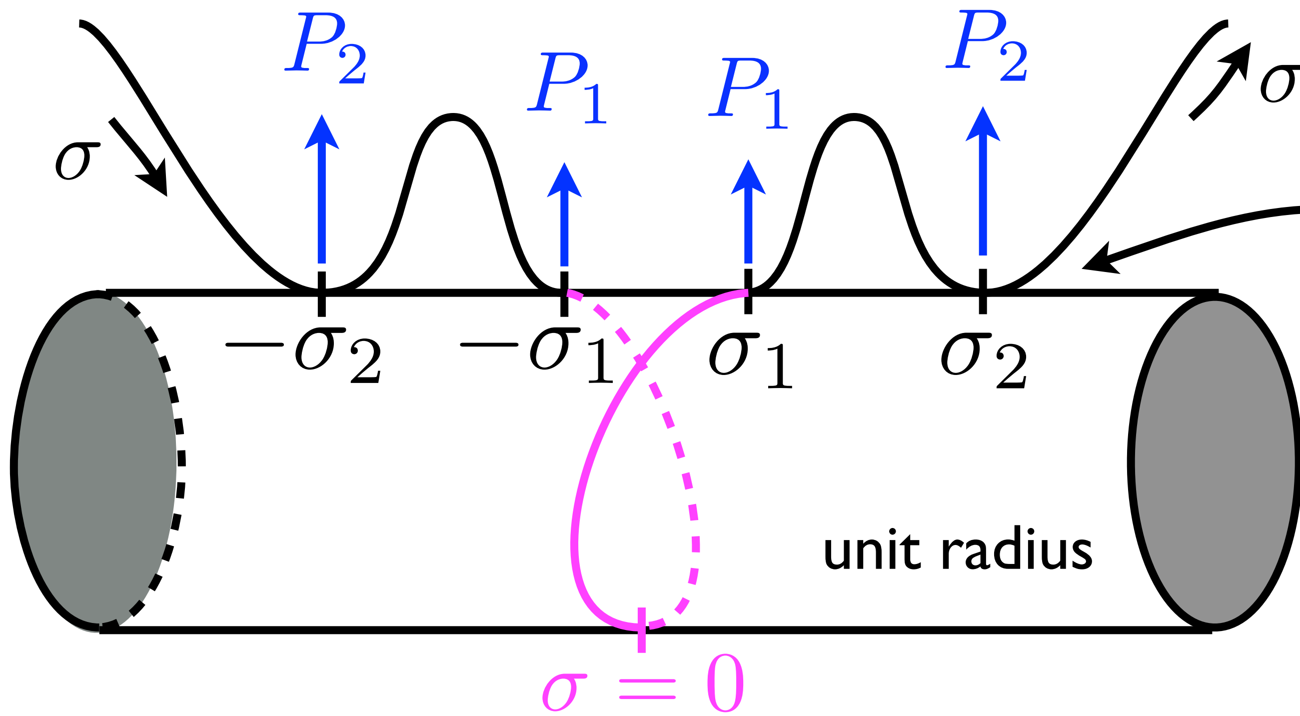
$$
\begin{aligned}
v(0) &= 0 \\
v'(0) &= 0.76 \\
v''(0) &= 0 \\
v'''(0) &= -0.38
\end{aligned}
$$

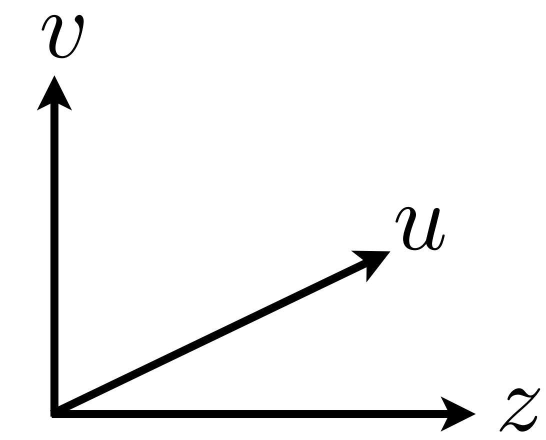




$\sigma_3 = -\sigma_1 = 2.66\ ;\ \sigma_2 = 0\ ;\ P_1 = P_3 = 0.32\ ;\ P_2 = 0.35$

Braid : third kind of solutions

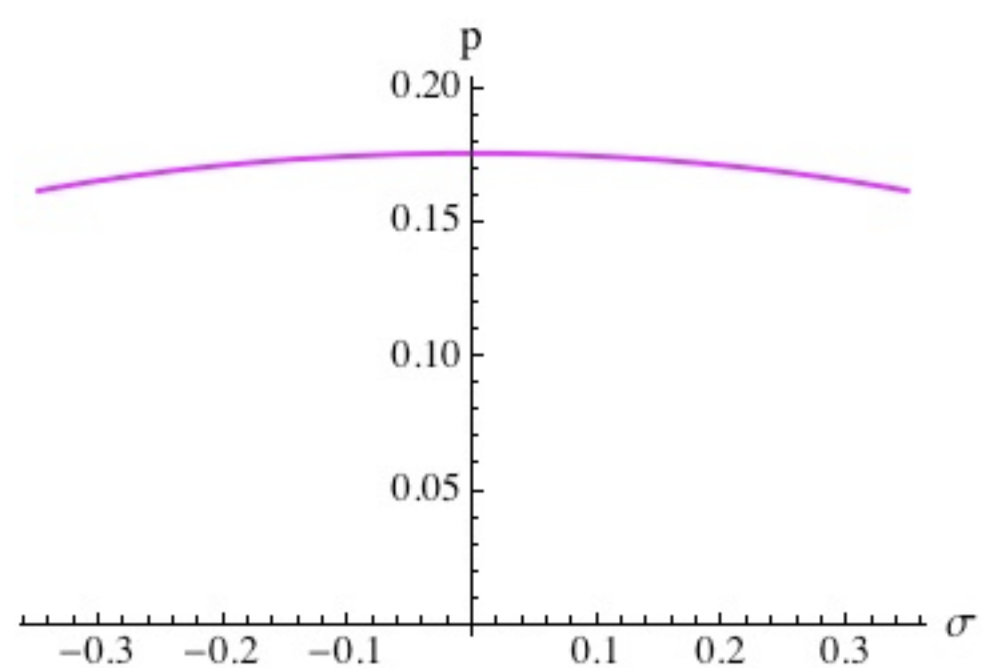


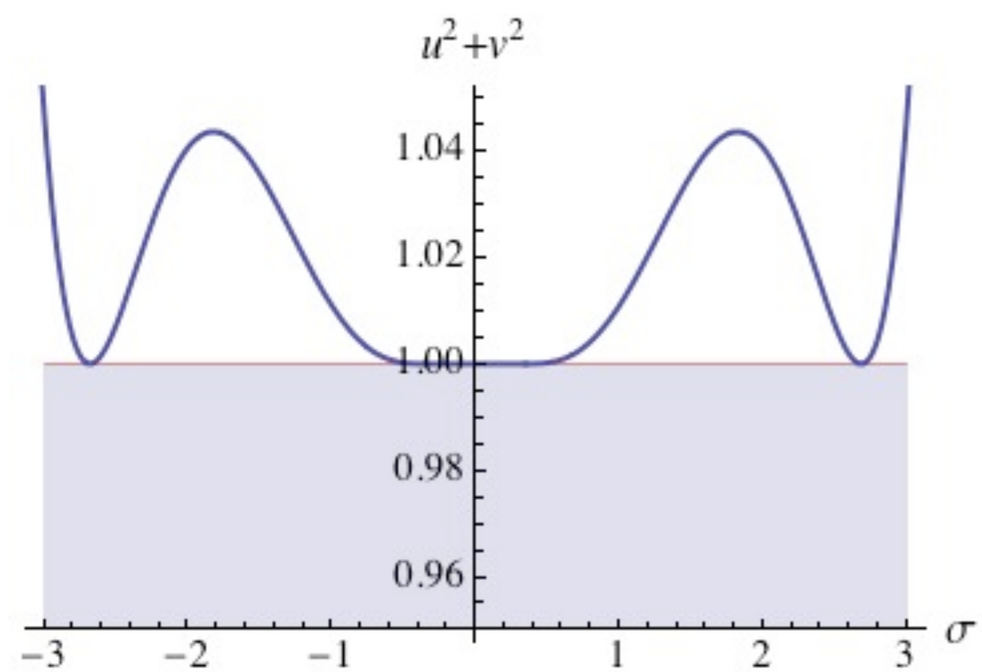


boundary conditions

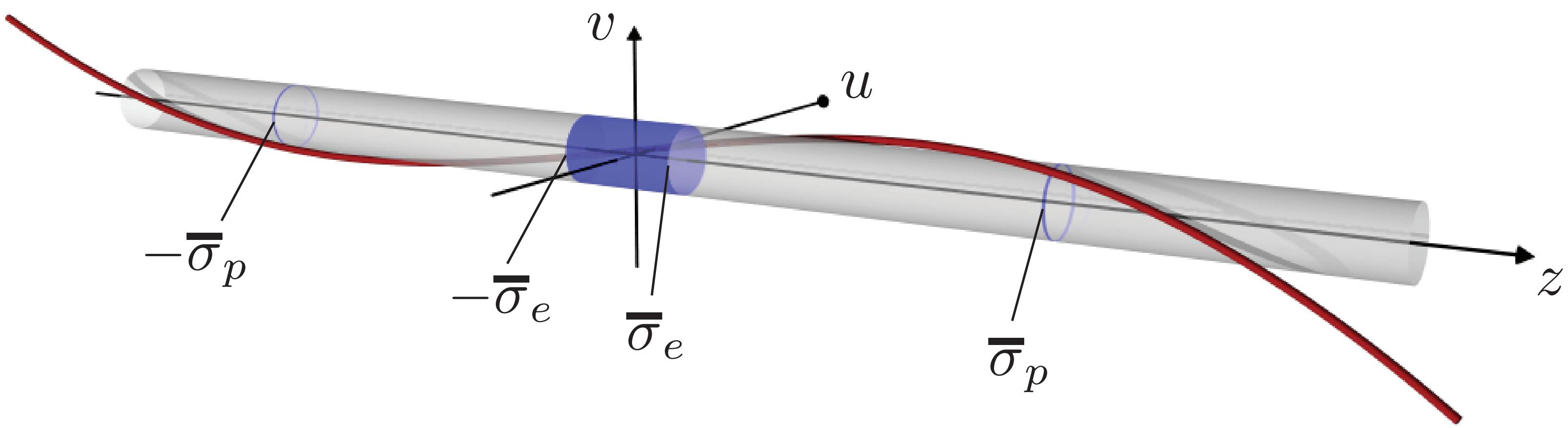
$$\begin{aligned} u''(+\sigma_2) &= 0 \\ v''(+\sigma_2) &= -1 \\ u'''(+\sigma_2) &= 0 \\ v'''(+\sigma_2) &= 0 \end{aligned}$$

$$\begin{aligned} \sigma_1 &= 0.35 \\ \sigma_2 &= 2.68 \\ P_1 &= 0.12 \\ P_2 &= 0.31 \end{aligned}$$





Braid : variational formulation

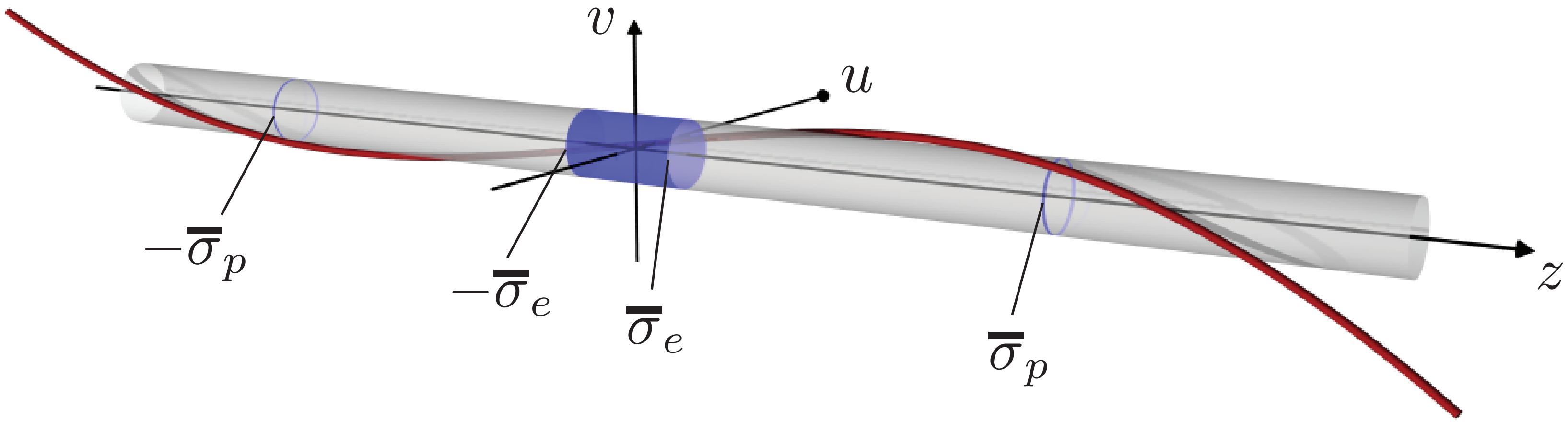


Kirchhoff equations => minimizing an energy

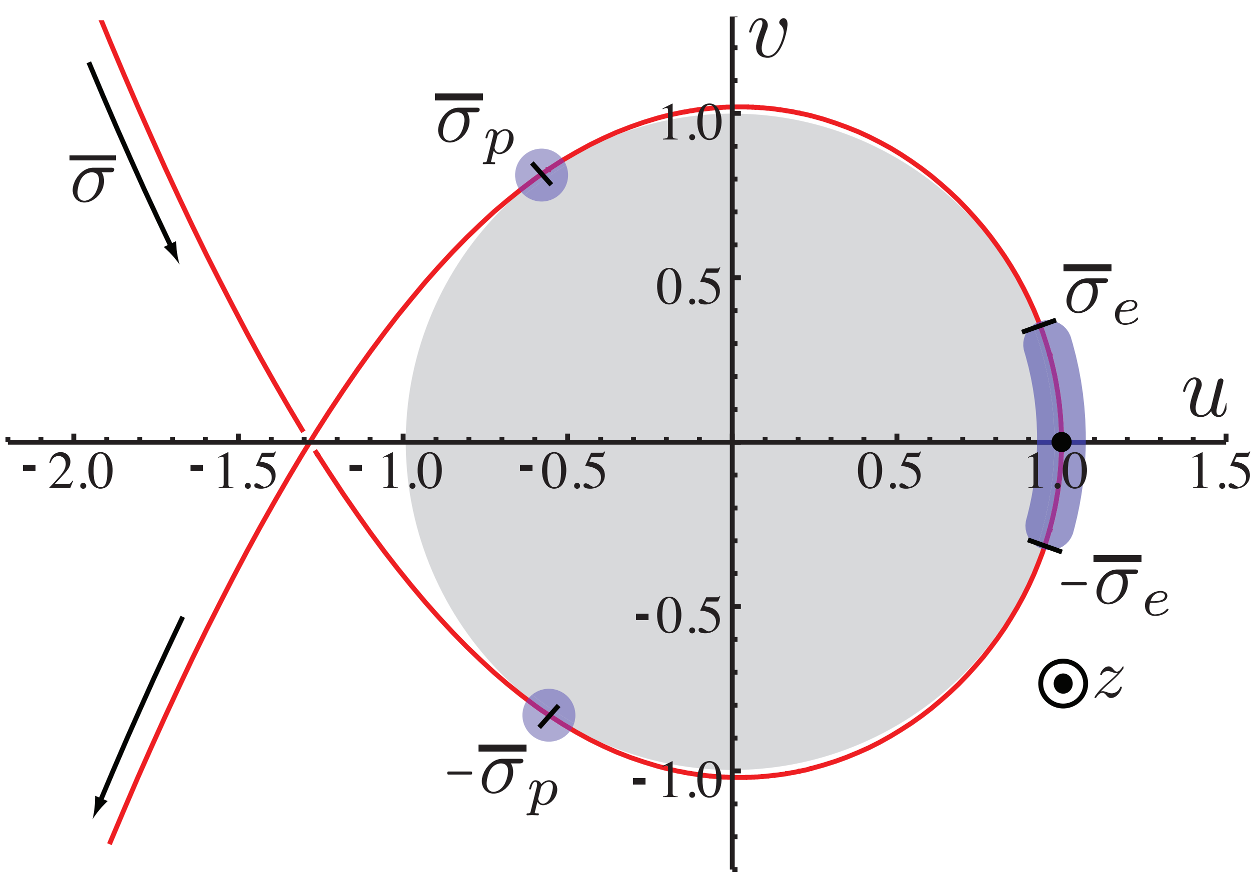
$$V = \frac{1}{2}\int_{-\infty}^{+\infty}\left(u''^2 + v''^2\right)d\sigma \; + \underbrace{v'(+\infty) + v'(-\infty)}_{\text{work of external applied moments}}$$

with constraint:
$u^2(\sigma) + v^2(\sigma) \geq 1 \; , \; \forall\sigma$

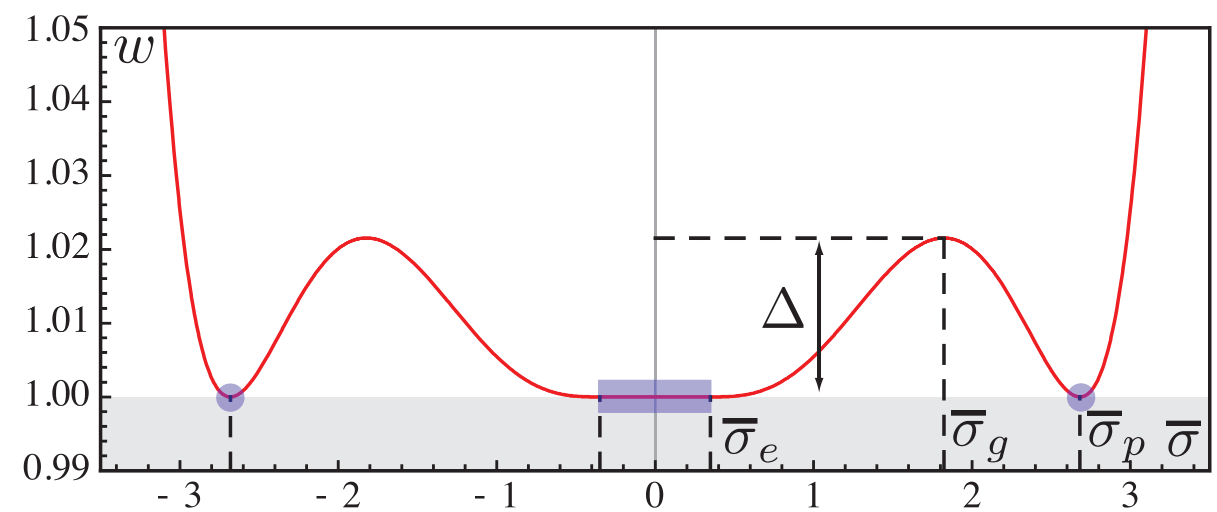
Braid : contact topology



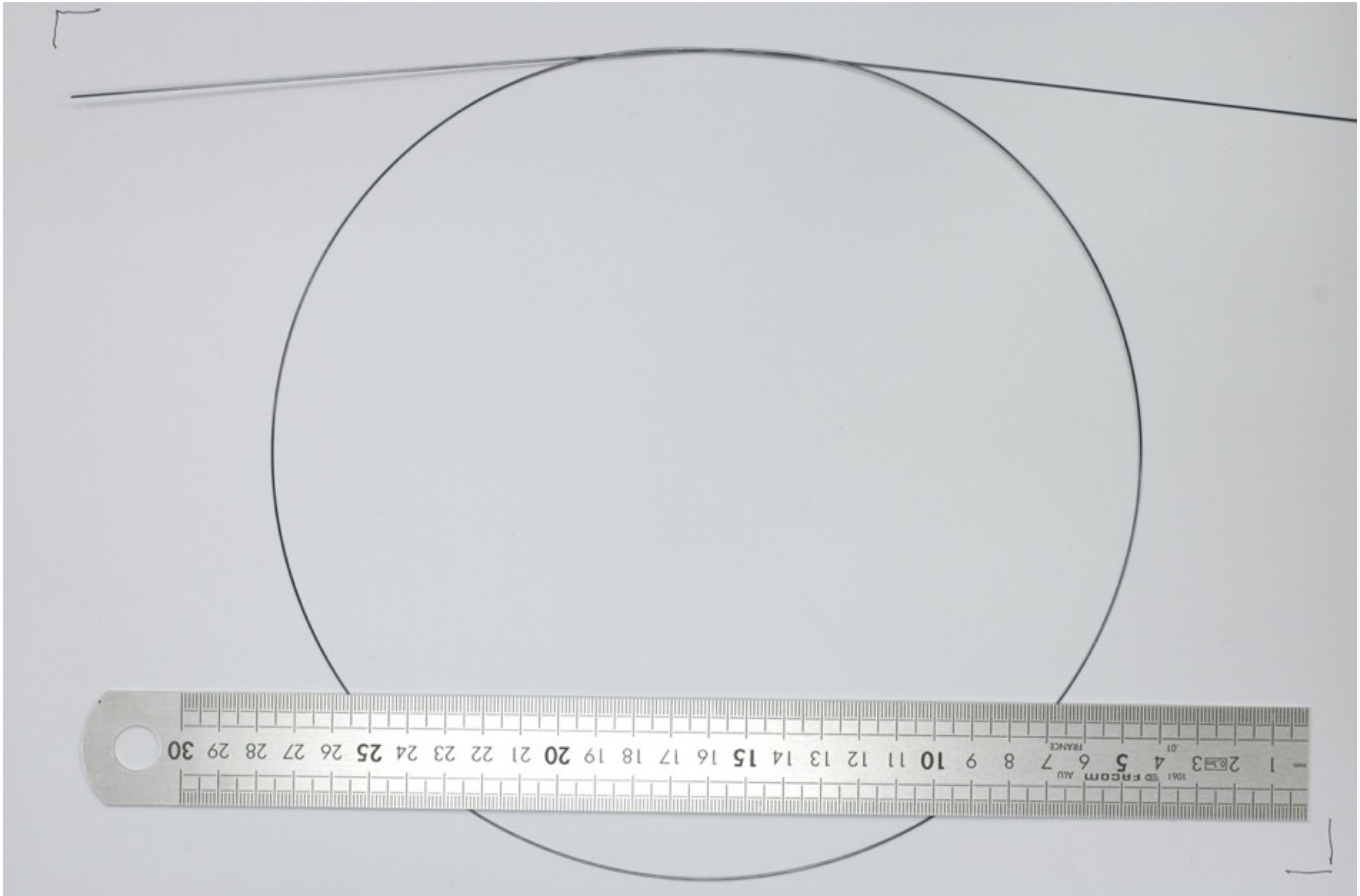
side view



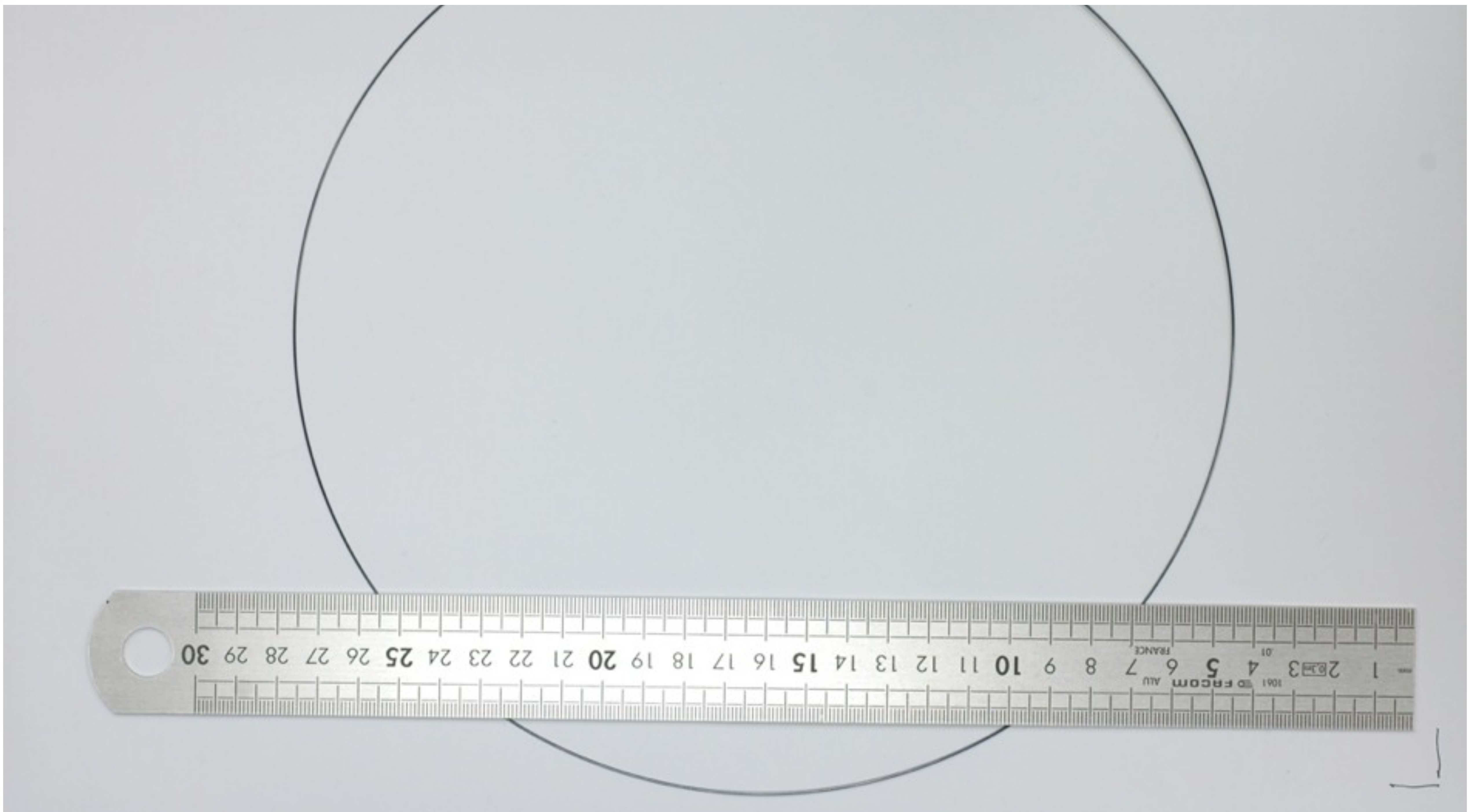
inter-strand distance



Braid : contact topology



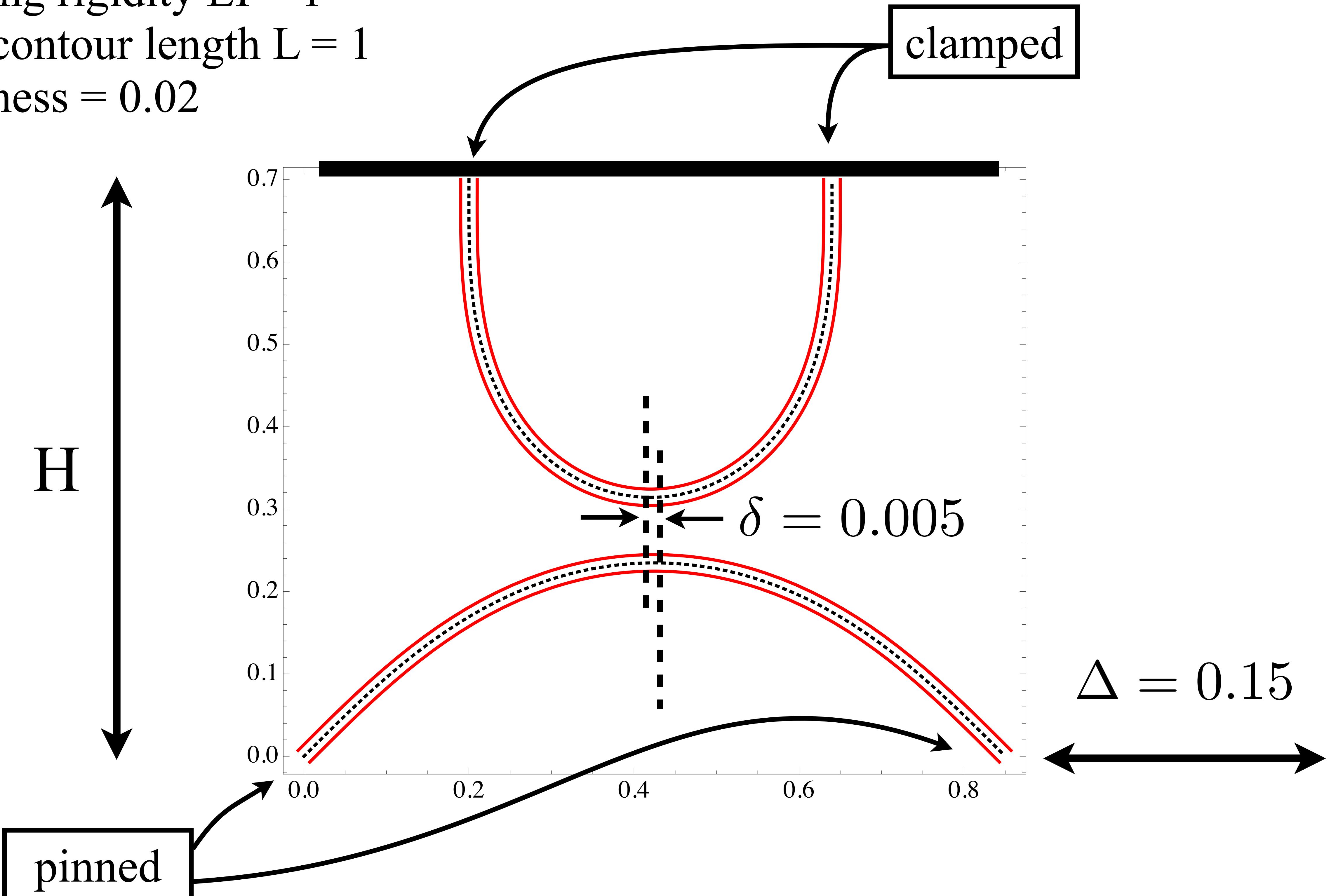
Braid : contact topology



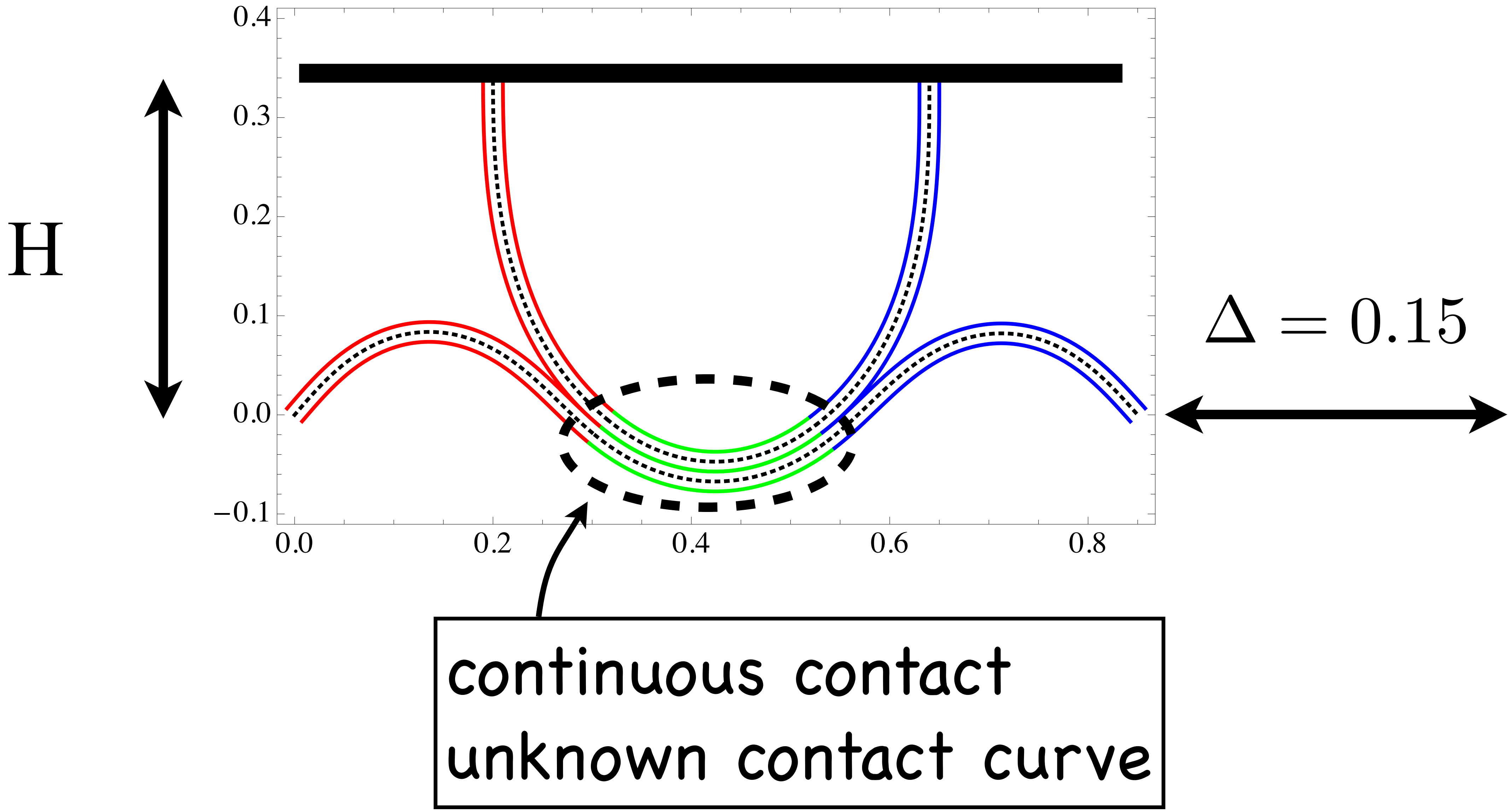
3rd `experiment'

Close packing

both rods have same:
- bending rigidity EI = 1
- total contour length L = 1
- thickness = 0.02



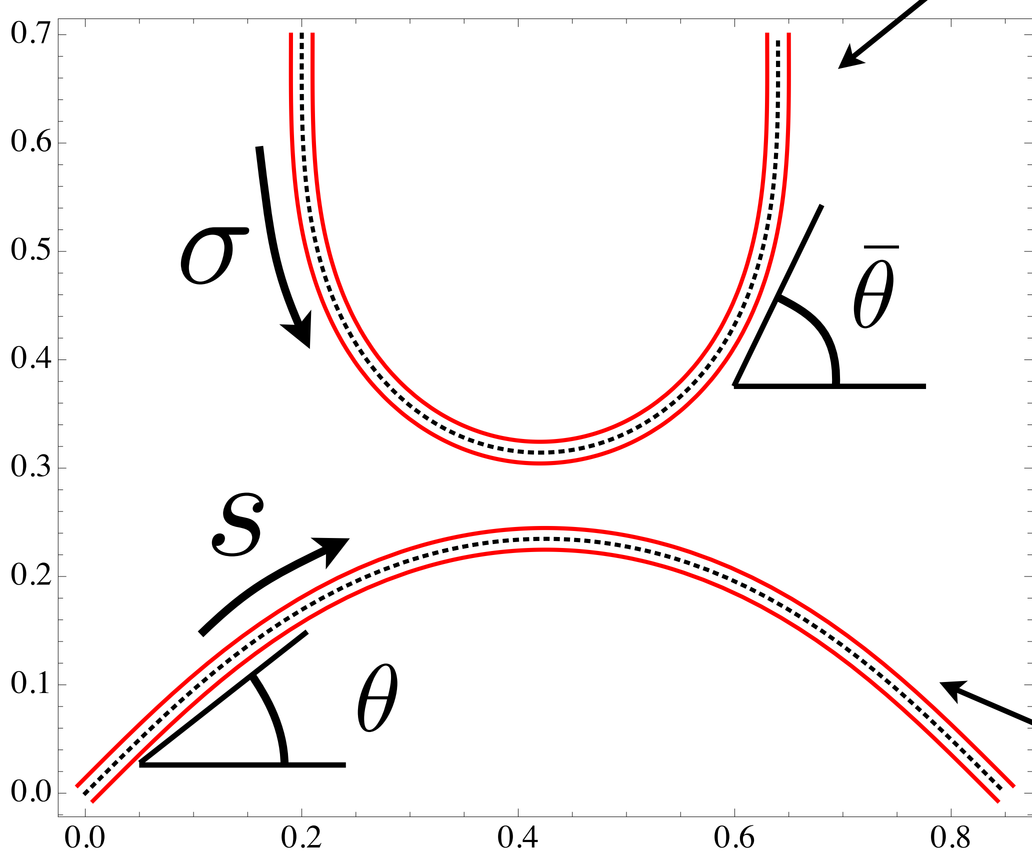
Close packing



Close packing

EI = 1
L = 1

rod #2
$$\begin{cases} \bar{x}'(\sigma) = \cos\bar{\theta} \\ \bar{y}'(\sigma) = \sin\bar{\theta} \\ \bar{\theta}'(\sigma) = \bar{m} \\ \bar{m}'(\sigma) = \bar{n}_x \sin\bar{\theta} - \bar{n}_y \cos\bar{\theta} \\ \bar{n}_x'(\sigma) = 0 \\ \bar{n}_y'(\sigma) = 0 \end{cases}$$



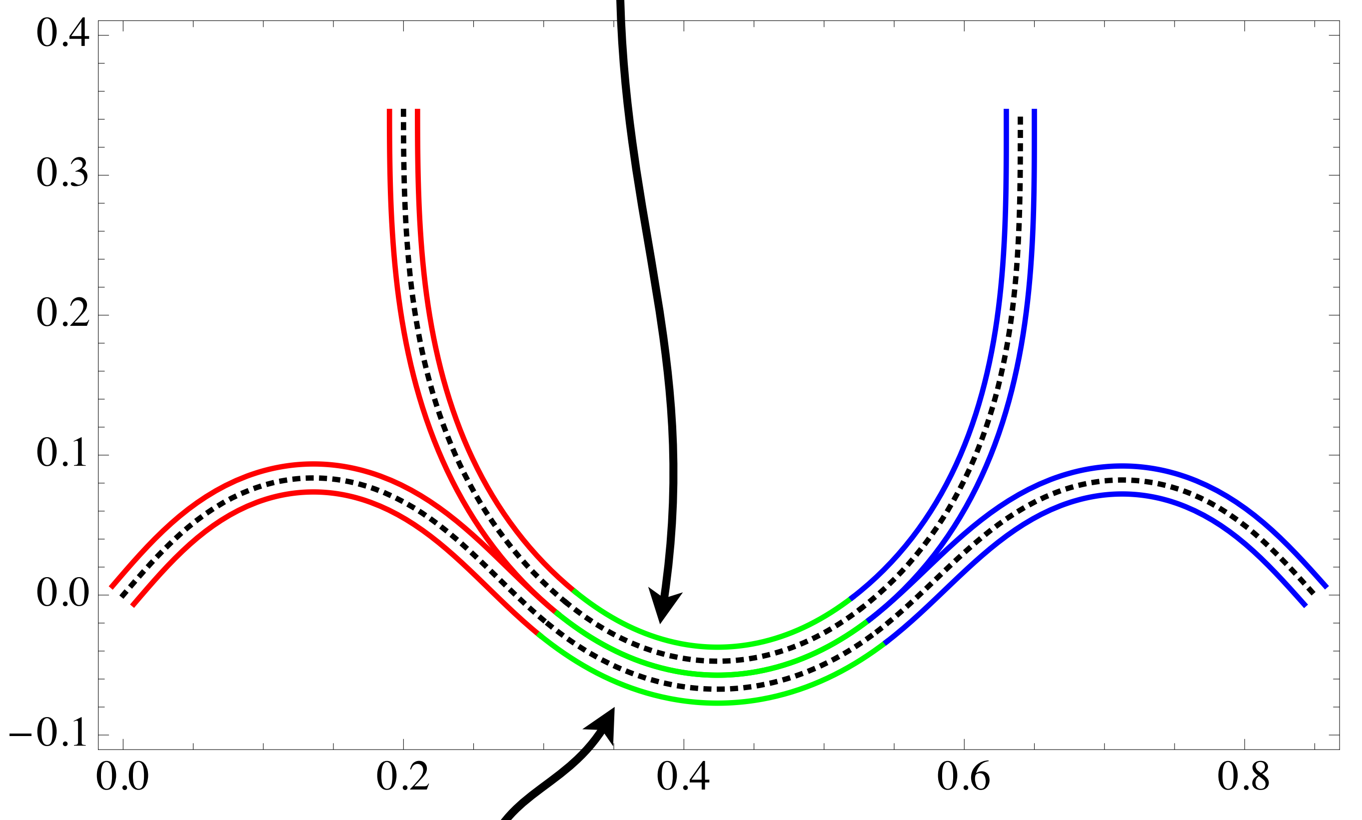
no contact: planar elastica

rod #1
$$\begin{cases} x'(s) = \cos\theta \\ y'(s) = \sin\theta \\ \theta'(s) = m \\ m'(s) = n_x \sin\theta - n_y \cos\theta \\ n_x'(s) = 0 \\ n_y'(s) = 0 \end{cases}$$

Close packing

$$\bar{x}'(\sigma) = \cos\bar{\theta}$$
$$\bar{y}'(\sigma) = \sin\bar{\theta}$$
$$\bar{\theta}'(\sigma) = \bar{m}$$

$$\bar{m}'(\sigma) = \bar{n}_x \sin\bar{\theta} - \bar{n}_y \cos\bar{\theta}$$
$$\bar{n}_x'(\sigma) = -\bar{p}(\sigma)\,\bar{u}_x(\sigma)$$
$$\bar{n}_y'(\sigma) = -\bar{p}(\sigma)\,\bar{u}_y(\sigma)$$



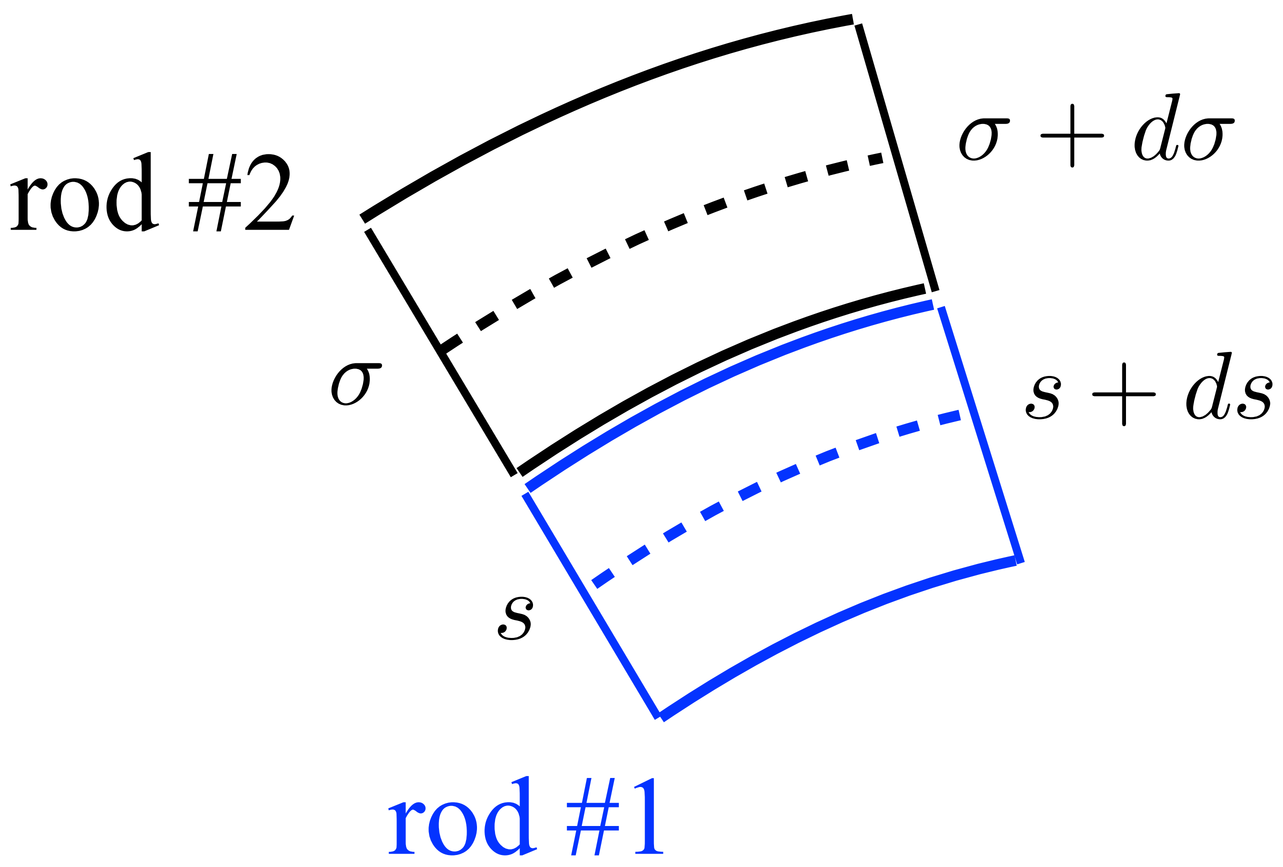
unknown functions

$$p(s), u_x(s), u_y(s)$$
$$\bar{p}(\sigma), \bar{u}_x(\sigma), \bar{u}_y(\sigma)$$

$$x'(s) = \cos\theta$$
$$y'(s) = \sin\theta$$
$$\theta'(s) = m$$

$$m'(s) = n_x \sin\theta - n_y \cos\theta$$
$$n_x'(s) = -p(s)\,u_x(s)$$
$$n_y'(s) = -p(s)\,u_y(s)$$

Close packing



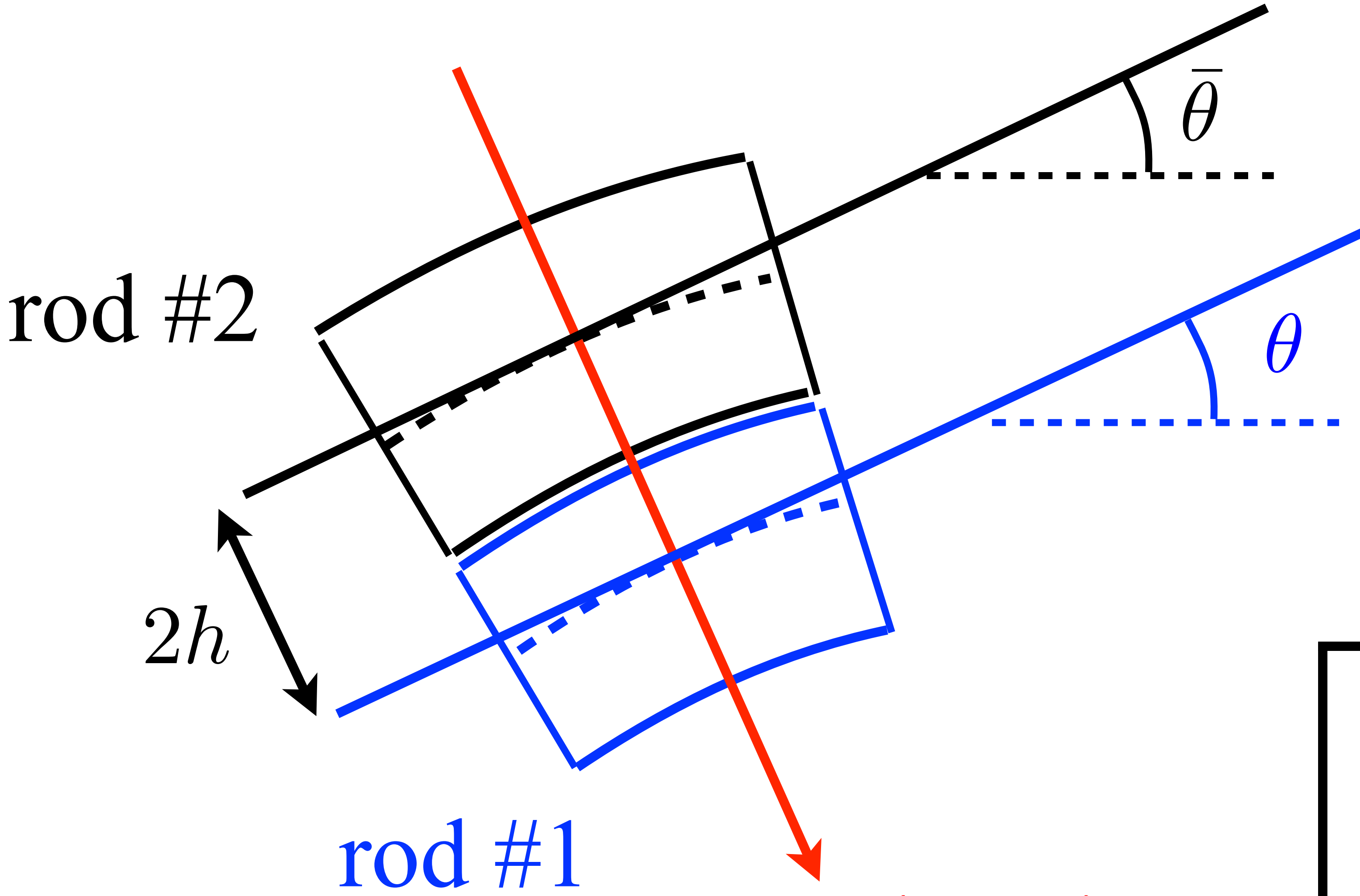
force balance & action-reaction

$$\bar{p}(\sigma)d\sigma = p(s)ds$$

$$\bar{u}_x(\sigma) = -u_x(s)$$

$$\bar{u}_y(\sigma) = -u_y(s)$$

Close packing



$$\vec{u}(s) = \begin{pmatrix} u_x \\ u_y \end{pmatrix}$$

contact conditions

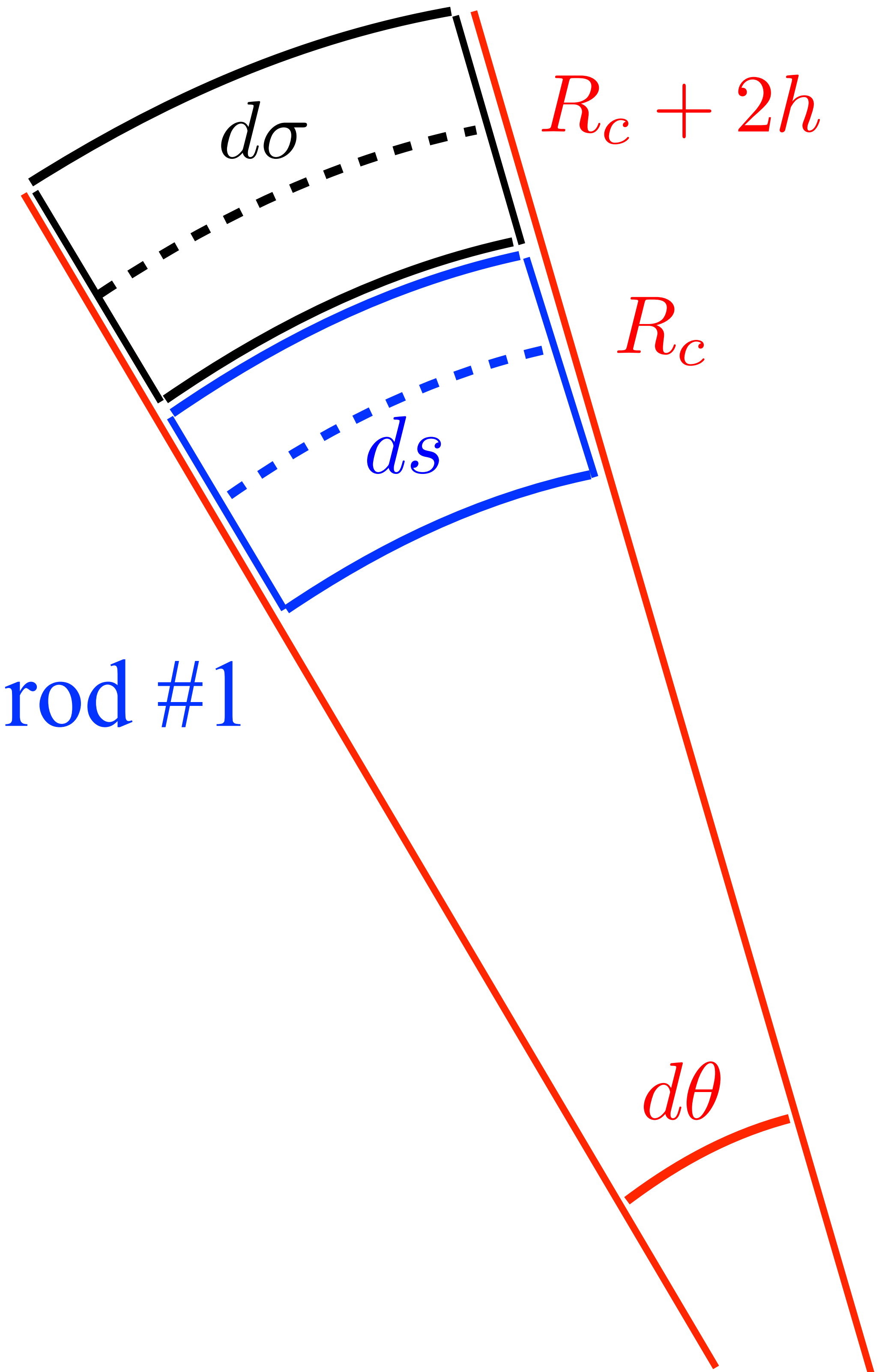
$$\theta(s) = \bar{\theta}(\sigma)$$

$$u_x = \sin\theta$$

$$u_y = -\cos\theta$$

$$(x-\bar{x})^2 + (y-\bar{y})^2 = 4h^2$$

Close packing



arc-length relation

$$d\sigma = (1 - 2h)\,\theta'(s)\,ds$$

where $$\frac{1}{R_c} = \theta'(s)$$
$$= m(s)$$

slave variable

$$\sigma = \sigma(s)$$

Close packing

full set of equilibrium equations

$$\begin{aligned} x'(s) &= \cos\theta \\ y'(s) &= \sin\theta \\ \theta'(s) &= m \\ m'(s) &= n_x \sin\theta - n_y \cos\theta \\ n_x'(s) &= -p\sin\theta \\ n_y'(s) &= p\cos\theta \end{aligned}$$

$$\begin{aligned} \bar{x}'(s) &= (1-2hm)\cos\theta \\ \bar{y}'(s) &= (1-2hm)\sin\theta \\ \bar{m}'(s) &= (1-2hm)(\bar{n}_x \sin\theta - \bar{n}_y \cos\bar{\theta}) \\ \bar{n}_x'(s) &= p\sin\theta \\ \bar{n}_y'(s) &= -p\cos\theta \end{aligned}$$

$$\frac{d\sigma}{ds} = 1 - 2hm$$

$$\frac{d\theta}{ds} = m$$

$$\frac{d\bar{\theta}}{d\sigma} = \bar{m}$$

$$m = (1-2hm)\bar{m}$$

$$\downarrow \; d^2/ds^2$$

$$p(s) = \frac{6h(1-2hm)^2 v\bar{v} + mt - m(1-2hm)^3\bar{t}}{1+(1-2hm)^3}$$

where

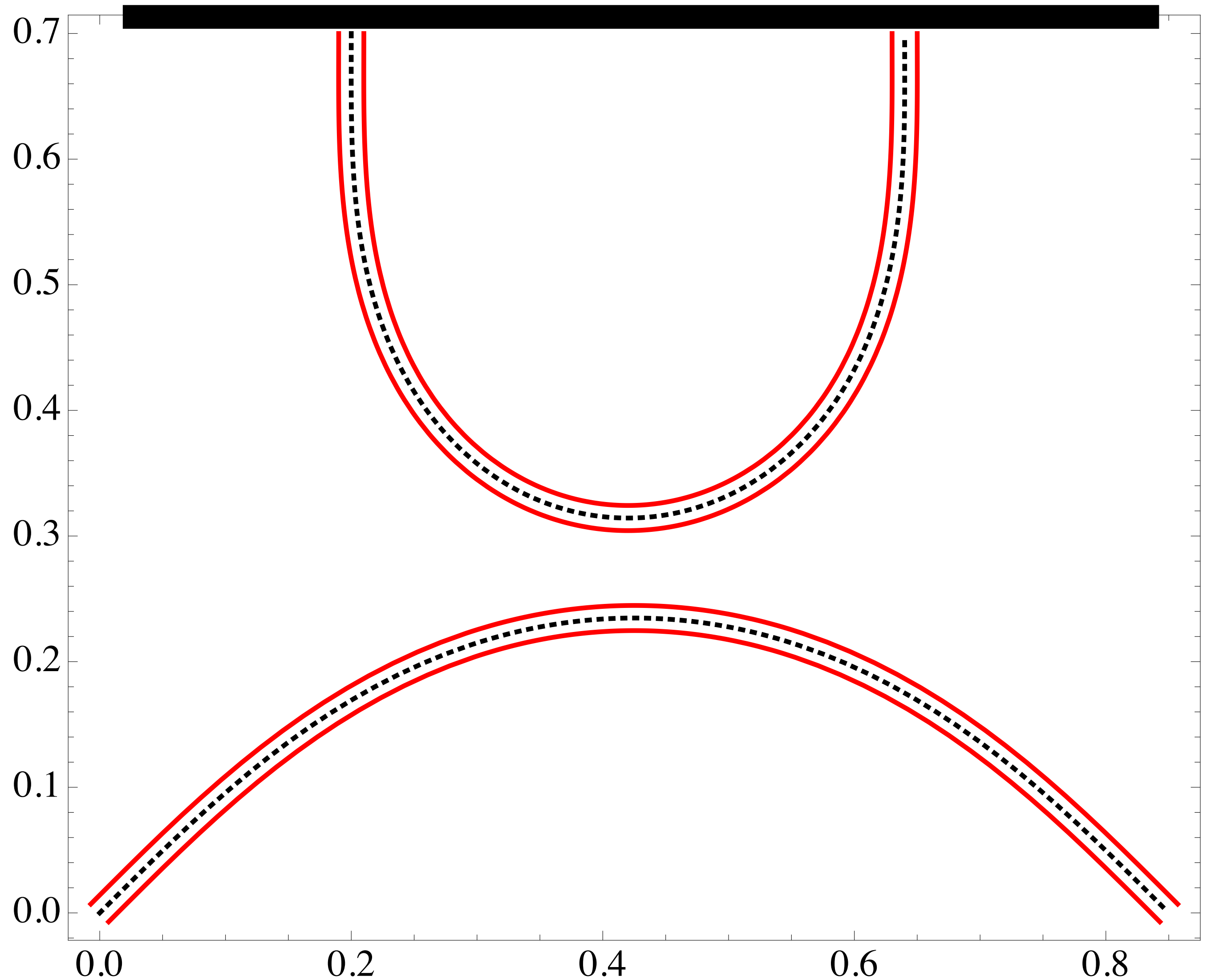
$$v = n_x \sin\theta - n_y \cos\theta$$

$$t = n_x \cos\theta + n_y \sin\theta$$

Close packing

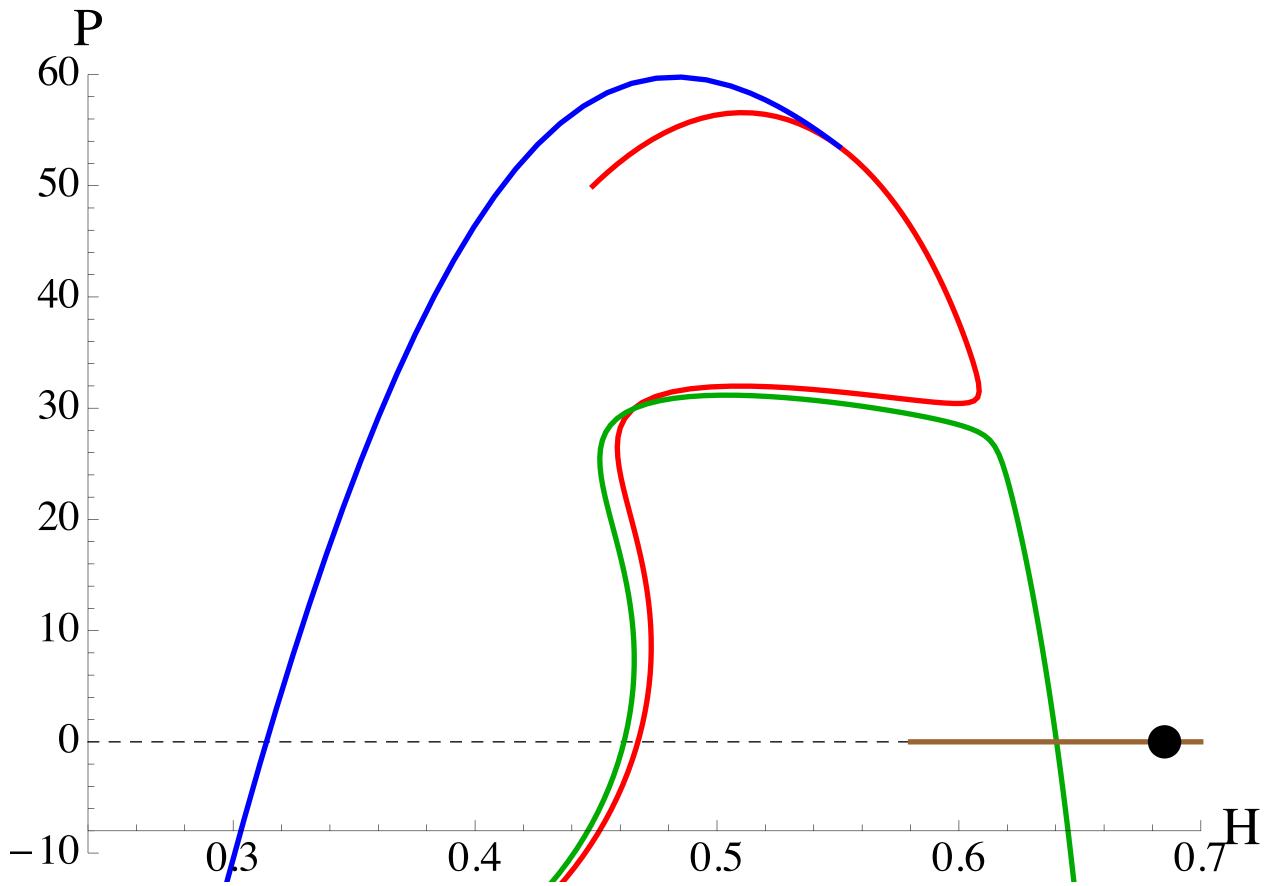
$$\bar{\theta}(0) = -\frac{\pi}{2} \qquad \bar{\theta}(1) = +\frac{\pi}{2}$$

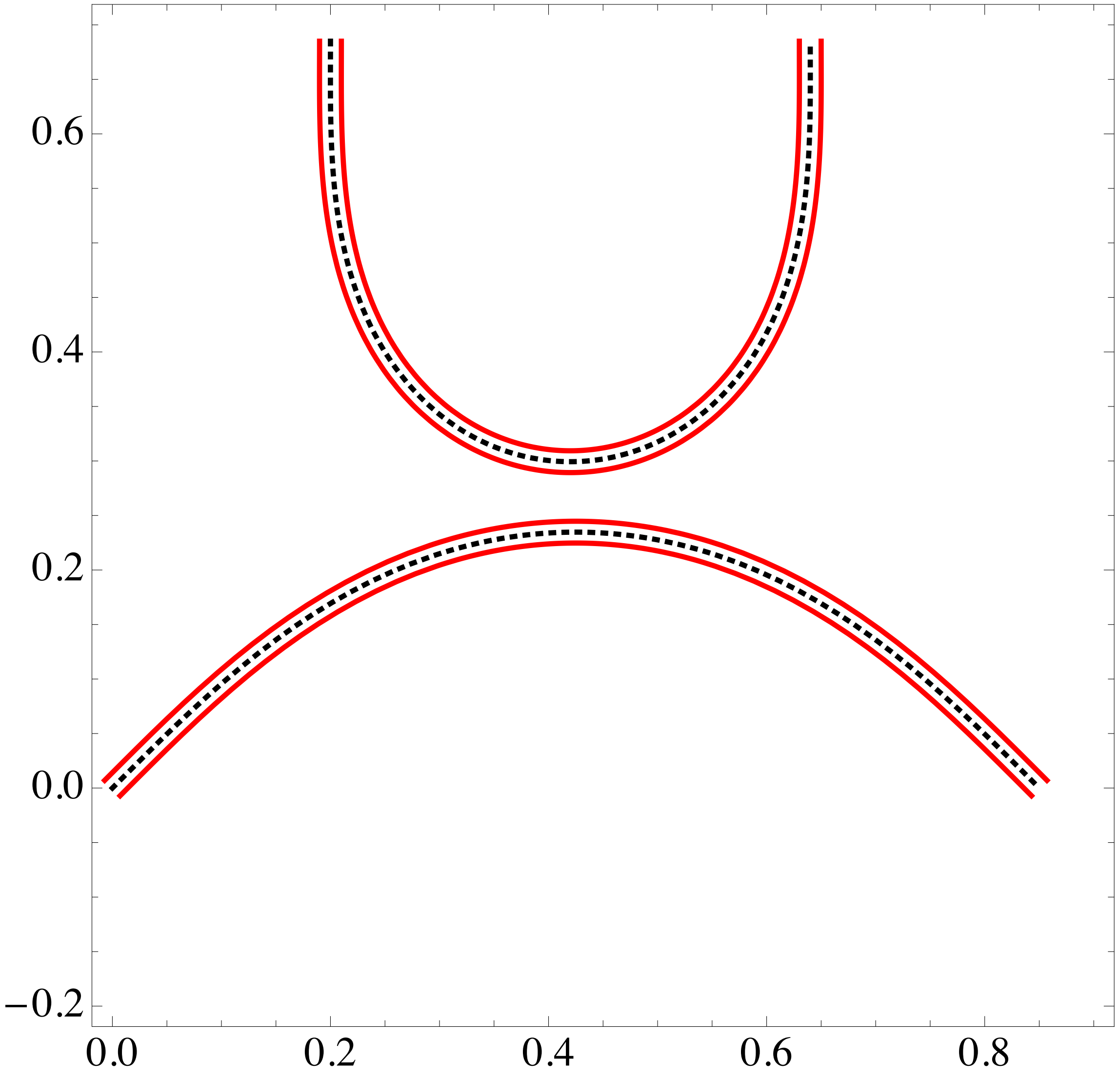
boundary
value
problem



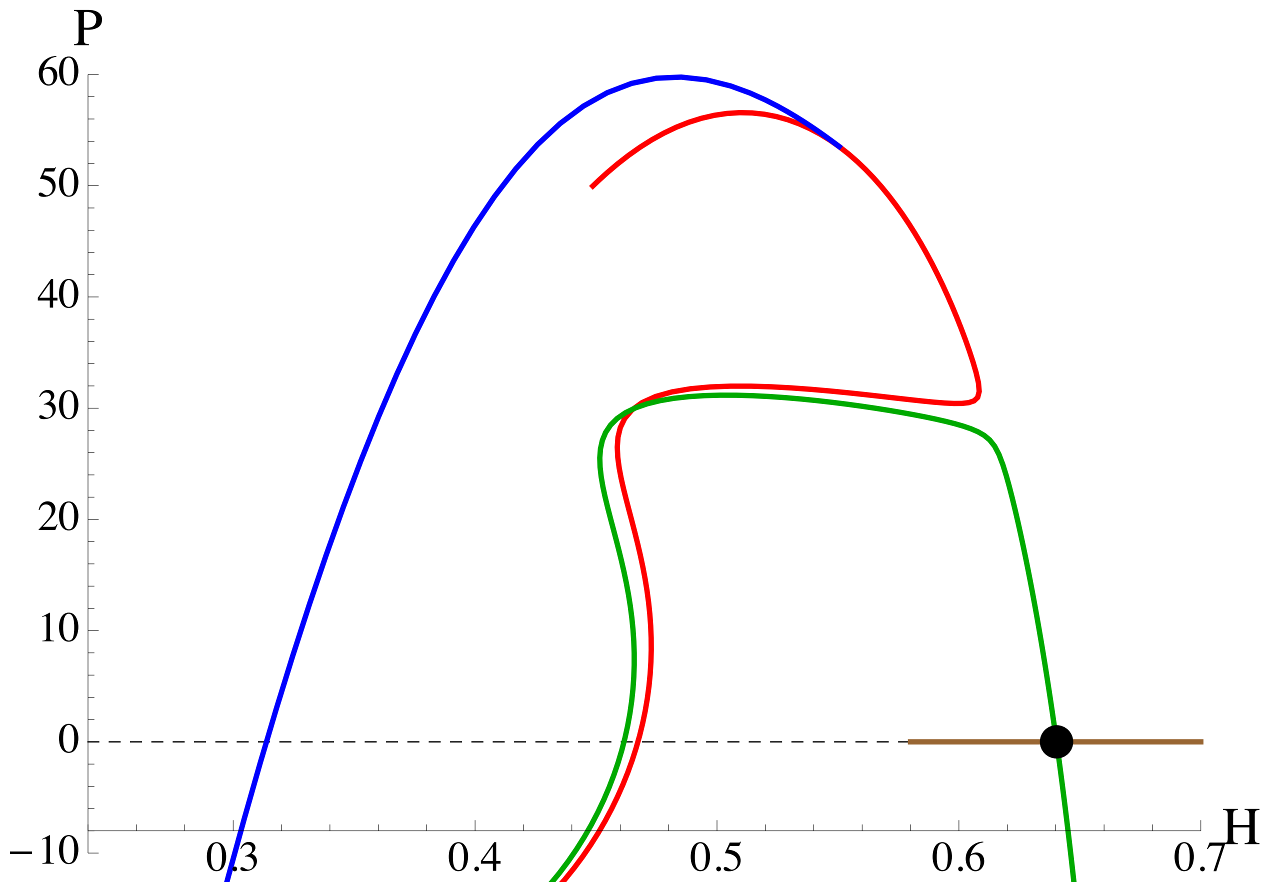
$$m(0) = 0 \qquad m(1) = 0$$

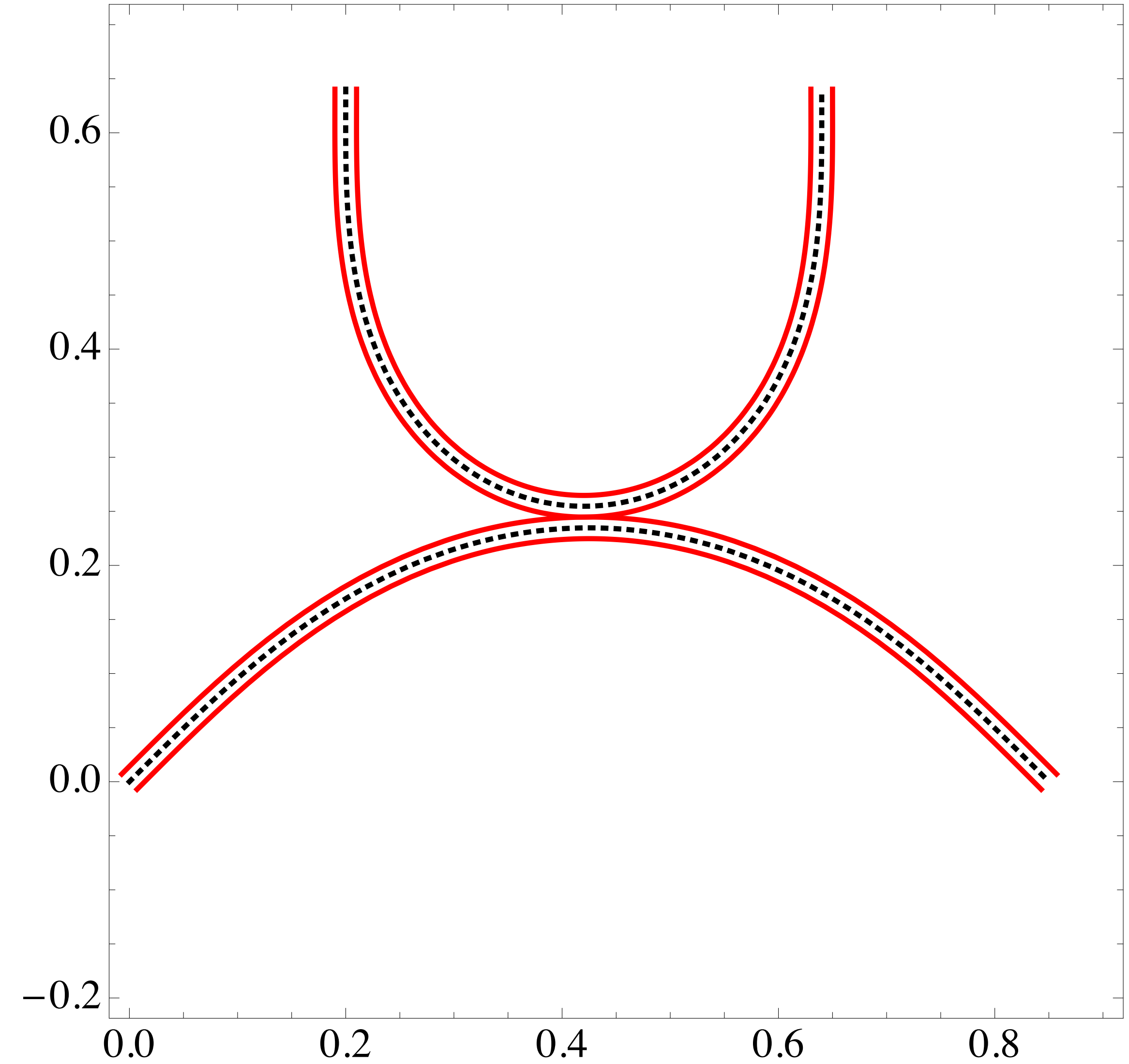
Bifurcation curves



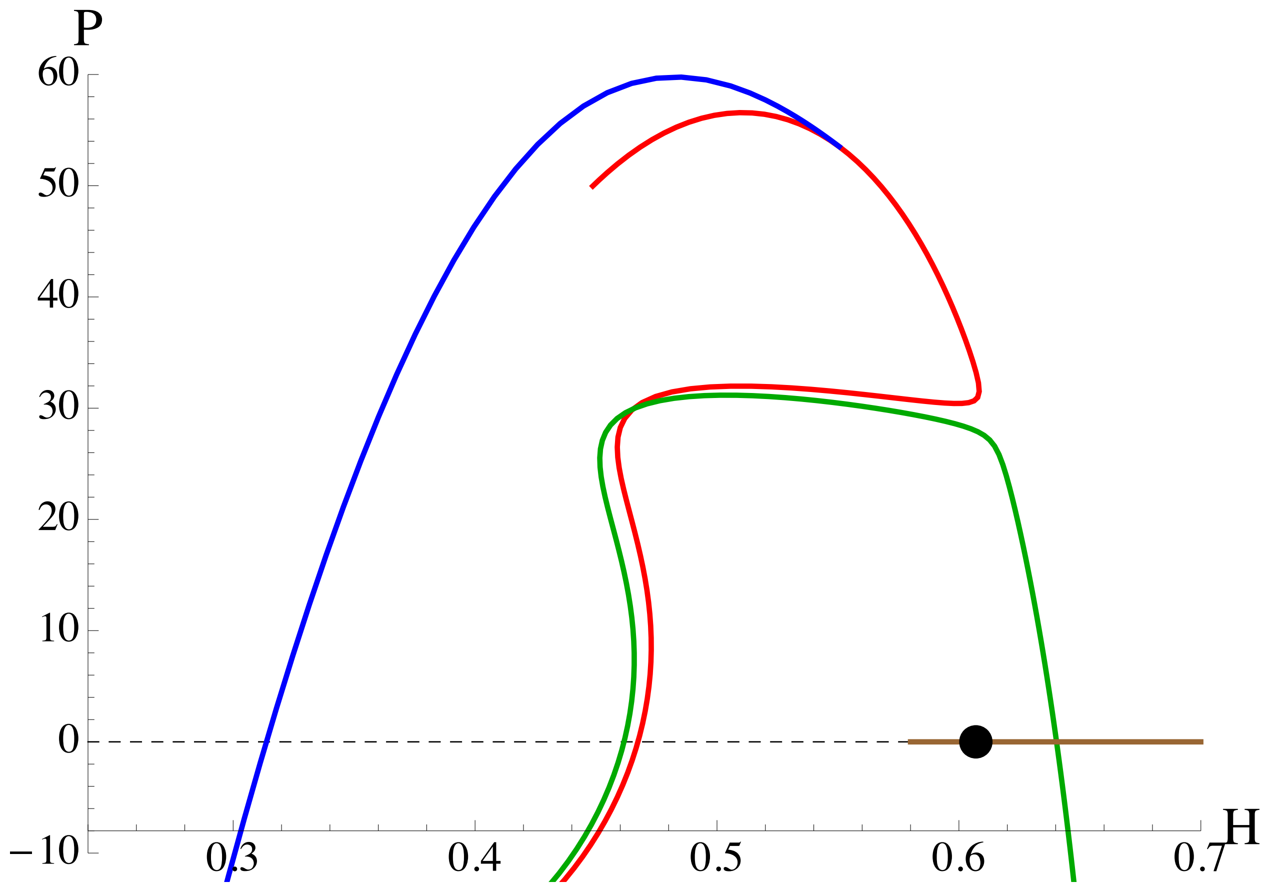


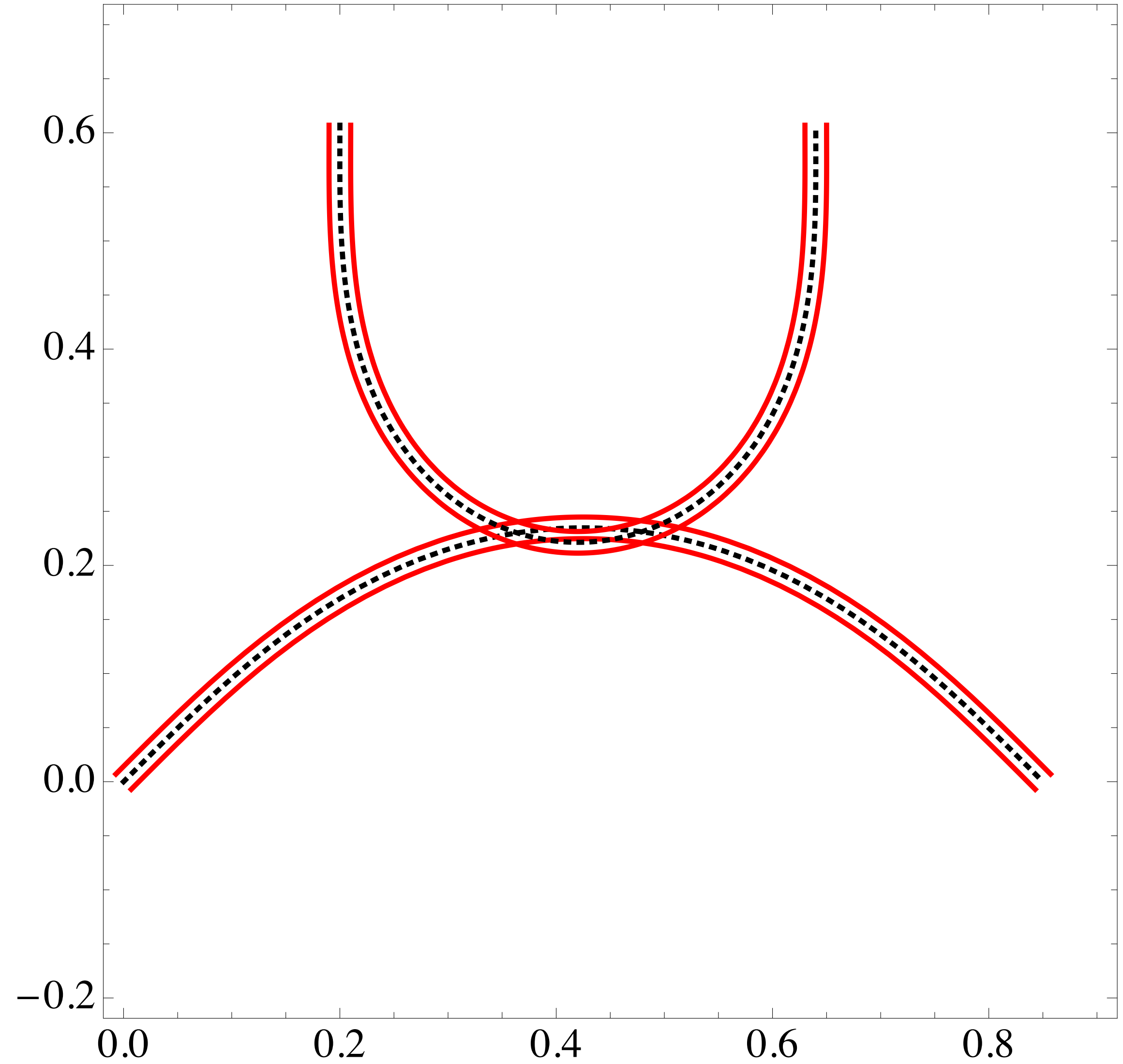
Bifurcation curves



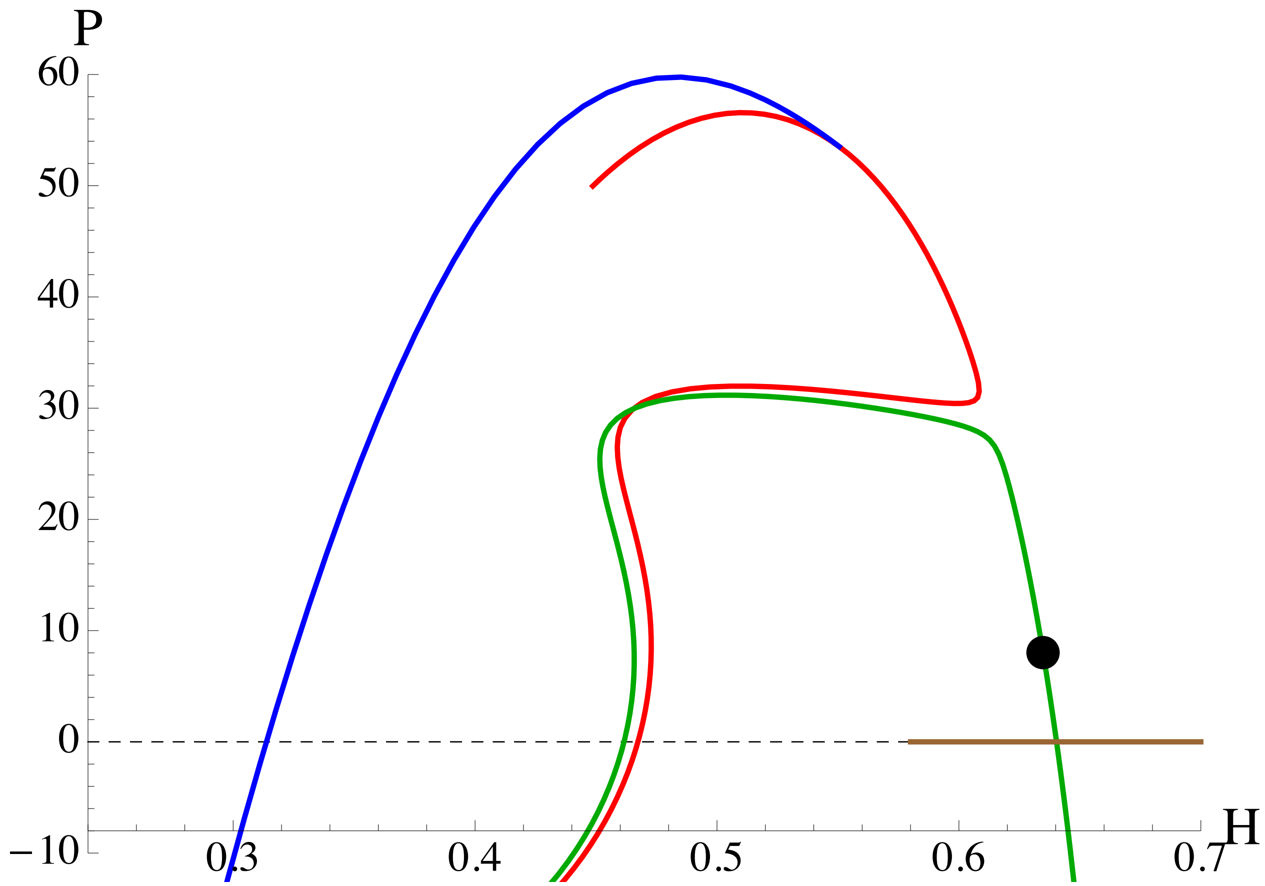


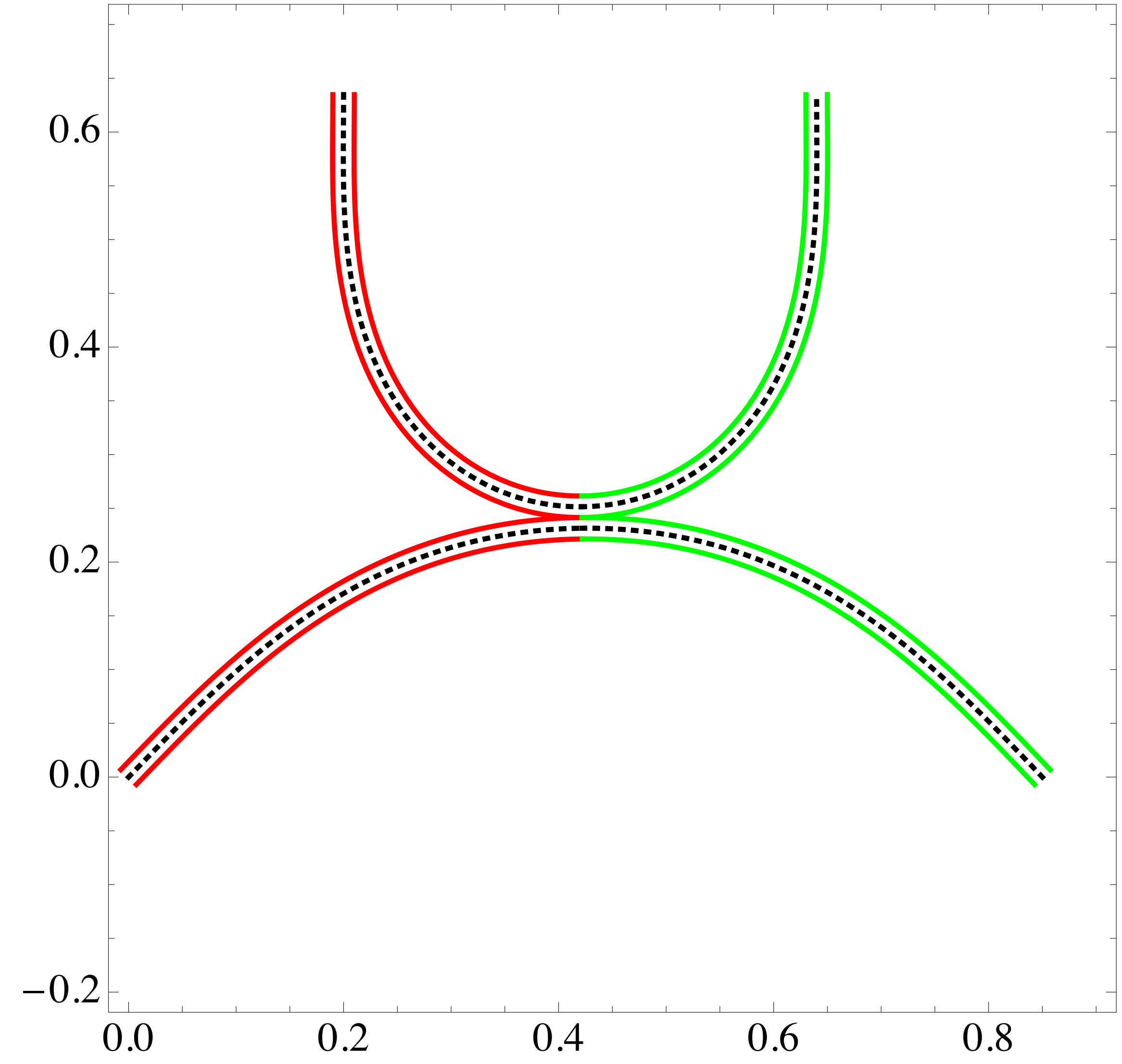
Bifurcation curves



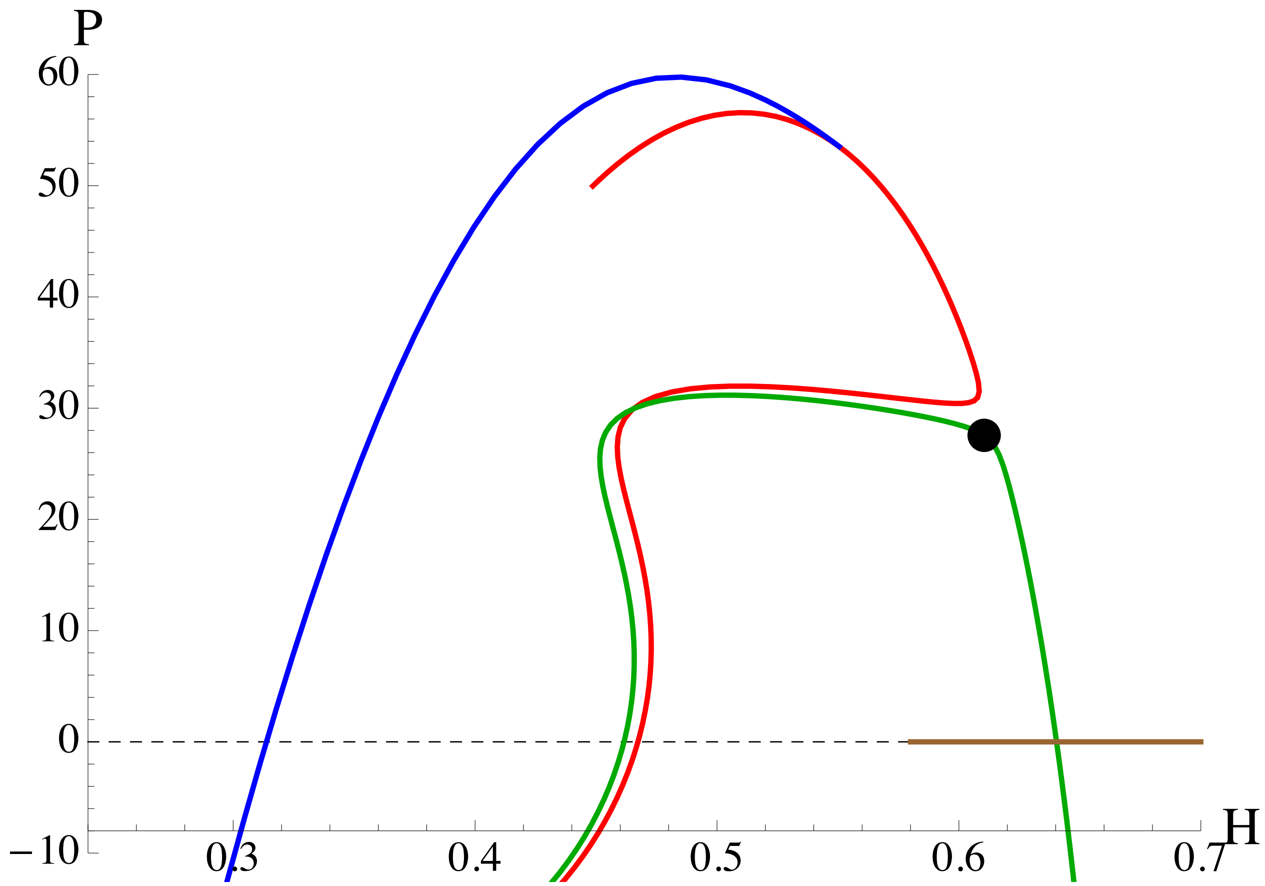


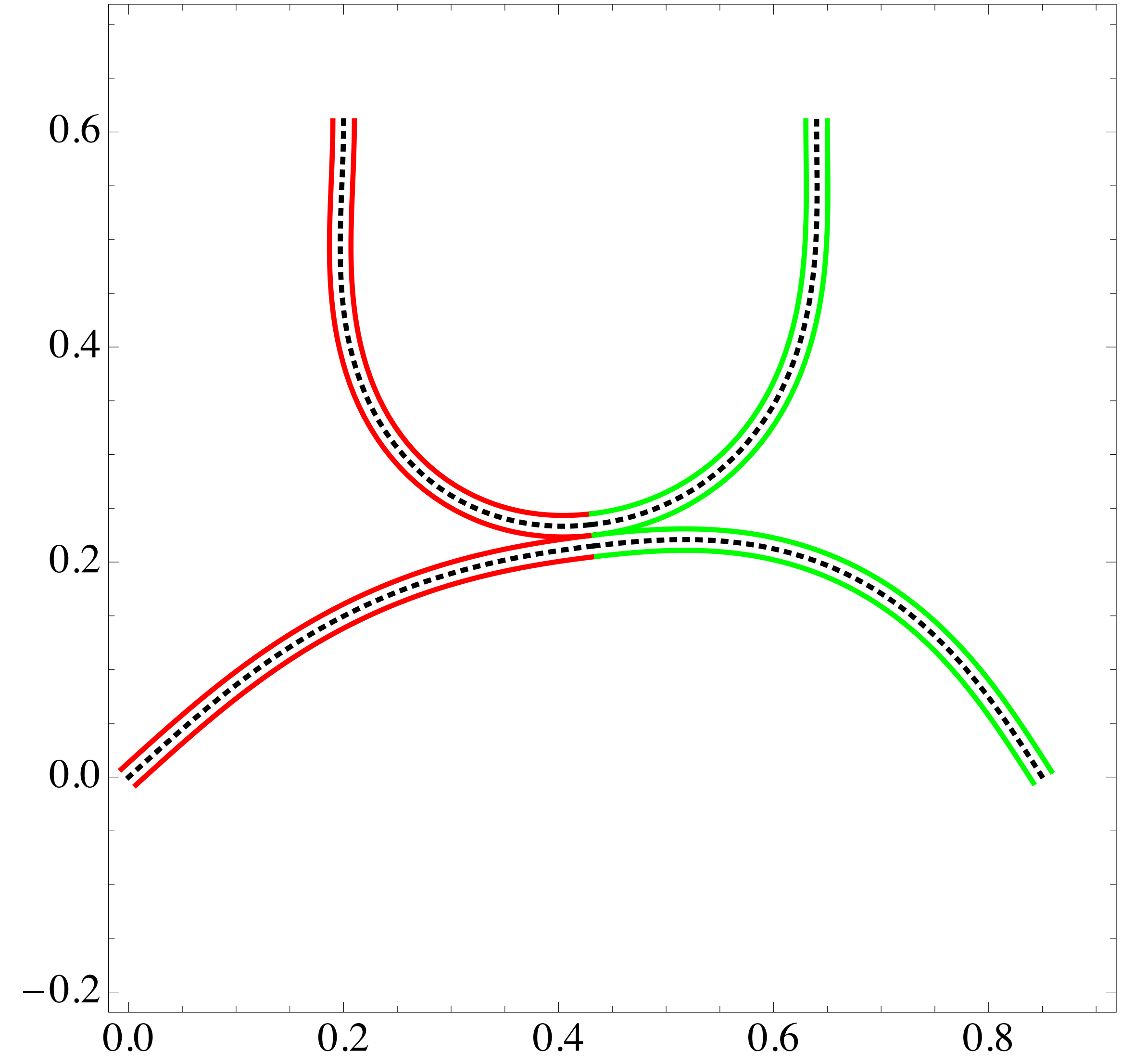
Bifurcation curves



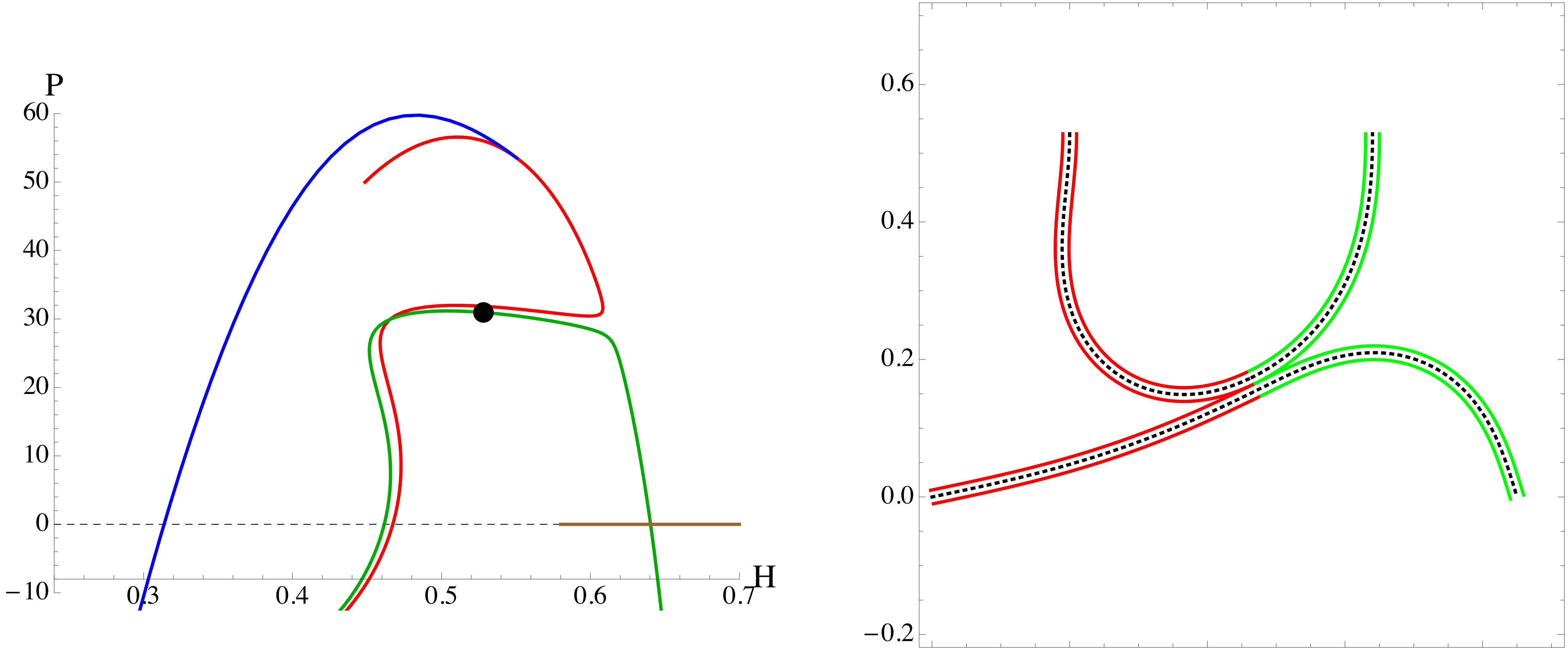


Bifurcation curves

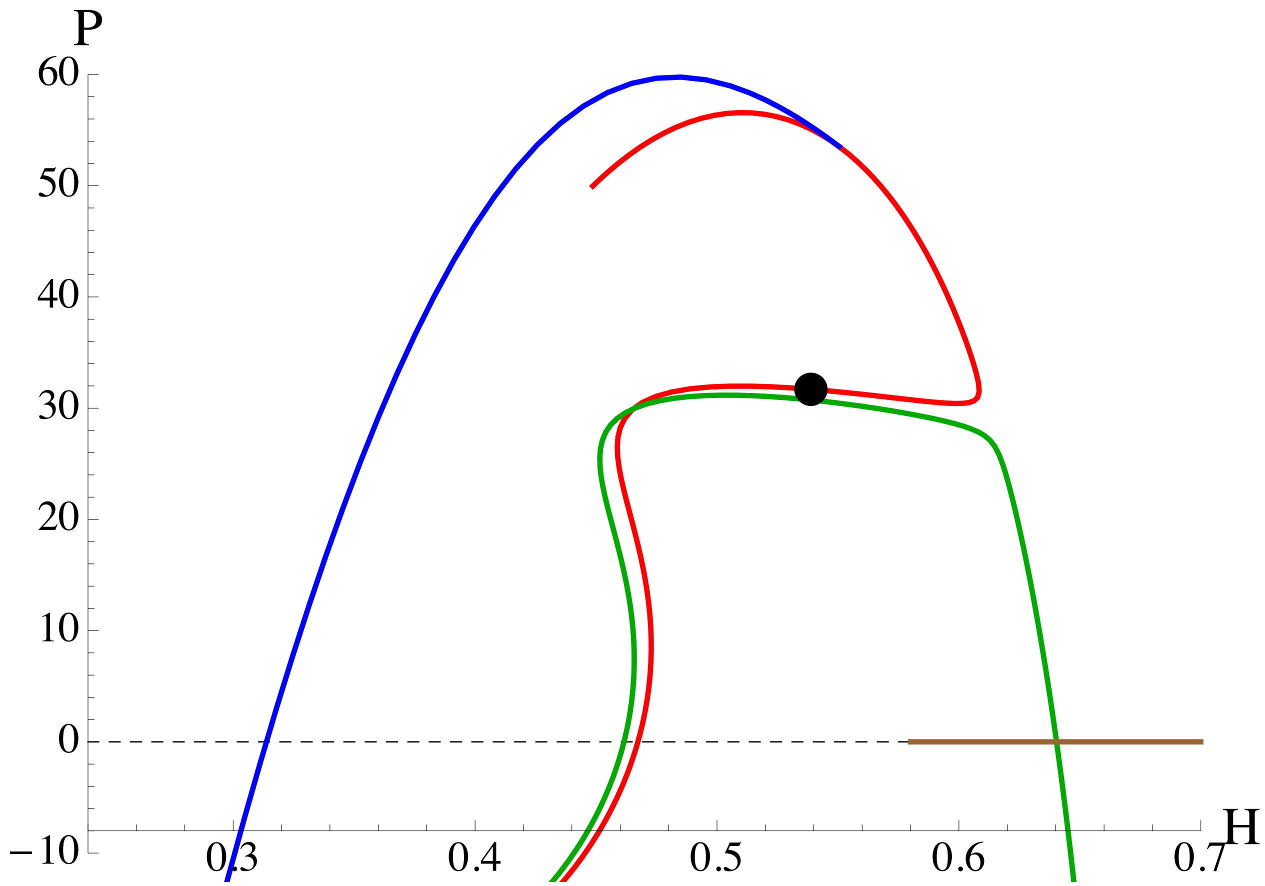


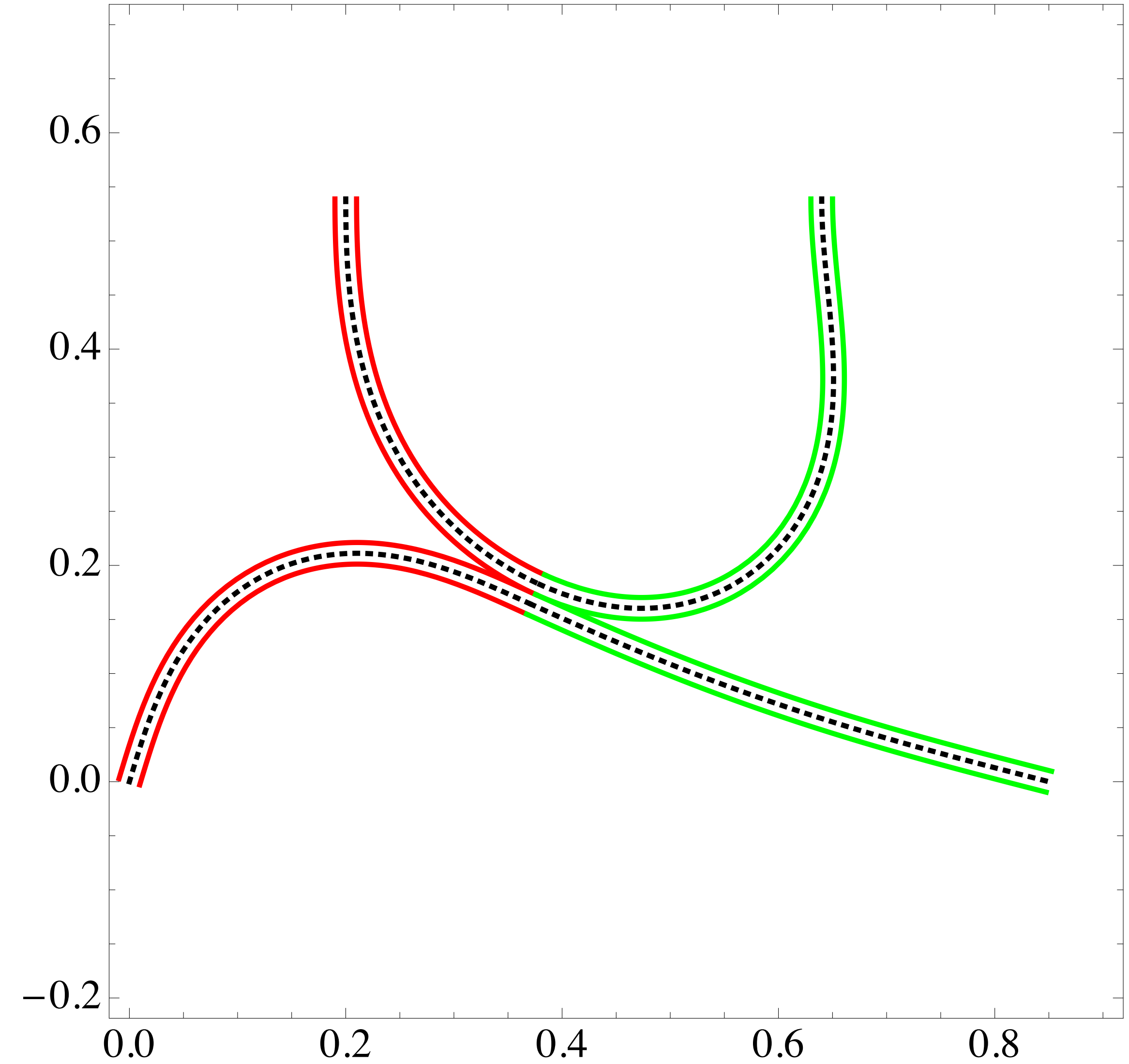


Bifurcation curves

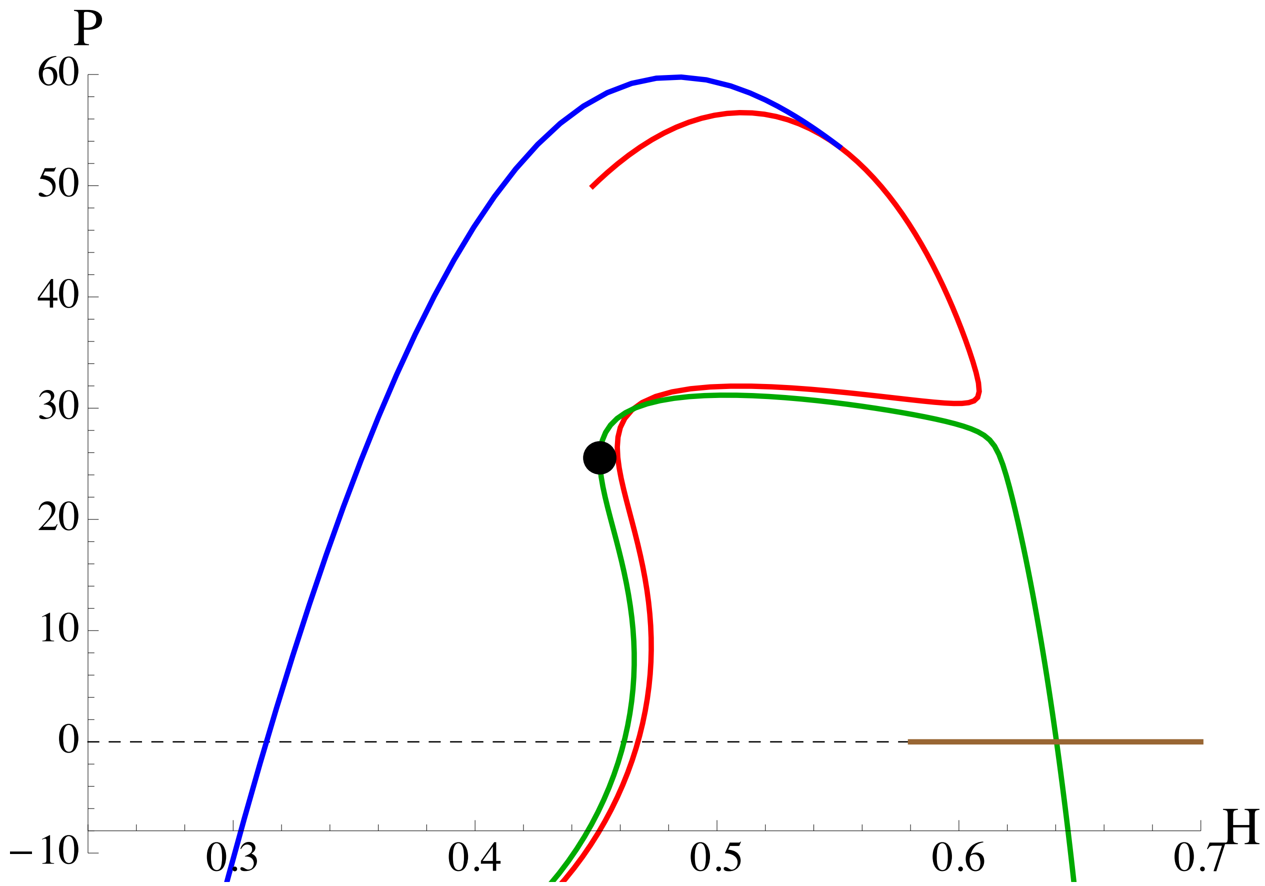


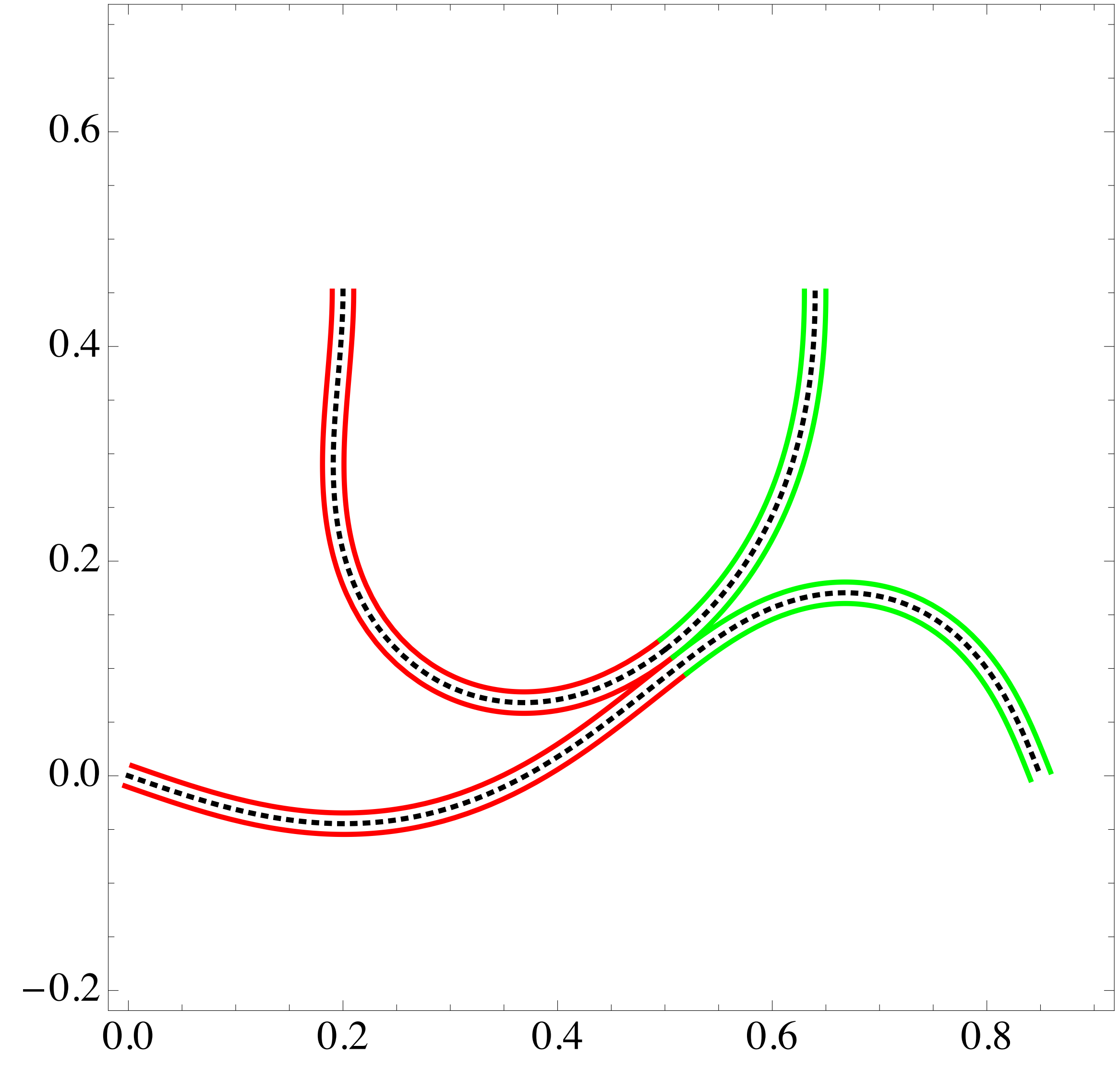
Bifurcation curves



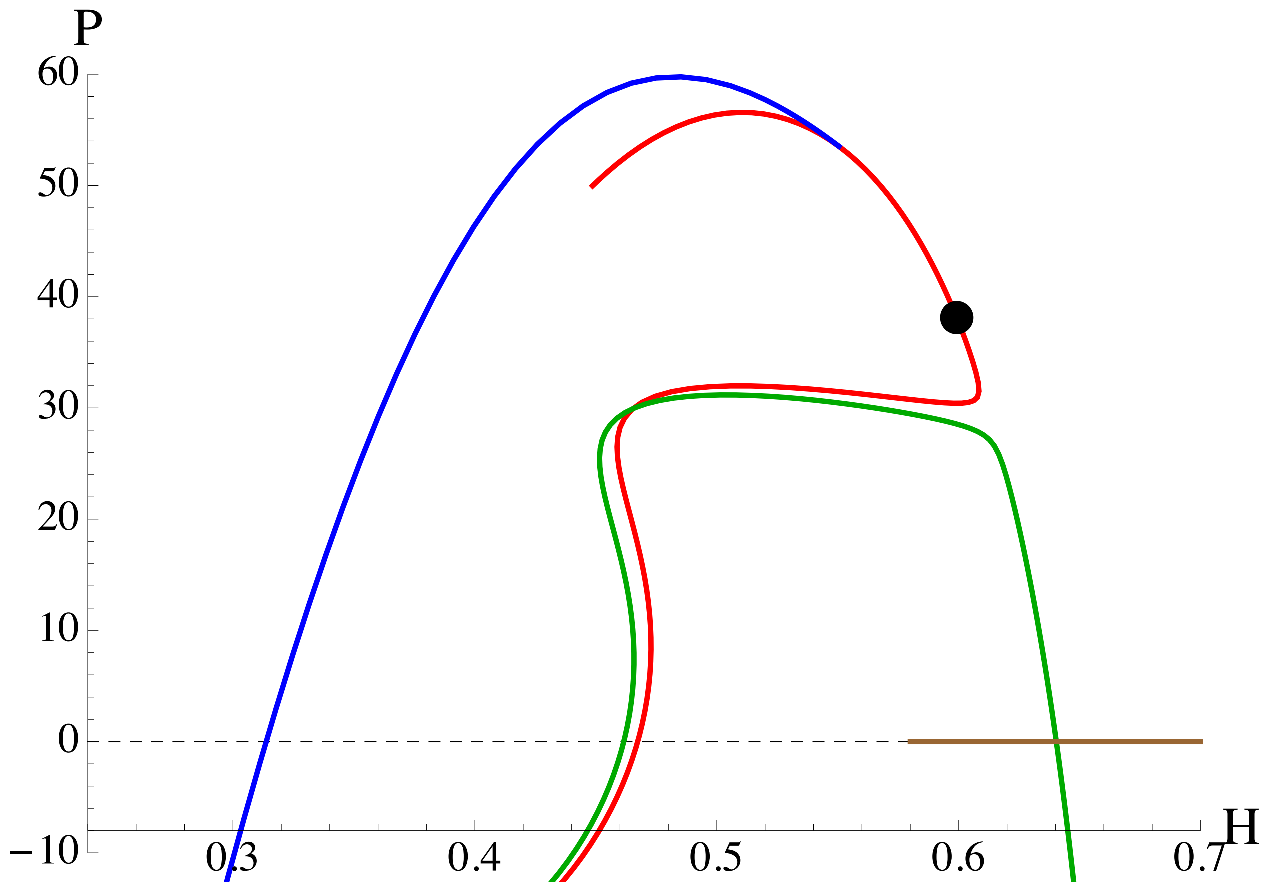


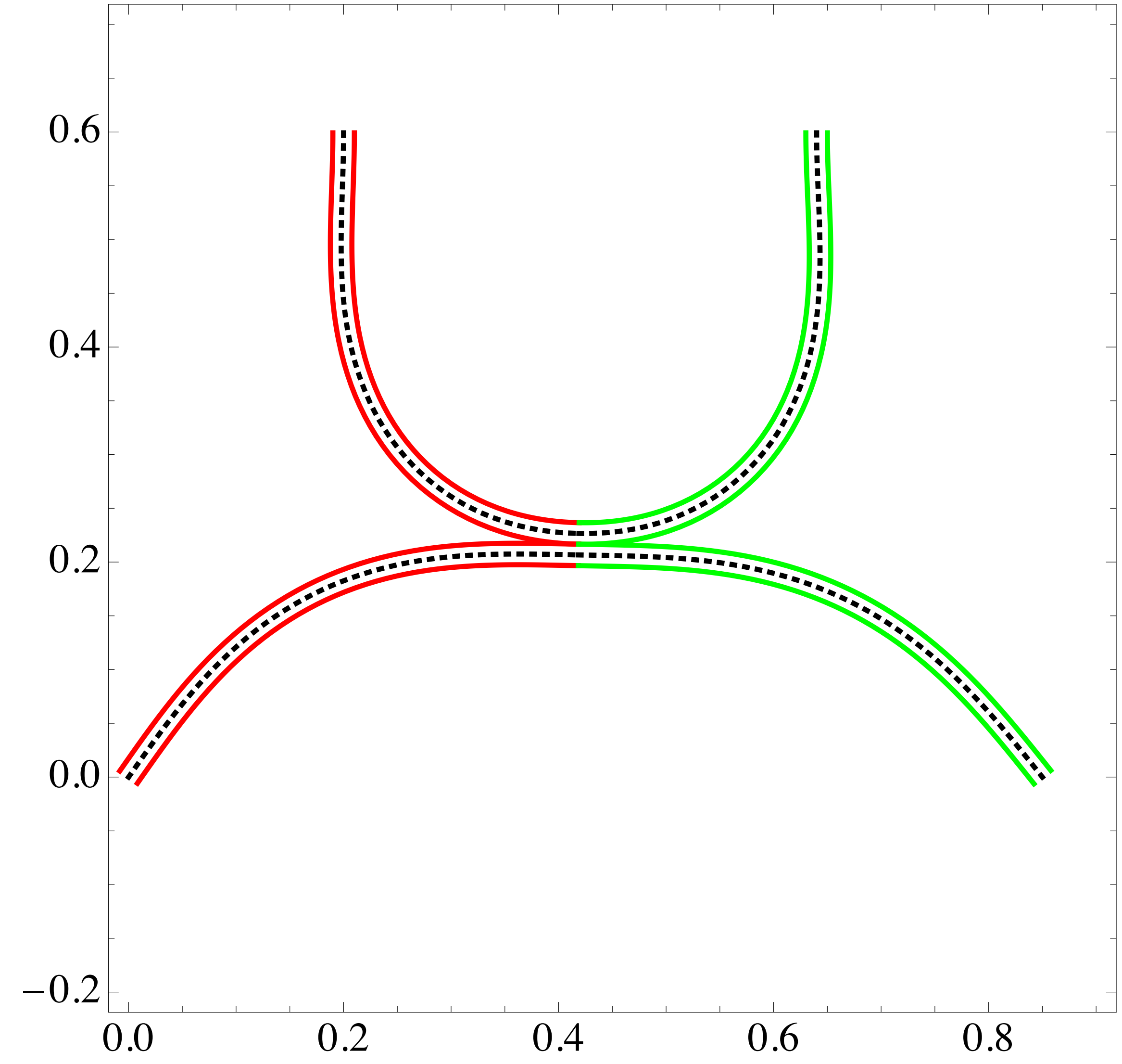
Bifurcation curves



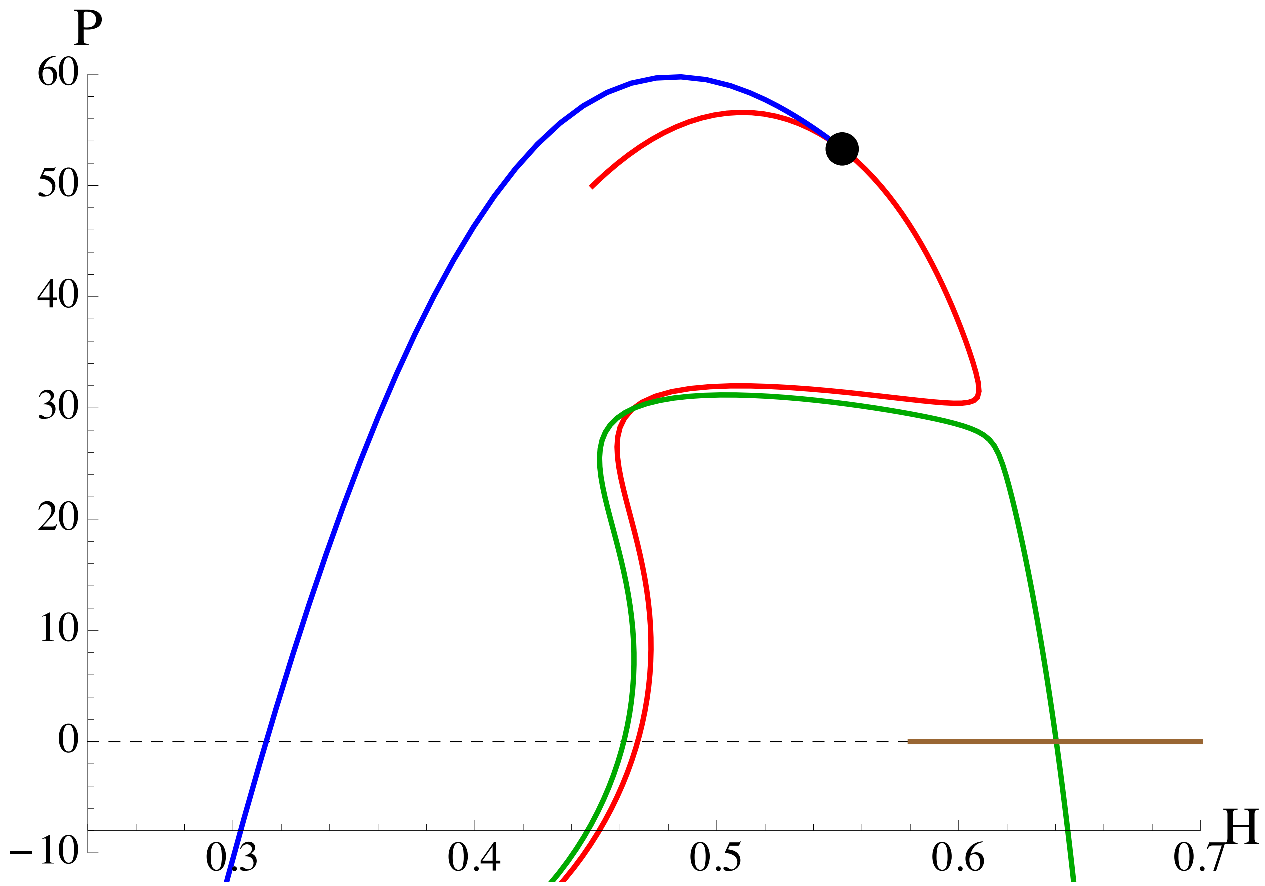


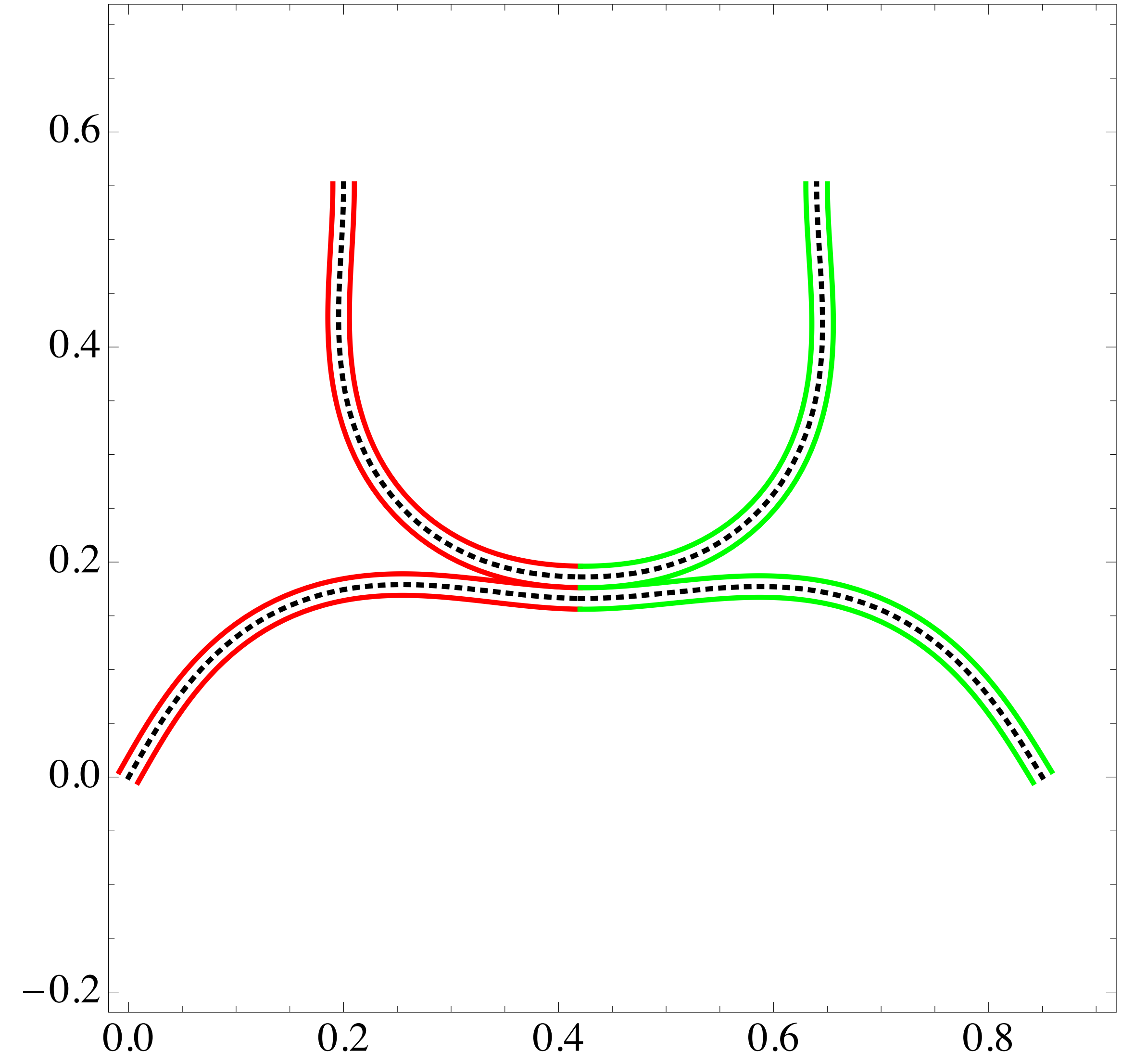
Bifurcation curves



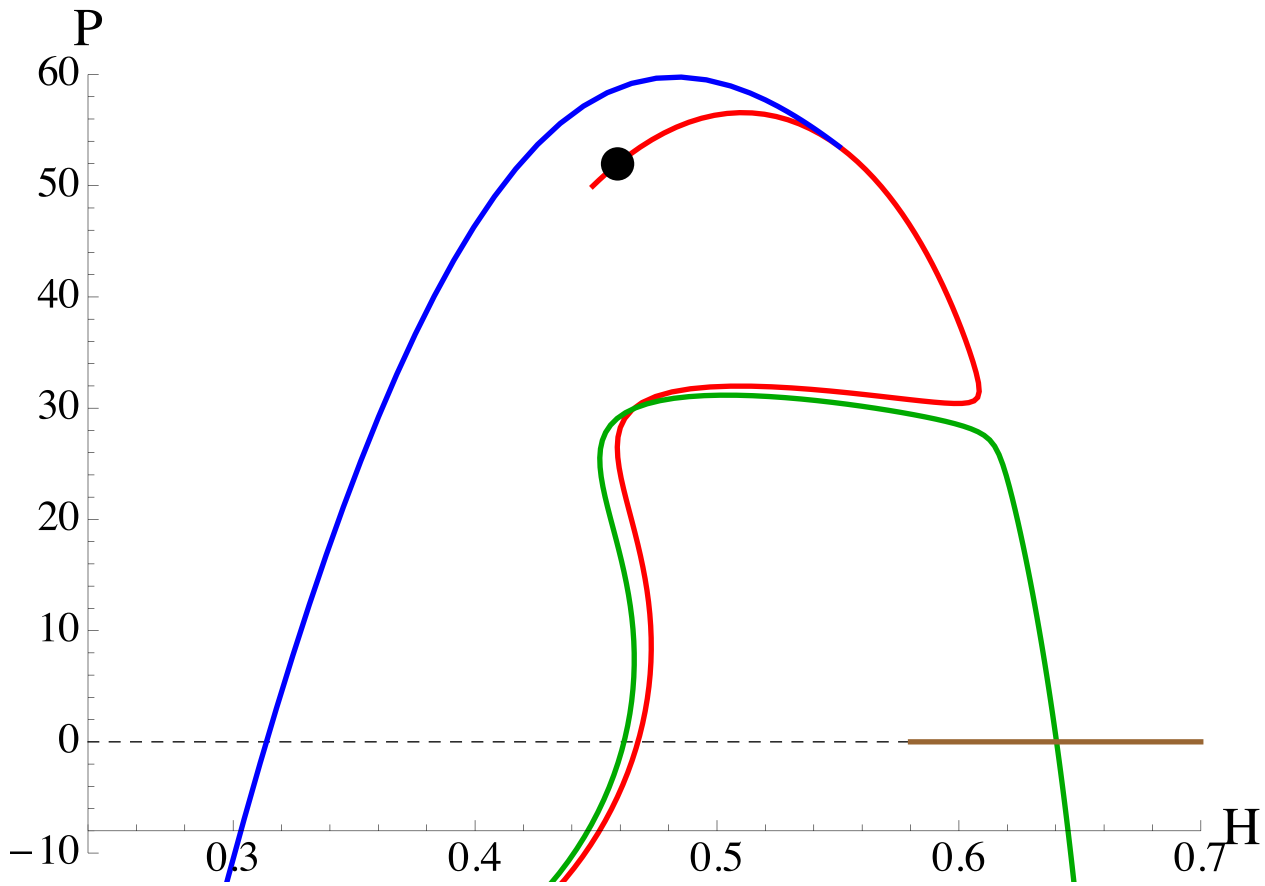


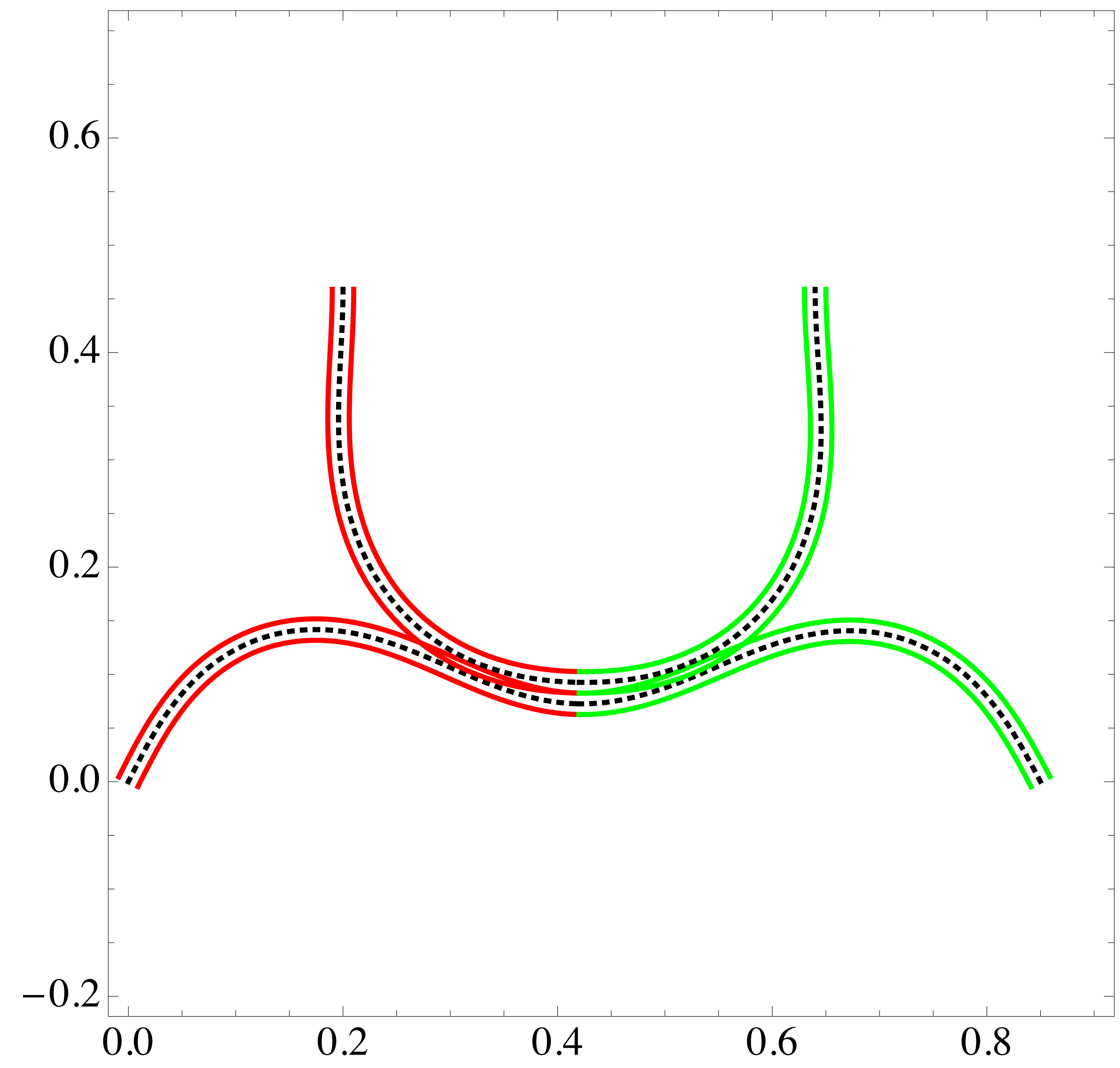
Bifurcation curves



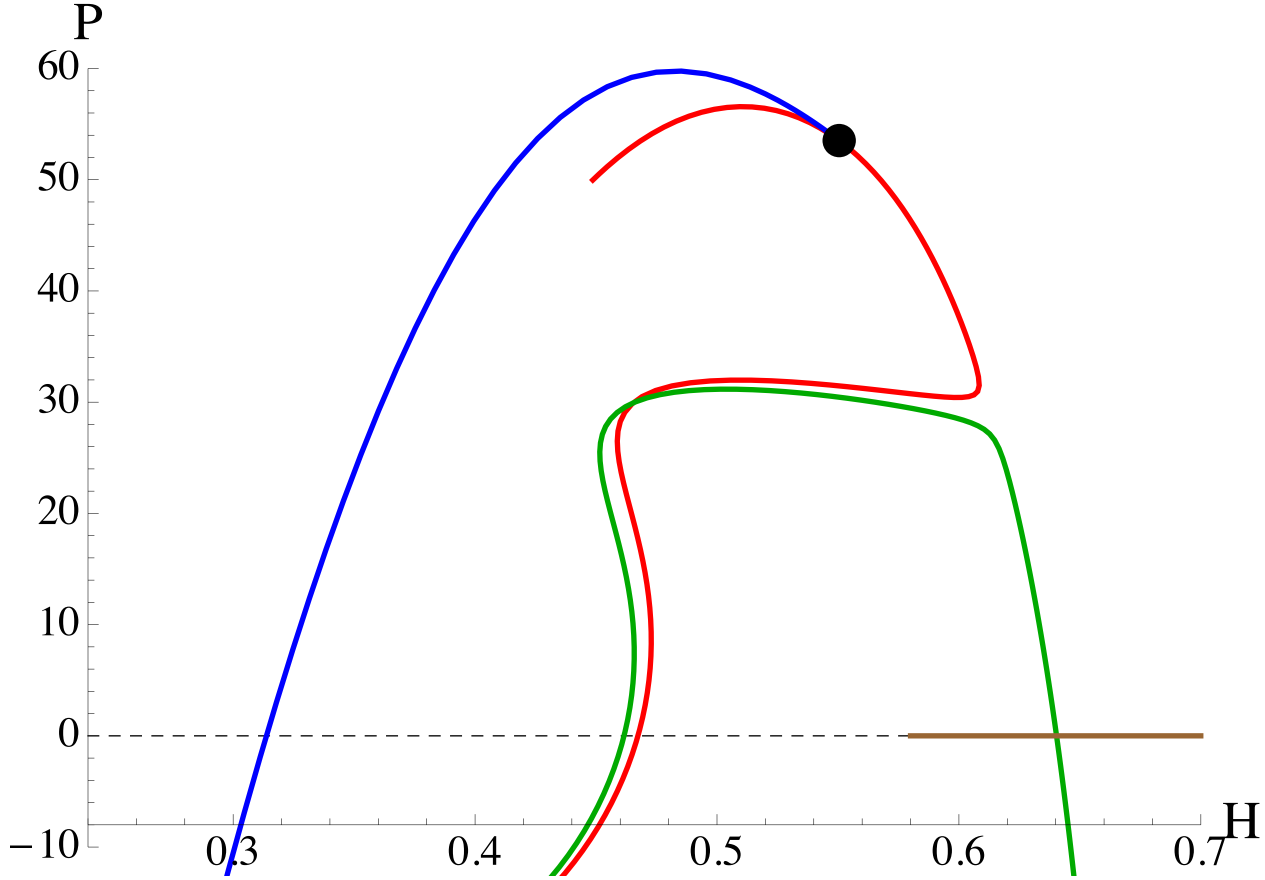


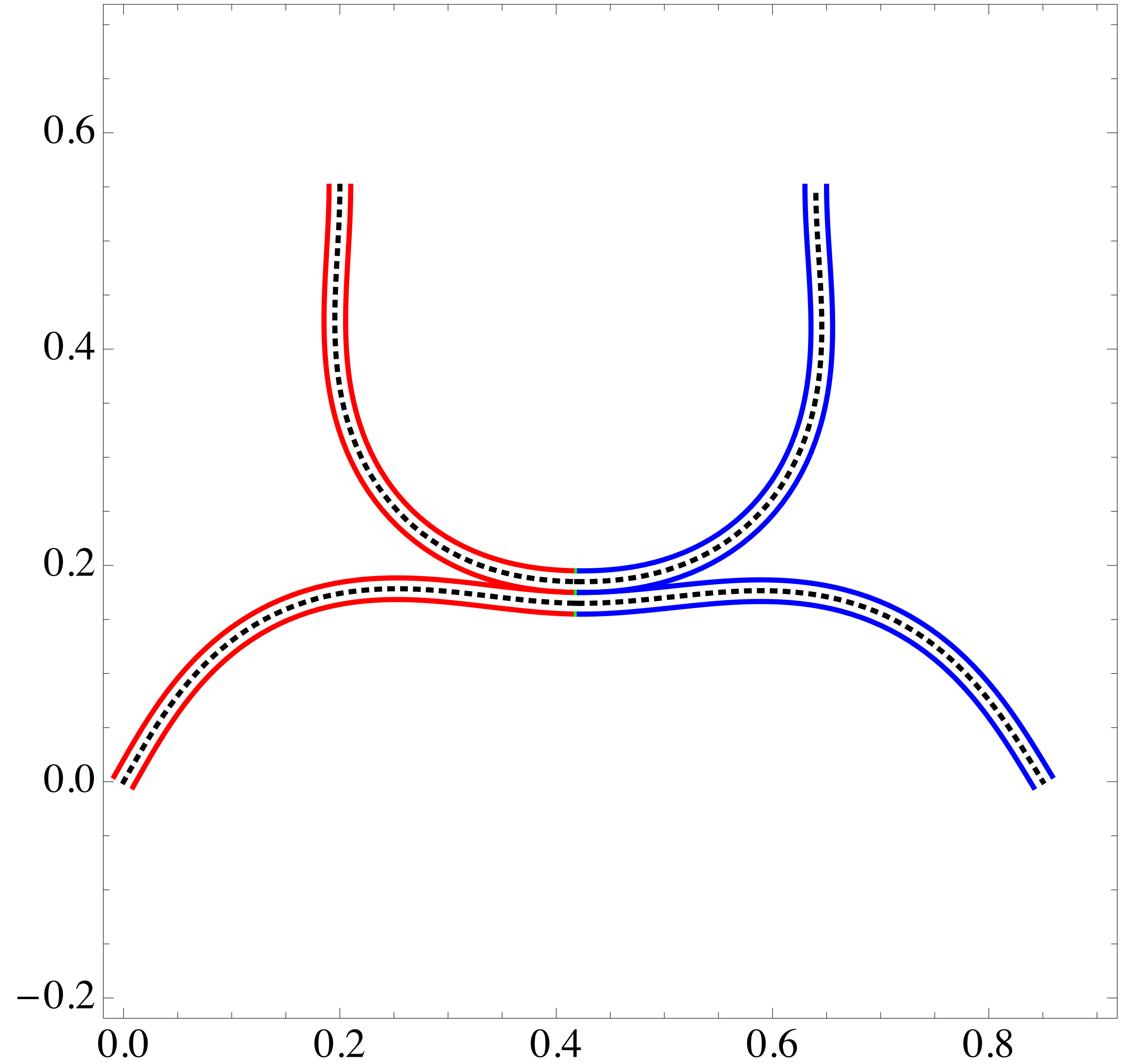
Bifurcation curves



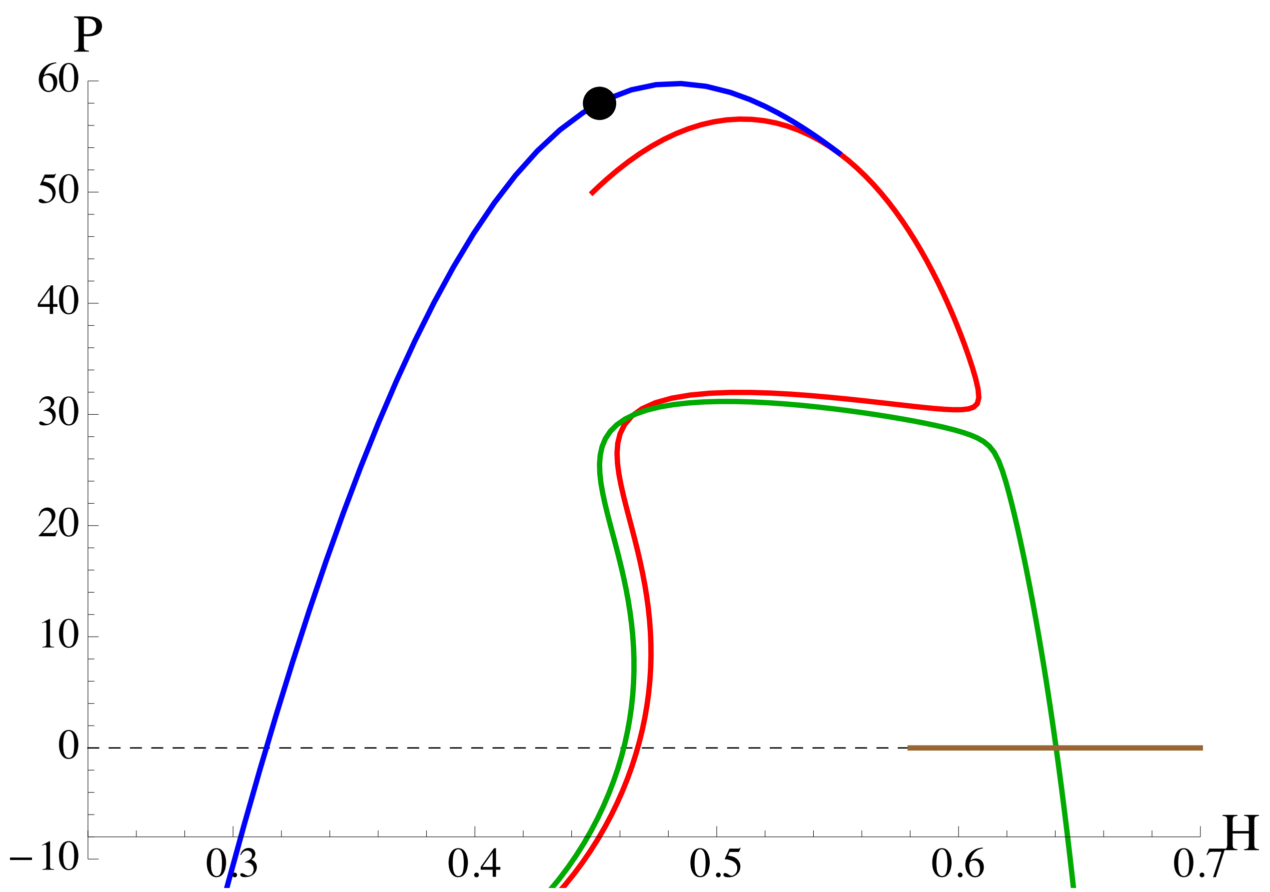


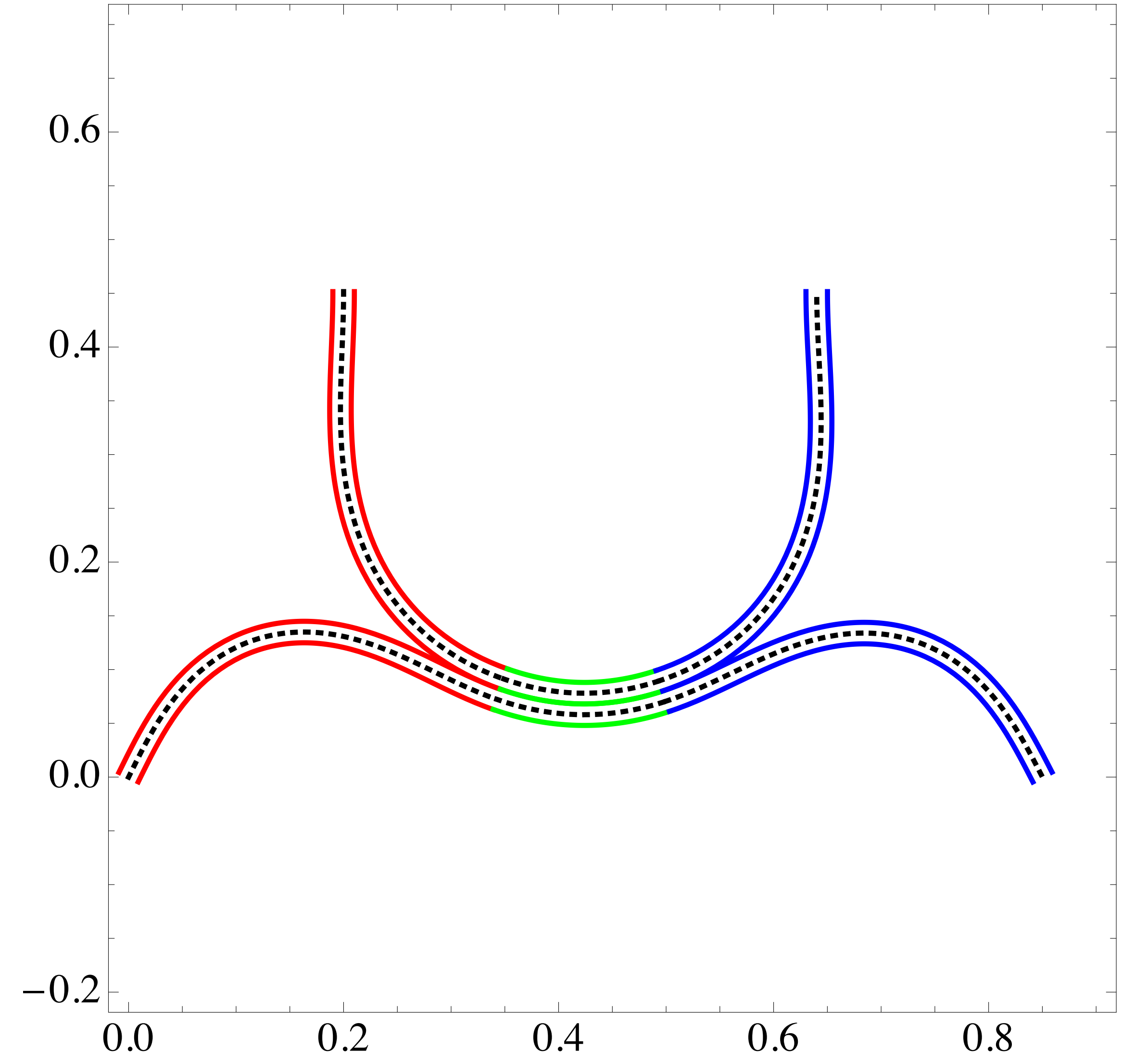
Bifurcation curves



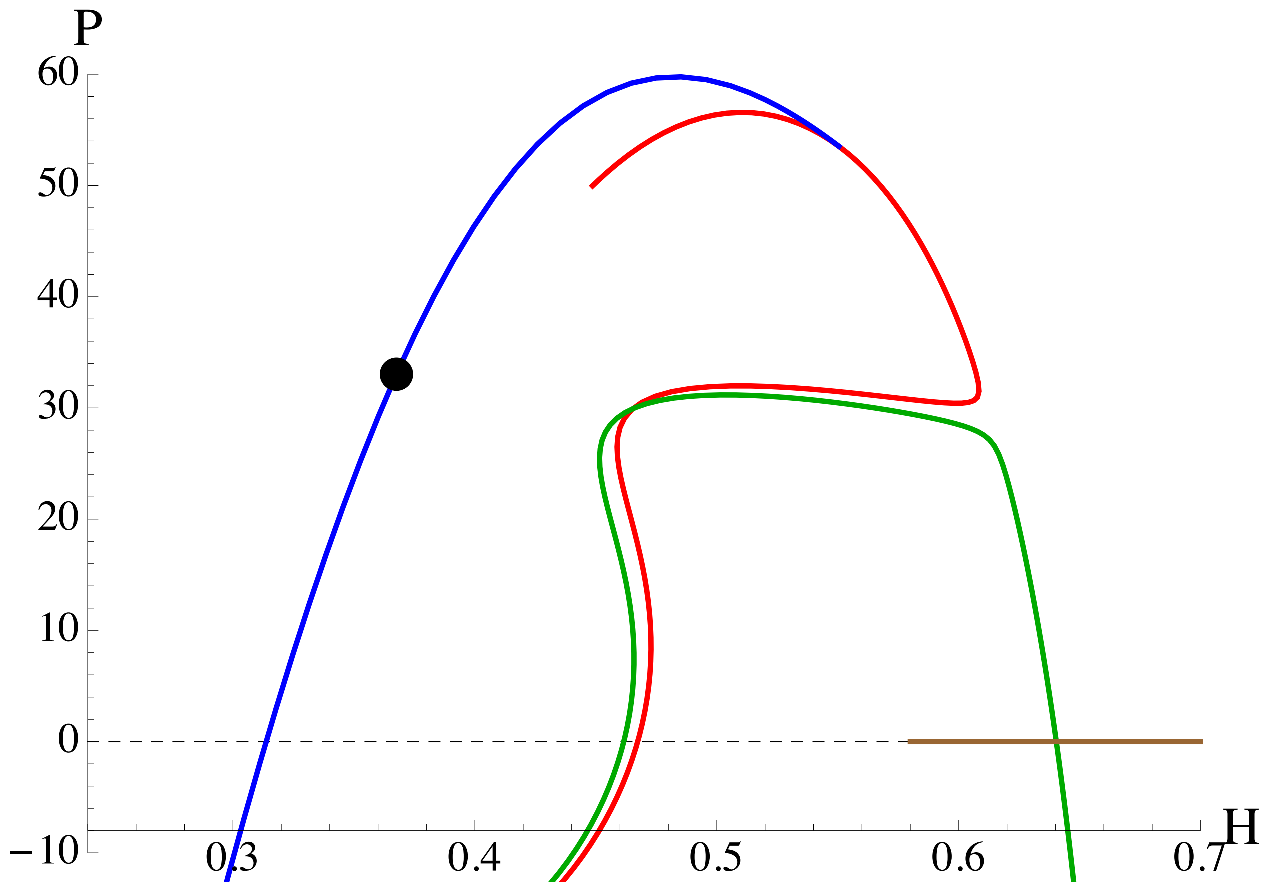


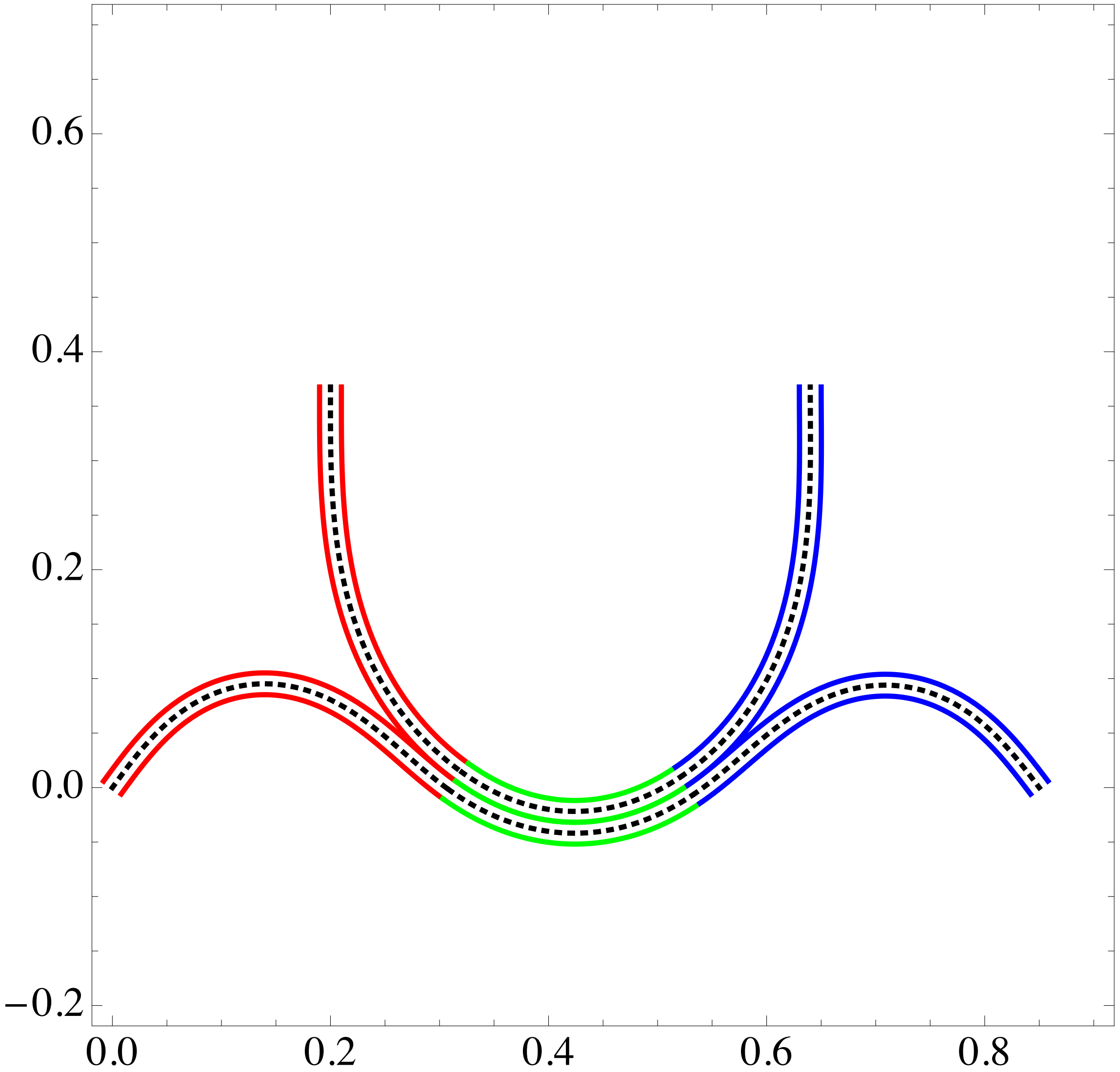
Bifurcation curves



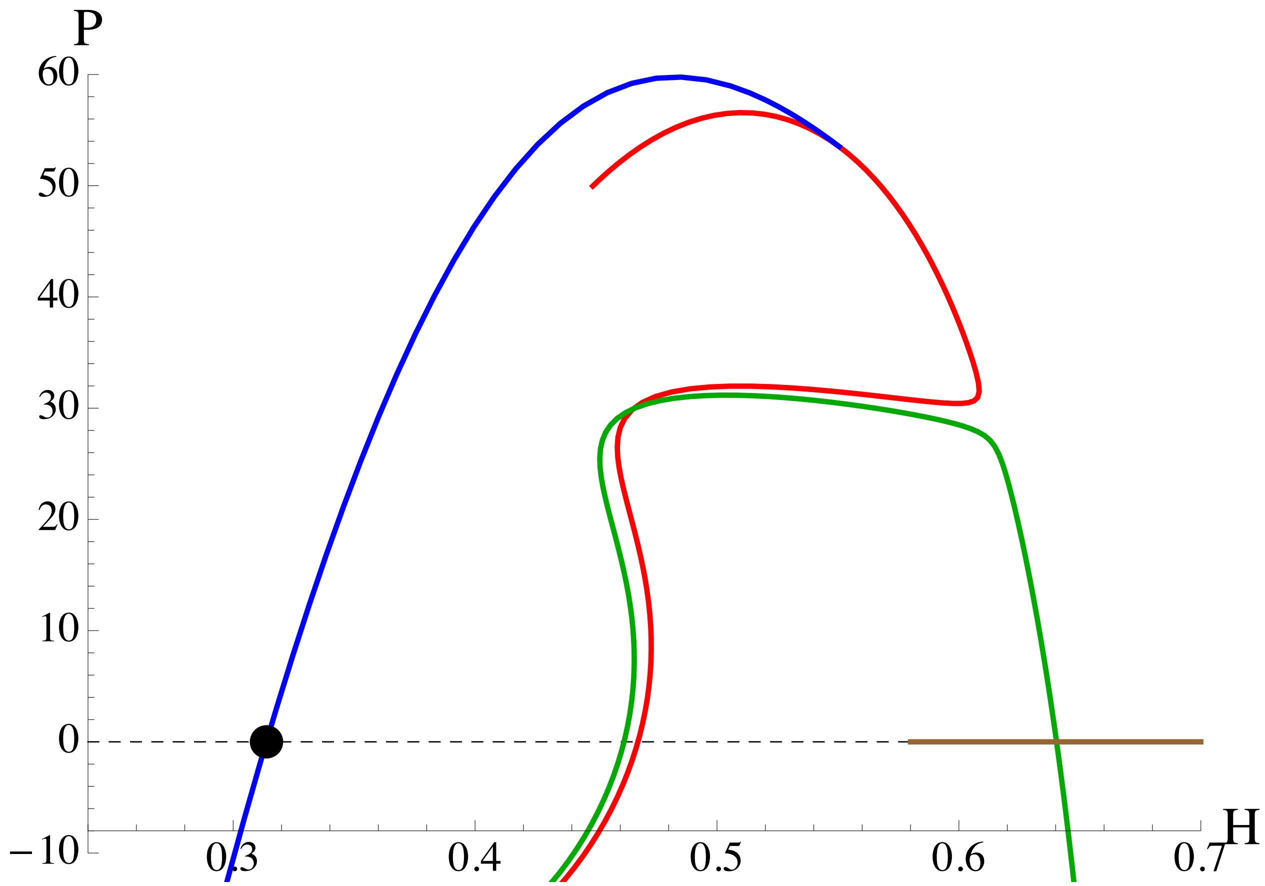


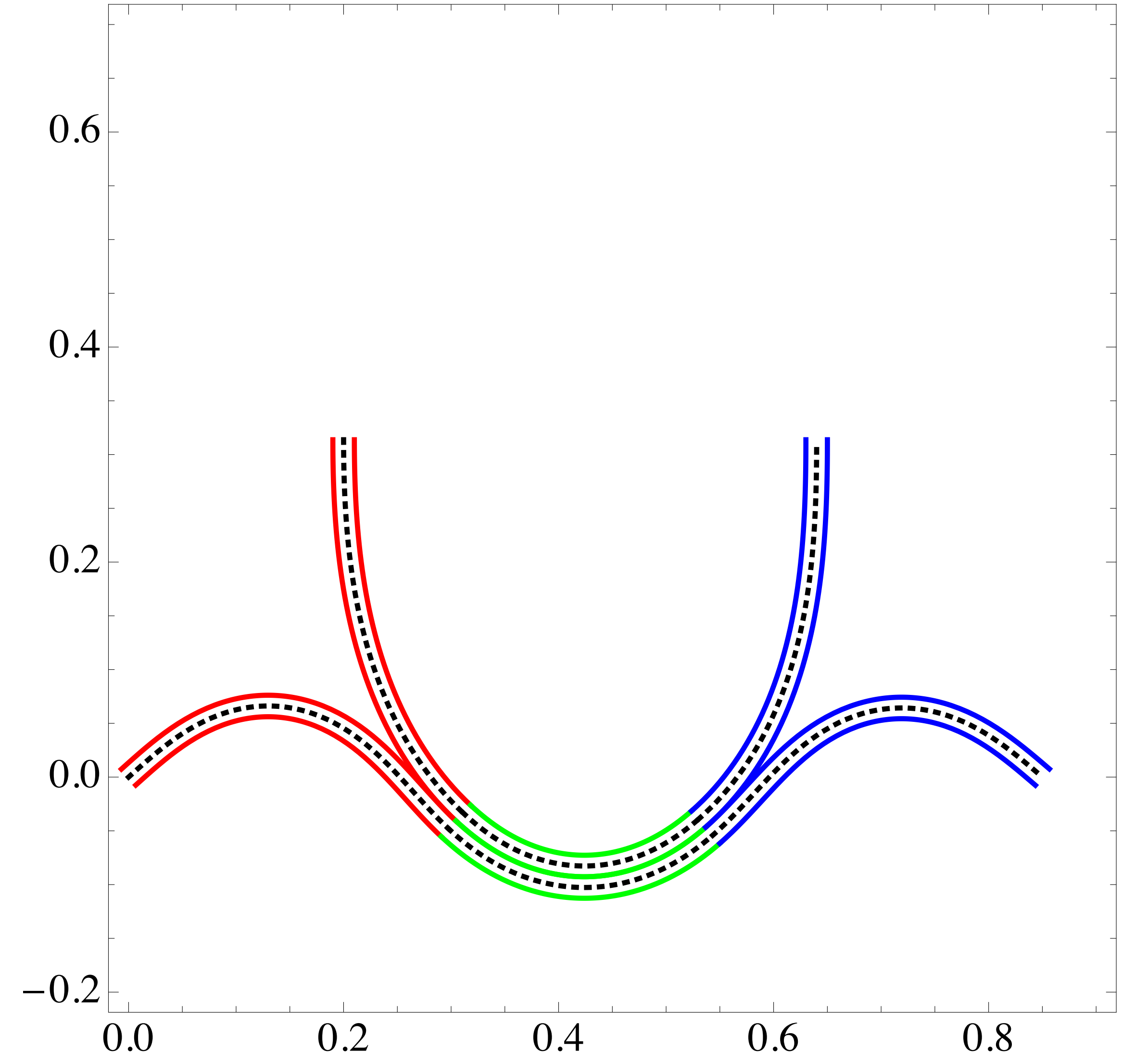
Bifurcation curves



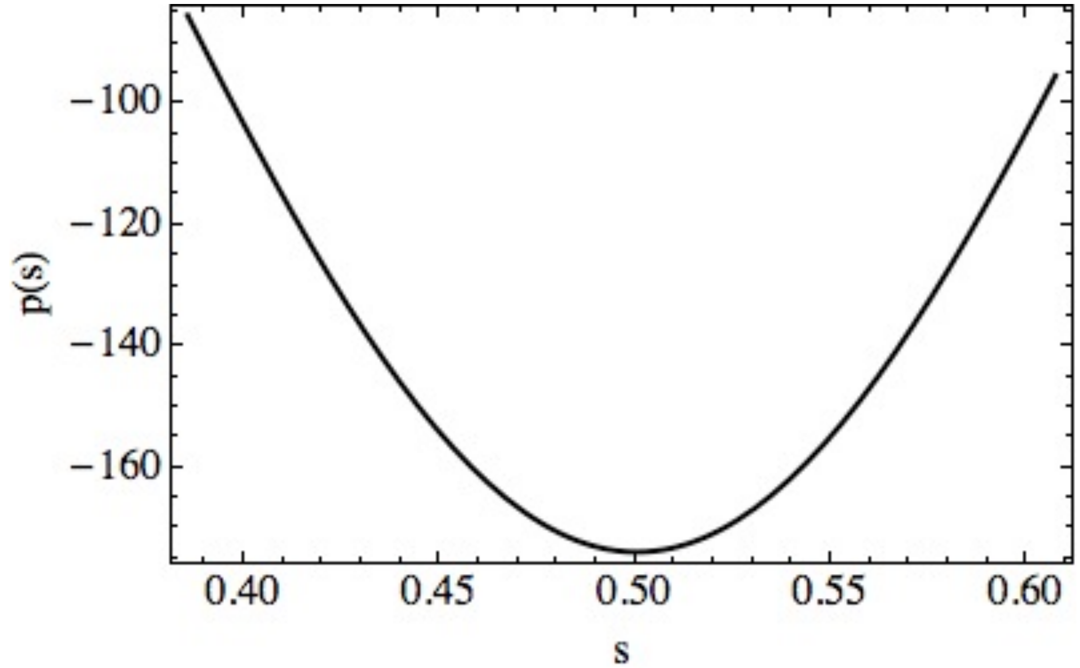


Bifurcation curves

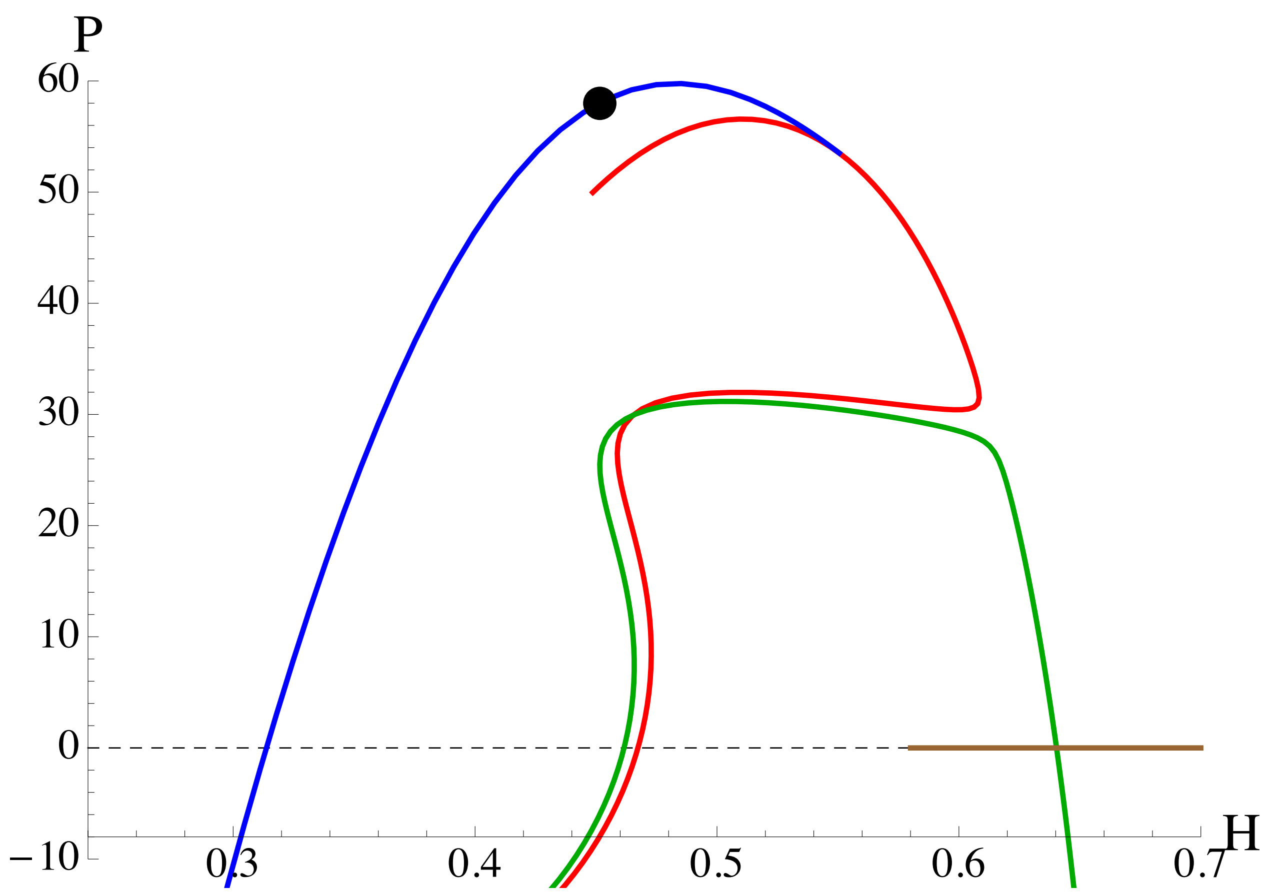


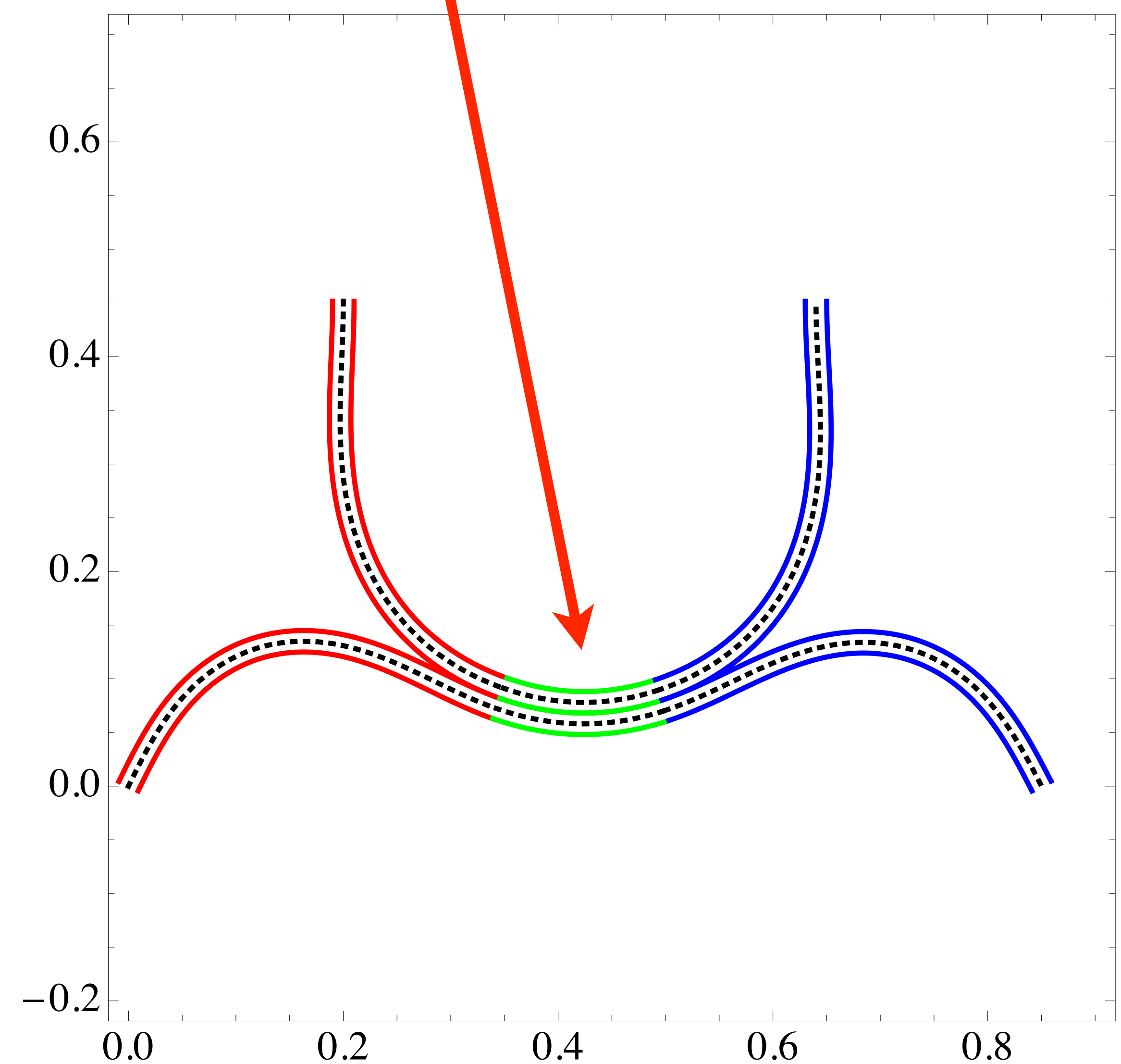


Contact pressure



negative !

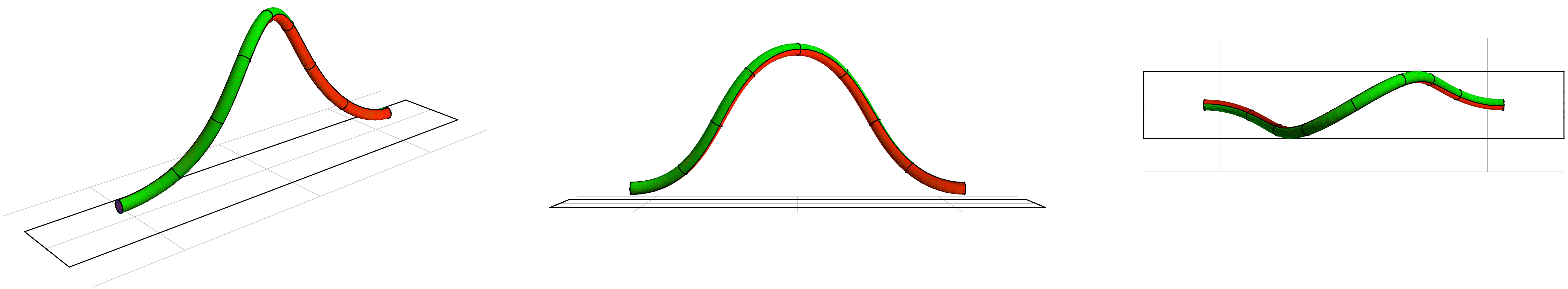




fin

Tige encastrée : symétrie des solutions

extrémités encastrées => solutions symétriques



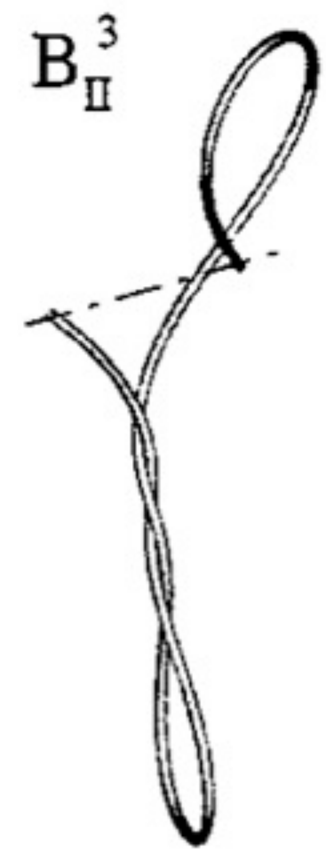
Généralisation :

contact ?

autres conditions de bords ?

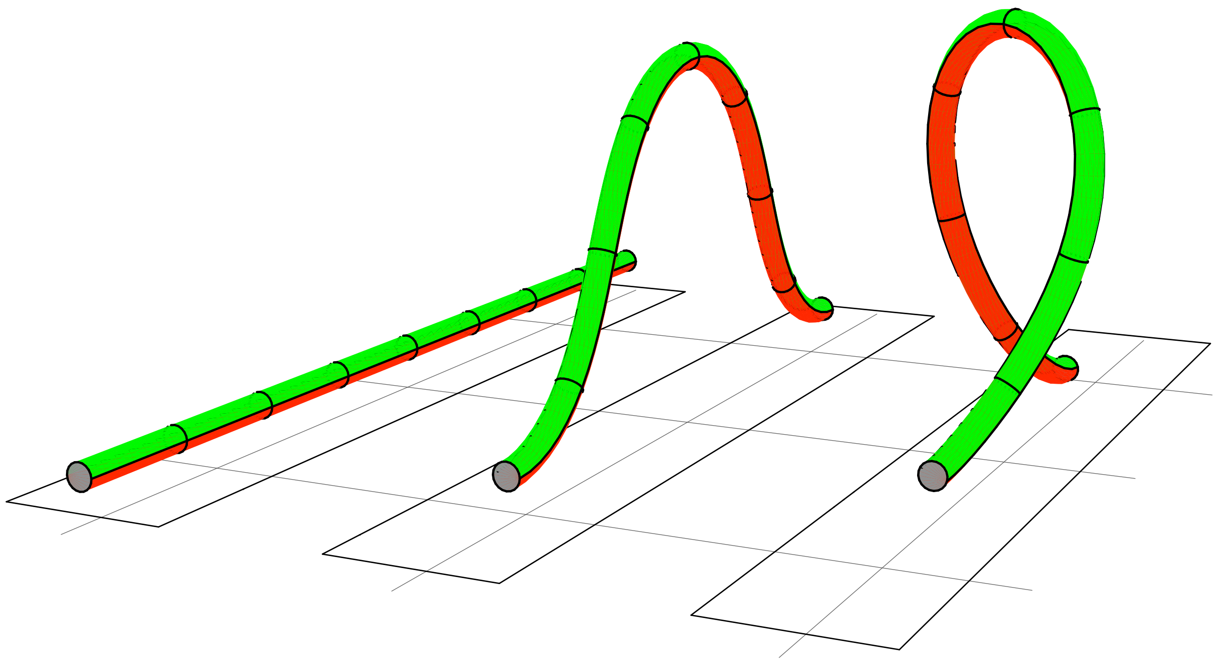
tige à section anisotrope ?

D. Swigon
(PhD Rutgers)
1999



Un cas d'école : tige encastrée

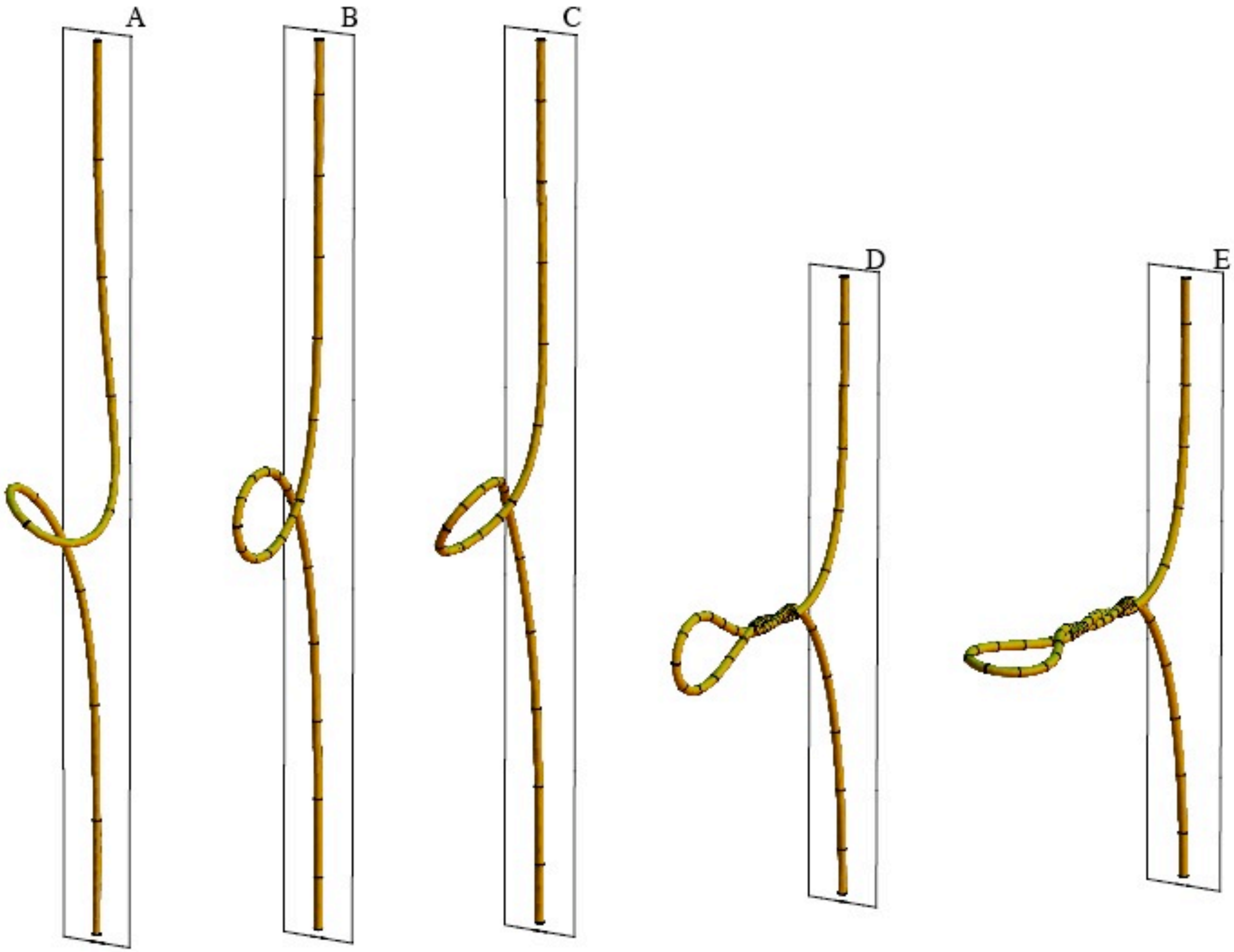
tige uniforme, isotrope, naturellement droite, ...



réduction système
21 D => 6D

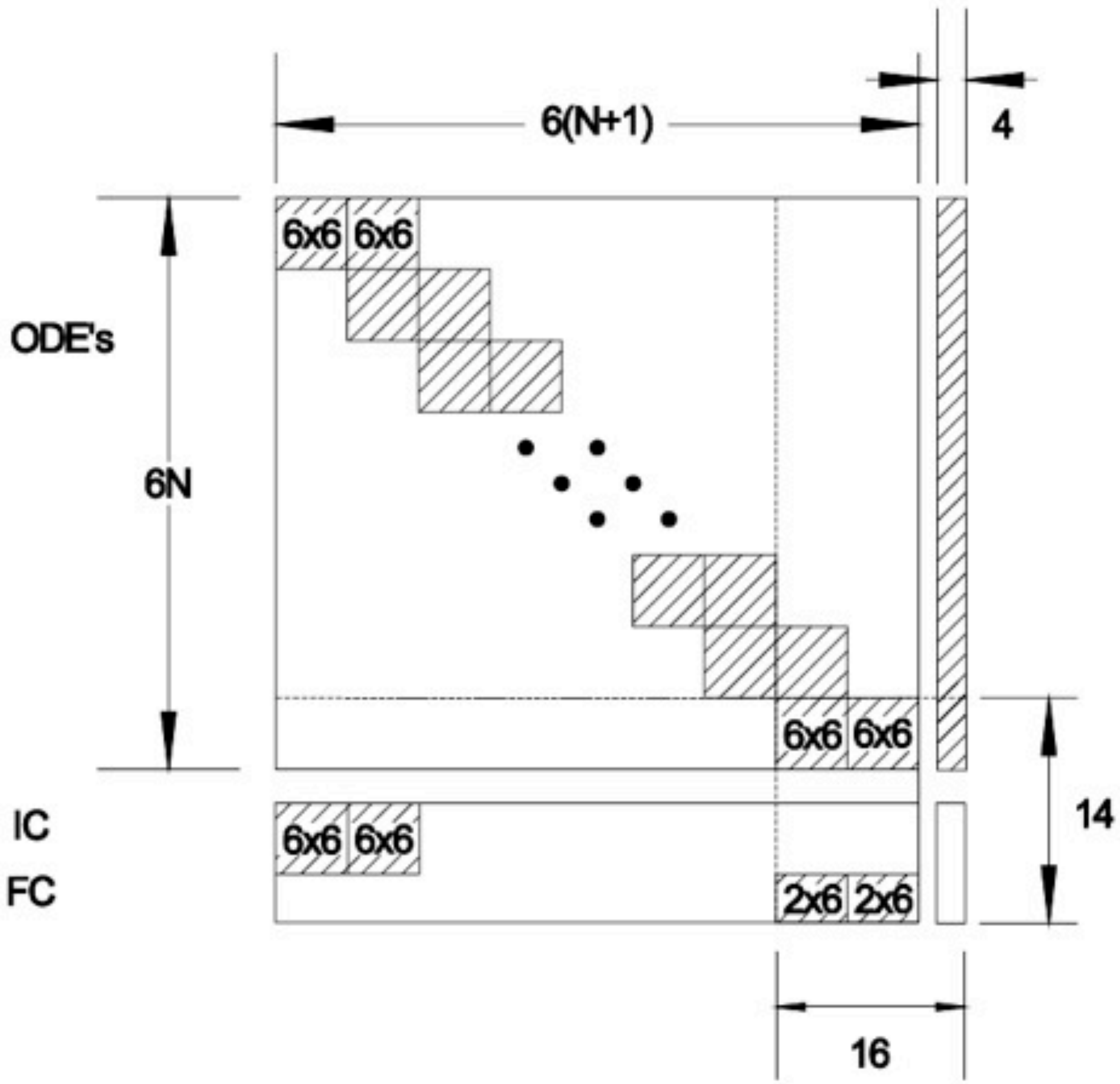
$$\begin{aligned} r' &= d_3 \\ d_3' &= (F \times r + M_0) \times d_3 \end{aligned}$$

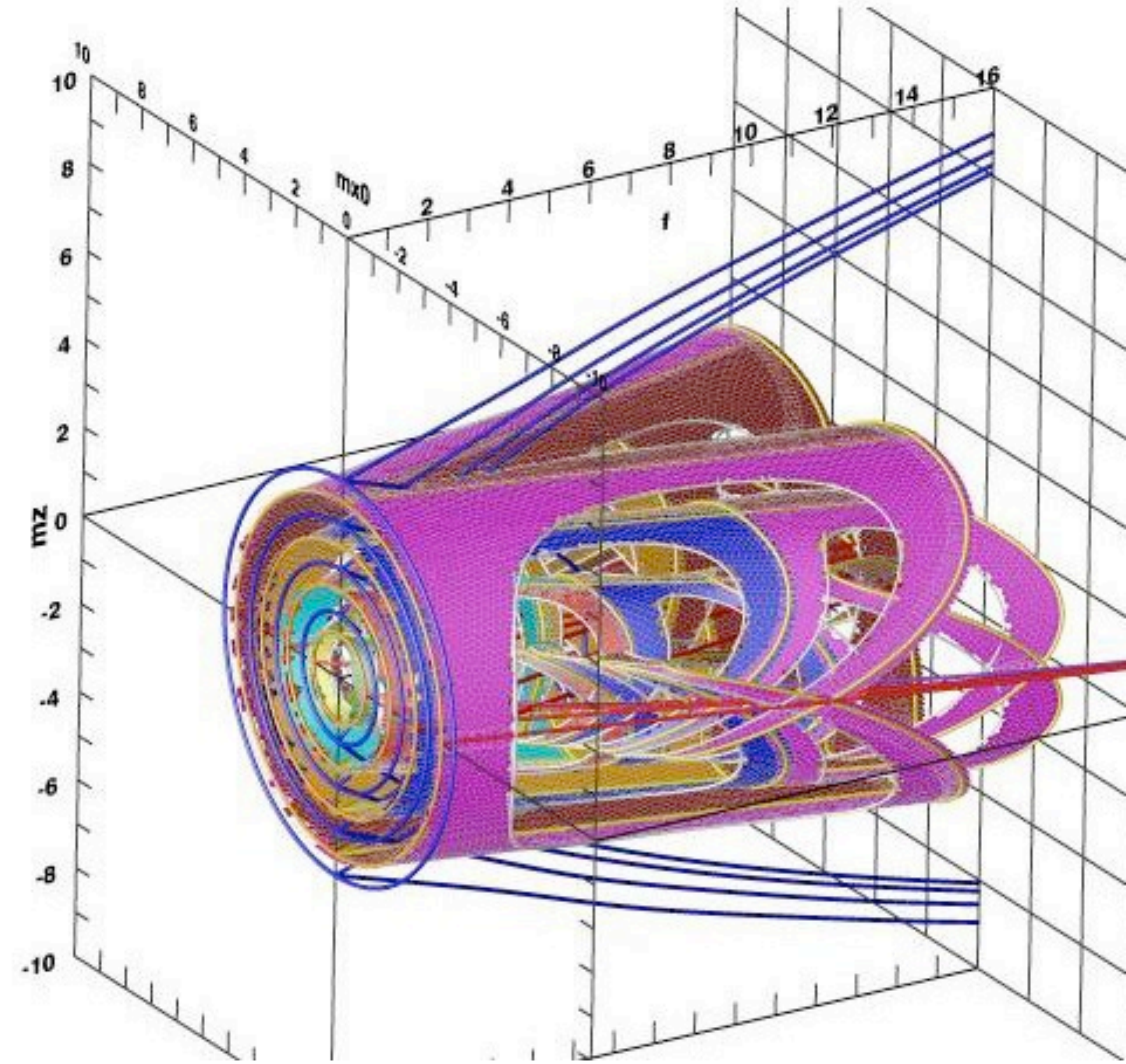
Prise en compte de l'auto-contact



$$\dot{x} = d_{3y}$$
$$\dot{y} = d_{3y}$$
$$\dot{z} = d_{3z}$$
$$\dot{d}_{3x} = fxd_{3z} - m_z d_{3y}$$
$$\dot{d}_{3y} = fyd_{3z} - m_{x0}d_{3z} + m_z d_{3x}$$
$$\dot{d}_{3z} = -fxd_{3x} - fyd_{3y} + m_{x0}d_{3y}$$

2D solution manifold





Michael Henderson (IBM)