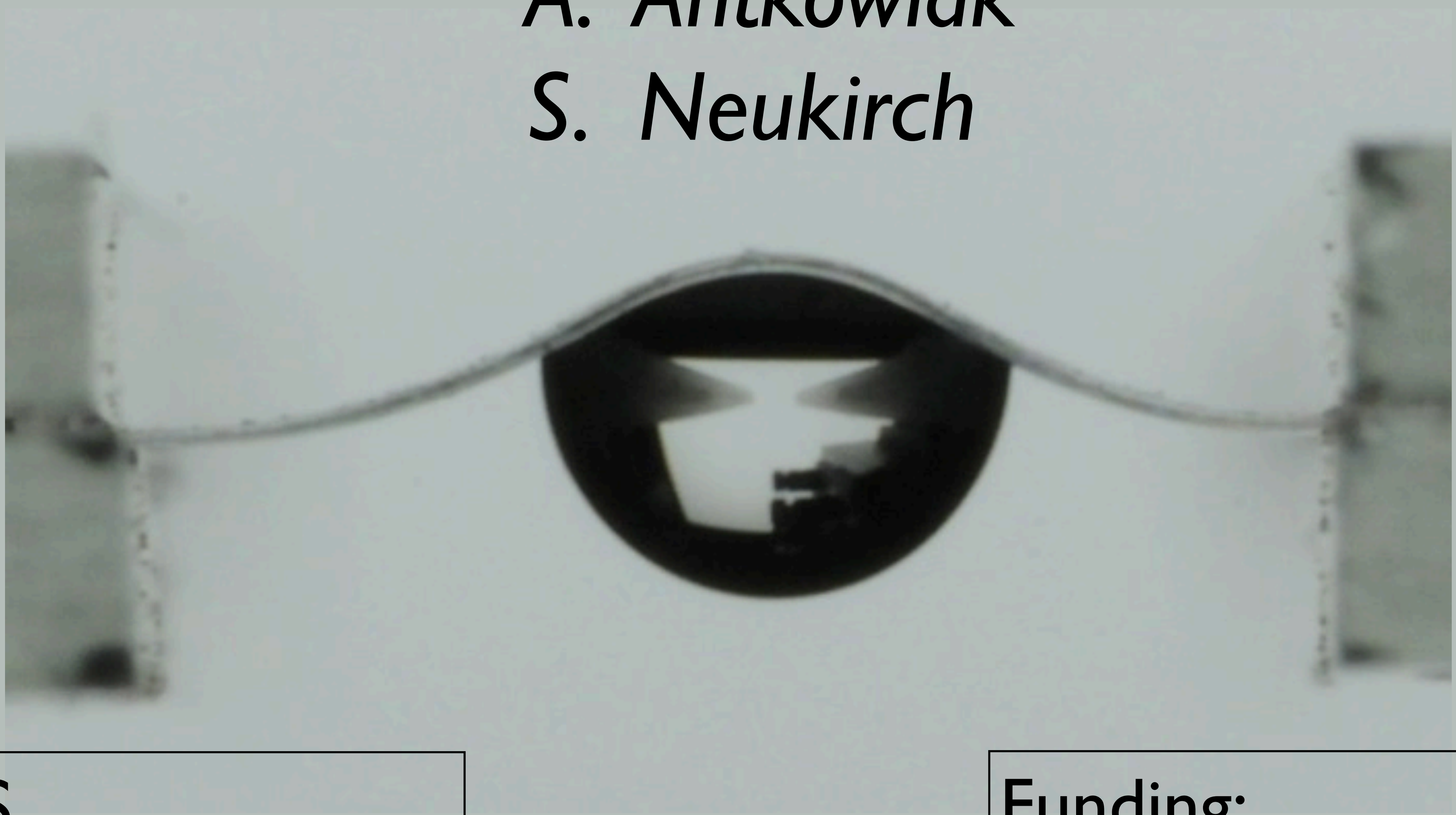
Snap-through induced by surface tension

A. Fargette ← PhD work
A. Antkowiak
S. Neukirch



CNRS
Univ. P. & M. Curie
ENS Paris
France

Funding:
CNRS
ANR
Ville de Paris

Elastocapillarity: state of the art (incomplete)

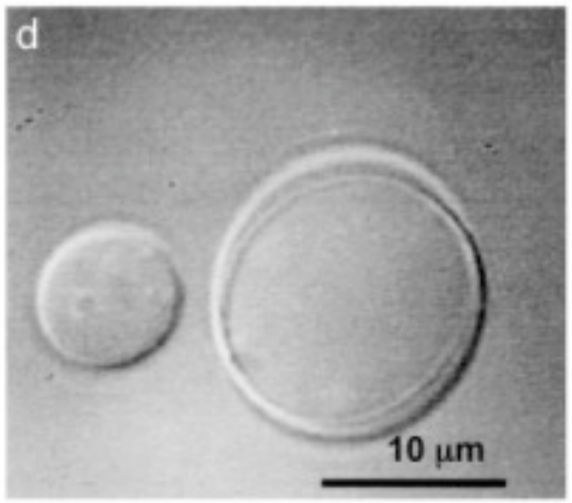
Bico et al., Nature (2004)

review article:

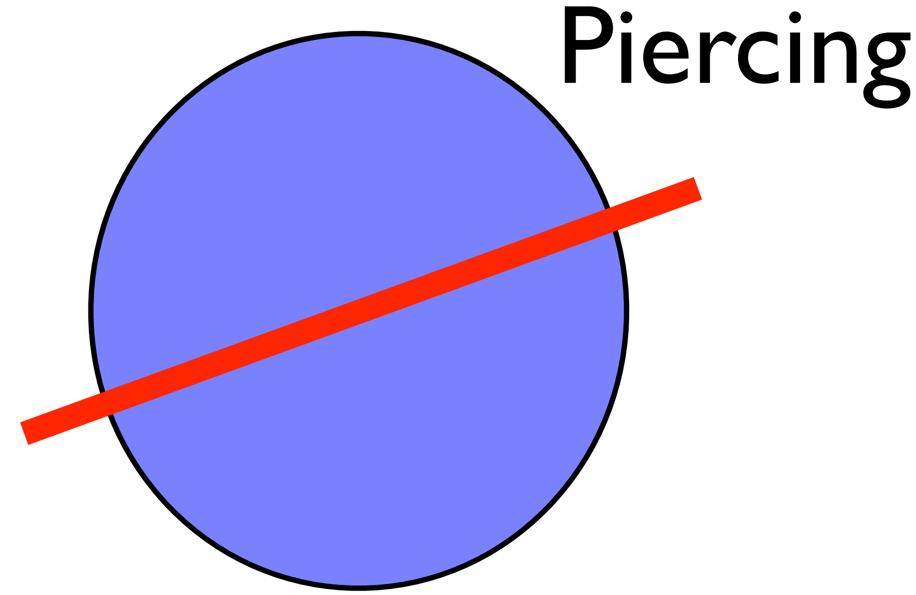
Roman + Bico
Journal of Physics: Condensed Matter 2010

The Elastocapillary lengthscale

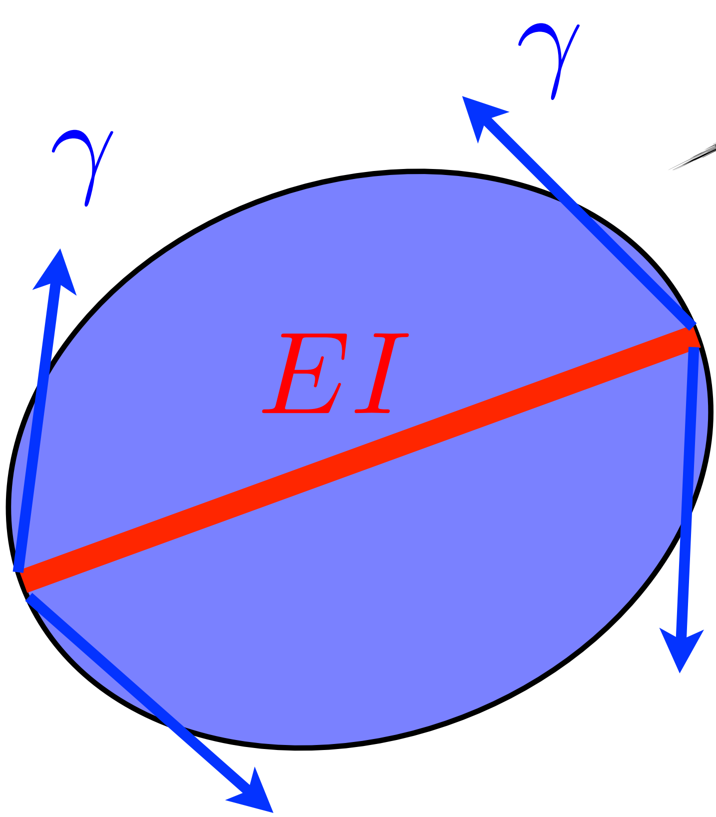
A tubulin rod growing inside a lipid vesicle



Cohen & Mahadevan, PNAS (2003)

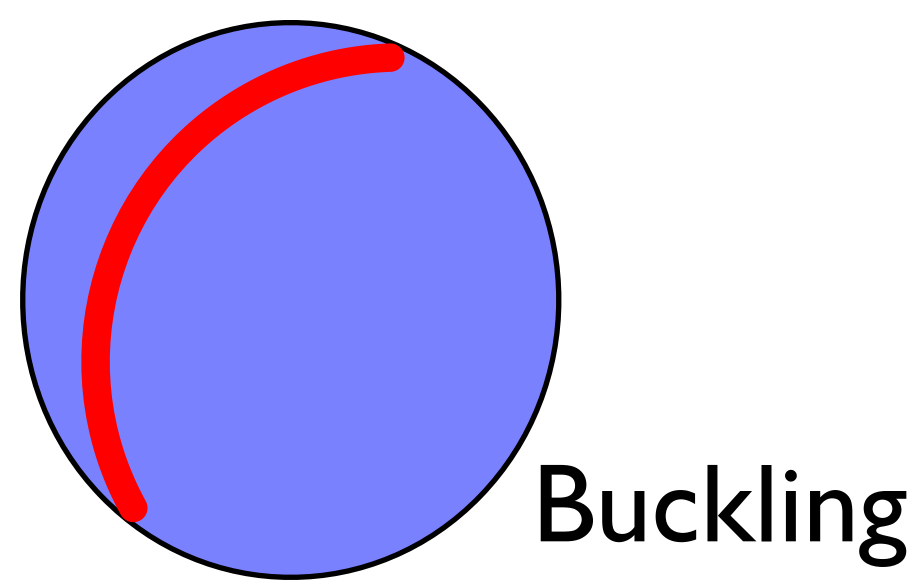


$$\frac{EI}{L^2} \gg \gamma$$

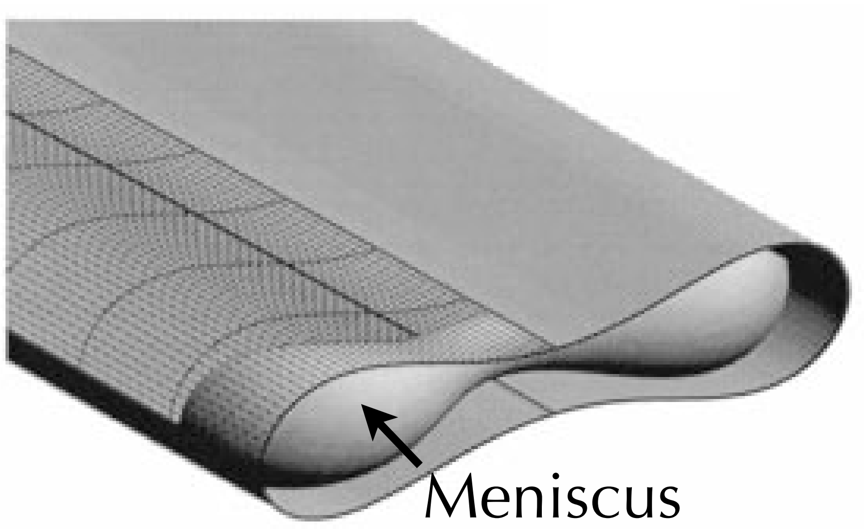


$$\frac{EI}{L^2} \sim \gamma \longrightarrow L_{\mathrm{EC}} = \sqrt{\frac{EI}{\gamma}}$$

$$\frac{EI}{L^2} \ll \gamma$$

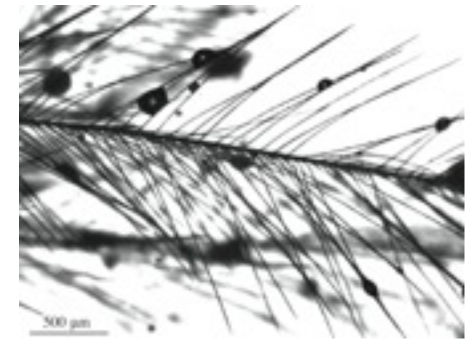


Elastocapillarity in Biology



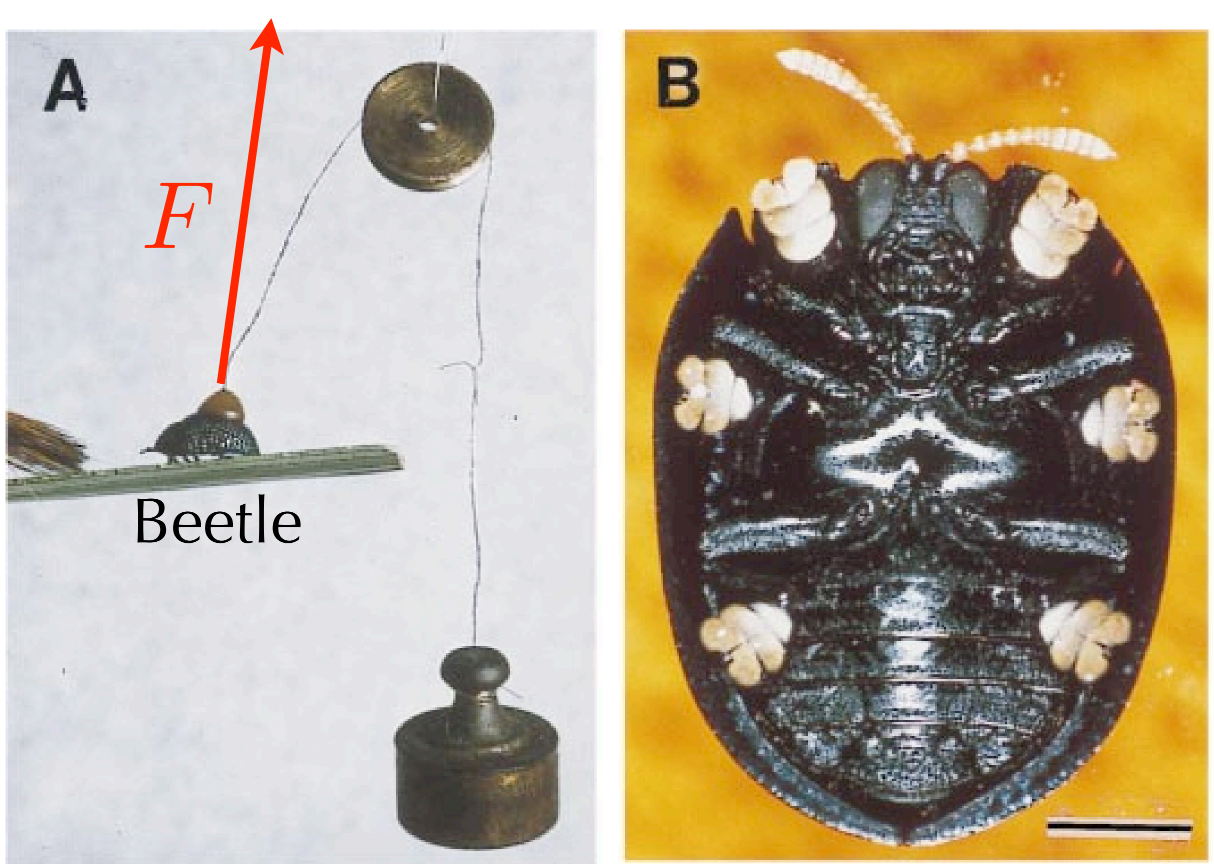
Lung's airway closure

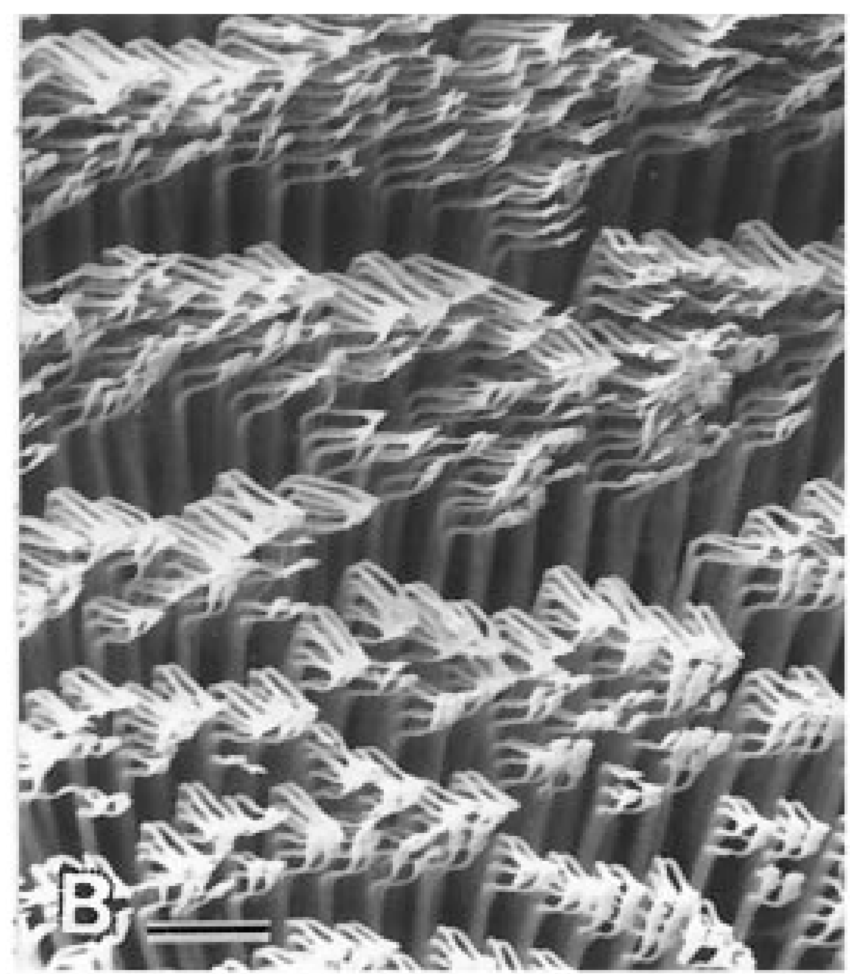
e.g. Heil, J. Fluid Mech 380, 1999



Wet feathers

Duprat, Protière, Beebe and Stone, *Nature* (2012)

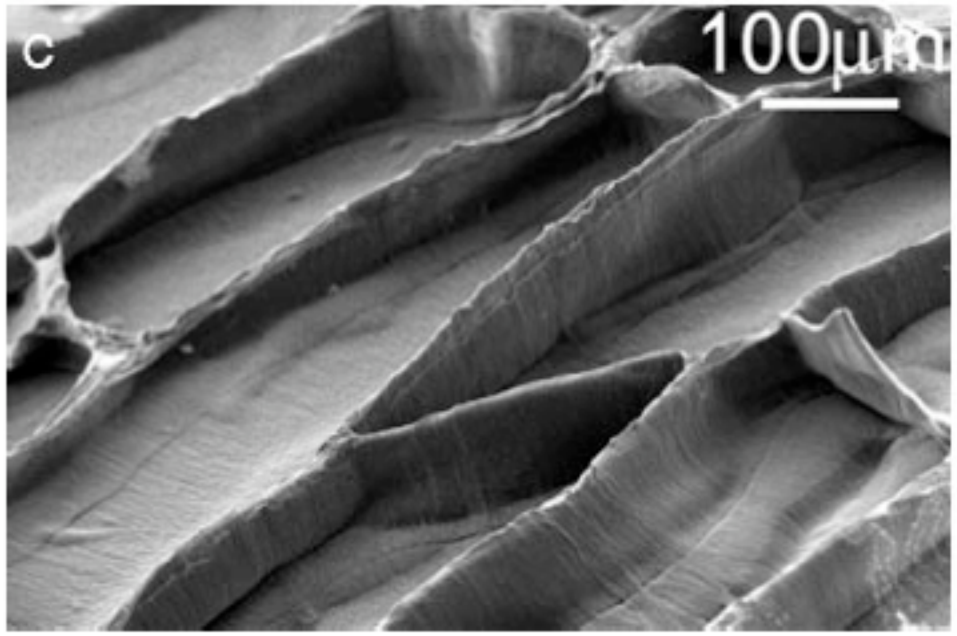




Insect adhesion

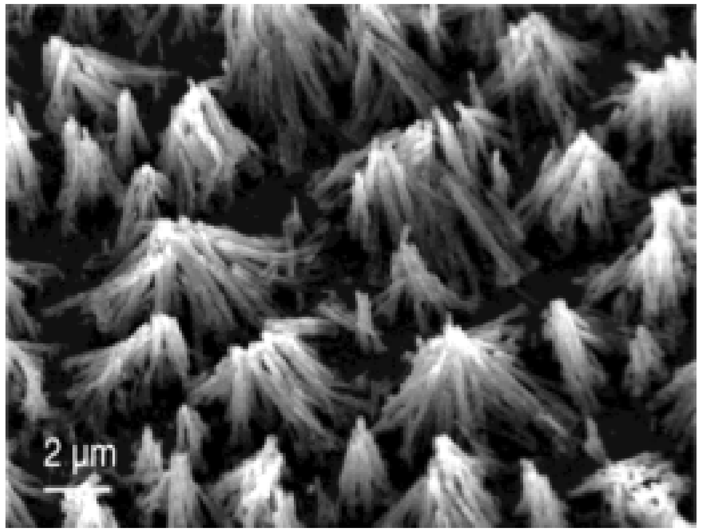
Eisner et al., PNAS, 2000

Elastocapillarity with Carbon Nanotubes



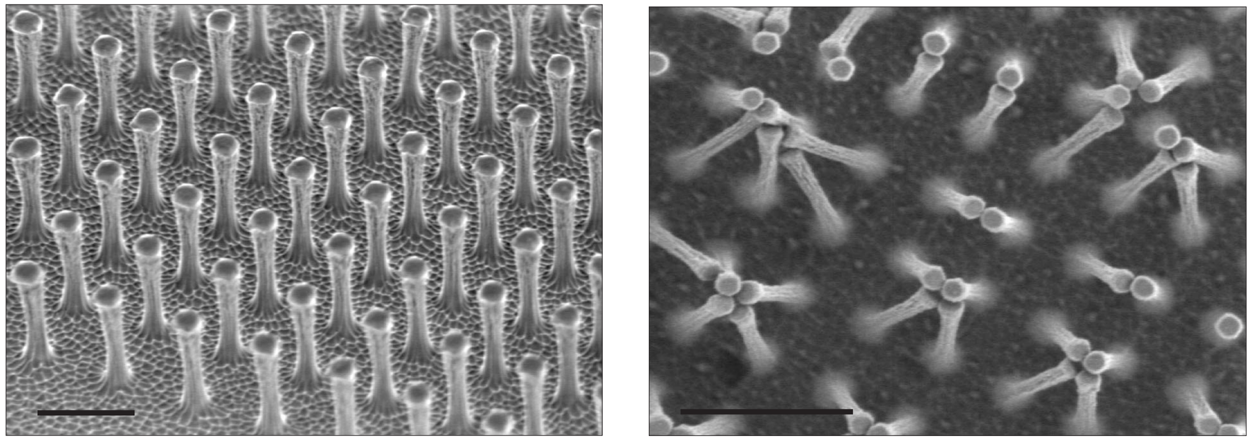
Cellular patterns

Chakrapani et al., PNAS, 2004



Teepee formation

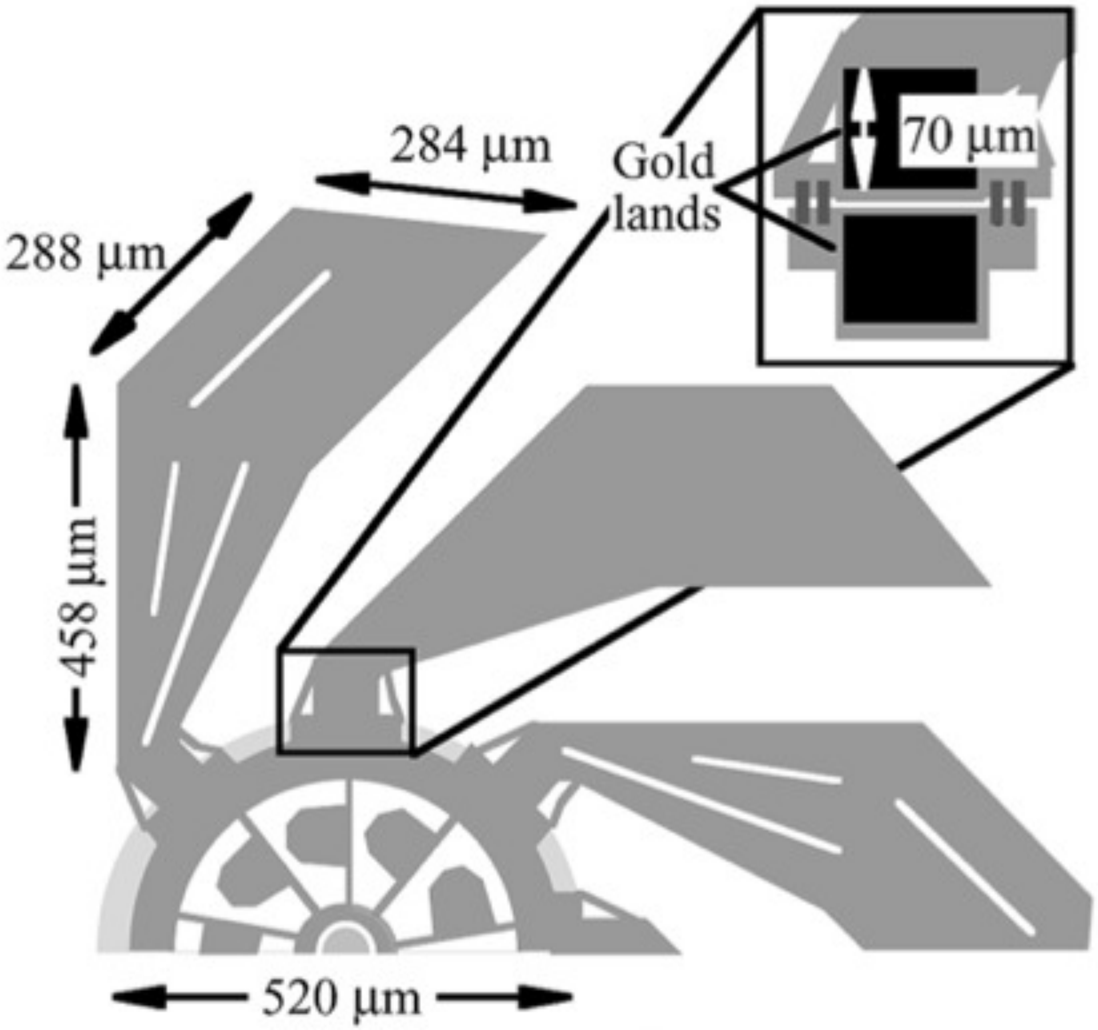
Lau et al., Nano Lett., 2003

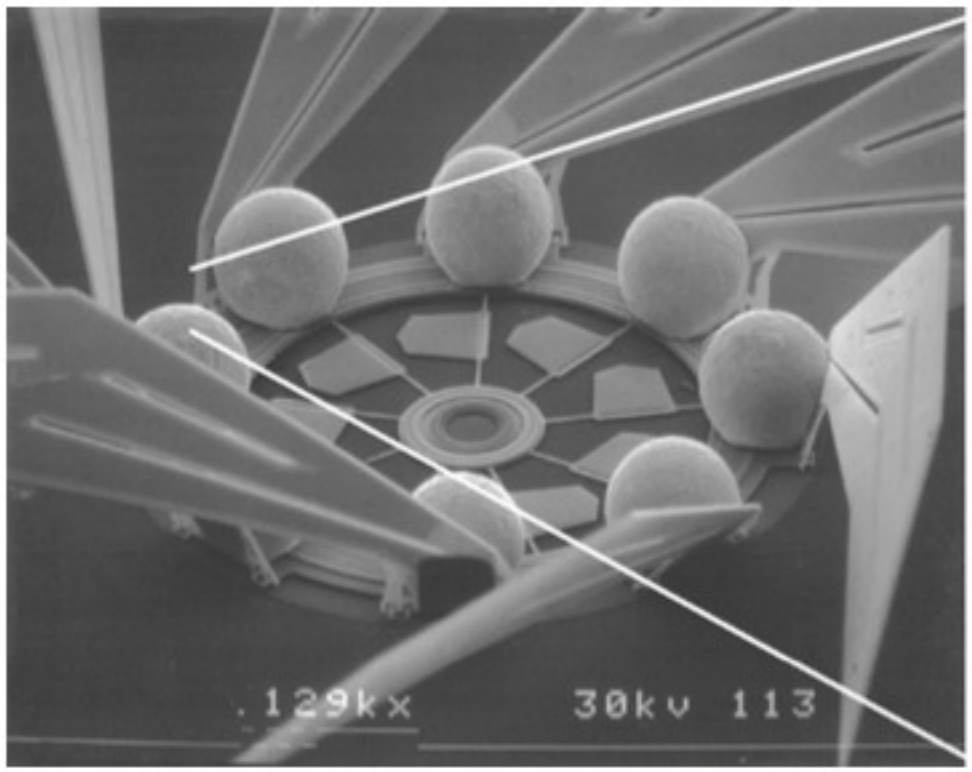


Bio-mimetism

Geim et al., Nature Mat., 2003

Elastocapillarity in Industry: Microfabrication





folding by surface tension of Pb:Sn solder spheres

microfan with polysilicon
180 rpm
micro-fluidic systems

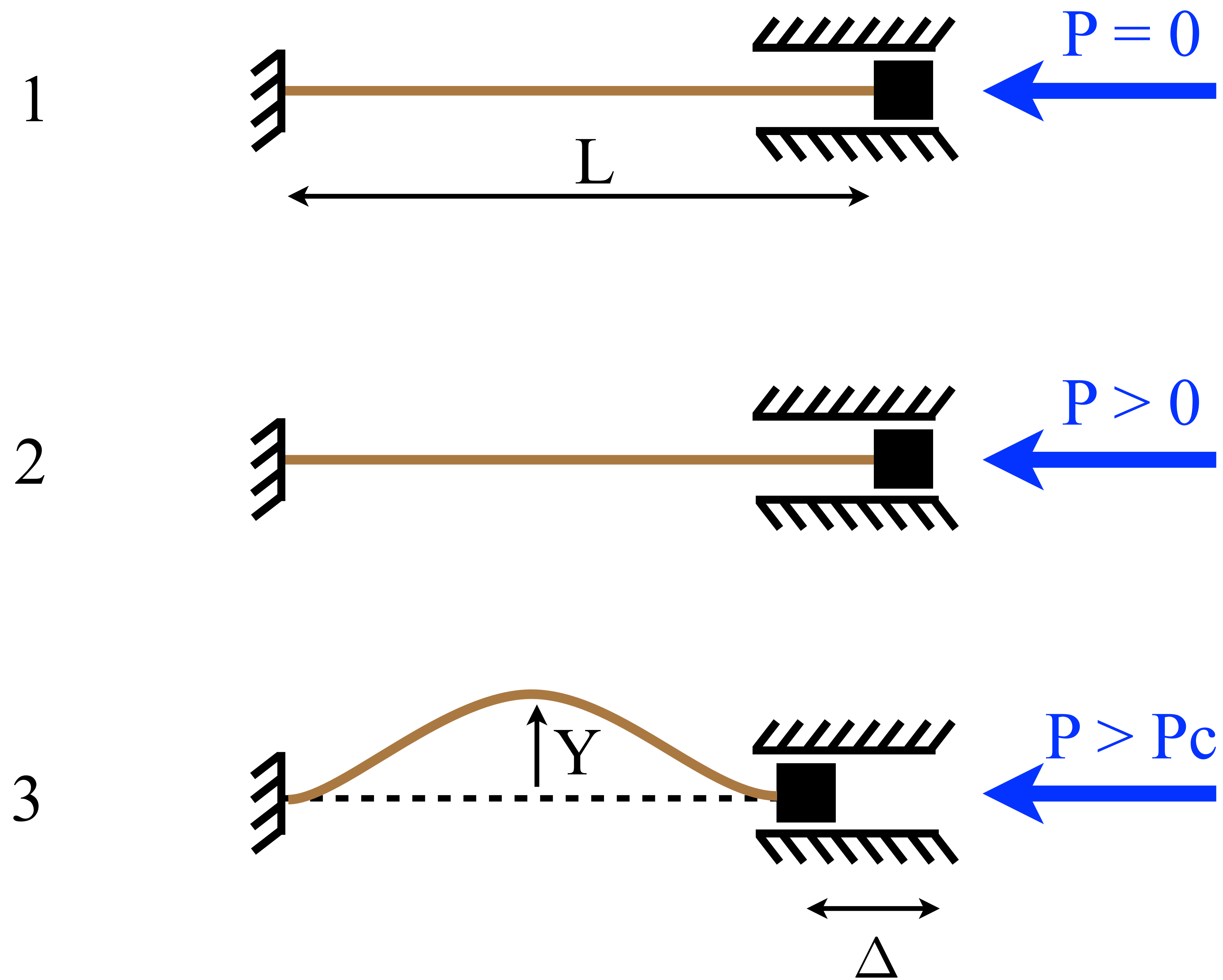
Linderman et al, Sens. Actuators (2002)

Here :
snap-through of
an elastic beam
induced by a drop

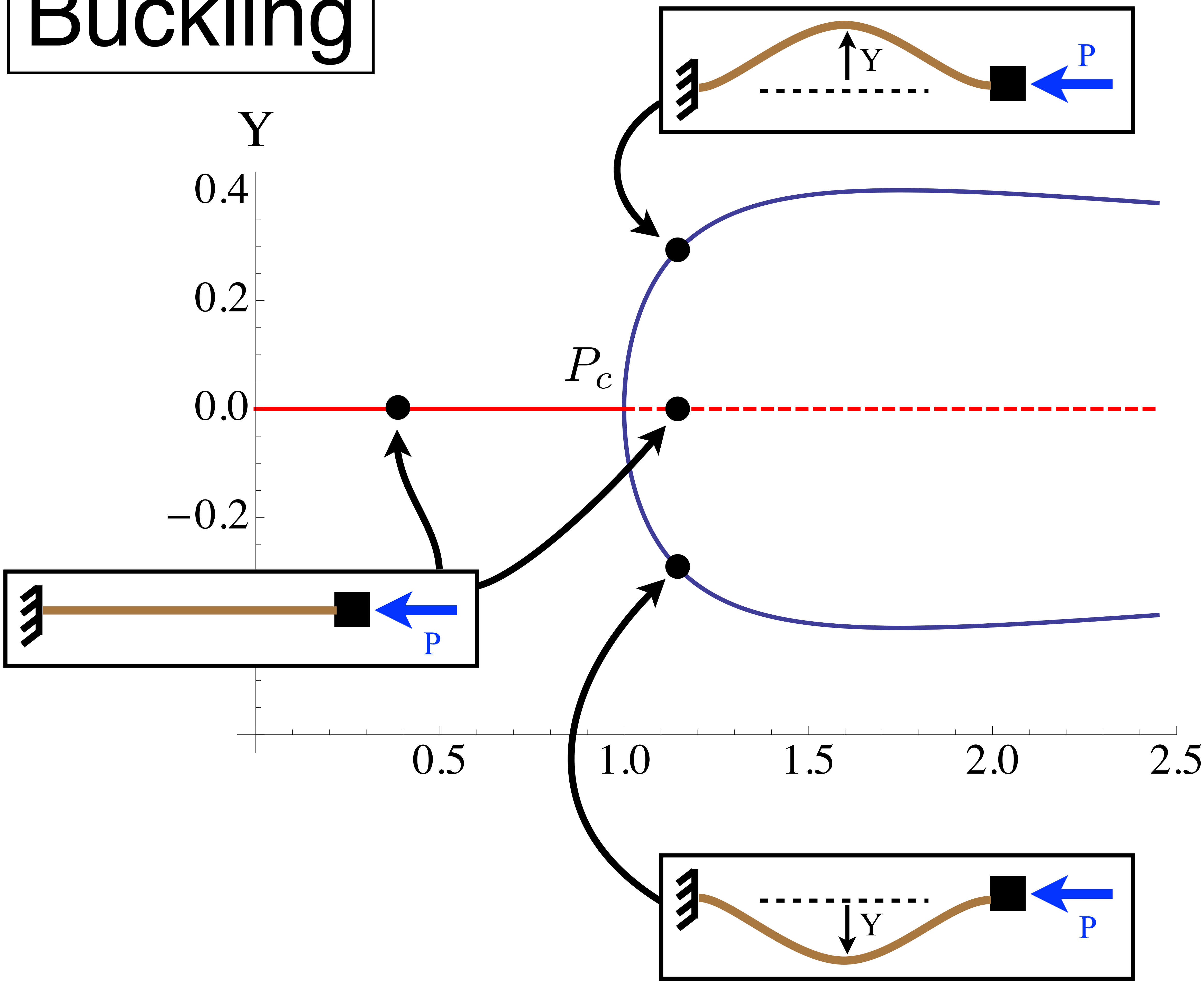
(classical)

Buckling
and
snap-through

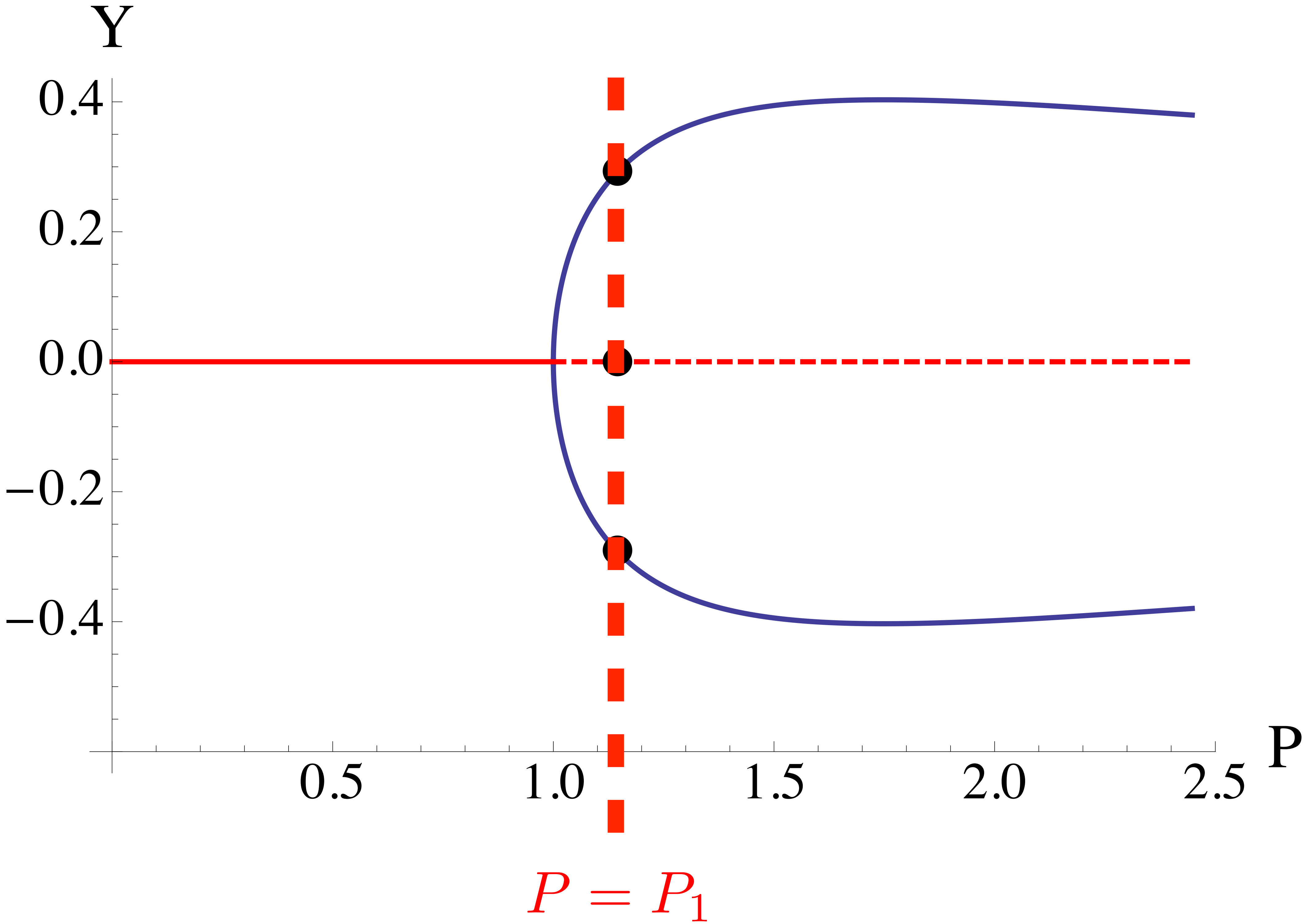
Buckling



Buckling

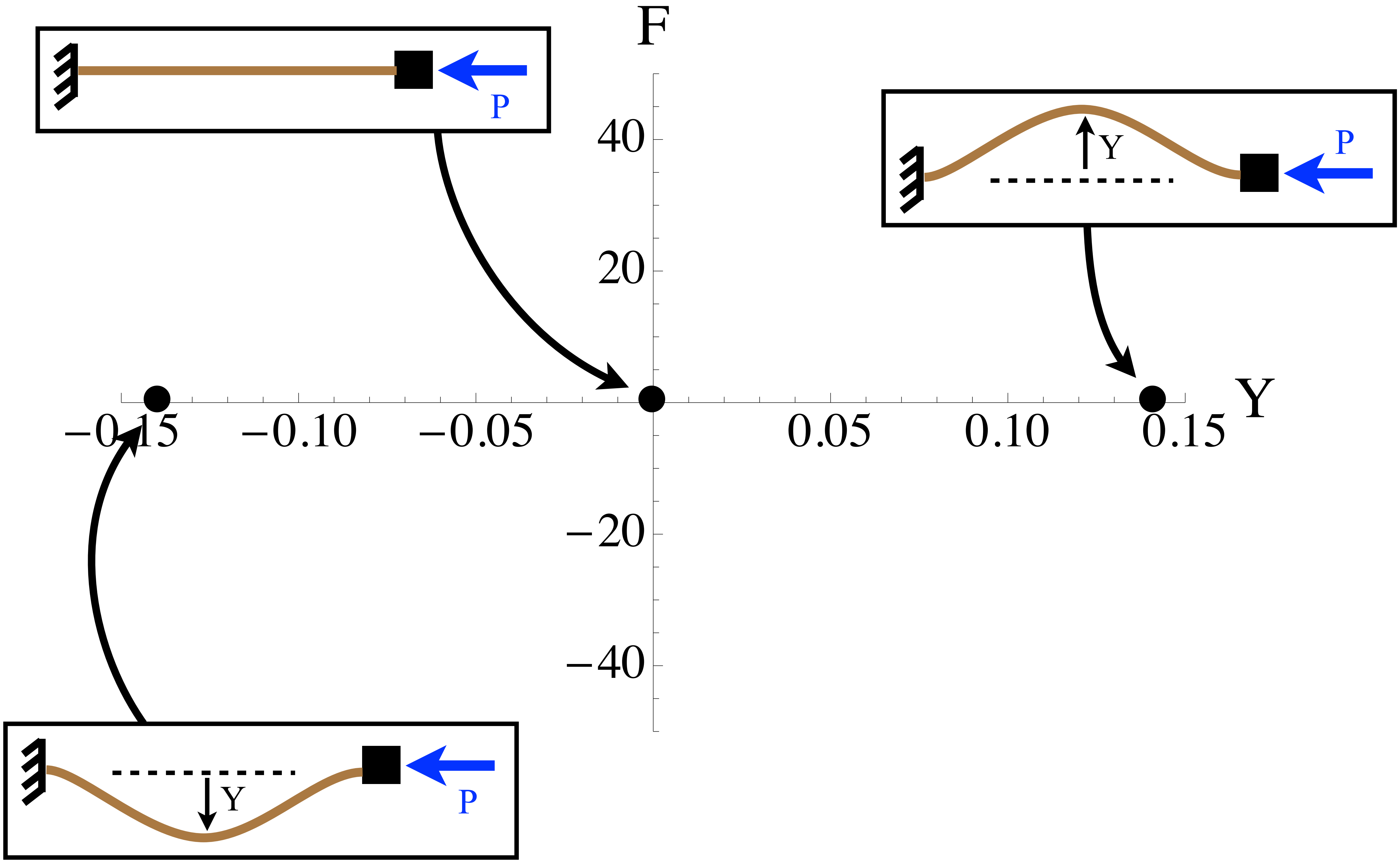


Buckling



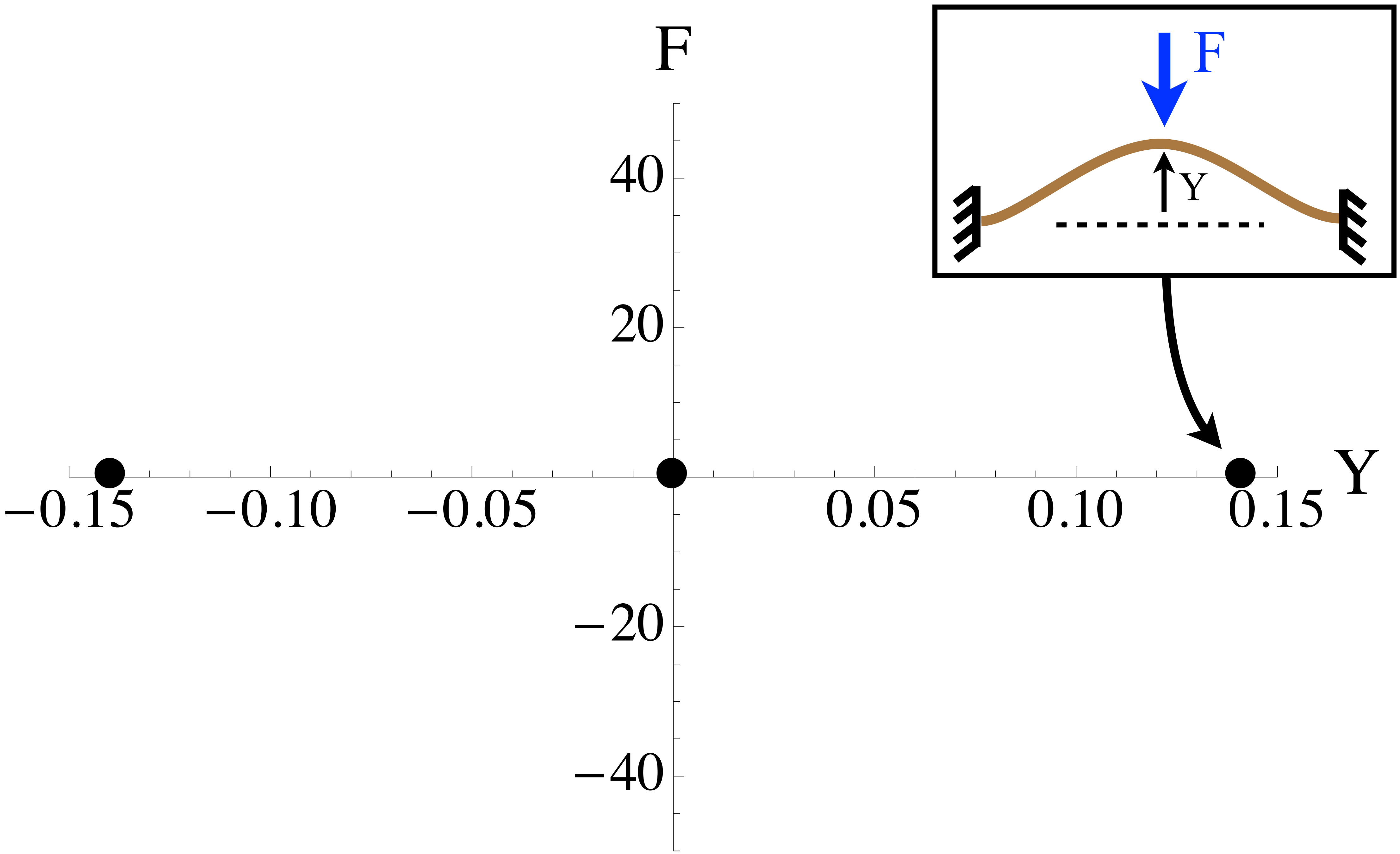
Snap-through

$$P = P_1$$

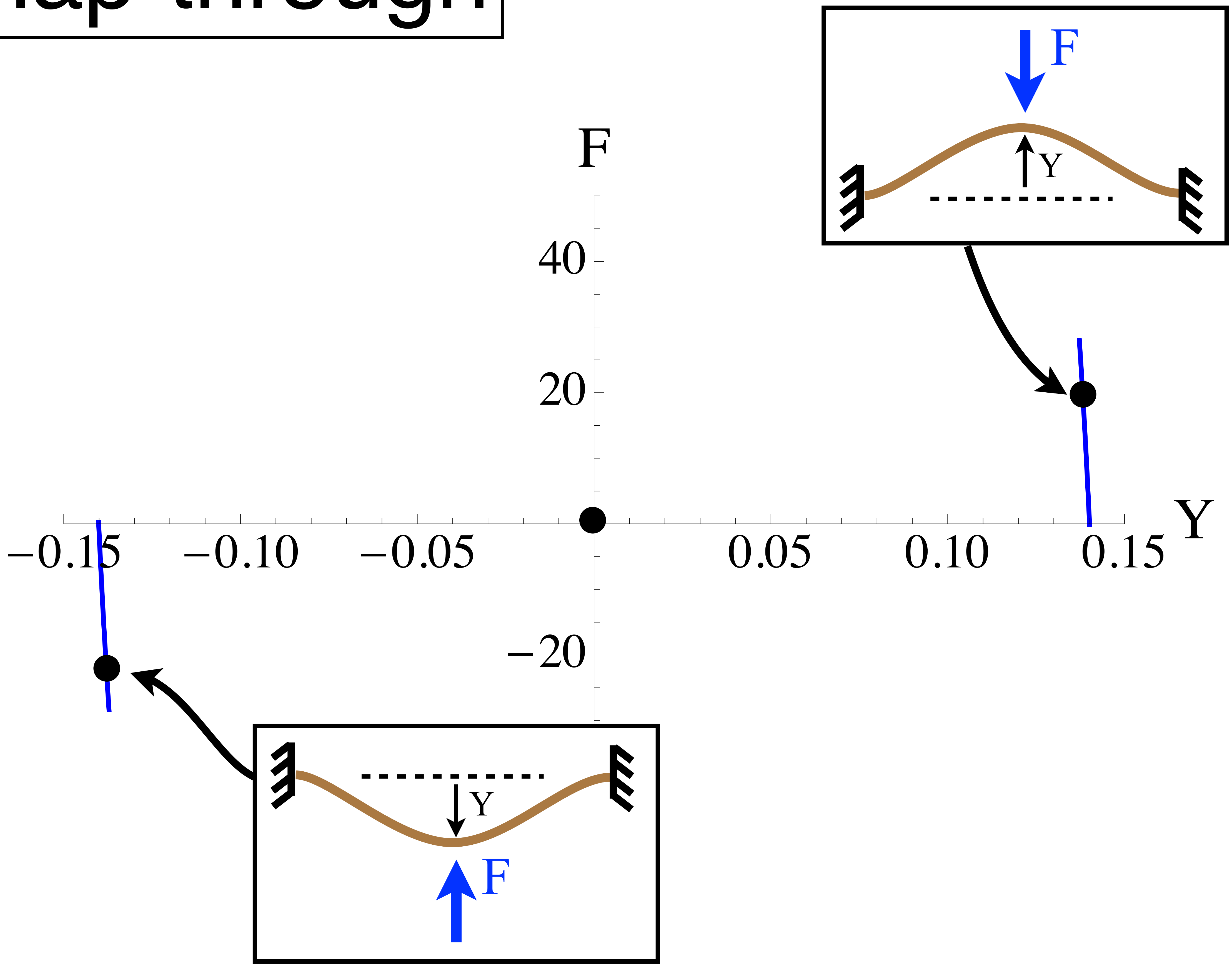


Snap-through

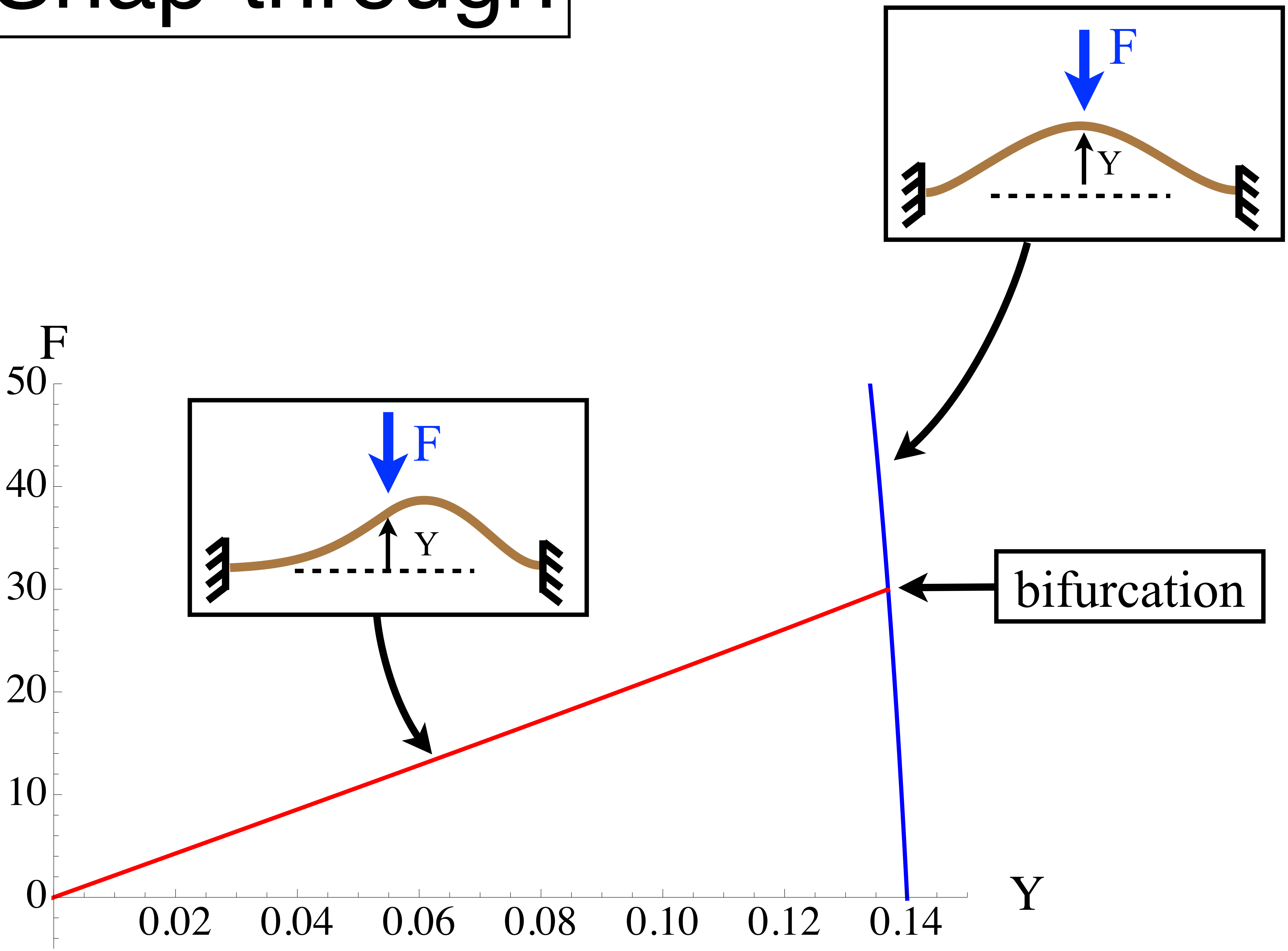
fixed ends



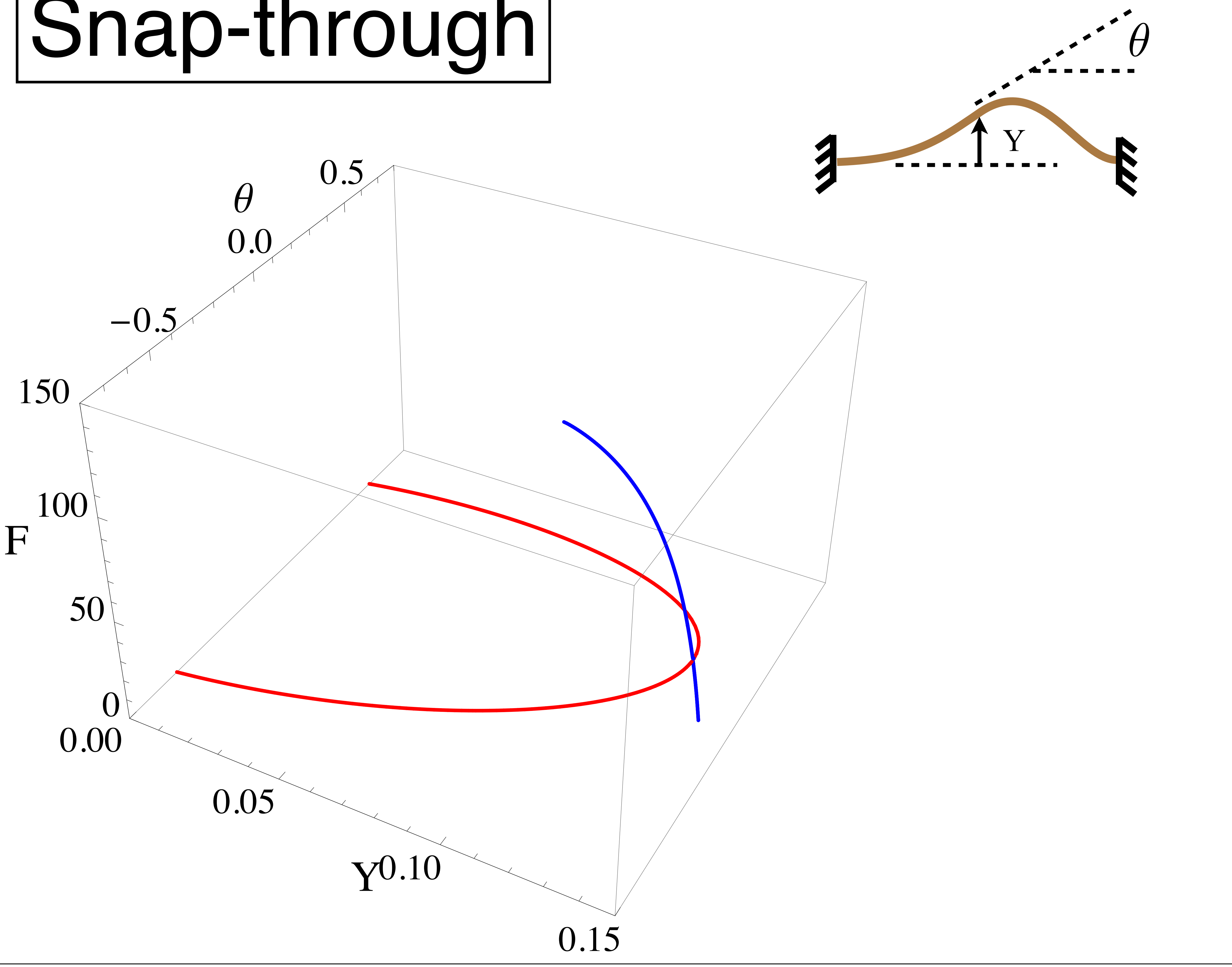
Snap-through

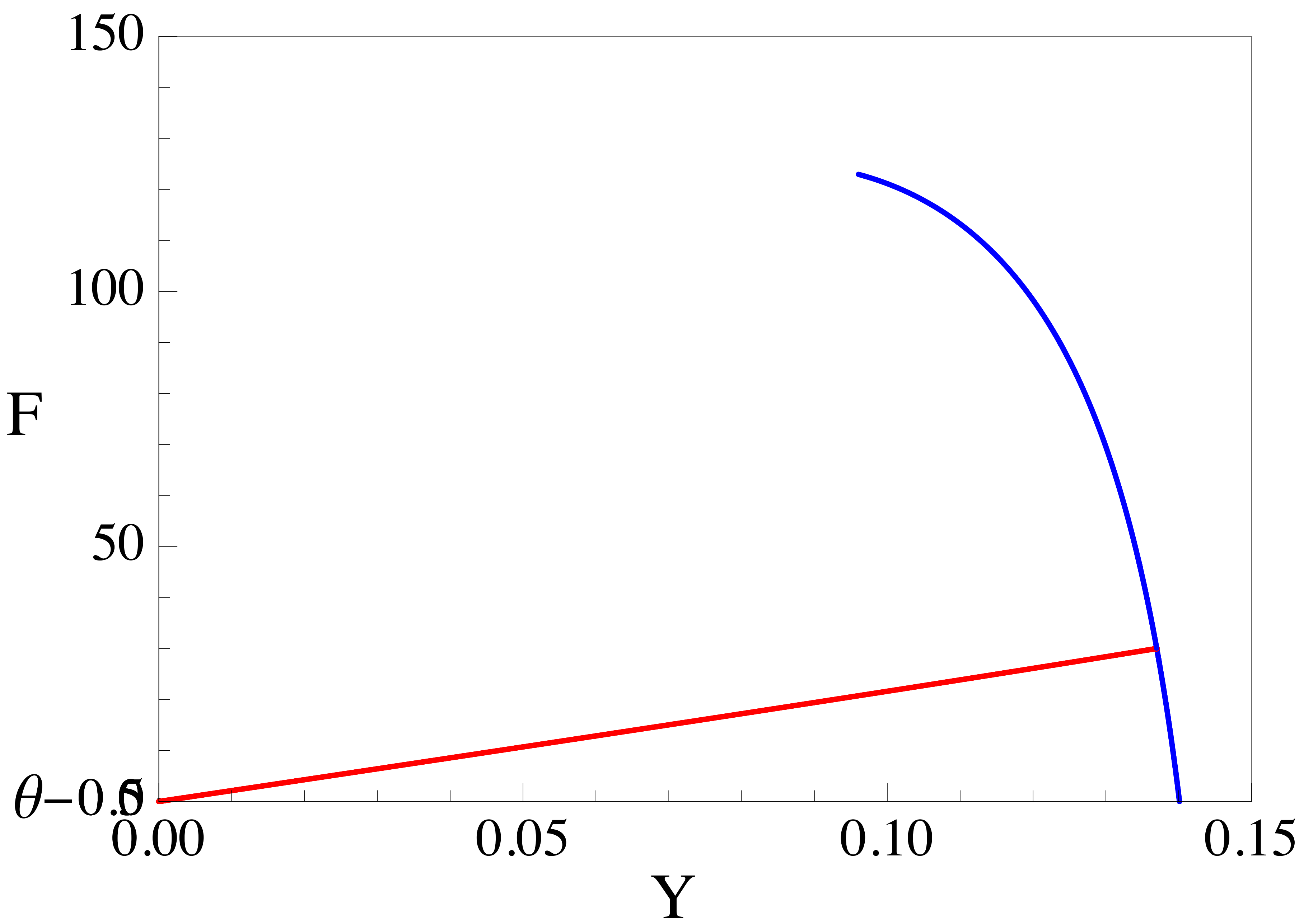


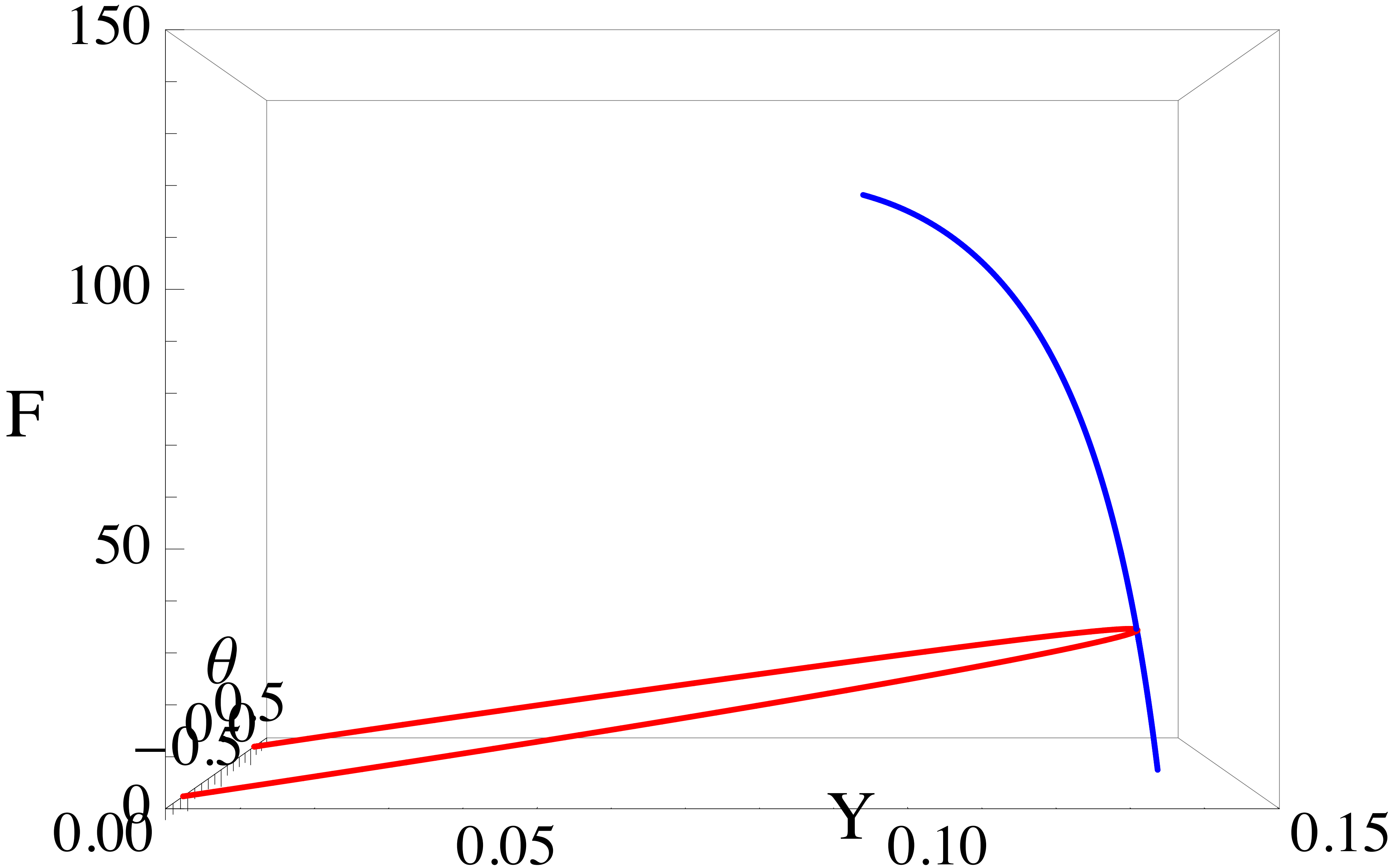
Snap-through

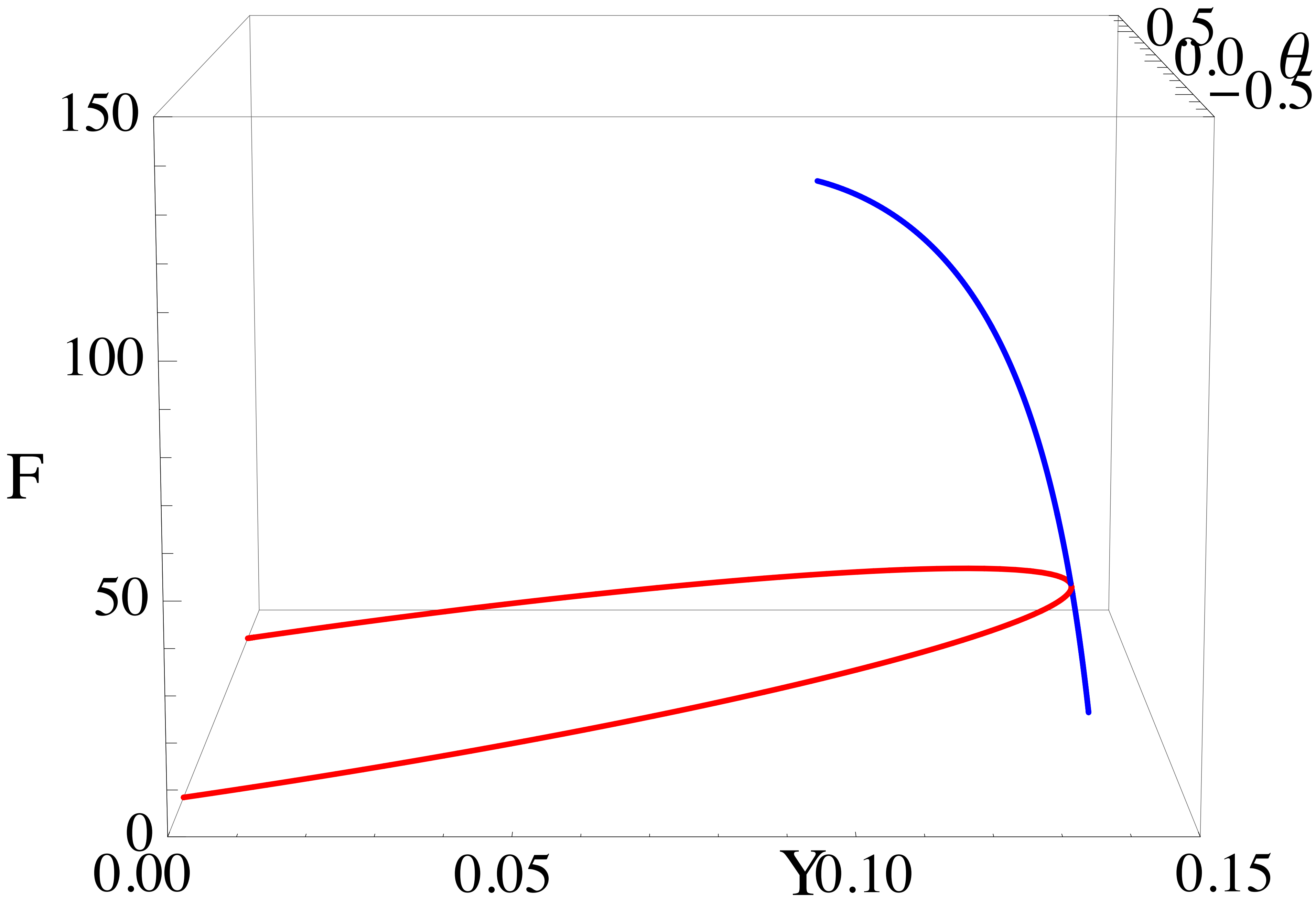


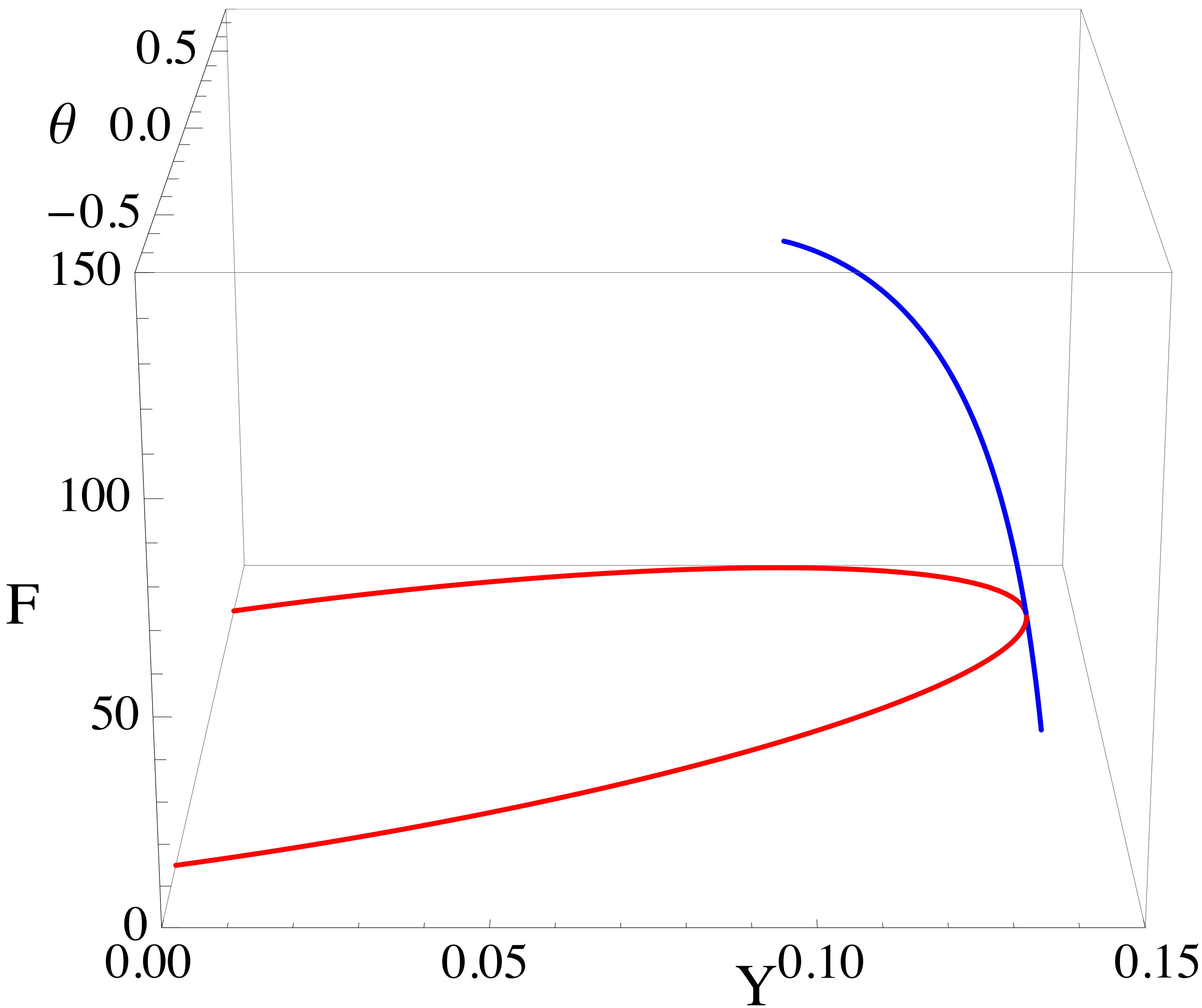
Snap-through

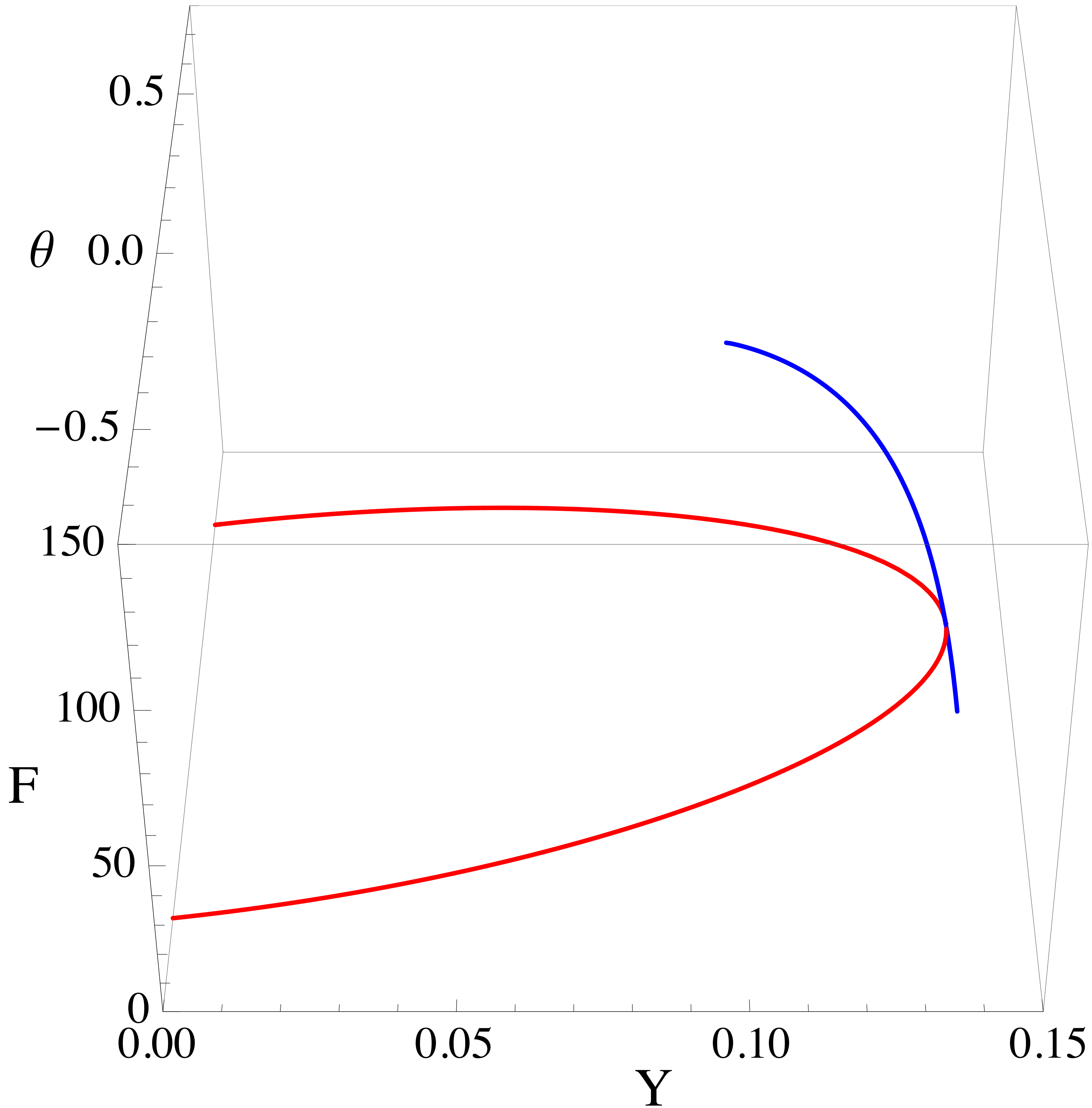


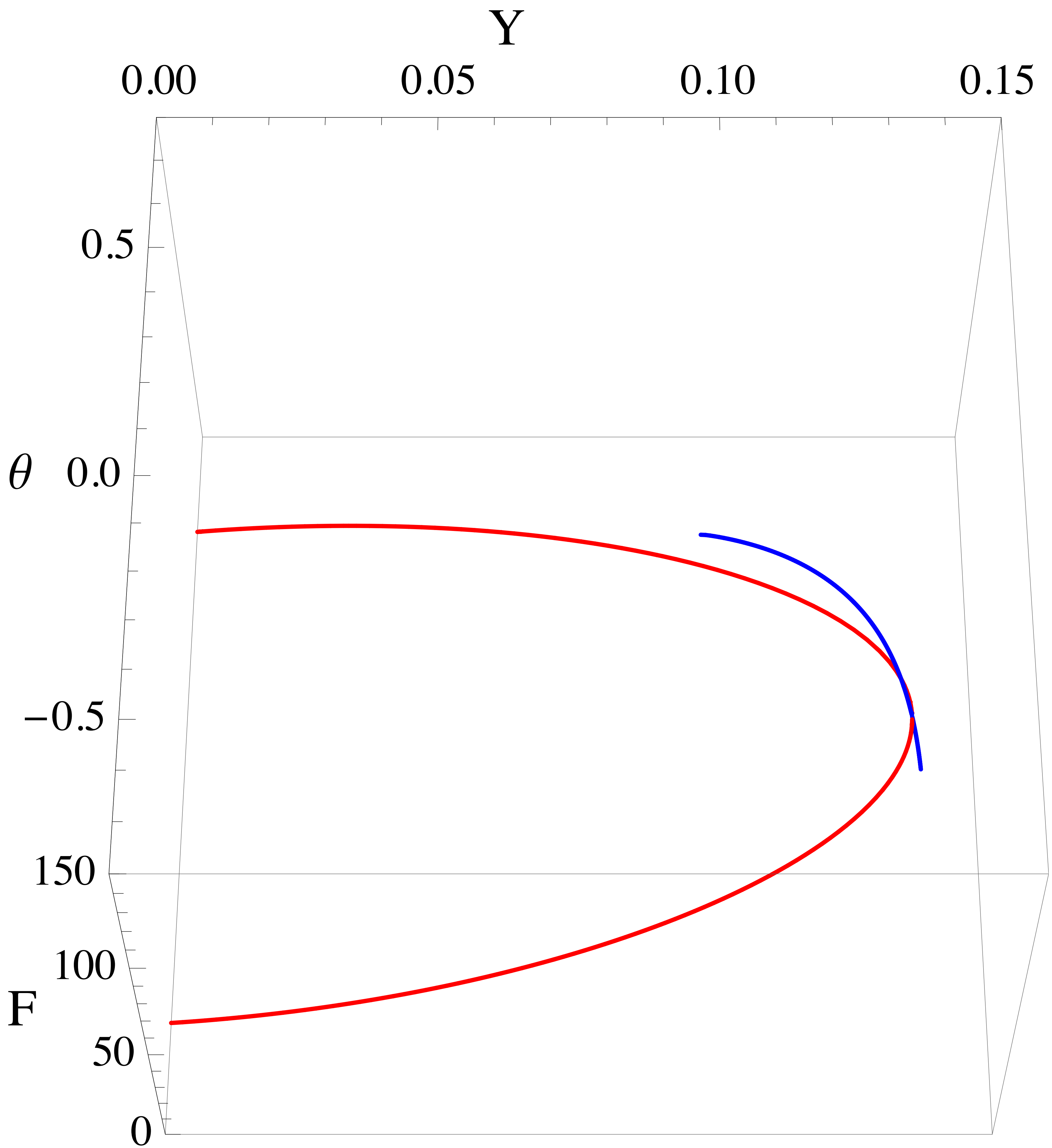

150
100
50
F
θ−0.5
0.00
0.05
0.10
0.15
Y

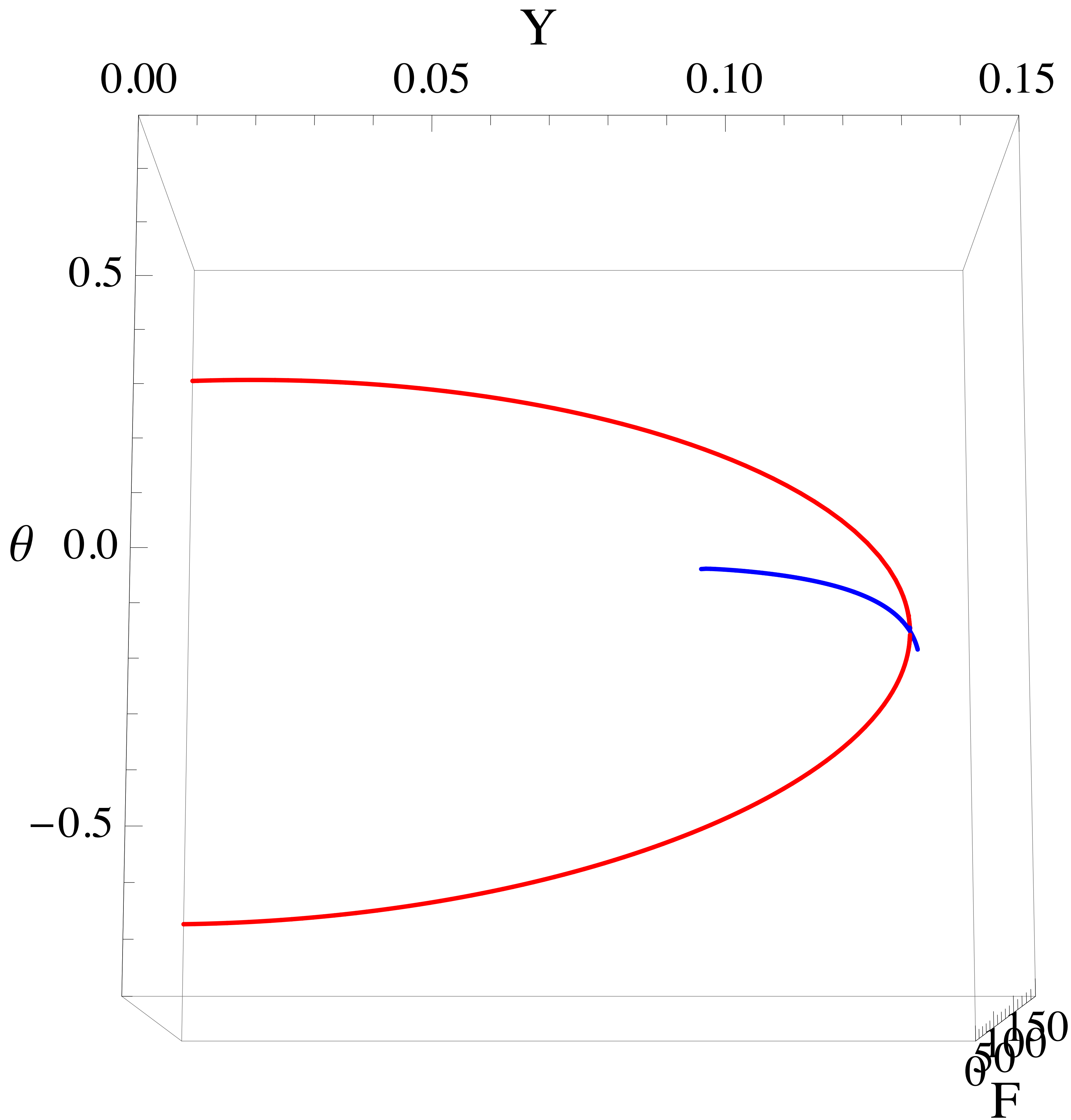

150
100
50
0
F
θ
0.5
0.0
−0.5
Y
0.00
0.05
0.10
0.15

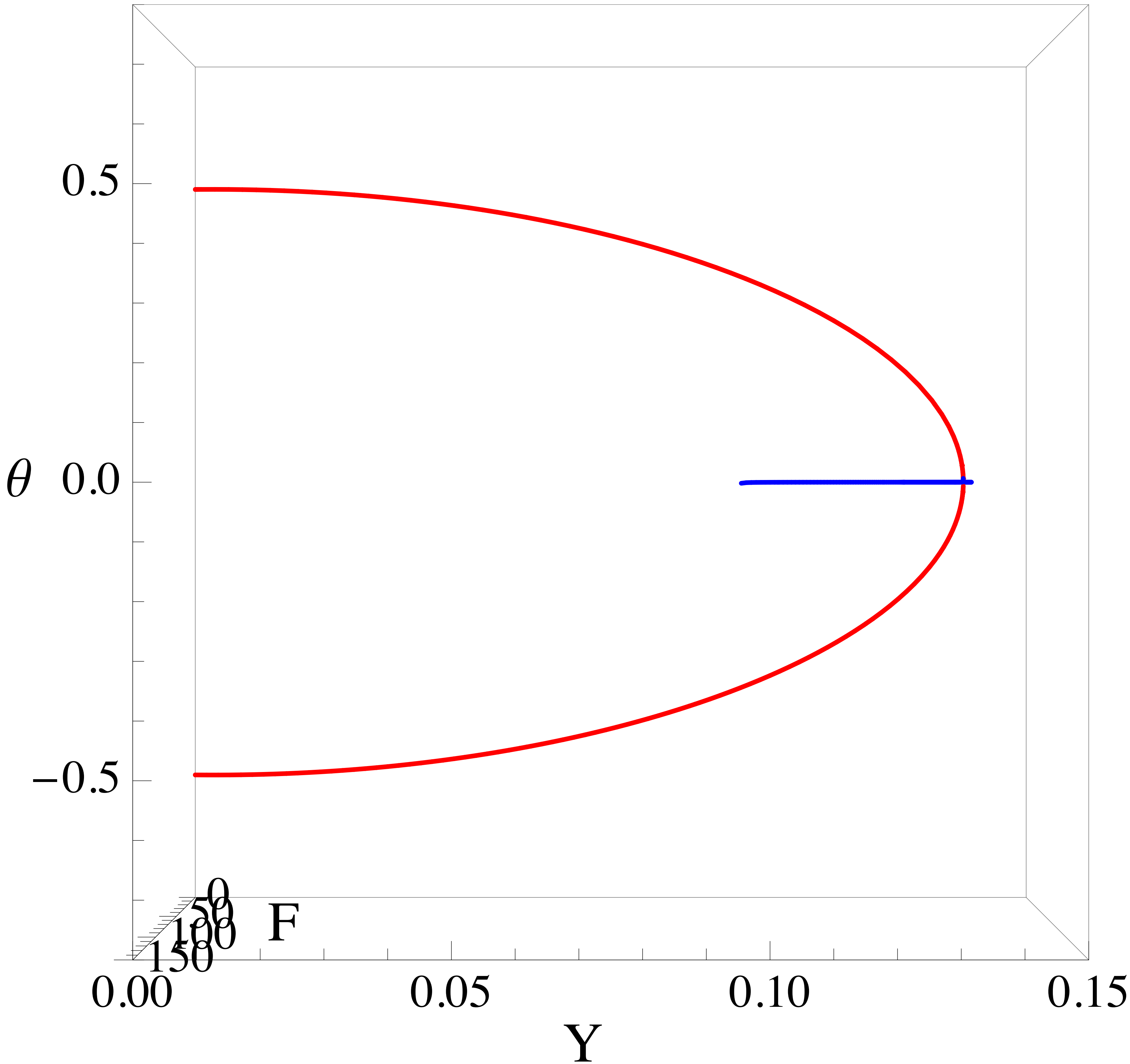

0.5
0.0
−0.5
θ
150
100
F
50
0
0.00
0.05
Y
0.10
0.15


0.5
θ 0.0
−0.5
150
100
F
50
0
0.00
0.05
Y
0.10
0.15

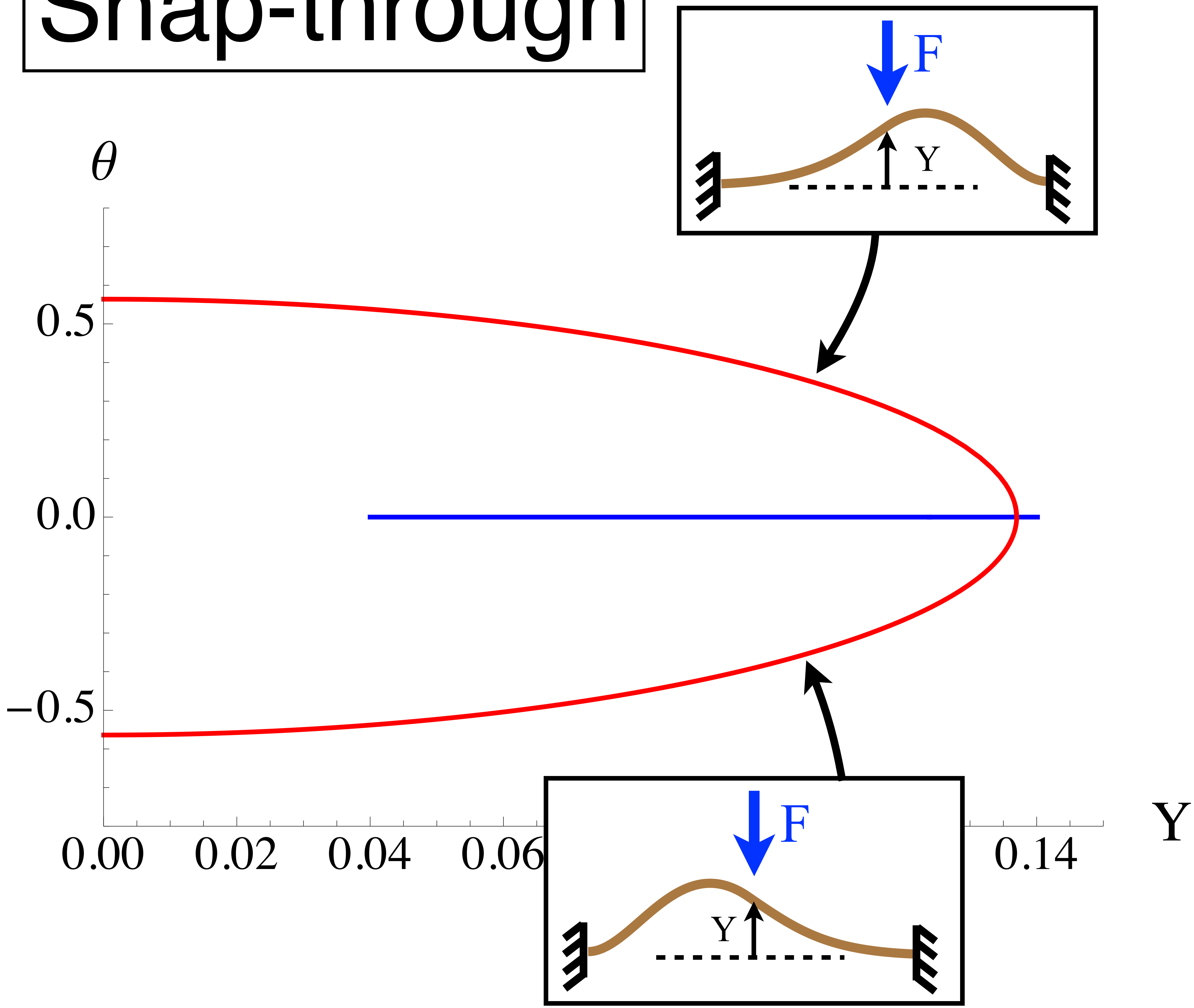

0.5
θ 0.0
−0.5
150
100
F
50
0
0.00
0.05
0.10
0.15
Y


Y
0.00
0.05
0.10
0.15
θ
0.5
0.0
−0.5
F
150
100
50
0


Y
0.00
0.05
0.10
0.15
θ
0.5
0.0
−0.5
0
50
100
150
F

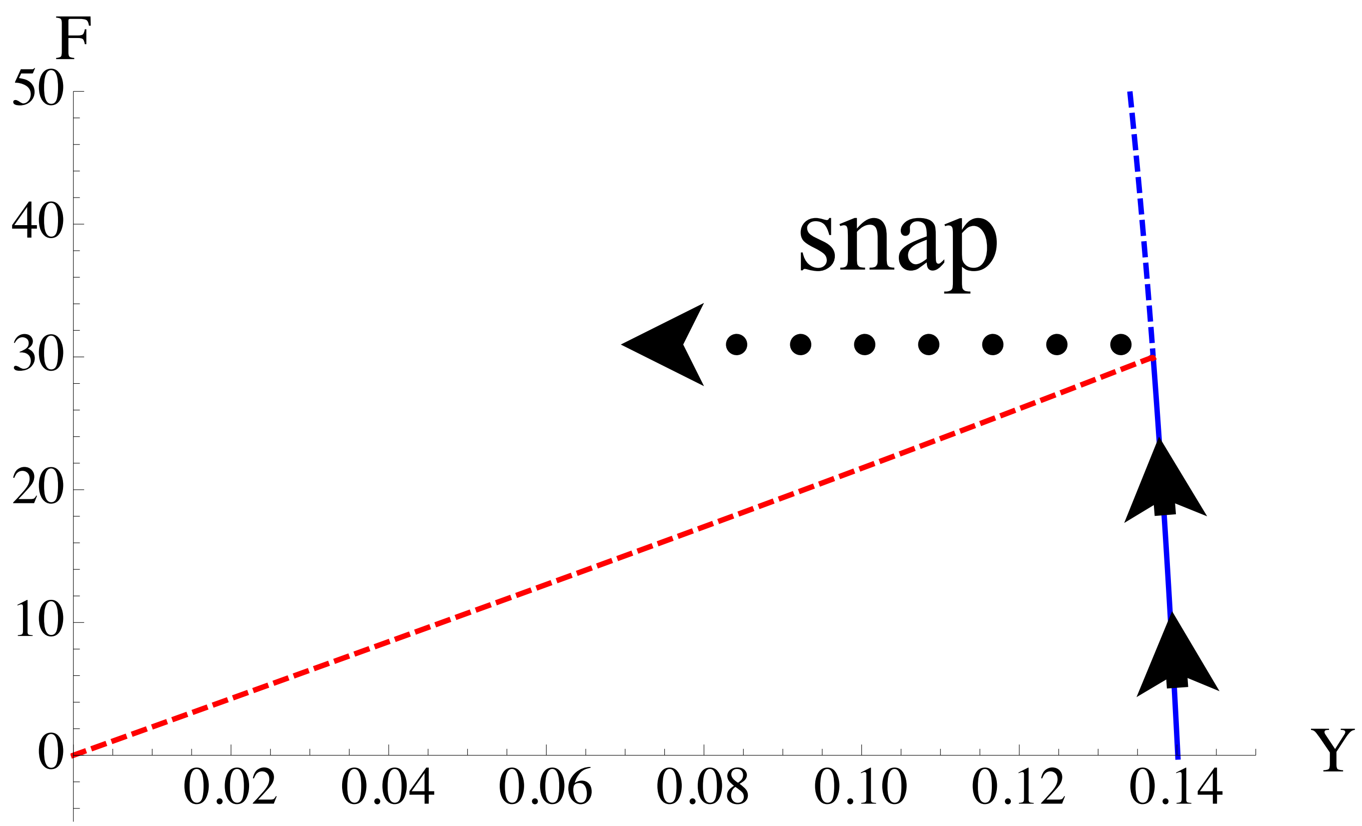

0.5
θ
0.0
−0.5
0
50
100
150
F
0.00
0.05
0.10
0.15
Y

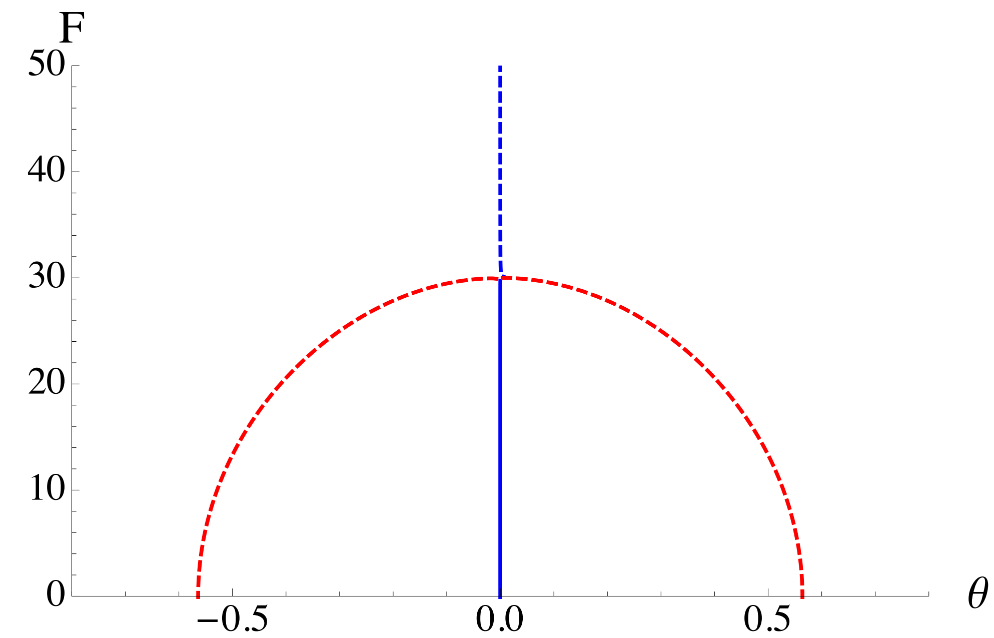
Snap-through



Snap-through

stability when force F is controlled

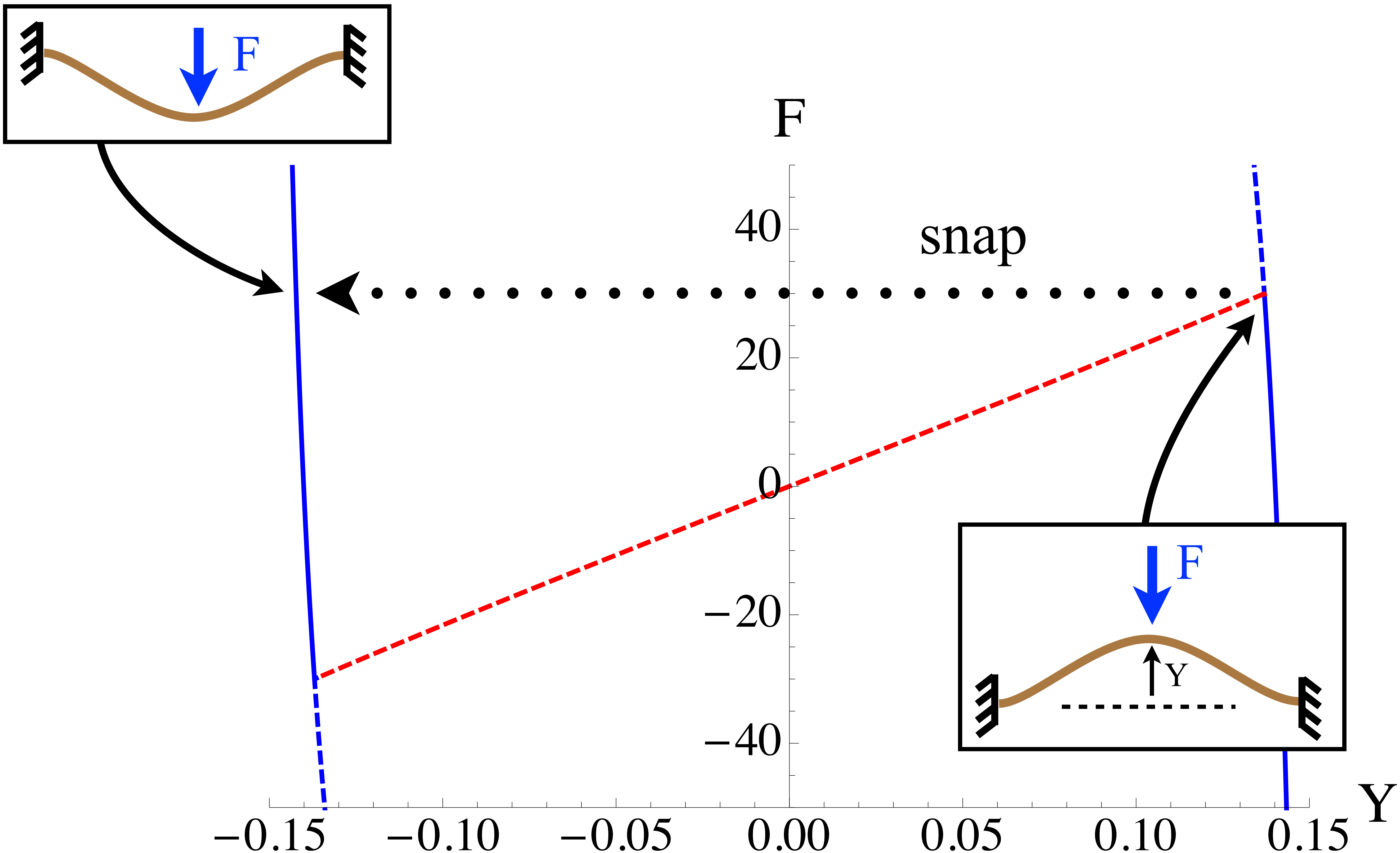




sub-critical
pitchfork

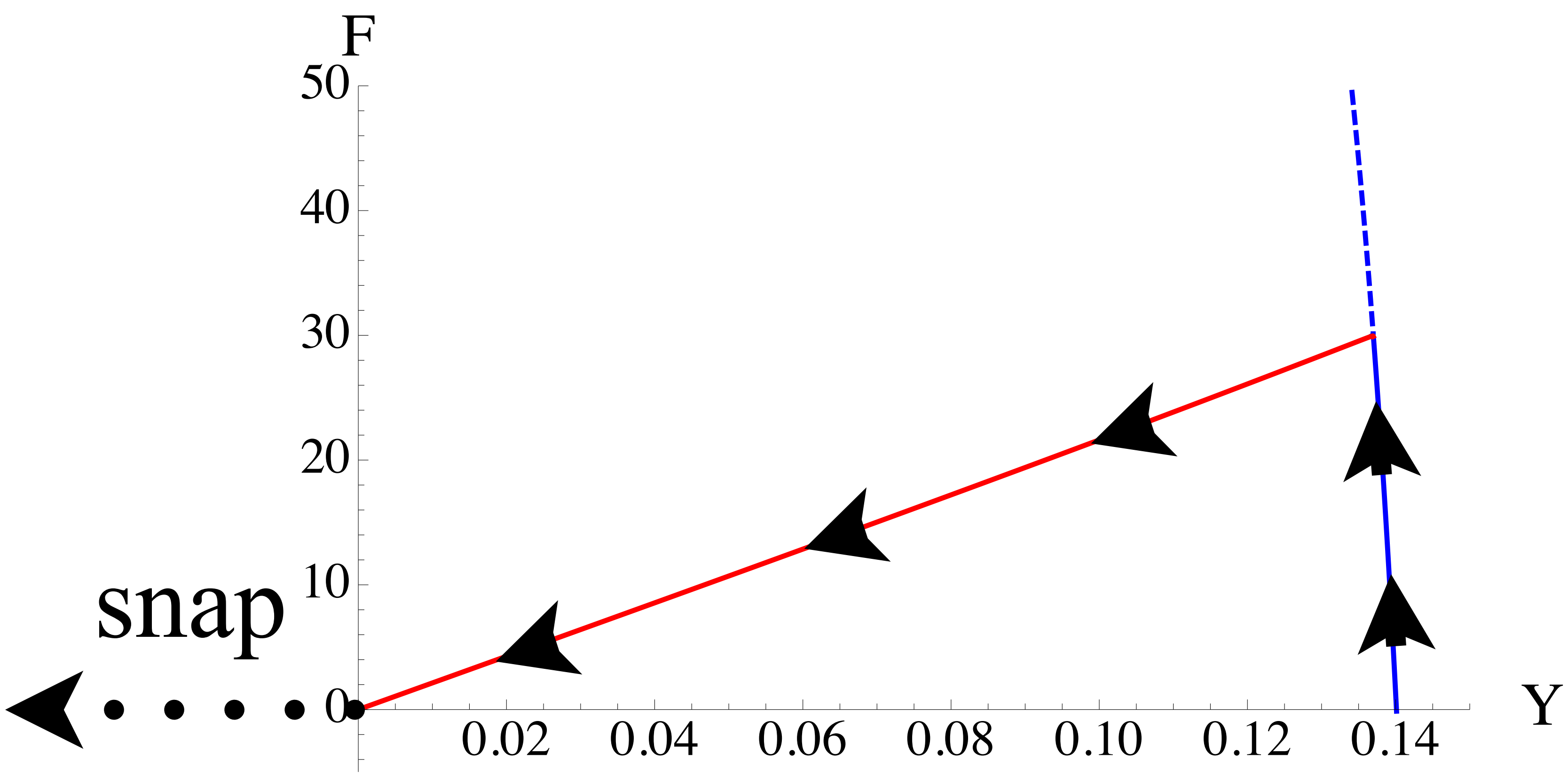
Snap-through

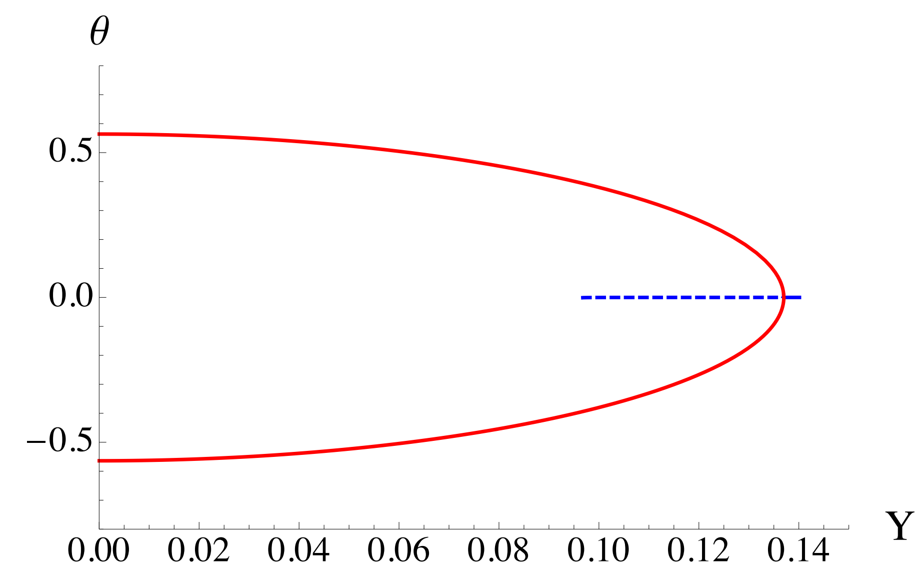
force F is controlled



Snap-through

stability when displacement Y is controlled

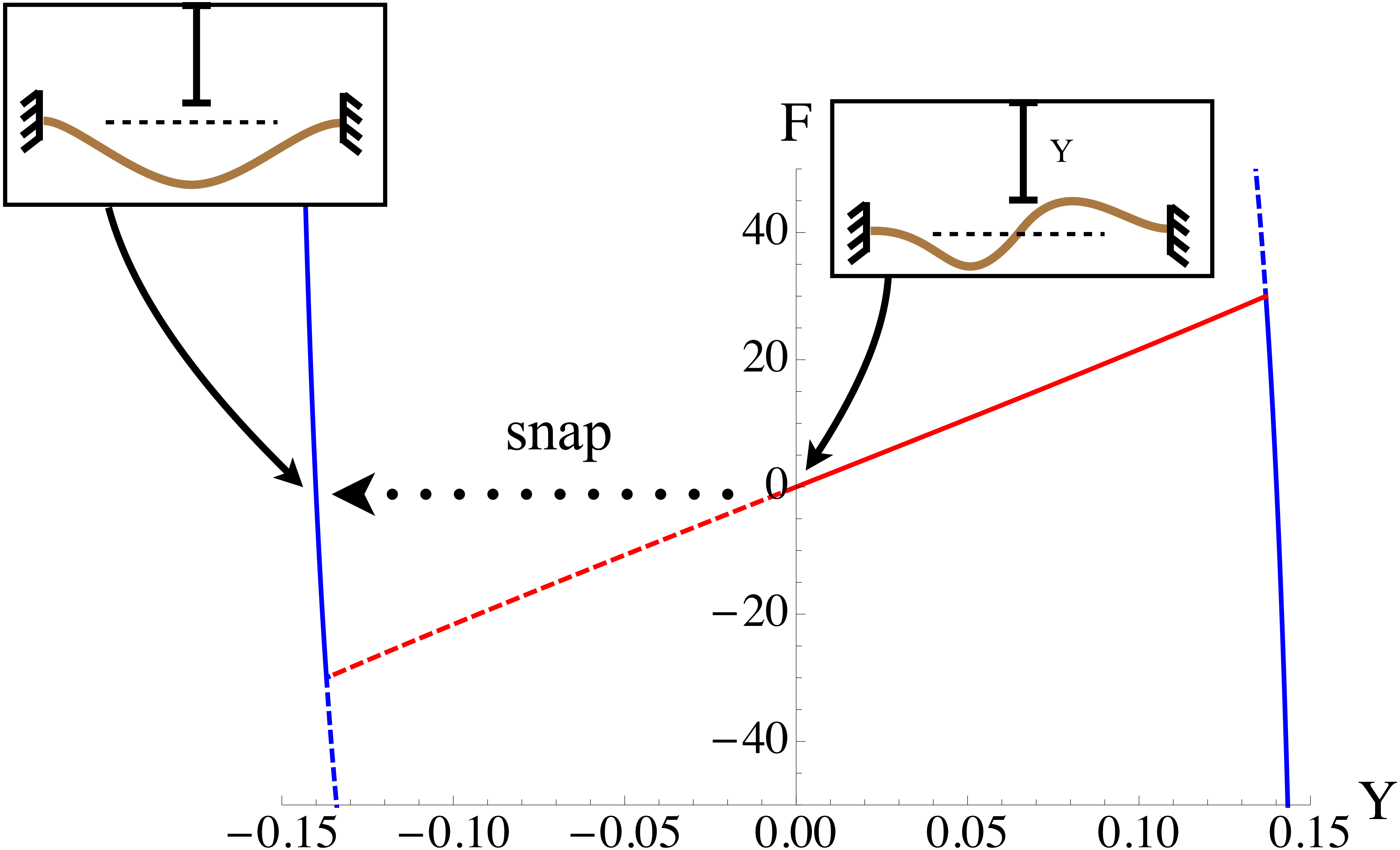




super-critical
pitchfork

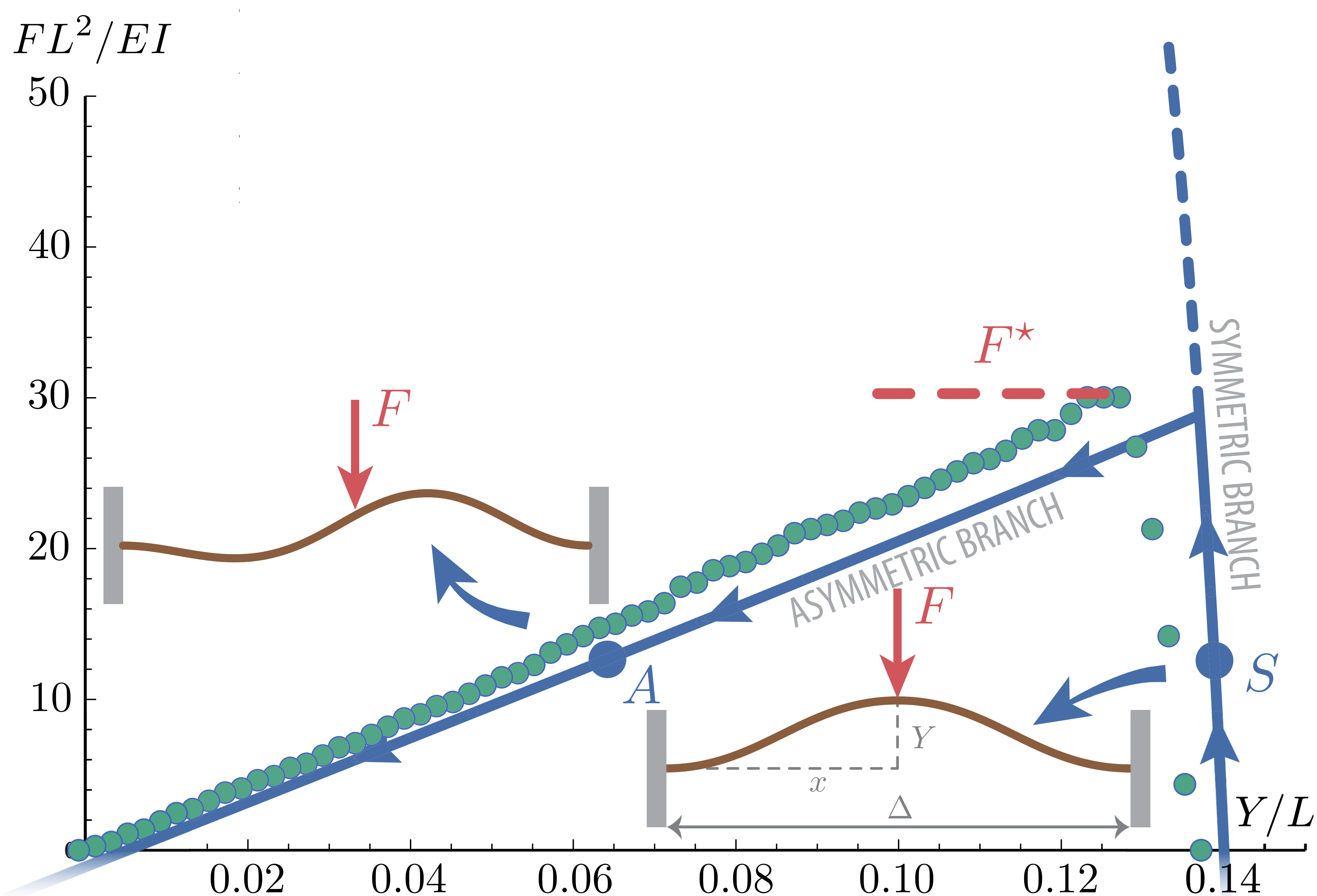
Snap-through

displacement Y is controlled



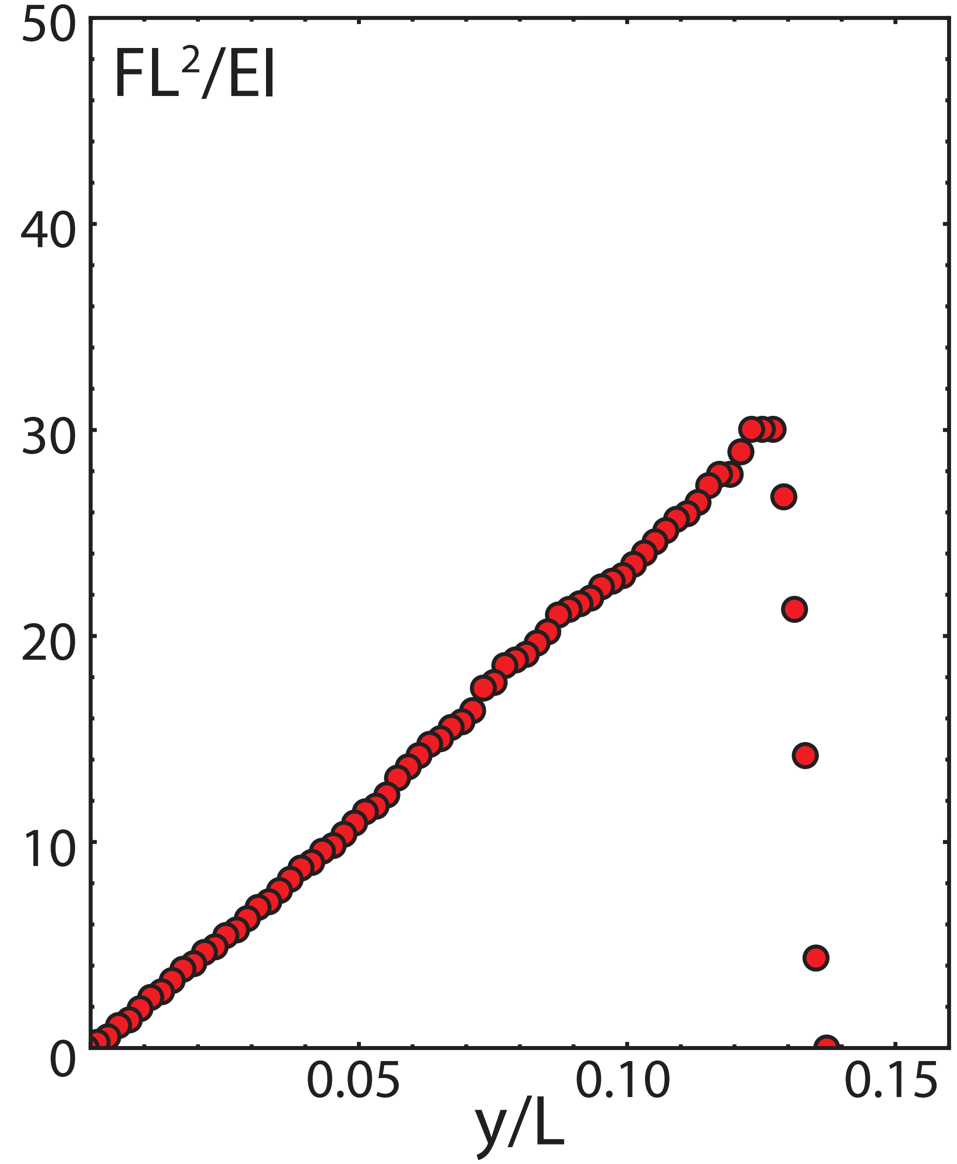
Snap-through

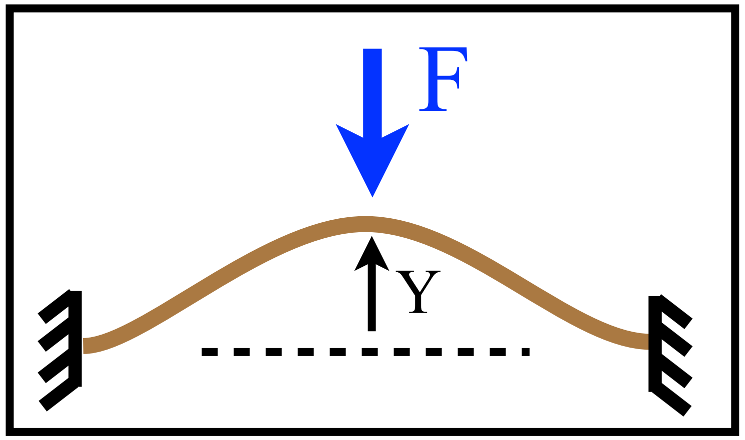
comparison with experiments



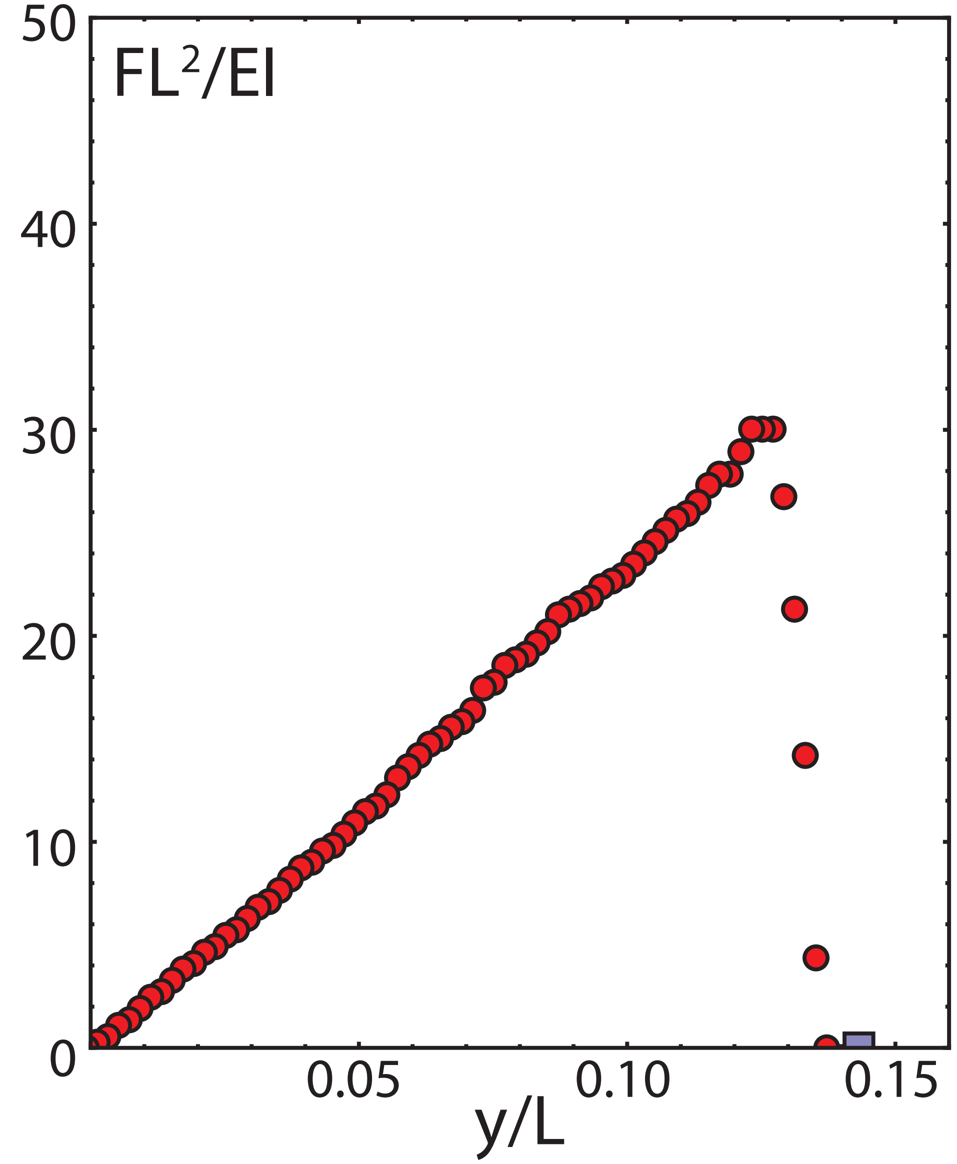
Snap-through with liquid drop

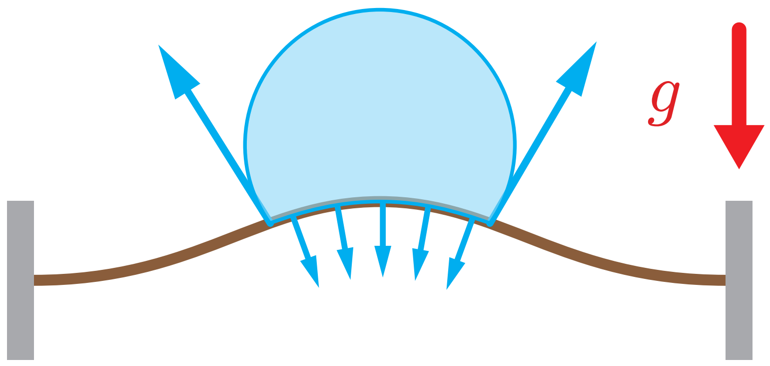
Elastocapillary Snapping vs Classical Snapping



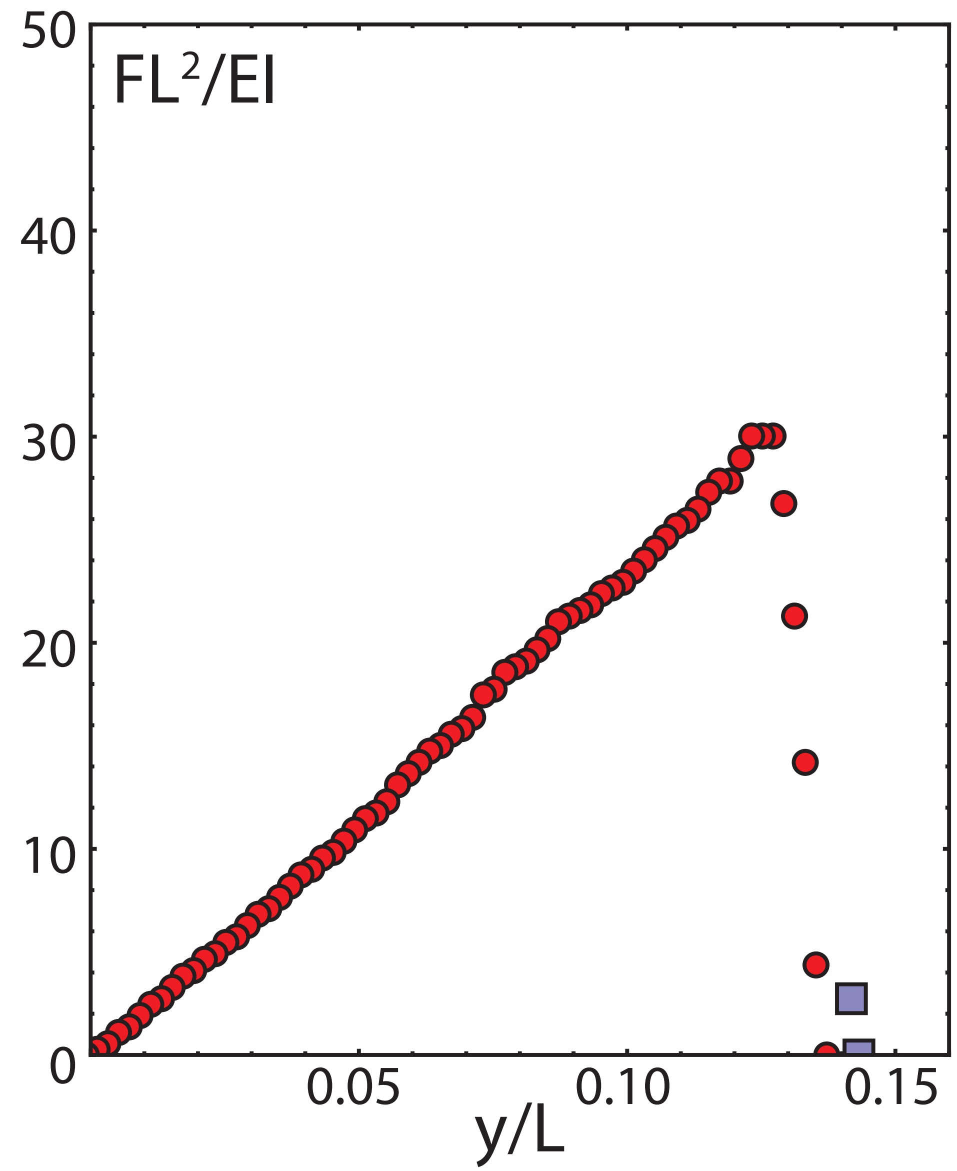


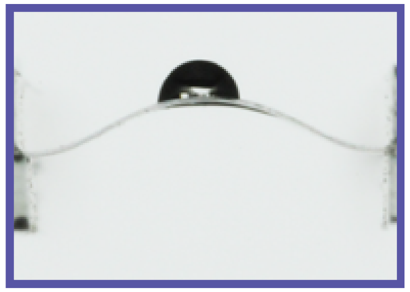
Elastocapillary Snapping vs Classical Snapping

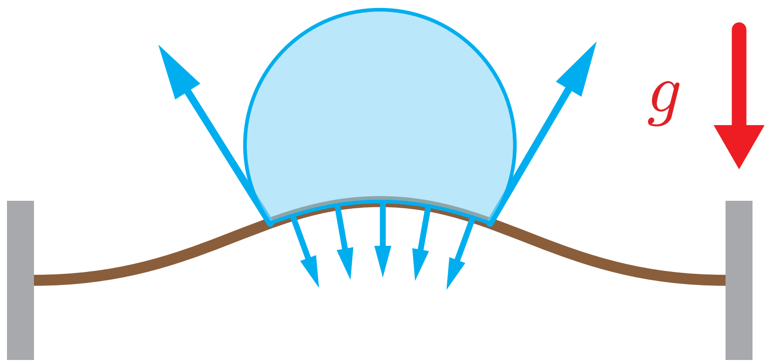




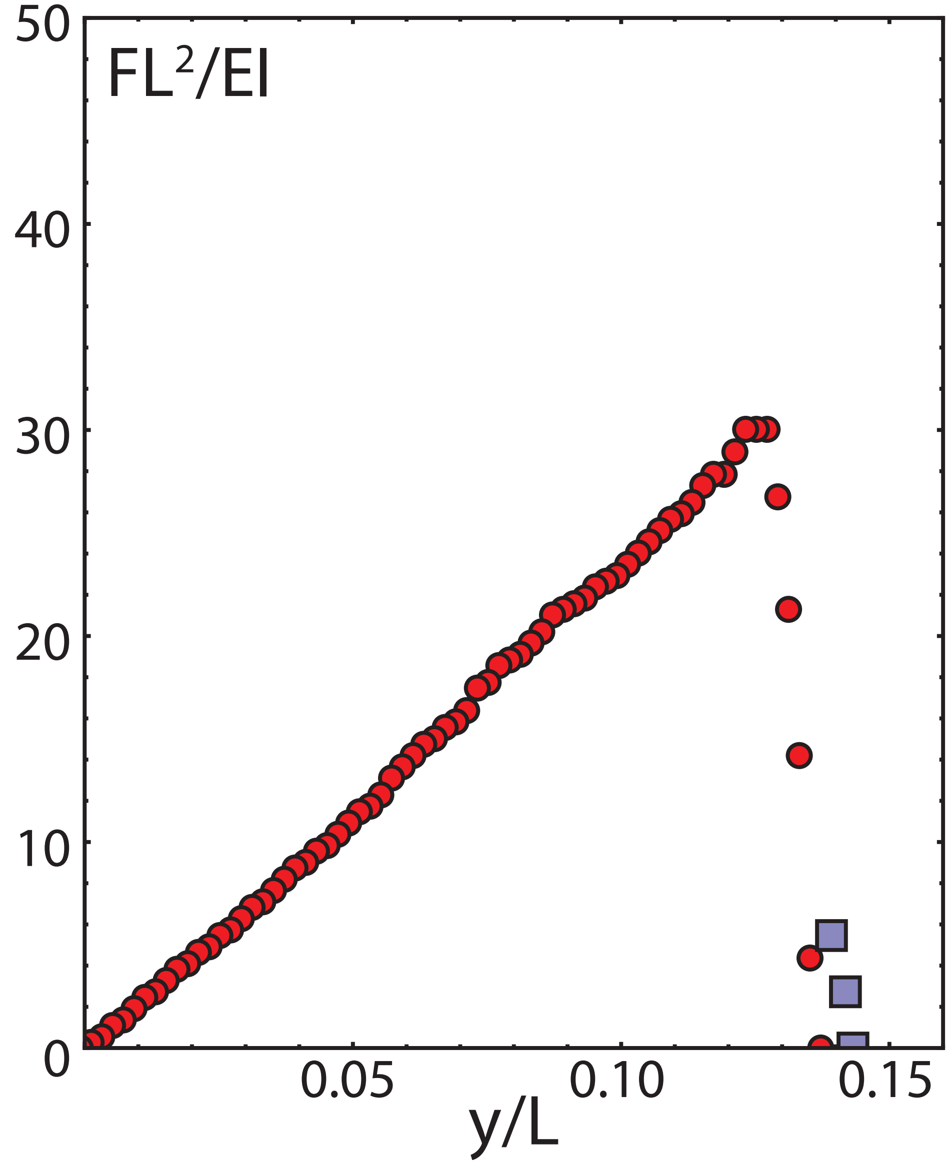
Elastocapillary Snapping vs Classical Snapping

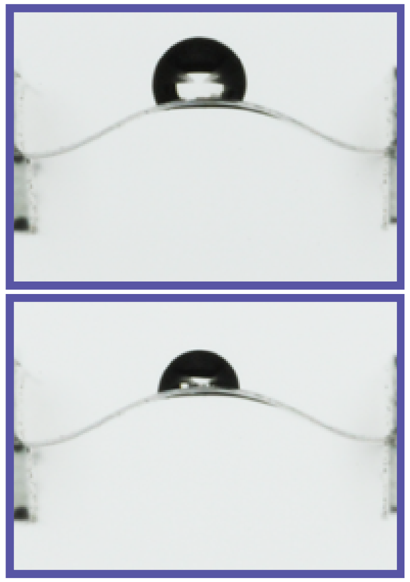


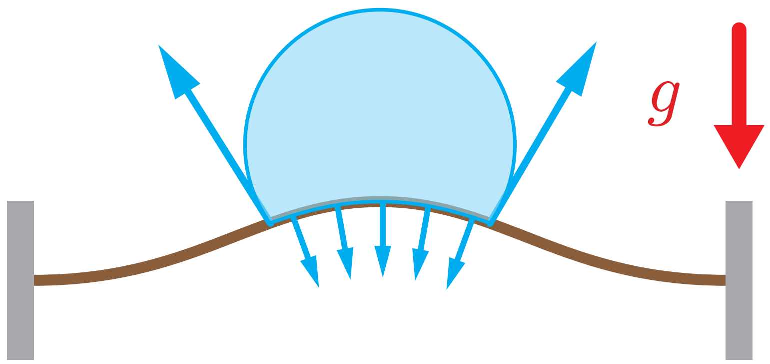




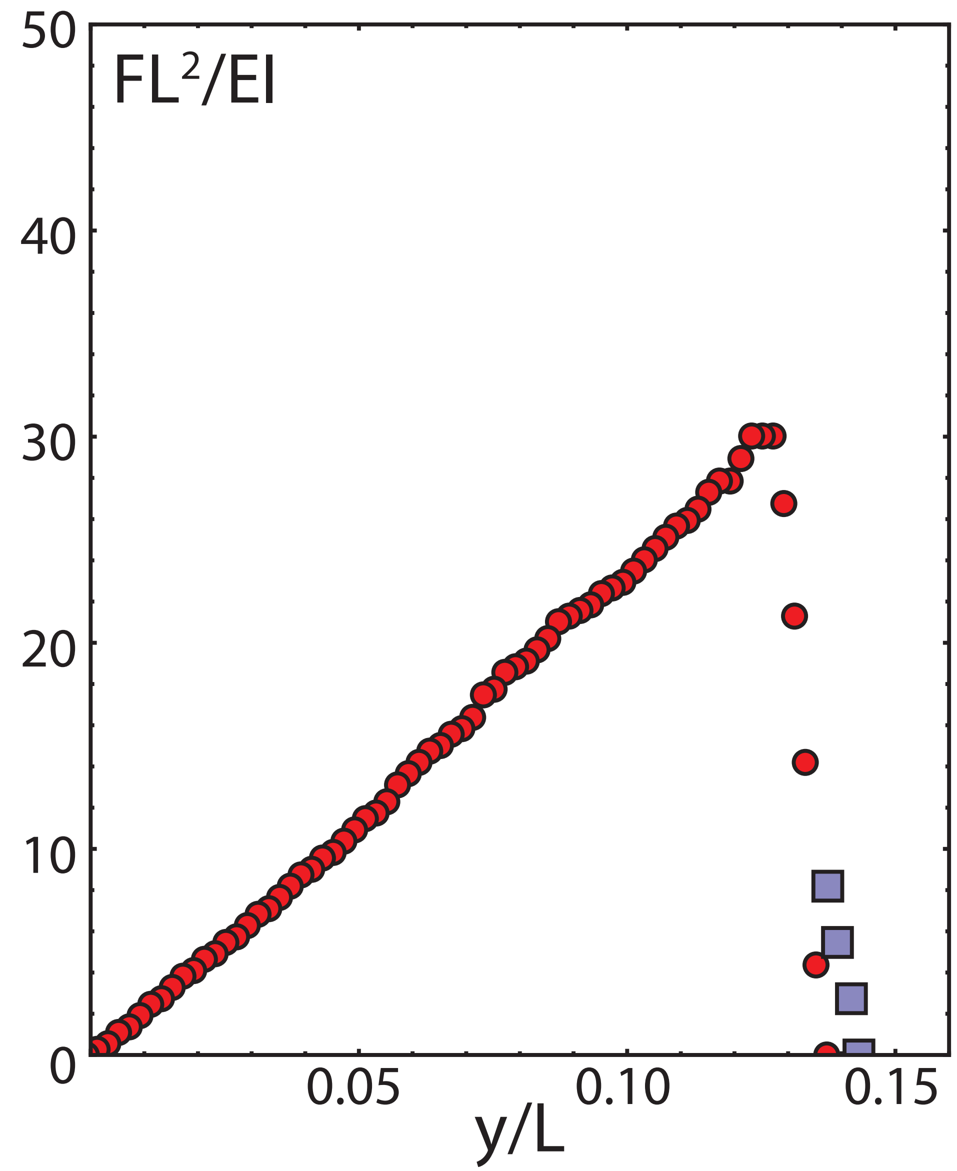
Elastocapillary Snapping vs Classical Snapping

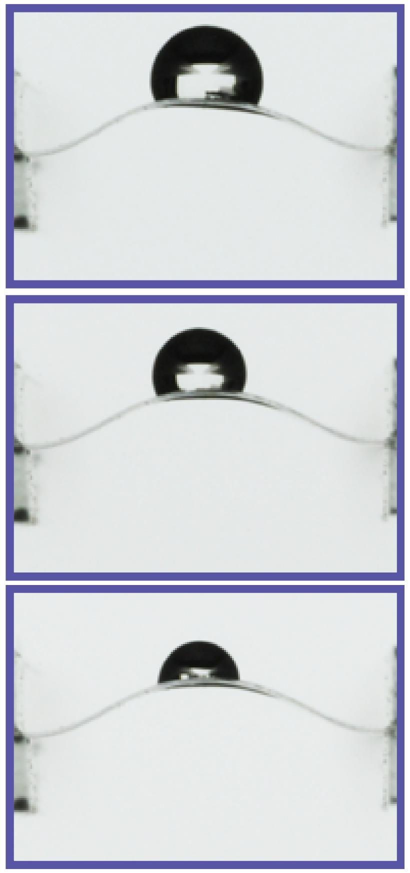


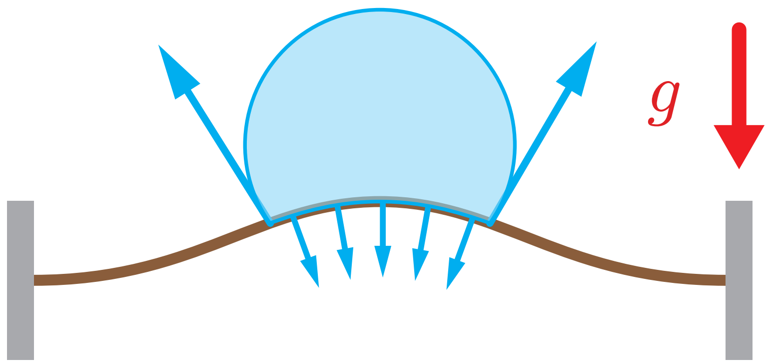




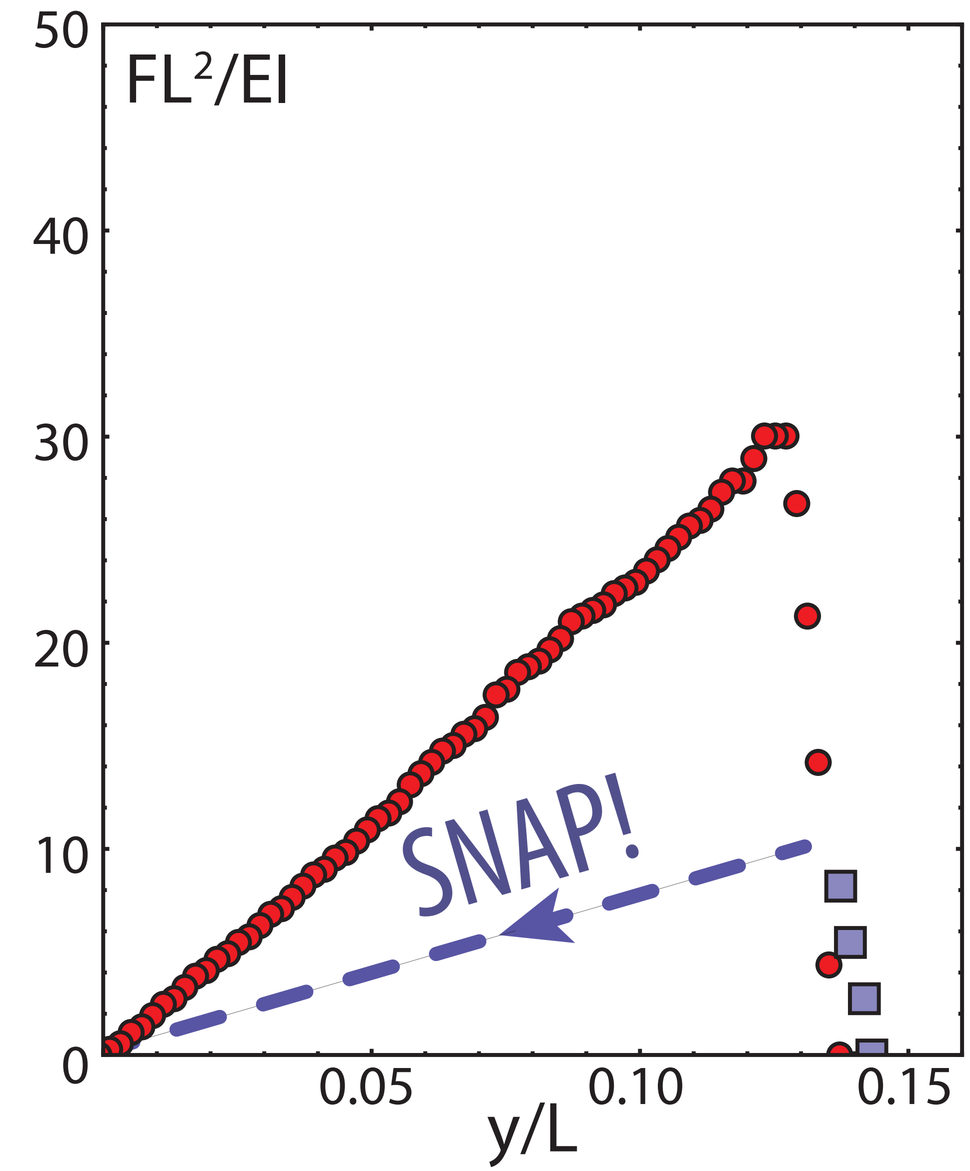
Elastocapillary Snapping vs Classical Snapping

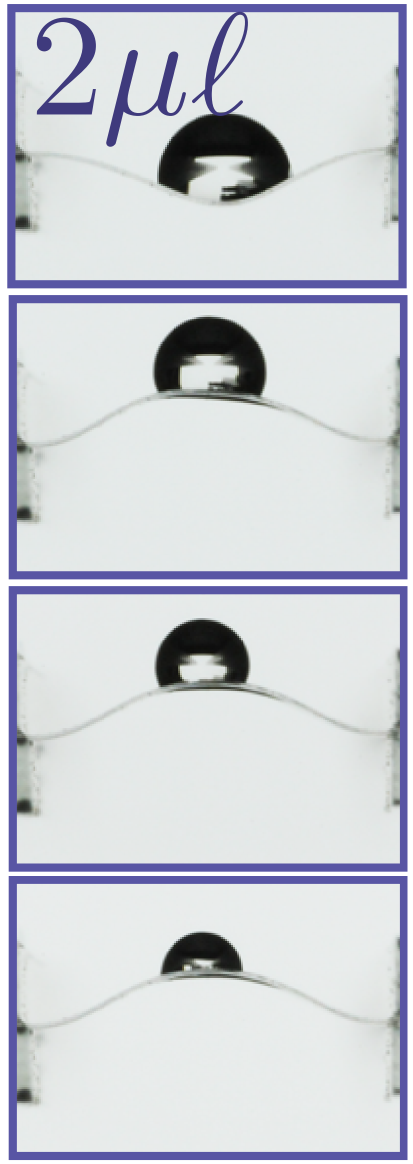


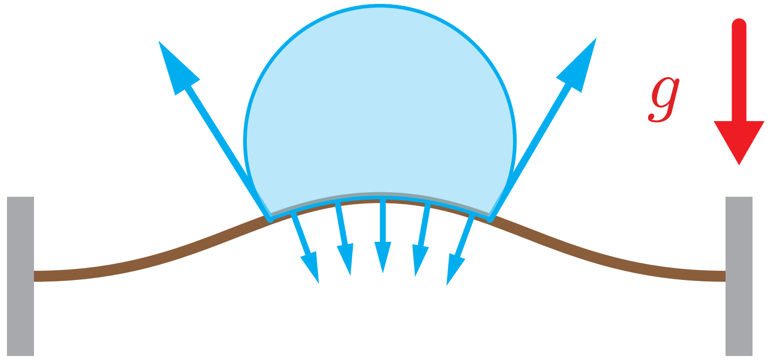




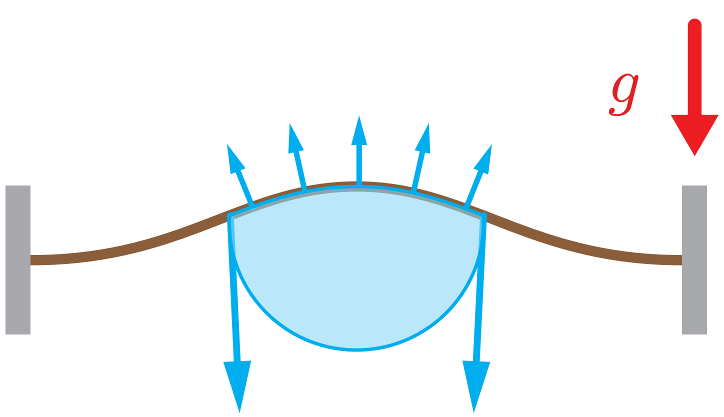
Elastocapillary Snapping vs Classical Snapping

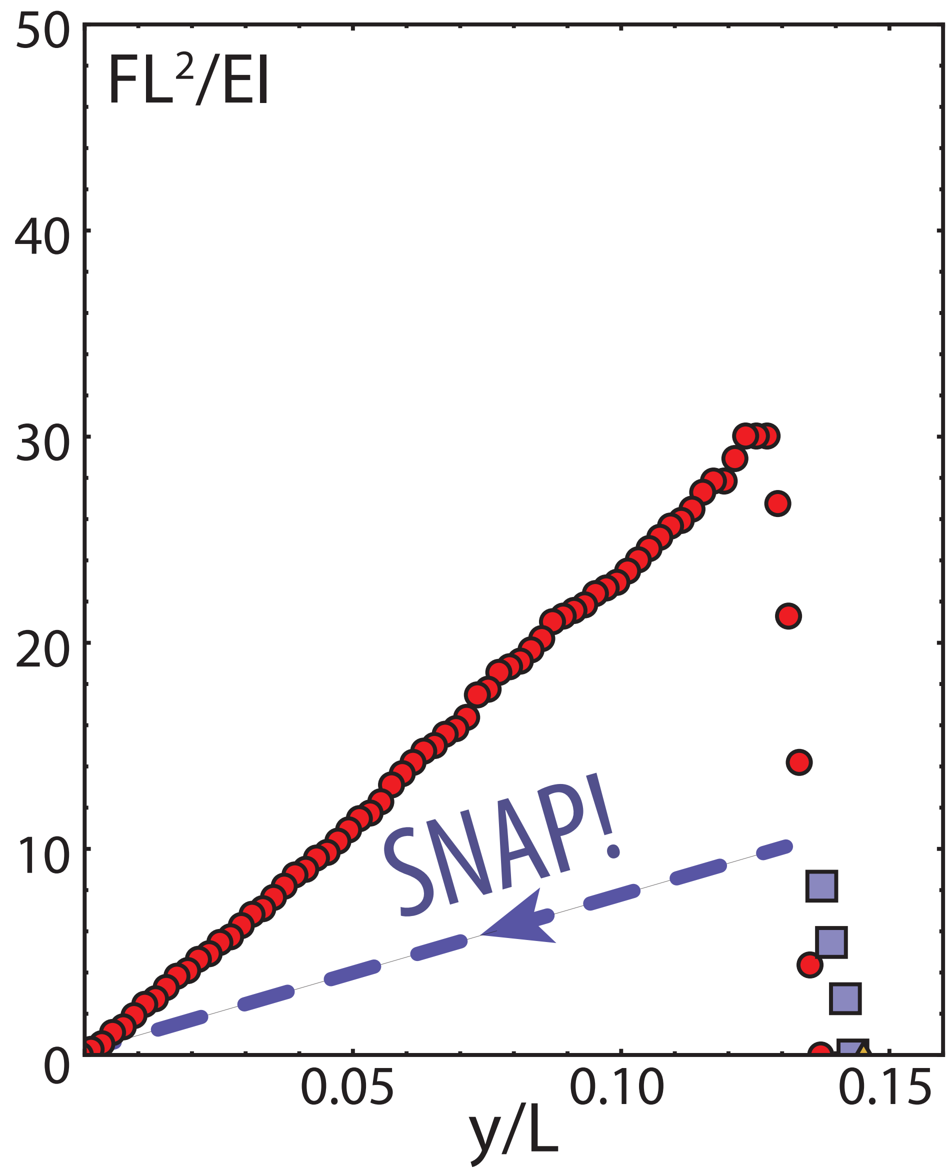


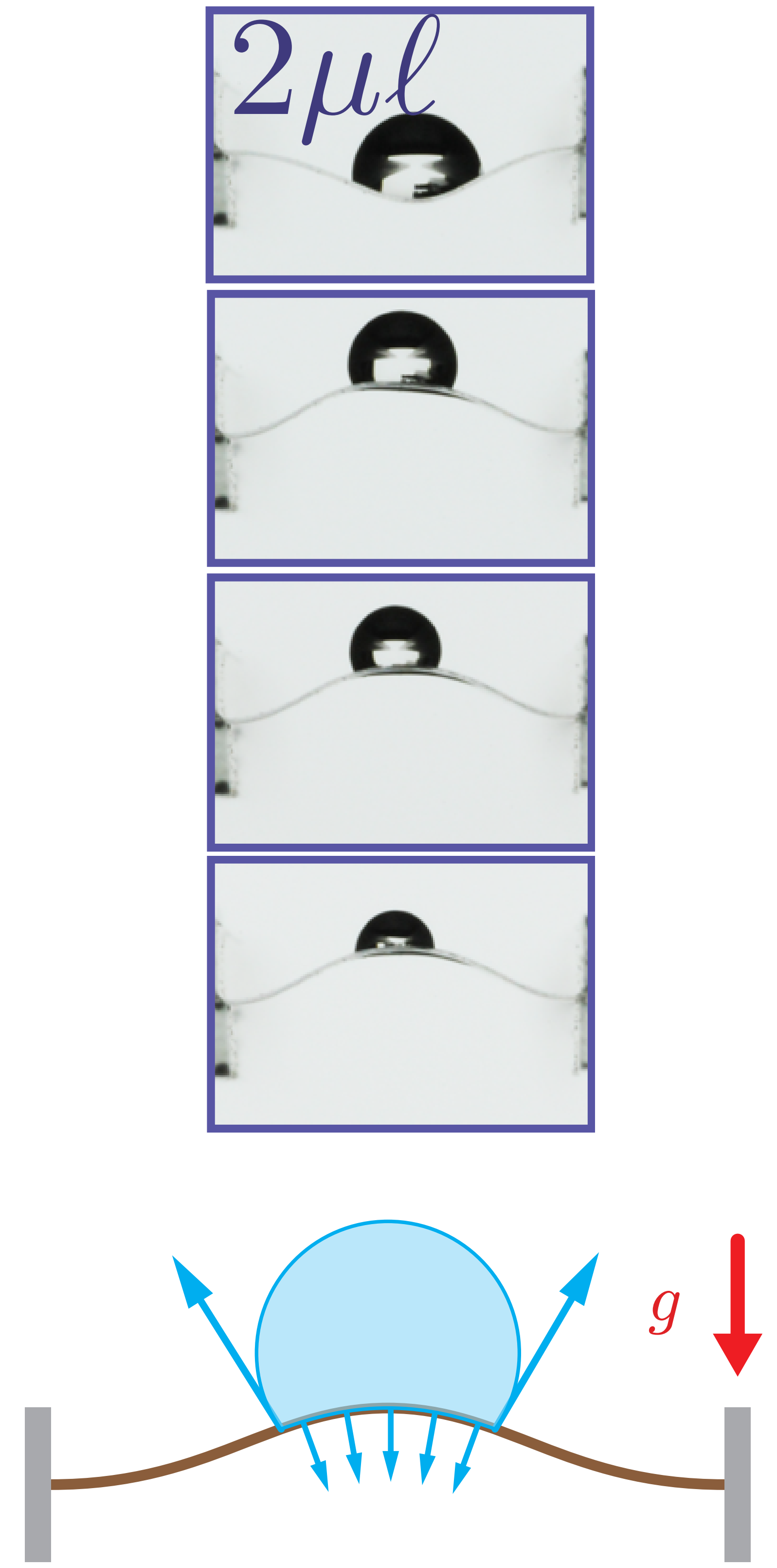




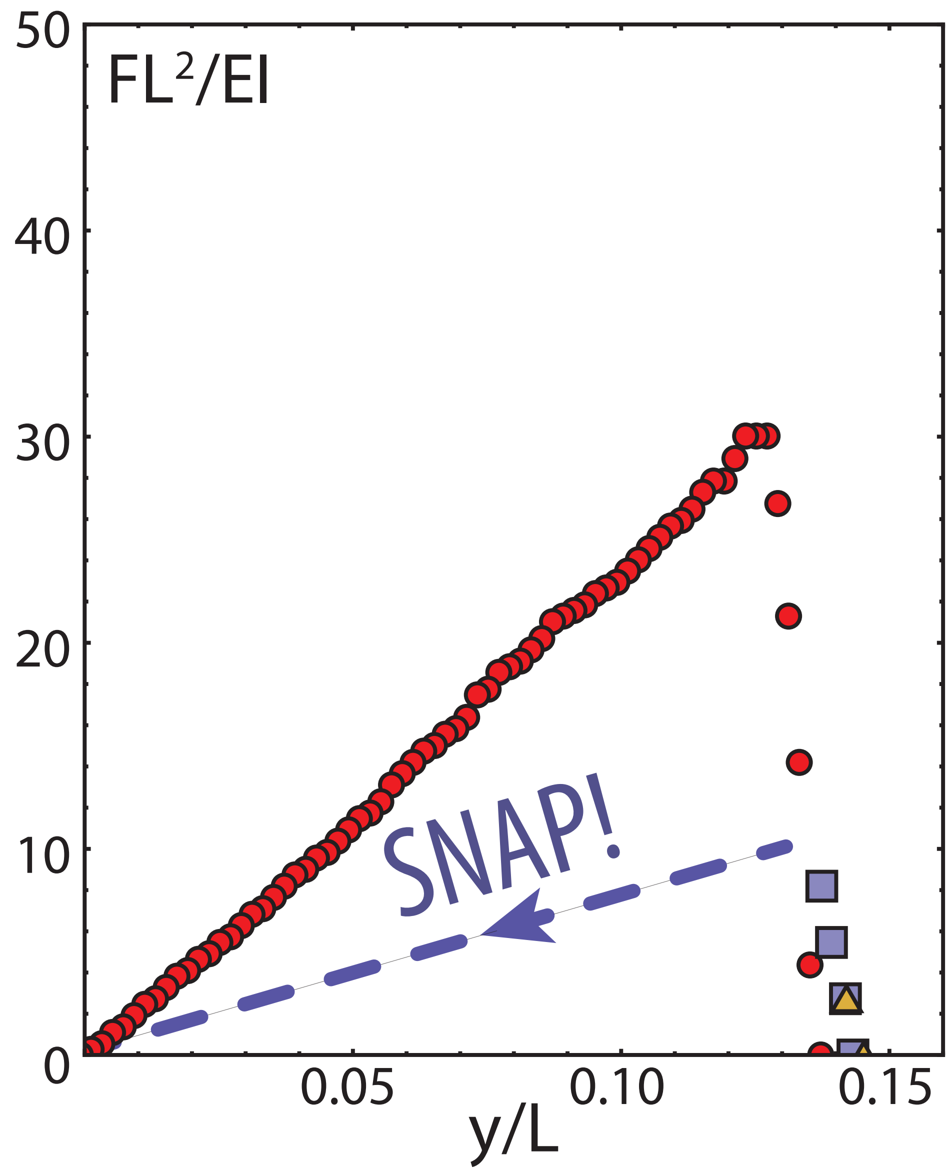
Elastocapillary Snapping vs Classical Snapping

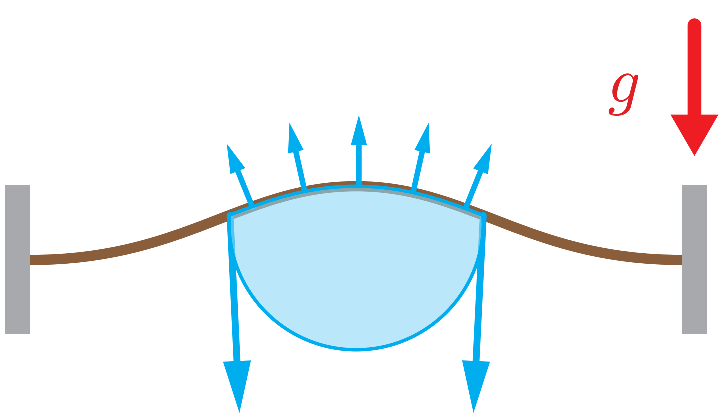


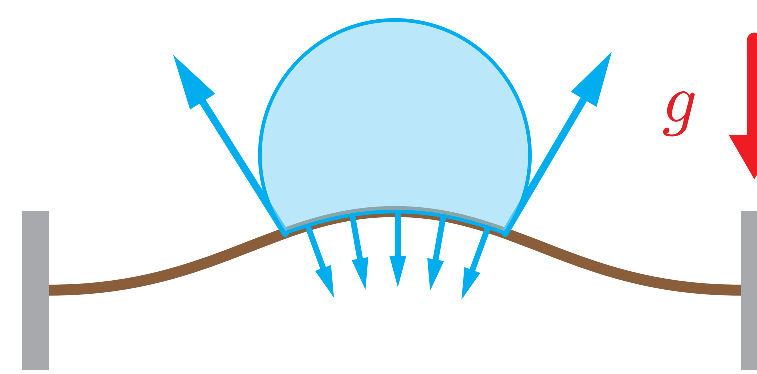




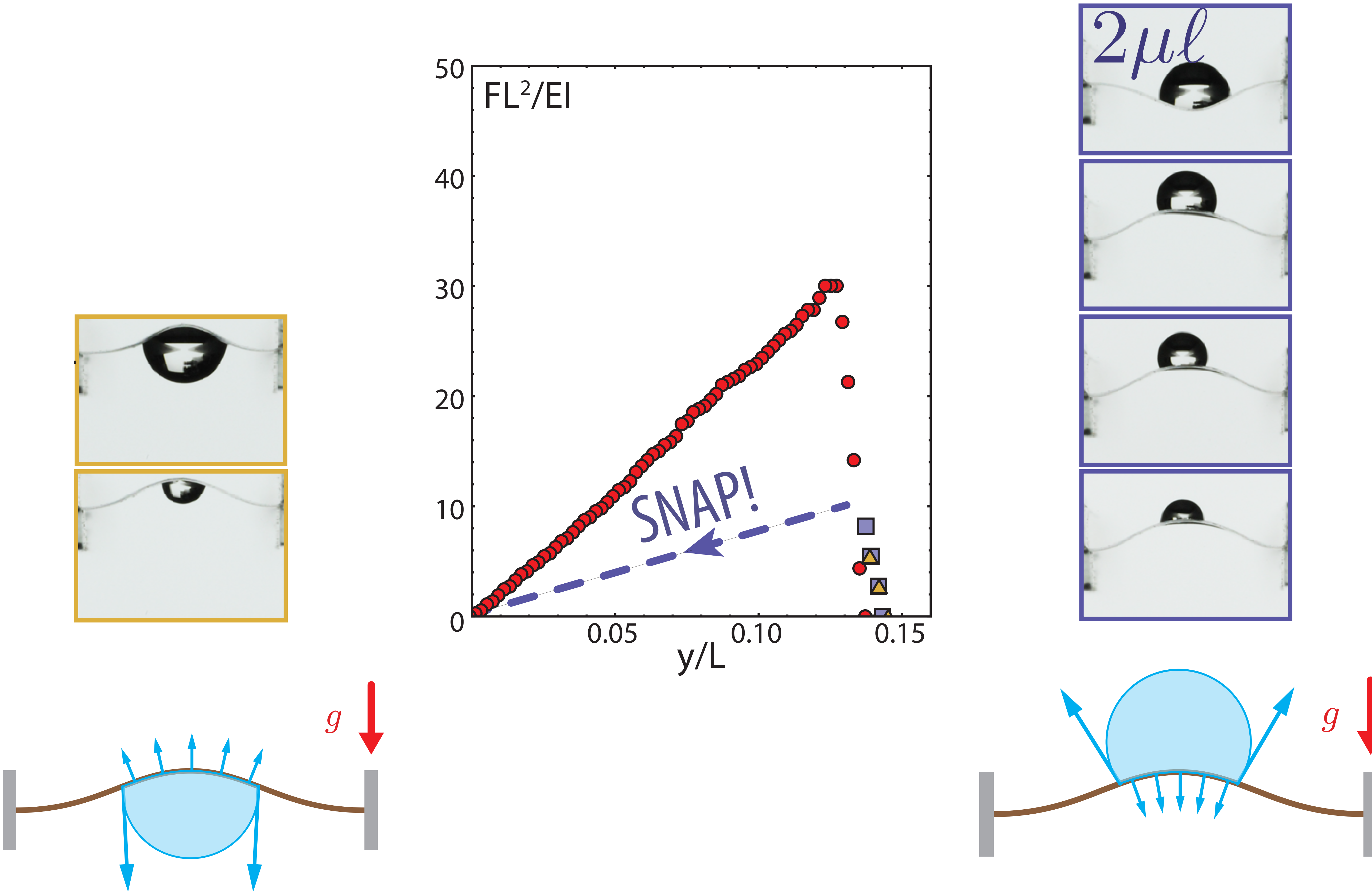
Elastocapillary Snapping vs Classical Snapping



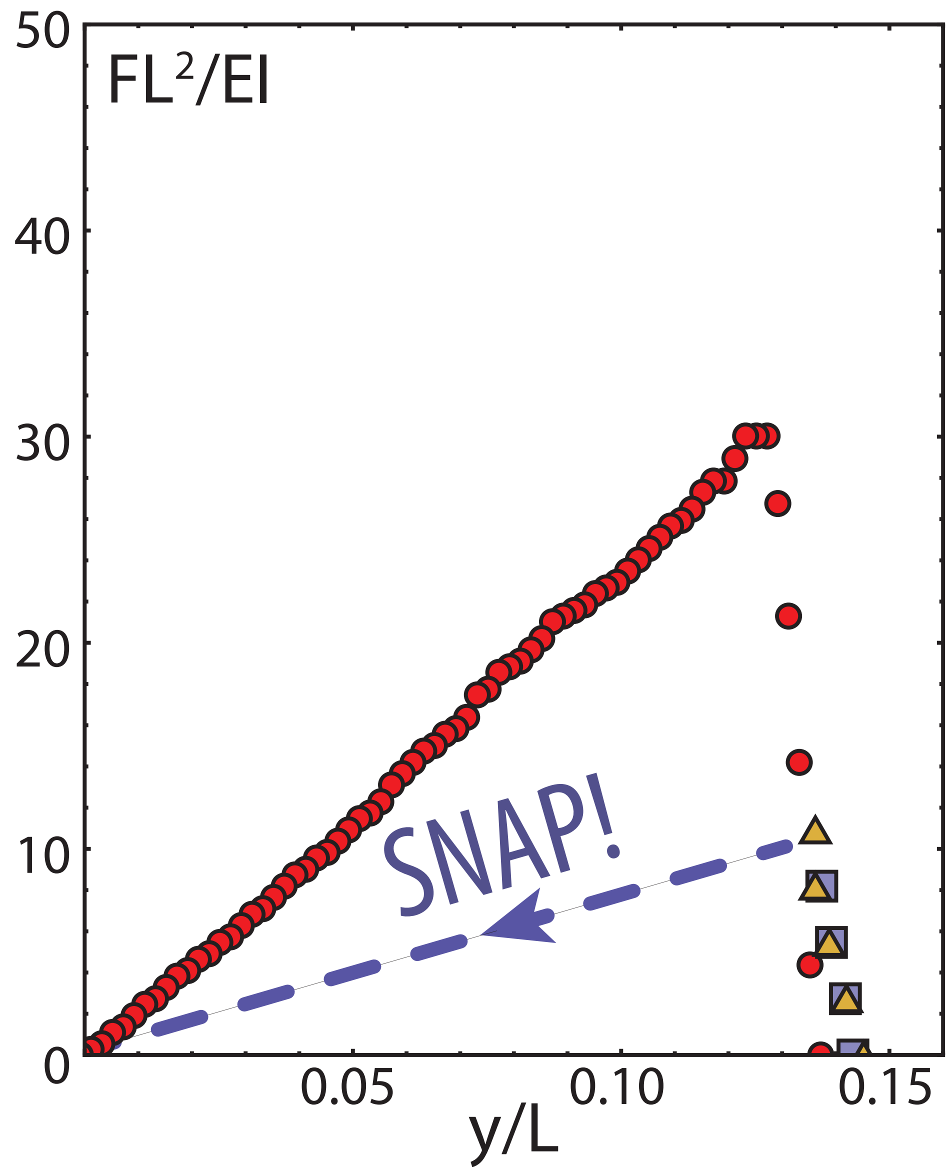


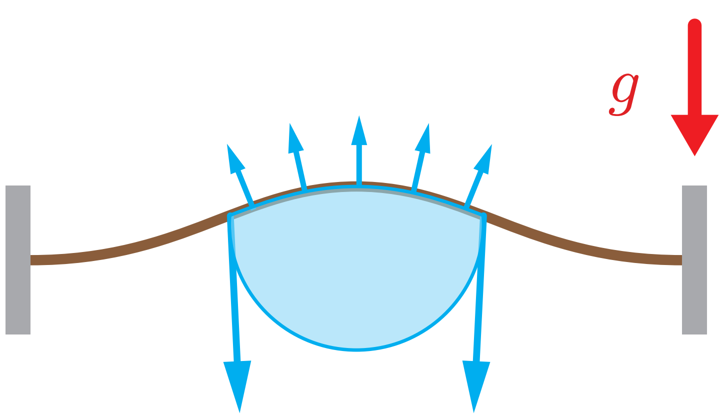


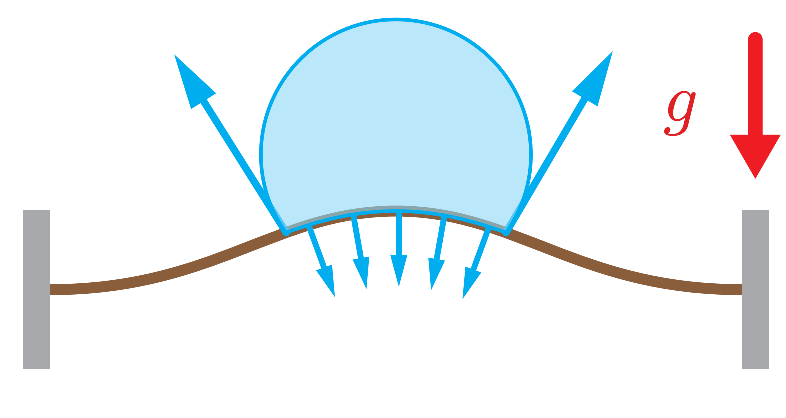
Elastocapillary Snapping vs Classical Snapping



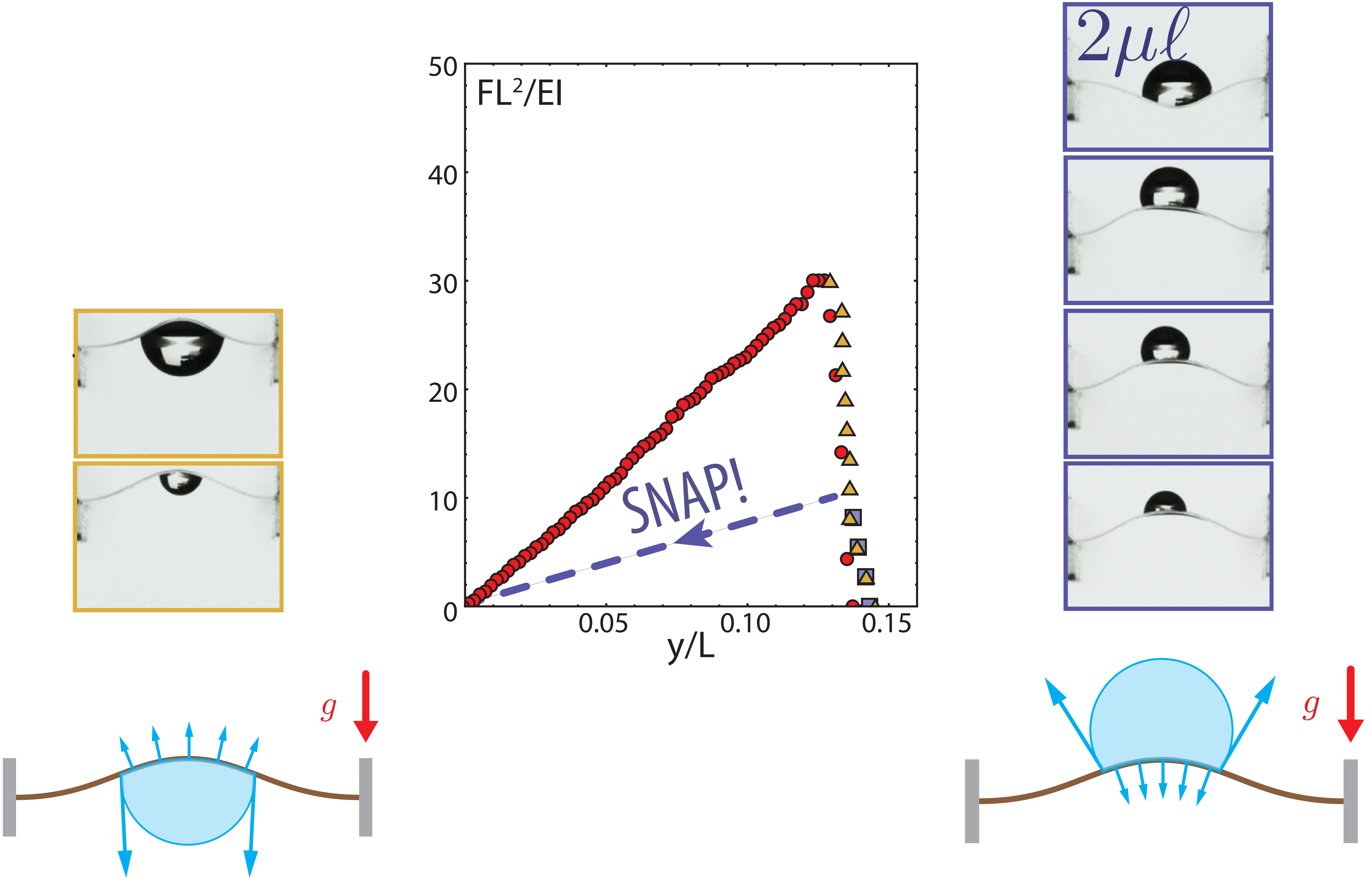
Elastocapillary Snapping vs Classical Snapping



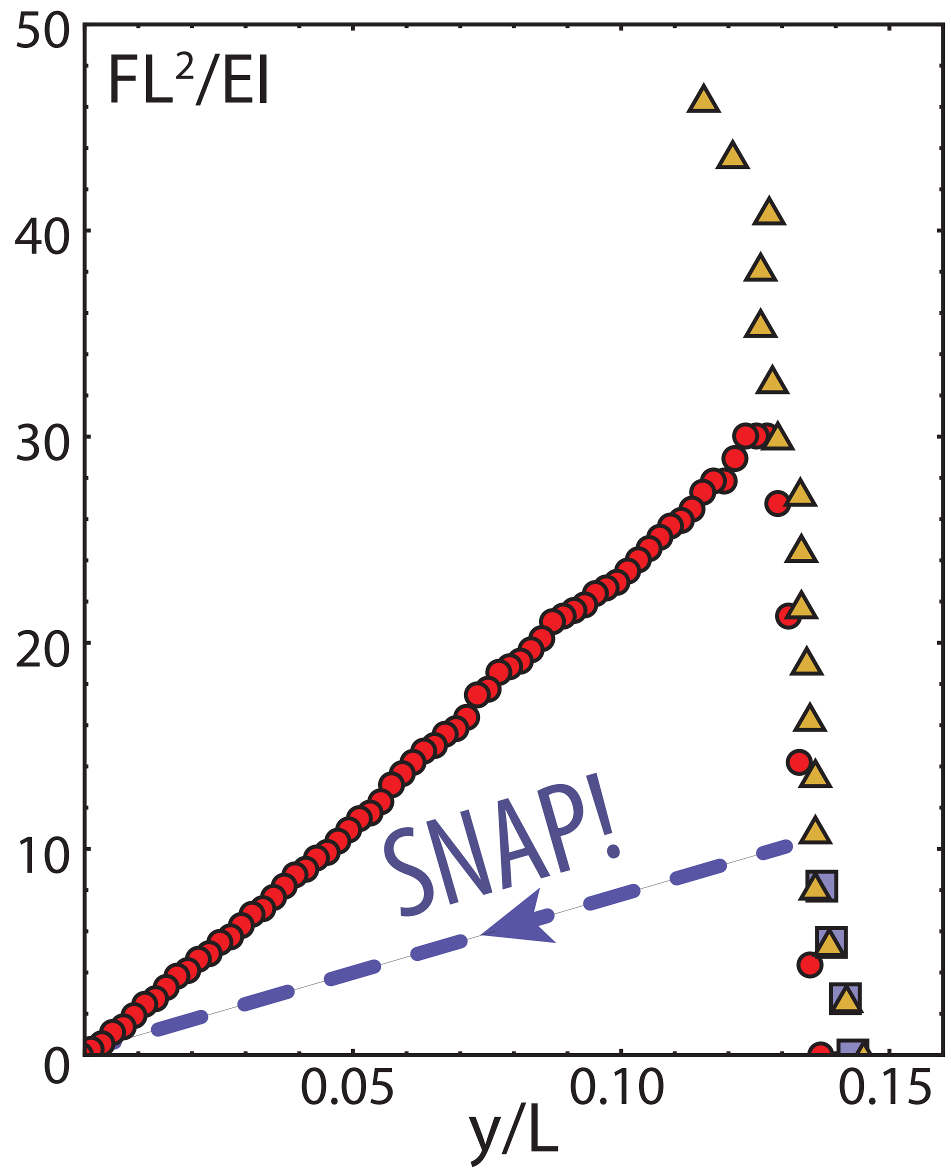


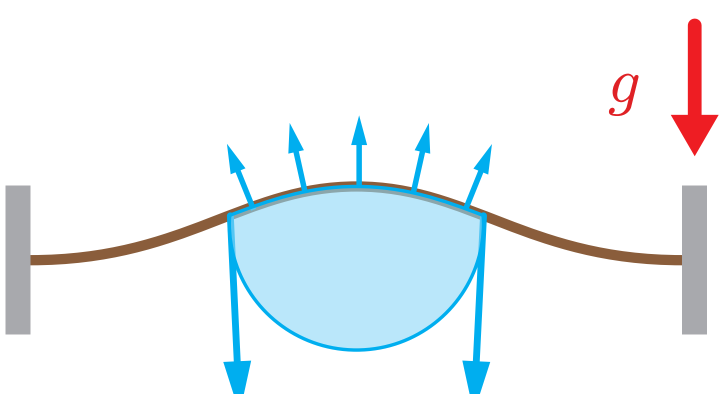


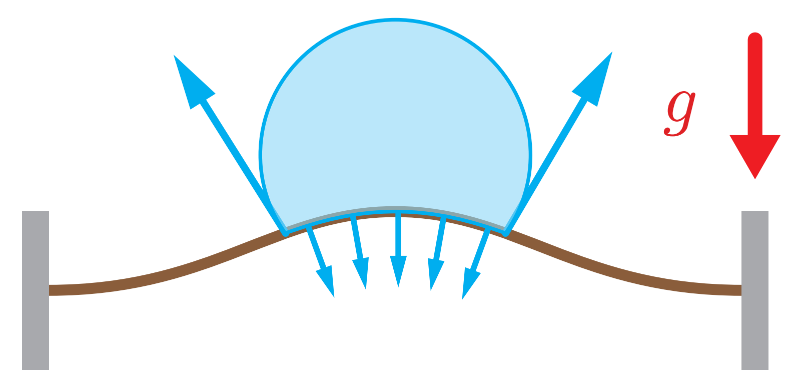
Elastocapillary Snapping vs Classical Snapping



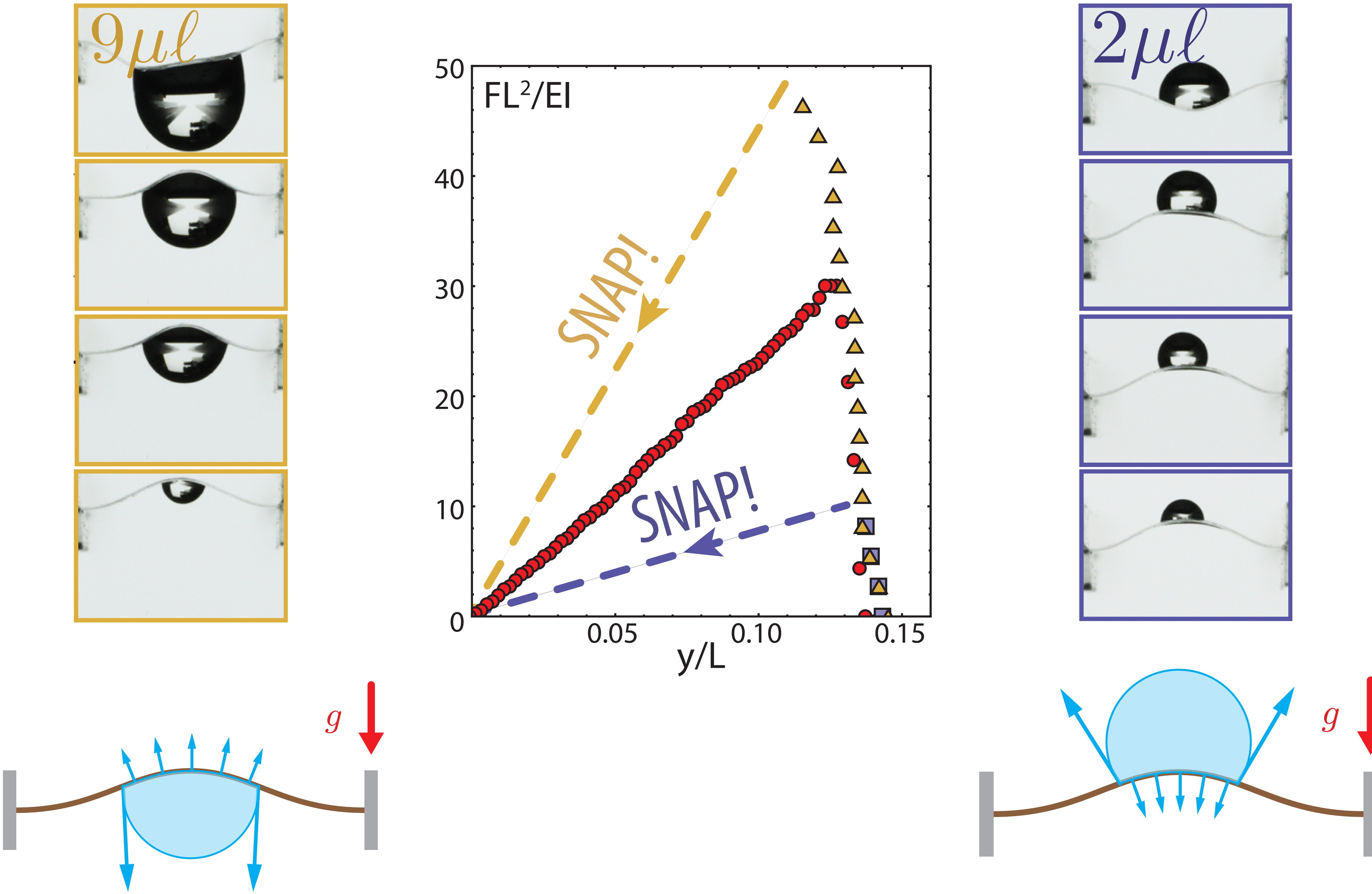
Elastocapillary Snapping vs Classical Snapping



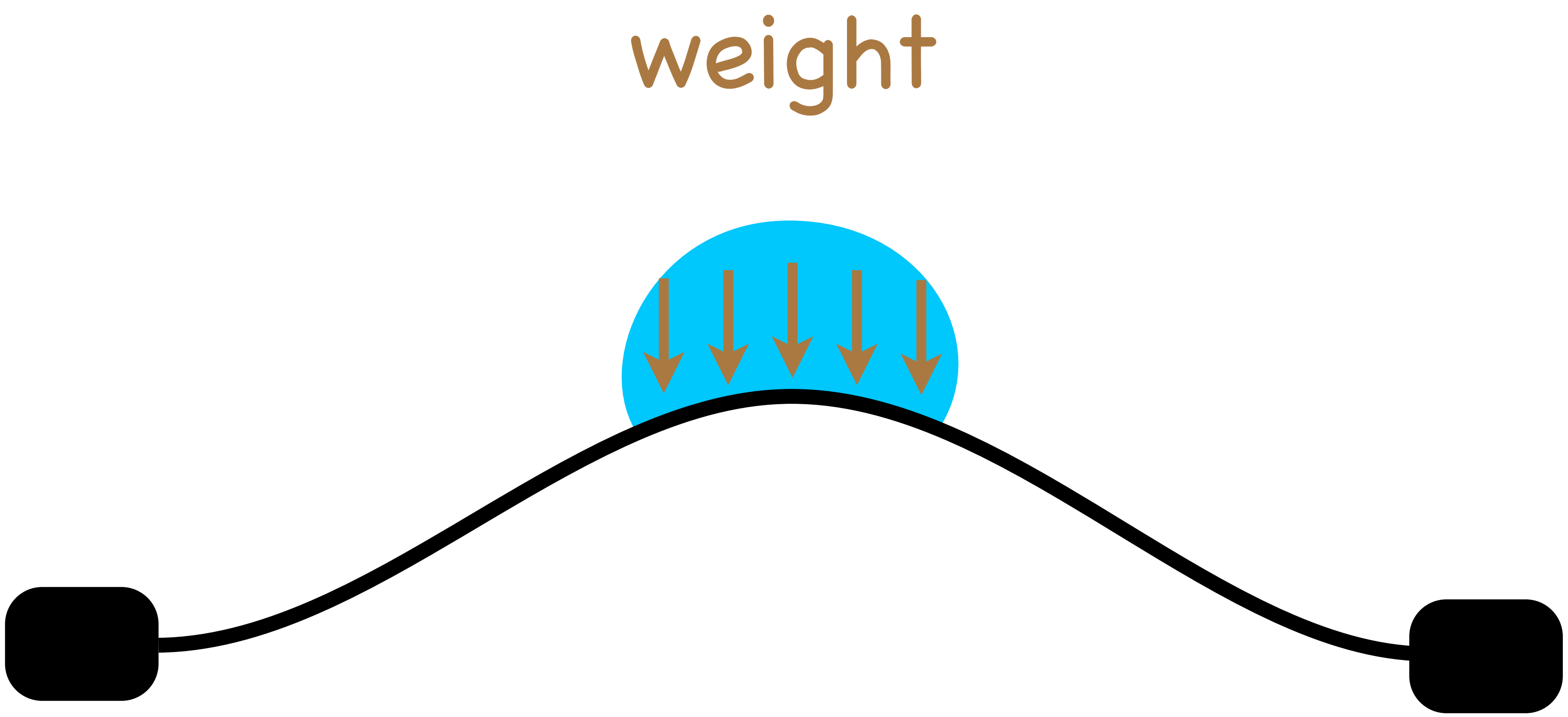




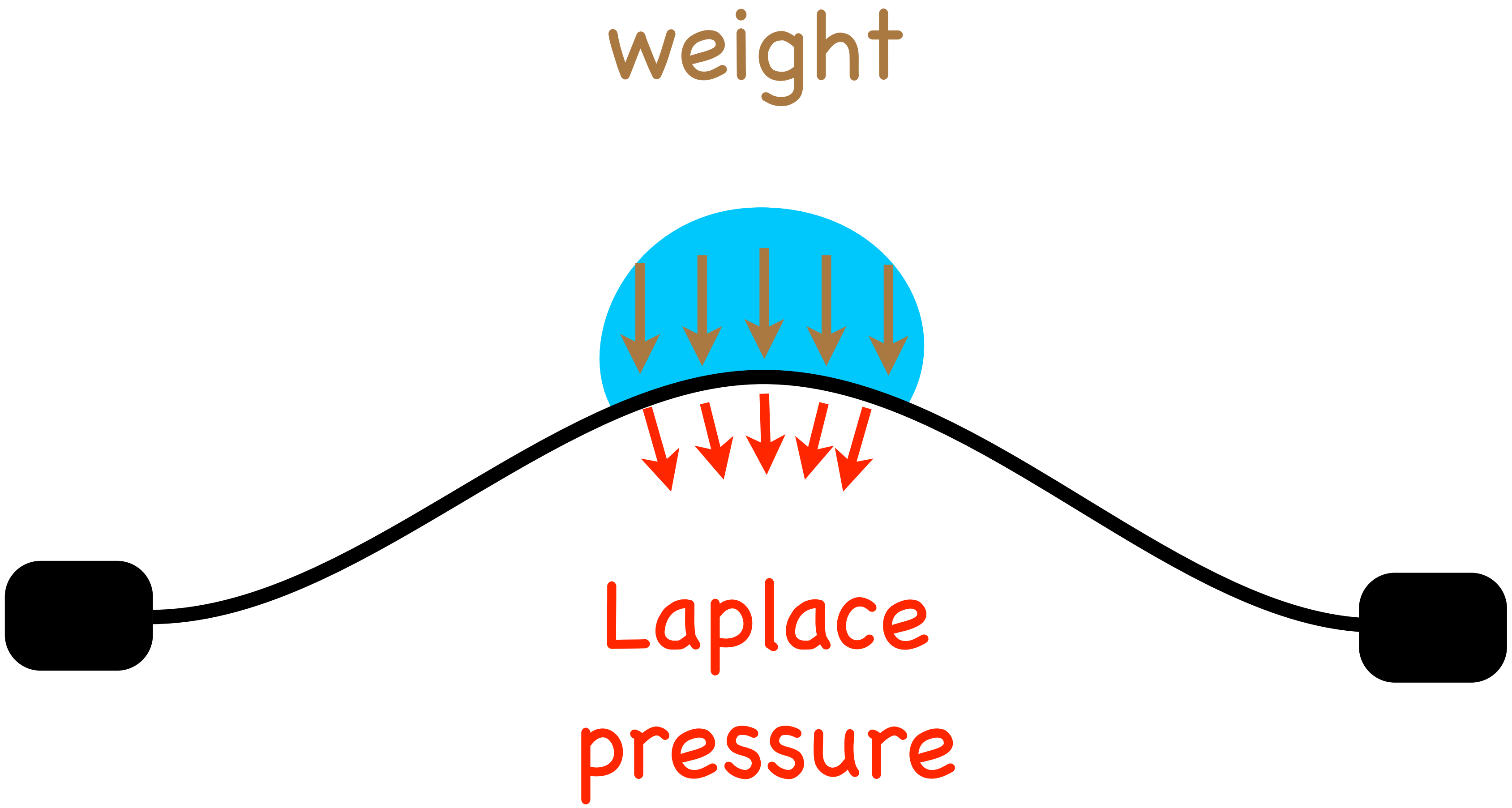
Elastocapillary Snapping vs Classical Snapping



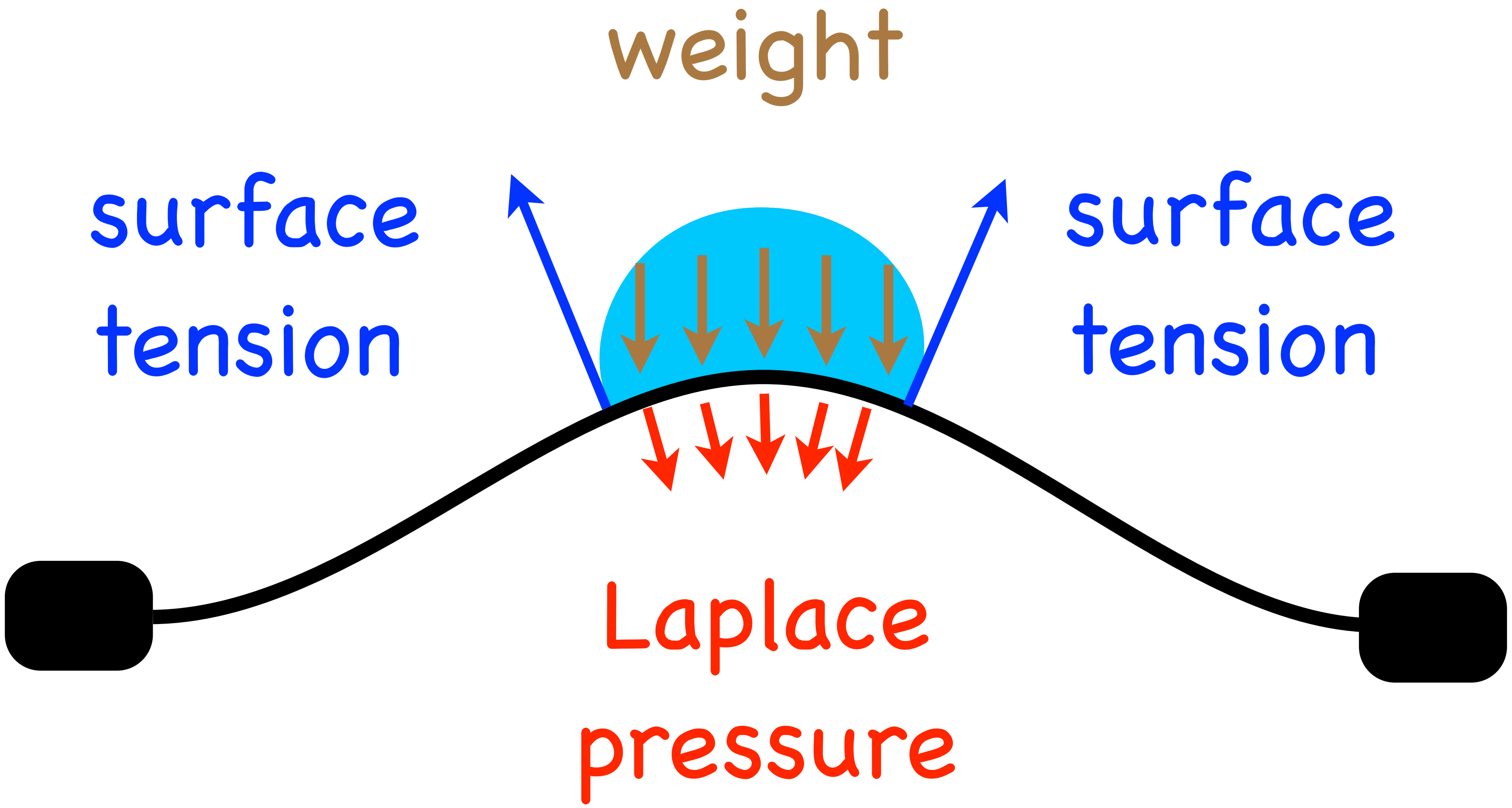
Capillary induced snap-through



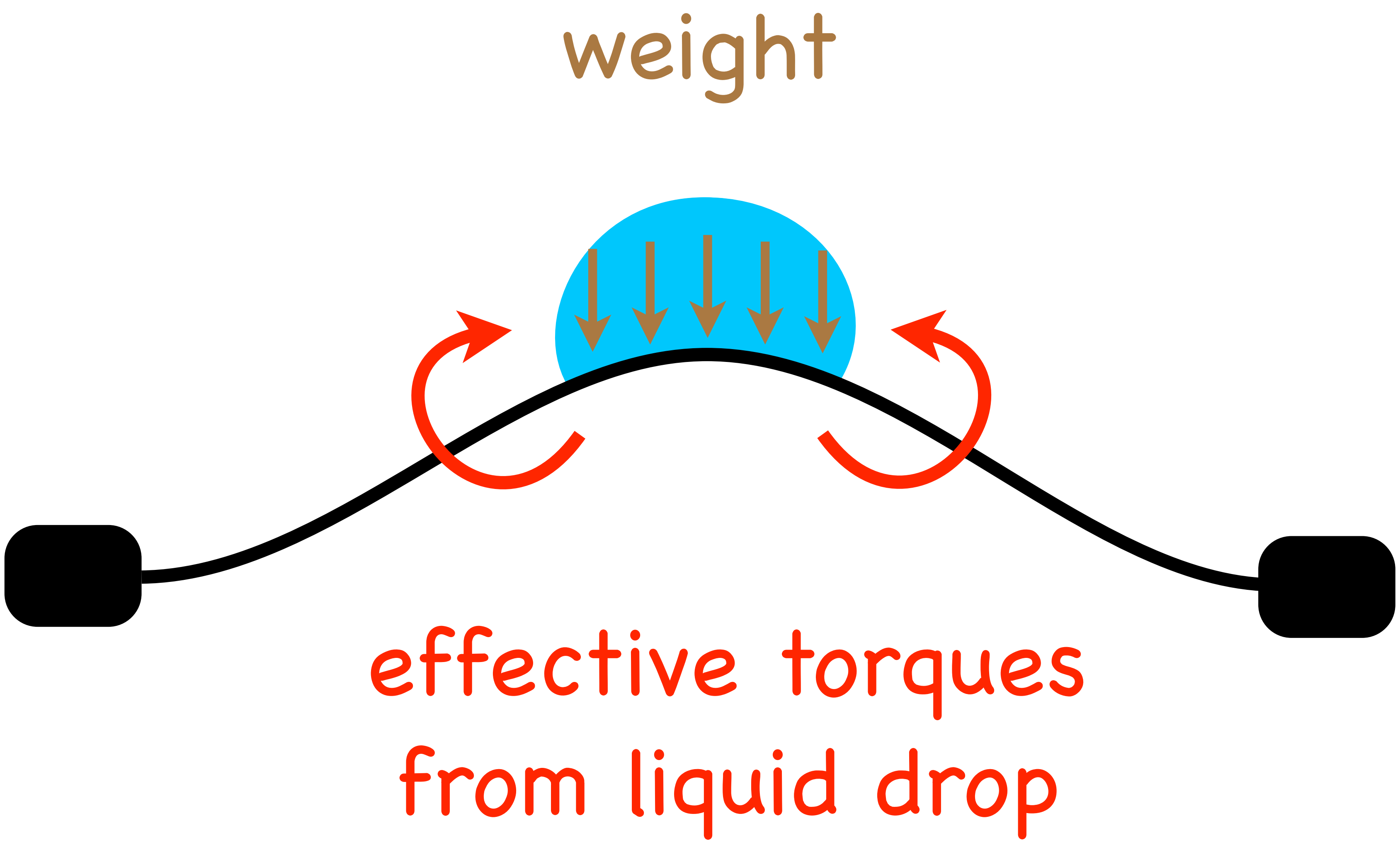
Capillary induced snap-through



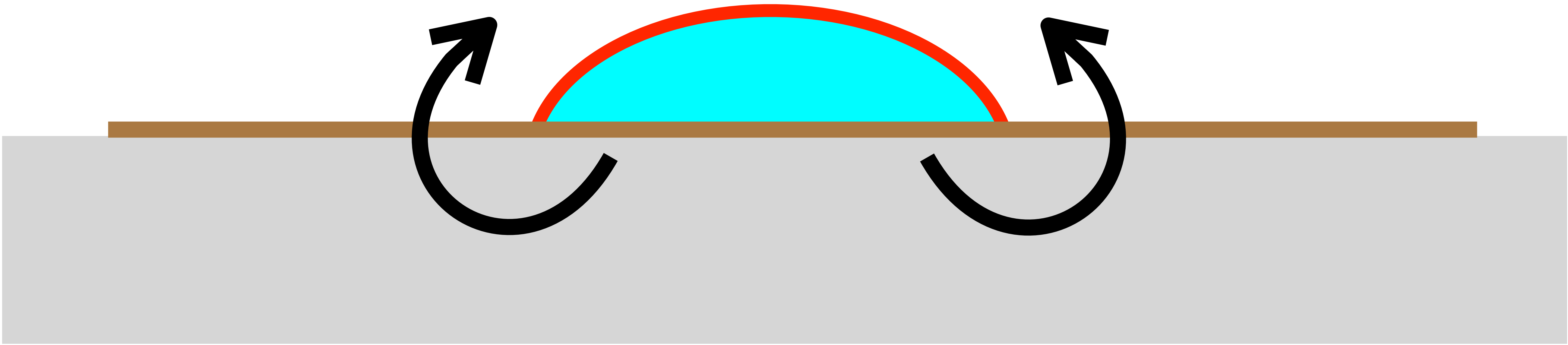
Capillary induced snap-through

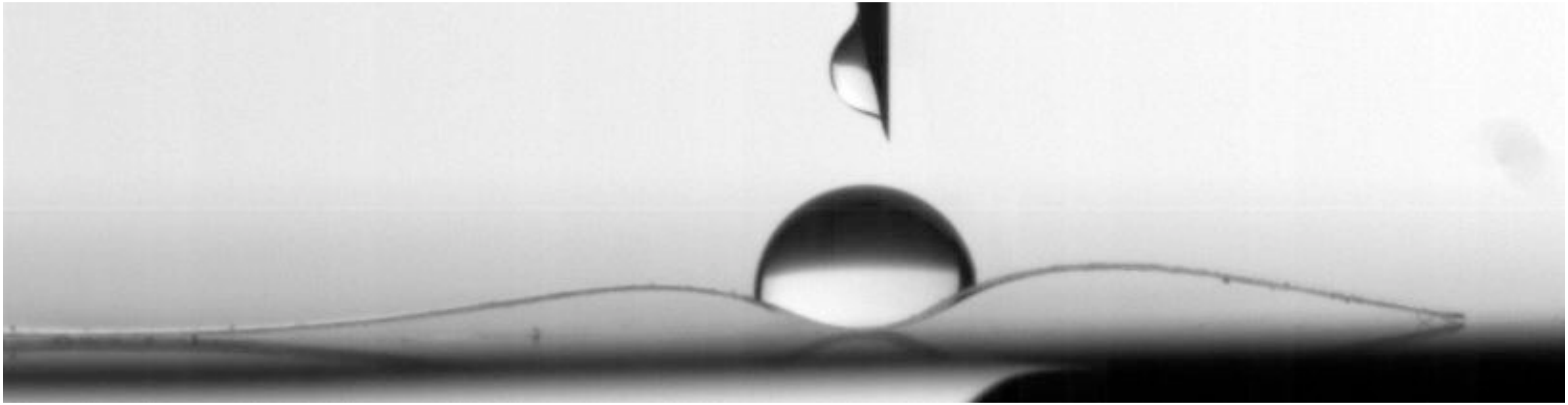


Capillary induced snap-through

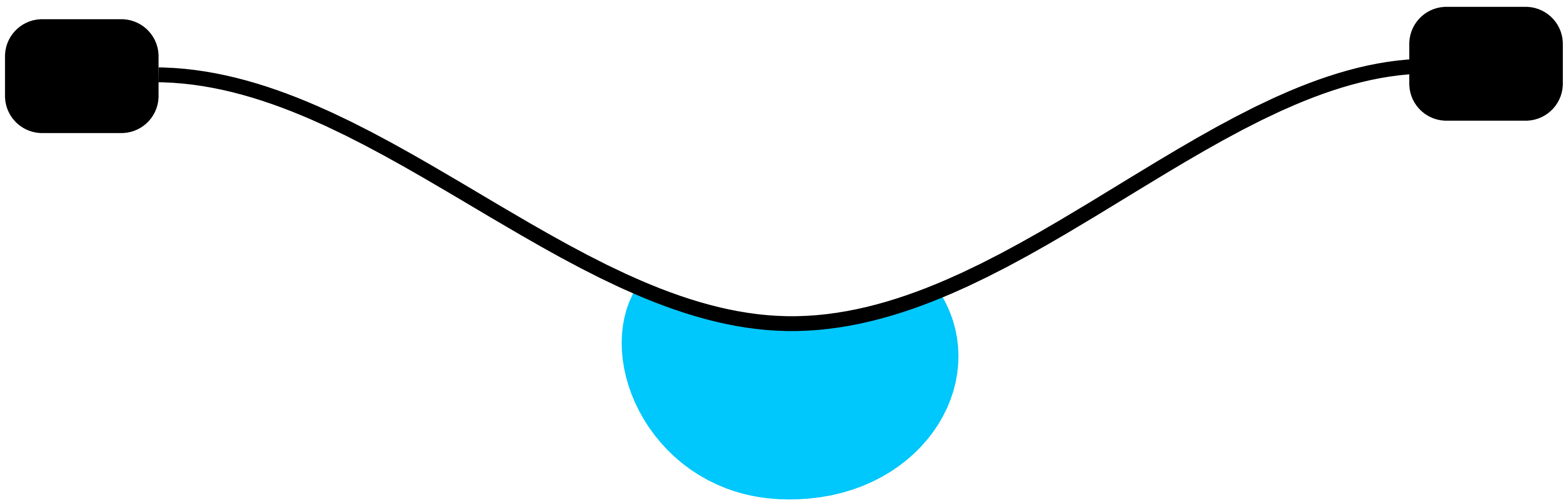


effective bending moments

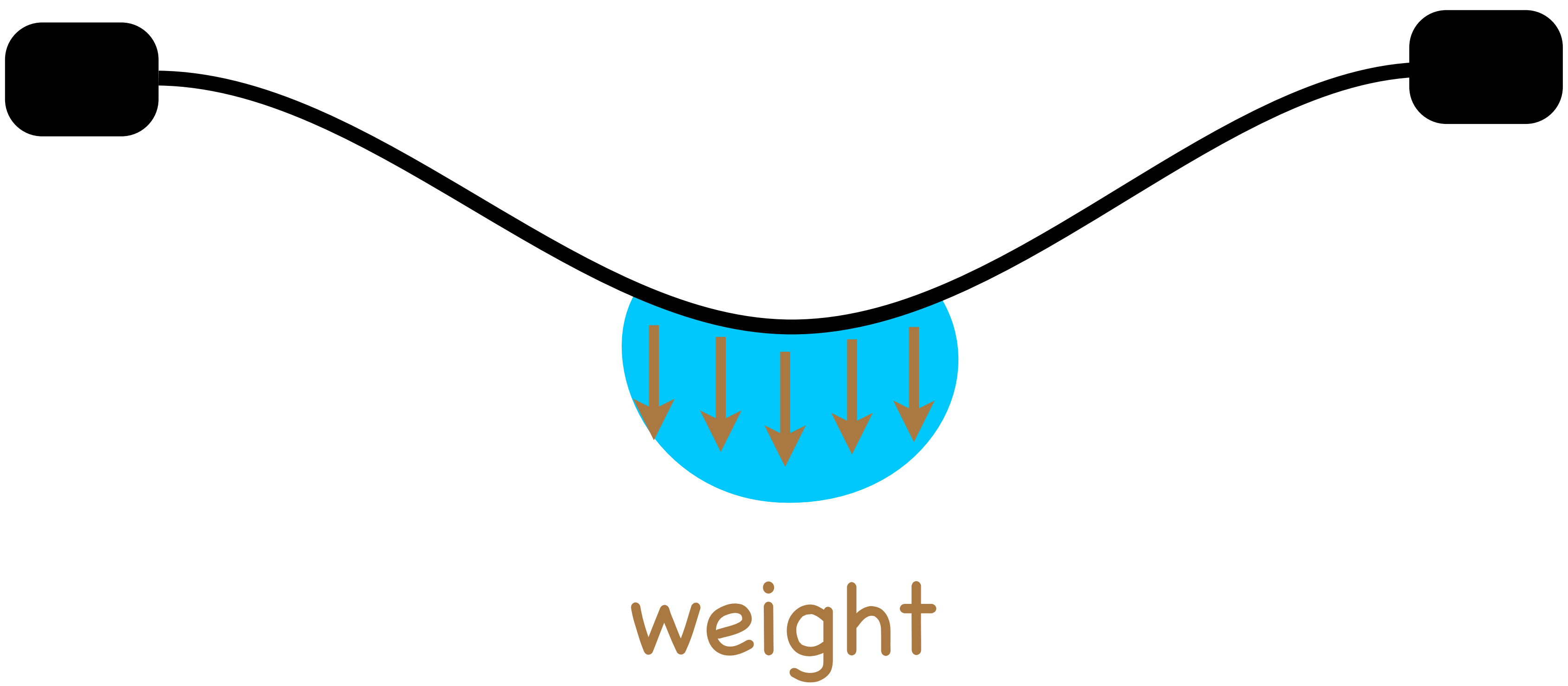




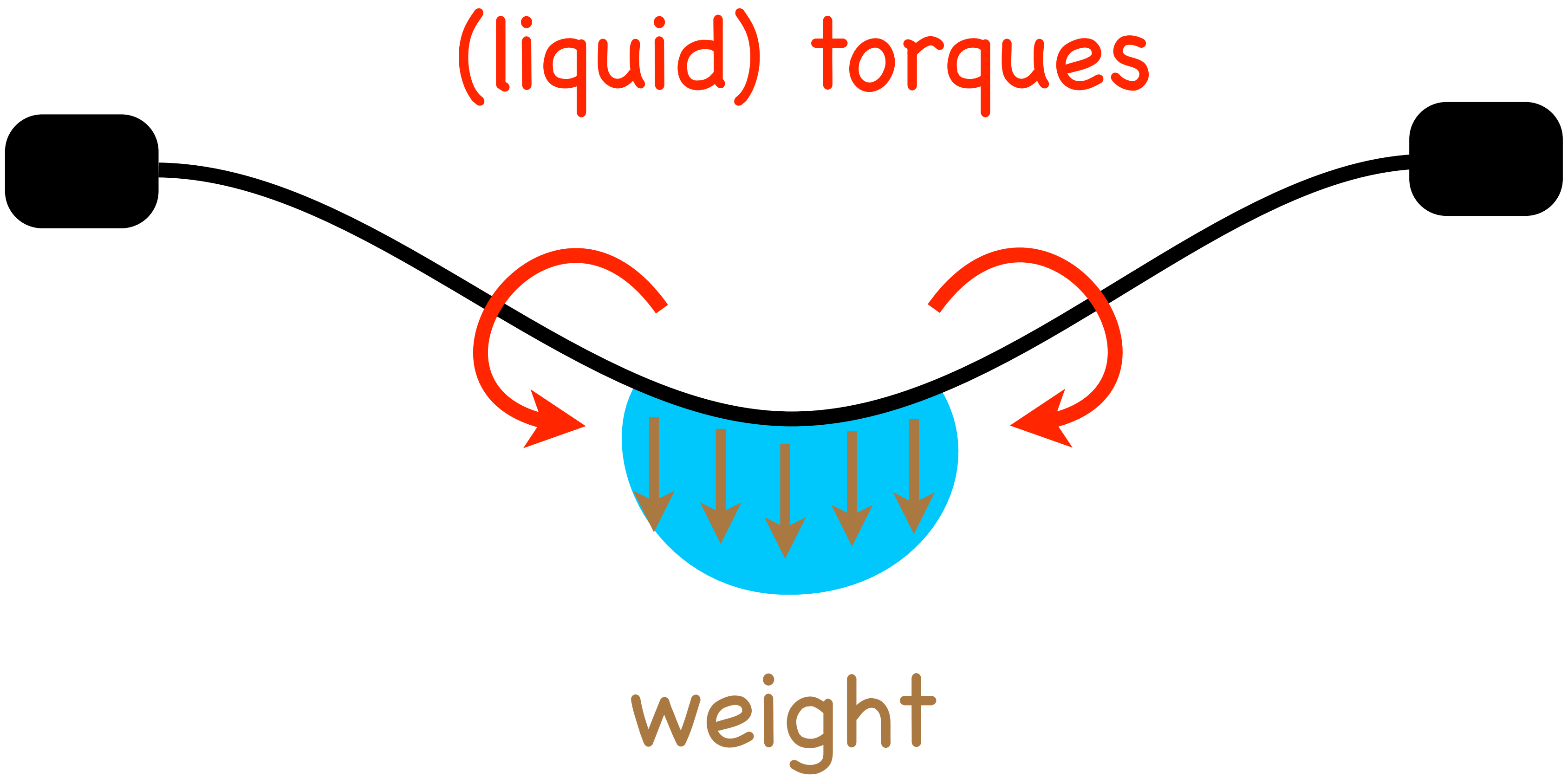
Capillary induced snap-through: against gravity



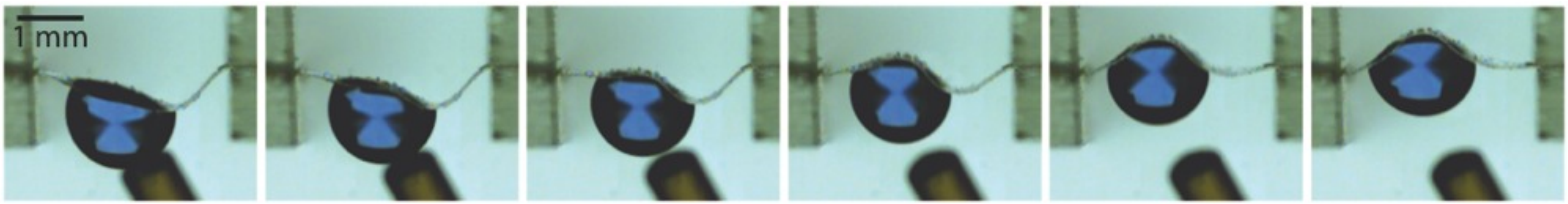
Capillary induced snap-through: <u>against gravity</u>



Capillary induced snap-through: <u>against gravity</u>



Capillary induced snap-through: against gravity



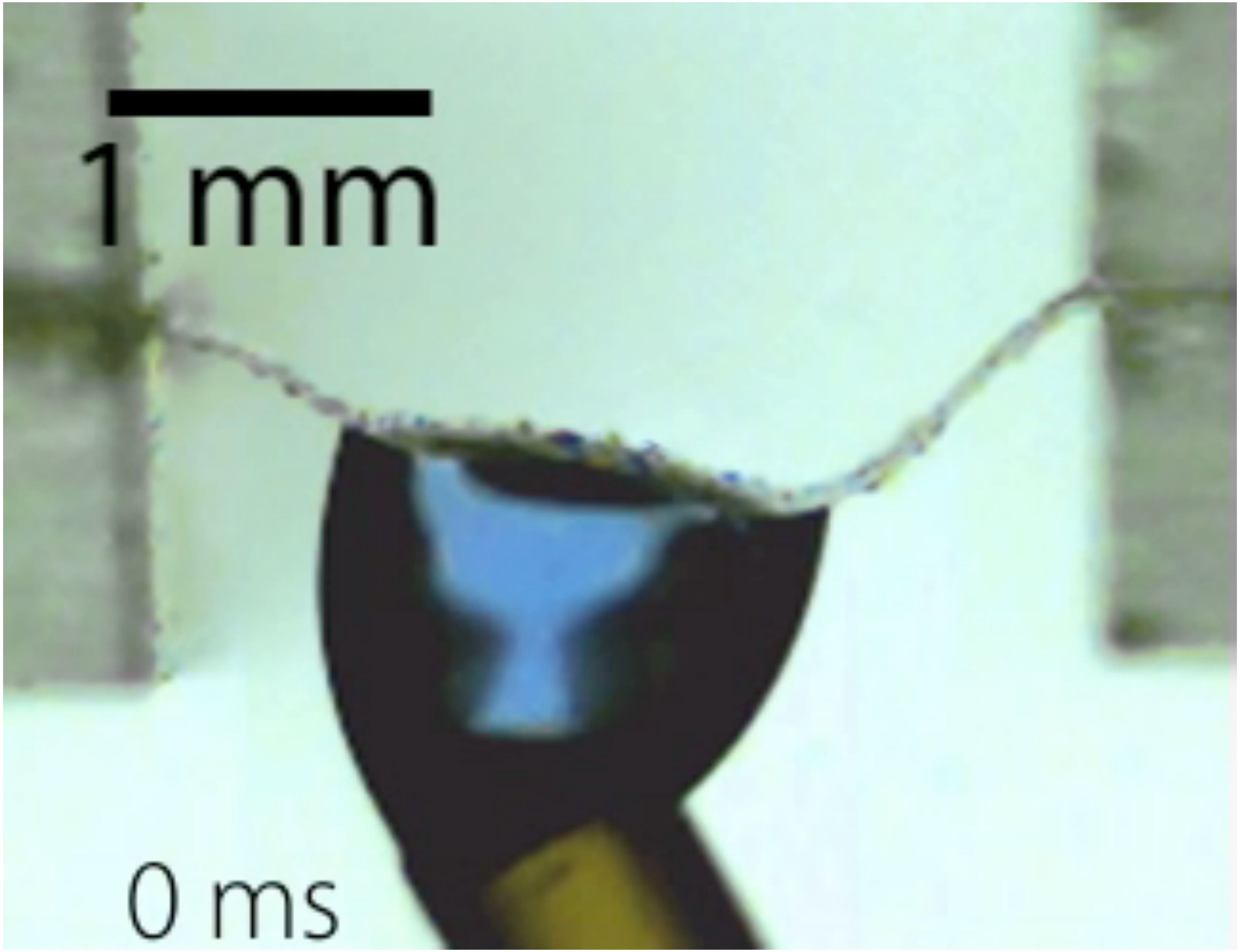
PDMS strip

dimensions: 34 microns by 1 mm by 3.5 mm

time interval between frames: 5 ms

Capillary induced snap-through: against gravity

Capillary induced snap-through: against gravity



Snap-through with soap bubble

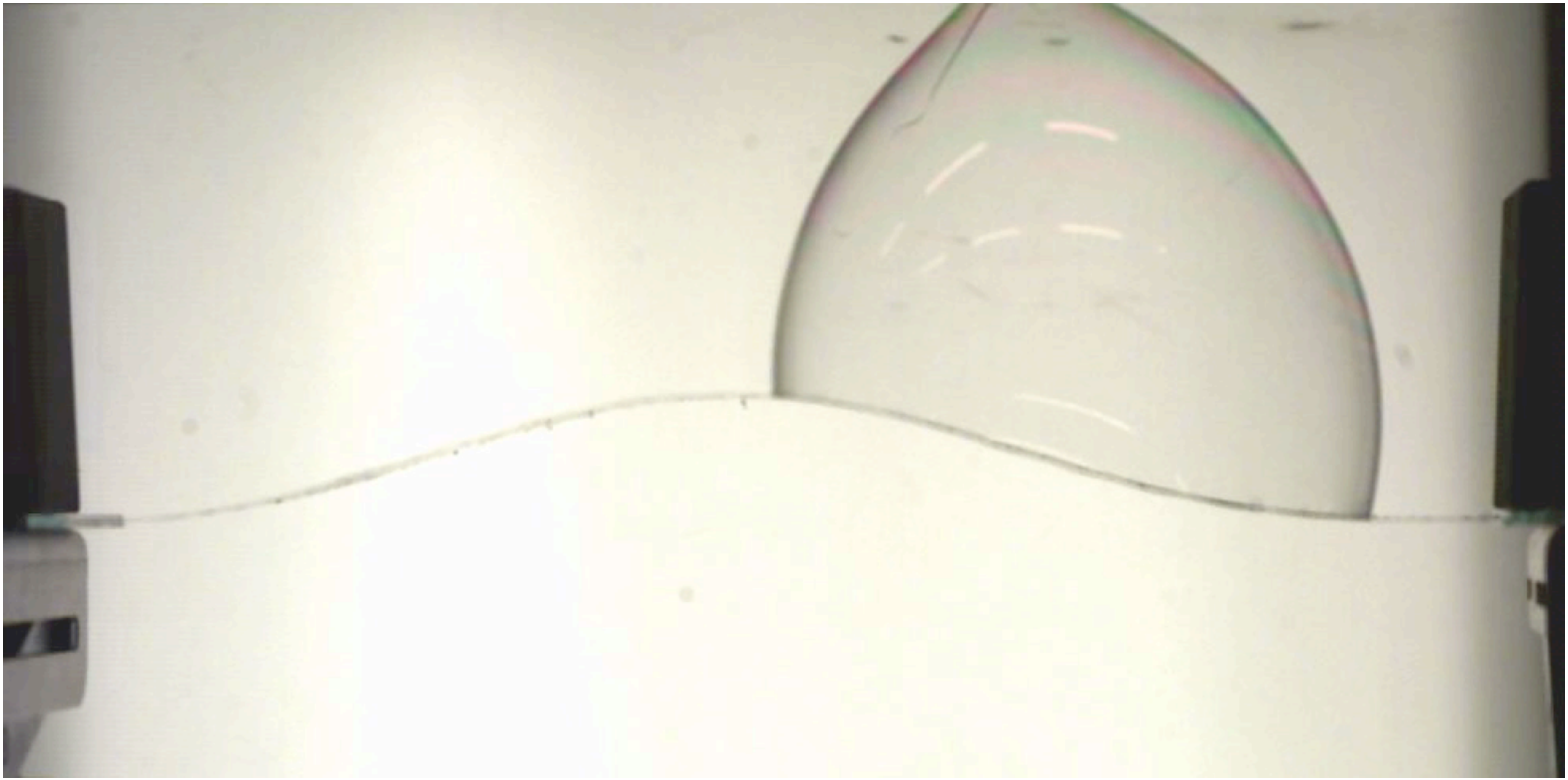
Elastocapillary snapping of a long metallic strip with a bubble

L = 24 cm   (width 8 cm)

duration ~ 1 sec

Snap-through with soap bubble

Elastocapillary snapping of a long metallic strip with a bubble



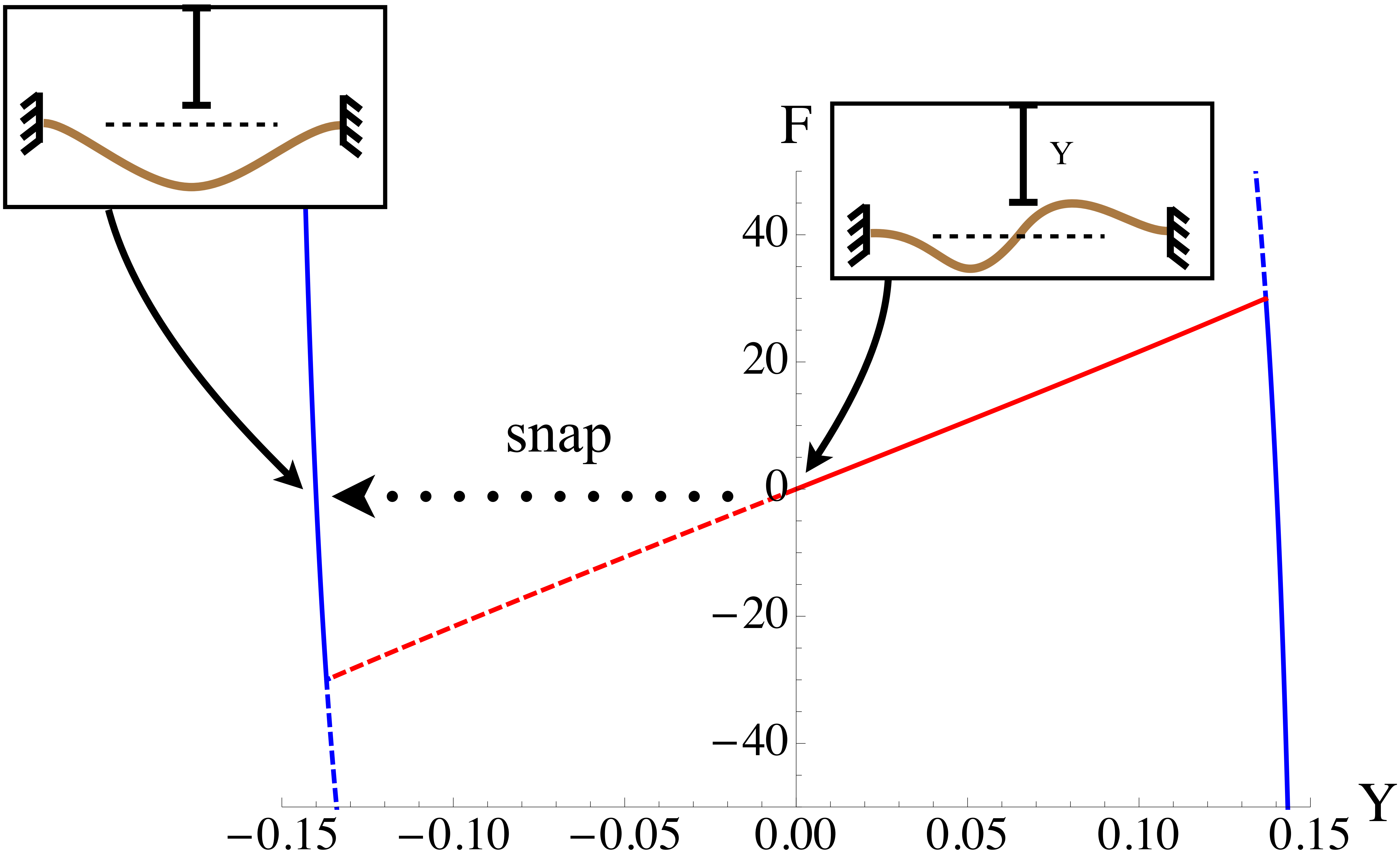
L = 24 cm   (width 8 cm)

duration ~ 1 sec

Snap-through: dynamics

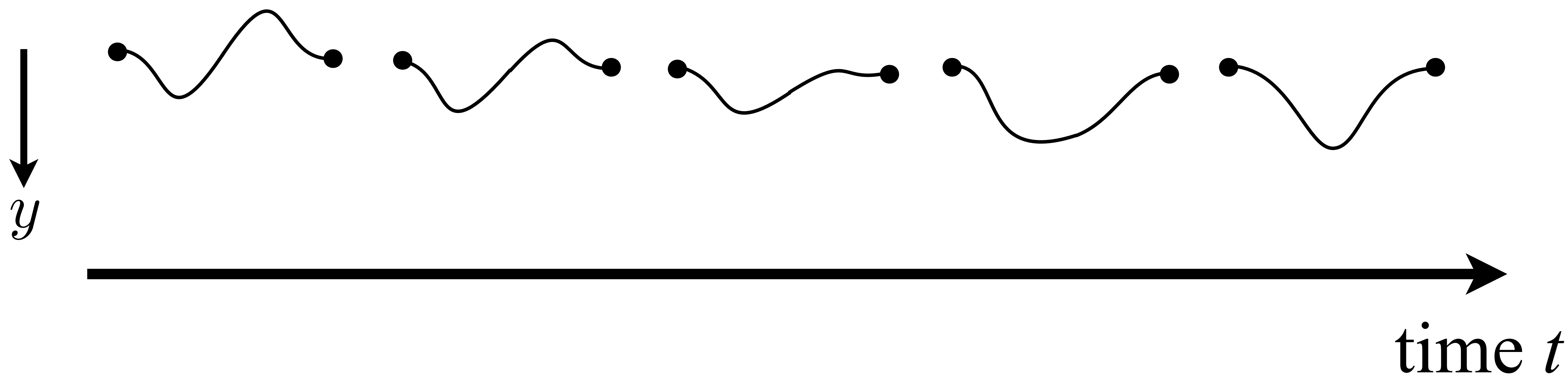
Snap-through

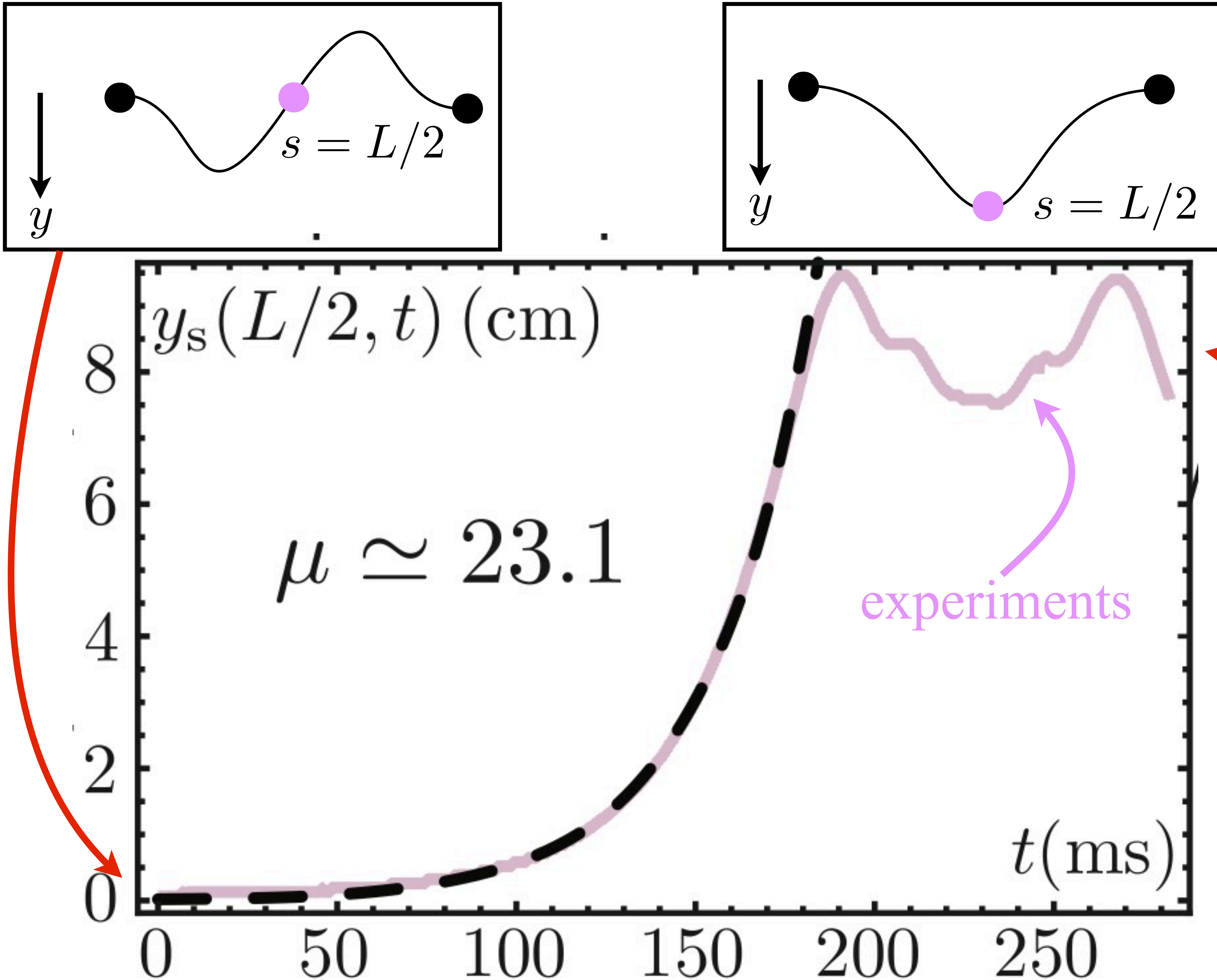
displacement Y is controlled



mettre ici les video

Snap-through dynamics: the dry case

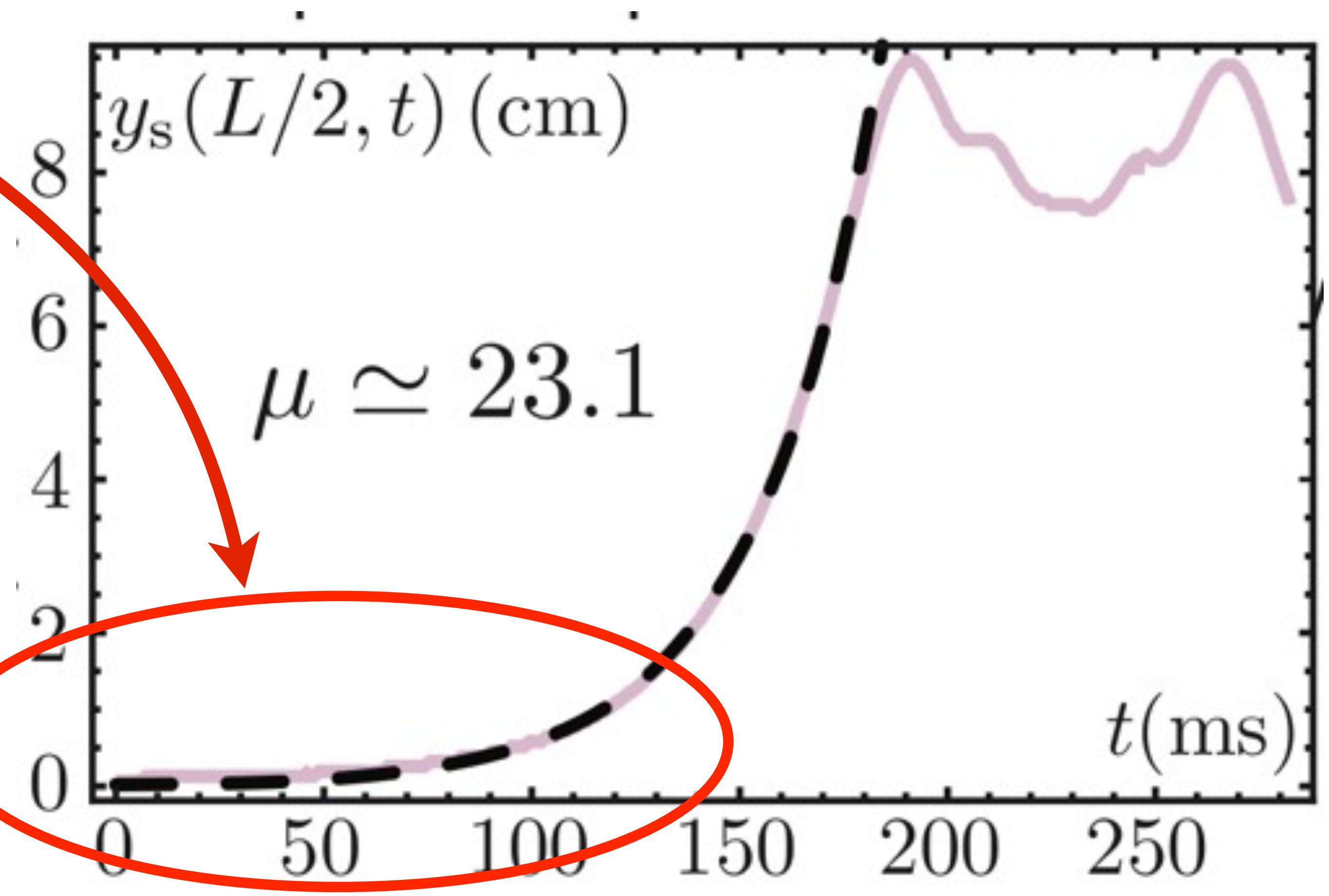




y
$s = L/2$
y
$s = L/2$
$y_s(L/2, t)$ (cm)
8
6
4
2
0
$\mu \simeq 23.1$
experiments
t(ms)
0
50
100
150
200
250

Snap-through dynamics: the dry case

Experiments:



near the unstable solution

$$y(s,t) = y_{EQ}(s) + \bar{y}(s)\, \mathrm{e}^{\mu\, t/T}$$

with scaling time:

$$T = L^2 \sqrt{\lambda/(EI)}$$

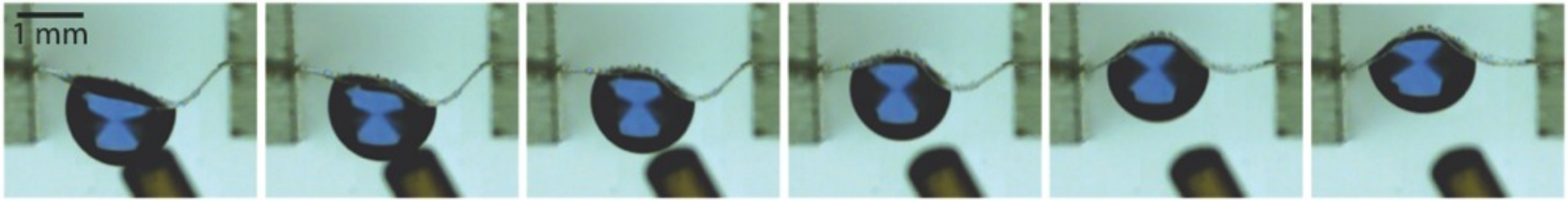
we fit and find

$$\mu \simeq 23.1$$

Theory says:

$$\mu = 24.3$$

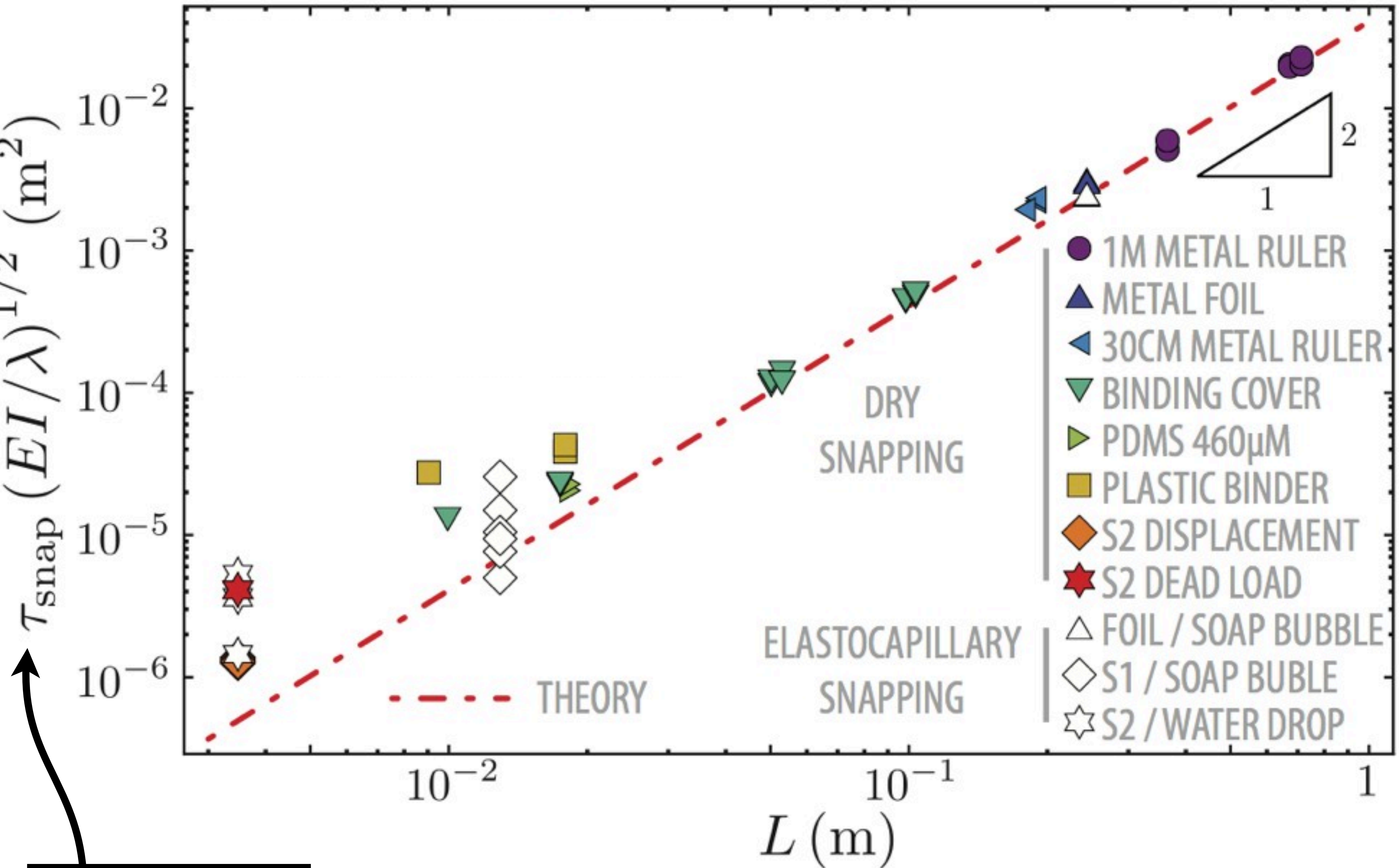
Snap-through dynamics: the wet case



Is the dynamics ruled by:

- inertia of drop (m) ?
- gravity (g) ?
- other effects (e.g. viscous) ?
- or just beam bending dynamics ?

Snap-through dynamics



$$\boxed{\tau_{\rm snap} = 1/\mu}$$

fin

Growth time at the bifurcation point

1D sys : $\ddot{x} = -V'(x)$ (conservative system)

Equilibrium x_e : $V'(x_e) = 0$

Stability $x = x_e + \delta x(t)$

$$\delta\ddot{x} = -V''(x_e)\,\delta x + O(\delta x^2)$$

stable : $V''(x_e) > 0$

unstable : $V''(x_e) < 0$

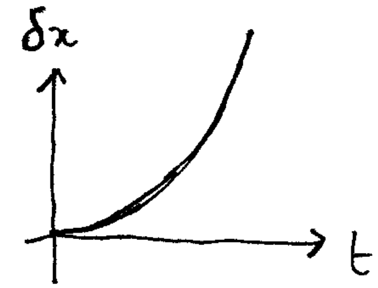
instability threshold : $V''_* = 0$

hence dynamics is $\delta\ddot{x} = 0\,\delta x + O(\delta x^2)$

UNLESS unilateral contact (inequality constraint)

Instability happens when $V''_* \neq 0$, $V''_* < 0$

$$\delta\ddot{x} = \underbrace{-V''_*}_{\oplus}\,\delta x$$



$\delta x(t) \propto e^{\mu t}$ where $\frac{1}{\mu} = +\sqrt{-V''_*}$

Elastocapillarity in Industry: Microfabrication

melting of solder

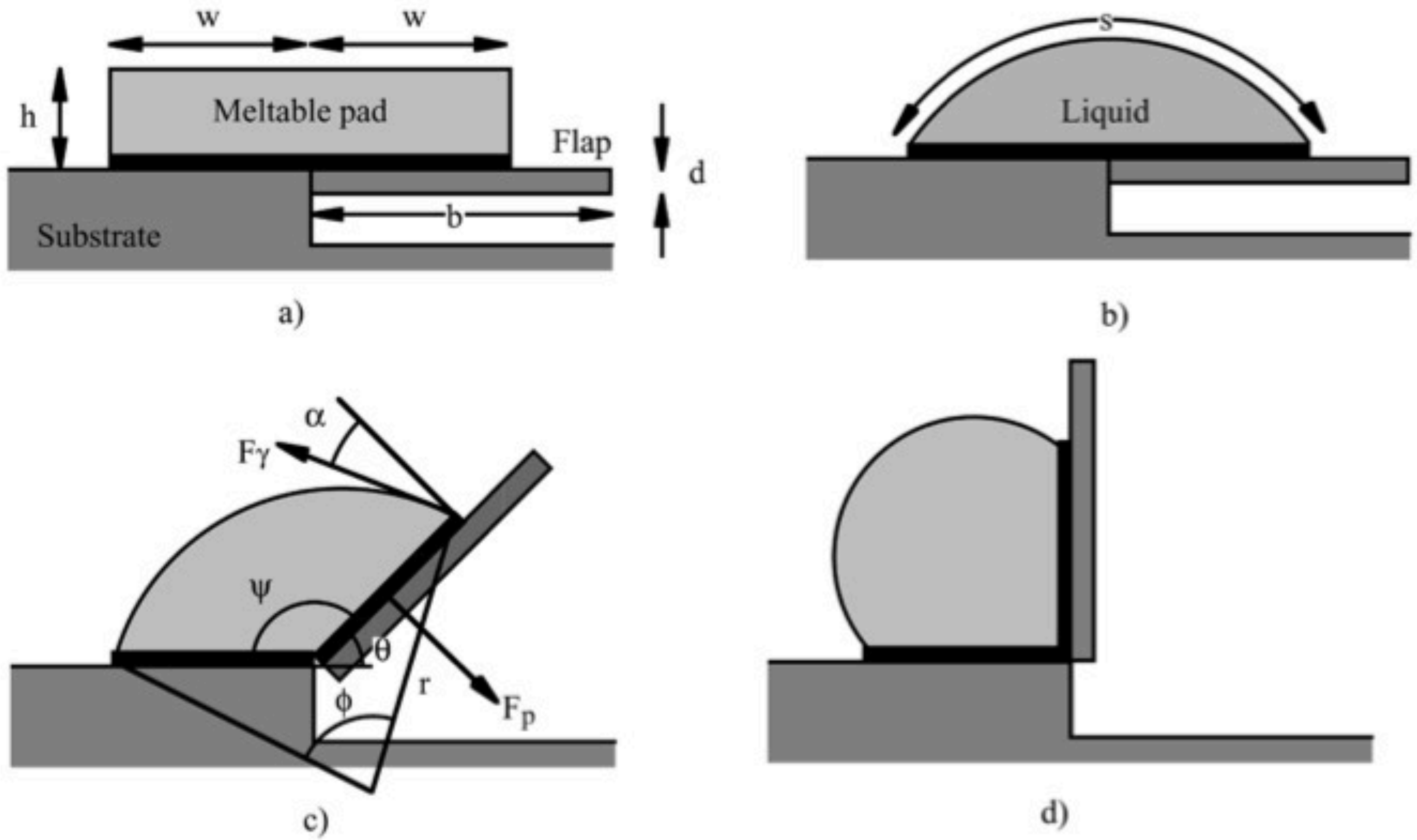


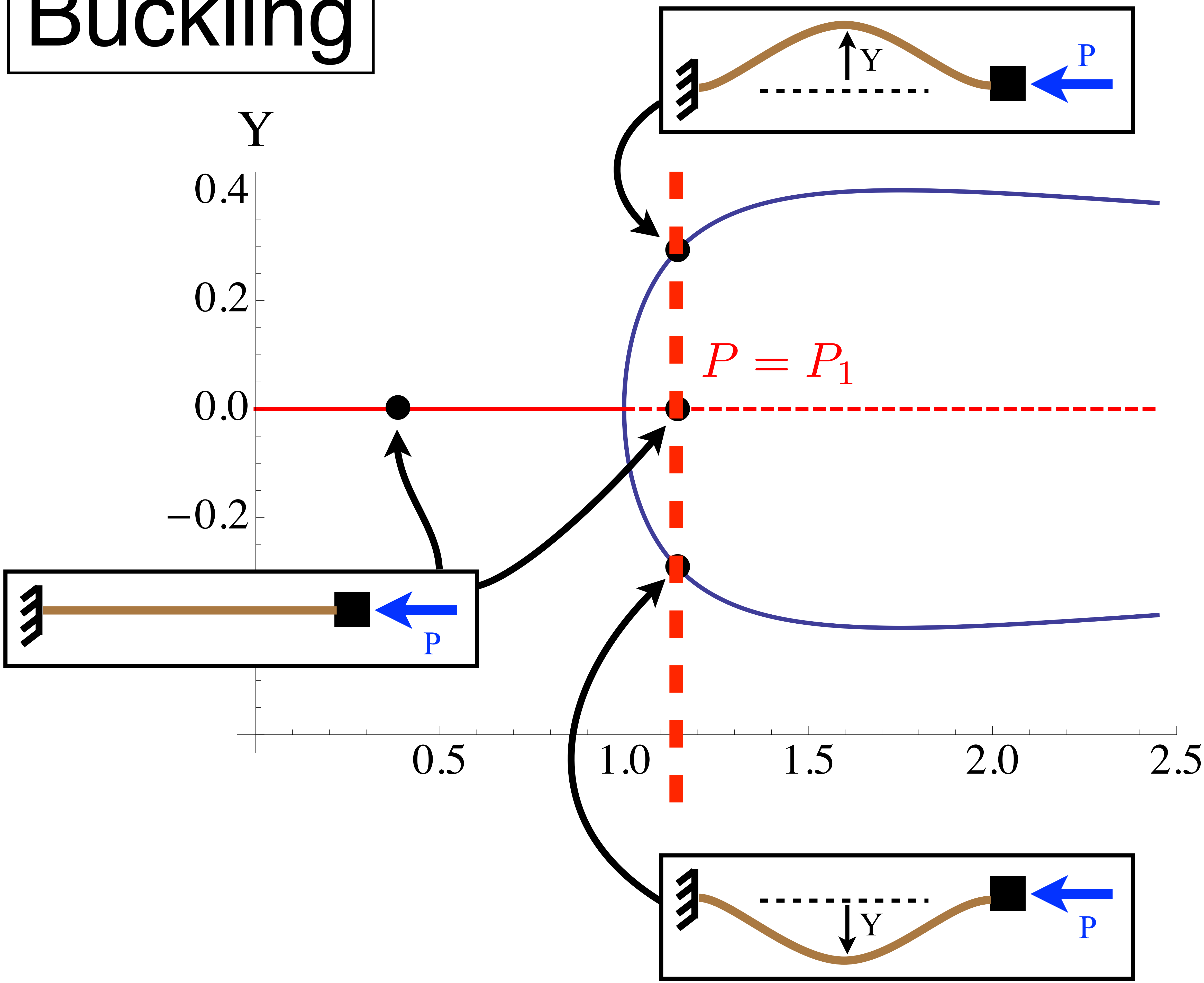
Fig. 3. Two-dimensional geometry for surface tension powered rotation.

re-solidification

rotate hinged joints for the
self-assembly of 3D microstructures

R. Syms, Journal of MEMS (1995)

Buckling



Capillary induced snap-through